ADELPHI UNIVERSITY, GARDEN CITY, NEW YORK 11530

Quarterly Technical Progress Report

PROJECT NAME: Development of a Phenomenological Model For Coal Slurry Atomization

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Principal Investigator: DR. JOHN DOOHER

ADDRESS: DEPARTMENT OF PHYSICS
ADELPHI UNIVERSITY
BLODGETT HALL ROOM 210
GARDEN CITY, NY 11530

TELEPHONE: (516) 877-4883

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PROGRESS SUMMARY

A theoretical derivation is presented below with a presentation of the various pressure drops occurring in the extensional viscometer. In particular, by using energy considerations the meaning of each pressure drop is clearly delineated. This allows for unambiguous data analysis. In the coming quarter, pressure transducers will be added to the viscometer and a series of tests performed on fluids with extensional flow properties.

Energy Dissipation and Extensional Flow

The total kinetic energy of an incompressible fluid is given by:

$$ E_{\text{Kin}} = \frac{1}{2} \rho \int v^2 \, dV $$

where $\rho$ is the fluid density and $v$ is the fluid velocity at a point $\mathbf{x} = (x,y,z)$ at time $t$. Subscript notation will also be used for coordinates, i.e. $x_1 = x, \ x_2 = y, \ x_3 = z$, $\mathbf{x} = (x,y,z) = x_i, \ i=1,2,3$.

Taking the time derivative of the energy yields

$$ \frac{d}{dt} E_{\text{Kin}} = \rho \int \frac{\delta v_i}{\delta t} \, v_i \, dV \quad (1) $$

where summation of repeated indices is implied (Einstein summation convention).

The Navier Stokes equation is

$$ \frac{\delta v_i}{\delta t} = -v_k \frac{\delta v_i}{\delta x_k} - \frac{1}{\rho} \frac{\delta P}{\delta x_i} + \frac{1}{\rho} \frac{\delta \sigma_{ij} \, k'}{\delta x_k} \quad (2) $$

where $P$ is the hydrostatic pressure and $\sigma_{ij} \, k'$ is the extra stress tensor.
Using Eq. 2 in Eq. 1 and the fact that $\nabla \cdot \mathbf{v} = 0$ and

the identity $\nabla \cdot (\mathbf{v} \left( \frac{1}{2} \mathbf{v}^2 + \frac{P}{\rho} \right)) = \nabla \cdot \frac{\mathbf{v}}{\rho} \left( \frac{1}{2} \mathbf{v}^2 + \frac{P}{\rho} \right) + \mathbf{v} \cdot \nabla \left( \frac{1}{2} \mathbf{v}^2 + \frac{P}{\rho} \right)$ yields

$$
\frac{d}{dt} E_{\text{Kin}} = - \int dV \left( \rho \mathbf{v} \cdot \left( \frac{1}{2} \mathbf{v}^2 + \frac{P}{\rho} \right) - \nabla \cdot \mathbf{v} \right) - \int \mathbf{v} \cdot \nabla \left( \frac{1}{2} \mathbf{v}^2 + \frac{P}{\rho} \right) dV
$$

(3)

where $\nabla \cdot \mathbf{v} = \mathbf{v}_i \sigma_{i,k}$ and $\nabla \cdot (\mathbf{v} \cdot \mathbf{v}) = \frac{\delta v_i}{\delta x_k} \sigma_{i,k} + v_i \frac{\delta \sigma_{i,k}}{\delta x_k}$ have been used.

Using the divergence theory transforms Eq. 3 into

$$
\frac{d}{dt} E_{\text{Kin}} = - \oint_{S} \mathbf{h} \cdot \mathbf{n} \left[ \rho \mathbf{v} \cdot \left( \frac{1}{2} \mathbf{v}^2 + \frac{P}{\rho} \right) - \nabla \cdot \mathbf{v} \right] - \int_{V} \mathbf{v} \cdot \frac{\delta v_i}{\delta x_k} dV
$$

(4)

where the first integral is over a close surface $S$ bounding the volume $V$ and $\mathbf{h}$ is a unit normal outward to the surface $dA$ on $S$. Equation 4 will be applied to steady flow through a contracting region shown in Figure 1.

Figure 1. Contracting Flow Geometry
The surface integral will vanish on the sides of the cylinders in Figure 1 since the velocities vanish there. Assuming that steady Poiseuille flow has been established for 1-2 and 3-4 on the cross sectional area \( A_0 \) at 1 and \( A_1 \) at 4 only \( v_3 \) exists and \( \hat{n} \) is the 3 direction so that the term \( \mathbf{v} \cdot \mathbf{\sigma} \cdot \hat{n} \) becomes \( v_3 \sigma_{33} \) which is zero. For steady flow, \( \frac{d}{dt} E_{\text{Kin}} = 0 \) since the velocity distribution is independent of time. Eq. (4) thus gives

\[
0 = + \int \int dA \rho v \left( \frac{1}{2} v^2 \right) + P_1 \int \int dA v - \int \int dA \rho v \left( \frac{1}{2} v^2 \right) - P_4 \int \int dA v \\
- \int \mathbf{V} \cdot \frac{\mathbf{\sigma}}{\mathbf{\delta x}} \cdot \mathbf{V} \cdot \mathrm{d}V
\]

\( (5) \)

Now \( \int \int dA v = Q_1 = Q_4 = \int \int dA v = Q \) the volumetric flow rate.

Eq. 5 can be rewritten \( (P_1 - P_4) Q = \Delta(KE) Q + \int \mathbf{V} \cdot \frac{\mathbf{\sigma}}{\mathbf{\delta x}} \cdot \mathbf{V} \cdot \mathrm{d}V \)

where \( \Delta(KE) = \left( \frac{1}{2} \rho v^2 \right)_4 - \left( \frac{1}{2} \rho v^2 \right)_1 \)

is the change in the value of the kinetic energy density averaged over cross sectional area, i.e.

\[
\int dA \rho v \left( \frac{1}{2} v^2 \right) = Q \left( \frac{1}{2} \rho v^2 \right)
\]

The volume integral represents the viscous energy dissipation and may be broken into Poiseuille pressure drops between 1-2 and 3-4 and the pressure drop across the contraction as flows.

\[
\int \mathbf{V} \cdot \frac{\mathbf{\sigma}}{\mathbf{\delta x}} \cdot \mathbf{V} = \int \int \int = \Delta P_{12} Q + \Delta P_{23} Q + \Delta P_{34} Q
\]
Defining $\Delta P_{23} = \Delta P_c$, the final equation for pressure drops becomes

$$ (P_1 - P_4 ) = \Delta P_T + \Delta KE + \Delta P_{12} + \Delta P_{34} + \Delta P_c \quad (6) $$

It is the study of $\Delta P_c$ from Eq. 6 that yields information on contracting flow. The basic equation is

$$ E_c = + \int_c \sigma_i k \frac{\delta v_i}{\delta x_k} \, dV = + \frac{1}{2} \int_c \sigma_i k \left( \frac{\delta v_i}{\delta x_k} + \frac{\delta v_k}{\delta x_i} \right) \, dV = Q \, \Delta P_c \quad (7) $$

The integral in Eq. 7 is over the contracting region 2-3.

As an example of applying the energy dissipation integral consider a tube of radius $R$ and length $L$ through which flows a fluid of viscosity $\eta$. Using the standard velocity distribution and applying Eq. 4 yields

$$ \Delta P \, Q = \frac{8 \, \eta Q^2 L}{\pi \, R^4} \quad \text{or} \quad \frac{\Delta P}{L} = \frac{8 \, \eta Q}{\pi \, R^4} $$

which is the result for Poiseuille flow.

It can be shown that for any fluid the flow field is such as to minimize the energy dissipation in a volume, $V$. (see A.M. Freudenthal and H. Geisinger in the Encyclopedia of Physics, Volume VI pages 251-256.)