Assessment of California Reformulated Gasoline Impact on Vehicle Fuel Economy

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**ASSESSMENT
OF
CALIFORNIA REFORMULATED GASOLINE IMPACT
ON
VEHICLE FUEL ECONOMY**

**S. Aceves, R. Glaser, and J. Richardson
Lawrence Livermore National Laboratory**

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ABSTRACT

Fuel economy data contained in the 1996 California Air Resources Board (CARB) report with respect to the introduction of California Reformulated Gasoline (CaRFG) has been examined and reanalyzed by two additional statistical methodologies. Additional data has also been analyzed by these two statistical approaches. Within the assumptions of the analysis, point estimates for the reduction in fuel economy using CaRFG as compared to conventional, non-reformulated gasoline were 2-4 %, with a 95% upper confidence bound of 6 %. Substantial variations in fuel economy are routine and inevitable due to additional factors which affect mileage, even if there is no change in fuel reformulation. This additional analysis confirms the conclusion reached by CARB with respect to the impact of CaRFG on fuel economy.

INTRODUCTION

The introduction of California Phase II Reformulated Gasoline (CaRFG) was initiated early in 1996 as part of a comprehensive regulatory program administered by the Air Resources Board (ARB) to reduce emissions from both on-road and off-road motor vehicles. Resultant emissions reductions are needed if California is to meet its obligations in the State Implementation Plan which is required by the amendments to the federal Clean Air Act. Under the ARB regulation, retail distribution of the new gasoline was required statewide on June 1, 1996.

In preparation for introduction of CaRFG, the ARB formed the California Reformulated Gasoline Advisory Committee consisting of more than 70 representatives from industry, public interest groups and government agencies in 1994. The function of the Committee is to advise the Board on issues concerning the compatibility of CaRFG with vehicles and equipment, the transition of CaRFG into the distribution and marketing systems and the public's acceptance of CaRFG. Of three subcommittees formed to research and monitor these issues, the Performance Subcommittee was given the task to help design fuel testing plans to evaluate the performance and compatibility of CaRFG in on-road motor vehicles, fuel storage systems and off-road vehicles and equipment.

The Performance Subcommittee, among other tasks, reviewed fuel economy data from test vehicles on the road and from laboratory chassis dynamometer vehicle tests to evaluate vehicle fuel economy trends for CaRFG in comparison to other gasolines. Their findings were published in March of 1996 in a report titled "CaRFG Performance and Compatibility Test Program", subtitled "Report of the Performance Subcommittee of the California Reformulated Gasoline Advisory Committee" with appendices.

The purpose of this current study, sponsored by the California Legislature and administered by the California Energy Commission, was to review and apply rigorous scientific and analytical skills to assess the reasonableness of data, procedures, and methodologies applied to reach CARB's conclusions regarding the impact of California Phase II gasoline on the fuel economy of vehicles in California. It was agreed that the following five tasks would constitute this current study:

- (1) Identify and review the sources of data utilized by ARB staff for accuracy, completeness, and integrity. Identify sources of data that were not used and any new data that has become available since ARB published their findings.
- (2) Review methods utilized by ARB staff for data aggregation and comparison with conventional gasoline, including statistical methods applied and the basis for rejection of data not utilized. If necessary, recompute fuel economy comparisons and resultant differences and compare with ARB findings.
- (3) Incorporate new relevant data into the analysis, recompute fuel economy and compare results with Task (2) findings.
- (4) Through a literature review, identify factors other than gasoline formulation (e.g., on-road driving habits, maintenance practices, climatic conditions, and others) which could explain fuel economy effects as reported by some California drivers. Compare these effects with the findings of Tasks (2) and (3).
- (5) Prepare a final report of findings of the study. Include a quantitative estimate of the confidence in the fuel economy results.

Consequently, no new experimental or test program data were generated by this study, and thus no new fuel economy data was generated as a result of this study.

BACKGROUND

The motivation for the development and introduction of reformulated gasoline is derived from the desire to reduce automobile emissions. California Phase 2 Reformulated Gasoline (CaRFG) has specifications for the following fuel properties:

1. Reid vapor pressure (RVP)
2. Aromatic content
3. Olefin content
4. Oxygen content
5. 90% distillation temperature (T90)
6. 50% distillation temperature (T50)
7. Sulfur content
8. Benzene content

The introduction of additional oxygen-containing compounds into gasoline both effects the energy content and the physical/chemical properties of the gasoline. The effect of variations in gasoline composition has been the focus of considerable research and analysis by the Auto/Oil Air Quality Improvement Research Program (AQIRP), a consortium of major U.S. automakers and oil companies (Chrysler, Ford, General Motors, Amoco, ARCO, Ashland, BP America, Chevron, Conoco, Exxon, Marathon, Mobil, Philips, Shell, Sun, Texaco, Unocal, Elf France, Ethyl Corp., and UOP). Although the primary focus has been on the effect of fuel variations on emissions, some data has been reported on fuel economy in the SAE literature. [1,2] One of these studies by AQIRP, although not using CaRFG, does calculate, based on emission data and for a series of “older” (1983-1985) and “current” (1989) small fleets, a range of -1.7 % to -4.9 % reduction in variously defined fuel economies in using an MTBE oxygenated gasoline compared to conventional gasoline. [1] The MTBE gasolines are similar to but not identical to CaRFG. The quality of maintenance has been identified as a major component in aggregated vehicle emissions. [3]

Current gasoline is formulated to meet a number of specifications, related to engine performance, volatility and seasonal effects, performance, etc. Surprisingly, the energy content, which has been shown to be proportional to fuel economy, [1] is not explicitly specified. Octane is indicative of engine performance (i.e., the ability of the fuel to be used in high performance, high compression ratio engines without “knocking”), not energy content, and is satisfied by additives and fuel composition; gasolines of significantly different energy content can have identical octane ratings. Representative analysis and specifications for CaRFG, Federal RFG, and “conventional” gasoline are shown in Table I. [4-7] The energy values shown are measured for different batches of the appropriate.

TABLE I. Typical Gasoline Parameters

PROPERTY	CONVENTIONAL	FEDERAL RFG	CA RFG
RVP, psi	8.7	7	6.8 - 7
Aromatics, v %	32	27	22
Olefin, v %	9	8.5	4
Oxygen, wt %	0	2	2
T90, °F	330	329	290
T50, °F	220	210	200
Sulfur, ppm	340	130	30
Benzene, v %	1.5	0.8	0.8
Energy, BTU/gal	117,900-115,000	115,600	111,400

gasoline. The conventional properties are measured for a representative 1988 standard gasoline; the Federal RFG and CaRFG parameters are regulated specifications

However, gasoline composition has varied considerably over the years; hence “conventional” is an ambiguous term. The industry standard is 1988 conventional gasoline, but the composition of gasoline, not counting the introduction of oxygenated compounds, has varied considerably since 1988. [8] Furthermore, due to a desire for different summer and winter physical properties (e.g., vapor pressure), energy content shows a seasonal dependence. Finally, probably because it is not specified, energy content can vary considerably even within a given nominal gasoline seasonal composition. Table II shows averages and typical ranges for variation in energy content of conventional gasolines documented by the EPA. [9]

TABLE II. Energy Content (BTU/gal) of Conventional Gasolines: EPA Data

	MINIMUM	MAXIMUM	AVERAGE	% RANGE
Summer	113,000	117,000	114,500	± 3.5
Winter	108,500	114,000	112,500	± 4.9

Table III shows similar data in a survey of 1990-1991 California gasoline. [10]

TABLE III. Energy Content (BTU/gal) of 1990-1991 California Conventional Gasolines: CARB Data

	MINIMUM	MAXIMUM	AVERAGE	% RANGE
Summer	109,900	120,300	115,800	+3.9 % to -5.1 %
Winter	110,900	114,300	117,700	± 3.0 %

It is clear that a major source of on-road variability in fuel economy, in addition to all the variability in the driving conditions (e.g., wind, load, speed, air conditioning, reproducibility of fill-up - *vida infra*), is the expected variation in the energy content of ostensibly identical fuels.

CARB Report

The Air Resources Board, under the guidance of the Performance Subcommittee of the California Reformulated Gasoline Advisory Committee, evaluated the performance and compatibility of CaRFG in on-road motor vehicles, fuel storage systems, and off-road vehicles and equipment. These findings are presented in the recent CARB report. [10] This CARB report, with respect to fuel economy, presents results from fleet on-road trials, a small dynamometer study, and catalogues earlier fuel economy studies done by various sources. The CARB report also examines maintenance issues related to the use of CaRFG in the CARB on-road fleet study. Finally, the CARB report also presents industry-reported results with respect to materials compatibility and off-road vehicles.

1) On-road fleet study. Any sampling of a large population has to consider the level of fidelity with which the sampling represents the true population. In this case important factors include the vehicle type, age and mileage distribution, and driving patterns. The fleets chosen by CARB are representative of large industrial or government fleets; these are the only organizations which have fleets, but these fleets are not the most representative imaginable of the overall California vehicle population. Table IV lists salient characteristics of the fleets and corresponding gasoline energy content which participated in

the study. More detailed characterization with respect to fleet and gasoline composition is in the CARB report.

TABLE IV. CARB Test (with CaRFG) and Control (with Conventional Fuel) Fleet Description and Corresponding Gasolines:

FLEET	TEST VEHICLES	CONTROL VEHICLES	AVG FUEL ENERGY BTU/GAL		FUEL ECONOMY CHANGE
			TEST	CONTROL	
City of Sacramento	106	81	110,400	115,500	-2.3 %
Sacramento County	173	241	110,700	115,600	-2.4 %
CSU Fresno	112	0	110,500	None	NA
Bank of America	20	10	110,300	113,700 ¹	NA ²
GTE	254	157	110,400	113,600 ¹	NA ³
Pacific Bell, North	84	110	110,400	115,400	NA ³
Pacific Bell, South	55	38	110,300	113,100 ¹	NA ³
Caltrans	25	0	110,200	None	NA
TOTAL	829	637	110,400		

¹ Federal Reformulated Gasoline

² Insufficient data to analyze

³ Auxiliary equipment used fuel from vehicle tank

Principle strengths and weaknesses are readily apparent from Table IV. A number of fleets containing a large number of vehicles were chosen. However, only two of the fleets resulted in useable data with respect to fuel economy (the other fleets did provide data concerning maintenance and hence materials compatibility). Even in those cases, some of the data was trimmed from the initial set on the basis of not being realistic (i.e., abnormal deviations in mileage, probably due to data entry error). The major reason for excluding six of the fleets was the use of auxiliary equipment, but three of the excluded fleets also used Federal RFG. Although that was the only gasoline available in southern California prior to the introduction of CaRFG, it does introduce additional ambiguity. Of the two fleets providing useable data, one consisted of police cars in the city of Sacramento, and the other contained 30% medium and heavy duty trucks in the county of Sacramento (the state-wide percentage is only 8 %). Thus, neither fleet was exceptionally representative of the California state vehicle population nor did at least the city fleet represent "typical" driving patterns. Table V presents the fuel analysis from the conventional gasoline used as the control for the two Sacramento fleets.

It is apparent that the conventional fuel available in Sacramento for the two fleets was neither identical (e.g., the county gasoline was substantially oxygenated) nor representative of the industry standard, circa-1988 conventional fuel. However, the energy contents were nearly identical and similar to that expected for industry standard gasoline.

The observations concerning the fidelity of the fleet representation of the California vehicle population coupled with the variability of the control fuel merely illustrate the difficulty in doing a large, representative, on-road fuel economy study with control of potentially significant variables. The CaRFG was controlled in that there was a winter and summer composition, but each was a single homogenous batch from Phillips 66. The "conventional" control fuels were sampled, but subject to the vagaries of commercial

supply. Thus, not unexpectedly, there was significant variation in the composition of the eight measured parameters of the control fuel, thereby representing real world conditions; energy content was not one of the directly measured parameters.

TABLE V. Composition Analysis of Control Fuel

PROPERTY	City of Sacramento	County of Sacramento
RVP, psi	10.9	11.7
Aromatics, v %	24	24
Olefins, v %	10.2	10.4
Oxygen, wt %	0.2	1.9
T90, °F	330	336
T50, °F	199	188
S, ppm	134	158
Benzene, v %	1.1	1.2
Energy, BTU/gal	115,500	115,600

The CARB conclusion with respect to the use of CaRFG, based on the two on-road fleet studies, was that about a 2.4 % reduction in fuel economy would be expected. The variation about the fleet-average relative difference was $\pm 3\%$. This is less than the relative difference in fuel energy content (-4.3 %).

2) CARB dynamometer study. CARB performed a limited dynamometer study using four vehicles (1995 Dodge Caravan, 1995 Ford Taurus, 1995 Chevrolet Lumina, and a 1995 Honda Accord LX) with three fuels (conventional, Federal RFG, and CaRFG). The results are summarized in Table VI. [10]

TABLE VI. CARB Dynamometer Fuel Economy Results

VEHICLE	CaRFG vs. Conventional	Federal RFG vs. Conventional
Dodge	-0.8 %	-2.0 %
Ford	-3.7 %	-2.6 %
Chevrolet	-5.0 %	-2.7 %
Honda	-4.3 %	0.0 %
AVERAGE	-3.5 %	-1.8 %

3) Cited historical data. The CARB report cited the study by Battelle using a limited number of Federal Express on-road test data for delivery vans using a CaRFG and a conventional gasoline over a two year period. [2] The on-road data has quite a bit of scatter, no doubt due to the variability in the drive cycle plus some variation (2.5 %) in the RFG energy content during the test. The dynamometer results are, as expected, more tightly grouped. The on-road results for the change in fuel economy of CaRFG relative to “conventional” fuel available at that time in the South Coast Air Basin ranged from +1 % to -7 %; the dynamometer results varied from + 1 % to -3 %. These results are per equivalent

energy, however, and the CaRFG had 1 to 3 % less energy/volume than the industry standard.

The CARB report also cites the Southeastern Wisconsin Fuel Efficiency Study, conducted jointly between the Wisconsin Department of Natural Resources and U.S. EPA. [10] This was a limited (eight vehicles) study comparing four fuels (“conventional” and three oxygenated RFG, although none were CaRFG). There was a 2.2 % reduction in average fuel economy between the conventional fuel and the MTBE-containing RFG. The report does conclude the following: 1) for any vehicle using the same gasoline, mileage may vary by more than 10 % between tests; 2) the drop in mileage between warm and cold weather was at least 5 % for all four fuels, which is a greater drop than the difference in mileage among fuel types; 3) there are virtually no meaningful mileage differences among the three RFG used in the study that can be distinguished from the other factors that affect mileage.

Thus, based on its own on-road and dynamometer testing, plus cited external studies, CARB concluded “that use of CaRFG will reduce the average miles per gallon (fuel economy) by 1 to 3 percent. The 1 percent reduction results from comparing CaRFG to an oxygenated conventional gasoline; since oxygenates are already in widespread use in California, the 1 percent reduction is the expected average fuel economy change when CaRFG is introduced.” [10]

Additional Data Sets

1) EPA. The US EPA reviewed the available literature from the perspective of the effect of oxygenates of fuel economy. An NRC report concluded that the fuel-economy penalty associated with the use of oxygenated fuels is approximately 2% to 3% and is related to changes in volumetric energy content. [11]

2) CA AAA Northern California. The California State Automobile Association

TABLE VII. CSAA Average Fuel Economy Change with CaRFG

MONTH	AVERAGE MPG	% CHANGE
Apr 96	22.72	
Apr 95	23.01	-1.26 %
May 96	22.57	
May 95	22.69	-0.53 %
Jun 96	22.26	
Jun 95	22.79	-2.33 %
Jul 96	22.05	
Jul 95	22.92	-3.80 %
Aug 96	22.20	
Aug 95	22.56	-1.60 %
Sep 96	22.37	
Sep 95	22.52	-0.67%
AVERAGE		-1.70%

(Northern California) compared their vehicle fleet fuel economy before and after the introduction of CaRFG; prior to the introduction of CaRFG, the fleet used conventional gasoline (i.e., non-oxygenated). They have summary data comparing average monthly fuel economy between equivalent time periods. These results are presented in Table VII. [12] The raw data which was the basis for these averages was not available.

3) CA AAA Southern California. The Automobile Club of Southern California compared CaRFG to Federal RFG (“regular”) using 3 vehicles under the Federal Test Procedure and Highway Fuel Economy Tests; thus, both urban and highway mileage data was obtained. [13] The three vehicles were a 1991 Plymouth Acclaim (2.5 liter 4 cylinder), a 1992 Plymouth Acclaim (2.5 liter 4 cylinder), and a 1989 Pontiac Bonneville (3.8 liter V6). Unfortunately, in this case the fuel economy with CaRFG was compared with Federal RFG, but “some of the constituents of CaRFG were present in the samples (of Federal RFG). This indicated that Chevron had already started (Feb 96) to convert their refinery by the time the most recent shipment of gasoline was delivered (to the Automobile Club of Southern California).” These results are presented in Table VIII.

TABLE VIII. Automobile Club of Southern California Fuel Economy Results: Reduction with CaRFG

VEHICLE	URBAN	HIGHWAY
91 Plymouth	-3.3 %	-1.2 %
92 Plymouth	-3.2 %	-1.6 %
89 Pontiac	-0.8 %	-5.8 %
AVERAGE	-2.4 %	-2.9 %

4) State-wide gasoline sales and Caltrans total mileage. CARB has released state-wide data documenting via gasoline tax receipts collected by the State Board of Equalization that 37.7 million gallons of gasoline (CaRFG) were consumed per day in Apr - Jul 96, compared to 36.9 million gallons per day in Apr-Jul 95 (a mixture of conventional (Northern California) and Federal RFG (Southern California)). [14] This increase of 2.2 % is compared to a Caltrans reported increase of 1.9 percent greater average daily traffic on state highways between Apr - Jul 96 compared to the same four months in 1995. Caltrans claims survey data, taken from 17 sites, is accurate to within 1 percent for total vehicle-miles on state highways. Thus, this data also supports the contention that the reduction in fuel economy attributable to the introduction of CaRFG is minimal.

5) Consumer Reports. Anecdotal data was provided by Consumer Reports; however, they refused to release the data from which they based their conclusions. They did describe the several different types of gasoline, where they are available, and concluded that there was “no meaningful differences in either acceleration or fuel economy between the various new fuels and the old-fashioned 1990 fuel.” [15]

6) BDM-Oklahoma, for the Department of Energy. BDM-Oklahoma has compared CaRFG with a variety of fuels, including the 1988 industry average for conventional gasoline; this industry average is the US EPA reference fuel for RFG certification. [4] They tested five 1994 cars having an initial mileage ranging between 18,000 and 30,000 accumulated miles. After 30,000 additional accumulated miles on CaRFG, dynamometer tests were done using 2 driving cycles (FTP, the Federal Test Procedure, and US75, a modified cycle including more severe speeds and accelerations). These specific results are

summarized in Table IX. Further data was provided and was analyzed in more statistical detail (*vida infra*). This data also illustrates the potential effect variation in driving cycle will have on mileage.

TABLE IX. BDM-OK Fuel Economy Results:
Percent Reduction with CaRFG vs. Conventional Gasoline

VEHICLE	FTP	US75
94 Honda Accord	-4.0 %	-2.6 %
94 Toyota Camary	-5.9 %	-5.3 %
94 Nissan Maxima	-5.7 %	-5.3 %
94 Ford Taurus	-3.6 %	-4.4 %
94 Chevy Lumina	-3.8 %	0.1 %
AVERAGE	-4.6 %	-3.5 %

7) Auto/Oil AQIRP. The Auto/Oil AQIRP performed a dynamometer test evaluation of CaRFG vs. conventional gasoline (i.e., industry average, non-oxygenated, representing 1988 national average composition, and measured to have 3% higher volume energy density than CaRFG). A preliminary report [10] of these results was included in the CARB report (Appendix 41), and a public document has been issued. [5] Detailed statistical analysis has been performed on this data (*vida infra*). The test fleets consisted of seven 1983-1985 vehicles, ten 1989 vehicles, and six 1994 vehicles; both passenger cars and light duty trucks were included in the test fleet composition. The reported conclusions from this FTP test of 23 vehicles are summarized in Table X.

TABLE X. Auto/Oil AQIRP Fuel Economy Results:
Percent Reduction with CaRFG vs. Conventional Gasoline

FLEET	% REDUCTION
1983-1985	-1.9 %
1989	-2.6 %
1994	-3.6 %

Summary. Table XI summarizes all the cited data which quantitatively measured the reduction in fuel economy due to the use of CaRFG. This table contains both data referenced to and obtained by CARB, as well as data unavailable to CARB at the time their report was published. Despite the variability in the reference vs. test fuel, the variations in the type of test, the variations in the vehicles, and the inherent variations in on-road driving conditions, all of the literature values, both before and after the CARB report, indicate a reduction in mileage due to the introduction of CaRFG which would be approximately 2 to 4 %. As with any on-road mileage parameter, there is significant scatter in the data, so larger (and smaller) reductions are expected in a certain fraction of all measurements.

TABLE XI. Summary of Literature Results

FUEL ECONOMY STUDY	REDUCTION DUE TO CaRFG
Auto/Oil AQIRP Emission	-1.7 to -4.9 % ¹
CARB On-road	-2.3 %
CARB Dynamometer	-1.8 %
Battelle On-road	+1 to -7 % ²
Battelle Dynamometer	+1 to -3 % ²
Wisconsin	-2.2 % ³
EPA	-2 to -3 %
AAA Northern California	-1.7 %
AAA Southern California	-2 to -3 % ⁴
California Gasoline Use	0 to -1 %
Consumer Reports	Insignificant
BDM-OK/DOE	-3.5 to -4.6 %
Auto/Oil AQIRP Dynamometer	-1.9 to -3.6 %

¹ Oxygenated but not CaRFG

² Compared per energy equivalent

³ Oxygenated RFG but not CaRFG

⁴ Federal RFG (not CaRFG)

STATISTICAL ANALYSIS OF CARB AND NEW DATA

Statistical Models for Mileage

Two statistical models will be developed to quantify the effect of CaRFG on fuel economy. The first model is based on large sample considerations and will be called the normal model. The second model uses relaxed assumptions and will be called the nonparametric model.

The Normal Model.

Gasoline fuel theory and experimentation support the contention that a reduction in energy content causes a proportionate reduction in fuel economy. [1] It is reasonable therefore to model mileage multiplicatively. Consider a particular vehicle and fuel type. We assume the model,

$$T = \mu\epsilon, \quad (1)$$

where T (a random variable) denotes the measured number of miles traveled with one gallon of fuel, μ (a parameter) is the mean mpg, and ϵ (a random variable) has mean 1 and standard deviation σ . The parameters μ and σ depend on such things as the vehicle, the fuel, and the manner of driving, and are unknown. The parameter σ in particular characterizes variation in mileage due to changes in speed, terrain, route, traffic, weather, number of passengers, energy content, and so on.

Consider N gallons of fuel: $T_1, \dots, T_N$ are generated, and

$$X = (1/N)\sum T_j \quad (2)$$

is the measured fuel economy (mpg). Regard $T_1, \dots, T_N$ as a random sample. Since T has mean μ and standard deviation $\mu\sigma$, it follows from large sample statistics that for sufficiently large N , $\log X$ is approximately normally distributed with mean $\log \mu$ and standard deviation $\sigma/\sqrt{N}$. [16] For brevity, denote this distribution as $N(\log \mu, \sigma/\sqrt{N})$. The size of the sample is effectively the total volume of gasoline, N gallons, and since a volume is infinitely divisible, the approximate normality of X is achieved if the sum $\sum T_j$ can be viewed as arising additively from a large number of relatively small and uniform contributions. For selected vehicles with extensive mileage data, the approximate normality conjecture can (and will in this report) be tested statistically, but for limited data, as in the dynamometer experiments, normality will be an untested assumption. [For this reason, an alternative model with no claim of normality will be developed.]

Compare now the vehicle's mileage for two scenarios. In scenario 1, fuel 1 (namely, conventional fuel) will be used; in scenario 2, fuel 2 (CaRFG) will be used. Ideally the only systematic difference between the scenarios is the fuel formulation difference. In dynamometer testing professional drivers replicate a rigid EPA protocol. In CARB on the road testing scenario 1 represents a 5 month time period in 1994, and scenario 2 represents the same 5 month period in 1995; hopefully, driving patterns and external factors "average out" over the comparable 5 month periods.

Adopt the model $T = \mu\epsilon$ for scenario 1 and the model $T = \mu\theta\epsilon$ for scenario 2, where

$$\theta = 1 - \delta, \quad (3)$$

and δ is the fuel economy reduction factor for CaRFG. For example, if $\delta = 0.03$, then CaRFG reduces mileage on the average by 3% relative to conventional fuel for this vehicle. If the vehicle uses N gallons in scenario 1 and X is the measured mileage, and if the vehicle uses M gallons in scenario 2 and Y is the measured mileage, then $\log X$ is approximately $N(\log \mu, \sigma/\sqrt{N})$, and $\log Y$ is approximately $N(\log \mu\theta, \sigma/\sqrt{M})$.

We are interested in $\delta = 1 - \theta$ but not μ . The nuisance parameter μ is eliminated by considering

$$Z = \log X - \log Y, \quad (4)$$

which is approximately $N(-\log \theta, \sqrt{(1/N + 1/M)} \sigma)$. If the value of δ is small, then $-\log \theta \approx \delta$.

In applications mileage data from several vehicles are assimilated. Consider a

collection of k vehicles and the corresponding data $Z_1, \dots, Z_k$, where for vehicle i there are N_i gallons for scenario 1 and M_i gallons for scenario 2. The Z 's are independent, with Z_i being distributed approximately as $N(-\log\theta_i, \sqrt{(1/N+1/M)} \sigma_i)$.

The intrinsic effects of CaRFG on mileage, measured by $\delta_i = 1 - \theta_i$, need not be the same for each vehicle, nor should the variability parameters σ_i necessarily be the same either. However, the available data preclude estimating differences in the δ_i and σ_i , because of limited replication in dynamometer testing and unreliable individual fill-up data in on the road testing. It is necessary then to make these assumptions:

- common underlying σ
- common reduction factor δ .

Thus, the relative variation in mileage is assumed to be same for all vehicles, i.e., $\sigma_i \equiv \sigma$, and the fuel effect is isolated in a single parameter and assumed to be the same for all vehicles, i.e., $\delta_i \equiv \delta$. By making these assumptions it is possible to develop maximum likelihood estimates of the parameters δ and σ as well as corresponding confidence bounds and intervals for δ . In essence these inferences apply to the corresponding means of the distributions of δ and σ over the population of all California vehicles.

For the Sacramento City and Sacramento County CARB data sets, there were control vehicles as well as test vehicles. For the control vehicles conventional fuel was used in both 1994 and 1995. The above modeling can be used for the control vehicles with the interpretation that δ represents the mean change proportion in mpg from 1994 to 1995. In fact, however, it may safely be assumed that $\delta = 0$ for each control population, since as shown in Appendix 1, hypothesis tests of $\delta = 0$ are accepted at the usual levels of significance. Consequently the use of the control vehicle data is merely to help in the estimation of the parameter σ .

We now develop the maximum likelihood estimates of δ and σ for the normal model. First consider the CARB on the road testing programs. Assume the following data:

- Log mileage differences $Z_1, \dots, Z_Q$ based on Q test vehicles; for vehicle i , N_i gallons for scenario 1 (conventional fuel) and M_i gallons for scenario 2 (CaRFG)
- Log mileage differences $W_1, \dots, W_C$ based on C control vehicles; for vehicle j , K_j gallons for scenario 1 (conventional fuel) and L_j gallons for scenario 2 (also conventional fuel).

Thus, Z_i is distributed as $N(\beta, \sqrt{r_i} \sigma)$, where

$$\beta = -\log(1 - \delta) \quad (5)$$

and

$$r_i = 1/N_i + 1/M_i, \quad (6)$$

and W_j is distributed as $N(0, \sqrt{s_j} \sigma)$, where

$$s_j = 1/K_j + 1/L_j. \quad (7)$$

The likelihood function is therefore

$$L = \prod_{i=1}^Q \frac{1}{\sigma \sqrt{2\pi r_i}} \exp\left\{-\frac{1}{2r_i \sigma^2} (z_i - \beta)^2\right\} \prod_{j=1}^C \frac{1}{\sigma \sqrt{2\pi s_j}} \exp\left\{-\frac{1}{2s_j \sigma^2} w_j^2\right\} \quad (8)$$

which is maximized by

$$\hat{\beta} = \frac{\sum_i z_i / r_i}{\sum_i 1 / r_i} \quad (9)$$

and

$$\hat{\sigma}^2 = \frac{1}{Q+C} \left[\sum_i \frac{1}{r_i} (z_i - \hat{\beta})^2 + \sum_j \frac{1}{s_j} w_j^2 \right] \quad (10)$$

In on the road testing a vehicle is observed in normal operating conditions over a period of time. Fuel economy, i.e., the number of miles driven relative to the number of gallons pumped, is the measured variable. In dynamometer testing, on the other hand, a fixed

driving protocol over a fixed distance is employed, and fuel consumption, i.e., the number of gallons consumed over this distance, is the measured variable. Thus, it is appropriate to use the multiplicative model,

$$T^* = \mu^* \epsilon^*, \quad (11)$$

where T^* (a random variable) denotes the measured number of gallons consumed in one mile of driving, μ^* (a parameter) is the mean gpm, and ϵ^* (a random variable) has mean 1 and standard deviation σ^* . The development for the reciprocal quantity mpg follows in analogous fashion to the previous development for mpg. Consider N^* miles of driving: $T_1^*, \dots, T_N^*$ are generated, and

$$X^* = (1/N^*) \sum T_j^* \quad (12)$$

is the measured fuel consumption (“gallage”, gpm). Regarding $T_1^*, \dots, T_N^*$ as a random sample allows $\log X^*$ to be approximated as $N(\log \mu^*, \sigma^*/\sqrt{N^*})$. We consider two scenarios, the first a dynamometer test of a vehicle using conventional fuel and the model eq. (11), the second a replicated dynamometer test of the same vehicle but with CaRFG as fuel, i.e., $T^* = \mu^* \theta^* \epsilon^*$, where

$$1/\theta^* = 1 - \delta, \quad (13)$$

and δ (precisely as before) is the fuel economy reduction factor for CaRFG. If the vehicle is driven N^* miles in scenario 1 and X^* is the measured gallonage, and if the vehicle is driven M^* gallons in scenario 2 and Y^* is the measured gallonage, then $\log X^*$ is approximately $N(\log \mu^*, \sigma^*/\sqrt{N^*})$, $\log Y^*$ is approximately $N(\log \mu^* \theta^*, \sigma^*/\sqrt{M^*})$, and

$$Z^* = \log Y^* - \log X^* \quad (14)$$

is approximately $N(\log \theta^*, \sqrt{(1/N^* + 1/M^*)} \sigma^*)$.

Assume the following dynamometer test data:

- Log gallonage differences $Z_1^*, \dots, Z_Q^*$ based on Q test vehicles; for vehicle i , N_i^* miles for scenario 1 (conventional fuel) and M_i^* miles for scenario 2 (CaRFG).

Under assumptions of a common gallonage variation parameter σ^* and a common fuel economy reduction factor δ , Z_i^* is distributed as $N(\beta, \sqrt{r_i^*} \sigma^*)$, where

$$\beta = -\log(1 - \delta) \quad (15)$$

and

$$r_i^* = 1/N_i^* + 1/M_i^*. \quad (16)$$

The likelihood function is therefore

$$L = \prod_{i=1}^Q \frac{1}{\sigma^* \sqrt{2\pi r_i^*}} \exp \left\{ -\frac{1}{2r_i^* \sigma^{*2}} (z_i^* - \beta)^2 \right\} \quad (17)$$

which is maximized by

$$\hat{\beta} = \frac{\sum_i z_i^* / r_i^*}{\sum_i 1/r_i^*} \quad (18)$$

and

$$\hat{\sigma}^{*2} = \frac{1}{Q} \sum_i \frac{1}{r_i^*} (z_i^* - \hat{\beta})^2. \quad (19)$$

Confidence limits for $\delta = 1 - e^{-\beta}$ may be constructed from $\hat{\beta}$ and its standard error. The estimated variance of $\hat{\beta}$ is

$$\text{var}(\hat{\beta}) = \hat{\sigma}^2 / \sum_i \frac{1}{r_i^*} \quad (20)$$

for the on the road (mpg) development eqs. (1) through (10), and is

$$\text{var}(\hat{\beta}) = \hat{\sigma}^{*2} / \sum_i \frac{1}{r_i} \quad (21)$$

for the dynamometer (gpm) development eqs. (11) through (19); the associated standard errors are the square roots. Thus, an approximate level $100(1 - \alpha)\%$ confidence interval for the on the road β , according to large sample statistics [16], is given by

$$[\underline{\beta}, \bar{\beta}] = \hat{\beta} \pm z_{\frac{\alpha}{2}} \hat{\sigma} / \sqrt{\sum_i \frac{1}{r_i}}, \quad (22)$$

where $\hat{\beta}$ is defined in eq. (9) and z_γ is derived from the equation

$$\Phi(z_\gamma) = 1 - \gamma, \quad (23)$$

with Φ denoting the standard normal cumulative distribution function. Similarly, an approximate level $100(1 - \alpha)\%$ upper confidence bound for β is

$$\tilde{\beta} = \hat{\beta} + z_\alpha \hat{\sigma} / \sqrt{\sum_i \frac{1}{r_i}}. \quad (24)$$

Since $\delta = 1 - e^{-\beta} \approx \beta$ for small δ , the point estimate eq. (9), confidence interval eq. (22), and upper bound eq. (24) apply approximately to the CaRFG mileage reduction factor δ . However, for best precision the following point estimate, confidence interval, and upper bound are recommended:

$$\hat{\delta} = 1 - e^{-\hat{\beta}}, \quad (25)$$

$$[\underline{\delta}, \bar{\delta}] = \left[1 - e^{-\underline{\beta}}, 1 - e^{-\bar{\beta}} \right], \quad (26)$$

and

$$\tilde{\delta} = 1 - e^{-\tilde{\beta}}. \quad (27)$$

The corresponding results for the dynamometer situation are obtained in obvious fashion: eqs. (25), (26), and (27) are valid based on the point estimates eqs. (18) and (19) along with the definition eq. (16).

The reduction in fuel economy (mpg) due to CaRFG is expected to be consistent with the reduction in energy content (BTU/gal). Let δ_0 denote the proportionate reduction in energy content; it is desired to test whether the mileage reduction factor δ is no more than δ_0 . Formally, we wish to test the hypothesis

$$H_0: \delta \leq \delta_0 \quad (\text{the CaRFG mileage reduction is no more than expected from the energy content reduction})$$

against the alternative

$$H_1: \delta > \delta_0 \quad (\text{the CaRFG mileage reduction is more than expected from the energy content reduction})$$

An appropriate test procedure may be made based on the above normal asymptotics. The P-value, or significance, of the observed data is the probability that the maximum likelihood mileage reduction estimate, $\hat{\delta}$, would exceed the observed measured value, if the actual reduction factor were δ_0 . This quantity is most conveniently calculated in terms of $\hat{\beta}$ and $\beta_0 = -\log(1 - \delta_0)$ as

$$P = 1 - \Phi\left(\frac{\hat{\beta} - \beta_0}{\text{S.E.}(\hat{\beta})}\right), \quad (28)$$

where $\text{S.E.}(\hat{\beta})$ is the square root of the estimated variance, eq. (20) or (21). The P-value gives the probability of a measured mileage reduction at least as high as observed, if the true CaRFG mileage reduction is no more than expected from the energy content reduction. In essence P measures the plausibility of the observed data in the context of the hypothesis H_0 being true. Formally, H_0 is accepted at level of significance α if $P \geq \alpha$.

The Nonparametric Model.

The validity of the normal model in analyzing mileage (or gallonage) data depends on the ability to treat, in log space, miles driven (or gallons consumed) as the sum of a large number of relatively small and uniform contributions. Limited by the existing experimental data we find it necessary in addition to assume the mileage reduction effect is the same for all vehicles and the relative variation in mileage (or gallonage) is the same for all vehicles. Although (as will be demonstrated later) examination of residuals reveals the Auto Oil dynamometer test data to conform to all the normal model assumptions, such is not the case for the CARB on the road test data. This may be the result of certain vehicles having different duties and therefore different driving profiles from one year to the next. We introduce here an alternative, nonparametric formulation which does not assume normality or constant relative variation.

A nonparametric approach which does not make assumptions of normality is based on the model

$$Z_i = \beta + e_i, \quad (29)$$

where as before $Z = \log X - \log Y$, and $\beta = -\log(1 - \delta)$, but the e_i are independent with distributions continuous and symmetric about 0, and not necessarily the same. In effect each Y_i is distributed as $(1 - \delta)X_i$, but the distributions may vary from vehicle to vehicle. We are assuming as before that the mileage reduction effect is the same for each vehicle, i.e., $\delta_i \equiv \delta$, but we are no longer assuming a common relative variation in mileage, i.e., that $\sigma_i \equiv \sigma$.

The maximum likelihood inferences presented earlier for the normal model have well-founded nonparametric counterparts. [17] The Hodges-Lehmann point estimator of β is the sample median, β_h , of the collection of all $B = Q(Q + 1)/2$ averages of Z -pairs. That is, denote the ordered values of $\{ (Z_i + Z_j)/2, i \leq j \}$ as $U^{(1)} \leq \dots \leq U^{(B)}$. Then, if B is odd, say $B = 2b + 1$, we have

$$\hat{\beta}_h = U^{(b+1)}, \quad (30)$$

and if B is even, say $B = 2b$, we have

$$\hat{\beta}_h = [U^{(b)} + U^{(b+1)}]/2. \quad (31)$$

The associated nonparametric estimate, $\hat{\delta}_h$, of δ is given by

$$\hat{\delta}_h = 1 - e^{-\hat{\beta}_h}. \quad (32)$$

Tukey confidence intervals and upper confidence bounds based on inverting the Wilcoxon signed rank test are functions of the U 's. In particular, a level $100(1 - \alpha)\%$ confidence interval for β is given by

$$[\beta_{-h}, \bar{\beta}_h] = [U^{(C_\alpha)}, U^{(B+1-C_\alpha)}], \quad (33)$$

and a level $100(1 - \alpha)\%$ upper confidence bound for β is given by

$$\tilde{\beta}_h = U^{(B+1-C_{2\alpha})}, \quad (34)$$

where the integers C_α and $C_{\alpha/2}$ may be extracted from tables in [17] for $Q \leq 15$. For larger Q (as in the CARB on the road applications), a large sample approximation is adequate:

$$C_\alpha \approx Q(Q + 1)/4 - z_{\alpha/2} [Q(Q + 1)(2Q + 1)/24]^{1/2} \quad (35)$$

The corresponding confidence limits for δ are obtained from eqs. (33) and (34) by applying the transformation

$$\delta = 1 - e^{-\beta}. \quad (36)$$

The nonparametric analog of the test whether the mileage reduction factor δ is no more than the value δ_0 consistent with the energy content reduction is the Wilcoxon signed rank test. [17] This procedure investigates the ranks of the absolute differences $|Z_i - \beta_0|$, where $\beta_0 = -\log(1 - \delta_0)$. Let R_i denote the rank of $|Z_i - \beta_0|$ in the joint ranking from least to greatest of $\{|Z_i - \beta_0|, 1 \leq i \leq Q\}$. Let H^+ denote the sum of those ranks for which $Z_i - \beta_0 \geq 0$, i.e., there is evidence that the mileage reduction is excessive. The hypothesis H_0 is accepted if H^+ is sufficiently small. Implementation of the Wilcoxon signed rank test and the associated calculation of P-values may be carried by using existing statistical software [19] or by using tables in [17] if $Q \leq 15$.

The control group of vehicles in the on the road CARB study is obviated nonparametrically by testing the hypothesis $H_0: \beta = 0$ against $H_1: \beta \neq 0$. See Appendix 1.

All nonparametric inferences developed above in terms of mileage and Z [eq. (4)] have obvious analogs in terms of gallonage and Z^* [eq. (14)]. Thus, nonparametric point estimates and confidence limits for the dynamometer testing are obtained by substituting Z^* for Z in eqs. (30) through (34).

Statistical Analysis of Existing Data Sets

Table XII provides an overview of the 13 data sets available for study of the effect of CaRFG on fuel economy. Of these, ten involve on the road tests and three involve dynamometer tests. Only two of the on the road data sets, Sacramento City and Sacramento County, provide data suitable for statistical analysis. The reasons for rejection of the road

TABLE XII. Overview of Data Sets

DATA SET	TESTING	DEPTH	VEHICLES	COMMENTS
Sacramento City	R	2	122	culled data 187-->122
Sacramento County	R	2	110	culled data 414-->110
Auto Oil AQIRP	D	2	32	most representative data set
CARB	D	2	4	city and highway results
Oklahoma DOE	D	2	5	most recent study
Wisconsin	R	0	8	vehicles combined; fed rfg
Battelle	R	0	30	dedicated vehicles
CSU Fresno	R	1	112	mostly poor quality data
Bank of America	R	0	30	poor quality data
GTE	R	0	411	auxiliary equipment used
Pacific Bell, North	R	0	194	auxiliary equipment used
Pacific Bell, South	R	0	93	auxiliary equipment used
Caltrans	R	0	25	irrelevant data

The results of applying the statistical estimation methods of the previous section to the Sacramento City and Sacramento County data sets are presented in Table XIII. The normal model point estimates of the CaRFG mileage reduction factor, δ , are obtained from eqs. (9) and (25); the normal confidence intervals are obtained from eqs. (9), (10), (22), and (26); and the normal upper confidence bounds are obtained from eqs. (9), (10), (24), and (27). The nonparametric results are based on the log differences, Z , of eq. (4). The nonparametric point estimates are obtained from eqs. (30), (31) and (32); the nonparametric confidence intervals are obtained from eqs. (33) and (36); and the nonparametric upper confidence bounds are obtained from eqs. (34) and (36). All calculations were performed with Microsoft Excel spreadsheets, which are displayed in Appendix 2.

TABLE XIII. Estimates of CaRFG Mileage Reduction Factor, δ

Normal Model

DATA SET	δ	90% CONFIDENCE INTERVAL	95% UPPER CONF. BOUND
Sacramento City	.021	(.009, .033)	.033
Sacramento County	.037	(.012, .062)	.062
Auto Oil	.029	(.023, .034)	.034
CARB city driving	.041	(.026, .055)	.055
CARB hwy driving	.033	(.017, .049)	.049
Oklahoma DOE	.047	(.040, .054)	.054

Nonparametric Model

Data Set	δ_h	90% CONFIDENCE INTERVAL	95% UPPER CONF. BOUND
Sacramento City	.023	(.010, .036)	.036
Sacramento County	.028	(.004, .057)	.057
Auto Oil	.031	(.026, .036)	.036
CARB city driving	.043	(.015, .061)	.061
CARB hwy driving	.044	(.003, .050)	.050
Oklahoma DOE	.048	(.038, .059)	.059

The dynamometer data sets are the most helpful in assessing the effect of CaRFG on fuel economy in that the large number of unavoidable extraneous factors that are nuisances in on the road testing because they cannot be replicated from one time period to the next, such as speed, traffic, weather, terrain, route, and fill-up precision, are eliminated. The only systematic difference in dynamometer testing between the mileage measured for a particular vehicle with conventional fuel and its mileage measured with CaRFG is the fuel formulation. Random variation is controlled through the use of professional drivers performing rigidly defined EPA driving protocols of changes in speed; measurements of fuel consumed and miles traveled are extremely accurate and unbiased.

The results of applying the statistical estimation methods of the previous section to the dynamometer data sets are presented in Table XIII. The normal model point estimates of δ are obtained from eqs. (18) and (25); the normal confidence intervals are obtained from eqs. (18), (19), (22), and (26); and the normal upper confidence bounds are obtained

from eqs. (18), (19), (24), and (27). The nonparametric results are based on the log differences, Z^* , of eq. (14). The nonparametric point estimates are obtained from eqs. (30), (31) and (32); the nonparametric confidence intervals are obtained from eqs. (33) and (36); and the nonparametric upper confidence bounds are obtained from eqs. (34) and (36). All calculations were performed with Microsoft Excel spreadsheets, which are displayed in Appendix 2.

The energy content (BTU/gal) was measured for all the tests. For the on the road tests, which required substantial quantities of fuel over a long period of time, average energy content values were calculated for conventional fuel use. This was not necessary for CaRFG, since a single batch of CaRFG was used. The hypothesis that the mileage reduction due to CaRFG was consistent with the associated energy content reduction was tested for the two Sacramento on the road data sets and the three dynamometer data sets. These results are presented in Table XIV. P-values, which measure the plausibility of the hypothesis of consistency, are given for both the normal and the nonparametric model methods. P-values for the normal model were calculated from eq. (28). Nonparametric model P-values were calculated with Stata software. [19]

TABLE XIV. Assessing Consistency of CaRFG Mileage Reduction and Energy Content Reduction

Normal Model

Data Set	Δ_{EC}	$\hat{\delta}$	P-VALUE
Sacramento City	.045	.021	1.
Sacramento County	.043	.037	.63
Auto Oil	.030	.029	.67
CARB city driving	.045	.041	.66
CARB hwy driving	.045	.033	.87
Oklahoma DOE	.055	.047	.97

Nonparametric Model

Data Set	Δ_{EC}	$\hat{\delta}_h$	P-VALUE
Sacramento City	.045	.023	.999
Sacramento County	.043	.028	.50
Auto Oil	.030	.031	.40
CARB city driving	.045	.043	.50
CARB hwy driving	.045	.044	.50
Oklahoma DOE	.055	.048	.89

Δ_{EC} denotes the energy content reduction

Discussion. All the statistical inferences presented in Tables XIII and XIV are based on models developed in the previous section and must be interpreted in the context of the underlying assumptions made. The normal model is based on the assumptions of a common intrinsic mileage reduction factor, δ , for all vehicles, a common measure of

relative mileage variability, σ , for all vehicles, and approximate normality of the difference in log mileage, Z or Z^* . The nonparametric model is based on the assumptions of a common intrinsic mileage reduction factor, δ , for all vehicles, and continuous, symmetric probability distributions, not necessarily the same for each vehicle, for the difference in log mileage, Z or Z^* . Both models have strong theoretical foundations; however, the pervasive assumption that the intrinsic mileage factor does not vary from vehicle to vehicle, an assumption crucial to the development of confidence intervals and bounds, is an assumption that cannot be tested from the available data, because of a lack of replications of mileage measurements for each vehicle studied. Thus, whatever variations in the δ_i that may exist from vehicle to vehicle are in effect absorbed into σ in the normal model and into the arbitrary distributions of Z or Z^* in the nonparametric model. The point estimates δ and δ_h are then estimates, or best guesses, of the common δ if indeed the assumption of a single δ is correct, and are estimates of the mean of the distribution of the δ 's if the assumption of a common δ is incorrect. If the common δ assumption is incorrect, the quality of δ or δ_h as an estimate of the mean of the distribution of the δ 's depends on how representative of the California population of vehicles is the sample of tested vehicles. If the common δ assumption is correct, the representativeness of the tested vehicles is not an issue.

Confidence intervals and confidence bounds put limits on the uncertainty of point estimates. Such uncertainty is unavoidable due to statistical variation in the data. The confidence level, e.g., 95%, reflects the degree of confidence in the statistical method used to generate the interval or bound. For example, the 95% upper confidence bound, $\tilde{\delta} = 0.057$, for δ based on the nonparametric model analysis and the Sacramento County data set has the interpretation that, under the nonparametric model assumptions (common δ and continuous, symmetric Z), we have 95% confidence that the true value of δ is no greater than 0.057. To say our level of confidence is 95% means that if (1) the nonparametric model assumptions are satisfied, and (2) we could apply this nonparametric model method to many similar data sets and compute a $\tilde{\delta}_h$ each time, then 95% of these $\tilde{\delta}_h$'s would be an upper bound for δ , i.e., greater than δ . The caveat here is that if the nonparametric model assumptions are not valid, then the nominal confidence coefficient, 95%, may not be achieved. Moreover, if the common δ assumption is not correct, then the confidence bound ought to be interpreted as bounding the mean of the distribution of δ 's; but again, the achieved confidence level need not be the nominal level.

It is apparent from Table XIII that remarkably consistent results have been obtained, whether the normal model or the nonparametric model is used, and regardless of the data set analyzed: point estimates are in the 2 to 4% range, 90% confidence intervals in the 0 to 6% vicinity, and 95% upper confidence bounds around 6% or less. Furthermore, as Table XIV attests, the estimated reductions in mileage are quite consistent with the associated reductions in energy content. The somewhat high point estimates for the Oklahoma study are to be expected, because of the similarly high energy content reduction due to the use of a 1988 conventional fuel formulation with relatively high (117,900 BTU/gal) energy content.

The P-values in Table XIV quantify the plausibility of the hypothesis that the mileage reduction is consistent with the energy content reduction. For example, $P = 0.999$ for the nonparametric model and the Sacramento City data set has the following interpretation: if the nonparametric model assumptions are satisfied, and if the true mileage reduction factor is no greater than the energy content reduction, then the probability that the nonparametric point estimator $\tilde{\delta}_h$ would exceed the value calculated from the data set

(namely 0.023) is 0.999. Hence it is quite plausible (there is a 99.9% chance) of observing data like we saw, if the hypothesis of consistency is correct. Typically, a P-value greater than 0.10 is considered sufficient evidence to support the null hypothesis. All the Table XIV P-values comfortably exceed 0.10.

From Tables XIII and XIV it is clear that the statistical method used to analyze the data has little bearing on the results. So it is mostly of academic interest to post-check for reasonableness of the normal model. For a given data set, consider the normal model residuals,

$$D_i = Z_i - \hat{\beta}, \quad (37)$$

for on the road tests (based on mileage) or

$$D_i = Z_i^* - \hat{\beta}, \quad (38)$$

for dynamometer tests (based on gallonage). The standard error of D_i is

$$\text{S.E.}(D_i) = \hat{\sigma} \sqrt{r_i - a}, \quad (39)$$

for on the road tests, and

$$\text{S.E.}(D_i) = \hat{\sigma}^* \sqrt{r_i^* - a^*} \quad (40)$$

for dynamometer tests, where

$$a = \sum_i \frac{1}{r_i} \quad (41)$$

and

$$a^* = \sum_i \frac{1}{r_i^*}. \quad (42)$$

The studentized residuals $D_i / \text{S.E.}(D_i)$, if the normal model is applicable, should be approximately standard normally distributed. The Shapiro-Wilk test of normality has been invoked for the data sets with a large enough number of vehicles to give meaningful results, namely the Sacramento City, Sacramento County, and the Auto Oil data sets. The computed P-values, obtained from Stata software [19], are $P = 0.01$, 0.02 , and 0.001 , respectively. That is, there is scant support for the normal model. However, it is interesting to note that the auto oil data set of 32 vehicles contains two vehicles whose mileage actually improved with CaRFG. If these two vehicles are removed as outliers, the P-value for analysis of the remaining 30 vehicles is $P = 0.40$. Thus there is strong support for the normal model restricted to those vehicles showing mileage reduction in the dynamometer testing.

The Auto Oil dynamometer data set is probably the best available in terms of providing reliable mileage data that are relevant to the California vehicle population. If the modeling assumption of a common δ is abandoned, it is instructive to fit a distribution to the collection of Auto Oil individual vehicle mileage reduction estimates. This fit will not necessarily represent the actual distribution of California vehicle δ 's because the selection of Auto Oil vehicles most definitely was not a random sample from the California population, although the selection was diverse in that it captured a range of automobile types and ages. Moreover, if in truth the distribution is degenerate, i.e., δ is the same for all vehicles, the fit is unnecessary and misleading. Thus, as an illustrative exercise of what might be done if a random sample of vehicles were available, we fit a distribution to the Auto Oil individual vehicle mileage reduction estimates, $1 - \exp(-Z_i^*)$. For simplicity we remove the two outlier vehicles with improved mileage with CaRFG and fit a normal distribution to the remaining 30 individual estimates. The normal fit is good (the Shapiro-Wilk P-value is 0.37), with an estimated mean of $\mu_f = 0.0326$ and an estimated standard deviation of $\sigma_f = 0.0132$. Important features of the vehicle population are then easily calculated. For example, the percentage of vehicles with δ greater than p , e.g., $p = 0.05$, would be estimated by $100\{1 - \Phi([p - \mu_f]/\sigma_f)\}\%$, and the percentage with δ approximately equal to p by $100\{\Phi([p + 0.005 - \mu_f]/\sigma_f) - \Phi([p - 0.005 - \mu_f]/\sigma_f)\}\%$. We emphasize that these formulas are only illustrative of a method; the existing data are insufficient to justify

that a fit is appropriate or that the given fit is representative.

Resolution of Results with Individual Mileage Records

Tables XIII and XIV show the effect of CaRFG on mileage to be in the 2 to 4% range and consistent with the energy content reduction. Yet there has been a perception among the media and the public that the mileage reduction is much greater, say in the 10 to 15% range, or more. An explanation for this misperception is that individual motorists will experience large fluctuations in their mileage from fill-up to fill-up, including dramatic reductions, even if there is no change in fuel formulation; a large reduction may be interpreted as the result of CaRFG, when in fact it is the inevitable consequence of random variation. The random variation in mileage from fill-up to fill-up, or one time period to the next, is due to a number of uncontrolled factors, such as terrain, route, speed, traffic, weather, number of passengers, energy content of the fuel, and fill-up precision. The mileage reduction effect of CaRFG is masked by these bigger players and is nearly impossible to detect in an individual vehicle's mileage history. Only through dynamometer testing, where extraneous factors are controlled, or on the road testing with a large number of vehicles, where extraneous factors essentially cancel out, can the CaRFG mileage reduction effect be reliably estimated.

In this section we will model the mileage history for an individual vehicle and illustrate the tendency for large fluctuations in recorded mileage. Figures 1 through 4 show the meticulously recorded mileage histories of four vehicles. On the horizontal axis are the fill-up numbers indexed sequentially in time, and on the vertical axis are the corresponding recorded mileages in mpg. It is apparent from each figure that mileage varies considerably over time, with swings of 20% or more occurring occasionally from one fill-up to the next. Figure 1 depicts the mileage record of a 1993 Honda Prelude and Figure 2 portrays that of a 1970 Cadillac on its second engine. These vehicles are privately owned and driven by individuals who work at the California Energy Commission. The 1990 Ford 15 passenger van of Figure 3 is used exclusively as a commute vehicle in a Lawrence Livermore National Laboratory van pool. It is driven each work day, nearly always by the same driver, over the same route, a 50 mile round trip between Danville and Livermore. Yet its mileage time history still exhibits considerable variability. The most extreme degree of variation we have encountered is shown in Figure 4, which chronicles the mileage history of a Sacramento police car (from the Sacramento City data base). The mileages in this figure are averaged over ten consecutive fill-ups, yet still show remarkable variation.

We model the recorded mileage over time for a particular vehicle by the normal model method detailed earlier. Let X denote the recorded mileage for a fill-up of N gallons, and define

$$H = \log X. \quad (43)$$

From eqs. (1) and (2) and large sample statistical theory, it follows that H is distributed approximately as $N(\log \mu, \sigma/\sqrt{N})$, where μ is the intrinsic mean mpg for the vehicle and σ is a parameter that characterizes the relative variation in recorded mileage from gallon to gallon. [16] The parameters μ and σ have unknown values which must be estimated by a sample, namely $H_1, \dots, H_k$, which is the sequential time history of recorded mileages over k fill-ups. Let N_i denote the number of gallons in fill-up i , and define

$$\theta = \log \mu. \quad (44)$$

The likelihood function of the sample $h_1, \dots, h_k$, assuming independence, is

$$L = \prod_{i=1}^k \frac{\sqrt{N_i}}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{N_i}{2\sigma^2}(h_i - \theta)^2\right\}, \quad (45)$$

which is maximized by

$$\hat{\theta} = \sum_{i=1}^k \frac{N_i}{N} h_i, \quad (46)$$

and

$$\hat{\sigma}^2 = \frac{1}{k} \sum_{i=1}^k N_i (h_i - \hat{\theta})^2, \quad (47)$$

where

$$N = \sum_{i=1}^k N_i. \quad (48)$$

We wish to compare the measured mileage for a particular vehicle in two consecutive time periods. Let X denote the mileage based on N gallons consumed in time period 1, and let Y denote the mileage based on N gallons consumed in time period 2. The assumption of the same number of gallons for both periods is made for convenience in graphical display and tabulation. If N is small, say $N = 20$, the time periods would correspond to consecutive fill-ups. For larger N , the time periods would involve averages over a number of fill-ups: e.g., $N = 50$ would involve about 3 fill-ups, and $N = 500$ nearly a year's worth of fill-ups. We wish to compare consecutive recorded mileages for two situations, one in which there is no change in fuel formulation from period 1 to period 2, and the other in which there is an intrinsic reduction of magnitude δ from time period 1 to time period 2, i.e., the mean mpg shifts from μ to $\mu(1 - \delta)$ from period 1 to period 2. Thus, $\delta = 0.03$ would be a reduction that we believe is expected if a batch of conventional fuel is used in time period 1 and a batch of CaRFG is used in time period 2, and $\delta = 0.10$, say, would be a reduction of the magnitude suspected by some, which would be appropriate if CaRFG were in fact reducing mileage significantly for the vehicle, or if a systematic change in driving patterns or vehicle operating efficiency occurred between the two time periods. If the normal distribution model is valid, then the probability that the recorded mileage drops by a large amount, say at least 20%, from time period 1 to time period 2 is estimated by

$$P\left\{\frac{Y}{X} \leq 0.80\right\} \approx \Phi\left(\sqrt{\frac{N}{2}} \frac{\log(0.80) - \log(1 - \delta)}{\hat{\sigma}}\right), \quad (49)$$

since a 20% reduction means that $(X - Y)/X$ is at least 0.20, and since $\log X$ being approximately $N(\log \mu, \sigma/\sqrt{N})$ and $\log Y$ being approximately $N(\log \mu(1 - \delta), \sigma/\sqrt{N})$ implies that $\log Y/X$ is approximately $N(\log(1 - \delta), \sigma\sqrt{2}/\sqrt{N})$. If there is no change in fuel formulation, then $\delta = 0$ and the $\log(1 - \delta)$ term vanishes. More generally, the probability, $G(\Delta, \delta, \sigma, N)$, of a drop in recorded mileage of at least Δ percent is estimated by

$$G(\Delta, \delta, \sigma, N) = P\left\{\frac{Y}{X} \leq 1 - \frac{\Delta}{100}\right\} \approx \Phi\left(\sqrt{\frac{N}{2}} \frac{\log(1 - \frac{\Delta}{100}) - \log(1 - \delta)}{\hat{\sigma}}\right). \quad (50)$$

Estimation of $\hat{\sigma}$ from available vehicle time histories shows $\hat{\sigma}$ to range from about 0.15 for low variation driving patterns, like a van pool, to 0.85 for high variation driving patterns, like a police car. Calculations of $\hat{\sigma}$ have been made from 24 vehicle time histories extracted from the Sacramento County data set and for the vehicles of the four figures. As an illustration, the calculation of $\hat{\sigma} = 0.272$ is presented in Appendix 2 for the Cadillac of Figure 2. In addition, Shapiro-Wilk tests of normality of the sequential log mileage records, $H_1, \dots, H_k$, have been performed with Stata software [19] for the above 28 vehicles. The results along with the associated $\hat{\sigma}$ estimates are presented in Table XV. P-values of at least 0.10 show support for the normality assumption. The normality assumption appears adequate for the carefully recorded Honda Prelude, Cadillac, van pool, and police car time histories as well as for many of the Sacramento County histories. Recording inaccuracies and systematic changes in driving patterns may explain the lack of

normality for certain vehicles.

Various probabilities $G(\Delta, \delta, \sigma, N)$, calculated from eq. (50), are plotted and tabulated as percentages in Figures 5 and 6 and Tables XVI and XVII. The number of gallons, N , in each time period (the batch size) ranges from 10 to 100 in the figures and 10 to 500 in the tables. The percentage reduction, Δ , in mpg experienced from batch 1 to batch 2 is taken to be 20% in the figures and ranges from 10% to 40% in the tables. The intrinsic mileage reduction factor, δ , from batch 1 to batch 2 is allowed to be 0 (no reduction), 0.03 (the estimated CaRFG effect), 0.06 (the upper bound CaRFG effect), and 0.10 (a large effect). The relative variation in recorded mileage from gallon to gallon as estimated by $\hat{\sigma}$ is given two representative values, $\hat{\sigma} = 0.20$, for a low variation driving pattern and $\hat{\sigma} = 0.60$ for a high variation driving pattern. One inescapable conclusion from the figures

TABLE XV. Estimated σ and Shapiro-Wilk Test of Normality P-Values

Vehicle ID	N	$\hat{\sigma}$	P-VALUE
102905	31	0.158	0.90812
107013	26	0.155	0.15523
107015	34	0.193	0.01119
107101	47	0.272	0.00864
107103	29	0.236	0.00029
107116a	106	0.260	0.09576
107116b	40	0.187	0.19292
107117	32	0.202	0.07420
107121	57	0.304	0.02585
107608	55	0.613	0.18589
107805	26	0.253	0.01555
110002	23	0.195	0.73702
110006a	46	0.148	0.00007
110006b	40	0.190	0.26711
110012	41	0.329	0.00006
110206	29	0.399	0.06678
110904	42	0.327	0.00000
110922	52	0.158	0.40119
132008	182	0.467	0.41135
132102a	37	0.265	0.16997
132102b	22	0.281	0.04623
134005a	84	0.498	0.00444
134005b	96	0.646	0.00321
134305	56	0.385	0.03984
Prelude	53	0.349	0.38464
Cadillac	97	0.272	0.17906
Van Pool	98	0.256	0.10035
Police Car	36	0.842	0.37306

and tables is that a dramatic reduction in recorded mileage from one fill-up to the next is quite possible, even if there is no change in the fuel formulation. For example, it is seen from Table XVI and Figure 5 that $G(20\%, 0, 0.20, 20) = 5.7\%$, i.e., there is about a 6% chance that a reduction in mileage of 20% or more will be experienced from one 20 gallon fill-up to the next, even if there is no difference in the fuel formulation and one is driving under low variation conditions. (Also note that there is a similar chance of experiencing a 20% *increase* in mileage.) If the first fill-up is conventional fuel and the second is CaRFG, and $\delta = 0.03$, then the probability of this dramatic reduction is estimated to be $G(20\%, 0.03, 0.20, 20) = 8.7\%$. Under high variation driving conditions these probabilities are substantially higher: 18.1% if there is no difference in fuel formulation, and 21.6% if there is a 3% intrinsic mileage reduction factor. None of these probabilities is negligible. The

TABLE XVI. Probability Second Batch Gets Delta (%) Lower Mileage Than First Batch, Based on Number of Gallons in Each Batch, Difference in Intrinsic Fuel Economy from Batch to Batch, and Mileage Variation in Driving

DELTA (%)	GALLONS	LOW VARIATION		HIGH VARIATION	
		NO DIFF	3% DIFF	NO DIFF	3% DIFF
10	20	22.8%	29.8%	33.4%	38.0%
10	30	18.1%	25.8%	29.9%	35.4%
10	40	14.6%	22.7%	27.1%	33.3%
10	50	11.9%	20.1%	24.8%	31.4%
10	75	7.5%	15.3%	20.2%	27.7%
10	100	4.8%	11.8%	16.8%	24.7%
10	500	0.0098%	0.40%	1.6%	6.3%
20	20	5.7%	8.7%	18.1%	21.6%
20	30	2.7%	4.8%	13.2%	16.8%
20	40	1.3%	2.7%	9.9%	13.3%
20	50	0.63%	1.6%	7.5%	10.7%
20	75	0.11%	0.42%	3.9%	6.4%
20	100	0.021%	0.12%	2.1%	3.9%
20	500	0%	0%	0.00026%	0.0042%
30	20	0.58%	1.1%	7.3%	9.1%
30	30	0.10%	0.24%	3.7%	5.1%
30	40	0.018%	0.055%	2.0%	3.0%
30	50	0.0033%	0.013%	1.1%	1.8%
30	75	0.00005%	0.00040%	0.24%	0.50%
30	100	0%	0.00001%	0.057%	0.15%
30	500	0%	0%	0%	0%
40	20	0.015%	0.034%	1.9%	2.5%
40	30	0.00049%	0.0016%	0.53%	0.82%
40	40	0.00002%	0.00008%	0.16%	0.28%
40	50	0%	0%	0.049%	0.097%
40	75	0%	0%	0.0027%	0.0073%
40	100	0%	0%	0.00016%	0.00058%
40	500	0%	0%	0%	0%

message is that motorists experience occasional dramatic drops in mileage as a fact of life, whether there is any change in fuel formulation or not. Even when we compare large batches of fuel, say 50 gallons (about 3 fill-ups), the probabilities are nonnegligible. For example, $G(20\%, 0, 0.20, 50) = 0.6\%$; hence nearly one in a hundred 50 gallon comparisons show a 20% reduction. The impact of occasional substantial reductions in experienced mileage is perhaps best understood in the context of the California vehicle population, whose size is approximately 20 million. Vehicles may experience dramatic changes in mileage, but their drivers must actually calculate, i.e., record, their mileage in order to perceive a problem. Figures 7 and 8 and Tables XVIII and XIX provide the number of California motorists expected to record a dramatic reduction in mileage, as a

TABLE XVII. Probability Second Batch Gets Delta (%) Lower Mileage Than First Batch, Based on Number of Gallons in Each Batch, Difference in Intrinsic Fuel Economy from Batch to Batch, and Mileage Variation in Driving

DELTA (%)	GALLONS	LOW VARIATION		HIGH VARIATION	
		6% DIFF	10% DIFF	6% DIFF	10% DIFF
10	20	37.9%	50.0%	43.0%	50.0%
10	30	35.3%	50.0%	41.4%	50.0%
10	40	33.2%	50.0%	40.1%	50.0%
10	50	31.3%	50.0%	38.9%	50.0%
10	75	27.6%	50.0%	36.6%	50.0%
10	100	24.6%	50.0%	34.6%	50.0%
10	500	6.2%	50.0%	18.7%	50.0%
20	20	12.7%	20.2%	25.5%	31.5%
20	30	8.1%	15.4%	21.0%	27.8%
20	40	5.3%	11.9%	17.6%	24.8%
20	50	3.6%	9.4%	14.9%	22.4%
20	75	1.4%	5.3%	10.1%	17.6%
20	100	0.54%	3.1%	7.0%	14.1%
20	500	0%	0.0016%	0.050%	0.81%
30	20	1.9%	3.8%	11.4%	15.2%
30	30	0.53%	1.5%	7.0%	10.4%
30	40	0.16%	0.60%	4.4%	7.3%
30	50	0.049%	0.25%	2.9%	5.2%
30	75	0.0027%	0.029%	1.0%	2.3%
30	100	0.00016%	0.0035%	0.36%	1.1%
30	500	0%	0%	0%	0.00001%
40	20	0.075%	0.21%	3.3%	4.9%
40	30	0.0051%	0.022%	1.2%	2.1%
40	40	0.00036%	0.0025%	0.48%	1.0%
40	50	0.00003%	0.00029%	0.19%	0.44%
40	75	0%	0%	0.019%	0.067%
40	100	0%	0%	0.0021%	0.011%
40	500	0%	0%	0%	0%

function of the percentage of motorists making mileage calculations. These numbers, $E(\Delta, \delta, \sigma, N)$, are given by

$$E(\Delta, \delta, \sigma, N) = (V/100) G(\Delta, \delta, \sigma, N), \quad (51)$$

where $V = 20000000$ is the California vehicle population size. $E(\Delta, \delta, \sigma, N)$ is the expected number of California motorists who would both experience and record a mileage reduction of at least Δ percent in comparing two consecutive batches of N gallons of fuel, if δ is the intrinsic mileage reduction effect between the two batches, σ is the relative mileage variation parameter, and exactly 1% of all motorists record their mileage. We can only conjecture what percentage of California motorists actually record their mileage. If the

TABLE XVIII. Expected Number of Motorists Recording Second Batch to Get Delta (%) Lower Mileage Than First Batch, Based on Number of Gallons in Each Batch, Difference in Intrinsic Fuel Economy from Batch to Batch, and Mileage Variation in Driving, Per One Percent of California Population Recording Mileage

DELTA (%)	GALLONS	LOW VARIATION		HIGH VARIATION	
		NO DIFF	3% DIFF	NO DIFF	3% DIFF
10	20	45626	59637	66710	75977
10	30	36153	51656	59833	70803
10	40	29206	45385	54299	66542
10	50	23881	40236	49644	62875
10	75	14910	30507	40487	55375
10	100	9573	23630	33615	49413
10	500	20	809	3150	12629
20	20	11460	17305	36231	43150
20	30	5330	9518	26454	33534
20	40	2565	5400	19763	26594
20	50	1260	3122	14976	21358
20	75	225	833	7771	12768
20	100	42	231	4165	7858
20	500	0	0	1	8
30	20	1167	2107	14536	18294
30	30	201	473	7453	10287
30	40	36	111	3947	5965
30	50	7	27	2132	3523
30	75	0	1	481	991
30	100	0	0	113	290
30	500	0	0	0	0
40	20	30	68	3703	4987
40	30	1	3	1065	1631
40	40	0	0	319	555
40	50	0	0	98	193
40	75	0	0	5	15
40	100	0	0	0	1
40	500	0	0	0	0

recording percentage is 2%, then the $E(\Delta, \delta, \sigma, N)$ values should be multiplied by 2; if the recording percentage is 3%, then the $E(\Delta, \delta, \sigma, N)$ values should be multiplied by 3, etc. To illustrate the use of these figures and tables, consider the scenario described earlier: a 20% reduction in mileage from one 20 gallon batch to the next, with no difference in fuel formulation, and low variation driving. From Table XVIII we see that $E(20\%, 0, 0.20, 20) = 11460$. Hence for every one percent who record mileage, about 11500 California motorists would be expected to record a 20% reduction in their mileage from one fill-up to the next, even if there is no change in fuel formulation and low variation driving conditions prevail. If, as some believe, $\delta = 0.10$, this number is seen from Table XIX to climb to around 40000. Viewing the recording of a dramatic reduction as a potential complaint to be

TABLE XIX. Expected Number of Motorists Recording Second Batch to Get Delta (%) Lower Mileage Than First Batch, Based on Number of Gallons in Each Batch, Difference in Intrinsic Fuel Economy from Batch to Batch, and Mileage Variation in Driving, Per One Percent of California Population Recording Mileage

DELTA (%)	GALLONS	LOW VARIATION		HIGH VARIATION	
		6% DIFF	10% DIFF	6% DIFF	10% DIFF
10	20	75847	100000	85909	100000
10	30	70648	100000	82788	100000
10	40	66367	100000	80177	100000
10	50	62684	100000	77894	100000
10	75	55155	100000	73101	100000
10	100	49173	100000	69139	100000
10	500	12419	100000	37474	100000
20	20	25415	40493	51030	63063
20	30	16253	30771	42005	55592
20	40	10681	23886	35181	49649
20	50	7138	18789	29788	44708
20	75	2723	10679	20233	35177
20	100	1078	6256	14098	28228
20	500	0	3	100	1621
30	20	3711	7556	22878	30490
30	30	1068	2952	14048	20891
30	40	320	1197	8875	14679
30	50	98	496	5705	10475
30	75	5	58	1977	4694
30	100	0	7	712	2178
30	500	0	0	0	0
40	20	150	414	6683	9786
40	30	10	45	2478	4263
40	40	1	5	954	1923
40	50	0	1	376	886
40	75	0	0	39	135
40	100	0	0	4	21
40	500	0	0	0	0

voiced publicly, it is clear that the enormity of the California vehicle population and the nonnegligibility of the probabilities $G(\Delta, \delta, \sigma, N)$ combine to create some rather significant numbers of recorded problems, whether or not there is in fact an issue in terms of an unacceptably large intrinsic mileage reduction factor, δ . Since the introduction of CaRFG has been made a fuel economy issue, the percentage of motorists who record mileage has undoubtedly increased. Novice recorders who are unaccustomed to the usual variations in mileage will look for an explanation for a recorded dramatic drop in mileage. Our contention is that the intrinsic mileage reduction due to CaRFG is modest and is masked by considerable random variation caused by a number of uncontrolled factors. We offer as anecdotal evidence Figures 1, 2, and 3. Each of these time histories begins with conventional fuel and ends with CaRFG. The locations of the shift points are anything but apparent. (The switch to CaRFG occurs at fill-up number 47 for the Honda, fill-up number 78 for the Cadillac, and fill-up number 69 for the Ford van.)

ADDITIONAL FACTORS

A number of additional factors besides the nominal fuel composition and corresponding energy content affects fuel economy. Vehicle parameters and driving conditions have a significant effect on vehicle fuel economy. This effect results in considerable deviations from the average fuel economy for any vehicle, and makes it difficult to evaluate fuel economy with accuracy. The purpose of this section is to give an estimate of the expected changes in fuel economy that result from changes in vehicle parameters and driving conditions.

Changes in fuel economy are evaluated with HVEC, a vehicle evaluation code that has been developed at LLNL [22,23]. HVEC has been validated against vehicular data and results from other codes. HVEC calculates vehicle fuel economy by simulating a second-by-second drive of the vehicle along the EPA Urban and Highway driving cycles.

An “average” vehicle is selected for the analysis, with the characteristics listed in Table XX. An engine performance map was selected from the literature, corresponding to a current 4-cylinder engine [24]. This vehicle is evaluated with HVEC, and then some parameters are changed to reflect common driving conditions that result in changes in fuel economy, including weight changes resulting from adding passengers, turning on the air conditioner, driving against a head wind, and driving faster in the highway. The results are given in Table XXI.

TABLE XX. Vehicle Characteristics for Fuel Economy Sensitivity Analysis.

VEHICLE PARAMETER	VALUE
Empty vehicle weight, kg	1200
Frontal area, m ²	2.2
Aerodynamic drag coefficient	0.3
Coefficient of rolling friction	0.01
Transmission efficiency	0.94
Transmission gears	5
Accessory load, W	1000

TABLE XXI. Fuel Economy and Changes in Fuel Economy with Respect to the Base Case “Average” Vehicle Described in TABLE XII. Results Obtained from HVEC.

VEHICLE DESCRIPTION	FUEL ECONOMY	CHANGE
	mpg	%
Base case vehicle	28.7	--
Base case with two 70 kg passengers added	28.0	- 2.4
Base case with 2000 W accessories (A/C on)	26.7	- 7.0
Base case driven against a 10 mph head wind	26.9	- 6.2
Base case driven 25% faster in highway cycle	25.0	- 13.0

The Environmental Protection Agency has also evaluated changes in fuel economy due to driving conditions [25]. Their results are shown in Table XXII. The conditions used by

EPA for calculating the fuel economy losses listed in Table XXII are not indicated with detail. [25] All the information available is listed in the table. Lack of the precise data used in generating Table XXII makes it impossible to compare these results with the HVEC results shown in Table XXI. However, both sets of results show that common driving conditions can result in very large changes in fuel economy, and these changes dwarf the ~3% change in fuel economy due to the change in fuel energy content.

TABLE XXII. Average and Maximum Fuel Economy Reduction Resulting from Effects Listed [25].

EFFECT	CONDITIONS	FUEL ECONOMY REDUCTION %	
		AVERAGE	MAXIMUM
Temperature	20 °F vs. 70 °F	5.3	13
Head wind	20 mph	2.3	6
Hills/Mountains	7% grade	1.9	25
Poor road conditions	Gravel, snow, etc.	4.3	50
Traffic congestion	20 vs. 27 mph	10.6	15
Highway speed	70 vs. 55 mph	N/A	25
Acceleration rate	“Hard” vs. “Easy”	<1	20
Wheel alignment	1/2 inch	<1	10
Tire type	Non-radial vs. radial	3.3	4
Tire pressure	15 psi vs. 26 psi	3.3	6
Air conditioning	Extreme heat	21	N/A
Defroster	Extreme use	Analogous to A/C on some vehicles	
Idling/warm-up	Winter vs. summer	Variable	20
Windows	Open vs. closed	Unknown but likely small	

CONCLUSIONS

Fuel economy data contained in the 1996 California Air Resources Board (CARB) report on the performance and compatibility of California Reformulated Gasoline (CaRFG) has been examined and reanalyzed by two additional statistical methodologies. Additional, more recent data obtained from outside sources and not analyzed by CARB has also been analyzed by these two statistical approaches. No new experimental data was generated by this review. Within the assumptions of the analysis, point estimates for the reduction in fuel economy using CaRFG as compared to conventional, non-reformulated gasoline were 2-4 %, with a 95% upper confidence bound of 6 %. Many additional factors affect mileage in addition to the fuel: variation in energy among ostensibly identical fuels, reproducibility of tank fill-up, speed, terrain, traffic, wind and weather, road conditions, number of passengers, type of vehicle, vehicle maintenance. The relative effect of some of these additional factors were estimated from the literature and from a vehicle evaluation code; in most cases they can easily be expected to exceed the reduction due to using CaRFG. Thus, substantial variations in fuel economy are routine and inevitable, even if there is no change in fuel reformulation. This additional analysis confirms the conclusion reached by CARB with respect to fuel economy.

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FIGURE CAPTIONS

Figure 1. Mileage Time History of a 1993 Honda Prelude

Figure 2. Mileage Time History of a 1970 Cadillac

Figure 3. Mileage Time History of a 1990 15 Passenger Ford Van

Figure 4. Mileage Time History of a Sacramento City Police Car

Figure 5. Probability Second Batch Gets 20% Lower Mileage Than First Batch, Based on Number of Gallons in Each Batch, Difference in Intrinsic Fuel Economy from Batch to Batch, and Mileage Variation in Driving

Figure 6. Probability Second Batch Gets 20% Lower Mileage Than First Batch, Based on Number of Gallons in Each Batch, Difference in Intrinsic Fuel Economy from Batch to Batch, and Mileage Variation in Driving

Figure 7. Expected Number of Motorists Recording Second Batch to Get 20% Lower Mileage Than First Batch, Based on Number of Gallons in Each Batch, Difference in Intrinsic Fuel Economy from Batch to Batch, and Mileage Variation in Driving, Per One Percent of California Population Recording Mileage

Figure 8. Expected Number of Motorists Recording Second Batch to Get 20% Lower Mileage Than First Batch, Based on Number of Gallons in Each Batch, Difference in Intrinsic Fuel Economy from Batch to Batch, and Mileage Variation in Driving, Per One Percent of California Population Recording Mileage

FIGURE 1

FIGURE 2

FIGURE 3

FIGURE 4

FIGURE 5

FIGURE 6

FIGURE 7

FIGURE 8

APPENDIX 1.

ANALYZING THE CONTROL VEHICLE DATA.

The Sacramento City and Sacramento County data sets include control vehicles which used conventional fuel in both 1994 and 1995. We show here that there is no significant mileage change effect from 1994 to 1995 for the control vehicles, whether the normal model or the nonparametric model is used. As a result the control vehicle data are used for the normal model merely to help in the estimation of the relative mileage variation parameter, σ ; the control vehicle data are not used at all for the nonparametric model.

Let W_j denote the log mileage difference for control vehicle j ,

$$W_j = \log U_j - \log V_j, \quad (A.1)$$

where U_j is the 1994 mileage based on K_j gallons, and V_j is the 1995 mileage based on L_j gallons, $j = 1, \dots, C$. Define

$$s_j = 1/K_j + 1/L_j. \quad (A.2)$$

The normal model has W_j distributed as $N(\lambda, \sqrt{s_j} \sigma)$, where λ is the intrinsic mileage change effect from 1994 to 1995. We wish to test the hypothesis

$$H_0: \lambda = 0 \quad (\text{there is no mileage change effect from 1994 to 1995})$$

against the alternative

$$H_1: \lambda \neq 0 \quad (\text{there is a mileage change effect from 1994 to 1995}).$$

The maximum likelihood estimates of λ and σ based on $w_1, \dots, w_C$ are

$$\hat{\lambda} = \frac{\sum_j w_j / s_j}{\sum_j 1/s_j} \quad (A.3)$$

and

$$\hat{\sigma}^2 = \frac{1}{C} \sum_j \frac{1}{s_j} (w_j - \hat{\lambda})^2. \quad (A.4)$$

The standard error of $\hat{\lambda}$ is

$$\text{S.E.}(\hat{\lambda}) = \hat{\sigma} / \sqrt{\sum_j \frac{1}{s_j}}. \quad (A.5)$$

The P-value of the control vehicle data for this hypothesis is therefore

$$P = 2 \left[1 - \Phi \left(\frac{|\hat{\lambda}|}{\text{S.E.}(\hat{\lambda})} \right) \right]. \quad (A.6)$$

The hypothesis of no mileage change effect is accepted at significance level α if $P \geq \alpha$. Since P is calculated to be $P = 0.20$ for the Sacramento City control vehicles and $P = 0.72$ for the Sacramento County control vehicles (see Appendix 2), it is judicious, since each P exceeds the usual nominal cutoff of 0.10, to accept H_0 and claim that there is no significant mileage change effect from 1994 to 1995 for either set of control vehicles under the normal model.

Consider now the nonparametric model, which assumes W_j to be distributed as

$$W_j = \lambda + e_j, \quad (A.7)$$

where e_j has an unspecified continuous distribution symmetric about 0. The P-values for the Wilcoxon signed rank test of H_0 are computed from Stata software [19] to be $P = 0.19$ for the Sacramento City control vehicles and $P = 0.97$ for the Sacramento County control vehicles. Thus we accept H_0 and claim that there is no significant mileage change effect from 1994 to 1995 for either set of control vehicles under the nonparametric model.

APPENDIX 2A

FUEL ECONOMY ANALYSIS OF SACRAMENTO CITY FLEET

Note: all vehicles with less than 1000 miles in either year have been removed

1994 mpg	1994 miles	1995 mpg	1995 miles	1994 gal	1995 gal	gal factor	del log mpg	weight	term	sdel	sigma	std residual	sigma0	student res
8.49	8295	7.7	6823	977	886.1	464.6662552	0.097668671	0.018380866	0.001795235	0.10512925	5.135565443	2.058624924	4.432530116	1.912533012
8.29	7435	8.29	6686	896.9	806.5	424.6506105	0	0.016797962	0	0.007460579	0.023636152	0.139659822	0	0
8.88	7422	8.25	2208	835.8	267.6	202.7008157	0.073588357	0.008018264	0.000590051	0.081048935	1.331527452	1.04823379	1.097674829	0.951743557
8.32	9256	8.69	4569	1112.5	525.8	357.0484649	-0.043510684	0.014123814	-0.000614537	-0.036050106	0.4640238	-0.618804155	0.675956892	-0.746865834
8.6	11003	8.07	9193	1279.4	1139.2	602.618242	0.063608721	0.023837851	0.001516295	0.0710693	3.043731547	1.584843071	2.438235219	1.418472409
7.59	9976	7.42	9791	1314.4	1319.5	658.4725312	0.022652534	0.026047287	0.000590037	0.030113113	0.597102607	0.70195226	0.337886821	0.528042306
7.57	10237	7.45	5501	1352.3	738.4	477.6095662	0.015979035	0.018892866	0.00030189	0.023439614	0.262406095	0.465339642	0.121947841	0.317227005
7.73	8782	7.8	2594	1136.1	332.6	257.2798121	-0.009014871	0.010177252	-9.17466E-04	-0.001554292	0.000621543	-0.022647417	0.02090859	-0.131354655
8.35	11861	8.43	7163	1420.5	849.7	531.6707118	-0.009535233	0.02103137	-0.000200539	-0.002074654	0.002288412	-0.043456049	0.048339858	-0.19972654
7.95	10598	8.12	10532	1333.1	1297	657.4011254	-0.021158226	0.026004905	-0.000550218	-0.013697647	0.123345234	-0.31903937	0.294299095	-0.49280778
9.01	3199	9.45	1876	355	198.5	127.3125565	-0.04767967	0.00503612	-0.000240121	-0.040219091	0.205937646	-0.412240793	0.289426117	-0.488710817
8.55	6943	8.5	7551	812	888.4	424.2418255	0.005865119	0.016781791	9.84272E-04	0.013325698	0.075334416	0.249333024	0.01459376	0.10974044
8.4	9906	8.03	6991	1179.3	870.6	500.8530075	0.045047178	0.01981231	0.000892489	0.052507757	1.380884046	1.067484797	1.016355082	0.915810934
8.02	8673	8.41	7760	1081.4	922.7	497.8832294	-0.047483052	0.019694834	-0.000935171	-0.040022473	0.79750855	-0.811242634	1.122547562	-0.962466159
8.11	10606	8.46	9348	1307.8	1105	598.9385776	-0.042251305	0.023692294	-0.00100103	-0.034790727	0.724952064	-0.773459785	1.069208867	-0.939321728
7.95	9628	8.54	9255	1211.1	1083.7	571.9317893	-0.071589079	0.022623983	-0.00161963	-0.0641285	2.352049221	-1.393176999	2.931148276	-1.555256363
9.47	5971	8.53	1023	630.5	119.9	100.7422042	0.104539546	0.003985073	0.000416598	0.112000124	1.263713015	1.021191807	1.100962852	0.953167938
8.29	10196	8.4	1785	1229.9	212.5	181.1936703	-0.013181737	0.007167502	-9.44801E-04	-0.005721158	0.005930768	-0.069958187	0.031483883	-0.16118597
7.87	8839	7.71	9902	1123.1	1284.3	599.1515037	0.020539875	0.023700717	0.00048681	0.028000454	0.469749996	0.622610564	0.252773906	0.45671914
8.47	11760	8.26	7292	1388.4	882.8	539.6616414	0.025105921	0.021347468	0.000535948	0.0325665	0.572352676	0.687250321	0.340152659	0.52980985
8.57	8972	8.01	5999	1046.9	748.9	436.5872647	0.067576972	0.017270141	0.001167064	0.07503755	2.458263071	1.424286216	1.993739958	1.282677124
8.85	13009	8.23	11306	1469.9	1373.8	710.1130991	0.072631444	0.02809004	0.00204022	0.080092023	4.555185326	1.938813871	3.746078596	1.758213196
8.7	9293	8.51	9907	1068.2	1164.2	557.0679269	0.022081083	0.022036011	0.000486579	0.029541662	0.486158627	0.633391296	0.271611965	0.473431926

1994 mpg	1994 miles	1995 mpg	1995 miles	1994 gal	1995 gal	gal factor	del log mpg	weight	term	sdel	sigma	std residual	sigma0	student res
8.19	6479	8.14	9779	791.1	1201.4	477.0025295	0.006123718	0.018868853	0.000115548	0.013584297	0.088022761	0.269513539	0.017887557	0.121495056
7.79	9063	8.71	11761	1163.4	1350.3	624.9508772	-0.111630931	0.024721266	-0.002759658	-0.104170352	6.781630883	-2.365648234	7.787803328	-2.535073645
10.32	11536	10.7	9426	1117.8	880.9	492.6552359	-0.036159981	0.01948803	-0.000704687	-0.028699403	0.405778302	-0.578665383	0.644168524	-0.729092856
10.79	8589	11.02	8103	796	735.3	382.2234703	-0.021092024	0.015119665	-0.000318904	-0.013631446	0.071023356	-0.242093812	0.170041091	-0.374593325
10.7	12593	10.75	5389	1176.9	501.3	351.5552199	-0.004662013	0.013906517	-6.48324E-04	0.002798566	0.00275337	0.047666729	0.00764083	-0.079406007
10.75	11575	11.32	10574	1076.7	934.1	500.1718072	-0.051665318	0.019785364	-0.001022217	-0.04420474	0.97736522	-0.898072779	1.335111158	-1.049643463
10.68	11125	10.44	8603	1041.7	824	460.0743957	0.022728251	0.018199225	0.000413637	0.03018883	0.419295905	0.588224898	0.237662193	0.442856622
10.38	8227	11.12	9102	792.6	818.5	402.6709081	-0.068864411	0.015928508	-0.001096907	-0.061403832	1.518242728	-1.11931854	1.909589112	-1.255315981
10.51	11327	10.87	11200	1077.7	1030.4	526.7596793	-0.033679516	0.020837104	-0.000701784	-0.026218938	0.362111822	-0.546643822	0.597508674	-0.702190905
10.74	11025	10.68	12774	1026.5	1196.1	552.414582	0.005602256	0.021851938	0.00012242	0.013062834	0.094262719	0.278902929	0.017337679	0.119613053
10.72	3228	10.44	5194	301.1	497.5	187.5748184	0.026466573	0.007419922	0.00019638	0.033927152	0.215908301	0.422102343	0.131392314	0.329282063
11.16	7674	11.09	4469	687.6	403	254.0828902	0.006292156	0.010050791	6.32411E-04	0.013752734	0.048056653	0.19914062	0.010059452	0.091110883
10.33	3608	10.05	2804	349.3	279	155.1085469	0.027479649	0.00613565	0.000168605	0.034940227	0.189359536	0.395299881	0.117127286	0.310893851
10.74	12362	10.81	10726	1151	992.2	532.858436	-0.006496543	0.021078353	-0.000136936	0.000964036	0.00049522	0.020215399	0.022489325	-0.136229544
10.46	11407	10.65	10887	1090.5	1022.3	527.6496356	-0.018001434	0.020872308	-0.000375731	-0.010540855	0.058626951	-0.219953941	0.170985713	-0.375632365
11.75	10124	11.43	8225	861.6	719.6	392.111915	0.027611763	0.015510824	0.000428281	0.035072341	0.482324764	0.630888881	0.298949827	0.496686374
10.08	11625	10.65	12754	1153.3	1197.6	587.5163044	-0.05500663	0.023240462	-0.001278379	-0.047546051	1.328155192	-1.046905556	1.777665291	-1.211178322
10.49	13011	10.74	10097	1240.3	940.1	534.767029	-0.023552667	0.021153852	-0.00049823	-0.016092088	0.138480754	-0.338047558	0.296650302	-0.494772428
10.44	11231	10.57	7774	1075.8	735.5	436.8414398	-0.012375217	0.017280196	-0.000213846	-0.004914639	0.010551326	-0.093311809	0.066900522	-0.234962117
10.99	13705	11.1	10953	1247	986.8	550.8727729	-0.00995934	0.021790949	-0.000217023	-0.002498761	0.003439544	-0.053276233	0.054640217	-0.212343661
10.76	11378	10.62	11149	1057.4	1049.8	526.7931473	0.013096539	0.020838428	0.000272911	0.020557118	0.222620194	0.428613032	0.090355209	0.273061008
10.88	10856	10.19	7100	997.8	696.8	410.2838664	0.065519394	0.016229654	0.001063357	0.072979973	2.185203234	1.342854658	1.761262895	1.205577643
10.48	12070	11.06	8548	1151.7	772.9	462.5111348	-0.053866317	0.018295615	-0.000985517	-0.046405739	0.996014292	-0.90660034	1.342013118	-1.052353072
10.59	1574	11.47	6164	148.6	537.4	116.4105539	-0.079824772	0.004604868	-0.000367583	-0.072364193	0.60959276	-0.709255958	0.741767369	-0.782378585
10.64	7479	10.87	11201	702.9	1030.5	417.8714953	-0.021386217	0.016529799	-0.00035351	-0.013925639	0.081035065	-0.258594679	0.191122006	-0.397135253
10.38	3356	11.24	9430	323.3	839	233.3723651	-0.079597967	0.009231542	-0.000734812	-0.072137388	1.214423757	-1.001078697	1.478609104	-1.10461206
10.62	3565	11.15	10849	335.7	973	249.5882173	-0.048700482	0.009872995	-0.00048082	-0.041239903	0.424482078	-0.591851526	0.591957599	-0.698921488
11.27	4844	11.15	9015	429.8	808.5	280.6212549	0.01070483	0.011100573	0.00011883	0.018165409	0.092599985	0.276432149	0.03215734	0.162900777
10.46	3687	10.63	8778	352.5	825.8	247.0461682	-0.016121734	0.009772439	-0.000157549	-0.008661155	0.018532318	-0.123665332	0.064209843	-0.230188646

1994 mpg	1994 miles	1995 mpg	1995 miles	1994 gal	1995 gal	gal factor	del log mpg	weight	term	sdel	sigma	std residual	sigma0	student res
10.35	2718	10.53	10603	262.6	1006.9	208.2803781	-0.017241806	0.008238975	-0.000142055	-0.009781228	0.019926687	-0.128233248	0.061917568	-0.226042466
10.27	5535	11.16	3793	538.9	339.9	208.4343537	-0.083108933	0.008245066	-0.000685239	-0.075648354	1.192801755	-0.992126906	1.439675829	-1.08997227
10.9	5552	10.91	11691	509.4	1071.6	345.27074	-0.000917011	0.013657921	-1.25245E-04	0.006543568	0.0147839	0.110453022	0.000290341	-0.015478802
11.52	5686	11.1	6986	493.6	629.4	276.6445592	0.037139547	0.010943266	0.000406428	0.044600126	0.550293391	0.673876391	0.381588552	0.561152318
9.89	3852	10.8	9189	389.5	850.8	267.1826171	-0.088021988	0.010568979	-0.000930303	-0.08056141	1.734052792	-1.196228789	2.070096306	-1.307008367
10.77	8469	11.89	8860	786.4	745.2	382.6229303	-0.09893322	0.015135467	-0.0014974	-0.091472641	3.201499428	-1.625398322	3.745029802	-1.757967054
10.88	3366	10.62	8884	309.4	836.5	225.86011	0.024187226	0.008934379	0.000216098	0.031647804	0.226217763	0.432062371	0.132133107	0.330209008
10.53	6015	10.99	10402	571.2	946.5	356.2237596	-0.042757442	0.014091191	-0.000602503	-0.035296864	0.44380799	-0.605174543	0.651247875	-0.733088239
8.36	10908	8.45	11121	1304.8	1316.1	655.21282	-0.010708014	0.025918342	-0.000277534	-0.003247436	0.006909768	-0.075511802	0.07512773	-0.248990758
61	in control group					25279.8894		1	-0.007460579	-0.03480413	0.930035696		0.942355218	
									aging effect		0.00584941	0.202153214	control sigma	
											std err ae	p value ae		
1994 MPG	1994 miles	1995 mpg	1995 miles	1994 gal	1995 gal	gal factor	del log mpg	weight	term	sdel	sigma	std resid		student res
9.72	1340	7.46	4500	137.9	603.2	112.2402915	0.264630204	0.005088992	0.001346701	0.243129925	6.634766093	2.339892476		2.345869144
8.65	9250	8.32	2564	1069.4	308.2	239.2487515	0.038897066	0.010847574	0.000421939	0.017396786	0.072408199	0.24444264		0.245779329
9.09	2463	9.01	4978	271	552.5	181.8184578	0.008839837	0.008243676	7.28727E-04	-0.012660443	0.029143102	-0.155078283		-0.15572147
10.17	12582	8.86	8673	1237.2	978.9	546.4983891	0.137895445	0.024778319	0.003416817	0.116395166	7.403869794	2.471795168		2.502999743
8.22	7206	8.41	9644	876.6	1146.7	496.8107646	-0.022851265	0.022525475	-0.000514736	-0.044351545	0.977256337	-0.898022753		-0.90831109
8.2	8728	7.83	3792	1064.4	484.3	332.8526635	0.046171644	0.01509159	0.000696804	0.024671365	0.202599505	0.408886039		0.412006777
8.16	7610	8.04	9001	932.6	1119.5	508.7694069	0.014815086	0.023067681	0.00034175	-0.006685194	0.022737829	-0.136980136		-0.138587913
8.73	6875	7.85	7007	787.5	892.6	418.3813463	0.106251838	0.018969473	0.002015541	0.084751558	3.005160685	1.574769308		1.589921496
8.38	7807	9.69	8934	931.6	922	463.3875701	-0.145246511	0.021010062	-0.003051638	-0.166746791	12.88425614	-3.260714447		-3.295517738
7.79	6135	7.73	6535	787.5	845.4	407.7117398	0.007731997	0.018485711	0.000142931	-0.013768282	0.07728812	-0.252545402		-0.254912512
10.56	4866	10.6	2299	460.8	216.9	147.4804781	-0.003780723	0.006686787	-2.52809E-04	-0.025281003	0.094259063	-0.27889752		-0.279834687
8.31	3712	7.84	4171	446.7	532	242.8163891	0.058220775	0.011009332	0.000640972	0.036720495	0.327412342	0.519793227		0.522678359

1994 MPG	1994 miles	1995 mpg	1995 miles	1994 gal	1995 gal	gal factor	del log mpg	weight	term	sdel	sigma	std resid		student res
8.29	2529	8.91	7393	305.1	829.7	223.0714399	-0.072124272	0.010114093	-0.000729472	-0.093624552	1.955345363	-1.270266458		-1.276739397
7.69	2502	7.71	8240	325.4	1068.7	249.447658	-0.002597404	0.011309994	-2.93766E-04	-0.024097684	0.144853846	-0.34573879		-0.347710685
7.8	6761	8.24	7288	866.8	884.5	437.7802775	-0.05487661	0.019849023	-0.001089247	-0.07637689	2.553760302	-1.451687571		-1.46631295
8.37	8999	7.84	5773	1075.1	736.4	437.0431355	0.06541505	0.019815601	0.001296239	0.04391477	0.842840775	0.833980434		0.84236821
7.76	7355	7.26	9553	947.8	1315.8	550.9432939	0.066602505	0.024979852	0.001663721	0.045102226	1.120734778	0.961688709		0.973929952
8.24	8189	8.23	9104	993.8	1106.2	523.495981	0.001214329	0.023735386	2.88226E-04	-0.02028595	0.215428953	-0.421633518		-0.426728211
7.91	6990	7.52	9267	883.7	1232.3	514.6424905	0.050561644	0.023333968	0.001179804	0.029061364	0.434647947	0.598896685		0.606008712
8.64	6599	7.71	7818	763.8	1014	435.6469794	0.113884395	0.019752299	0.002249479	0.092384116	3.718170649	1.751651687		1.769211834
8.95	7680	8.13	2731	858.1	335.9	241.4035092	0.096092609	0.010945271	0.00105176	0.074592329	1.34317288	1.052807694		1.058617059
8.47	5248	8.65	3873	619.6	447.7	259.9034198	-0.021028812	0.01178406	-0.000247805	-0.042529092	0.470093465	-0.622838141		-0.626540677
9.16	5367	8.21	6613	585.9	805.5	339.1853169	0.109493255	0.015378713	0.001683865	0.087992976	2.626231776	1.472141713		1.483593795
10.51	10325	10.33	7379	982.4	714.3	413.5842046	0.017274902	0.01875197	0.000323938	-0.004225378	0.007384057	-0.078060373		-0.078802723
10.47	8302	10.18	5416	792.9	532	318.3808589	0.028089014	0.014435436	0.000405477	0.006588734	0.013821364	0.106796876		0.107576153
10.42	8705	10.12	9932	835.4	981.4	451.2668208	0.029213372	0.020460505	0.00059772	0.007713093	0.026846676	0.148842978		0.150389452
10.55	9883	10.27	10598	936.8	1031.9	491.0265251	0.026898836	0.022263216	0.000598855	0.005398556	0.014310679	0.108670885		0.109901148
10.23	8920	10.39	9049	871.9	870.9	435.6998566	-0.015519225	0.019754697	-0.000306578	-0.037019505	0.597102139	-0.701951985		-0.708989856
9.48	5070	9.76	8339	534.8	854.4	328.9181687	-0.029108084	0.014913199	-0.000434095	-0.050608364	0.842427348	-0.833775868		-0.840063414
10.93	8798	10.7	8676	804.9	810.8	403.9196138	0.021267561	0.018313776	0.000389489	-0.000232719	2.18755E-04	-0.004248761		-0.004288209
10.49	9487	10.25	10598	904.4	1034	482.4337598	0.023144717	0.021873619	0.000506259	0.001644437	0.001304585	0.032810981		0.033175825
10.58	6131	10.37	8638	579.5	833	341.7511504	0.020048404	0.015495048	0.000310651	-0.001451875	0.000720392	-0.024381895		-0.024573018
10.45	9950	10.14	11123	952.2	1096.9	509.7204529	0.03011398	0.023110802	0.000695958	0.008613701	0.037819136	0.176660272		0.178737731
10.6	10296	10.18	10409	971.3	1022.5	498.121301	0.04042899	0.022584895	0.000913084	0.01892871	0.178474907	0.383770577		0.388179102
10.69	8584	10.68	9752	803	913.1	427.2590758	0.000935892	0.019371991	1.81301E-04	-0.020564388	0.180685325	-0.38613977		-0.389935151
12.9	9785	11.07	10082	758.5	910.7	413.8305476	0.152988565	0.018763139	0.002870546	0.131488285	7.154786311	2.429860977		2.452982778
10.73	8551	10.81	7158	796.9	662.2	361.6662189	-0.007428075	0.016398001	-0.000121806	-0.028928355	0.302660268	-0.499759201		-0.503907819
10.52	10004	11.01	7904	951	717.9	409.0855653	-0.045525743	0.018548001	-0.000844412	-0.067026023	1.8378119	-1.231497749		-1.243080032
9.23	3682	10.53	2972	398.9	282.2	165.2761415	-0.131769278	0.007493645	-0.000987432	-0.153269557	3.882593931	-1.789963101		-1.796707705
10.23	4905	10.72	9378	479.5	874.8	309.7294543	-0.046786576	0.01404318	-0.000657032	-0.068286855	1.44429775	-1.091720486		-1.099467794
10.47	5722	9.91	10671	546.5	1076.8	362.5153699	0.054969677	0.016436501	0.000903509	0.033469397	0.406089909	0.578887526		0.583704431

1994 MPG	1994 miles	1995 mpg	1995 miles	1994 gal	1995 gal	gal factor	del log mpg	weight	term	sdel	sigma	std resid		student res
10.34	8895	10.36	10352	860.3	999.2	462.2811293	-0.001932368	0.020959896	-4.05022E-04	-0.023432647	0.253833467	-0.457675362		-0.462548516
11.7	11521	13.25	9733	984.7	734.6	420.7297272	-0.124408711	0.019075949	-0.002373214	-0.14590899	8.957097527	-2.718734161		-2.745042373
8.45	1840	7.04	6401	217.8	909.2	175.7087489	0.182558271	0.007966661	0.00145438	0.161057992	4.557828129	1.939376214		1.947147857
7.51	10762	7.57	6305	1433	832.9	526.7424423	-0.007957602	0.023882581	-0.000190048	-0.029457881	0.457089589	-0.614163154		-0.621631089
7.68	7897	7.38	9347	1028.3	1266.5	567.5187162	0.039845909	0.025731384	0.00102529	0.018345629	0.19100529	0.397013972		0.402222568
9.83	6249	10.11	9070	635.7	897.1	372.0553693	-0.028086099	0.016869046	-0.000473786	-0.049586379	0.914813067	-0.868858936		-0.876281388
10.4	7312	10.35	3587	703.1	346.6	232.1562923	0.004819286	0.010526001	5.07278E-04	-0.016680993	0.064598773	-0.230884739		-0.232109564
10.23	1877	10.49	11124	183.5	1060.4	156.4300989	-0.025097842	0.007092564	-0.000178008	-0.046598122	0.339669968	-0.529433805		-0.531321373
12.47	1783	12.96	3572	143	275.6	94.14906832	-0.038541931	0.004268733	-0.000164525	-0.060042211	0.339413708	-0.529234054		-0.530367263
10.15	1011	10.26	10935	99.6	1065.8	91.08776386	-0.010779134	0.004129933	-4.45171E-04	-0.032279414	0.094909858	-0.279858662		-0.280438357
10.49	8819	10.25	10676	840.7	1041.6	465.2144292	0.023144717	0.021092892	0.000488189	0.001644437	0.001258021	0.032220106		0.032565385
10.12	6112	10.1	12220	604	1209.9	402.8775566	0.00197824	0.018266529	3.61356E-04	-0.01952204	0.153540679	-0.355954773		-0.35925103
10	2866	9.83	10079	286.6	1025.3	223.9888559	0.017146159	0.010155688	0.000174131	-0.004354121	0.004246463	-0.059196602		-0.059499502
11.05	8761	10.09	9909	792.9	982.1	438.7082197	0.090885594	0.019891096	0.001807814	0.069385314	2.112082542	1.320196375		1.333525635
10.18	2800	10.55	5819	275	551.6	183.510767	-0.035700849	0.008320406	-0.000297046	-0.057201128	0.600441559	-0.703912155		-0.706858974
9.69	7153	10.3	10669	738.2	1035.8	431.0189177	-0.061049469	0.019542462	-0.001193057	-0.082549749	2.93716163	-1.556850877		-1.572289887
10.34	6500	10.2	8820	628.6	864.7	363.9927811	0.013632149	0.016503487	0.000224978	-0.007868131	0.022533877	-0.136364415		-0.137503781
10.3	5018	10.25	11204	487.2	1093.1	336.9982408	0.00486619	0.015279551	7.43532E-04	-0.01663409	0.093245038	-0.277393293		-0.279537115
10.22	4372	9.88	12885	427.8	1304.1	322.1282869	0.033834073	0.014605345	0.000494158	0.012333793	0.049002947	0.201091721		0.202576513
10.76		7816		8.99		6189	726.4	688.4	353.4448403	0.179722706	0.016025242	0.0028801	0.158222427	8.848256986 2.70
61		in test group							22055.50709		1	0.02150028	-0.019188762	1.24845889 1.10
												0.028960858		0.00840651 0.01
NON- PARAMETRIC		0.022786847	0.022529187			delta								
61										model with aging			1.106047684	sigma:no aging
945.5		1891								CA RFG effect	0.028960858		0.007447583	
139.2219451										std error	0.010142939			

				beta		delta					
CONFIDENCE		lo index	hi index	conf. interval for rfg effect		conf. interval for rfg effect		model w/o aging	beta		
50	0.674490366	852	1040	0.018041323	0.028021612	0.017879553	0.027632649	CA RFG effect	0.02150028	CA RFG BTU/g	110400
75	1.150349362	785	1107	0.01444412	0.031723273	0.014340305	0.031225369	std error	0.007447583	conv BTU/g	115500
90	1.644853	717	1175	0.009907965	0.03636557	0.009859043	0.035712285				
95	1.959961082	673	1219	0.007731997	0.039744168	0.007702182	0.038964728	delta	0.021270796	0.045160396	energy effect
99	2.575834515	587	1305	0.002874168	0.046750671	0.002870041	0.045674691			1	p value
		beta		delta							
CONFIDENCE		conf. interval rfg effect		conf. interval rfg effect							
50	0.674490366	0.016476957	0.026523602	0.016341954	0.026174941						
75	1.150349362	0.012932958	0.030067602	0.012849686	0.029620068						
90	1.644853	0.009250101	0.033750458	0.009207451	0.033187265						
95	1.959961082	0.006903308	0.036097252	0.006879535	0.035453515						
99	2.575834515	0.002316539	0.04068402	0.002313858	0.039867535						

APPENDIX 2B.

FUEL ECONOMY ANALYSIS OF SACRAMENTO COUNTY FLEET

NOTE: all vehicles with ridiculous mpg or less than 1000 miles in either year have been removed

1994 mpg	1994 miles	1995 mpg	1995 miles	1994 gal	1995 gal	gal factor	del log mpg	weight	term	sdel	sigma	std residual	sigma0	student res
10.89	1139	13.63	2218	104.6	162.7	63.66786382	-0.224428309	0.011357293	-0.002548898	-0.218668622	3.044340426	-1.351361479	3.206827153	-1.38695606
9.32	1209	9.97	2854	129.7	286.3	89.26228365	-0.067417955	0.015922914	-0.00107349	-0.061658268	0.339352179	-0.451180425	0.405713208	-0.49332656
17.59	2086	17.9	3914	118.6	218.7	76.8983694	-0.017470154	0.013717396	-0.000239645	-0.011710467	0.010545461	-0.079534896	0.023469866	-0.11865341
39.17	1175	37.14	1170	30	31.5	15.36585366	0.053216592	0.002741014	0.000145867	0.058976279	0.053445534	0.179052603	0.043516184	0.16156613
12.28	5569	13.44	5087	453.5	378.5	206.3097957	-0.090263412	0.036802252	-0.003321897	-0.084503726	1.473233417	-0.940071114	1.68090568	-1.00414539
33.39	1516	43.77	1685	45.4	38.5	20.83313468	-0.270692197	0.003716286	-0.00100597	-0.26493251	1.462261788	-0.936564071	1.526532645	-0.95692516
15.97	4959	16.69	3112	310.5	186.5	116.5155936	-0.044097775	0.020784453	-0.000916548	-0.038338089	0.171255673	-0.3205143	0.226577831	-0.36866646
8.99	991	9.03	4384	110.2	485.5	89.81383247	-0.004439519	0.016021301	-7.11269E-04	0.001320168	0.000156531	0.009690043	0.00177017	-0.03258610
14.15	1937	14.15	14179	136.9	1002	120.4441127	0	0.021485236	0	0.005759687	0.003995612	0.048957202	0	0
11.07	1031	10.89	4599	93.1	422.3	76.28275126	0.01639381	0.01360758	0.00022308	0.022153497	0.037437851	0.149858156	0.020501523	0.11089653
15.19	5062	14.17	1366	333.2	96.4	74.76834264	0.069510263	0.013337435	0.000927089	0.07526995	0.42360493	0.504086922	0.361256455	0.46551398
15.07	3708	12.99	2695	246.1	207.5	112.5788139	0.148526182	0.020082197	0.002982732	0.154285869	2.679840644	1.267883682	2.483491644	1.22055197
14.54	2346	14.85	1970	161.3	132.7	72.80445578	-0.021096393	0.01298711	-0.000273981	-0.015336706	0.017124668	-0.10135286	0.032402191	-0.13941583
6.57	2193	8.48	1070	333.8	126.2	91.57730435	-0.255196617	0.016335875	-0.00416886	-0.24943693	5.697828361	-1.848756053	5.964000654	-1.89144522
21.36	1038	29.22	4021	48.6	137.6	35.91493018	-0.313333392	0.006406629	-0.002007411	-0.307573705	3.397609293	-1.427616785	3.52604936	-1.45435062
6.54	2989	7.44	6016	457	808.6	291.9802465	-0.128933683	0.052084443	-0.006715439	-0.123173997	4.429875663	-1.630124204	4.853848875	-1.70634974
8.63	2975	8.92	1491	344.7	167.2	112.5880836	-0.033051441	0.02008385	-0.0006638	-0.027291755	0.083860094	-0.224286224	0.122990973	-0.27161987
5.59	3435	5.82	2977	614.5	511.5	279.1445382	-0.040320975	0.049794765	-0.002007773	-0.034561288	0.333433297	-0.447228437	0.453827884	-0.52175968
10.54	3696	8.29	4127	350.7	497.8	205.7495109	0.240127574	0.036702307	0.008813236	0.245887261	12.43972757	2.731683392	11.86377435	2.66769617
10.2	1769	8.05	1004	173.4	124.7	72.53599463	0.236715629	0.012939221	0.003062916	0.242475316	4.264701486	1.599444763	4.064502882	1.56145202
20.02	1790	19.09	1277	89.4	66.9	38.26525912	0.047567136	0.006825889	0.000324688	0.053326823	0.108816833	0.255489448	0.086580217	0.22789471
14.98	4356	17.21	3403	290.8	197.7	117.6891709	-0.138774632	0.0209938	-0.002913407	-0.133014945	2.082271638	-1.117618596	2.266504956	-1.16601265
20.36	1452	18.8	2831	71.3	150.6	48.39017575	0.079715322	0.008632006	0.000688103	0.085475009	0.353537516	0.460513834	0.307496946	0.42948236
13.5	1266	13.16	5336	93.8	405.5	76.17844983	0.02550776	0.013588974	0.000346624	0.031267446	0.074476106	0.211365162	0.049565188	0.17243019

1994 mpg	1994 miles	1995 mpg	1995 miles	1994 gal	1995 gal	gal factor	del log mpg	weight	term	sdel	sigma	std residual	sigma0	student res
25.35	1402	21.63	1192	55.3	55.1	27.59990942	0.15869749	0.004923367	0.000781326	0.164457177	0.74647165	0.669162487	0.695100775	0.64572680
23.57	4188	29.28	2044	177.7	69.8	50.1149899	-0.216929972	0.008939685	-0.001939286	-0.211170285	2.234772202	-1.157821361	2.358341906	-1.18940103
10.41	1856	10.59	3877	178.3	366.1	119.9038024	-0.017143277	0.021388854	-0.000366675	-0.01138359	0.015537869	-0.09654297	0.035238762	-0.14539023
6.23	1085	6.35	3579	174.2	563.6	133.0701003	-0.01907848	0.023737503	-0.000452875	-0.013318793	0.023605339	-0.118995366	0.048435973	-0.17045468
7.06	1533	6.94	2759	217.1	397.6	140.4245323	0.017143277	0.025049412	0.000429429	0.022902964	0.073659092	0.210202612	0.041269639	0.15734040
6.72	1892	5.93	2278	281.5	384.1	162.4461388	0.125063942	0.028977702	0.003624066	0.130823628	2.780236708	1.291414963	2.540818347	1.23455867
31.65	3276	24.03	3170	103.5	131.9	57.99341546	0.275435099	0.010345065	0.002849394	0.281194786	4.585568792	1.658523136	4.399641101	1.62455175
22.24	3154	22.26	1505	141.8	67.6	45.77688634	-0.000898876	0.008165839	-7.34008E-05	0.00486081	0.001081592	0.02547163	3.69868E-04	-0.00471029
13.8	4513	13.59	15163	327	1115.7	252.8827199	0.015334364	0.045110091	0.000691735	0.021094051	0.112522437	0.259803198	0.05946353	0.18886447
9.82	2594	9.42	3790	264.2	402.3	159.4713578	0.041586034	0.02844705	0.001183	0.047345721	0.357473748	0.463070382	0.27578948	0.40673708
6.42	5606	6.66	7677	873.2	1152.7	496.8348092	-0.036701367	0.088627106	-0.003252736	-0.03094168	0.475663467	-0.534164259	0.669231683	-0.6335970
42.54	3450	30.02	4005	81.1	133.4	50.43701632	0.348580984	0.008997129	0.003136228	0.35434067	6.332736129	1.949039424	6.128536391	1.91735845
28.17	3141	34.66	2769	111.5	79.9	46.54571578	-0.207328702	0.008302985	-0.001721447	-0.201569016	1.891155599	-1.065095523	2.000776475	-1.09552984
5.1	1715	5.65	2411	336.3	426.7	188.0723591	-0.102415005	0.033548996	-0.003435921	-0.096655319	1.757019112	-1.026628155	1.97265963	-1.08780488
23.22	973	29.35	2694	41.9	91.8	28.76903515	-0.23427857	0.00513192	-0.001202299	-0.228518883	1.502344331	-0.949313526	1.579030162	-0.97324042
26.64	1649	23.91	1977	61.9	82.7	35.40200553	0.108117064	0.006315132	0.000682774	0.113876751	0.459090178	0.524775956	0.413824648	0.49823370
36.47	1138	44	1254	31.2	28.5	14.89447236	-0.187699629	0.002656927	-0.000498704	-0.181939942	0.493038948	-0.54383298	0.524749401	-0.56104914
8.69	1511	8.82	2288	173.9	259.4	104.1072236	-0.014848931	0.018571005	-0.00027576	-0.009089244	0.008600751	-0.071827891	0.022954679	-0.11734390
10.05	5045	9.72	4569	502	470.1	242.763296	0.033387016	0.043304953	0.001445823	0.039146703	0.372026095	0.472401894	0.270606508	0.40289701
9.07	3396	8.8	2120	374.4	240.9	146.5837153	0.030220543	0.026148108	0.00079021	0.035980229	0.189763894	0.337389621	0.133872151	0.28338055
24.48	7497	24.08	6615	306.3	274.7	144.820327	0.016474837	0.025833549	0.000425604	0.022234524	0.071595413	0.20723711	0.039307171	0.15355388
22.52	1518	21.97	1672	67.4	76.1	35.74313589	0.024725917	0.006375984	0.000157652	0.030485604	0.033218671	0.141161431	0.021852315	0.11449160
26.85	3855	22.88	556	143.6	24.3	20.78308517	0.160002654	0.003707358	0.000593187	0.165762341	0.571060027	0.585282863	0.532064634	0.56494624
19.88	7077	19.37	2632	356	135.9	98.35413702	0.025988724	0.01754475	0.000455966	0.031748411	0.099137191	0.243861495	0.066429738	0.19962098
12.94	6748	13.3	6869	521.5	516.5	259.4939788	-0.027440746	0.04628943	-0.001270217	-0.021681059	0.121979902	-0.27050112	0.195397552	-0.34236115
51.95	4151	39.38	2721	79.9	69.1	37.0542953	0.277023644	0.006609873	0.001831091	0.282783331	2.963099059	1.333208313	2.843624418	1.30605373
50	in control					5605.901327		1	-0.005759687	0.148766184	1.193777121		1.195333936	control sigma

	group													
									aging effect	SE of ae ->	0.015944119	0.717918599	<- p value of ae	
1994 mpg	1994 miles	1995 mpg	1995 miles	1994 gal	1995 gal	gal factor	del log mpg	weight	term	sdel	sigma	std residual		student res
5.05	1805	5.79	2032	357.4	350.9	177.0600875	-0.136744048	0.026422218	-0.003613081	-0.174936198	5.418512015	-1.802872225		-1.82717279
7.93	2606	7.74	2550	328.6	329.5	164.5246923	0.024251348	0.024551594	0.000595409	-0.013940801	0.031974707	-0.138493125		-0.14022519
17.4	2748	18.37	3537	157.9	192.5	86.74586187	-0.054248693	0.01294486	-0.000702242	-0.092440843	0.741270226	-0.666827046		-0.67118539
22.38	1978	22.77	1029	88.4	45.2	29.90778443	-0.017276177	0.004463061	-7.71046E-04	-0.055468327	0.092018335	-0.234942786		-0.23546882
6.23	2547	5.69	2038	408.8	358.2	190.9154628	0.090666085	0.02848982	0.00258306	0.052473935	0.525688374	0.561550882		0.56972520
4.36	1299	5.78	1318	297.9	228	129.1523103	-0.281931625	0.019273065	-0.005433686	-0.320123775	13.23542947	-2.817694775		-2.84524648
22.95	1889	23.96	2348	82.3	98	44.73322241	-0.043067837	0.006675423	-0.000287496	-0.081259986	0.29538176	-0.420936673		-0.42234871
12.62	5795	12.66	2999	459.2	236.9	156.2770866	-0.00316456	0.023320825	-7.38001E-04	-0.041356709	0.267292796	-0.400422575		-0.40517495
5.32	2941	4.94	2929	552.8	592.9	286.0741206	0.074107972	0.0426901	0.003163677	0.035915823	0.369020258	0.470489604		0.48086565
6.17	1950	6.62	2480	316	374.6	171.4068926	-0.070396532	0.025578607	-0.001800645	-0.108588682	2.021144676	-1.101092054		-1.11545029
9.58	1496	9.29	1261	156.2	135.7	72.61507366	0.030739039	0.010836159	0.000333093	-0.00745311	0.004033684	-0.049189892		-0.04945859
6.63	4805	6.73	4633	724.7	688.4	353.0418796	-0.014970339	0.052683525	-0.00078869	-0.053162489	0.997784695	-0.773647892		-0.79486944
17.37	2828	18.71	5433	162.8	290.4	104.3184466	-0.07431356	0.015567172	-0.001156852	-0.11250571	1.320414356	-0.889979589		-0.89698876
11.65	6160	11.87	3030	528.8	255.3	172.1752838	-0.018708029	0.025693272	-0.00048067	-0.056900178	0.557439911	-0.578261073		-0.58583606
23.41	1948	20.92	4683	83.2	223.9	60.65932921	0.112457642	0.009052035	0.00101797	0.074265493	0.334558245	0.447982239		0.45002368
10.84	2609	7.94	3663	240.7	461.3	158.1693875	0.311329721	0.023603208	0.00734838	0.273137571	11.80008999	2.660526489		2.69249196
48.86	1461	50.71	1937	29.9	38.2	16.77209985	-0.037164064	0.002502857	-9.30163E-04	-0.075356214	0.095241357	-0.239021912		-0.23932159
5.49	2398	6.81	1401	436.8	205.7	139.8439844	-0.215463865	0.020868556	-0.00449642	-0.253656014	8.997754038	-2.323228818		-2.34785616
5.32	3121	6.07	2446	586.7	403	238.900778	-0.131885302	0.035650544	-0.004701783	-0.170077451	6.910524994	-2.03601254		-2.07330516
24.62	1891	20.32	5760	76.8	283.5	60.42964197	0.191953498	0.009017759	0.00173099	0.153761349	1.42871097	0.92575724		0.92995981
28.94	4110	22.05	5150	142	233.6	88.31522897	0.271912119	0.013179053	0.003583544	0.23371997	4.824221526	1.701134094		1.71245579
25.31	3462	21.34	4280	136.8	200.6	81.33396562	0.170616329	0.012137257	0.002070814	0.132424179	1.426285697	0.924971158		0.93063608
26.14	1085	17.73	1574	41.5	88.8	28.28242517	0.388208588	0.004220513	0.001638439	0.350016439	3.464922537	1.441689358		1.44474135
27.46	1700	24.39	2688	61.9	110.2	39.63614178	0.118557188	0.005914799	0.000701242	0.080365039	0.255991584	0.39186617		0.39303024
35.76	3508	26.94	4637	98.1	172.1	62.48338268	0.283217779	0.009324233	0.002640789	0.24502563	3.75134979	1.500094809		1.50713771

1994 mpg	1994 miles	1995 mpg	1995 miles	1994 gal	1995 gal	gal factor	del log mpg	weight	term	sdel	sigma	std residual		student res
18.44	1800	18.4	3000	97.6	163	61.04681504	0.002171554	0.009109858	1.97825E-04	-0.036020596	0.079207225	-0.217975322		-0.2189750
25.38	4302	24.54	3710	169.5	151.2	79.91393826	0.033657023	0.01192535	0.000401372	-0.004535127	0.00164362	-0.031399708		-0.03158862
12.21	5821	11.87	1812	476.7	152.7	115.6531459	0.02824108	0.017258619	0.000487402	-0.00995107	0.011452413	-0.082884522		-0.08360915
12.16	1808	11.47	1004	148.7	87.5	55.08573243	0.058416945	0.008220301	0.000480205	0.020224796	0.022532398	0.11625955		0.11674036
20.25	1296	17.84	1358	64	76.1	34.76374019	0.126711666	0.005187703	0.000657342	0.088519517	0.272398408	0.404228761		0.40528136
7.84	5969	7.39	5514	761.4	746.1	376.8361791	0.059111099	0.056234287	0.003324071	0.02091895	0.16490444	0.314514811		0.32374942
25.66	4541	23.97	5571	177	232.4	100.4758183	0.068130311	0.014993746	0.001021529	0.029938161	0.090055822	0.232423922		0.23418621
32.66	993	23.5	3464	30.4	147.4	25.20224972	0.329150666	0.003760866	0.001237892	0.290958517	2.133543289	1.131294452		1.13342779
11.1	1801	8.14	7418	162.3	911.3	137.7645212	0.310154928	0.020558243	0.00637624	0.271962779	10.18958103	2.472310943		2.49812280
7.42	3837	6.97	3432	517.1	492.4	252.2239128	0.062563832	0.037638721	0.002354823	0.024371683	0.149815689	0.299780635		0.30558674
19.59	3303	20.12	1320	168.6	65.6	47.22527754	-0.026695113	0.007047306	-0.000188129	-0.064887263	0.198835272	-0.345359687		-0.34658308
19.92	2637	18.8	3875	132.4	206.1	80.61341211	0.057867382	0.012029731	0.000696129	0.019675233	0.031206644	0.136819645		0.13765009
14.62	1295	16.37	4366	88.6	266.7	66.50610751	-0.113059937	0.009924534	-0.001122067	-0.151252087	1.521473104	-0.955338032		-0.96011425
18.22	3568	17.58	2595	195.8	147.6	84.15864881	0.035758	0.012558777	0.000449077	-0.00243415	0.000498647	-0.017295043		-0.01740467
12.73	5392	13.15	5110	423.6	388.6	202.6729377	-0.032460346	0.030244357	-0.000981742	-0.070652496	1.011697731	-0.779023064		-0.79107772
26.6	1524	21.21	2609	57.3	123	39.08985025	0.226438447	0.005833277	0.001320878	0.188246298	1.385214068	0.911556043		0.91422641
21.21	3166	19.06	3899	149.3	204.6	86.31472167	0.10688087	0.012880522	0.001376681	0.068688721	0.407244973	0.49425696		0.49747118
22.05	3180	21.08	3250	144.2	154.2	74.51621984	0.044987878	0.011119862	0.000500259	0.006795729	0.003441303	0.045434564		0.04568930
21.91	3727	21.62	4438	170.1	205.3	93.02485349	0.013324341	0.013881858	0.000184967	-0.024867808	0.057527303	-0.185764162		-0.18706711
22.1	1452	20.86	2951	65.7	141.5	44.86751931	0.057744159	0.006695464	0.000386624	0.019552009	0.017152003	0.10143372		0.10177500
29.74	4940	20.43	7090	166.1	347	112.330345	0.375488532	0.016762766	0.006294227	0.337296383	12.77969414	2.768759014		2.79226092
22.65	2969	22.52	1736	131.1	77.1	48.54855908	0.005756049	0.007244776	4.17013E-04	-0.032436101	0.05107797	-0.175041789		-0.17567932
13.04	1549	13.88	2094	118.8	150.9	66.46985539	-0.062427399	0.009919124	-0.000619225	-0.100619548	0.672960322	-0.635359689		-0.63853443
17.53	8955	17.27	3491	510.8	202.1	144.806677	0.014942807	0.021609125	0.000322901	-0.023249343	0.078272634	-0.216685525		-0.21906535
19.38	1572	18.28	4086	81.1	223.5	59.50705844	0.05843404	0.008880084	0.000518899	0.020241891	0.024382074	0.120937295		0.12147786
19.77	1054	19.07	8398	53.3	440.4	47.54571602	0.036049218	0.007095124	0.000255774	-0.002142932	0.000218337	-0.011444294		-0.01148511
19.33	7417	18.79	7795	383.7	414.8	199.3221791	0.02833348	0.029744332	0.00084276	-0.00985867	0.019372794	-0.10780057		-0.10944047
13.19	5366	14.43	1772	406.8	122.8	94.32598187	-0.089850406	0.014076022	-0.001264736	-0.128042556	1.546464667	-0.963152217		-0.97000330
15.74	8642	14.21	4307	549	303.1	195.2844737	0.102259301	0.029141796	0.00298002	0.064067151	0.801564627	0.693416545		0.70374658

1994 mpg	1994 miles	1995 mpg	1995 miles	1994 gal	1995 gal	gal factor	del log mpg	weight	term	sdel	sigma	std residual		student res
22.29	1716	23.08	2818	77	122.1	47.22099448	-0.034828294	0.007046667	-0.000245423	-0.073020444	0.251781644	-0.388630571		-0.39000712
18.93	4412	17.84	4392	233.1	246.2	119.7354893	0.059304838	0.017867817	0.001059648	0.021112689	0.05337157	0.178928662		0.18054894
20.63	1782	25.73	3388	86.4	131.7	52.17276479	-0.220911299	0.007785607	-0.001719928	-0.259103449	3.502597546	-1.449506099		-1.45518190
11.57	2052	8.94	1423	177.4	159.2	83.90398099	0.257879952	0.012520774	0.003228857	0.219687802	4.049435227	1.558555083		1.56840483
14.36	3192	12.56	5497	222.3	437.7	147.4253182	0.133929403	0.021999898	0.002946433	0.095737253	1.351244684	0.900309695		0.91037951
13.74	2384	13.04	4003	173.5	307	110.8522373	0.05228973	0.016542192	0.000864987	0.014097581	0.022030971	0.114958675		0.11592147
60	in test group			-0.055993751		6701.181802		1	0.03819215	0.842898444	1.366997626	1.291145162	sigma:aging	
										0.043951836		0.016699061	0.023369758	
		beta	delta											
	non- parametric	0.028190847	0.027797193										sigma:no aging	
	60									model with aging		1.291799701		
	915	1830								CA RFG effect	0.043951836		0.015780453	
	135.8399794			beta		delta				std error	0.023369758			
confidence		lo index	hi index	conf. interval for rfg effect		conf. interval for rfg effect								
50	0.674490366	823	1008	0.019579677	0.041342694	0.01938924	0.040499742			model w/o aging	beta			
75	1.150349362	759	1072	0.010973518	0.047590733	0.010913528	0.046476046			CA RFG effect	0.03819215	CA RFG BTU/g	110700	
90	1.644853	692	1139	0.003911907	0.058819969	0.003904266	0.057123499			std error	0.015780453	conv BTU/g	115600	
95	1.959961082	649	1182	0.001809345	0.074205122	0.001807709	0.071518777							
99	2.575834515	565	1266	-0.008284701	0.079426091	-0.008319114	0.076353717			delta	0.037472026	0.043312117	energy effect	
											0.627201222	p value		

		beta		delta	
confidence		conf. interval for rfg effect		conf. interval for rfg effect	
50	0.674490366	0.027548386	0.048835913	0.02717239	0.047662617
75	1.150349362	0.020039115	0.056345184	0.019839666	0.054787193
90	1.644853	0.012235623	0.064148676	0.012161072	0.062134449
95	1.959961082	0.007263075	0.069121224	0.007236762	0.066786455
99	2.575834515	-0.002455687	0.078839986	-0.002458705	0.075812205

APPENDIX 2C.

FUEL ECONOMY ANALYSIS OF AUTO OIL FLEET

NORMAL MODEL

industry mpg	industry miles	rfg miles	rfg miles	mile factor	del log mpg	weight	term	sdel	sigma	std residual	student resid
22.62	22.196	22.30	22.196	11.098	-0.014247792	0.03125	-0.000445244	0.014917765	0.002469746	0.734621999	0.746376697
24.16	22.196	22.84	22.196	11.098	-0.056184988	0.03125	-0.001755781	-0.027019431	0.008102091	-1.330565875	-1.35185628
21.40	22.196	20.52	22.196	11.098	-0.041990902	0.03125	-0.001312216	-0.012825345	0.001825504	-0.631581246	-0.641687187
21.54	22.196	21.04	22.196	11.098	-0.023486284	0.03125	-0.000733946	0.005679273	0.000357956	0.279674537	0.284149614
19.11	22.196	18.73	22.196	11.098	-0.020085242	0.03125	-0.000627664	0.009080315	0.000915054	0.447158096	0.454313079
19.73	22.196	19.26	22.196	11.098	-0.024109914	0.03125	-0.000753435	0.005055643	0.00028366	0.248964026	0.252947704
15.00	22.196	14.62	22.196	11.098	-0.025659747	0.03125	-0.000801867	0.00350581	0.000136402	0.17264284	0.175405301
20.27	22.196	19.39	22.196	11.098	-0.044384491	0.03125	-0.001387015	-0.015218934	0.002570474	-0.749453026	-0.761445036
15.26	22.196	14.70	22.196	11.098	-0.037387532	0.03125	-0.00116836	-0.008221975	0.000750235	-0.404889333	-0.411367974
26.07	22.196	24.81	22.196	11.098	-0.04953843	0.03125	-0.001548076	-0.020372873	0.004606269	-1.003257605	-1.019310746
22.51	22.196	21.39	22.196	11.098	-0.051036132	0.03125	-0.001594879	-0.021870575	0.005308418	-1.077011586	-1.094244866
19.50	22.196	19.46	22.196	11.098	-0.002053389	0.03125	-6.41684E-04	0.027112168	0.008157803	1.335132677	1.356496157
22.77	22.196	22.44	22.196	11.098	-0.014598799	0.03125	-0.000456212	0.014566758	0.002354889	0.717336728	0.728814844
20.72	22.196	19.95	22.196	11.098	-0.037870274	0.03125	-0.001183446	-0.008704717	0.000840919	-0.428661854	-0.43552088
29.57	22.196	28.34	22.196	11.098	-0.0424861	0.03125	-0.001327691	-0.013320543	0.001969194	-0.655967155	-0.666463296
26.23	22.196	25.32	22.196	11.098	-0.035309197	0.03125	-0.001103412	-0.00614364	0.000418886	-0.302542166	-0.307383148
14.51	22.196	14.04	22.196	11.098	-0.032927668	0.03125	-0.00102899	-0.003762111	0.000157075	-0.185264335	-0.188228753
26.63	22.196	27.39	22.196	11.098	0.028139583	0.03125	0.000879362	0.05730514	0.03644449	2.821978867	2.867133396
21.84	22.196	21.40	22.196	11.098	-0.020352229	0.03125	-0.000636007	0.008813328	0.000862034	0.434010378	0.440954985
26.19	22.196	25.32	22.196	11.098	-0.033783061	0.03125	-0.001055721	-0.004617504	0.000236624	-0.22738797	-0.231026409
21.13	22.196	20.33	22.196	11.098	-0.038596204	0.03125	-0.001206131	-0.009430647	0.000987024	-0.464410112	-0.471841146
28.93	22.196	27.46	22.196	11.098	-0.052148719	0.03125	-0.001629647	-0.022983162	0.005862249	-1.131800676	-1.149910637
22.24	22.196	21.35	22.196	11.098	-0.04084073	0.03125	-0.001276273	-0.011675173	0.001512765	-0.574941281	-0.584140926

industry mpg	industry miles	rfg miles	rfg miles	mile factor	del log mpg	weight	term	sdel	sigma	std residual	student resid
25.07	22.196	24.63	22.196	11.098	-0.0177067	0.03125	-0.000553334	0.011458857	0.001457227	0.564288854	0.573318049
20.02	22.196	19.25	22.196	11.098	-0.039220713	0.03125	-0.001225647	-0.010055156	0.001122076	-0.495163927	-0.503087053
24.39	22.196	23.62	22.196	11.098	-0.032079401	0.03125	-0.001002481	-0.002913845	9.42275E-04	-0.143491624	-0.145787636
26.00	22.196	24.67	22.196	11.098	-0.052508607	0.03125	-0.001640894	-0.023343051	0.006047278	-1.149523322	-1.167916863
18.02	22.196	17.66	22.196	11.098	-0.020180057	0.03125	-0.000630627	0.0089855	0.000896044	0.442488942	0.449569214
27.71	22.196	27.29	22.196	11.098	-0.015273024	0.03125	-0.000477282	0.013892533	0.002141941	0.684134681	0.695081531
19.56	22.196	18.77	22.196	11.098	-0.041226814	0.03125	-0.001288338	-0.012061257	0.00161447	-0.593953916	-0.603457782
17.15	22.196	16.50	22.196	11.098	-0.038637793	0.03125	-0.001207431	-0.009472236	0.000995749	-0.466458139	-0.473921943
13.40	22.196	13.87	22.196	11.098	0.034473527	0.03125	0.001077298	0.063639084	0.044946157	3.133892524	3.184037988
32n test group				355.13 6		1	-0.029165557	-3.60822E-15	0.067649125		
			beta						0.003589755		
		CARFG effect	0.029165557		0.030768938	energy effect					
		std error	0.003589755		0.672437755	p value					
	delta	0.0287 44347		CARFG BTU/g	111370						
				conv BTU/g	114850						
			beta		delta						
	confidence	conf. interval for rfg effect			conf. interval for rfg effect						
	50	0.674490366	0.0267	0.0316	0.0264	0.0311					
	75	1.150349362	0.0250	0.0333	0.0247	0.0327					
	90	1.644853	0.0233	0.0351	0.0230	0.03446					
	95	1.959961082	0.0221	0.0362	0.0219	0.0356					
	99	2.575834515	0.0199	0.0384	0.0197	0.0377					

APPENDIX 2D

FUEL ECONOMY ANALYSIS OF AUTO OIL FLEET

NONPARAMETRIC MODEL

industry mpg	industry miles	rfg mpg	rfg miles	del log mpg
22.62	22.196	22.3	22.196	0.014247792
24.16	22.196	22.84	22.196	0.056184988
21.4	22.196	20.52	22.196	0.041990902
21.54	22.196	21.04	22.196	0.023486284
19.11	22.196	18.73	22.196	0.020085242
19.73	22.196	19.26	22.196	0.024109914
15	22.196	14.62	22.196	0.025659747
20.27	22.196	19.39	22.196	0.044384491
15.26	22.196	14.7	22.196	0.037387532
26.07	22.196	24.81	22.196	0.04953843
22.51	22.196	21.39	22.196	0.051036132
19.5	22.196	19.46	22.196	0.002053389
22.77	22.196	22.44	22.196	0.014598799
20.72	22.196	19.95	22.196	0.037870274
29.57	22.196	28.34	22.196	0.0424861
26.23	22.196	25.32	22.196	0.035309197
14.51	22.196	14.04	22.196	0.032927668
26.63	22.196	27.39	22.196	-0.028139583
21.84	22.196	21.4	22.196	0.020352229
26.19	22.196	25.32	22.196	0.033783061
21.13	22.196	20.33	22.196	0.038596204
28.93	22.196	27.46	22.196	0.052148719
22.24	22.196	21.35	22.196	0.04084073

industry mpg	industry miles	rfg mpg	rfg miles	del log mpg
25.07	22.196	24.63	22.196	0.0177067
20.02	22.196	19.25	22.196	0.039220713
24.39	22.196	23.62	22.196	0.032079401
26	22.196	24.67	22.196	0.052508607
18.02	22.196	17.66	22.196	0.020180057
27.71	22.196	27.29	22.196	0.015273024
19.56	22.196	18.77	22.196	0.041226814
17.15	22.196	16.5	22.196	0.038637793
13.4	22.196	13.87	22.196	-0.034473527
32	in test group			

		beta	delta				
	non-parametric	0.031471402	0.030981332				
	32						
	264	528					delta

	53.47896783			beta		conf. interval for rfg effect	
confidence		lo index	hi index	conf. interval for rfg effect	0.033710871	0.02896983	0.033148991
50	0.674490366	228	301	0.02939774	0.035585297	0.027779109	0.034959584
75	1.150349362	202	327	0.028172246	0.0369527	0.026267152	0.036278282
90	1.644853	176	353	0.026618296	0.037886981	0.025466038	0.037178249
95	1.959961082	159	370	0.02579591	0.039548544	0.021550167	0.038776709
99	2.575834515	126	403	0.021785763			

		beta		delta			
	non-parametric	0.031471402		0.030981332			
	32						
	264	528					delta
	53.47896783			beta		conf. interval for rfg effect	
confidence		lo index	hi index	conf. interval for rfg effect	0.033710871	0.02896983	0.033148991
50	0.674490366	228	301	0.02939774	0.035585297	0.027779109	0.034959584
75	1.150349362	202	327	0.028172246	0.0369527	0.026267152	0.036278282
90	1.644853	176	353	0.026618296	0.037886981	0.025466038	0.037178249
95	1.959961082	159	370	0.02579591	0.039548544	0.021550167	0.038776709
99	2.575834515	126	403	0.021785763			

APPENDIX 2E

FUEL ECONOMY ANALYSIS OF CARB FLEET

CITY DRIVING

NORMAL MODEL

industry mpg	industry miles	rfg mpg	rfg miles	mile factor	del log mpg	weight	term	sdel	sigma	std residual	student resid
22.476	21.958	21.558	22.013	10.9927328	-0.041665274	0.237890581	-0.009911776	-0.000215041	5.08333E-06	-0.011630622	-0.013322762
20.956	22.056	20.636	32.957	13.21323309	-0.015368862	0.285943792	-0.004394631	0.02608137	0.008988142	1.546548533	1.830194954
21.591	22.042	20.511	22.083	11.03124048	-0.05130336	0.238723915	-0.012247339	-0.009853127	0.001070958	-0.533844629	-0.611848337
26.223	21.964	24.629	21.924	10.97199089	-0.062737446	0.237441712	-0.014896487	-0.021287214	0.004971908	-1.150244442	-1.317205758
4	in test group			46.20919725		1	-0.041450232	-0.005274012	0.061301542		
									0.009017937		
		beta							beta		delta
	CARFG effect	0.041450232	0.045160396	energy effect	confidence			conf. interval for rfg effect		conf. interval for rfg effect	
	std error	0.009017937	0.659617833	p value	50	0.674490366	0.0354	0.0475	0.0347	0.0464	
					75	1.150349362	0.0311	0.0518	0.0306	0.0505	
delta	0.040602919		CA RFG BTU/g	110400	90	1.644853	0.0266	0.0563	0.0263	0.0547	
			conv BTU/g	115500	95	1.959961082	0.0238	0.0591	0.0235	0.0574	
					99	2.575834515	0.0182	0.0647	0.0181	0.0626	

city mpg												
	Taurus	Caravan	Lumina	Accord			d rfg	mpg rfg	d conv	mpg conv	reduction	
d rfg	22.013	32.957	22.083	21.924		Taurus	22.013	21.558	21.958	22.476	.0408	
mpg rfg	21.558	20.636	20.511	24.629		Caravan	32.957	20.636	22.056	20.956	.0153	
d conv	21.958	22.056	22.042	21.964		Lumina	22.083	20.511	22.042	21.591	.0500	
mpg conv	22.476	20.956	21.591	26.223		Accord	21.924	24.629	21.964	26.223	.0608	
reduction	.0408	.0153	.0500	.0608								

APPENDIX 2F

FUEL ECONOMY ANALYSIS OF CARB FLEET

HIGHWAY DRIVING

NORMAL MODEL

industry mpg	industry miles	rfg mpg	rfg miles	mile factor	del log mpg	weight	term	sdel	sigma	std residual	student resid
37.608	40.991	35.882	41.042	20.50824207	-0.046985885	0.259032084	-0.012170852	-0.013463462	0.003717423	-0.671699381	-0.780324506
31.508	40.908	31.407	61.285	24.53247072	-0.003215959	0.30986064	-0.000996499	0.030306465	0.022532628	1.653712114	1.990634421
36.379	40.904	34.787	41.071	20.49366495	-0.044768168	0.258847965	-0.011588149	-0.011245745	0.002591768	-0.560856842	-0.651475922
38.235	20.448	36.338	40.952	13.63821655	-0.050893756	0.172259311	-0.008766923	-0.017371333	0.004115512	-0.70675016	-0.776817075
4	in test group			79.17259428		1	-0.033522424	-0.011774075	0.090770769		
									0.010201372		
		beta						beta		delta	
	CARFG effect	0.033522424	0.045160396	energy effect		confidence		conf. interval for rfg effect		conf. interval for rfg effect	
	std error	0.010201372	0.873028419	p value		50	0.674490366	0.0266	0.0404	0.0263	0.0396
						75	1.150349362	0.0218	0.0453	0.0216	0.0442
delta	0.032966773		CA RFG BTU/g	110400		90	1.644853	0.0167	0.0503	0.0166	0.0491
			conv BTU/g	115500		95	1.959961082	0.0135	0.0535	0.0134	0.0521
						99	2.575834515	0.0072	0.0598	0.0072	0.0580

hwy mpg											
	Taurus	Caravan	Lumina	Accord			d rfg	mpg rfg	d conv	mpg conv	reduction
d rfg	41.042	61.285	41.071	40.952		Taurus	41.042	35.882	40.991	37.608	.0459
mpg rfg	35.882	31.407	34.787	36.338		Caravan	61.285	31.407	40.908	31.508	.0032
d conv	40.991	40.908	40.904	20.448		Lumina	41.071	34.787	40.904	36.379	.0438
mpg conv	37.608	31.508	36.379	38.235		Accord	40.952	36.338	20.448	38.235	.0496
reduction	.0459	.0032	.0438	.0496							

APPENDIX 2G

FUEL ECONOMY ANALYSIS OF CARB FLEET

CITY DRIVING

NONPARAMETRIC MODEL

industry mpg	industry miles	rfg mpg	rfg miles	del log mpg		
22.476	21.958	21.558	22.013	0.041665274		
20.956	22.056	20.636	32.957	0.015368862		
21.591	22.042	20.511	22.083	0.05130336		
26.223	21.964	24.629	21.924	0.062737446		
4	in test group					
	0.041665274	0.015368862	0.05130336	0.062737446		
0.041665274	0.041665274					
0.015368862	0.028517068	0.015368862				
0.05130336	0.046484317	0.033336111	0.05130336			
0.062737446	0.05220136	0.039053154	0.057020403	0.062737446		
0.015368862			beta			
0.028517068		nonparametric	0.0441			
0.033336111						
0.039053154				88% confidence interval for rfg effect:	0.0154	0.0627
0.041665274						
0.046484317				94% upper bound for rfg effect:	0.0627	

0.05130336						
0.05220136						
0.057020403			delta			
0.062737446		nonparametric	0.0431			
				88% confidence interval for rfg effect:	0.0153	0.0608
				94% upper bound for rfg effect:	0.0608	

APPENDIX 2H

FUEL ECONOMY ANALYSIS OF CARB FLEET

HIGHWAY DRIVING

NON-PARAMETRIC MODEL

industry mpg	industry miles	rfg mpg	rfg miles	del log mpg		
37.608	40.991	35.882	41.042	0.046985885		
31.508	40.908	31.407	61.285	0.003215959		
36.379	40.904	34.787	41.071	0.044768168		
38.235	20.448	36.338	40.952	0.050893756		
4	in test group					
	0.046985885	0.003215959	0.044768168	0.050893756		
0.046985885	0.046985885					
0.003215959	0.025100922	0.003215959				
0.044768168	0.045877027	0.023992064	0.044768168			
0.050893756	0.048939821	0.027054858	0.047830962	0.050893756		
0.003215959			beta			
0.023992064		nonparametric	0.0453			
0.025100922						
0.027054858				88% confidence interval for rfg effect:	0.0032	0.0509
0.044768168						
0.045877027				94% upper bound for rfg	0.0509	

				effect:		
0.046985885						
0.047830962						
0.048939821			delta			
0.050893756		nonparametric	0.0443			
				88% confidence interval for rfg effect:	0.0032	0.0496
				94% upper bound for rfg effect:	0.0496	

APPENDIX 2I

FUEL ECONOMY ANALYSIS OF OKLAHOMA FLEET

NORMAL MODEL

industry mpg	industry miles	rfg mpg	rfg miles	mile factor	del log mpg	weight	term	sdel	sigma	std residual	student resid
24.80	22	23.75	22	11	-0.043284753	0.175438596	-0.007593816	0.004789022	0.000252282	0.424852342	0.467871419
23.63	33	22.18	22	13.2	-0.063268788	0.210526316	-0.013319745	-0.015195012	0.003047727	-1.476667851	-1.661934835
21.20	22	20.00	22	11	-0.058268908	0.175438596	-0.010222615	-0.010195132	0.001143348	-0.904448862	-0.996030221
22.13	33	21.26	33	16.5	-0.040062646	0.263157895	-0.010542802	0.00801113	0.00105894	0.870423358	1.014012525
20.95	22	20.20	22	11	-0.036450346	0.175438596	-0.006394798	0.01162343	0.001486145	1.031158561	1.135570104
5	in test group			62.7		1	-0.048073776	-0.000966562	0.037385672		
									0.004721407		
		beta							beta		delta
	CA RFG effect	0.048073776		0.05670948	energy effect	confidence		conf. interval for rfg effect		conf. interval for rfg effect	
	std error	0.004721407		0.966304239	p value	50	0.674490366	0.0449	0.0513	0.0439	0.0500
						75	1.150349362	0.0426	0.0535	0.0417	0.0521
delta	0.046936529			CA RFG BTU/g	111400	90	1.644853	0.0403	0.0558	0.03951	0.0543
				conv BTU/g	117900	95	1.959961082	0.0388	0.0573	0.0381	0.0557
						99	2.575834515	0.0359	0.0602	0.0353	0.0585

APPENDIX 2J

FUEL ECONOMY ANALYSIS OF OKLAHOMA DOE FLEET

NONPARAMETRIC MODEL

industry mpg	industry miles	rfg mpg	rfg miles	del log mpg		
24.80	22	23.75	22	0.043284753		
23.63	33	22.18	22	0.063268788		
21.20	22	20.00	22	0.058268908		
22.13	33	21.26	33	0.040062646		
20.95	22	20.20	22	0.036450346		
5	in test group					
	0.043284753	0.063268788	0.058268908	0.040062646	0.036450346	
0.043284753	0.043284753					
0.063268788	0.053276771	0.063268788				
0.058268908	0.050776831	0.060768848	0.058268908			
0.040062646	0.0416737	0.051665717	0.049165777	0.040062646		
0.036450346	0.03986755	0.049859567	0.047359627	0.038256496	0.036450346	
0.036450346						
0.038256496			beta			
0.03986755		nonparametric	0.0492			
0.040062646						
0.0416737				88% confidence interval for rfg effect:	0.0383	0.0608
0.043284753						

0.047359627				94% upper bound for rfg effect:	0.0608	
0.049165777						
0.049859567						
0.050776831			delta			
0.051665717		nonparametric	0.0480			
0.053276771						
0.058268908				88% confidence interval for rfg effect:	0.03753	0.0590
0.060768848						
0.063268788				94% upper bound for rfg effect:	0.0590	

APPENDIX 2K
CADILLAC DIARY

fill-up	mpg	gallons	odometer	mpg	ln(mpg)	c*f	term
			1198				
1	9.84295026	22.859	1423	9.84	2.286755489	52.27294373	9.55900E-04
2	10.50036973	20.285	1636	10.50	2.351410469	47.69836136	0.090245478
3	9.54885137	21.678	1843	9.55	2.256420872	48.91469166	0.017349048
4	9.614411497	19.762	2033	9.61	2.26326317	44.72660677	0.009090336
5	9.550840204	22.197	2245	9.55	2.25662913	50.0903968	0.01750382
6	10.49882754	18.764	2442	10.50	2.351263588	44.11910997	0.083111481
7	9.949738435	19.498	2636	9.95	2.297546263	44.79755703	0.003212396
8	9.854655087	22.223	2855	9.85	2.287943941	50.84497821	0.000232335
9	9.400884191	19.679	3040	9.40	2.240803748	44.09677695	0.037937343
10	8.884596813	20.710	3224	8.88	2.184319082	45.23724819	0.20872469
11	9.530999339	21.194	3426	9.53	2.254549575	47.78292369	0.019279869
12	9.02571108	20.497	3611	9.03	2.200077291	45.09498424	0.146815724
13	9.831460674	19.224	3800	9.83	2.285587517	43.93813442	1.47841E-04
14	9.014617626	40.157	4162	9.01	2.19884744	88.29911667	0.296056516
15	9.465918055	21.551	4366	9.47	2.247697775	48.44013474	0.029523718
16	9.206798867	16.944	4522	9.21	2.219942218	37.61470095	0.071079039
17	10.92128028	18.496	4724	10.92	2.390713205	44.21863144	0.207831413
18	8.977320454	38.096	5066	8.98	2.194701447	83.60934634	0.308640117
19	9.214243897	39.287	5428	9.21	2.220750536	87.24662633	0.160718602
20	9.726564438	20.151	5624	9.73	2.274860744	45.84071886	0.001955029
21	9.431715334	20.993	5822	9.43	2.244077982	47.10992907	0.034659584
22	9.104704097	20.429	6008	9.10	2.208791214	45.1233957	0.117747602
23	9.39942614	20.214	6198	9.40	2.240648639	45.29247158	0.039244536
24	8.924006507	21.515	6390	8.92	2.188745006	47.0908488	0.198139989
25	10.92950469	20.129	6610	10.93	2.391465985	48.13781881	0.229404577

fill-up	mpg	gallons	odometer	mpg	ln(mpg)	c*f	term
26	10.2399832	19.043	6805	10.24	2.326299979	44.29973049	0.032938285
27	11.3681891	19.968	7032	11.37	2.430819025	48.5385943	0.426270525
28	9.806313119	20.497	7233	9.81	2.283026374	46.79519159	5.81396E-04
29	10.44620911	21.156	7454	10.45	2.346239148	49.63703542	0.080091681
30	10.50326272	22.374	7689	10.50	2.351685944	52.6166213	0.100363086
31	10.93202523	21.405	7923	10.93	2.391696576	51.1942652	0.245001775
32	10.1010101	20.691	8132	10.10	2.312635429	47.85073966	0.016134802
33	10.49536304	22.105	8364	10.50	2.350933544	51.967386	0.096941106
34	10.0940897	21.894	8585	10.09	2.311950075	50.61783494	0.01624515
35	9.47567909	20.579	8780	9.48	2.24872842	46.27658216	0.026643933
36	10.31230145	20.461	8991	10.31	2.333337498	47.74241855	0.048381646
37	10.62351072	40.288	9419	10.62	2.363069538	95.20334554	0.24737351
38	10.26534073	21.821	9643	10.27	2.328773243	50.81616094	0.042365902
39	9.968260266	20.164	9844	9.97	2.299406072	46.36522403	0.004354576
40	9.964279	21.276	10056	9.96	2.299006598	48.91366437	0.004348316
41	9.89911429	21.113	10265	9.90	2.292445287	48.40039735	0.001263105
42	10.08680471	20.621	10473	10.09	2.311228106	47.65983477	0.014500275
43	10.22679101	18.872	10666	10.23	2.325010847	43.8776047	0.030650257
44	9.725373964	21.593	10876	9.73	2.274738343	49.11842503	0.00214732
45	10.1208139	18.872	11067	10.12	2.314594086	43.68101959	0.016853166
46	9.807286814	20.393	11267	9.81	2.283125662	46.55978162	5.12255E-04
47	9.952958653	20.195	11468	9.95	2.297869859	46.4054818	0.003497109
48	9.644570645	20.426	11665	9.64	2.26639513	46.29338692	0.006852006
49	10.01754224	21.662	11882	10.02	2.30433778	49.91656499	0.008344801
50	10.17183488	20.252	12088	10.17	2.319622615	46.97699719	0.024684177
51	10.01798099	19.465	12283	10.02	2.304381578	44.85478741	0.007531958
52	10.68619474	18.435	12480	10.69	2.368952697	43.67164297	0.130828347
53	10.78658326	20.303	12699	10.79	2.378303071	48.28668726	0.177845296
54	9.816370385	20.476	12900	9.82	2.284051439	46.76823727	8.89569E-05

fill-up	mpg	gallons	odometer	mpg	ln(mpg)	c*f	term
55	11.04337398	20.012	13121	11.04	2.401830608	48.06543413	0.2745067
56	8.841189542	21.151	13308	8.84	2.179421431	46.09694269	0.234475786
57	10.09715097	18.322	13493	10.10	2.312253301	42.36510499	0.013899113
58	11.35442011	22.194	13745	11.35	2.429607105	53.9227001	0.465963203
59	9.525660964	20.576	13941	9.53	2.253989311	46.37808407	0.019419533
60	9.927705936	19.642	14136	9.93	2.295329428	45.08486062	0.002214837
61	9.382623381	21.316	14336	9.38	2.238859402	47.72352702	0.044813245
62	10.72322736	21.169	14563	10.72	2.37241217	50.22159323	0.162822883
63	9.804841434	21.316	14772	9.80	2.282876288	48.66179094	7.17192E-04
64	10.153743	20.879	14984	10.15	2.317842406	48.39423158	0.022919274
65	9.289919058	21.744	15186	9.29	2.22892984	48.46585044	0.06765622
66	10.65274151	19.150	15390	10.65	2.365817278	45.30540088	0.125974429
67	9.28248871	19.930	15575	9.28	2.228129691	44.40662474	0.063803806
68	10.94871113	18.815	15781	10.95	2.393221745	45.02846713	0.221540537
69	9.726770614	20.459	15980	9.73	2.274881941	46.54180964	0.001976377
70	9.771292029	23.436	16209	9.77	2.279448702	53.42115978	0.000648877
71	9.936859538	19.322	16401	9.94	2.296251029	44.36816238	0.002573349
72	10.39818648	20.292	16612	10.40	2.341631414	47.51638465	0.065745739
73	9.842105263	19.000	16799	9.84	2.286669638	43.44672312	7.29214E-04
74	9.677419355	18.290	16976	9.68	2.26979527	41.51455549	0.004068903
75	9.49022455	19.283	17159	9.49	2.250262274	43.39180743	0.02288284
76	10.2298977	18.182	17345	10.23	2.32531458	42.27886969	0.029976411
77	8.997429306	20.228	17527	9.00	2.196938904	44.43968015	0.155833766
78	9.835386686	19.318	17717	9.84	2.285986768	44.16069239	3.14632E-04
79	10.21015067	20.176	17923	10.21	2.32338239	46.87656309	0.030173412
80	9.992046132	20.116	18124	9.99	2.30178939	46.30279536	0.005867562
81	10.03923967	19.623	18321	10.04	2.306501381	45.2604766	0.00931778
82	9.336338132	21.743	18524	9.34	2.233914112	48.57199455	0.056103016
83	9.400892288	18.828	18701	9.40	2.240804609	42.18986918	0.036295354

fill-up	mpg	gallons	odometer	mpg	ln(mpg)	c*f	term
84	9.055708219	16.012	18846	9.06	2.203395301	35.28076556	0.105874096
85	9.147735935	19.677	19026	9.15	2.21350641	43.55516563	0.09976301
86	9.287320364	40.916	19406	9.29	2.228650068	91.18744619	0.128589952
87	9.169199595	19.740	19587	9.17	2.215849997	43.74087895	0.09360269
88	9.363769803	19.757	19772	9.36	2.236847966	44.19340527	0.045259894
89	9.638731514	40.773	20165	9.64	2.265789514	92.38303586	0.014596983
90	9.494482936	19.485	20350	9.49	2.250710886	43.85510162	0.022524233
91	9.444388627	19.906	20538	9.44	2.245420769	44.69734583	0.030728653
92	9.676125539	19.946	20731	9.68	2.269661567	45.27066961	0.004517217
93	9.43632987	19.923	20919	9.44	2.24456712	44.71851072	0.032105837
94	9.939033762	19.519	21113	9.94	2.296469809	44.8247942	0.002699084
95	9.447081955	20.853	21310	9.45	2.245705906	46.82970526	0.031724989
96	10.68469254	18.344	21506	10.68	2.368812114	43.45348941	0.129748409
97	8.935485793	16.787	21656	8.94	2.190030517	36.76404228	0.150483894
		total miles	20458		2079.082	2.284710564	0.272
		avg mpg	9.839919734		97	9.822842725	sigma hat

APPENDIX 2L
PRELUDE DIARY

case	gallons	miles	mpg	ln(mpg)	b*e	sigma
1	11.008	270	24.53	3.199799678	35.22339485	0.038125801
2	12.300	295	23.98	3.177376094	39.08172596	0.081248751
3	13.100	333	25.42	3.23553026	42.3854464	0.007002772
4	11.587	331	28.57	3.352234595	38.84234226	0.101477941
5	12.800	389	30.39	3.414134173	43.70091741	0.309440727
6	25.250	749	29.66	3.389912828	85.5952989	0.435049951
7	12.814	324	25.28	3.230205193	41.39184934	0.010368534
8	13.560	390	28.76	3.359022457	45.54834451	0.13660961
9	14.050	404	28.75	3.358792482	47.19103437	0.140898206
10	12.328	266	21.58	3.071623211	37.86697094	0.431225378
11	25.900	564	21.78	3.080811283	79.79301222	0.819137215
12	13.250	382	28.83	3.361423056	44.53885549	0.139948114
13	11.250	286	25.42	3.235623682	36.40076642	0.005965329
14	11.770	352	29.91	3.398077254	39.99536928	0.228805458
15	13.260	368	27.75	3.323330953	44.06736844	0.055473381
16	12.250	321	26.20	3.265915186	40.00746103	0.000646436
17	13.450	332	24.68	3.206155863	43.12279636	0.037064515
18	12.900	347	26.90	3.292097469	42.46805734	0.014430905
19	14.250	401	28.14	3.337204521	47.55516442	0.087932129
20	13.950	346	24.80	3.210959267	44.79288177	0.031729124
21	13.000	319	24.54	3.200241745	41.60314269	0.044351139
22	13.250	315	23.77	3.168575086	41.98361989	0.107505825
23	13.650	284	20.81	3.035234717	41.43095388	0.681336727
24	10.780	294	27.27	3.305887202	35.63746404	0.0240531
25	13.340	385	28.86	3.362476294	44.85543376	0.143801441
26	11.700	355	30.34	3.412528948	39.92658869	0.277038007
27	13.390	339	25.32	3.231491948	43.26967718	0.00987656
28	11.800	328	27.80	3.324914077	39.23398611	0.051811585
29	12.270	324	26.41	3.273586257	40.16690337	0.002737016
30	12.850	335	26.07	3.26078672	41.90110936	5.86198E-04
31	11.995	330	27.51	3.314602758	39.75866008	0.03755171
32	12.350	310	25.10	3.222916234	39.80301549	0.015770509
33	12.670	357	28.18	3.338498787	42.29877964	0.080779987
34	12.690	311	24.51	3.19897863	40.59503882	0.045186253
35	10.275	227	22.09	3.095236257	31.80355254	0.274387056
36	12.900	394	30.54	3.419123598	44.10669441	0.332194295
37	13.638	386	28.30	3.342977355	45.59152517	0.096979215
38	12.979	353	27.20	3.30313539	42.87139423	0.025683784
39	12.684	308	24.28	3.189758426	40.45889588	0.060200409
40	12.790	296	23.14	3.141695839	40.18228978	0.174947757
41	13.188	355	26.92	3.292810464	43.4255844	0.015388784
42	12.500	333	26.64	3.282413846	41.03017307	0.007058486

case	gallons	miles	mpg	ln(mpg)	b*e	sigma
43	10.060	284	28.23	3.340407073	33.60449516	0.067241809
44	13.800	393	28.48	3.34914102	46.21814607	0.113000842
45	11.480	306	26.66	3.282978711	37.6885956	0.006794366
46	11.744	268	22.82	3.127644509	36.73105711	0.201558364
47	10.465	244	23.32	3.149131869	32.95566501	0.125521521
48	13.330	349	26.18	3.265054788	43.52318032	0.000546665
49	13.900	335	24.10	3.182241692	44.23315951	0.081153243
50	13.170	316	23.99	3.177800698	41.85163519	0.086089006
51	10.164	236	23.22	3.14497974	31.96557407	0.131330328
52	11.460	289	25.22	3.227563977	36.98788317	0.011074887
53	11.860	303	25.55	3.240561412	38.43305835	0.003880931
total	18011		689.149	3.25865		
miles				0872		
total	689.15		53	26.0144	sigma:	0.349
gallons				1668		
avg mpg	26.13513188					

APPENDIX 2M
VANPOOL DIARY

case	gallons	miles	mpg	ln(mpg)	b*e	sigma
1	27.3	301	11.03	2.400223563	65.52610326	0.010300973
2	22.9	226	9.87	2.289398089	52.42721623	0.191308271
3	17.9	200	11.17	2.413516654	43.2019481	0.019161307
4	17.6	227	12.90	2.557051115	45.00409963	0.546742445
5	28.6	287	10.03	2.306075498	65.95375924	0.159689756
6	23.1	252	10.91	2.38959647	55.19967846	0.001787955
7	25.7	278	10.82	2.381130122	61.19504413	2.82279E-05
8	25.4	294	11.57	2.448830593	62.20029707	0.117559775
9	25.3	288	11.38	2.432156084	61.53354894	0.06673078
10	25.4	276	10.87	2.385651692	60.59555297	0.000598207
11	20.9	233	11.15	2.411289295	50.39594626	0.019430227
12	25.7	291	11.32	2.426832275	62.36958947	0.054460599
13	28.6	307	10.73	2.37344103	67.88041345	0.001548273
14	27.3	285	10.44	2.345602478	64.03494765	0.033818544
15	26.3	289	10.99	2.396857749	63.0373588	0.006782582
16	26.1	285	10.92	2.390553866	62.3934559	0.002483758
17	25.0	283	11.32	2.426571073	60.66427682	0.052377738
18	27.3	293	10.73	2.373285907	64.79070526	0.001540871
19	19.9	232	11.66	2.45601764	48.87475103	0.11259197
20	28.9	310	10.73	2.372730702	68.5719173	0.001881179
21	25.7	269	10.47	2.348220388	60.34926396	0.027276614
22	22.0	279	12.68	2.540169328	55.88372523	0.558777892
23	29.0	309	10.66	2.366045447	68.61531796	0.006312101
24	24.7	295	11.94	2.480172113	61.26025118	0.243914324
25	21.3	233	10.94	2.392331381	50.95665841	0.002832955
26	25.0	276	11.04	2.401525041	60.03812602	0.010739523
27	27.2	310	11.40	2.433355324	66.18726482	0.075131787
28	27.3	288	10.55	2.356073778	64.32081414	0.016689093
29	26.7	297	11.12	2.409068573	64.32213091	0.021338249
30	26.6	270	10.15	2.317510743	61.64578577	0.106542744
31	28.1	315	11.21	2.416803062	67.91216606	0.036426413
32	32.5	358	11.02	2.399292897	77.97701915	0.01111614
33	26.1	283	10.84	2.383511583	62.20965232	0.000192088
34	25.0	293	11.72	2.461296784	61.5324196	0.161998512
35	26.1	291	11.15	2.411387953	62.93722557	0.024421822
36	29.3	313	10.68	2.368615675	69.40043926	0.00434889
37	22.6	234	10.35	2.337371209	52.82458933	0.042622414
38	26.6	299	11.24	2.419532358	64.35956071	0.039907866
39	25.6	291	11.37	2.430730916	62.22671144	0.063826573
40	26.6	296	11.13	2.409448239	64.09132315	0.021833165
41	28.2	303	10.74	2.374410828	66.95838534	0.001150701
42	30.2	330	10.93	2.39125073	72.21577205	0.003299193

case	gallons	miles	mpg	ln(mpg)	b*e	sigma
43	26.7	291	10.90	2.388659702	63.77721404	0.001649933
44	31.2	338	10.83	2.382627801	74.33798738	0.000104382
45	23.8	254	10.67	2.367648686	56.35003873	0.004115568
46	25.9	290	11.20	2.415637954	62.56502302	0.031436725
47	23.2	281	12.11	2.494202391	57.86549546	0.298361177
48	23.5	274	11.66	2.456127685	57.7190006	0.133349693
49	31.8	348	10.94	2.39273619	76.08901084	0.004531612
50	28.3	306	10.81	2.380723297	67.37446931	1.60930E-06
51	19.2	230	11.98	2.48316903	47.67684537	0.201209915
52	26.9	290	10.78	2.377754636	63.96159972	0.000249265
53	26.9	274	10.19	2.32100182	62.43494895	0.09618546
54	27.4	317	11.57	2.44835876	67.08503004	0.125063488
55	28.7	302	10.52	2.353529895	67.54630798	0.021340979
56	21.6	224	10.37	2.338952737	50.52137912	0.037823439
57	26.3	285	10.84	2.382920241	62.67080234	0.000118374
58	31.1	360	11.58	2.448896212	76.1606722	0.144219106
59	27.6	307	11.12	2.409031975	66.48928251	0.022000441
60	32.4	359	11.08	2.405163966	77.92731249	0.019234774
61	22.8	234	10.26	2.328560579	53.09118121	0.06221714
62	25.4	288	11.34	2.428211306	61.67656718	0.057098047
63	31.2	328	10.51	2.352595514	73.40098002	0.024817107
64	16.5	177	10.73	2.372789352	39.1510243	0.001058471
65	27.0	282	10.44	2.346070205	63.34389553	0.032563858
66	55.0	584	10.62	2.362567798	129.9412289	0.018280132
67	56.7	585	10.32	2.333837636	132.328594	0.125042897
68	23.7	238	10.04	2.306795626	54.67105633	0.129792007
69	25.9	277	10.69	2.369774537	61.37716052	0.003147687
70	29.4	324	11.02	2.399748841	70.55261594	0.010557764
71	29.8	311	10.44	2.345284519	69.88947866	0.037585475
72	27.8	284	10.22	2.323938217	64.60548245	0.089880603
73	17.3	182	10.52	2.353300186	40.71209321	0.013081718
74	26.6	278	10.45	2.346709898	62.42248328	0.030910447
75	25.8	276	10.70	2.370026374	61.14668044	0.002993913
76	15.8	175	11.08	2.404776034	37.99546134	0.009083613
77	22.5	228	10.13	2.31583032	52.10618219	0.094970054
78	23.9	252	10.54	2.355550629	56.29766002	0.015235424
79	25.2	273	10.83	2.382627801	60.04222058	8.43087E-04
80	29.2	289	9.90	2.292257979	66.93393298	0.228912246
81	31.3	314	10.03	2.305774888	72.17075401	0.176174344
82	32.3	338	10.46	2.347978665	75.83971089	0.03479211
83	18.1	189	10.44	2.345835077	42.45961489	0.022126443
84	26.9	307	11.41	2.434721461	65.4940073	0.078216147
85	32.2	332	10.31	2.333168516	75.12802623	0.073050048
86	34.4	370	10.76	2.375446441	81.71535758	0.000985448
87	26.9	273	10.15	2.317345509	62.33659418	0.108307694

case	gallons	miles	mpg	ln(mpg)	b*e	sigma
88	26.4	277	10.49	2.350653496	62.0572523	0.02399057
89	26.0	282	10.85	2.383810533	61.97907386	0.000235849
90	31.2	310	9.94	2.296154203	71.64001112	0.223538392
91	28.3	294	10.39	2.340717963	66.24231834	0.045462989
92	28.1	313	11.14	2.410433614	67.73318456	0.0246782
93	29.4	285	9.69	2.271494506	66.78193847	0.351253804
94	15.6	169	10.83	2.382627801	37.16899369	5.21911E-04
95	26.5	282	10.64	2.364762338	62.66620196	0.006814876
96	26.3	302	11.48	2.440858078	64.19456746	0.094867469
97	43.3	446	10.30	2.332166317	100.9828015	0.102409233
98	19.7	225	11.42	2.435481766	47.9789908	0.05890767
total miles		28372	2620.500		2.380798707	
total gallons		2620.50	98		10.81353626	sigma: 0.256
avg mpg		10.82694142				