Statistical Analysis of the Particulation of Shaped Charge Jets

A.J. Schwartz, R.W. Minich, E.L. Baker

This article was submitted to 18th International Symposium on Ballistics, San Antonio, TX, November 15-19, 1999

August 12, 1999
This document was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor the University of California nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or the University of California, and shall not be used for advertising or product endorsement purposes.

This is a preprint of a paper intended for publication in a journal or proceedings. Since changes may be made before publication, this preprint is made available with the understanding that it will not be cited or reproduced without the permission of the author.

This report has been reproduced
directly from the best available copy.

Available to DOE and DOE contractors from the
Office of Scientific and Technical Information
P.O. Box 62, Oak Ridge, TN 37831
Prices available from (423) 576-8401
http://apollo.osti.gov/bridge/

Available to the public from the
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Rd.,
Springfield, VA 22161
http://www.ntis.gov/

OR

Lawrence Livermore National Laboratory
Technical Information Department’s Digital Library
http://www.llnl.gov/tid/Library.html
A statistical analysis of shaped charge jet break-up was carried out in order to investigate the role of nonlinear instabilities leading to the particulation of the jet. Statistical methods generally used for studying fluctuations in nonlinear dynamical systems are applied to experimentally measured velocities of the individual particles. In particular we present results suggesting the deviation of non-Gaussian behavior for interparticle velocity correlations, characteristic of nonlinear dynamical systems. Results are presented for two silver shaped charge jets that differ primarily in their material processing. We provide evidence that the particulation of a jet is not random, but has its origin in a deterministic dynamical process involving the nonlinear coupling of two oscillators analogous to the underling dynamics observed in Rayleigh-Benard convection and modeled in the return map of Curry and Yorke.

INTRODUCTION

The ability of a shaped charge jet to penetrate a designated target is degraded when it breaks up into particles smaller than the original spatially coherent jet. A fundamental understanding of the dynamical instability leading to particulation may lead to the design of jets optimized for greater penetration. The data for shaped charge jets is in the form of an ordered sequence of particles and their corresponding masses, lengths, and velocities. Nonlinear dynamical systems can be characterized in terms of statistical correlations in a sequence of values of a dynamical variable. The statistical fluctuations reflect the existence of an attractor in phase space representing the underlying nonlinear dynamics. An elegant example of a physical system having an underlying attractor is the “leaky faucet” [1,2]. The leaky faucet results in an ordered sequence of droplets of different sizes and different time intervals between each pair of drops. Statistical analysis of the droplet sizes or time intervals can
determine whether the fluctuations reflect an underlying attractor or reflect random Gaussian statistics such as occurs in an unbiased random walk process. One measure to test for non-Gaussian behavior of the ordered shaped charge jet particle data was successfully applied to the leaky faucet and is referred to as the mean fluctuation function:

\[ F(m) = \sqrt{V(n + m) - V(n)} \]  

(1)

where \( F(m) \) is the mean fluctuation function, \( V \) is the particle velocity, \( n \) is the particle number, and \( m \), the number of particles away from \( n \). This mean fluctuation function quantifies the correlation between a pair of particles located at \( m \) away from \( n \).

The average is taken over all values of \( n \). If the sequence exhibits Gaussian statistics having a well-defined length scale and short-range behavior one expects the scaling relation:

\[ F(m) \sim m^\alpha \]  

\[ \alpha = 1/2 \]  

(2)

Non-Gaussian behavior is indicative of a nonlinear dynamical system and would have \( \alpha \neq 1/2 \). The leaky faucet has a value \( \alpha \ll 1/2 \), indicative of long range correlations in sequences of interval times between the droplets and indicates the nonlinear dynamical origin of the droplets.

RESULTS AND DISCUSSION

Two high purity silver liners were fabricated in the standard, 81-mm shaped charge design. The metallurgical processing of these two jets differed only in the heat treatment. The two liners were loaded with Octol high explosive, tested using a relatively long standoff (20 charge diameters), and recorded using flash x-ray radiography. A high-precision digitizing light table and specialized software was used for data reduction. A plot of the particle number versus particle velocity for the two jets is presented in Figure 1 and indicates that both jets have greater than 100 particles. The average break-up times for the two liners were 175 and 179 ms.

Analysis of the shaped charge jets was accomplished following the approach to the “leaky faucet”. Shaped charge jets are typified by a nearly constant velocity gradient along the entire length of the jet. The deviations in velocity for each particle from the values expected if the velocity gradient were truly constant are shown in Figure 2. The corresponding mean fluctuation function is shown in Figure 3.
Figure 1. Velocity versus particle number for two silver jets.

Figure 2. Velocity fluctuation as a function of the particle number.

Figure 3. Log-log plot of mean fluctuation function, $F(m)$ for a silver shaped charge jet. Two lines of slope 0.50 and 0.60 are included for comparison.
The value of $\alpha$ is determined from the slope of $F(m)$ in the log-log plot of Figure 3. The fitted value is $\alpha \sim 0.590$ and may be suggestive of weakly non-Gaussian statistics or perhaps an underlying attractor. Other sources of additive noise will tend to reduce the value of $\alpha$. Noise reduction techniques have been developed by others [3], but will not be addressed in this paper. In order to explore further the possibility of an attractor, it is instructive to plot phase portraits derived from the ordered sequence of observed fluctuations. The phase portraits are visual representations of the return maps of the underlying dynamics. Shown below are the phase portraits for $\Delta V(n+1)$ versus $\Delta V(n)$ in Fig.4 and $\Delta V(n+4)$ versus $\Delta V(n)$ in Fig. 5.

![Figure 4. Phase portrait of velocity fluctuations for silver shaped charge jet corresponding to data in Figure 2.](image)

![Figure 5. Phase portrait of velocity fluctuations of silver shaped charge jet corresponding to data in Figure 2.](image)
The phase portraits appear to have structure similar to attractors observed in Rayleigh-Benard Convection and also seen in the Curry-Yorke return map [4]. In these cases the attractor indicates the presence of two coupled nonlinear oscillators. There are three regions of closely clustered points in the phase portraits of Figures 4 and 5 suggesting a near frequency locking of the two frequencies in a ratio of 3 to 1. Further evidence of the presence of the two frequencies in the shaped charge jet can be seen in the discrete Fourier transform of the velocity fluctuations for two silver jets (Figure 6). The frequencies for both silver jets are in the ratio of 3 to 1. The frequencies in the case of Rayleigh-Benard convection have a purely dynamical origin and the degree of locking and corresponding shape of the attractor depend on the thermal gradient that drives the instability as well as the degree of nonlinearity. It would be interesting to identify a similar drive parameter for shaped charge jets. Had the velocity fluctuations been purely random, the phase portraits would have had little discernible structure while the phase coordinates would be more uniformly distributed. In that case, the value of \( \alpha \) would not have departed from 1/2 and the Fourier Transform would not exhibit strong resonant peaks.

![Figure 6. Discrete Fourier Transform of velocity fluctuations for two different silver jets. Zero frequency corresponds to a value of 1.](image)

**SUMMARY**

The methods used in characterizing nonlinear dynamical systems have been applied to the measured velocity fluctuations for two silver shaped charge jets. We observe evidence that the fluctuations exhibit a non-Gaussian behavior indicative of an underlying nonlinear dynamical system. The use of phase portraits give visual clues that the origin of the fluctuations may be due to a quasiperiodic route to chaos involving two non-linearly coupled resonators analogous to Rayleigh-Benard
Convection. Further corroboration of these results can be obtained by analyzing other measurable jet variables such as the time interval between particles. Using noise reduction techniques the phase portraits will better expose the dynamical attractor. It is hoped that the ideas presented in this paper will allow useful information to be extracted from shaped charge jet data that will result in a better understanding of jet break-up.

ACKNOWLEDGMENTS

The authors acknowledge the support of the Joint DoD/DOE Munitions Technology Development Program. This work performed under the auspices of the U.S. Department of Energy and Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48.

REFERENCES