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Magnetic Bearing Update

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Abstract

Stabilization of whirl instability by floppy, viscous bearing mounts is discussed and required material properties are estimated for the new tilt-whirl mode in eddy-current stabilized magnetic bearings. A relatively low Young's modulus $Y \sim 10^5$ and high viscosity $\eta \sim 10^7$ are required (both in MKS units), suggesting the need for careful mounting design. New information on periodic bearings shows that, thus far, Earnshaw's Theorem cannot be defeated by periodicity, despite the author's earlier claims.

Two topics identified in Ref. 1 have led to further work involving students in Mechanical Engineering at Berkeley. One result concerns the forces due to eddy-currents in eddy-current stabilized bearings.² Recently, these forces were shown to produce a tilting whirl-type instability,³ later confirmed by Ryutov.⁴ The other subject is periodic bearings.⁵

Review of Eddy-Current Stabilized Bearing

In Ref. 3, it was shown that a whirl instability involving tilt motion may exist in eddy-current stabilized magnetic bearings, and that this tilt-whirl would be more difficult to stabilize than the usual translational whirl instability encountered in many mechanical systems. Recently Ryutov has recovered these results in a more reliable way by reformulating the stability analysis for small angular perturbations in the proper way to linearize the equation.⁴ His equations for tilting motion, for a single rotor midway between identical axisymmetric bearings, are:

$$I_1 \ddot{\theta}_x + I_3 \ddot{\theta}_y = 2K\ell^2 \theta_x + 2K_w \ell^2 \theta_y - 2R_f \ell^2 \dot{\theta}_x \quad (1)$$

$$I_1 \ddot{\theta}_y + I_3 \ddot{\theta}_x = -2K\ell^2 \theta_y + 2K_w \ell^2 \theta_x - 2R_f \ell^2 \dot{\theta}_y \quad (2)$$

and for translation,

$$M\ddot{x} = -2Kx + 2K_w y - 2R_f \dot{x} \quad (3)$$

$$M\ddot{y} = -2Ky - 2K_w x - 2R_f \dot{y} \quad (4)$$

Here M is the rotor mass; I_3 is the principal moment of inertia (about z , the rotor axis); I_1 is the other moment, about x, y ; K, K_w and R_f are the bearing restoring force, whirl force and friction per bearing (hence multiplied by 2); Ω is the rotor speed (angular frequency); and 2ℓ is the length of the shaft between bearings. As is noted in Ref. 3, we are most interested in magnetic bearings stabilized by eddy-currents and mounted on "floppy" bearings. Then, effectively, the bearings and shaft are rigidly attached so that K and R_f are actually those of the mounting. There is, however, a residual bearing whirl force, shown in Ref. 3 to be:

$$K_w = (K_w)_{\text{bearing}} \left(\frac{K_{\text{mount}}}{K_{\text{bearing}}} \right)^2 \quad (5)$$

Since there is no slip relative to the mounting, the mounting friction itself does not produce a whirl torque.

The results of Ref. 3. are easily recovered by letting θ 's vary as $\exp pt$, and (for different p 's) x, y also, translation and tilt being decoupled.^{3,4} Defining frequencies in units of $(2K\ell^2/I_1)^{1/2}$ for tilt and $(2K/m)^{1/2}$ for translation, both tilt and translation eigenfrequency equations can be written as:

$$(\omega^2 + c\omega + 1)^2 = -(b - a\omega)^2 \quad (6)$$

where $c = R_f/K$ (a reciprocal frequency), $b = K_w/K$ and $a = (I_3\Omega/2K\ell^2)$ (a frequency) in the appropriate frequency units for the two cases. Solving Eq. (6) gives:

$$\omega = -\frac{c \pm i a}{2} \pm \sqrt{\left(\frac{c \pm i a}{2}\right)^2 - 1 \pm i b} . \quad (7)$$

For translational whirl, $a = 0$ and $b, c \ll 1$ so, expanding with -1 as the large term, we obtain

$$\omega = \pm i + \frac{1}{2} (b - c) . \quad (8)$$

Then, recalling that $\omega \propto p$ is the Laplace (not the Fourier) frequency, stability requires,

$$c > b . \quad \text{Translation} \quad (9)$$

This is Eqs. (36) and (53) in Ref. 3.

For tilt-whirl, $a > 1$, $b, c < 1$ so, expanding with a as the large term gives a stable root $\omega = -i a - c$ for the negative sign, and for the positive sign

$$\omega = i a^{-1} + a^{-1} \left(b - \frac{c}{a} \right) . \quad (10)$$

Hence stability if

$$c' > b' a' . \quad \text{Tilt} \quad (11)$$

which is Eq. (52) of Ref. 3, and we have added primes to remind us that the frequency units in Eqs. (9) and (11) differ by $(I_1/M\ell^2)^{1/2}$, which is order unity. When the system is tilt-whirl unstable, the growth constant from Eq. (10) is b'/a' , which is Eq. (31) of Ref. 3. However, the implication in Ref. 3 that the mode is purely growing is incorrect since in Eq. (10) the real part is smaller by the factor $b < 1$.

Comparing Eqs. (9) and (11) shows that, in agreement with Refs. 3 and 4, the tilt-whirl is the more difficult to stabilize, by the large factor a' (though its growth rate in the absence of frictional stabilization would be less than that for translational whirl by this same large factor). Note also that, the growth constant b for translational whirl is actually the absolute value from $(b^2)^{1/2}$ when $a = 0$, and the tilt-whirl growth rate b'/a' depends on the relative sign for K_w and Ω . As Ryutov has pointed out,⁴ this means that in principle the tilt-whirl can be eliminated by design. The unstable case is the unusual one shown in Fig. 1, taken from Ref. 2, with the magnetic element inside and the eddy-current element outside. Inverting this arrangement should give tilt-whirl stabilization independent of mounting friction while the translational whirl can be stabilized either by the mounting friction or by clamping the bearing (oval shape) to create an asymmetry in the restoring force.^{1,3}

Frictional Stabilization of Tilt-Whirl

The work of Ref. 2 provides numbers whereby the tilt-whirl frictional stabilization criterion, Eq. (11), can be evaluated for a specific case. This work, from the Ph.D. thesis of W. Lorimer at U.C. Berkeley,² provides values for K_w and K for the magnetic bearing itself. The commercial code utilized for the calculations could presumably be used to calculate these numbers for other bearing designs as well.

Restoring the units, Eq. (11) becomes:

$$\Omega < \omega_0 (K_b/K_{wb}) (K_b/K_m) . \quad (12)$$

The code calculates K_b/K_{wb} , while the other factor is a frequency relating rotor and mounting parameters:

$$\omega_0 = \frac{R_m \ell^2}{I_3} \simeq \frac{R_m}{M} . \quad (13)$$

The factor $K_b/K_m \gg 1$ for a "floppy" bearing, and Lorimer finds $K_b/K_{wb} \simeq 3$ (see Fig. 2). Hence, the stability requirement would be

$$\Omega < 3 \frac{K_b}{K_m} \omega_o \quad (14)$$

The coefficient in Eq. (14) could be increased another order of magnitude or more by an improved eddy-current stabilization design. Then

$$\Omega < 100 \omega_o \quad (15)$$

where the larger coefficient corresponds to the improved design.

From Eq. (13), we can see that ω_o is the effective frequency appearing in the viscous frictional force, $R_m \dot{x}$, for the mounting:

$$R_m = \eta \frac{A}{d} \equiv \eta t_m \quad (16)$$

where A is the area of surfaces moving relative to each other and d is their separation distance, the ratio being roughly the thickness t_m of the mounting material; and η is the coefficient of viscosity of the material. Combining Eqs. (12) - (16) gives us the required viscosity for stabilization

$$\eta > \left(\frac{K_{wb}}{K_b} \right) \left(\frac{K_m}{K_b} \right) \left(\frac{M\Omega}{t_m} \right) \equiv 10^7 \text{ (MKS)} = 10^8 \text{ poise}, \quad (17)$$

valid for $\Omega = 2 \cdot 10^6$ (200,000 RPM) and a typical rotor mass of 4 - 5 kg and the mounting thickness $t_m = .01$ m. The range of values corresponds to that in Eq. (15), the lower value of η corresponding to the higher value of K_b/K_{wb} for the improved bearing design.

To be "floppy" we also require $K_m \ll K_b = \frac{\alpha(Mg)}{dx}$ or $K_m \sim \frac{Mg}{dx} \cong 10^3$ to 10^4 for a displacement $dx \ll 1$ cm, $M = 4$ kg, $\alpha = 0.3$ to 3 , $g = 10$ (gravity, MKS). The spring constant is related to the Young's modulus by

$$K_m = Y_m \frac{A_m}{t_m} \quad (18)$$

or, for typical dimensions, $Y_m \cong 10^5 - 10^6$ (MKS: newton/m²).

Thus we would like a mounting having effective bulk properties of a Young's modulus around 10^5 and viscosity around 10^7 in MKS units. Solid rubber has $Y = 10^6$ and a highly viscous fluid like pitch has $\eta = 10^{10}$, suggesting that a spongy synthetic, or a composite with springy wires and a viscous matrix, might be required.

To conclude this section, we note that it is not clear that the tilt-whirl instability has ever been observed. Attempting to do so would seem to be a high priority as a test of Ryutov's theory, and as a prudent investigation into future problems if the tilt-whirl theory is correct.

Periodic Bearings

We conclude this note with an acknowledgment that, despite implications to the contrary in Ref. 1, we have in fact not found any periodic bearing system that is stable without the aid of eddy-current damping. So far as we know now, at best periodic bearings would be neutral with no net restoring force, or unstable. The case in Ref. 1 that was claimed to be stable was in fact a case of neutral forces, as the drawing for this case clearly indicated despite the wrong statements in the text (see Fig. 4A of Ref. 1). As usual, the issue revolves around the ratio of lateral (to the shaft) and axial forces, a ratio 1:2 being the origin of Earshaw's theorem often cited as the proof that no stable bearing exists based on the frozen-in fields of permanent magnets above. In Ref. 1 these ratios were estimated very crudely. However, Thomas Chang, a U.C. Berkeley graduate student, has carried out accurate numerical calculations for various cases in Ref. 1, always finding the fixed 1:2 ratios for circular magnets and corresponding values for non-circular ones that lead to the

negative conclusion reported above, in agreement with the actual drawings in Ref. 1 (but not the optimistic text, for which the present author takes full responsibility). One of the more interesting of Mr. Chang's results concerns an array of four circular magnets mounted "vertically" on the rotor (magnet face parallel to the axis), offset axially from a corresponding fixed set. The results are shown in Table 1, which gives effective spring constants obtained from the numerically calculated forces due to a displacement. The Ks are given in units relative to K_y , and d is the diameter of the array. The combination of forces for the array gives (similar to the drawings in Ref. 1):

$$K_{\text{lateral}} = 2(K_x - K_y) \quad (17)$$

$$K_{\text{axial}} = 4 K_z \quad (18)$$

As can be seen from the table, within the accuracy of the calculation $K_{\text{axial}} = 2 K_{\text{lateral}}$ for all cases.

Table 1.

Z-offset	Kx	Ky	Kz	$K_{\text{axial}}/K_{\text{lateral}}$
0.0*d	2.0	1.0	1.0	2.0
0.1*d	1.94	1.0	0.54	2.0
0.2*	1.77	1.0	0.76	2.0
0.3*d	1.54	1.0	0.53	2.0
0.4*d	1.27	1.0	0.27	2.0

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4. T. K. Fowler, R. F. Post, and D. D. Ryutov, "Linear Stability of Two Elastically Coupled Rotors (axisymmetry) System," in preparation.
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Figure 1. Radial flux passive eddy-current bearing

Figure 2. Effect of velocity on moderate displacement stiffness

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