Superdeformation in the A=150 and A=190 Regions

M. P. Carpenter and R. V. F. Janssens

Argonne National Laboratory, Argonne, IL 60439

Abstract. Superdeformation has been established for over a decade in the mass 150 region and nearly as long in the A=190 region. The first measurements directed at nuclei in these regions concentrated on mapping out the superdeformed (SD) islands by identifying SD rotational bands in γ-ray coincidence data. These early studies provided new insights into the physics of superdeformation, but also raised unexpected issues. The new gamma-ray arrays (Gammasphere, Eurogam/Euroball and Gasp) have provided a wealth of new data on properties of SD states in these two mass regions. This paper highlights some of the more recent results from the large arrays which have addressed the outstanding issues in the field, namely, ΔI = 4 staggering, identical bands, SD vibrational bands, and questions about the feeding into and the decay out of the SD well.

INTRODUCTION

Excited states associated with superdeformed (SD) shapes were first observed over thirty years ago in nuclei of the actinide region, around mass 240. These examples are the well known fission isomers, and a summary of both the experimental and theoretical work on this subject can be found in the review article by Bjornholm and Lynn [1]. While much information was obtained on the properties of these isomers in the 1960's and 1970's, little is known about the excited states built on these isomeric SD band heads.

This situation changed dramatically in 1986, with the identification of a SD rotational band in 152Dy [2]. A subsequent lifetime measurement on the transitions in this SD band confirmed that the average quadrupole deformation of the band (β2) was as large as that of the fission isomers, i.e., the deformation corresponds to a major to minor axis ratio of 2:1 for the prolate nucleus [3]. Superdeformation in the A=190 region was first reported in 1989 by Moore et al. [4] when a rotational SD band was identified in 191Hg. The measured quadrupole moment of this band confirmed a large deformation corresponding to a major to minor axis ratio of 1.7:1. By now islands of superdeformation in these two mass regions have been well established and can be traced to large gaps in the calculated single-particle energies at large deformation. For the A=150 region, these gaps are calculated to occur at Z=66 and N=86 (152Dy) while for A=190 they are calculated to be at Z=80 and N=112 (192Hg). As a result, 152Dy and 192Hg are often referred to as doubly-magic SD nuclei. Figure 1 shows the extent of the experimentally established islands of superdeformation in both mass regions.

Many SD bands have been observed in both the A=150 and A=190 regions, and as a result, features common to the bands of a particular mass region have become apparent. For example, bands in the A=150 region have on average more coincident γ rays and the transition energies are twice as large as those observed in mass 190 SD bands. The latter observation is due primarily to the fact that the spin region over which the bands are observed is different. For the A=150 region, the SD bands cover a spin range from ~ 25 – 65h while the SD bands in the A=190 region are observed from I ~ 10 – 45h. The population pattern of the bands in the two regions exhibit the same general features: as the spin decreases the intensity in the SD band increases, reaches a maximum, and then drops rapidly over two or three transitions as the band depopulates and decays to the yrast levels. The transitions linking the SD band to the known yrast states are in general not observed and, thus, crucial information such as spins, parities and excitation energies of states in the SD well are not known. Isotopic identification comes from observing coincidences between known yrast transitions and members of the SD band.

Most of the studies on SD nuclei in these two mass regions carried out in the late 1980's and early 1990's focused on mapping out the two islands of superdeformation. A summary of this early work can be found in
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, make any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.
the reviews by Nolan and Twin [5] and Janssens and Khoo [6]. The measurements were performed with so-called second generation $\gamma$-ray arrays which consisted of 10-20 Compton-suppressed Ge (CSGe) spectrometers coupled to a sum-energy and multiplicity array. Since the spins of the SD bands were not known, it became a common practice to present the dynamical moment of inertia ($I^{(2)}$) as a function of rotational frequency ($\hbar\omega$) in order to study the response of the nucleus under the stress of rotation. The quantity $I^{(2)}$ is defined as the second derivative of the excitation energy with respect to spin. Experimentally, it is represented as $4/\Delta E_{\gamma}$ where $\Delta E_{\gamma}$ is the energy difference between successive transitions in a rotational band. Therefore, this quantity is independent of spin.

For the mass 150 region, the evolution of the $I^{(2)}$ moment as a function of $\hbar\omega$ was found to be characterized by pronounced isotopic and isotonic variations. These variations have been attributed to differences in the occupation of specific high-N intruder orbitals, namely $j_{15/2}$ neutrons and $i_{13/2}$ protons [7]. This realization represents one of the early theoretical successes in understanding the underlying single-particle nature of SD bands. In contrast, the vast majority of SD bands in the mass 190 region were found to display the same smooth and rather pronounced increase in $I^{(2)}$ with increasing $\hbar\omega$. It is generally accepted that this rise results from quasiparticle alignments of the same high-N, $j_{15/2}$ and $i_{13/2}$ intruder orbitals. This contrast between the $I^{(2)}$ moments of inertia in the two mass regions highlights one of the more interesting differences between $A=150$ and $A=190$ SD bands: pair correlations have a major influence on the properties of the SD bands in the $A=190$ region while “single” particle features dominate near $A=150$.

With the construction of the present generation of powerful $\gamma$-ray spectrometers, Gammasphere, Eurogam/Euroball and Gasp, a new era in the study of nuclei at high angular momentum has begun. These new arrays are characterized by a gain in detection sensitivity of several orders of magnitude over earlier detection systems, and thus, are especially suitable for the study of SD bands which are weakly populated in heavy-ion induced fusion reactions. In the majority of nuclei where superdeformation had been established, multiple SD bands have now been observed. As examples of this fact, it is interesting to note that up to nine SD bands have been reported in a single nucleus in the $A=190$ region ($^{192}$Tl [8]), while in the $A=150$ region,
thirteen SD bands have recently been observed in $^{149}$Gd [9]. Configurations for all thirteen bands have been proposed in ref. [9], underlying the fact that the intrinsic structure of SD bands is well understood in both mass regions. A review of the recent theoretical work which has led to this understanding can be found in the contribution of J. Dobaczewski to these proceedings [10].

In the remainder of this contribution, experimental results which address outstanding questions associated with superdeformation in the A=150 and 190 regions will be reviewed. Specifically, we will discuss the status of $\Delta I = 4$ staggering, of identical bands, of the presence of collective excitations in the SD well, and of feeding and decay of SD bands.

**$\Delta I = 4$ BIFURCATION IN SUPERDEFORMED NUCLEI**

Rotational states in nuclei are usually classified by two quantum numbers: parity and signature. The signature quantum number ($\alpha$) is associated with the fact that the intrinsic Hamiltonian is invariant under a rotation by 180 degrees around an axis perpendicular to the symmetry axis ($C_2$ symmetry). As a result, rotational bands built on configurations with $K \neq 0$ are characterized by two $\Delta I = 2$ rotational sequences with each of these labeled by its signature quantum number (0 or 1 for even nuclei, 1/2 or -1/2 for odd nuclei). If these so-called signature partner bands do not undergo Coriolis mixing, they form a regular sequence of states which differ in spin by 1 $\hbar$. However, strong Coriolis mixing can cause the two sequences to decouple, and levels in one band are then pushed up in energy while levels in the other are pushed down relative to their respective unperturbed states. The splitting of the two signature partner bands is often referred to in the literature as $\Delta I = 1$ staggering or $\Delta I = 2$ bifurcation.

Recently, a new phenomenon, $\Delta I = 4$ bifurcation, has been observed in several A=150 SD nuclei. The first reported case was in $^{149}$Gd from data taken with Eurogam [11]. In this experiment, the $J^{(2)}$ moment for the yrast SD band in $^{149}$Gd was observed to have an anomalous staggering for $\hbar \omega > 0.49$ MeV. In order to quantify this staggering, the differences in energy between two consecutive $\gamma$ rays in the SD band were plotted after subtracting a smooth reference given by $E_{\gamma}(1) = |E_{\gamma}(1 + 2) + 2E_{\gamma}(1) + E_{\gamma}(1 - 2)|/4$. When this quantity is plotted as a function of rotational frequency (see top panel in fig. 2), it was observed that the SD band splits into two sequences where successive members differ in spin by 4 $\hbar$. In analogy to $\Delta I = 2$ bifurcation, it was noted in ref. [11] that this bifurcation could be accounted for if levels in the SD band were alternatively pushed up and down by 58±18 eV relative to the unperturbed positions defined by the reference band. This small perturbation of the levels can be compared with that observed in $\Delta I = 2$ bifurcation which can be on the order of 10 to 100 keV.

In ref. [11], it was suggested that this effect might reflect the fact that the intrinsic Hamiltonian of the system is invariant under a rotation by $\pi/2$ ($C_4$ symmetry), and this possibility was linked to the hexadecapole deformation. Several theoretical papers began to explore this possibility [12–14]. Hamamoto and Mottelson assumed that the $C_4$ symmetry was brought about by the $Y_{44}$ deformation which produces hexadecapole distortions in the plane perpendicular to the symmetry axis of the prolate nucleus [12]. The Hamiltonian studied

$$H = A I_z^2 + B_1 (I_1^2 - I_2^2)^2 + B_2 (I_1^2 + I_2^2)^2,$$

(1)

is invariant with respect to the $C_{4v}$ point symmetry. By choosing appropriate values for the $A$, $B_1$ and $B_2$ coefficients ($B_2 = 0$, and $A/B_1 \sim 100$), a regular $\Delta I = 2$ staggering pattern occurs for the calculated transition energies. This pattern results from tunneling between the four equivalent minima present in the classical phase space.

Following the initial observation of $\Delta I = 4$ bifurcation in $^{149}$Gd, more examples were reported in both the 150 and 190 SD regions [15–17]. Unfortunately, the presence of bifurcation in all of the SD bands involved has been brought into question by subsequent measurements with larger data sets [8,18,19]. On the theoretical side, it has been demonstrated that the magnitude of hexadecapole deformation necessary to induce the staggering is much larger than is reasonable [20]. Other interpretations have been put forth as well. For example, several papers have suggested that the effect could be brought about by mixing between two or more bands [21,22], another discusses staggering in terms of intrinsic vertical motion [23], while yet another interpretation has suggested that $\Delta I = 4$ bifurcation is associated with the quadrupole deformation as defined within the interacting boson model [24].

A recent report by Haslip et al. [19] from data taken at Gammasphere identifies two more examples of $\Delta I = 4$ bifurcation in nuclei neighboring $^{149}$Gd, i.e. $^{148}$Eu and $^{148}$Gd. Figure 2 presents the staggering plots
for SD bands $^{148}\text{Gd}(\text{yrast})$ and $^{148}\text{Eu}(\text{yrast})$, the two newly discovered examples. Also plotted in the figure are $^{149}\text{Gd}(\text{yrast})$ (from ref. [11]) and $^{148}\text{Gd}(\text{yrast})$ where no staggering occurs. The magnitude of the effect in the two new examples is comparable to that in $^{149}\text{Gd}(\text{yrast})$. In addition, the $^{148}\text{Gd}(6)$ has identical γ-ray energies to those of $^{149}\text{Gd}(\text{yrast})$ in the frequency range where staggering is observed. Interestingly, the phase of the oscillation is opposite in sign. The structure of $^{148}\text{Gd}(6)$ has been proposed to be the same as that of $^{149}\text{Gd}(\text{yrast})$, except for a hole in the [411]1/2 neutron orbital. The $^{148}\text{Eu}$ yrast band is proposed to have the configuration $^{149}\text{Gd@n}[301]1/2^{+}$, and its moment of inertia is identical to that of $^{149}\text{Gd}(\text{yrast})$ in the staggering region.

One intriguing aspect of these new results is that they can be understood with the Hamiltonian given in (1). As pointed out in ref. [12], the tunneling amplitude can be expressed as a function of the angular momentum, $\cos[\frac{\pi}{2}(I - I_0)]$, where $I_0 = \sqrt{1/2} (\frac{A - 3}{2\hbar^2})$. By assuming the relative spins (for the 3 bands), ref. [19] has shown that the staggering as a function of spin can be reproduced by assuming a common value for $I_0 = 0.76 \pm 0.08$. In addition, the model accounts for the fact that the staggering in the isospectral bands ($^{149}\text{Gd}(\text{yrast})$ and $^{148}\text{Eu}(\text{yrast})$) is opposite in phase. While this adds support to the Hamiltonian suggested by Hamamoto and Mottelson, it is fair to state that the mechanism responsible for the $\Delta I = 4$ bifurcation remains unclear. This is a topic where more experimental examples would be very useful or even required to help solve the puzzle.
IDENTICAL BANDS IN SUPERDEFORMED NUCLEI

One of the more fascinating properties associated with the study of SD nuclei is that of identical bands. This classification not only includes pairs of SD bands which share identical transition energies (isospectral), but also encompasses SD pairs sharing identical moments of inertia. When first reported, this phenomenon was very surprising since mass differences between neighboring nuclei were expected to result in differences between the transition energies of SD bands which are an order of magnitude larger than those observed in identical bands.

The first report of an isospectral SD pair came from an experiment which observed an excited SD band in $^{151}$Tb whose transition energies were identical to the original $^{152}$Dy SD band [25]. It was shown in ref. [26] that this phenomenon could be accounted for if the SD band in $^{151}$Tb (i) was built on an intrinsic excitation with $\Omega=1/2$, (ii) had a decoupling parameter of $a=+1$, and (iii) had an identical moment of inertia to that of the core. Requirements (i) and (ii) were shown to be satisfied in this case by attributing the SD band in $^{151}$Tb to the configuration $^{152}$Dy(yrast)$\otimes[301]1/2^-$ and assuming that the pseudo-SU(3) asymptotic limit was nearly reached at large deformations [26]. In this model, it is the decoupling and alignment of the spin of the valence nucleon which allows for isospectral bands to be observed in neighboring nuclei. However, no explanation was proposed in [26] as to why the bands were isospectral to such a high-degree of precision.

Shortly after this report, examples of identical bands were also reported in the A=190 region. The first examples in $^{191,193,194}$Hg all showed an identical band relationship with the yrast SD band of $^{192}$Hg. More surprising still was the suggestion that these bands exhibited an alignment of 1 $\hbar$ relative to $^{192}$Hg [27]. The latter observation was based on the best estimates for the spins of these bands, since these quantities had not yet been measured experimentally. Stephens et al. [27] suggested that this phenomenon could be explained if the nucleonic spins from a quasiparticle pair decoupled and aligned with the axis of rotation while the orbital angular momentum remained strongly coupled to the symmetry axis. While this suggestion accounted for the relative spins of the bands, it offered no explanation for the fact that the bands were identical.

In the following years, many examples of identical bands have been uncovered, and there is at present no universally accepted explanation of the phenomenon. Theoretical suggestions have ranged from the possible manifestation of a new symmetry [28] to subtle cancellation effects between mass and pairing terms in the collective Hamiltonian [29], from the continuous readjustment of the self-consistent mean field with angular momentum [30] to a balance between residual p-n and pairing interactions [31]. A thorough review of both experimental data and theoretical work associated with identical bands is given in ref. [32].

With the current generation of $\gamma$-ray arrays much additional information has been obtained with regards to identical bands in the A=150 and A=190 regions. For example, many new pairs are now known, and the energies of these SD bands are determined with a high degree of accuracy. Several studies have tried to quantify the preponderance of identical bands in both normal deformed (ND) and SD nuclei [33,34]. In ref. [33], this was done by evaluating the fractional change (FC) in the $J^{(2)}$ moment between two rotational bands where $FC(\omega) = \frac{\langle J^{(2)}(\omega) - J_Y^{(2)}(\omega) \rangle}{\langle J_X^{(2)}(\omega) \rangle}$ and the mass of the nucleus corresponding to band X is always the largest. In addition, it can be shown that $FC = \frac{d\omega}{d\omega}$, where $i_{eff}$ is the difference in spins between the two bands at fixed $\omega$. Thus, if $i_{eff}$ is a linear function of $I$, the average slope of this function yields the average fractional change, $\overline{FC}$. This is the quantity presented in figure 3 for SD bands in the A=130, 150 and 190 regions. In panels a-c, all possible pairs are presented.

For A=150 SD nuclei (3-b), the distribution shows a number of sharp peaks. The one at $\overline{FC} = 0$ represent the identical bands. The other peaks in the distribution can be correlated with a change in the the number of occupied high-$N$ intruder orbitals. For example, an analysis of the intruder content of the bands producing the large peak at 0.1 shows that 85% of the pairs tested differ in intruder content by one $\frac{1}{2}$ proton and one $\frac{1}{2}$ neutron, irrespective of their actual mass difference. Additionally, the $\overline{FC}$ values are predominantly positive which indicates that the $J^{(2)}$ moment increases with mass. This again can be traced to the filling of the intruder orbitals as N and Z increase. In contrast, the A=190 (and A=130) SD nuclei show a broader distribution centered around $\overline{FC} = 0$, followed by secondary distributions on either side which are greatly reduced relative to the major distribution. The full distribution has been interpreted as resulting from the static pair correlations present in the A=190 SD nuclei. Support for this interpretation comes from the fact that ND nuclei show similar (although broader) distributions. This is illustrated in panels e and f of figure 3 where a subset of SD distributions are compared with normal deformed distributions. For SD nuclei, this subset corresponds to pairs of bands where the reference is the yrast band in a nucleus with mass A and the other member of the pair resides in the neighboring nucleus with A-1. For the normal deformed distribution odd-A
FIGURE 3. Panels a-c give the distributions for the average fractional change \( \langle FC \rangle \) in the \( J^{(2)} \) moment of SD bands in the mass 130, 150 and 190 regions. Panels e and f give the \( FC \) distributions for a subset of SD bands in the \( A=150 \) (e) and \( A=190 \) regions (f). In these cases, the comparison is performed for neighboring nuclei where the band in the heavier mass nucleus is always the yrast SD band. The stars show the corresponding distributions for normally deformed nuclei where odd-A nuclei are compared with yrast bands in even-even nuclei which are heavier in mass by one unit. The figure is taken from ref. 33.

Rare-earth nuclei are referenced to the yrast band in the neighboring even-even nucleus. The distribution for ND nuclei is similar to that for \( A=190 \) SD bands, but is significantly wider as would be expected because of increased pair correlations.

While the work of ref. [33] does not offer an explanation for identical bands, it has shown that they are more prevalent for SD nuclei than ND nuclei. This fact appears to be correlated with the decrease in the pairing strength in the SD well. In addition, the \( A=150 \) SD bands are observed to fall into distinct families defined by the number of high-\( N \) intruders occupied. A similar correlation cannot be made in the \( A=190 \) SD region.

Progress has also been made in characterizing the quadrupole moments of identical bands. This is accomplished by using the Doppler Shift Attenuation Method (DSAM) to measure the lifetimes of the SD states to high precision. The first such measurement using the new arrays was performed in the \( A=150 \) region by Savajols et al. [35] with the Eurogam spectrometer. Quadrupole moments were deduced for SD bands in \( ^{148,149}\text{Gd} \) nuclei produced in the reaction \( ^{30}\text{Si} + ^{124}\text{Sn} \) and for SD band 1 in \( ^{152}\text{Dy} \) produced in the \( ^{120}\text{Sn}(^{38}\text{S},4n) \) reaction. These quadrupole moments were found to vary from 14.6 eb to 17.5 eb. Two interesting conclusions were drawn from the data: (i) a direct relationship was observed between the number of occupied high-\( N \) orbitals and the magnitude of the quadrupole moments, and (ii) identical bands were found to also be characterized by identical quadrupole moments, e.g., \( ^{152}\text{Dy}(1) \) and \( ^{149}\text{Gd}(4) \). A similar measurement by Nisius et al. [36] with Gammasphere on SD bands of \( ^{151,152}\text{Dy} \) and \( ^{151}\text{Tb} \) reached the same conclusions, and quantified the
contributions of individual high-N orbitals to the quadrupole moments.

This link between identical SD bands in the A=150 and their high-N intruder content was examined in a recent theoretical investigation using the cranked Hartree-Fock (HF) Skyrme model [37]. In this study, an attempt was made to calculate relative quadrupole moments between SD bands by considering only contributions from the effective single-particle multipole moment $q_1(i)$ relative to a common core ($^{152}$Dy, yrast). Once the individual $q_1(i)$ values were determined, quadrupole moments in SD bands were constructed by directly adding the $q_1(i)$’s to produce the desired SD configurations relative to $^{152}$Dy(yrast). Both quadrupole ($\lambda = 2$) and hexadecapole ($\lambda = 4$) contributions were considered. The calculated relative quadrupole moments ($\delta Q_2$) using this extreme shell model approach were then compared to the experimental results of references [35] and [36], and excellent agreement between theory and experiment was found. The largest contributions to the quadrupole moments originate from the high-N intruder configurations. For identical bands, the difference in single-particle occupations come from orbitals which contribute very little to the quadrupole moment and apparently also to the moment of inertia.

More recent experimental lifetime measurements have attempted to test the limits of this concept of additivity of quadrupole moments as more and more particles are removed/added to the $^{152}$Dy SD core. The work of Hackman et al. [38] reports on lifetime measurements in SD bands of $^{142}$Sm, a nucleus lying at the lower corner in N and Z of the A=150 SD region, ten nucleons away from $^{152}$Dy. In $^{142}$Sm two SD bands are known and are found to have quadrupole moments of 11.7 eb and 13.2 eb. The two bands differ in intruder content by one $i_{13/2}$ proton and one $j_{15/2}$ neutron. The difference of 1.5 eb between the bands can be accounted for fully by this difference in intruder content. In addition, the quadrupole moments of these bands can be accounted for by starting from the $^{152}$Dy core and subtracting the contributions from the single-particle orbitals. It was concluded that additivity of quadrupole moments applies when as many as 10 particle are removed from the core.

In contrast, lifetime measurements in $^{153}$Dy appear to violate the additivity concept. Based on the assigned configuration for SD bands 1-3, additivity predicts that these bands have quadrupole moments between 17.3 and 17.9 eb. The experimental quadrupole moments are found to be $\sim 16.2$ eb for all 3 bands [39]. Thus, the deformation of the core appears to shrink immediately above the N=86 single-particle gap. Interestingly, SD bands 2 and 3 are identical to $^{152}$Dy(yrast), and as a result, this represents a case where identical bands do not have identical quadrupole moments.

The concept of additivity has also been recently extended to relative alignments between SD bands [40]. This approach builds on the pioneering work on effective alignments by Ragnarsson [41]. In this study, incremental alignments of specific orbitals were extracted from the data. These alignments were then used to calculate transitions energies for known SD bands by adding these effective alignments to a common core. The bands calculated in this way showed excellent agreement with the experimentally measured bands. This again stresses the concept of an extreme shell model picture for the SD bands of the A=150 region.

Precision lifetime measurements have also been made on SD bands in the A=190 region. One recent measurement has focused on the yrast SD band of $^{193}$Hg and on the three SD bands of $^{194}$Hg [42]. In this study, all SD bands were found to have the same quadrupole moment within errors ($Q_2 \sim 17.5$ eb). The $J^{(2)}$ moment of inertia of all 4 bands are similar, and the two excited bands in $^{194}$Hg are identical to the yrast SD band in $^{192}$Hg. On the other hand, a recent lifetime measurement in $^{190}$Hg shows that the yrast and first excited SD band have quadrupole moments which are also identical to the heavier Hg isotopes, even though the $J^{(2)}$ moments for these bands differ markedly from those measured in the heavier isotopes [43]. The fact that large deviations are not observed in the quadrupole moments in contrast to what is observed in the A=150 region can be explained by the presence of static pair correlations which minimize the polarizing effects of the high-N orbitals.

Somewhat puzzling are the results recently obtained for $^{193}$Hg by Busse et al. [44] where the quadrupole moments measured for five SD bands in $^{193}$Hg were found to be smaller than that for $^{192}$Hg yrast by $\sim 15\%$. There is no apparent explanation for this observation, and it is unclear whether the effect is due to a real difference in quadrupole moments or to a difference in the properties of the sidefeeding. Another measurement of the relative quadrupole moments using a different reaction in $^{192,193}$Hg is highly desirable.

With all these studies, have we moved any closer to understanding the identical band phenomenon? In the A=150 region, the characterization of the bands is well founded. For many cases quadrupole moments and transition energies can be described by the additivity of single-particle contributions. Current theoretical calculations using cranked-HFB or cranked relativistic mean field are able to reproduce the observables (moments of inertia, quadrupole moments) extremely well (see ref. [10] for a review), and one may be tempted to dismiss the phenomenon as a natural consequence of the mean field involving no new physics. However, there has
been a recent theoretical realization that pseudo-spin symmetry can be linked to explicit symmetries in the relativistic mean field [45]. If so, it is interesting to wonder whether or not such symmetries could also be related to identical SD bands in the A=150 region. No such link have yet been established, but it is clearly an avenue which should be pursued.

In the A=190 region, the characterization of the bands is also well founded, and in a few instances, excitation energies and spins have now been determined (see below). However, pair correlations do not allow for a simple single-particle description of SD bands. Attempts to apply a so-called extreme quasiparticle picture have been made but found to work only in isolated cases [17]. Theoretical calculations are doing well in reproducing the moments of inertia of the bands, but not as well as in the A=150 region, due to the fact that one must deal with the consequences of pairing [46]. In addition, it is difficult to point to a specific common feature or structure for the identical bands in the A=190 region. For example, a grouping of identical bands is found to include excitations built on SD ground states, quasiparticle excitations and octupole vibrations (see following discussion). Thus, an understanding of identical bands based on some fundamental symmetry is less clear. The idea that this linking commonality could be pseudo-spin has been revived in a recent paper by Schuck et al., where a pair of newly observed SD bands in 191Au is suggested to show pseudo-spin alignment for excitations with a pseudo orbital angular momentum (A) of one [47]. Evidence for such alignments in normally deformed nuclei with A = 0, 1 has recently been presented by Stephens et al. [48].

**VIBRATIONAL EXCITATIONS IN THE SUPERDEFORMED WELL**

The intrinsic structure of nearly all SD bands in the A=150 and A=190 regions can be understood in terms of either single-particle or quasiparticle excitations. In contrast, until recently, there had been no evidence for SD bands built on collective excitations. This came as somewhat of a surprise since rotational bands associated with quadrupole and octupole vibrations are commonplace features in nuclei at normal prolate deformation. Several recent theoretical investigations have suggested that collective octupole vibrations play a significant role in the SD wells of the A~150 and A~190 nuclei due to the presence near the Fermi surface of pairs of orbitals with opposite parity and \( \Delta l = 3 \) [49–51].

Strong evidence for an SD band built on an octupole vibrational state was found at Gammasphere in its early implementation phase. In this study, states in the SD well of 199Hg were populated, and a newly identified SD band was found to decay over at least three levels to the yrast SD states rather than to the states of normal deformation [52]. Based on the branching to the yrast SD band, it was suggested that the newly observed band was built on an octupole vibration. A subsequent experiment with Eurogam measured the energies of the transitions linking the excited and yrast SD bands. The angular distributions of these connecting transitions were found to be consistent with \( \Delta l = 1 \) dipoles [53]. Assuming an E1 character results in transition matrix elements on the order of \( 10^{-3} \) Weisskopf units (W.u.) for these linking transitions. However, these rates were only estimates, since the quadrupole moment for either band was not known. In addition, the anomalously large \( J^{(2)} \) moment for the excited SD band left open the possibility that the deformation in the two SD bands was significantly different. Recently, the quadrupole moments of both bands have been measured [43] and found to be identical (~ 17.6 eb). With the quadrupole moments determined, the \( B(E1) \) rates were extracted from the measured intensities. The \( B(E1) \) values are found to range between 1.5 and 2.5 \( \times 10^{-3} \) W.u. These numbers are orders of magnitude larger than those observed in heavy deformed nuclei, but are similar to those observed in actinide nuclei with strong octupole correlations. Further support for the octupole character of the band came from random-phase approximation (RPA) calculations based on the cranked Nilsson model (see figure 4).

Figure 4 presents both the partial SD level scheme for 199Hg and a comparison of the RPA calculations with the data. In the upper panel, the \( J^{(2)} \) moment of the excited SD band is compared with the RPA calculation for the lowest lying octupole excitation. In the region where the band is observed, there is excellent agreement between the data and the calculations. In the lower panel, theoretical Routhians relative to the SD vacuum are plotted for the four lowest excited states of both signatures. Both vibrational and non-vibrational states are calculated in the RPA. However, the lowest calculated states are all members of the octupole multiplet. The experimental Routhian for the excited SD band relative to the yrast SD band (i.e. its energy and slope with \( \hbar \omega \)) match nicely with the lowest lying calculated Routhian. This lowest vibrational state at \( \hbar \omega > 0.3 \) MeV corresponds to the completely aligned octupole phonon. At low frequency, this state corresponds to the K=2 component of the multiplet and one would not expect decays to the K=0 yrast SD band. However, at the higher frequencies a substantial K=0 component is mixed into the wavefunction due to the Coriolis force.
FIGURE 4. On the left, partial level structure for SD bands in $^{190}$Hg. All members of the SD band proposed to be built on an octupole vibration are shown together with the transitions linking it with the yrast SD band. On the right, comparison of the data with results from RPA calculations based on the Nilsson potential. See text for details.

thus enhancing the E1 decays from the excited to the yrast SD band.

While the RPA calculations are able to account satisfactorily for the properties of the excited SD band in $^{190}$Hg, the work of ref. [51] predicts that the lowest lying excited SD bands in $^{192,194}$Hg should also be associated with octupole vibrations. In the case of $^{194}$Hg, this was surprising since the two excited SD bands observed can be understood as signature partners of a two-quasiparticle excitation [54]. Of additional interest is the fact that these two excited bands exhibit an identical band relationship with the yrast SD band in $^{192}$Hg. Direct transitions from the SD states to ND yrast states have recently been observed for both SD band 1 (yrast) and SD band 3 (see next section) [55,56]. In addition, transitions linking SD band 3 and SD band 1 have been measured. (see fig. 6). Therefore, $^{194}$Hg represents the first case where the absolute spins, parities and excitation energies of the two bands have been established, i.e. band 1 has even spin, positive parity and band 3 has odd spins and negative parity. In addition, band 3 is estimated to lie only 0.8 MeV above the yrast SD band at $\hbar \omega = 0$, and the measured B(E1) values for the decay of band 3 to band 1 are $\sim 10^{-5}$ W.u., i.e. they are smaller than in $^{190}$Hg. All these observations are in good agreement with the RPA calculations. In particular, the decrease in the B(E1) rate is predicted. A larger separation in energy between the low lying $K^\pi = 2^-$ component and the remaining members of the octupole multiplet results in reduced admixtures from the $K^\pi = 0^-$ and $1^-$ components, thus quenching the B(E1) rate. While the decay of band 2 is not observed to go through band 1 or band 3 nor are direct links to normal states observed, this band is assumed to be the signature partner to band 3. Assuming no signature splitting between bands 2 and 3 at low spins, it has been pointed out that the small increase in signature splitting with increasing spin is consistent with the octupole interpretation of these two bands [57]. Thus, there is strong experimental support for the theoretical interpretation of ref. [51] that bands 2 and 3 in $^{194}$Hg are built on the K=2 octupole vibrational state.

Additional support for this interpretation comes from new data obtained with Eurogam II on $^{196}$Pb, the isotope to $^{194}$Hg. Three excited SD bands have been identified, all of which are reported to decay to the yrast SD band [58]. Excited SD bands 2 and 3 appear to be strongly coupled signature partners with extracted B(E1) rates on the order of $10^{-5} - 10^{-6}$ W.u., i.e. these bands appear to be analogues to SD bands 2 and 3 in $^{194}$Hg. The interpretation of these structures as K=2 octupole vibrational bands is again consistent with the RPA calculations. Band 4 lies $\sim 280$ keV higher in energy than band 3 and its B(E1) decay rate to SD band
1 is $\sim 10^{-4}$ W.u. It has been interpreted as the $K=0$ member of the octupole multiplet.

In summary, there is now strong evidence that the lowest lying excited SD bands in even-even nuclei of the $A=190$ SD region correspond to structures built on octupole vibrations. All measured observables are consistent with RPA calculations which include the octupole degree of freedom. Finally, in the $A=150$ region, collective excitations have been invoked to account for a small number of SD bands, i.e., band 6 in $^{152}$Dy [59], band 5 in $^{150}$Gd [60] and band 3 in $^{148}$Gd [15]. However, information such as branching ratios, relative excitation energies and spins are not available, thus making it difficult to perform detailed comparisons between experiment and theory as is the case for $A=190$.

FEEDING AND DECAY

The discovery of a SD band in $^{152}$Dy [2] opened up a number of avenues of inquiry most of which were directed towards mapping out the $A=150$ and $A=190$ SD regions. However, a few investigations did attempt to understand how SD bands were fed and how they decayed. Schiffer and Herskind [61] presented some of the first calculations describing the feeding of SD bands. Moore et al. [63] added critical experimental information to this topic by measuring the average entry points in the energy-spin $(E,I)$ plane for both ND and SD states in $^{192}$Hg and $^{152}$Dy. Lauritsen et al. [62] measured, for the first time, the entry distribution of a SD band, and showed that this distribution originates from the higher spin components of the total entry distribution. Monte Carlo calculations were able to reproduce all the observables associated with the feeding of the SD band in the $A=190$ region. Recent work on $^{143}$Eu has focused on feeding of SD bands in the $A=150$ region. This work has attempted to account for all of the feeding into the SD well [64]. Indeed, it appears that a significant amount of intensity that gets trapped in the SD well bypasses the SD yrast band. The transitions which account for this intensity have been interpreted as coming from rotationally damped states in the second well [65].

In the $A=150$ SD region, it was observed that mass-symmetric reactions enhance the population of SD bands relative to mass-asymmetric reactions forming the same compound system at the same excitation energy [66,67], and it was suggested that such effects could be explained by an increase in the fusion time for mass-symmetric reactions. The same effect could not be confirmed in the $A=190$ SD region [68]. This subject has recently been re-examined in the mass 150 region using Gammasphere [69]. In this study, the yrast SD band in $^{147}$Gd was populated in both the $^{76}$Ge + $^{76}$Ge and $^{28}$Si + $^{124}$Sn reactions. The beam energy for each reaction was chosen such that the compound nucleus $^{152}$Dy was produced with an excitation energy of 87 MeV. Relative to the ND states, a population enhancement by a factor of $\sim 5$ was found in the mass-symmetric reaction for the $^{147}$Gd yrast SD band. In addition, the feeding of the SD quasicontinuum in coincidence with the SD band was measured to be at least $12$ times stronger in the mass-symmetric reaction. Interestingly enough, the relative population intensities as a function of $\gamma$-ray energy remained identical in the two reactions indicating no enhancement in the feeding of the highest-spin states. Rather, the additional feeding occurs over the entire band for the mass-symmetric reaction. While dissipative collision calculations suggest that the fusion time for the mass-symmetric reaction is about 6 times longer than that for the mass-asymmetric one, it is far from clear that this time difference is sufficient to account for the observed changes in the feeding.

The ability to link the SD states to known yrast levels of definite spin, parity and excitation energy has been a long outstanding challenge in the study of superdeformation. In the mass 130 region, the observed SD bands lie only $\sim 0.8$ MeV above the ND yrast states, and it has been possible to identify many of the decay pathways between the SD and less deformed yrast states. In contrast, only two examples of direct links have been reported in the $A=190$ region and no examples have yet been observed conclusively in the $A=150$ region.

The first report that the excitation energy of a SD band in either region had been determined was in $^{143}$Eu where 5-6 “sum peaks” were observed in coincidence with the yrast SD band [70]. These sum peaks were extracted from 3-fold coincidence events where one of the three detected $\gamma$ rays was required to be a member of the SD band and the other two transitions were summed together. The sum peaks seen in this procedure were associated with the two-step decay of the SD band towards the yrast line, placing the lowest SD state at 3.635 MeV above the yrast 35/2$^+$ level. However, two recent follow-up studies performed on $^{143}$Eu at Eurogam II [71] and Gammasphere [72] are in contradiction over this result. Both works report discrete high-energy $\gamma$ rays ($E_{\gamma} \geq 2.5$ MeV) in coincidence with the SD band, however, none of these lines can be established as a direct link between the SD band and the yrast structure. Furthermore, the Gammasphere measurement does not find evidence of the sum peaks discussed above while the Eurogam data reproduce only a subset of the sum-peaks proposed originally.

In the mass 190 region, a different approach was adopted. Here, a total spectrum in coincidence with
FIGURE 5. Deconvolved components of the total spectrum in coincidence with the SD yrast band in $^{192}\text{Hg}$. See text and ref. 73 for details.

The known $^{192}\text{Hg}$ SD band was extracted from the data by placing pairwise coincidence gates on SD lines. This spectrum was then properly background subtracted and corrected for neutron interactions, coincidence summing, detector response and photo-peak efficiency [73]. In order to extract the total spectrum of γ rays connecting SD to ND states, transitions which feed the SD band, connect SD band members, and connect ND states at the end of the cascade were removed. The various components extracted from this deconvolution are shown in fig. 5, and panel 3 is the decay spectrum. This spectrum is characterized by a quasicontinuum of mostly dipole character with a statistical distribution which supports the suggestion of ref. [74] that an SD level decays to ND states when it acquires a small component of a hot compound state, and decays through the admixture of this component.

Other quantities extracted from the decay spectrum are: (i) the average number of transitions in the decay of the SD band to the yrast line ($3.2 \pm 0.6$), and (ii) the excitation energy and spin of the SD band at the point of decay ($4.3 \pm 0.9$ MeV and $10.1 \pm 0.7\hbar$). The decay spectrum also contains a noticeable bump between 1.3 and 2.2 MeV which sits on top of the statistical spectrum. A recent calculation by Dossing based on a model which treats self-consistently the weakening of pair correlations with increasing number of quasiparticle excitations reproduces the decay spectrum including this bump [75]. In the calculation, the bump arises from a combination of transitions from the sequential steps of the de-excitation cascade and from the last step of the decay where the statistically fed continuum states must cross the pair gap to the ground band.

While the extraction of the decay spectrum in $^{192}\text{Hg}$ revealed much about the nature of the decay process, it could only provide limits on the excitation energy and spins of the SD band. In order to ascertain these physical quantities directly, it is necessary to observe so-called “one-step” linking transitions between SD states and states of known spin, parity and excitation energy. The first unambiguous observation of these one-step transitions reported three γ rays with energies of 3489, 4195, and 4485 keV which were shown to decay directly from SD band 1 to a member of the yrast negative parity band in $^{194}\text{Hg}$ [55] (see fig. 6). Angular distribution information confirmed that these transitions had dipole character and, based on the fact that the decay spectrum showed a statistical distribution [73], the γ rays were assumed to be electric dipoles and thus parity changing. With this information, the lowest observed member of the SD band was assigned a spin of...
FIGURE 6. Linking transitions for $^{194}\text{Hg}$ and $^{194}\text{Pb}$

8, an excitation energy of 6419 keV, and positive parity. These “one-step” transitions only carried 3% of the decay strength. In a followup measurement, two one-step transitions of dipole character from SD band 3 to known positive parity states in $^{194}\text{Hg}$ were observed, thus establishing odd-spins and negative parities for the levels in this SD band [56] (see discussion above).

Immediately after the linking transitions were established for SD band 1 in $^{194}\text{Hg}$, the observation of one-step decays in $^{194}\text{Pb}$ was reported from data taken at Eurogam [76] and subsequently confirmed by a study at Gammasphere [77]. The known placement of single-step decay transitions in $^{194}\text{Hg}$ and the single-step links deduced in ref. [76] are summarized in the partial decay schemes of fig. 6. Interestingly, there are significant differences between the two cases:

- The yrast SD band in $^{194}\text{Hg}$ lies ~1.4 MeV higher in excitation energy than the yrast SD band in $^{194}\text{Pb}$.
- The one-step decay intensity in $^{194}\text{Hg}$ accounts for only ~3% of the decay strength while for $^{194}\text{Pb}$ it is ~21%.
- For $^{194}\text{Hg}$, the one-step transitions are all of E1 multipolarity while in $^{194}\text{Pb}$ both E1 and M1/E2 one-step transitions are observed.

Based on these observations, lifetimes for the $8^+$ and $10^+$ levels of the yrast SD band in $^{194}\text{Pb}$ were measured using the recoil-distance Doppler shift method in order to ascertain whether the deduced electromagnetic properties are consistent with statistical decay [78]. These lifetime results were then used in a simple band mixing model in order to estimate the admixture of ND states into the SD band members at the points of decay. These admixtures are very small (~1%) and again consistent with the model suggested by Vigezzi et al. [74]. In addition, by correcting for this admixture, reduced transition probabilities were estimated for the ND-ND portion of the one-step decays. These are (i) $B(E1) \sim 10^{-6} - 10^{-5}$ W.u., (ii) $B(M1) < 5 \times 10^{-4}$ W.u., and (iii) $B(E2) < 3 \times 10^{-2}$ W.u.. These transition probabilities are all within the limits of statistical decays established, for example, from neutron resonance capture experiments. Therefore, even though variances in the SD decay properties are observed between $^{194}\text{Hg}$ and $^{194}\text{Pb}$, they both appear to be statistical in nature.

While the one-step decay pathways between superdeformed and normally deformed states have been unambiguously established for $^{194}\text{Hg}$ and $^{194}\text{Pb}$, the observation of more cases in the A=190 region has not been forthcoming. For example, data sets of equivalent size have been taken on $^{192}\text{Hg}$ and approximately 50 weak transitions with energies ranging between 1 and 3.2 MeV have been observed in coincidence with SD band 1 [79]. However, none of these $\gamma$ rays appears to represent a decay between the SD band and a known level, and
as a result, the precise excitation energy of this SD band has not yet been established. One obvious question to be asked is why the one step decays are observed in only a few examples. Recent work has attempted to show experimentally that the observed linking transitions are due to fluctuations based on a Porter-Thomas distribution [80]. While the analysis is consistent with such a conclusion, the rather low probability represented by such decays (10^-4) cannot rule out the possibility that there are also special selection rules associated with the decay of SD bands.

SUMMARY

The study of SD nuclei in the \( \Lambda = 150 \) and \( \Lambda = 190 \) regions has made great strides over the last decade. Part of this success has come from the construction of very large and powerful \( \gamma \)-ray spectrometers. It is safe to say that a "very good" understanding of the characteristics of SD bands in both regions exists. However, several open problems still remain such as, (i) a fundamental explanation for \( \Delta I = 4 \) staggering, (ii) fundamental understanding of identical bands, and (iii) precise knowledge of excitation energies, spins and parities of SD bands beyond a few isolated examples.

ACKNOWLEDGEMENT

The authors would like to acknowledge the numerous contributions of current and past Argonne Physics Division members, T.L. Khoo, T. Lauritsen, I. Ahmad, E.F. Moore, R.R. Chasman, G. Hackman, D. Nisius, S.M. Fisher, I. Widenhoever, P. Reiter and B. Crowell and students H. Amro and S. Siem, who have collaborated with us on our own studies of superdeformed nuclei in these two mass regions. This work is supported by the U.S. Department of Energy, Nuclear Physics Division under contract W-31-109-ENG-38.

REFERENCES

8. S.M. Fischer et al., to be published.
10. J. Dobaczewski, proceedings to this conference.
39. D. Nisius et al., to be published.