A Chaotic System of Two-Phase Flow in a Small, Horizontal, Rectangular Channel*

Y. Cai, M. W. Wambsganss, and J. A. Jendrzejczyk

Energy Technology Division
Argonne National Laboratory
Argonne, Illinois 60439


*This work was supported by the U.S. Department of Energy, Energy Efficiency and Renewable Energy (Division of Advanced Industrial Concepts), and represents a U.S. contribution to the International Energy Agency (IEA) program on Research and Development in Heat Transfer and Heat Exchangers.
DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.
A Chaotic System of Two-Phase Flow in a Small, Horizontal, Rectangular Channel

Y. Cai, M. W. Wambsganss, and J. A. Jendrzejczyk

Energy Technology Division
Argonne National Laboratory
Argonne, Illinois 60439

Abstract

Various measurement tools that are used in chaos theory were applied to analyze two-phase pressure signals with the objective of identifying and interpreting flow pattern transitions for two-phase flows in a small, horizontal rectangular channel. These measurement tools included power spectral density function, autocorrelation function, pseudo-phase-plane trajectory, Lyapunov exponents, and fractal dimensions. It was demonstrated that the randomlike pressure fluctuations characteristic of two-phase flow in small rectangular channels are chaotic. As such, they are governed by a high-order deterministic system. The correlation dimension is potentially a new approach for identifying certain two-phase flow patterns and transitions.

1 Introduction

This study presents an application of chaos theory on two-phase flow in a small, horizontal, rectangular channel. The data analyzed in this study are from previous experiments (Wambsganss et al., 1991, 1992a, 1992b). In the experiments, horizontal two-phase flow was studied in a rectangular channel with a
small cross-sectional area (19.05 x 3.18 mm). Adiabatic flows of air/water mixtures were tested over a large mass flux range (50-2000 kg/m²s). The full quality covered the experimentally achievable range. The two-phase flow patterns and transitions had been identified by dynamic pressure measurements, together with visual observations and photographic data; flow pattern maps were also developed (Wambgsanns et al., 1992a, 1992b).

Usually, the flow pattern maps of two-phase flows are based on visual identification of phase distribution (Clarke and Blundell, 1989; Brauner and Maron, 1992; Galbiati and Andreini, 1992; Koizumi, 1992; Hibiki et al., 1992). Although identification of visual flow patterns may be adequate for some cases, in many situations these methods are not applicable or are too subjective. Several other methods have been developed to more objectively identify and interpret flow patterns and transitions of two-phase flow, e.g., pressure/time signals (Weisman et al., 1979), RMS of pressure/time series and friction pressure gradients (Wambgsanns et al., 1991, 1992a, 1992b), the power spectral density function (PSD), and probability density function (PDF) (Hubbard and Dukler, 1966; Matsui, 1984 and 1986; Tutu, 1982 and 1984; Vince and Lahey, 1982). These studies have contributed to our understanding of flow patterns and transitions of two-phase flows, but there is no accepted method for objectively distinguishing flow patterns.

The purpose of this study is to apply chaos theory to experimental data on dynamic pressure-to-time signals from two-phase flows in an attempt to identify and interpret flow pattern transitions. This new approach may present a promising way to identify flow patterns.

In this study, time history data of dynamic pressure p₁ from previous experiments (Wambgsanns et al., 1991, 1992a, 1992b) were processed with various measuring tools, such as PSD, autocorrelation function, pseudo-phase-plane trajectory, Lyapunov exponents, and fractal dimensions. It was demonstrated that
the randomlike pressure fluctuations characteristic of two-phase flow in small rectangular channels are chaotic. As such, they are governed by a high-order deterministic system. The correlation dimension is potentially a new approach for identifying certain two-phase flow patterns and transitions.

2 Experimental

Experiments on two-phase flow in a small, horizontal, rectangular channel were conducted several years ago (Wambsganss 1991, 1992a, 1992b). The flow apparatus (illustrated schematically in Fig. 1) was designed to allow adiabatic flow experiments with air/liquid mixtures in channels of small cross-sectional area. Air was supplied from a compressed air storage tank, flowed through a pressure regulator and preselected rotameter to an air/liquid mixer. Laboratory water was the liquid in the experiments. The water flowed through a control valve and preselected rotameter to the mixer, where air was injected into the liquid stream through a porous medium in opposing walls of the flow channel. The two-phase mixture then flowed through the transparent channel. The mixture exiting from the channel flowed through an expansion to a drain. A vane-type dry gas meter was utilized to calibrate the gas rotameters, and a weighing-technique-with-stop-watch was used to calibrate the liquid rotameters. The estimated uncertainty in flow rate measurements was ±3%. The flow channel was rectangular, 1.14 m in length, and had cross-sectional dimensions of 19.05 x 3.18 mm (aspect ratio, defined as the ratio of the height of the vertical side of the channel to the width of the horizontal side, was 6).

The measured dependent variable was pressure. Pressure taps, spaced at intervals of 114 mm along the entire length of the channel, were located at the center of the long (vertical) side. Differential pressure, over a specified channel length, and dynamic pressures, at two locations, were measured. Differential
pressure was measured with a strain-gauge-type transducer (Viatran Model 209), and dynamic pressure was measured with piezoresistive-type transducers (Endevco Model 8510B-50). The pressure transducers were calibrated against a known standard. Relative to the exit of the mixer, the pressure taps used with the dynamic-pressure-measuring transducers, $p_1$ and $p_2$, were located at $L/D_h$ of 79 and 142, respectively. The channel length over which the differential pressure measurement was made corresponded to an $L/D_h$ of 132. The estimated uncertainty in pressure measurements was $\pm 5\%$.

The test procedure consisted of establishing total mass flux $G$ and a mass quality $x$ in the test section. As overall test channel pressure drop allowed, tests were performed over a range of quality (typically $10^{-4}-1$) for each mass flux used ($50$-$2000$ kg/m²s). At steady state, visual and photographic observations were made and pressures were measured. Multiple photographs were taken and pressure measurements were recorded and averaged on a computer-controlled data acquisition system.

From experimental results, typical pressure/time history, RMS pressure, and frictional pressure gradient data in the form of two-phase frictional multipliers plotted as a function of both mass quality $x$ and Martinelli parameter $X$, together with visual observations and photographic data, were summarized in previous work (Wambsganss 1991, 1992a, 1992b). Such results were used to identify two-phase flow patterns and transitions and to develop flow pattern maps for two-phase flow in a small, horizontal, rectangular channel.

3 Measuring Chaos

In this study, data on dynamic pressure $p_1$ (see Fig. 1) in flow regimes of plug, slug, and annular flows, with a fixed mass flux $G = 500$ kg/m²s, were investigated with various chaos measuring tools. Typical pressure/time histories,
exhibiting different characteristics corresponding to the different flow patterns (plug, slug, and annular flows), are shown in Figs. 2a-c. It is demonstrated that different flow patterns of two-phase flows in the small channel always exhibit randomlike pressure fluctuation characteristics, even though plug and slug flow are characterized by low frequency, and annular flow is characterized by high frequency.

Various measurement tools that are used in chaos theory were applied in this study to analyze dynamic pressure signals of two-phase flows with the objective of identifying and interpreting flow pattern transitions. These measurement tools included PSD, autocorrelation function, pseudo-phase-plane trajectory, Lyapunov exponents, and fractal dimensions.

The experimental pressure data were obtained from original recorded tapes at a sampling rate of 1000 digitizations per second with a Macintosh computer. The data length of 65,000 points for each test case was utilized to calculate various measures of chaos, especially for the algorithm for computing the fractal dimension.

3.1 Power Spectral Density

Power spectral density makes it possible to distinguish between periodic and chaotic responses. For a chaotic motion, the power spectrum is known to be continuous.

The PSD analysis can give a first indication of the dimensional behavior of a time series. The PSD analysis suggested by Gorman and Robbins (1992) can reveal information complementary to a conventional dimensional analysis or an estimation of Lyapunov exponents. Therefore, this approach should be considered before performing an expensive and time-consuming dimensional analysis like the correlation integral or the "nearest neighbor" approaches.
The characteristics of the power spectrum of chaotic-dynamic systems in the low-to-moderate-frequency regimes are reflected as broad-band structures without dominant peaks. However, at the high-frequency limit of the asymptotic regimes, power spectra decay toward a noise level. That noise level may be determined by instrumentation limits for data originating from measurements. The manner in which the spectrum decays toward the noise level contains useful information on the underlying dynamics. Some researchers have suggested that the asymptotic analysis of power spectra is very useful in distinguishing between high- and low-to-moderate dimensional chaos (Ding and Tam, 1993).

The PSD of plug, slug, and annular flows in a small channel are shown in Figs. 3a-c on a semi-log scale. All of the flow regimes exhibit a broad band of frequencies, which is a characteristic of chaotic behavior. At the high-frequency range for all of the flow regimes (see Figs. 3), the power spectra show a clear power-law falloff by visual inspection. As suggested by Sigeti and Horsthemke (1987), the power-law falloff of the power spectrum should indicate high-dimensional chaos rather than a stochastic process because the data were generated from a deterministic system.

Therefore, the above analysis of the PSD for two-phase flows in the small channel indeed indicates the existence of high-dimensional chaotic behavior, and, as we will see later the high-dimensional attractors from correlation integral estimations. However, it is not possible to distinguish the flow patterns and transitions of two-phase flows from the analysis of power spectra.

### 3.2 Autocorrelation Function

Autocorrelation function is another signal processing tool used to identify chaotic motion. When a signal is chaotic, information about its past origins is lost. This means that the signal is only correlated with its recent past.
The autocorrelation functions of the plug, slug, and annular flows are shown in Figs. 4a-c. The autocorrelation functions for plug, slug, and annular flows have a peak at the origin delay time \( \tau = 0 \), and drop off rapidly with time, thus reflecting chaotic behavior.

It can be observed from Figs. 4 that the plug, slug, and annular flow have different periodic oscillations along the time delay axis, with the amplitude values of autocorrelation functions near zero. However, with limited analysis, we cannot claim that those differences can be used to identify flow patterns.

The importance and contribution of calculating the autocorrelation functions before performing other measurements of chaos is not only in that the autocorrelation function has provided a useful tool to distinguish chaotic behavior but also in that it can provide an optimum choice of time delay \( \tau \) for other measurements of chaos. To estimate the attractor dimension, it is necessary to construct the phase space. The phase space may be reconstructed by using the time-delay embedding method, while the calculated values of the correlation dimension and Lyapunov exponents from reconstructed phase space are very sensitive to the time delay \( \tau \). Our experience has indicated that it is very useful and practical to choose an optimum time delay value from the calculated autocorrelation functions. As shown in Figs. 4, the optimum time delay is estimated to be the smallest value of \( \tau \) at which the first minimum in the autocorrelation function occurs, which is also considered to be the most independent. This is the time delay used in subsequent measurements of chaos.

3.3 Pseudo-Phase-Planes

To obtain the fractal dimension of a strange attractor, many methods assume that the dimension of the phase space wherein the attractor lies is known, or that all the stable variables can be measured. However, in our experiments, the time
history of only one state variable is available, i.e., dynamic pressure $p_1(t)$. Also, the number of degrees of freedom, or minimum number of significant modes contributing to the chaotic dynamics of two-phase flows, are not known a priori. In this case, pseudo-phase-space (embedding space) trajectories of the motion, or a strange attractor, can be reconstructed with the time delay embedding method (Packard et al., 1980). Given a scalar time series, one can reconstruct $D$-dimensional vectors from the following equation:

$$x_i = \{x(t_i), x(t_i + \tau), x(t_i + 2\tau), \ldots, x[t_i + (D - 1)\tau]\}$$

where $D$ is the embedding dimension (the minimum dimension of the subspace), and $\tau$ is the time delay. The reconstructed geometrical structure has the same dimensional characteristics, for example, the correlation dimension of the attractor generated from the dynamics underlying the original scalar time series.

Figures 5a-c present pseudo-phase-plane plots of the measured dynamic pressure for plug, slug, and annular flows, respectively. It is obvious from these three flow patterns that the pseudo-phase-plane plots, indeed, indicate chaotic behavior. However, the difference of pseudo-phase-plane signatures among the three flow patterns is not sufficiently clear to allow one to distinguish flow patterns.

### 3.4 Lyapunov Exponents

Chaos in dynamics implies that the outcome of a dynamic process is sensitive to changes in initial conditions. If one imagines a set of initial conditions within a sphere or radius $\varepsilon$ in phase space, the chaotic motion trajectories originating in the sphere will map the sphere into an ellipsoid whose major axis grows as $d = \varepsilon e^{\lambda t}$, where $\lambda > 0$ is known as a Lyapunov exponent. Lyapunov
exponents, the average exponential rates of divergence or convergence of nearby orbits in phase space (Moon, 1987), have been shown to be the most useful dynamic diagnostic tool in quantifying chaotic systems.

Several investigators of chaotic dynamics have developed algorithms to calculate the Lyapunov exponent. For regular motion $\lambda \leq 0$; but for chaotic motion, $\lambda > 0$. Thus, the sign of $\lambda$ is a criterion for chaos. Any system containing at least one positive Lyapunov exponent is defined as chaotic.

We used the algorithms proposed by Wolf et al. (1985) to determine the Lyapunov exponents from a time series of absolute pressure for different flow patterns. Figures 6a-c show Lyapunov exponents for plug, slug, and annular flows. The exponents are positive, which shows that the attractor had an expanding direction, constituting convincing evidence for chaotic behavior of plug, slug, and annular flows. The results provide further evidence of chaotic behavior in two-phase flows in small channels.

3.5 Correlation Dimensions

Another approach for predicting chaotic motion quantitatively is the use of fractal dimensions. A noninteger fractal dimension of the orbit in a phase space implies the existence of a strange attractor. The basic idea is to characterize the "strangeness" of the chaotic attractor. Although practical use of fractal dimensions in measuring and characterizing chaotic motion has yet to be fully established, various definitions have been developed, including the capacity dimension, correlation dimension, and information dimension.

Of the several methods available to estimate the attractor dimensions, we used measurement of the correlation dimension, which has been used successfully by many investigators in other fields (Moon, 1987). In particular, we used
Grassberger and Procaccia's method (1983) to calculate the correlation dimensions of measured pressure data of two-phase flow.

The correlation function is defined as: 
\[ C(r) = \lim_{N \to \infty} \frac{1}{N} \] (number of pairs of points on the attractor whose distance is less than r). Here, N is the number of D-dimensional vectors constructed from the scalar time series. For some range r, called the scaling region, \( C(r) \) scales as \( r^{d_c} \), where \( d_c \) is the correlation dimension (Grassberger and Procaccia, 1983). A saturation of \( d_c \) as the embedding dimension D increases indicates that the time series has a nonrandom component.

Correlation integral analysis is applied to the experimental data of pressure in two-phase flows. Figures 7a-c show the correlation functions \( C(r) \) versus \( \log_2 r \) for plug, slug, and annular flows, respectively, with embedding dimensions increasing from 5 to 30 in intervals 5. The correlation dimensions at different embedding dimensions are determined by the slopes of the least-squares fitted straight line of the correlation functions. The estimated correlation dimensions versus the embedding dimensions for plug, slug, and annular flows are plotted in Fig. 8, which shows that the correlation dimensions tend to saturate at values of \( d_c \approx 6.6 \) as the embedding dimension reaches 15 or greater for the plug flow, at values of \( d_c \approx 5.4 \) as the embedding dimension reaches 20 or greater for the slug flow, and at values of \( d_c \approx 9.3 \) as the embedding dimension reaches 20 or greater for the annular flow. The results from Figs. 7 and 8 provide evidence that two-phase flow in the small channel is deterministic chaotic motion, not a random oscillation (the dimension of random oscillation will increase as the embedding dimension increases). It appears that the correlation dimension may provide a new tool for the identification of two-phase flow patterns and transitions in a small, horizontal, rectangular channel.
4 Conclusions

Various measurement tools that are used in chaos theory were applied to analyze two-phase pressure signals with the objective of identifying and interpreting flow pattern transitions. These tools included power spectral density function (PSD); autocorrelation function, pseudo-phase-plane trajectory, Lyapunov exponents, and fractal dimensions.

Application of the pseudo-phase-plane trajectory, the PSD and autocorrelation functions, indeed, indicated the existence of chaotic behavior in the two-phase pressure measurement. In particular, the pseudo-phase-plane trajectory displayed complicated patterns, the PSD was continuous, and the autocorrelation function showed a peak at $\tau = 0$ and then dropped off rapidly. The Lyapunov exponents were shown to be $>0$ in quality ranges corresponding to the various flow patterns of interest. This provides the most convincing evidence for chaotic behavior and distinguishes two-phase flow in a small, horizontal, rectangular channel as a deterministic chaos rather than a random system. However, the results obtained with the various measurement tools of chaotic motion, such as the PSD function, autocorrelation function, pseudo-phase-plane trajectory, Lyapunov exponents, cannot be easily adopted as criteria for identifying flow pattern transition.

The theory of fractal dimensions, in the form of correlation dimensions, was used to quantify the identified chaotic behavior. Embedding techniques and an approach to obtain optimum time delay were employed to calculate the correlation dimensions. In general, high dimensions were found for two-phase pressure signals in ranges corresponding to the various flow patterns of interest.

It appears that fractal dimensions offer a promising way to objectively classify flow patterns. However, the time for data processing is such that it is not a practical tool for online analysis. The software we used for estimating dimensions
is not well developed. Algorithms should be improved so that more data points can be involved and processing time can be reduced. Then, not only will the measurement of chaos be more accurate, but the method of estimating fractal dimension will be easily applied to broad practical purposes.

It was demonstrated that the randomlike pressure fluctuations characteristic of two-phase flow in small rectangular channels are chaotic. As such, they are governed by a high-order deterministic system. The correlation dimension is potentially a new approach to identifying certain two-phase flow patterns and transitions. However, more work will be required before objective techniques are available for the identification and classification of two-phase flow patterns and transitions. We hope this study will stimulate future research relating to the application of chaos theory in this important area.

Acknowledgments

This work was supported by the U.S. Department of Energy, Office of Energy Efficiency and Renewable Energy (Division of Advanced Industrial Concepts), and represents a U.S. contribution to the International Energy Agency (IEA) program on Research and Development in Heat Transfer and Heat Exchangers.

References


Figure Captions

Fig. 1  Schematic illustration of adiabatic two-phase flow apparatus

Fig. 2  Time histories of dynamic pressure $p_1(t)$ for (a) plug, (b) slug, and (c) annular flows

Fig. 3  PSD of dynamic pressure $p_1(t)$ for (a) plug, (b) slug, and (c) annular flows

Fig. 4  Autocorrelation functions of dynamic pressure $p_1(t)$ for (a) plug, (b) slug, and (c) annular flows

Fig. 5  Pseudo-phase planes of dynamic pressure $p_1(t)$ for (a) plug, (b) slug, and (c) annular flows

Fig. 6  Lyapunov exponents of dynamic pressure $p_1(t)$ for (a) plug, (b) slug, and (c) annular flows

Fig. 7  Correlation dimensions of dynamic pressure $p_1(t)$ for (a) plug, (b) slug, and (c) annular flows

Fig. 8  Correlation dimensions $d_C$ vs. embedding dimension $d_E$ for plug, slug, and annular flows
(a) $G = 500 \text{ kg/m}^2\text{s}$, $x = 0.00130$

(b) $G = 500 \text{ kg/m}^2\text{s}$, $x = 0.00266$

(c) $G = 500 \text{ kg/m}^2\text{s}$, $x = 0.03991$
Fig. 3
Fig. 4
(a) $G = 500 \text{ kg/m}^3\text{s}, \ x = 0.00130$

(b) $G = 500 \text{ kg/m}^3\text{s}, \ x = 0.00266$

(c) $G = 500 \text{ kg/m}^3\text{s}, \ x = 0.03991$
Fig. 6
Figure 7

(a) $G = 500 \text{ kg/m}^3\text{s}, \ x = 0.0013$

(b) $G = 500 \text{ kg/m}^3\text{s}, \ x = 0.0044$

(c) $G = 500 \text{ Kg/m}^3\text{s}, \ x = 0.03991$
\[ G = 500 \, \text{kg/m}^2\text{s} \]

- \( x = 0.00130 \)
- \( x = 0.00266 \)
- \( x = 0.03991 \)

Correlation Dimension \( d_c \)

Embedding Dimension \( d_E \)