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To: HIF Note Mailing List
From: Dave Judd
Subject: Comments on Steering Errors and Corrections ...

COMMENTS ON STEERING ERRORS AND CORRECTIONS

IN A SMALL RECIRCULATOR

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July 1994

I. COMMENTS ON EFFECTS OF ERRORS

Analyses of the effects of steering errors (discrepancies in bending fields) often tend to become somewhat elaborate, making it unnecessarily difficult for everyone to have a good "feel" for the magnitudes of the physical quantities involved. We try to overcome this tendency by dealing with a simpler problem.

Particles in an ideal ring tend to oscillate in the transverse directions (betatron oscillations) relative to the desired path (optic axis, or reference trajectory). The simplest version of this motion (considering the in-plane transverse coordinate x) results from applying the smooth approximation which replaces the actual focusing forces of alternating sign (alternating-gradient or AG focusing) by a constant restoring force that produces the same average frequency, measured by σ , the phase advance per spatial period $2L$ of the structure. [L is the half-lattice-period (HLP).] In terms of the radian wave number $k = 2\pi/\lambda$ (with λ the wavelength of the oscillation),

$$x(s) = a \cos(ks + \delta)$$

with a and δ constants defining the amplitude and phase of the oscillation, and s the distance along the reference path. The phase advance in a distance λ is 2π , so $\sigma = 2\pi(2L/\lambda) = 2kL$. [σ may refer to one particle alone (σ_0), to the behavior of one particle in the presence of many others (usually denoted by σ), or to the coherent motion of the entire beam in the presence of steering errors, a "coherent" $\sigma = \sigma_c$, which may differ from σ_0 because of wall image effects that are important for intense beams.]

Let us ask what happens if $a = 0$ (no oscillation) upstream of a certain station s_0 at which a one-time error "kick" ΔF_x is applied. The equation of motion will be $\Delta F_x = m a_x$, or

$$m d^2x/dt^2 = mv^2 d^2x/ds^2 = \Delta F_x,$$

using $ds = v dt$. If the kick is due to a bending-field error ΔE (or ΔB for a magnetic bending field) then

$$\Delta F_x/mv^2 = (\Delta F_x/F_0)/\rho = (\Delta E/E)/\rho$$

with $F_0 = mv^2/\rho$ the desired bending force and ρ the radius of curvature in the bend. If a fraction f of the ring's circumference is devoted to bending, then $\rho = fR$ with R the radius of the ring.

We imagine the error kick to be applied over the length fL of a bending element, assuming that every HLP contains one bending element filling a fraction f of its length. Then

$$(\Delta F_X/mv^2) fL = (\Delta E/E)(fL/fR) = (L/R)(\Delta E/E).$$

We take the error force to be applied locally, over a distance short enough that the oscillation's phase does not change much; that is, $fL \ll \lambda$. This requirement is the same as $f \ll 4\pi/\sigma_0$. [For the small ring described in John Barnard's Note # 26, an approximate value of f is $\approx 1/3$, and a typical value of σ_0 is ≈ 1 radian, so the requirement becomes $1 \ll 4\pi/f \approx 40$, which is well satisfied.] During this kick, the displacement x will not change from zero, but the slope will change;

$$\begin{aligned} \int_{\text{kick}} d^2x/ds^2 ds &= (dx/ds)_{\text{after}} - (dx/ds)_{\text{before}} = x'_{\text{after}} \\ &= (\Delta F_X/mv^2) \int_{\text{kick}} ds = (\Delta F_X/mv^2) (fL) = (L/R)(\Delta E/E). \end{aligned}$$

Note that we use a prime for d/ds . For distances beyond s_0 , the slope will be

$$x'(s) = x'_{\text{after}} \cos [k(s-s_0)]$$

and the displacement will be

$$x(s) = (x'_{\text{after}}/k) \sin [k(s-s_0)],$$

corresponding to the new "initial conditions" just after s_0 : $x = 0$, $x' = x'_{\text{after}}$. Therefore the kick has induced an oscillation of amplitude

$$a = (L/R)(\Delta E/E)/k;$$

using $k = \sigma_0/2L$,

$$a = (2L/\sigma_0) (L/R)(\Delta E/E).$$

Let us find the numerical magnitude of this oscillation for the small-ring parameters in Barnard's Note # 26: $2\pi R/L$ is the number $N = 40$ of LHP's in the ring, $L = 36$ cm, and σ_0 is about one radian on the average, so

$$a = [(4\pi \times 36 \text{ cm})/(40 \times 1)] (\Delta E/E) = 11 \text{ cm} \times (\Delta E/E).$$

From this we learn that (subject to our approximations) a fractional error of 1 % in a single bending field will induce an oscillation of amplitude 1 mm. A simple-minded estimate of the effect of applying M random (uncorrelated) errors with 1 % rms gives a rms displacement of $M^{1/2}$ mm. After 15 laps of 40 errors

each, $M = 40 \times 15 = 600$, and $x_{rms} = 2.7$ cm. Along the way, extreme values exceeding the rms by a factor of three have a non-negligible probability, giving with some odds a maximum displacement of 8 cm.

With a pipe radius of 3.5 cm and a maximum beam radius of 1.4 cm, the largest tolerable displacement would seem to be about 2 cm, showing that the assumed rms fractional bending field error of 1 % is intolerably large by a factor of about four; it should not exceed $(0.01/4) = \underline{0.0025}$. At this stage we consult the last entry on page 4 of Note # 26, finding

$$(dE/E)_{rms} \text{ (dipole field strength)} = \underline{0.00244}.$$

The degree to which this close agreement is coincidental is not clear, but at the least our confidence in the qualitative validity of these simple analytic estimates is strengthened.

[We must point out that the kicks will be multiplied by a sinusoidal factor; the square root of the average of its square is $2^{-1/2}$, so that our simple-minded estimate is pessimistic by this factor; but, on the other hand, the estimate in Note # 26 is no doubt based on a more accurate treatment of AG oscillations, and may depend on other considerations as well.]

Proceeding more systematically, we assume that every bending field has an error, whose combined effect at a distance s is

$$x(s) = (4\pi L/N) \sum_{n=1}^M (1/\sigma_{0n})(\Delta E_n/E_n) \sin [(\sigma_{0n}/2)(s/L - n)]$$

in which N is the number of half-periods in the ring and M is the largest integer less than s/L . We assume that the errors at different bending elements have statistically independent values, and also that the errors at each bend are statistically independent on each successive turn, because the bending fields vary with time according to prescribed functions which will not be perfectly realized. Notice that the effects of errors on the later laps are more than twice as serious as those on early laps because σ_0 falls from about 1.4 to about 0.6 radians during acceleration. By squaring this equation and making the usual assumptions one obtains the conventional estimates about rms values. The uncertainties in our crude estimate are due not only to our cavalier treatment of AG oscillator waveforms but also to uncertainties in the form of the distribution function from which the individual random bending errors are assumed to be drawn. The Gaussian distribution (defined by its standard deviation) is often assumed, but a distribution of errors uniform over a symmetrical range, and zero outside it (defined by the magnitude of its boundaries) may better represent the result of efforts to limit errors in real electrical or mechanical systems. It is well known that the half-width of this distribution is $3^{1/2}$ times its standard deviation. The factor $3^{1/2} \approx 2$ may not seem very important to a theorist, but it may loom very large in the problems of an engineer seeking to make a real-world system meet a tolerance.

Real systems do not exist in an abstract world of exactly symmetric error distributions. It is of interest to examine the importance of a limited succession of errors of the same size and sign; such an episode might be termed a "local systematic error region" to distinguish it from random uncorrelated errors. We assume the succession of identical error kicks to persist only over a region Δs within which the sine factor in the equation above does not change sign; that is, $\sigma_0 \Delta n / 2 = \pi$, or $\Delta n = 2\pi / \sigma_0$. For $\sigma_0 = 0.6$ radian, Δn is about ten kicks. Then the sum of the trigonometric factors is

$$\sum_{n=1}^{10} \sin(\pi n / 10) \approx 6,$$

and

$$\Delta x = (4\pi L / \sigma_0 N) (6 \Delta E / E) \approx 50 (\Delta E / E) \text{ cm.}$$

It is our perception that an rms error of 0.2 % may be unattainably small, perhaps by an order of magnitude, in a practical system whose design is tightly constrained by a minimal budget. For a reasonable estimate $\Delta E / E = 2 \%$, this Δx is a centimeter, the same order of magnitude as the total allowable displacement. If there is approximately one such episode per lap and fifteen laps, one might expect $\Delta x = 15^{1/2} =$ four times this large if the individual episodes are uncorrelated. Even for 1 % errors the displacement is intolerably large, as we also found for individual random errors. Thus we are forced to contemplate the problem of steering the beam, which will no doubt require solution in any power plant driver, and in intermediate-scale systems as well.

II. COMMENTS ON STEERING

Employing our idealized simple-harmonic model of transverse oscillations, imagine that an unknown set of error kicks upstream of a certain station at $s = s_a$ has led to a displacement there of

$$x(s_a) = A \cos \psi,$$

beyond which we assume that no further error kicks occur. Measuring stations are to be established both at s_a and at another station s_b further along by a distance $\Delta s_m = s_a - s_b$; $m =$ "measuring".

We abbreviate $k\Delta s_m$ as p_m . It is considered practical to measure the displacements x_a and x_b , but not the slopes. [The displacements considered are of the beam centroids, but we may think of a single particle having $\sigma = \sigma_{\text{coherent}}$ of a beam.]

Let the phase between x_a and the point of maximum displacement be denoted by ψ . Then

$$x_a = A \cos \psi \quad \text{and} \quad x_b = A \cos (p_m - \psi).$$

Solving for A and ψ ,

$$A = [x_a^2 - 2x_ax_b \cos p_m + x_b^2]^{1/2} / [\sin p_m],$$

$$\tan \psi = [x_b/x_a - \cos p_m] / [\sin p_m], \quad \cos \psi = \pm x_a/A.$$

Attention must be paid to the signs. We choose to represent the displacement due to this oscillation, for $s > s_a$, by specifying two constants, A and ψ , in the form

$$x(s) = A \cos [k(s-s_a) - \psi];$$

$x(s)$ attains a maximum at $s = s_m$, where $k(s_m - s_a) = \psi$. If the maximum occurs upstream from s_a , $s_m - s_a$ is negative, so ψ is taken as negative. Similarly, if the maximum occurs for $s_m > s_a$ we take ψ to be positive. [These assignments are arbitrary because we could have chosen the argument of $x(s)$ to be $k(s - s_a) + \psi$.] Further, $k\Delta s - \psi$ is taken as positive for $s_m < s_b$, and negative for $s_m > s_b$. All these signs are irrelevant in the equations

$$x_a = A \cos \psi, \quad x_b = A \cos (p_m - \psi)$$

because the cosine is an even function, but a consistent set of choices must be made, as we now explain. The phase angle ψ is determined only by specifying its tangent. For a given value of x_b/x_a , and thence for $\tan \psi$, the solutions ψ and $\psi \pm \pi$ all satisfy the equation defining $\tan \psi$, while an ordinary calculator will always give the "principal value", lying between $-\pi/2$ and $+\pi/2$. It is evident that on passing from a positive value of x_a to a negative value for a fixed positive value of x_b this principal value of ψ will jump from $+\pi/2$ to $-\pi/2$, although the physical situation has not changed. Similarly, in passing from a negative value of x_a to a positive value for a fixed negative value of x_b the principal value will again jump from $+\pi/2$ to $-\pi/2$. We denote this "twice-jumping" function of principal values by ψ_p on Fig. 1 below. The situation is clarified by representing x_a and x_b as Cartesian coordinates in a plane. Because the physical problem is linear, we may consider the point with these coordinates to lie on a circle of arbitrary radius. [See Fig. 1(a).] As the point traverses this circle in the counterclockwise direction, starting from $x_b = 0$, $x_a > 0$, one sees that in the first quadrant x_a and x_b are both positive; in the second, $x_b > 0$ and $x_a < 0$, etc. To eliminate the discontinuities just described we could invoke two rules: (1) to the principal values of ψ in the second and third quadrants, add π ; and (2) to the principal values in the fourth quadrant add 2π . As a result the only

discontinuity in ψ will occur when crossing the starting line at $x_a > 0$ and $x_b = 0$, where ψ just below this line will exceed ψ just above it by 2π . We denote this function with a single jump at the starting line by ψ_0 on Fig. 1. The phase angle ψ is, of course, only defined *modulo* 2π , so we expect a remaining element of arbitrariness in specifying a particular range of this angle. As adjusted, it will increase by 2π in going counterclockwise once around the circle just described, dropping discontinuously by this amount at some azimuth, chosen to be the starting line in the description above; but we may choose this azimuth as we wish. Because of the greater symmetry to be shown below, we suggest rotating this azimuth of discontinuity backward by 135° , placing it at the bisector of the third quadrant where $x_a = x_b$ with both negative. We denote this function by ψ_c . To accomplish this we replace the former two rules by these: (1) to the principal values in the second quadrant and the first (upper) half-sector of the third quadrant add π ; And (2) to the principal values in the second (lower) half-sector of the third quadrant subtract π . With these rules, ψ will have a discontinuous decrease of 2π at 225° and will be continuous elsewhere. The three functions (ψ_p , ψ_0 , and ψ_c) are graphed vs. the polar angle μ in Fig. 1(b).

FIGURE 1(a)

FIGURE 1(b)

The algebraic relations associated with the various regimes of the parameter space (x_a and x_b as Cartesian coordinates) turn out to be unexpectedly complicated. We display some of the connections in the graph of Fig. 2 and the sketches of Fig. 3.

In Fig. 2 we have again used x_a and x_b as Cartesian coordinates, and have used the rules leading to the angle ψ_c [shown in the right part of Fig. 1(a) and graphed in Fig. 1(b)] which undergoes a discontinuous decrease of 2π when passing the third-quadrant bisector in the counterclockwise direction. Here and in Fig. 3 we have replaced the angle $p_m - \psi$ by φ , so that

$$x_a = A \cos \psi, \quad x_b = A \cos \varphi.$$

It must be remembered that φ is not an independent variable, but only an abbreviation. However, the values of ψ and φ are found to be interchanged by reflection across the line with rotation angle $\mu = 45^\circ$ (and 225°), displaying the symmetry referred to above. The values of ψ and φ are marked on Fig. 2 along the coordinate axes, along the line $\mu = 225^\circ$, and along the first-quadrant lines $p_m = x_b/x_a$ where ψ changes sign and $p_m = x_a/x_b$ where φ changes sign.

The purpose of Fig. 3 is to aid in visualizing the various possible relations among the positions at which x_a and x_b are measured and that at which the maximum value A is attained. It consists of a number of sketches of displacement vs. angle ks , on which are shown ψ , $\varphi = p_m - \psi$, and $k \Delta s$ for different sectors of the plot of x_b vs. x_a in Fig. 2. In every sketch the relation

$$\varphi + \psi = k \Delta s = p_m$$

is illustrated, although in many of them φ or ψ is a negative angle so that $-\varphi$ or $-\psi$ is the labeled angle.

Consideration must be given to the spacing $s_b - s_a = \Delta s_m$ between the two measuring stations. As it is increased a larger number of error kicks will occur during the measurement process, lowering its accuracy. In particular, the scheme fails if p_m is near to π ($\sin p_m \rightarrow 0$). On the other hand if the spacing is too small relative to the wavelength of the oscillation being measured the error in determining its slope (and thence one of its two orthogonal components) will be too large. The value $p_m = \pi/2$ is optimum. Of course, it is not possible to use this value because the spacing must for practical reasons be a multiple of the HLP, but it will not be a multiple of the phase advance per half-period (which in any case will be changing during acceleration). If σ_0 ranges from 1.36 to 0.63 radian per period (as in Recirculator Note # 26) the wavelength goes from about 8 to about 4 periods (16 to 8 HLP's), so a spacing of 4 HLP's is unusable near the end of acceleration. One could choose 3 HLP's, but if many measurements are to be made in each turn this will not divide evenly into 40. Perhaps a reasonable practical compromise spacing is 3 HLP's. In preparing Figs. 2 and 3 the restriction $p_m < \pi/2$ has been used.

$$\psi = \psi_c$$

$$\phi = k\Delta S - \psi$$

FIGURE 2.

$\psi = \phi = \frac{RAS}{2}$
 $\mu = \pi/4$

MIDDLE LINE: $x_a/x_b < 0$.

BOTTOM LINE: $x_a < 0, x_b < 0$.

The information gained from such a pair of measurements will contain inaccuracies, both from the measuring process and from steering errors at bending elements within the distance Δs , of which there might be four if $\Delta s = 3$ HLP's as suggested above. Even if it were exact, what is to be done with it?

If we had complete freedom to locate a special correction kicker at a station of our choice for each error, we would choose one at which the error oscillation has zero displacement, and simply straighten it out with a kick that would cancel its slope.

However, correction kicks can be applied only at fixed stations where regular bending elements or special correction kickers are located, so we must consider a two-kick method for undoing error oscillations.

- If the first correction kick is to be applied at a station $s = s_c$, let us suppose that the error oscillation displacement and slope there are

$$x(s_c) = C \cos k s_c + D \sin k s_c, \quad x'(s_c) = -k [C \sin k s_c - D \cos k s_c].$$

The parameters C and D which define the "correction regime" are related to the parameters A and φ of the "measurement regime" defined above as follows:

$$C = A \cos (k s_a + \psi), \quad D = A \sin (k s_a + \psi);$$

it will be recalled that when evaluating ψ careful consideration must be given to the signs and relative magnitudes of x_a and x_b .

We apply a kick at s_c which increases the slope there by $\Delta x' = \theta_c$, to be determined. Then

$$x^*(s_c) = -k [C \sin k s_c - D \cos k s_c] + \theta_c = x'(s_c) + \theta_c ,$$

the asterisk denoting a value after application of the kick at s_c .

Proceeding downstream from s_c ,

$$x(s) = x(s_c) \cos k (s - s_c) + [x^*(s_c)/k] \sin k (s - s_c) ,$$

$$x^*(s) = -k x(s_c) \sin k (s - s_c) + x^*(s_c) \cos k (s - s_c) .$$

Our second kick will be applied at $s = s_d$, where

$$x^*(s_d) = x(s_c) \cos k (s_d - s_c) + [x^*(s_c)/k] \sin k (s_d - s_c) = x(s_d) + (\theta_c/k) \sin k (s_d - s_c) ,$$

$$x^*(s_d) = -k x(s_c) \sin k (s_d - s_c) + x^*(s_c) \cos k (s_d - s_c) .$$

At this station we choose θ_c to make $x^*(s_d)$ vanish, and we choose the second kick, θ_d , to cancel the slope $x^*(s_d)$. The expressions for the required kicks θ_c and θ_d may be written in several forms. The abbreviation p_r (r = "reckoning") for $k(s_c - s_d)$, the phase between the measuring and correcting intervals simplifies some of them. Explicitly, with $k(s_d - s_c)$ denoted by p_c (c = "correcting"), the first kick is required to be

$$\theta_c = -x'(s_c) - k x(s_c) \cot p_c ,$$

which may also be written as

$$\theta_c = -[k/\sin p_c] x(s_d) ,$$

or alternatively as

$$\theta_c = -[k/\sin p_c][C \cos k s_d + D \sin k s_d]$$

$$= [kA/\sin p_c] \cos [k(p_m + p_r + p_c) - \psi]$$

$$= [k/\sin^2 p_c][-x_a \sin (p_r + p_c) + x_b \sin (p_m + p_r + p_c)] .$$

For the second correction kick

$$\theta_d = k x(s_c)/\sin p_c ,$$

which may also be written as

$$\theta_d = -x^*(s_d) + k x^*(s_d) \cot p_c ,$$

or alternatively as

$$\theta_d = [k/\sin p_c][C \cos k s_c + D \sin k s_c]$$

$$= [kA/\sin p_c] \cos (p_m + p_r - \psi)$$

$$= [k/\sin^2 p_c] [-x_a \sin p_r + x_b \sin (p_m + p_r)] .$$

As mentioned above, the final result at $s = s_d$ will not be the perfect absence of unwanted oscillation; at every bending element from s_a to s_d another random error kick will occur. If $s_b - s_a = \Delta s$ is equal to $3L$, as suggested above, and if the same interval is suitable for $s_d - s_c$, this amounts to eight error kicks, to which must be added those in the open interval $s_c - s_b$. The choice of this distance will depend on the time required after receiving measurements to calculate the corrections and apply them. It seems plausible that this may require a similar interval of $3L$, and perhaps more, since the time to pass one HLP at the final energy is about $0.3\mu\text{sec}$ and the operation may well require at least one microsecond. Thus a number of uncompensated error kicks N_e of order ten or more may contaminate each correction operation. They will contribute a root-mean-square displacement of order

$$\Delta A_{\text{rms}} = 2^{-1/2} N_e^{1/2} (4\pi L/\sigma_0 N) (\Delta E/E)_{\text{rms}} .$$

With $L = 36$ cm, $\sigma_0 = 0.6$ (near the end of acceleration), $N = 40$, $N_e = 10$, and $\Delta E/E_{\text{rms}} = 2\%$, this number is 0.84 cm. We also need to consider the contributions to this amplitude from errors of measurement at s_a and s_b and errors in applying corrective kicks at s_c and s_d . Combining all these errors in quadrature,

$$A_{\text{rms}}^2 = (1/2) \{ [N_e (\Delta E/E)_e^2 + (\Delta E/E)_c^2 + (\Delta E/E)_d^2] [4\pi L/\sigma_0 N]^2 + (\Delta x_a)^2 + (\Delta x_b)^2 \} ,$$

in which N_e is the number of kick errors in $s_a \leq s \leq s_d$; the ratios $(\Delta E/E)_c$ and $(\Delta E/E)_d$ are the rms errors in the applied correcting fields as fractions of the desired bending fields; and Δx_a and Δx_b are the rms errors in measuring oscillation displacements at s_a and s_b . We use the parameters above, 2% for all the $(\Delta E/E)_{\text{rms}}$ values, $N_e = 10$, and $(\Delta x_a)_{\text{rms}}$ and $(\Delta x_b)_{\text{rms}} = 2$ mm. Then

$$A_{\text{rms}} = 2^{-1/2} \{ (4\pi \times 36 \text{ cm} / 0.6 \times 40) [12 \times (2 \times 10^{-2})^2] + 2 \times (0.2)^2 \}^{1/2}$$

$$= 2^{-1/2} [1.84 + 0.08]^{1/2} = 1 \text{ cm} .$$

Evidently, for the parameters assumed here the most important contribution is that of the random error kicks occurring between the start of the measurement and the end of the correction. Further, the amplitude thus induced will be approximately proportional to σ_0^{-1} .

To estimate the number of measurement and correction operations that will be needed we consider the rms amplitude induced by M error kicks:

$$A_{\text{rms}} = 2^{-1/2} (4\pi L/\sigma_0 N) M^{1/2} (\Delta E/E)_{\text{rms}} .$$

For the parameters above,

$$A_{\text{rms}} = 2^{-1/2} (4\pi \times 36 \text{ cm} / 0.6 \times 40) (2 \times 10^{-2}) M^{1/2} = 0.27 M^{1/2} \text{ cm} .$$

If we go for a full turn between corrections this value will be $0.27 \times 40^{1/2} = 1.7$ cm, which is clearly too large, considering that a value of twice A_{rms} has significant probability. Choosing $M = 10$ HLP's gives $A_{rms} = 0.85$ cm, but requires tightly spaced correction operations. [These numbers are pessimistic by about a factor of two at the start of acceleration when σ_0 is about 1.36 radian.] With four corrections per turn and 15 turns there will be 60 corrections; the probability of a residual random error after a correction operation exceeding twice the rms value (estimated here as of order 1 cm) is non-negligible.

It has been pointed out by Alex Friedman that the loss of effectiveness from choosing Δs_m and Δs_c smaller than optimum may be more than compensated by the introduction of fewer error kicks during a cycle of measuring and correcting. To illustrate this effect we consider the error in amplitude A after such a cycle owing to errors in the correcting kicks θ_c and θ_d . The errors in displacement and slope at s_d due to an error in θ_c are

$$\Delta x^*(s_d) = (\Delta\theta_c/k) \sin p_c, \quad \Delta x'^*(s_d) = \Delta\theta_c \cos p_c;$$

the error in slope at s_d due to an error in θ_d is

$$\Delta x'^*(s_d) = \Delta\theta_d.$$

The resulting oscillation, for $s < s_d$, is

$$\Delta x(s) = k^{-1} [\Delta\theta_c \sin k(s - s_c) + \Delta\theta_d \sin k(s - s_d)].$$

It can be shown with a little calculation that the amplitude of the error oscillation composed of these two components is

$$(\Delta A)^2 = (\Delta\theta_c)^2 + 2(\Delta\theta_c)(\Delta\theta_d) \cos p_c + (\Delta\theta_d)^2.$$

Each of the two angular errors will have terms in the two position measurement errors Δx_a and Δx_b ;

$$\Delta\theta_c = (k/\sin p_c) [-\Delta x_a \sin (p_r + p_c) + \Delta x_b \sin (p_m + p_r + p_c)],$$

$$\Delta\theta_d = (k/\sin p_c) [-\Delta x_a \sin p_r + \Delta x_b \sin (p_m + p_r)].$$

After inserting these expressions, the error amplitude will be of the form

$$(\Delta A)^2 = C_{11} (\Delta x_a)^2 + 2C_{12} (\Delta x_a)(\Delta x_b) + C_{22} (\Delta x_b)^2.$$

If the Δx 's are statistically independent random variables drawn from Gaussian distributions with the same rms value, the rms value of the product $\Delta x_a \Delta x_b$ will vanish, so that $\Delta A_{rms} = (C_{11}^2 + C_{22}^2)^{1/2} \Delta x_{rms}$. Collecting all the terms,

$$\begin{aligned} \Delta A_{rms}/\Delta x_{rms} = & (\sin^{-1} p_c) \{ \sin^2 p_r + \sin^2 (p_m + p_r) + \sin^2 (p_r + p_c) + \sin^2 (p_m + p_r + p_c) \\ & + 2[\sin p_r \sin (p_r + p_c) + \sin (p_m + p_r) \sin (p_m + p_r + p_c)] \cos p_c \}^{1/2}. \end{aligned}$$

The three arguments occurring here are $p_m = k(s_b - s_a)$, $p_r = k(s_c - s_b)$, and $p_c = k(s_d - s_c)$. Because $k = \sigma/2L$ and we may consider the s -values to be separated by multiples of L , the values of the p 's are integers times $\sigma/2$. The smallest possible values of the integers n_m and n_c are unity; other possible values mentioned above are 2, 3, and 4. The integer n_r may be larger. The ratio $\Delta A_{rms}/\Delta x_{rms}$ above is graphed vs. the integer n_r for three values of n_m (1, 2, and 3) assuming that $n_c = n_m$, and $\sigma = 0.63$ radian, the value at the end of acceleration in Note #26, in Figure 4(a), and again for $\sigma = 1.36$ radian, at the start in Figure 4(b).

FIGURE 4

Notice that for $\sigma = 0.63 = \pi/5$, in Figure 4(a), the angle $k(s_a - s_b) = p_m = (\sigma/2)n_m$ is exactly 90° , the value recommended as optimum above, for $n_m = 5$. For this choice of measuring interval the ratio $\Delta A_{rms}/\Delta x_{rms}$ is exactly $2^{1/2}$ for all values of n_r , as expected when two independent measurements contribute. For the least-favored choice ($n_m = 1$) the ratio is over five times as large for the reasonable range n_r from 2 to 6.

For the other value, $\sigma = 1.36 = \pi/2.3$ in Figure 4(b), the choices $n_m = 2$ (for 78°) and $n_m = 3$ (for 117°) bracket the optimum angle. For them the ratio is within about 30 % of $2^{1/2}$ for the range of values of n_r shown. For the other values, $n_m = 1$ and 4, the ratio is larger by a factor of up to about 2.5.

All of the discussion above has been based on consideration of a sequence of random error impulses having average value zero. There is also the possibility of additional error impulses that are uniform around the orbital path. The tolerances on these are perhaps tighter than the rms value of 2 % that we assumed above; for the parameters of Note # 26, a uniform error of 2 % will displace the orbit by 1.6 cm. If the 2 % is regarded as an rms value for a random (temporally) distributed error, one would expect some values twice as large as the rms, which are intolerable.

Such errors may be regarded in three ways: (1) they result in a reference path of a different radius; (2) the particle energy needs to have a different value to give a reference path of the desired radius; or (3) the position detectors all have the same zero-offset error. Ignoring the last possibility, a technical problem exists in determining whether it is more practical to correct the particle energy or to correct all the bending fields by the same amount. To implement either choice, information on the uniform component of error kicks will be needed, requiring three rather than two measuring stations for each measurement operation. Let us locate them at distances s_a , s_b , and s_c with equal separations $k \Delta s_m$, where displacements Δx_a , Δx_b , and Δx_c are measured. We could represent them in terms of A and ψ as in the treatment above and the new parameter ξ which measures the uniform displacement, and perform a more elaborate analysis similar to the one above:

$$\begin{aligned}x_a &= A \cos [-k \Delta s_m - \psi] + \xi \\x_b &= A \cos [-\psi] + \xi, \\x_c &= A \cos [k \Delta s_m - \psi] + \xi.\end{aligned}$$

By inverting these equations we find:

$$\begin{aligned}\tan \psi &= (x_c - x_a)(1 - \cos k \Delta s)/(2x_b - x_a - x_c), \\ \cos \psi &= (2x_b - x_a - x_c)/2A(1 - \cos k \Delta s), \\ \sin \psi &= (x_c - x_a)/2A.\end{aligned}$$

$$\begin{aligned}A^2 &= (x_a - 2x_b + x_c)^2/(1 - \cos k \Delta s) + (x_a - x_c)^2; \\ \xi &= (x_a + x_c - 2x_b \cos k \Delta s)/2(1 - \cos k \Delta s).\end{aligned}$$

We will not pursue this topic further here.

Finally some comments are in order on the differences between our approximate results and accurate ones that do not depend on the smooth approximation. The correct response to a kick is described in terms of the beta-function for the actual A.G. betatron orbits, well known to accelerator designers. This function may be described as the local radian wave length of the oscillation; $\beta(s) = \lambda(s)/2\pi$. The effect at distance s of an angular kick θ applied at distance s_0 is to produce a displacement

$$x(s) = \theta [\beta(s_0)\beta(s)]^{1/2} \sin\left[\int_{s_0}^s \beta^{-1}(s) ds\right]$$

for $s > s_0$, compared with the simpler equation for the smooth approximation;

$$x(s) = [\theta/k] \sin k(s - s_0).$$

The beta function for an orbit in a ring is the closed periodic solution of a certain nonlinear second-order differential equation which depends on the lattice function $K(s)$ in the betatron equation $d^2x/ds^2 + K(s)x = 0$. In the smooth approximation K is constant and $\beta = \text{constant} = k^{-1} = K^{-1/2}$. If $K(s)$ is piecewise constant, it can be shown that $\beta(s)$ is a sinusoidal function plus a constant in F regions, a hyperbolic function plus a constant in D regions, and a parabolic function in the drift regions between them. In the thin-lens limit of a symmetrical FODO lattice, this function consists of symmetric sections of parabolas.

In Figure 5 are graphed the beta-functions for three lattices which have the same phase advance per period, 1.36 radians. One is the smooth-approximation constant; the second displays the parabolas for the thin-lens FODO lattice; and the third is for a FODO lattice very similar to that in Note # 26. For this lattice the positions of electrodes providing the bending kicks are approximately midway between F and D regions, where all three beta functions are nearly equal. Therefore the largest actual displacements induced by error kicks, which occur in the F regions, will be larger by the factor $(\beta_F/\beta_D)^{1/2} \approx (86/55)^{1/2} = 5/4$ than estimated from the smooth approximation. On the other hand, the measuring stations are projected to be located inside the quadrupoles, where the beta function is a maximum or a minimum; here, when a detailed evaluation is needed, there is room for improvement over our approximate treatment.

None of the comments, assumptions, methods, or numerical values in this paper are presented as being accurate or conclusive. Rather, they are offered with the intention of stimulating discussion of the physical issues, drawing attention to some of the relevant numerical magnitudes, and indicating where the greatest concentration of effort may be needed and where the most difficult requirements for precision may be found. Obviously they are only scratching the surface of the problems of steering a beam in a small recirculator.

FIGURE 5

To: HIF Note Distribution
 From: Joshua Rintamaki, U. of Michigan Grad Student
 Subject: Evaluation of a Capacitive Beam Position Monitor
 Diagnostic for use on the Heavy Ion Recirculator

I. Introduction

One of the methods being looked at to measure the position of the beam inside the HIF recirculator is a capacitive probe. The probe would consist of a metal plate with a dielectric between the plate and the beam tube wall. (Figure #1) The probe would be a non-invasive way to measure beam offset. It may also be possible to use this probe as a beam current meter, and also for pulse shape analysis.

Figure #1:

II. Project and Goals

My project this summer was to construct working models of this probe, and answer the following:

- (1) What is the physics behind its operation?
- (2) Will it work?
- (3) What configuration works best?
- (4) What precision should it be able to measure? $\left(\frac{\text{mV}}{\text{mm}}\right)$
- (5) What materials can be used?
- (6) What future work should be done with this probe?

III. Theory and Calculations (Partially taken from Recirculator Notes #10, November 19, 1993)

A beam with current $I_b(t)$ flowing with a velocity of $\beta_b c$ down a beam tube will have a charge density of:

$$q_b(t) = \frac{I_b(t)}{\beta_b c}$$

An equal but opposite charge will appear on the inside of the beam tube that this beam is traveling down. For an electrode with length L in a tube of radius R and azimuthal angle ϕ , (Figure #2) this charge is given as:

$$Q_p(t) = - \frac{\phi L I_b(t)}{2 \pi \beta_b c}$$

Figure #2:

If there is a dielectric between the electrode and the beam pipe wall with some dielectric constant κ and thickness D , (Figure #2) a capacitance will exist given by:

$$C = \frac{\phi L R \epsilon \kappa}{D}$$

Where ϵ is the permittivity of free space.

Now with this capacitance, the current flowing into the capacitor is equal to the time derivative of the charge on the electrode (Figure #2):

$$I_s(t) = - \frac{dQ_p(t)}{dt} = \frac{\phi L dI_b(t)}{2 \pi C \beta_b c dt} = \frac{D dI_b(t)}{2 \pi \beta_b c R \epsilon \kappa dt}$$

The capacitor integrates this current, giving a voltage across this capacitor of:

$$V_p(t) = \frac{\phi L I_b(t)}{2 \pi C \beta_b c} = \frac{D I_b(t)}{2 \pi \beta_b c R \epsilon \kappa}$$

Now if two plates are used to measure X-direction offsets, and the beam is off-center on the X-axis, the charge on each plate will be multiplied by this distribution:

$$\left(\frac{2}{\sqrt{A^2 - B^2}} \tan^{-1} \frac{\sqrt{A^2 - B^2} \tan \frac{X}{2}}{A + B} - \frac{2}{\sqrt{A^2 - B^2}} \tan^{-1} \frac{\sqrt{A^2 - B^2} \tan \frac{-X}{2}}{A + B} \right) * \frac{R^2 - x^2}{2X}$$

Where $A = (-x^2 - R^2)$, $B = 2xR$, x = the distance offset, R = the inner radius of the tube, and X = half of the azimuthal angle of the electrode. If the beam is also off-center on the Y-axis, then the charge is ALSO multiplied by this distribution:

$$\left(\frac{2}{\sqrt{A^2-B^2}} \tan^{-1} \frac{\sqrt{A^2-B^2} \tan \frac{Y}{4}}{A+B} - \frac{2}{\sqrt{A^2-B^2}} \tan^{-1} \frac{\sqrt{A^2-B^2} \tan \frac{p-Y}{4}}{A+B} \right) * \frac{R^2-y^2}{Y}$$

Where $A = (-y^2 - R^2)$, $B = 2yR$, y = the distance offset, R = the inner radius of the tube, and Y = the azimuthal angle of the electrode. These distributions model the change in the electric field on the capacitor due to change in position of the beam center.

Now for the actual experiments, a "simulated" beam was used. This beam consisted of a copper rod with a pulse generator attached. In order to model the geometry of the rod, the signal voltage was multiplied by this approximation:

$$\frac{1}{\ln \frac{R_{out}}{R_{in}}}$$

where R_{out} is the Inner radius of the tube, and R_{in} is the radius of the rod. This equation came from looking at textbook modeling of cylindrical capacitors, and should be regarded as an approximation only.

IV. Experiments and Data Collected

Five different setups of the diagnostic were created during this project. The schematic for equipment setup is given as Figure #3. The end-view of each setup is given as Figure #4:

Figure #3:

Figure#4:

Setup #1:

Setup #1 was a very rough design just to see if the theory worked in reality. The diagnostic was set up using a 1 7/8 " ID pipe, with 30 mil teflon dielectric and 20 mil, ~ 135 degree copper plates. An actual beam could not be obtained, so a "simulated" beam consisting of a copper rod hooked up to a pulse generator was used. Two different diameter rods were looked at: 1/2" and 1/4". 5 Volts was run through a 50 Ohm termination resistor on the rod, generating a 100 mA current. The tube was fixed and the rod was attached at one end to an X-Y translational stage with 2 " of movement on the Y-axis and 1 " of movement on the X-axis.

Analysis #1:

The plots of the data from this setup showed some linearity of voltage versus position for 1cm offset from the center of the tube (Figures #5-#8), and so it was shown that the idea would work, but that further work was needed for precision. The center ± 10 mm was decided on as the region of interest due to its linearity. The 1/2 " rod generated a larger signal voltage, but since all it was doing was shortening the distance between the input pulse and the plates due to its larger diameter, the 1/4 " rod was decided upon as the one to use. At the center of the tube, the top plate gave 48 mV, the bottom plate 44 mV. Calculations estimated the signal voltage to be ~ 49.7 mV at the center.

Figure #5:

Figure #6:

Figure #7:

Figure #8:

Setup #2:

Setup #2 looked at the possibility of having 4 plates all in the same diagnostic. The diagnostic was set up using a 2 3/4 " ID pipe, with 30 mil teflon dielectric and 20 mil, 60 degree plates spaced equally along the inside of the tube. (Figure #4) The 1/4" copper rod was used with the same signal setup as setup #1. The translational setup remained the same.

Analysis #2:

The calculated voltage for this setup with a correction for the simulated beam gave an answer of 10.7 mV. Measured center voltages gave 10 mV, 9.5 mV, 9.5 mV, and 11 mV. These values were close enough to the calculated values to show that the calculations were modeling reality well, considering that all the plates were hand-fitted into place. Plots of the voltages observed for each pair of plates were made (Figures #9-10) and compared to calculated voltages (Figure #11) for the 20 mm x 20 mm center region of interest. The behavior was similar, and so the next step was to have a setup with the plates more exactly positioned using a printed PC board.

Figure #9:

Figure #10:

Figure #11:

Setup #3:

Setup #3 had the same geometry as setup #2 except that the plates were now printed as a PC board on 5 mil Kapton. The 1/4" copper rod was used, except a 10 Volt signal was used, generating a 200 mA current. The translational setup remained the same.

Analysis #3:

The top and left plates for this setup read higher than the bottom and right for no real apparent reason. The device was rotated, and the readouts from those positions remained the same. I have no explanation for those differences. The calculated voltage for this setup with a correction for the simulated beam gave an answer of 4.36 mV. Measured center voltages gave 10mV, 11 mV, 7 mV, and 6.5 mV. These voltages did not correspond well with the calculated voltage. Due to the high probe capacitance of the thin dielectric Kapton, the measured signal was very low. The next step was to look at making the plates a full 180 degrees and with a "double" dielectric to increase the signal and try and reduce the effect of the transverse offset on the measured signal.

Setup #4:

Setup #4 had another 4 plate configuration, in an attempt to remove some of the geometry effects that the shapes of the plates were having on the measured signal. The diagnostic was set up using a 2 3/4" ID pipe, with a 30 mil teflon dielectric and the plates printed on 5 mil Kapton. The 1/4" copper rod was used, with the same signal setup as setup #3. The tube was fixed and the rod was attached at both ends to X-Y translational stages, with 1 inch of movement on both axes.

Analysis #4:

This setup gave the highest signals of all, and also raised some questions on what outside signals could be affecting the plates. The calculated voltage for this setup with a correction for the simulated beam gave an answer of 27.70 mV. Measured center voltages gave 31mV, 32.5 mV, 32.5 mV, and 29.5 mV. These voltages came pretty close to what was calculated, and plots of the voltages observed for each pair of plates, (Figures #12-13) were compared to calculated voltages (Figure #14) for the 20 mm x 20 mm center region of interest. These show that the near 180 degree plates do not cancel out the effect of transverse offset on the measured signal, but that the effect is reversed; i.e. for a 60 degree plate the edges had lower voltages, but for the 180 degree plate the

edges had higher voltages. The "double" dielectric raised the signal to much more acceptable levels, and made it easier to read output on the oscilloscope. With what time was remaining, the next step was to look at altering the azimuthal length of the plates to see if a plate size somewhere between 60 and 180 degrees would minimize the effects of transverse offset on the measured signal.

Figure #12:

Setup #4 - Top-Bottom - ROI

Figure #13:

Setup #4 - Right-Left - ROI

Figure #14:

Setup #4 - Calculated - ROI

Setup #5:

Setup #5 was a two plate setup, looking at different size azimuthal length plates to see the geometry effects that the shapes of the plates were having on the measured signal. The diagnostic was set up using a 2 3/4 " ID pipe, with a 30 mil teflon dielectric and the plates printed on 5 mil Kapton. The plates were cut from the near 180 plates to 160, 150, 140, and 130 degree plates. The 1/4" copper rod was used in the same setup as setup #4. The translational setup remained the same.

Analysis #5:

The different azimuthal length of the plates does affect the transverse offset on the measured signal. It would seem that a maximized length is less than 130 degrees but greater than 60, it is probably around 100 degrees. There was not enough justification to cut up the rest of the PC board plates to keep lowering the length by 10 degrees to see when the crossover would happen. Since it is not known how the diagnostic will react in an actual vacuum with a beam, trying to maximize the signal for a rod is not worth the effort, since a beam is what will actually be measured.

V. Final Analysis + Suggestions for Future Work:

After all five setups are looked at, this project gives definite proof-of-principle but it needs much more work before it can be used as a useful diagnostic on the HIF recirculator. All of the work done with the diagnostic models was somewhat rough and needs to be refined for better accuracy of the diagnostic. Listed below are some of the things necessary for future work to have the diagnostic be a useful tool:

- 1) The diagnostic needs to be manufactured in a tube that is the exact size as that which will be on the recirculator. The closest model was mounted in a 2.75 inch ID tube, but where the diagnostic would be mounted is a 2.70 inch ID tube. If accuracy is to be measured in the sub-millimeter range, the sample diagnostic must be exactly to scale.
- 2) The diagnostic needs to be fitted onto an actual beam line. In all of the setups, there is the possibility of error in the alignment of the tube itself. If the tube was mounted onto a beam line, this error could be completely removed.
- 3) An actual particle beam should be run down the center of the tube in a vacuum. This is by far the most important suggestion in this list. There are far too many unknowns on how this diagnostic will react once it is used in vacuum with a beam. The materials used for the dielectric may become charged or have poor vacuum properties, the beam will most certainly act different than the pulsed rod will, and the behavior of the diagnostic in general is not known in the situation in which it is planned to be used. All of the data assembled here should be helpful preliminary work for the diagnostic, but the really useful data will be collected once the diagnostic is used to look at a real beam.
- 4) The size and shape of the plates needs some further looking into. This should be done after the diagnostic has been tested with a real beam, for then it will be known if a near 180 degree plate is the best choice, or if there is some in-between size that will produce a more linear signal over transverse offsets.
- 5) The actual materials used in the diagnostic and the methods of mounting the plates should be looked at more. Currently Teflon and copper plated Kapton were looked at, with the plates being held in by ring tension. How these materials and that method of mounting will work in a vacuum is not known.
- 6) The plates should be impedance matched to get rid of "ringing" in the output signal. Short cables from the plates to the analyzing hardware will also lower the capacitance of the diagnostic, giving it a higher signal voltage.
- 7) A computerized/digital system is needed to acquire the signal outputs and analyze it. In order to calibrate this diagnostic to sub-millimeter dimensions, a more accurate method than oscilloscope readings is needed. Using a CAMAC type setup to integrate the pulses and look at total collected voltage per plate and differences between opposed plates will give a much more precise diagnostic.
- 8) The diagnostic has the ability to replicate the beam waveform. Figure #15 is a digitized polaroid of input and output signals from setup #4 of the diagnostic. With the pulsed rod, the pulse length was exactly seen, and the relative amplitude of the input pulse was also observed. With a beam, different modulated pulses should be sent down the tube to see what the diagnostic sees.

Figure #15:

- 9) The diagnostic should be "calibrated" by using a target at the far end of the beam tube that can be damaged or have a mark left by the impact of the beam. The beam could be moved off center using the steering plates, and by looking at what readings the diagnostic's plates gave, the location of the hit on the target could be matched with the output voltages of the plates. By using a target, a better calibration offset curve (in $\frac{\text{mV}}{\text{mm}}$) would be generated that through calculation alone, for the diagnostic may react differently in reality than it does on paper.

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To: HIF Note Mailing List

From: Steven Lund

Subject: Beam steering with dipole biased electrostatic quadrupoles

Comment – An earlier version of this note was distributed on July 5 as recirculator note 29 (distributed to some incorrectly labeled as recirculator note 28). Although the conclusions of that note along with the final design equations presented were basically correct, it turned out that several results in the intermediate analysis suffered from numerical errors and errors in interpretation. This version corrects those problems and also expands and generalizes the analysis, including several important practical extensions to the theory. This corrected and expanded note supersedes the earlier version.

BEAM STEERING WITH DIPOLE BIASED ELECTROSTATIC QUADRUPOLES, WITH APPLICATION TO THE SMALL RECIRCULATOR MATCHING SECTION

INTRODUCTION

The small recirculator matching section will be constructed from a section of the SBTE electrostatic transport lattice loaned to us from Lawrence Berkeley Laboratory. Because this section has a 15.24 cm (6 in) half lattice period, and the small recirculator has a 36 cm half lattice period, only every other quadrupole will be used towards the end of the matching section. Several of these "extra" quadrupoles will be removed from the lattice to make room for beam diagnostics. Tom Fessenden has suggested that two of these "extra" electrostatic quadrupoles should be left in for use in beam steering – which will be needed to compensate for inevitable beam offsets due to lattice misalignments. This steering will be accomplished by biasing the quadrupole conductors so that they have appropriate dipole field components to deflect the beam and thereby correct the offsets. In this note we derive simple equations that describe the biasing potentials needed to correct a given misalignment of the beam centroid. These equations include a single numerical coefficient calculated from the full three dimensional lattice geometry that relates the biasing potentials to the bending dipole field component. This coefficient enables a concrete connection of the theory to the experimental apparatus and should also render the results presented here accurate. Later notes will include both envelope and detailed three dimensional particle-in-cell simulations of the matching section. In these simulations we will evaluate the influence of these steerers on beam quality, which is an issue of concern not

addressed in this note. Although the analysis presented below is performed specifically in reference to the small recirculator matching section, the results derived should be readily applicable to other machines with beam focusing provided by a lattice of alternating gradient electrostatic quadrupoles. Furthermore, many of the results can be carried over with straightforward modifications to analyze steering voltages required to correct particular beam centroid errors in the main ring of the small recirculator. It is also possible that the theory can be applied to enable a relaxation of the mechanical alignment criteria for linear and recirculating machines by allowing for compensation of these errors through the use of dipole steering fields, thereby offering the possibility of reduced machine fabrication costs.

ELECTROSTATIC POTENTIAL OF A DIPOLE BIASED QUADRUPOLE

In Fig. 1, a diagram of the matching section by S. Eylon and T. Fessenden is presented. The seven electrostatic quadrupoles of the matching section used for beam focusing and envelope manipulations are labeled Q1 - Q7 and the two quadrupoles to be biased for beam steering are labeled S1 and S2. Qualitatively, S1 will produce a dipole bending field to an offset beam centroid, making it converge back towards the machine axis, and S2 will zero that convergence as the beam crosses the axis, thereby correcting the misalignment as the beam exits S2. Several quadrupoles have been removed (between Q3 and Q4, Q6 and Q7, and after Q7) to make room for diagnostics. The full quadrupole lattice (i.e., with no quadrupoles removed) has a half lattice period of $L = 15.24$ cm (6 in), and each quadrupole has an axial length $\ell = 10.16$ cm (4 in), an aperture $a = 2.54$ cm (1 in), and a rod radius of 2.54 cm (1 in).

In an ordinary focusing quadrupole, the quadrupole conductors along the x and y transverse axes of symmetry are biased to potentials V_q and $-V_q$. Because the orientation of the misalignment errors are unknown apriori, the rod biases must be changed to obtain bending dipole field components along both the x and y transverse axes of symmetry. This can be accomplished by adding potentials V_x and V_y to the "quadrupole" conductors in the positive x and y directions and potentials $-V_x$ and $-V_y$ to the conductors in the negative x and y directions. A transverse cross-section of this structure is sketched in Fig. 2. To calculate the fields resulting from this bias configuration, note that the linearity of Maxwell's equations can be exploited to regard the fields as resulting from a superposition of three "quadrupoles," one (quadrupole 1) with the conductors along the y axis grounded and the conductors along the positive and negative x axes at potentials V_x and $-V_x$, another (quadrupole 2) with the conductors along the x axis grounded and the conductors along the positive and negative y axes at potentials V_y and $-V_y$, and a third conventional quadrupole (quadrupole 3) with conductors at potentials $\pm V_q$ with the usual quadrupolar symmetry. This situation is sketched in Fig. 3. The potential ϕ of the full dipole biased quadrupole is given simply the sum of the potentials of the three quadrupoles, i.e., $\phi = \phi_1 + \phi_2 + \phi_3$. We now analyze these three contributions to the potential in turn.

The electrostatic potential of quadrupole 1 (ϕ_1) is symmetric in the azimuthal angle θ and can therefore be multipole expanded (see Appendix A for a review of multipole

expansions) as

$$\phi_1 = V_x \sum_{l,k} m_{l,k}(z) \cos(l\theta) \left(\frac{r}{a}\right)^k. \quad (1)$$

Here, r is the radial coordinate, k and l are integers giving the radial and azimuthal order of the multipole moments $m_{k,l}$. The multipole moments $\{m_{l,k}\}$ are dimensionless and vary with the axial coordinate z . The $\{a^{-k}m_{l,k}\}$ can be interpreted as the usual multipole moments due to a quadrupole 1 configuration with $V_x = 1$. Furthermore, by symmetry, the potential along the axis of symmetry $r = 0$ is zero, and therefore $m_{0,0} = 0$ for all z . Likewise, under the transformation $x \rightarrow -x$ and $y \rightarrow y$ the potential transforms as $\phi \rightarrow -\phi$, whereas $\cos(l\theta) \rightarrow \cos[l(\pi - \theta)] = \cos(l\pi)\cos(l\theta)$. It follows that all even l moments vanish for all z in the expansion (1). Using similar arguments to those employed in recirculator note 24, it is also possible to show as a consequence of Maxwell's equations that for any given value of l , that the moments will only be nonvanishing when $k = l, l + 2, l + 4, l + 6, \dots$. Also, the nonvanishing higher-order moments $m_{l,k}$ with $k > l$ are related to axial derivatives of each "fundamental" moment $m_{l,l}$ (see Appendix A) and consequently, axial integrals of these so-called "pseudo-moments" with $k > l$ will vanish, i.e., $\int_{-\infty}^{\infty} dz m_{l,k>l} = 0$. The axial integrals of the pseudo-moments will also vanish in each half-space, i.e., for a lens axially centered at $z = 0$, $\int_{-\infty}^0 dz m_{l,k>l} = \int_0^{\infty} dz m_{l,k>l} = 0$. In summary, the nonvanishing moments of the expansion (1) with $\cos(\theta)$ azimuthal variation will be $m_{1,1}, m_{1,3}, m_{1,5}, \dots$, whereas the nonvanishing moments with $\cos(3\theta)$ azimuthal variation will be $m_{3,3}, m_{3,5}, m_{3,7}, \dots$, etc.

To obtain the electrostatic potential of quadrupole 2, note from Fig. 3 that quadrupole 2 can be obtained by rotating quadrupole 1 through an angle $-\pi/2$ about the axis of symmetry and replacing V_x with V_y . Therefore, the potential of quadrupole 2 can be expressed as $\phi_2(r, \theta) = (V_y/V_x)\phi_1(r, \theta - \pi/2)$, or

$$\phi_2 = V_y \sum_{l,k} m_{l,k}(z) \cos[l(\theta - \pi/2)] \left(\frac{r}{a}\right)^k. \quad (2)$$

Finally, the potential of quadrupole 3, ϕ_3 , is just the usual quadrupole potential due to conductors biased to potentials $\pm V_q$ with quadrupolar symmetry. We multipole expand ϕ_3 in a manner analogous to Eq. (1) as

$$\phi_3 = V_q \sum_{l,k} q_{l,k}(z) \cos(l\theta) \left(\frac{r}{a}\right)^k. \quad (3)$$

Using similar arguments to those following Eq. (2) it can be shown that the z -dependent multipole moments $\{q_{l,k}\}$ are nonvanishing only for $l = 2, 6, 10, \dots$, and $k = l, l + 2, l + 4, \dots$. Thus, there is no overlap between the $\{m_{l,k}\}$ and the $\{q_{l,k}\}$ moments. Also, as for the $m_{l,k}$, higher order pseudo-moments $q_{l,k}$ with $k > l$ will be related to axial derivatives of each fundamental moment $q_{l,l}$, and consequently axial integral of the pseudo-moments vanish, i.e., $\int_{-\infty}^{\infty} dz q_{l,k>l} = \int_{-\infty}^0 dz q_{l,k>l} = \int_0^{\infty} dz q_{l,k>l} = 0$. Combining these results, the

potential of the full dipole biased quadrupole $\phi = \phi_1 + \phi_2 + \phi_3$ is given by

$$\begin{aligned} \phi = & \sum_{l=1,3,\dots}^{\infty} \sum_{k=l,l+2,\dots}^{\infty} m_{l,k}(z) \{V_x \cos(l\theta) + V_y \cos[l(\theta - \pi/2)]\} \left(\frac{r}{a}\right)^k \\ & + V_q \sum_{l=2,6,\dots}^{\infty} \sum_{k=l,l+2,\dots}^{\infty} q_{l,k}(z) \cos(l\theta) \left(\frac{r}{a}\right)^k. \end{aligned} \quad (4)$$

The dipole component of the dipole biased quadrupole potential is given by the fundamental $l = 1, k = 1$ indexed term of Eq. (4), i.e., $\phi_d = m_{1,1}(V_x x + V_y y)/a$, and the quadrupole component is given by the $l = 2, k = 2$ indexed term, i.e., $\phi_q = V_q q_{2,2}(x^2 - y^2)/a^2$. Clearly, for a good quadrupole with dipole bending field components, all higher-order “quadrupole” multipole coefficients $q_{l,k}$ should be small compared to the fundamental quadrupole moment $q_{2,2}$, and all higher-order “dipole” multipole coefficients $m_{l,k}$ should be small compared to the fundamental dipole moment $m_{1,1}$. The SBTE quadrupoles have been carefully shaped to suppress higher-order multipoles with quadrupolar symmetry, and therefore the higher-order $q_{l,k}$ moments should be small compared to $q_{2,2}$, and we will neglect them in the present analysis. On the other hand, this detailed shaping will not necessarily suppress the higher order $m_{l,k}$ moments, and some analysis of these nonlinear focusing terms will be appropriate. However, it should also be stressed that the symmetry-based conclusions above on which moments are nonvanishing do not depend on the details of the shape of the quadrupole conductors. Provided, of course, that all conductors are of the same shape and are arranged such that they have reflection symmetry about the lines at angles $\theta = \pm\pi/4$ through the origin that divide the transverse quadrants. It is also clear from the analysis above that the leading-order fundamental error term of the potential will be the sextupole component given by the $l = 3, k = 3$ term of Eq. (4), i.e., $\phi_s = m_{3,3}[V_x \cos(3\theta) - V_y \sin(3\theta)](r/a)^3 = m_{3,3}[V_x(x^3 - 3xy^2) + V_y(y^3 - 3xy^2)]$. Finally, it should also be stressed that by symmetry there is no coupling between the dipole and quadrupole potential terms and their related error field components. For example, if $V_q = 0$ then only the fundamental dipole potential term and associated higher-order error terms exist, and no terms with quadrupolar symmetry exist. Conversely, if $V_x = V_y = 0$ and $V_q \neq 0$, then the fundamental quadrupole potential term and associated higher-order error fields exist, and no terms with dipolar symmetry exist. Because of this feature, the quadrupole bias components will not generate dipole and related error field components, and conversely the dipole bias components will not generate quadrupole and related error field components. This independent variability of the quadrupole and dipole field strengths for beam focusing and bending is, of course, a desirable feature in actual experiments.

Because the geometry of the matching section lattice is fixed, it is convenient to calculate the multipole moments $m_{k,l}$ numerically. To do this, we employ the detailed three-dimensional SOR Poisson solver in the particle-in-cell simulation WARP3d to calculate the electrostatic potential; and then extract the multipole moments from the potential using an algorithm based on a least square error fit. More specifically, this algorithm is based on a harmonic azimuthal decomposition of the potential for each azimuthal mode number l , followed by a linear least-square fit to the reduced radial expansion to extract the $m_{l,k}$.

moments with $k = l, l+2, l+4, \dots, k_{\max}$, where $k_{\max}$ is some maximum power cutoff on the radial expansion. This method is detailed in Appendix B. The code tools used in the three-dimensional field solver were developed primarily by Dave Grote, and the algorithm to extract the multipole moments was developed by the author. In these numerical calculations we regard the detailed “potato” shaped conductors of the SBTE quadrupoles as cylindrical rod sections with the characteristic dimensions quoted previously. Also, to obtain a realistic axial dependence of the multipole moments, we perform these calculations on a three quadrupole length of lattice with the middle quadrupole centered on the grid and dipole biased with $V_y = 0$, $V_x = 1$, and $V_z = 0$ and with all conductors of the two adjacent quadrupoles grounded. Four hundred axial gridpoints and an 80×80 rectangular array of transverse gridpoints are used. In Fig. 4 we contour the resulting electrostatic potential ϕ in the axial midplane of the dipole biased quadrupole. In this figure the transverse x and y coordinates are in meters. The maximum radial extent of the beam in the region of these steerers will be less than 1.4 cm. For coordinates within this characteristic radius, it is apparent from Fig. 4 that the potential contours are fairly flat and uniformly spaced, as one would expect for a good electrostatic dipole. The leading order multipole moments $m_{l,k}$ corresponding to this solution for the electrostatic potential are plotted in Fig. 5(a)-5(i) for $l = 1, 3$, and 5 , and $k \leq 7$. The oscillations in the pseudo-multipole moments $m_{l,k}$ with $k > l$ tend to mitigate, in part, their influence on the dynamics of the beam centroid. Evidently, the dipole moment $m_{1,1}$ dominates the field expansion, and as expected, the leading order error field component is the sextupole moment $m_{3,3}$. Although the sextupole moment appears disturbingly large, it is best to keep in mind that the maximum radial extent of the beam will be roughly $1/2$ the quadrupole aperture. Therefore, the peak field intensity associated with the dipole moment will scale directly with the peak value of $m_{1,1}$ (at all particle radii), whereas at the maximum particle radii, the peak field intensity associated with the quadrupole and sextupole moments will scale as $1/4$ the peak value of $m_{3,3}$. Furthermore, we will find that the dipole bias voltages needed to correct typical errors of the beam centroid are small, thereby further mitigating the intensity of the non-linear sextupole fields and their influence on the beam distribution. Finally, it should be pointed out that the actual SBTE quadrupole conductors are carefully shaped to suppress higher-order multipoles when biased in a quadrupole configuration, and these details may alter the multipoles plotted in Fig. 5. Nevertheless, we expect that the qualitative features of the multipoles will be insensitive to these details.

For purposes of conducting analytic calculations of the beam optics associated with a dipole biased quadrupole, it is convenient to construct a hard-edge approximation to the potential (4). In this regard, we define hard-edge multipole strengths $\bar{m}_{l,k}$ and $\bar{q}_{l,k}$ by

$$\begin{aligned}\bar{m}_{l,k}\ell &= \int dz m_{l,k}(z), \\ \bar{q}_{l,k}\ell &= \int dz q_{l,k}(z).\end{aligned}\tag{5}$$

Computing these quantities numerically from the results above, we obtain the “average” dipole, sextupole, and decapole moments $\bar{m}_{1,1} = 1.0644$, $\bar{m}_{3,3} = 0.2431$, and $\bar{m}_{5,5} = -0.0094$. Also, for the pseudo-multipole moments $m_{l,k}$ with $k > l$ we obtain for $l = 1$,

$\bar{m}_{1,3} = -5.6 \times 10^{-5}$, $\bar{m}_{1,5} = -1.5 \times 10^{-11}$, and $\bar{m}_{1,7} = -3.5 \times 10^{-18}$; for $l = 3$, $\bar{m}_{3,5} = -1.2 \times 10^{-3}$ and $\bar{m}_{3,7} = -4.1 \times 10^{-10}$; and for $l = 5$, $\bar{m}_{5,7} = -8.4 \times 10^{-4}$. These small average values of the $l = 1$ and $l = 3$ pseudomultipole moments compared to their fundamental moment values indicate consistency with general considerations that predict a zero average value for all pseudo-multipole moments, while the higher order $l = 5$ results appear to be suffering to some degree from numerical and truncation errors. On the other hand, deviations from ideal quadrupole behavior in the SBTE geometry should be small, so we approximate $\bar{q}_{2,2} = 1$. Taking this multipole to be unity will also enable a concrete correspondence to quadrupole focusing strengths obtained in earlier ideal analyses of the beam envelope performed in recirculator note 31. Motivated by the numerical analysis above, we neglect all but the fundamental dipole, quadrupole, and sextupole moment terms and approximate the electrostatic potential (4) as

$$\phi \simeq \bar{m}_{1,1} \frac{V_x x + V_y y}{a} + V_q \frac{x^2 - y^2}{a^2} + \bar{m}_{3,3} \frac{V_x(x^3 - 3xy^2) + V_y(y^3 - 3yx^2)}{a^3}, \quad (6)$$

within the dipole biased quadrupole lens, and zero outside. The hard edge potential (6) contains the dipole bending field, the quadrupole focusing field, and the leading order nonlinear error field associated with the sextupole moment. This approximate expression will be used in all subsequent analyses of the beam optics of the lens.

EQUATIONS OF MOTION OF THE BEAM CENTROID

We now write the equations of motion of a centroid particle in the field described by the hard-edge electrostatic potential (4). For a beam made up of particles of charge state q and energy $\mathcal{E}$, the nonrelativistic and paraxial equations of motion of the x - and y -coordinates of the beam centroid can be expressed as

$$\begin{aligned} x'' \pm k_q^2 x &= -k_{dx} - k_{sx}^3(x^2 - y^2) + 2k_{sy}^3 xy, \\ y'' \mp k_q^2 y &= -k_{dy} + k_{sy}^3(x^2 - y^2) + 2k_{sx}^3 xy. \end{aligned} \quad (7)$$

Here, primes denote derivatives with respect to the axial coordinate z , k_{dx} , k_q , and k_{sx} are the x -dipole, quadrupole, and x -sextupole couplings defined by $k_{dx} = qV_x \bar{m}_{1,1}/2\mathcal{E}a$, $k_q^2 = q|V_q|/\mathcal{E}a^2$, and $k_{sx}^3 = 3qV_x \bar{m}_{3,3}/2\mathcal{E}a^3$ within the lens and zero outside. Analogous expressions with V_y replacing V_x define the y -dipole and y -sextupole couplings k_{dy} and k_{sy} . The plus sign in Eq. (7) is for $V_q > 0$, and the minus sign is for $V_q < 0$, corresponding to focusing and defocusing quadrupole strengths along the x -axis. All so-called energy effects associated with the change in axial velocity as the centroid particle encounters differing electrostatic potentials are neglected in Eq. (7). For the 80 KeV heavy-ion beam and characteristic focusing and steering potentials associated with the matching section, such effects should be negligible for transverse displacements of the beam centroid expected from on-axis (corresponding to zero potential).

LINEAR OPTICS OF THE BEAM CENTROID

In calculating the beam optics of the dipole biased quadrupoles, we will first analyze the linear optics described by the equations of motion (7) by taking the sextupole couplings k_{sx} and k_{sy} to be zero. Investigation of nonlinear effects due to the sextupole field will be investigated after the linear optics has been thoroughly characterized. For present purposes, we take $k_{sx} = k_{sy} = 0$, and then the equation of motion (7) for the x -centroid coordinate reduces to

$$x'' \pm k_q^2 x = -k_{dx}. \quad (8)$$

For initial conditions $x = x_0$ and $x' = x'_0$ at an axial coordinate $z = 0$, the solution to Eq. (8) can be expressed as

$$\begin{pmatrix} x \\ x' \end{pmatrix} = -k_{dx} \begin{pmatrix} [1 - \cos(k_q z)]/k_q^2 \\ \sin(k_q z)/k_q \end{pmatrix} + \begin{bmatrix} \cos(k_q z) & (1/k_q) \sin(k_q z) \\ -k_q \sin(k_q z) & \cos(k_q z) \end{bmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} \quad (9a)$$

and

$$\begin{pmatrix} x \\ x' \end{pmatrix} = -k_{dx} \begin{pmatrix} [\cosh(k_q z) - 1]/k_q^2 \\ \sinh(k_q z)/k_q \end{pmatrix} + \begin{bmatrix} \cosh(k_q z) & (1/k_q) \sinh(k_q z) \\ k_q \sinh(k_q z) & \cosh(k_q z) \end{bmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} \quad (9b)$$

for focusing and defocusing quadrupole strengths within the lens, and as a simple drift

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} \quad (9c)$$

outside the lens ($k_q = k_{dx} = 0$). In writing these solutions, we have, of course, assumed that k_{dx} and k_q have not changed over the axial distance $|z|$ measured from the initial conditions at $z = 0$.

In the conceptual design of the matching section, it is currently planned to employ dipole biased quadrupole beam steerers with zero quadrupole beam focusing, i.e., $V_q = 0 = k_q$. To examine this situation, we take the limit of vanishing quadrupole strength ($k_q \rightarrow 0$), thereby reducing Eqs. (9a) and (9b) for the x -centroid trajectory to

$$\begin{pmatrix} x \\ x' \end{pmatrix} = -k_{dx} \begin{pmatrix} z^2/2 \\ z \end{pmatrix} + \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}. \quad (10)$$

Comparing Eqs. (9) and (10), it is interesting to note that $[1 - \cos(k_q z)]/k_q^2 < z^2/2$ and $\sin(k_q z)/k_q < z$, whereas $[\cosh(k_q z) - 1]/k_q^2 > z^2/2$ and $\sinh(k_q z)/k_q > z$. From these bounds one can conclude that the effective dipole bending strength is weakened by the presence of a quadrupole field component that is focusing in the plane of bending and strengthened by a quadrupole field component that is defocusing in the plane of bending. The dipole beam steerers will be constructed from two lenses separated by a drift distance d as sketched in Fig. 6. Denote the dipole coupling strengths of the first and second lens as k_{dx1} and k_{dx2} . Here, and in other similar quantities, we will use the additional subscripts 1

and 2 to refer to quantities associated with the first and second lens. For a beam entering the first lens with centroid coordinates x_0 and x'_0 , it is straightforward to show from Eqs. (9) and (10) that the x and x' centroid coordinates on exiting the second lens are

$$\begin{pmatrix} x \\ x' \end{pmatrix} = -k_{dx2} \begin{pmatrix} \ell^2/2 \\ \ell \end{pmatrix} - k_{dx1} \begin{pmatrix} \ell(d + 3\ell/2) \\ \ell \end{pmatrix} + \begin{bmatrix} 1 & d + 2\ell \\ 0 & 1 \end{bmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}. \quad (11)$$

For zero centroid error, we set $x = 0 = x'$ in Eq. (11) and solve for the dipole voltages V_{x1} and V_{x2} of the first and second lens. This solution yields

$$\begin{aligned} qV_{x1} &= \frac{2\mathcal{E}}{\bar{m}_{1,1}} \frac{[x_0 + (d + 3\ell/2)x'_0]a}{\ell(d + \ell)}, \\ qV_{x2} &= -\frac{2\mathcal{E}}{\bar{m}_{1,1}} \frac{(x_0 + \ell x'_0/2)a}{\ell(d + \ell)}. \end{aligned} \quad (12)$$

Some investigation of characteristic magnitudes predicted by Eq. (12) is instructive. For example, assuming a pure displacement centroid error with $x'_0 = 0$, and employing the lattice parameters consistent with the conceptual design sketched in Fig. 1, $\ell = 10.16$ cm (4 in), $d = 20.32$ cm (8 in), $a = 2.54$ cm (1 in), the beam energy $\mathcal{E} = 80$ KeV, and approximating $\bar{m}_{1,1} \simeq 1$, we obtain the correction voltages per centimeter of centroid displacement as

$$\frac{V_{x1}}{x_0} = -\frac{V_{x2}}{x_0} = 1.31 \frac{\text{KV}}{\text{cm}}. \quad (13a)$$

Therefore, to correct a characteristic 0.1 cm x -centroid displacement, we would need dipole biasing voltages on the order of ± 130 V on both lenses. On the other-hand, for the same situation and a centroid with a pure axial divergence error with $x_0 = 0$, Eq. (12) gives the correction voltage per radian of axial divergence as

$$\frac{V_{x1}}{x'_0} = 46.67 \frac{\text{KV}}{\text{rad}} \quad \frac{V_{x2}}{x'_0} = -6.67 \frac{\text{KV}}{\text{rad}}. \quad (13b)$$

Therefore, to correct a characteristic axial divergence $x'_0 \sim \sigma_0 x_0 / (2L) \sim 0.0003$, we would need dipole voltages on the order of ± 14 V on the first lens and ± 2 V on the second lens. Of course, for the correction of general centroid errors with both displacement and divergence components, the biasing voltages required will be a linear combination of the values indicated in Eqs. (13a) and (13b). Implementation of signals of this magnitude will pose no technical problems.

Because of the approximation of zero sextupole couplings, the voltages for linear optics corrections of centroid errors along the x and y principal axes can be calculated independently. Furthermore, the derivation of the corrections along the y axis parallels the derivation of Eq. (12). Therefore, we can immediately summarize the correction voltages to zero a beam centroid error with coordinates $x = x_0$ and $y = y_0$ and convergences $x' = x'_0$ and $y' = y'_0$.

and $y' = y'_0$ measured on entry to the first beam steering lens as

$$\begin{aligned}
 qV_{x1} &= \frac{2\mathcal{E}}{\bar{m}_{1,1}} \frac{[x_0 + (d + 3\ell/2)x'_0]a}{\ell(d + \ell)}, \\
 qV_{x2} &= -\frac{2\mathcal{E}}{\bar{m}_{1,1}} \frac{(x_0 + \ell x'_0/2)a}{\ell(d + \ell)}, \\
 qV_{y1} &= \frac{2\mathcal{E}}{\bar{m}_{1,1}} \frac{[y_0 + (d + 3\ell/2)y'_0]a}{\ell(d + \ell)}, \\
 qV_{y2} &= -\frac{2\mathcal{E}}{\bar{m}_{1,1}} \frac{(y_0 + \ell y'_0/2)a}{\ell(d + \ell)}.
 \end{aligned} \tag{14}$$

Here, V_{x1} and V_{y1} are the x and y correction voltages on the first “quadrupole,” etc., $\mathcal{E}$ and q are the energy and charge of a beam particle, ℓ is the quadrupole length, d is the drift length separating the lenses, and $\bar{m}_{1,1}$ is the hard-edge dipole strength parameter. For the SBTE lattice, $\bar{m}_{1,1} = 1.0644 \sim 1$.

Some discussion on extensions and limitations associated with Eq. (14) is appropriate. First of all, the conceptual design of the beam steerers has zero quadrupole bias, and therefore the zero quadrupole couplings prevent errors in the envelope manipulations of the matching section. This is a good feature of the design, but one might wish for more diagnostic space between the beam focusing quadrupoles. In light of the decoupling between the field components due to the quadrupole and dipole biases of the lens, one might imagine exploiting this feature by adding the dipole correction voltages to two quadrupoles used for beam focusing and envelope manipulations and thereby eliminate the need for extra steering lenses. Furthermore, in the conceptual design indicated in Fig. 1, there is a conventionally biased quadrupole lens for beam focusing between the two dipole biased quadrupoles for beam steering. Such a situation also exists in several other design alternatives, and since the voltages for beam focusing are much larger than the dipole bias voltages for beam steering, the intermediate quadrupole field may significantly influence the centroid trajectory of an offset beam. We now generalize the previous analysis for steering voltage specifications to include these effects.

First, we examine the situation where the first and second dipole biased quadrupoles forming the beam steerers have focusing and defocusing quadrupole couplings $k_{q1} = \sqrt{q|V_{q1}|/\mathcal{E}a^2}$ and $k_{q2} = \sqrt{q|V_{q2}|/\mathcal{E}a^2}$ in addition to the dipole biases denoted as previously, and as previous, the two lenses are separated by a drift distance d with no intermediate quadrupole lenses. Paralleling the development leading to Eq. (14), some analysis shows that the centroid correction voltages for this situation are specified by the matrix

equations,

$$\begin{aligned}
\begin{pmatrix} qV_{x1} \\ qV_{x2} \end{pmatrix} &= \frac{2\mathcal{E}a}{\bar{m}_{1,1}} \left\{ (1 \ 0)[Q_{D2}][D](B_{F1}) \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} + (1 \ 0)[B_{D2}] \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \right. \\
(0 \ 1)[Q_{D2}][D](B_{F1}) \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + (0 \ 1)[B_{D2}] \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \Big\}^{-1} [Q_{D2}][D][Q_{F1}] \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}, \\
\begin{pmatrix} qV_{y1} \\ qV_{y2} \end{pmatrix} &= \frac{2\mathcal{E}a}{\bar{m}_{1,1}} \left\{ (1 \ 0)[Q_{F2}][D](B_{D1}) \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} + (1 \ 0)[B_{F2}] \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \right. \\
(0 \ 1)[Q_{F2}][D](B_{D1}) \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + (0 \ 1)[B_{F2}] \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \Big\}^{-1} [Q_{F2}][D][Q_{D1}] \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix},
\end{aligned} \tag{15}$$

if the first and second lenses are defocusing and focusing along the x -axis. In this equation, $[Q_{F1}]$ and $[Q_{D1}]$ denote the usual 2×2 focusing and defocusing quadrupole advance matrices of the first lens,

$$\begin{aligned}
[Q_{F1}] &= \begin{bmatrix} \cos(k_{q1}\ell) & (1/k_{q1})\sin(k_{q1}\ell) \\ -k_{q1}\sin(k_{q1}\ell) & \cos(k_{q1}\ell) \end{bmatrix}, \\
[Q_{D1}] &= \begin{bmatrix} \cosh(k_{q1}\ell) & (1/k_{q1})\sinh(k_{q1}\ell) \\ k_{q1}\sinh(k_{q1}\ell) & \cosh(k_{q1}\ell) \end{bmatrix},
\end{aligned} \tag{16a}$$

(B_{F1}) and (B_{D1}) denote 2×1 bending advance vectors associated with the dipole fields in the presence of focusing and defocusing quadrupole fields of the first lens,

$$\begin{aligned}
(B_{F1}) &= \begin{pmatrix} [1 - \cos(k_{q1}\ell)]/k_{q1}^2 \\ \sin(k_{q1}\ell)/k_{q1} \end{pmatrix}, \\
(B_{D1}) &= \begin{pmatrix} [\cosh(k_{q1}\ell) - 1]/k_{q1}^2 \\ \sinh(k_{q1}\ell)/k_{q1} \end{pmatrix},
\end{aligned} \tag{16b}$$

$[D]$ denotes the 2×2 advance matrix for a simple drift of length d

$$[D] = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}, \tag{16c}$$

and $\{ [\dots] \}^{-1}$ denotes the inverse matrix of $[\dots]$. Expressions analogous to (16a) and (16b) are defined for the second dipole biased quadrupole lens by replacing k_{q1} with k_{q2} . Equation (15) is cumbersome to analytically expand, but it is straightforward to use numerically because the matrices and vectors given by Eqs. (16a)-(16c) are of dimension 2×2 and 2×1 and depend only on geometric factors and quadrupole couplings. Note that the quadrupole couplings k_{q1} and k_{q2} will be fixed by the envelope manipulations required by the matching section. If the first dipole biased quadrupole is defocusing and the second focusing, then Eq. (15) can be modified for this situation by replacing all focusing/defocusing matrices and vectors with defocusing/focusing matrices and vectors, i.e., $[Q_{F1}] \rightarrow [Q_{D1}]$ and $(B_{F1}) \rightarrow (B_{D1})$, etc. Also, if there are intermediate quadrupole and

drift spaces between the two dipole biased quadrupoles, then the drift matrix $[D]$ should be replaced by the appropriate 2×2 advance matrices. For example, if the drift space is replaced by an intermediate quadrupole that is focusing along the x -axis and flanked by two equal drift lengths of length d on either side, then we replace $[D] \rightarrow [D][Q_F][D]$ and $[D] \rightarrow [D][Q_D][D]$ in the x - and y -plane dipole bias equations. Here, $[Q_F]$ and $[Q_D]$ denote the usual 2×2 focusing and defocusing quadrupole advance matrices defined analogously to Eq. (16a). Similar substitution patterns are obvious to modify Eq. (15) appropriately for any conceivable combination of dipole biased quadrupole beam steerers with intermediate quadrupole lenses and drifts.

We now apply Eq. (15) and the substitution patterns discussed above to examine several situations of interest in the conceptual design of the small recirculator matching section. First, we check the validity of our general formula by checking to see if it gives the same correction voltages as Eq. (14). To do this, we set the quadrupole couplings of the first and second dipole focusing lenses to zero (i.e., $k_{q1} = k_{q2} = 0$) and regard the intermediate space between the lenses to be a free drift. Then applying Eq. (15) with the conceptual design lattice parameters, we obtain the correction voltages

$$\begin{aligned} \begin{pmatrix} qV_{x1} \\ qV_{x2} \end{pmatrix} &= \frac{2\mathcal{E}a}{\bar{m}_{1,1}} \begin{bmatrix} 0.003229 & 0.1148 \\ -0.003229 & -0.01640 \end{bmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}, \\ \begin{pmatrix} qV_{y1} \\ qV_{y2} \end{pmatrix} &= \frac{2\mathcal{E}a}{\bar{m}_{1,1}} \begin{bmatrix} 0.003229 & 0.1148 \\ -0.003229 & -0.01640 \end{bmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}. \end{aligned} \quad (17a)$$

Here, the initial centroid offsets x_0 and y_0 are measured in centimeters and all other quantities carry the dimensions discussed previously. Plugging lattice numbers in Eq. (14) obtains the same formula as Eq. (17a), verifying in part the more general formula (15). Next, applying the substitutions described above, we apply Eq. (15) to correct for the presence of the beam focusing quadrupole Q5 positioned between the two dipole biased beam steerers with zero quadrupole couplings. This intermediate quadrupole has the same characteristic dimensions as the beam steerers will be quadrupole biased to $V_q = 2.135$ KV, as required by the envelope solution of the matching section (see recirculator note 31). For this situation, we obtain the correction voltages

$$\begin{aligned} \begin{pmatrix} qV_{x1} \\ qV_{x2} \end{pmatrix} &= \frac{2\mathcal{E}a}{\bar{m}_{1,1}} \begin{bmatrix} 0.001812 & 0.1076 \\ -0.004567 & -0.02320 \end{bmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}, \\ \begin{pmatrix} qV_{y1} \\ qV_{y2} \end{pmatrix} &= \frac{2\mathcal{E}a}{\bar{m}_{1,1}} \begin{bmatrix} 0.004094 & 0.1192 \\ -0.002438 & -0.01239 \end{bmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}. \end{aligned} \quad (17b)$$

Note that for the effect of the intermediate quadrupole has tended to decrease the correction voltage in one plane while increasing it in the other, as should expected on qualitative grounds. In addition, note that the correction voltages given by Eq. (17b) are substantially different from those given by Eq. (17a). Evidently, even though a quadrupole field has a weaker bending strength than a dipole field, the voltages for quadrupole focusing are much larger than the voltages for dipole bending, and therefore the intermediate quadrupole must be taken into account to accurately calculate the dipole bias voltages for the correction of

centroid errors. Finally, we examine the possibility of adding dipole bias voltages to Q6 and Q7 to form a steering lens combination (see Fig. 1). This would allow the removal of the two single-purpose lenses for quadrupole steering, thereby creating additional room for experimental diagnostics. Furthermore, note that Q6 and Q7 are near the end of the matching section, and therefore if these lenses are dipole biased, then they could be used to correct centroid errors accumulated along the 3/4 of the matching section rather than 1/2 the matching section as the conceptual design steerers S1 and S2 would. To analyze this case, we take the first and second lenses to have quadrupole biases $V_{q1} = -2.096$ KV and $V_{q2} = 1.959$ KV, as required by the envelope solution of the matching section (see recirculator note 31). The drift distance between these two lenses is $d = 20.32$ cm (8 in) and contains no intermediate quadrupoles. For this situation, we the correction voltages

$$\begin{aligned} \begin{pmatrix} qV_{x1} \\ qV_{x2} \end{pmatrix} &= \frac{2\mathcal{E}a}{\bar{m}_{1,1}} \begin{bmatrix} 0.007075 & 0.1266 \\ -0.003450 & -0.01694 \end{bmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}, \\ \begin{pmatrix} qV_{y1} \\ qV_{y2} \end{pmatrix} &= \frac{2\mathcal{E}a}{\bar{m}_{1,1}} \begin{bmatrix} -0.0005976 & 0.1025 \\ -0.003025 & -0.01593 \end{bmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}. \end{aligned} \quad (17c)$$

Again, note that the correction voltages given by Eqs. (17c) and (17a) are substantially different, showing that the presence of alternating gradient quadrupole fields for beam focusing substantially alters the required dipole fields. Nevertheless, the characteristic voltages predicted by (17c) are still small, and their synthesis would pose no technical problems. So this alternative design for extra lattice space is also a valid option. Finally, it should be mentioned that Eq. (15) has been checked by employing the voltages predicted by Eq. (17) with several choices of centroid errors and then numerically integrating the equations of motion (7) (with zero sextupole couplings, i.e., $k_{sx1} = k_{sy1} = 0$, etc.) to verify that the final centroid error is zero.

An issue of concern in using dipole biased quadrupoles is the number of power supplies required. For a conventional quadrupole biased with single-polar power supplies, two high-voltage power supplies are required for each quadrupole. For each dipole biased lens, two single-polar power supplies will be required for each orthogonal direction, for a total of four supplies per lens, or one power supply per rod. By choosing appropriate bias voltages for each rod, this four supply dipole biased configuration can also be used to produce a quadrupole focusing component. Thus for pure dipole or combined function dipole/quadrupole lenses, a total of four power supplies per lens will be required. However, as in the conceptual design application discussed above, it is possible that the dipole bias voltages required for centroid correction with no quadrupole focusing component could be small enough to eliminate the need for special high-voltage power supplies. On the other hand, for combined function lenses, high-voltage power supplies will be necessary. Assuming identical high-voltage supplies are used for all biases, then the conceptual design would require 8 additional power supplies for beam steering, whereas the alternative design with dipole biases added to Q6 and Q7 would require only 4 additional supplies.

Another issue of practical concern in the use of combined function lenses might be that if all four conductors of the lenses are biased independently to voltages of high magnitude, then regulation errors in the power supplies could be significant compared to the component

of the voltages intended to correct centroid errors of the beam. For the small recirculator matching section, the quadrupole biases will be on the order of ± 2 KV, and the centroid correction biases should be smaller than ± 150 V. Thus, each rod in a combined function lens could be biased to a voltage on the order of 2.2 KV. The high-voltage power supplies on order for the beam steerers and quadrupole focusing lenses of the small recirculator have a programmable voltage outputs that can be adjusted over the range 0.5-5.6 KV to an accuracy of $0.1\% + 5$ V, and are regulated to 0.05% of full scale. Thus, for our characteristic 2.2 KV voltages, we would expect uncertainties on the order of 7 V, which is 5% of the 150 V contribution to the bias for the correction of centroid errors. Therefore, it is unlikely that the component of the biases intended for the correction of centroid errors in a combined function lens would be obscured by inaccuracies in the programming or regulation of the power supply. It should also be pointed out that the high-voltage supplies have unrated performance for outputs below 0.5 KV, and therefore would be unsuitable in the absence of voltage dividing for biasing the beam steerers of the conceptual design, which have no quadrupole component for beam focusing. Furthermore, because the power-supplies are programmable and computer interfaced, the voltage components required for beam focusing and dipole corrections can be clearly separated in the controlling program, thereby giving the experimenter distinct adjustment "knobs" to independently control the quadrupole focusing strength for beam focusing and envelope manipulations and the dipole bending strength for elimination of centroid errors. It is also relevant to point out that the "sense" of centroid errors will be apriori unknown, and therefore the polarity of the dipole biasing voltages needed to correct them will also be unknown. Therefore, because power supplies are typically only single polarity, it follows in a pure dipole biased lens that the polarity of the supplies may need to be flipped if the sense of the error changes. Needless to say, this could prove inconvenient in practice. On the other hand, for a dipole biased quadrupole lens, the dipole biasing voltages will be small compared to the quadrupole bias, and therefore any change in the sense of the errors will not require a change in the power supply polarity, only a consistent change on the magnitude of the bias voltages on particular quadrupole conductors.

MODIFICATIONS FOR NONSYMMETRIC DIPOLE BIASES

In the conceptual design configuration, the number of power supplies required per dipole biased lens could be lowered from four to two if the biases could be added to the quadrupole conductors in a nonsymmetric fashion, say rather than dipole biasing the conductors along the plus and minus x -axes to $+V_x$ and $-V_x$, bias them to $+V_x$ and zero, etc. A transverse cross-section of this structure is sketched in Fig. 7. Note that this asymmetric dipole bias configuration would simplify the wiring of the power supplies in response to a change in the sense of centroid corrections. Such a configuration would also increase the so-called energy effect, where beam particles change axial velocity when crossing potential lines on entry and exit to a lens. Fortunately, given the small characteristic dipole voltages of the steerers and that the heavy-ion beam has an energy of 80 KeV, it is clear that such effects will still be negligible. However, it is also clear that the field structure of such a configuration will be different than the previous configuration.

To understand precisely how it differs, we examine the fields of a dipole biased quadrupole with conductors along the positive and negative x -axes biased to potentials $V_q + V_x$ and V_q , and conductors along the positive and negative y -axes biased to potentials $-V_q + V_y$ and $-V_q$. Proceeding analogously to our previous field calculations, we refer to Fig. 8 and exploit the linearity of Maxwell's equations to regard the fields as resulting from a superposition of three "quadrupoles," one (quadrupole 1) a pure dipole biased configuration with $V_x \rightarrow V_x/2$ and $V_y \rightarrow V_y/2$, another (quadrupole 2) a conventional quadrupole with conductors at potentials $\pm V_q$ with the usual quadrupolar symmetry, and a third distorted quadrupole (quadrupole 3) with conductors along the x -axis at potential $V_x/2$ and conductors along the y -axis at potential $V_y/2$. Paralleling the analysis presented earlier, it is then straightforward to show that the electrostatic potential of this configuration can be expanded as

$$\begin{aligned} \phi = & \sum_{l=1,3,\dots}^{\infty} \sum_{k=l,l+2,\dots}^{\infty} \frac{m_{l,k}(z)}{2} \{V_x \cos(l\theta) + V_y \cos[l(\theta - \pi/2)]\} \left(\frac{r}{a}\right)^k \\ & + V_q \sum_{l=2,6,\dots}^{\infty} \sum_{k=l,l+2,\dots}^{\infty} q_{l,k}(z) \cos(l\theta) \left(\frac{r}{a}\right)^k \\ & + \sum_{l=0,2,\dots}^{\infty} \sum_{k=l,l+2,\dots}^{\infty} \frac{d_{l,k}(z)}{2} \{V_x \cos(l\theta) + V_y \cos[l(\theta - \pi/2)]\} \left(\frac{r}{a}\right)^k. \end{aligned} \quad (18)$$

In Eq. (18), the first, second, and third summation expressions correspond to the potential contributions from quadrupoles 1, 2, and 3. The moments $\{m_{l,k}\}$ and $\{q_{l,k}\}$ are precisely the moments introduced previously, and the moments $\{d_{l,k}\}$ are due to the distorted quadrupole 3. The $\{d_{l,k}\}$ can be thought of as the z -dependent moments due to a "quadrupole" configuration with the conductors along the x -axis held at unit potential, and the conductors along the y -axis grounded. Note that the $l = 0, k = 0$ term of this expansion with moment $d_{0,0}$ takes into account the nonzero on-axis electrostatic potential of this configuration. Furthermore, note that all $m_{l,k}$ moments enter the expansion with a multiplying factor of $1/2$. Therefore, the dipole bias V_x will need to be doubled in this configuration to achieve the same dipole bending field strength as the previous configuration. However, in contrast to the previous situation, this dipole bias will generate a quadrupole field component due to the moment $d_{2,2}$, and thereby modify the beam focusing properties of the lens. Clearly, this is not a completely desirable situation, but because the quadrupole bias component V_q still generates no dipole fields, it is possible to change V_q by an appropriate amount to compensate for the quadrupole fields generated by the dipole biases. To understand this further, it is first best to relate the $q_{l,k}$ moments to the $d_{l,k}$ moments. It is straightforward to show that

$$q_{l,k} = d_{l,k}[1 - \cos(l\pi/2)]. \quad (19)$$

Note that this equation shows explicitly that $q_{l,k} = 0$ for $l = 0, 4, 8, \dots$ and that $q_{l,k} = 2d_{l,k}$ for nonvanishing $q_{l,k}$ corresponding to $l = 2, 6, 10, \dots$. Using Eq. (19), Eq. (18) can be

expressed as

$$\begin{aligned} \phi = & \sum_{l=1,3,\dots}^{\infty} \sum_{k=l,l+2,\dots}^{\infty} \frac{m_{l,k}(z)}{2} \{V_x \cos(l\theta) + V_y \cos[l(\theta - \pi/2)]\} \left(\frac{r}{a}\right)^k \\ & + \sum_{l=0,2,\dots}^{\infty} \sum_{k=l,l+2,\dots}^{\infty} d_{l,k}(z) \{V_x/2 + V_q + (V_y/2 - V_q) \cos(l\pi/2)\} \cos(l\theta) \left(\frac{r}{a}\right)^k. \end{aligned} \quad (20)$$

It is evident from the $l = 2$ term of Eq. (20) that V_q can be adjusted such that $2V_q + V_x/2 - V_y/2$ remains constant to maintain fixed quadrupole focusing strength while V_x and V_y are varied. Such a prescription can be readily programmed in a modern, computer controlled power supply, thereby eliminating a potential problem associated with asymmetric dipole biases. For reference purposes, it is also clear from the analysis above that the potential for an asymmetrically biased quadrupole configuration with conductors along the x - and y -axes biased to potentials V_{qx} and V_{qy} is given by

$$\phi = \sum_{l=0,2,\dots}^{\infty} \sum_{k=l,l+2,\dots}^{\infty} d_{l,k}(z) \{V_{qx} + V_{qy} \cos(l\pi/2)\} \cos(l\theta) \left(\frac{r}{a}\right)^k. \quad (21)$$

Note that the conventional expression is obtained for the usual $\pm$ biased configuration when $V_{qx} = -V_{qy} = V_q$ and $q_{l,k} = d_{l,k}[1 - \cos(l\pi/2)]$ is substituted. Evidently, if $V_{qx} - V_{qy} = 2V_q$, the fundamental quadrupole field (labeled with $l = 2$) and all field components labeled by $l = 6, 10, 14, \dots$ will be identical to those resulting from the usual bias configuration, while on the other hand, there will be additional error fields labeled by $l = 0, 4, 8, \dots$, that do not exist in the usual bias configuration. Finally, it is worthwhile to mention that the moments $\{m_{l,k}\}$ and $\{d_{l,k}\}$ form a complete basis set for the quadrupole geometry and can therefore be used to construct the field for *any* bias configuration on the quadrupole conductors. Thus, a calculation of the $\{m_{l,k}\}$ and $\{d_{l,k}\}$ moments, which depend only on geometric factors, is equivalent to a calculation of the general field structure of a particular quadrupole geometry. In this regard, a thorough understanding of the structure and properties of these geometric moments is particularly valuable.

From the analysis above, there appears to be few detrimental consequences from using asymmetric dipole biases, provided the quadrupole focusing voltages are adjusted consistently to compensate for any quadrupole field components generated by the dipole correction voltages. For completeness, and to provide information that would allow a more concrete estimation of both intrinsic quadrupole field errors and quadrupole field errors due to any asymmetric dipole biases, we now present numerical calculations of the $d_{l,k}$ moments. These numerical calculations are performed analogously to those presented previously to calculate the $m_{l,k}$. In these calculations, we used the same numerical and lattice parameters as previous and set the central quadrupole conductors along the positive x -axis to unit potential, and grounded all other conductors. The electrostatic potential obtained is contoured in Fig. 9, and the corresponding $d_{l,k}$ moments are plotted in Fig. 10(a)-10(i) for $l = 0, 2$, and 4 , and $k \leq 6$. Analogously to Eq. (5), we defined a hard-edge multipole strength

$$\bar{d}_{l,k} = \int dz d_{l,k}(z), \quad (22)$$

and calculated $\bar{d}_{0,0} = 0.7504$, $\bar{d}_{2,2} = 0.6089$, and $\bar{d}_{4,4} = 0.0022$. This shows that $\bar{q}_{2,2} = 2\bar{d}_{2,2} = 1.2178$, which is considerably more than the ideal value $q_{2,2} = 1$ assumed in the previous analysis. With this in mind, it would be better to replace $k_{q1} = \sqrt{q|V_{q1}|/\mathcal{E}a^2}$ by $k_{q1} = \sqrt{2q\bar{d}_{2,2}|V_{q1}|/\mathcal{E}a^2}$, etc., in the previous formulas when a more precise correspondence to the actual experimental system is desired. (The envelope calculations, which determined the quadrupole focusing voltages, did not employ this correction factor, so it was better not to employ it in the previous numerical examples that employed that data.) As discussed for the $m_{l,k}$ and $q_{l,k}$ moments, general considerations dictate that $\bar{d}_{l,k} = 0$ for $k > l$. In our numerical calculations we obtain for $l = 0$, $\bar{d}_{0,2} = -9.1 \times 10^{-6}$, $\bar{d}_{0,4} = -2.2 \times 10^{-12}$, and $\bar{d}_{0,6} = -4.5 \times 10^{-19}$; for $l = 2$, $\bar{d}_{2,4} = -1.3 \times 10^{-4}$ and $\bar{d}_{2,6} = -3.9 \times 10^{-11}$; and for $l = 4$, $\bar{d}_{4,6} = -4.2 \times 10^{-3}$. Again, it appears that the lower order moments are consistent and that possible truncation and numerical errors are influencing the higher order terms. Finally, it should be stressed that the detailed shaping associated with the actual SBTE quadrupole conductors was not included in this calculation. This shaping was designed to suppress higher-order multipoles in a quadrupole configuration and may alter the multipoles plotted in Fig. 10.

NONLINEAR OPTICS OF THE BEAM CENTROID

We now analyze the influence of leading-order nonlinear field components on the dynamics of the beam centroid. In this analysis, we will employ the full equations of motion with sextupole couplings given by Eq. (7). The influence of the nonlinear sextupole terms will be evaluated in the conceptual design configuration sketched in Fig. 1 with two pure dipole biased lenses for beam steering (S1 and S2) and an intermediate quadrupole lens Q5 for beam focusing. Other possible configurations should yield similar results. The sextupole terms are analyzed by numerically integrating the full equations of motion (7) for a number of initial centroid errors, and then determining how the nonlinear sextupole terms influence the final centroid coordinates. In particular, for the given lens configuration and a set of initial centroid errors, we will employ the correction voltages predicted by Eq. (15) in the *absence* of sextupole couplings, and then we will numerically calculate the final centroid coordinate errors in the presence of sextupole couplings that depend on these correction voltages. Deviations of the final coordinates from zero will indicate how large an effect the nonlinear sextupole fields have on the dynamics of the beam centroid. If the deviations are small, then their concomitant influence on the dipole biasing voltages required to correct errors in the beam centroid should also be small. It should also be pointed out that nonlinear corrections to the centroid trajectory due to sextupole couplings can be calculated perturbatively and formula (which must be solved numerically) for the correction voltages in the presence of small sextupole couplings can be derived. However, these formula are complicated and difficult to interpret, while in contrast, the numerical calculations presented here are easy to understand. The influence of nonlinear field components on both the beam envelope and quality will be assessed in future studies by particle-in-cell simulations with WARP3d.

In our numerical calculations, we take both the dipole and quadrupole biases to be given in the usual $\pm$ configurations. In addition, we take the hard-edge dipole and sextupole moments to be the calculated values $\bar{m}_{1,1} = 1.0644$ and $\bar{m}_{3,3} = 0.2431$. While on the other hand, we take the hard-edge quadrupole moment to be the ideal value $\bar{q}_{2,2} = 1$, so that the quadrupole focusing strengths obtained from the beam envelope solution constructed in Recirculator note 31 can be applied directly. From that envelope solution, we set the quadrupole bias on the intermediate quadrupole to be $V_q = 2.135$ KV. Corresponding to this situation, we will employ the centroid correction voltages predicted by Eq. (17b), which are calculated in the absence of sextupole couplings. Using these voltages for a given initial centroid error entering the first steering lens, we then numerically integrate the full equations of motion (7) with nonlinear sextupole couplings $k_{sx1}^3 = 3qV_{x1}\bar{m}_{3,3}/2\mathcal{E}a^3$, etc., to obtain the final centroid coordinates on exiting the second steering lens. For nonzero sextupole couplings, these final coordinate will in general be nonzero, constituting a centroid error. In Fig. 11, we plot the errors in final centroid coordinate on exiting the second steering lens verses the initial error on entry to the first steering lens. In Figs. 11(a) and 11(b) we plot errors in the final centroid position and divergence as measured by $\sqrt{x^2 + y^2}$ and $\sqrt{x'^2 + y'^2}$ verses a pure coordinate error in the initial centroid. In Figs. 11(c) and 11(d) we plot errors in the final centroid position and divergence as measured by $\sqrt{x^2 + y^2}$ and $\sqrt{x'^2 + y'^2}$ verses a pure divergence error in the initial centroid. The centroid and divergence errors are taken to range from 0 to 0.5 cm and from 0 to 0.002 rad, respectively, along both the principal x - and y -axes. It is evident from these plots that even for errors significantly larger than the 0.1 cm and 0.0003 rad coordinate and divergence errors expected, that the final coordinate and centroid errors due to the nonlinear sextupole fields will be very small. For example, for a 0.2 cm initial error in the centroid coordinate, the final error will be less than 0.002 cm, or less than 1% of the initial error. Because these corrections are so small, there is little incentive to carry out the detailed perturbative analysis necessary to derive nonlinear corrections to the centroid correction voltages. Furthermore, for such small corrections it is likely that other effects neglected in the present analysis would enter at a similar order, making it questionable if such an analysis would even be relevant. In conclusion, for reasonable centroid errors, it appears that the nonlinear optics corrections to the dipole correction voltages are negligible.

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APPENDIX A THREE DIMENSIONAL MULTIPOLE EXPANSIONS

In an unbounded region of space with some charge density $\rho(\mathbf{x})$, the electrostatic potential ϕ can be expressed simply in terms of the free-space Greens function $1/|\mathbf{x} - \mathbf{x}'|$ as

$$\phi(\mathbf{x}) = \int d^3x' \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}. \quad (\text{A1})$$

In (r, θ, z) cylindrical-polar coordinates, $1/|\mathbf{x} - \mathbf{x}'|$ can be expanded as

$$\frac{1}{|\mathbf{x} - \mathbf{x}'|} = \frac{2}{\pi} \sum_{l=-\infty}^{\infty} \int_0^{\infty} dk e^{il(\theta - \theta')} \cos[k(z - z')] I_l(kr_{<}) K_l(kr_{>}). \quad (\text{A2})$$

Here, $I_m(x)$ and $K_m(x)$ are m th order modified Bessel functions of the first and second kind, and $r_{>}$ and $r_{<}$ denote the greater and lesser of r and r' . We will examine geometries where all charges are external to some lens aperture of radius $r = a$. Then for the fields of interest interior to the lens aperture (i.e., $r < a$), we can employ Eq. (A2) with $r_{>} = r'$ and $r_{<} = r$. Expanding $I_l(kr)$ in the resulting expression as

$$I_l(kr) = \sum_{p=0}^{\infty} \frac{1}{p!(l+p)!} \left(\frac{kr}{2}\right)^{l+2p} \quad (\text{A3})$$

shows that ϕ can in turn be expanded as

$$\phi = \sum_{l=0}^{\infty} \sum_{p=0}^{\infty} m_{l,p}^e(z) \cos(l\theta) \left(\frac{r}{a}\right)^{l+2p} + \sum_{l=1}^{\infty} \sum_{p=0}^{\infty} m_{l,p}^o(z) \sin(l\theta) \left(\frac{r}{a}\right)^{l+2p}. \quad (\text{A4})$$

Here, $m_{l,p}^e$ and $m_{l,p}^o$ are z -dependent multipole moments associated with the even and odd parts of the potential with respect to the azimuthal angle θ . The integers l and p that label each multipole coefficient are associated with the azimuthal ($\sim \cos(l\theta)$ for $m_{l,p}^e$ and $\sim \sin(l\theta)$ for $m_{l,p}^o$) and radial ($\sim r^{l+2p}$) variations. Note that the moments are nonvanishing only for radial variation $\sim r^k$ with k satisfying $k = l, l+2, l+4, \dots$. Explicit integral expressions for the multipole moments $m_{l,p}^e$ and $m_{l,p}^o$ are straightforward to derive. Except for very regular geometries, these integral expressions are of limited practical utility. Examples of their use and derivation have been presented in a slightly different context from the present in recirculator note 24. In the present note, we employ symmetry arguments to deduce the nonvanishing moments, and then calculate these moments with a numerical procedure detailed in appendix B. However, some useful results follow from the integral expressions for the $m_{l,p}^e$ and $m_{l,p}^o$. In particular, it is straightforward to derive the recursion relation

$$m_{l,p+1}(z) = \frac{-1}{4(p+1)(l+p+1)} \frac{\partial^2}{\partial z^2} m_{l,p}(z), \quad (\text{A5})$$

where $m_{l,p}$ denotes either $m_{l,p}^e$ or $m_{l,p}^o$. Using this recursion relation, all $m_{l,p}$ for $p > 0$ can be calculated from a $2p$ th order axial derivative of $m_{l,0}$ as

$$m_{l,p}(z) = \frac{(-1)^p l!}{4^p p! (l+p)!} \frac{\partial^{2p}}{\partial z^{2p}} m_{l,0}(z). \quad (\text{A6})$$

With this feature in mind, the moments with labeled with $p = 0$ (i.e., the $m_{l,0}$) are called fundamental multipole moments, and the moments labeled with $p > 0$ are called pseudomultipole moments. It follows from Eq. (A6) that all pseudomultipole moments exist only in three-dimensional geometries and are associated with the finite axial length of the lens. Assuming reasonable decay in field intensity at axial coordinates far from the lens, it also follows from Eq. (A6) that axial integrals of the pseudomultipole moments vanish, i.e.,

$$\int_{-\infty}^{\infty} dz m_{l,p}(z) = 0 \quad (A7)$$

for all $p > 0$. Furthermore, in the usual situation where the lens is formed from some transverse structure that is axially extruded over a finite length in z , then it also follows from Eq. (A6) that the axial integrals of the pseudomultipole moments also vanish over each longitudinal half-space, i.e., for a lens axially centered at $z = 0$

$$\int_0^{\infty} dz m_{l,p}(z) = \int_{-\infty}^0 dz m_{l,p}(z) = 0. \quad (A8)$$

Because of these average properties, the pseudomultipole moments of a lens $m_{l,p}$ with $p > 0$ are typically not as important as their corresponding fundamental moments $m_{l,0}$, because particles transversing the lens will sample less of their force on average insofar as the transverse coordinates of the particle do not rapidly vary while traveling through the lens.

In actual applications, detailed shaping of the transverse cross-section of a lens can be employed to suppress unwanted fundamental multipole moments $m_{l,0}$. These shaping, insofar as they reduce or eliminate a fundamental multipole produce a concomitant reduction or elimination of the corresponding pseudomultipole moments that depend on axial derivatives of the fundamental multipole moment. Beyond that, the pseudomultipole moments can be further reduced by increasing the aspect ratio (i.e., the ratio of axial length to aperture radius of the lens). For a small aspect ratio lens, the fundamental multipole moments vary strongly in z and the pseudomultipole moments associated with the dominant fundamental moment characterizing the lens can lead to error fields with unacceptably large amplitude. The only way to reduce such error field components is to increase the aspect ratio of the lens, thereby reducing three-dimensional effects. Finally, in the main text, we chose to label the multipole moments with the indices l and $k = l + 2p$ so that the radial dependence associated with each multipole moment of the field is more clearly expressed. In terms of this index relabeling, the multipole expansion (A4) becomes

$$\phi = \sum_{l=0}^{\infty} \sum_{k=l, l+2, \dots}^{\infty} m_{l,k}^e(z) \cos(l\theta) \left(\frac{r}{a}\right)^k + \sum_{l=1}^{\infty} \sum_{k=l, l+2, \dots}^{\infty} m_{l,k}^o(z) \sin(l\theta) \left(\frac{r}{a}\right)^k, \quad (A9)$$

and the recursion relations (A5) and (A6) become

$$m_{l,k+2}(z) = \frac{-1}{(k-l+2)(k+l+2)} \frac{\partial^2}{\partial z^2} m_{l,k}(z), \quad (A10)$$

$$m_{l,k}(z) = \frac{(-1)^{(k-l)/2} l!}{4^{(k-l)/2} [(k-l)/2]! [(k+l)/2]!} \frac{\partial^{k-l}}{\partial z^{k-l}} m_{l,l}(z).$$

APPENDIX B : NUMERICAL CALCULATION OF THE MULTIPOLE MOMENTS

To develop a numerical method for the calculation of the general multipole moments $m_{l,k}^e(z)$ and $m_{l,k}^o(z)$ in Eq. (A9), we first exploit the orthogonality associated with the trigonometric basis expanding the azimuthal angle dependence in θ to obtain

$$\begin{aligned} \int_0^{2\pi} d\theta \phi \cos(l\theta) &= \pi \sum_{k=l, l+2, \dots}^{\infty} m_{l,k}^e \left(\frac{r}{a}\right)^k \\ \int_0^{2\pi} d\theta \phi \sin(l\theta) &= \pi \sum_{k=l, l+2, \dots}^{\infty} m_{l,k}^o \left(\frac{r}{a}\right)^k \end{aligned} \quad (B1a)$$

for $l \neq 0$, and

$$\int_0^{2\pi} d\theta \phi = 2\pi \sum_{k=0, 2, \dots}^{\infty} m_{0,k}^e \left(\frac{r}{a}\right)^k \quad (B1b)$$

for $l = 0$. If the sums in k associated with the radial field expansions are cut-off at some value $k = k_{\max}$ and the truncated expressions are evaluated at n different radial data points at $r = r_1, r_2, \dots, r_n$, then the resulting set of equations can be expressed in matrix form as

$$\begin{bmatrix} (r_1/a)^l & (r_1/a)^{l+2} & \dots & (r_1/a)^{k_{\max}} \\ (r_2/a)^l & (r_2/a)^{l+2} & \dots & (r_2/a)^{k_{\max}} \\ \vdots & \vdots & \ddots & \vdots \\ (r_n/a)^l & (r_n/a)^{l+2} & \dots & (r_n/a)^{k_{\max}} \end{bmatrix} \begin{pmatrix} m_{l,l} \\ m_{l,l+2} \\ \vdots \\ m_{l,k_{\max}} \end{pmatrix} = \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{pmatrix}. \quad (B2)$$

Here, $m_{l,k}$ denotes either the even or odd multipole moment $m_{l,k}^e$ or $m_{l,k}^o$, and I_i denotes the integrals

$$I_i = \begin{cases} (1/\pi) \int_0^{2\pi} d\theta \phi(r=r_i) \cos(l\theta) & \text{for } l \neq 0, \\ (1/2\pi) \int_0^{2\pi} d\theta \phi(r=r_i) & \text{for } l = 0, \end{cases} \quad (B3a)$$

$m_{l,k} = m_{l,k}^e$, and

$$I_i = \frac{1}{\pi} \int_0^{2\pi} d\theta \phi(r=r_i) \sin(l\theta) \quad (B3b)$$

for $m_{l,k} = m_{l,k}^o$. If $n = (k_{\max} - l)/2$, then the matrix in Eq. (B2) is a square matrix of dimension $n \times n$, and the system can, in principal, be solved by the usual numerical methods for the vector of multipole moments $(m_{l,l}, m_{l,l+2}, \dots, m_{l,k_{\max}})$. In actual practice, this method of solution is not generally acceptable due to the possibility of numerical noise rendering the matrix nearly singular. To avoid this problem, n is chosen such that $n > (k_{\max} - l)/2$ and the *overconstrained* vector of multipole moments in Eq. (B2) is solved for using a linear least squares method.

Equation (B2) for the truncated vector of multipole moments $(m_{l,l}, m_{l,l+2}, \dots, m_{l,k_{\max}})$ forms the basis of our numerical calculations. In applying this

equation, ϕ was first calculated on a three-dimensional cartesian grid using a detailed SOR fieldsolver in the particle-in-cell simulation WARP3d. This fieldsolver returns ϕ_{i_x, i_y, i_z} , where i_x , i_y , and i_z denote the x -, y -, and z - grid indices labeled from 0 to n_x , n_y , and n_z , with corresponding grid increments dx , dy , and dz . Also, the minimum grid coordinates at $i_x = i_y = i_z = 0$ are $x = x_{\min}$, $y = y_{\min}$, and $z = z_{\min}$. The moments are calculated at each axial grid index i_z by numerically solving Eq. (B2) at each corresponding gridpoint. In this procedure, we first fix the n distinct radial data points by setting

$$r_i = (n_{\text{gmin}} + i - 1) \sqrt{\frac{dx^2 + dy^2}{2}} \quad (B4)$$

with $i = 1, 2, \dots, n$ and $n_{\text{gmin}} > 1$ an integer fixing the minimum data point radius. In the usual situation where the quadrupole is centered on a square transverse grid with $dx = dy$, note that the radial data points will correspond to transverse grid points along the principal x and y axes. Take the quadrupole center or other axis about which the moments are to be calculated to have transverse coordinates $x = x_c$ and $y = y_c$. Then using area weighting to interpolate the potential from the nearest grid points, the $l \neq 0$ azimuthal integral $I_i = (1/\pi) \int_0^{2\pi} d\theta \phi(r = r_i) \cos(l\theta)$ in Eq. (B3a) can be approximated as

$$I_i = \frac{2}{n_\theta} \sum_{i_\theta=1}^{n_\theta} \cos[2\pi(i_\theta - 1)/n_\theta] \{ \phi_{i_x, i_y, i_z} [(i_x + 1)dx - x_c][(i_y + 1)dy - y_c] + \phi_{i_x+1, i_y, i_z} [x_c - i_x dx][(i_y + 1)dy - y_c] + \phi_{i_x, i_y+1, i_z} [(i_x + 1)dx - x_c][y_c - i_y dy] + \phi_{i_x+1, i_y+1, i_z} [x_c - i_x dx][y_c - i_y dy] \} / (dx dy), \quad (B5)$$

and the expressions for $I_i = (1/\pi) \int_0^{2\pi} d\theta \phi(r = r_i) \sin(l\theta)$ and $I_i = (1/2\pi) \int_0^{2\pi} d\theta \phi(r = r_i)$ can be obtained by replacing $\cos[\dots]$ with $\sin[\dots]$ and $1/2$, respectively. In Eq. (B5), n_θ is the number of points used to discretize the azimuthal integral $\int_0^{2\pi} d\theta \dots$, and the grid indices i_x and i_y are given by

$$i_x = \text{Int}\{x_c - x_{\min} + r_i \cos[2\pi(i_\theta - 1)/n_\theta]\} / dx, \\ i_y = \text{Int}\{y_c - y_{\min} + r_i \sin[2\pi(i_\theta - 1)/n_\theta]\} / dy,$$

where $\text{Int}\{\dots\}$ denotes the integer truncation of $\dots$. To avoid interpolating the potential multiple times from the same set of transverse grid points in the discretized azimuthal integral in Eq. (B5), n_θ can be chosen such that

$$n_\theta = \text{Int} \left\{ 2\pi \frac{r_i}{\sqrt{(dx^2 + dy^2)/2}} \right\}. \quad (B6)$$

Furthermore, the minimum grid index n_{gmin} that determines the r_i through Eq. (B4) must be chosen appropriately. Setting $n_{\text{gmin}} = 1$ would result in a coarse approximation to the discretized azimuthal integral, at least for the first few radial data points. On the

other hand, taking n_{gmin} too large would result in data points at large radii being used in the calculation, rendering the radial cutoff at $k = k_{\text{max}}$ questionable. When $l \geq 1$, a suitable choice for n_{gmin} can be estimated as follows. The angular period associated with the azimuthal variations of the moment $m_{l,k}$ [$\sim \cos(l\theta)$ and $\sim \sin(l\theta)$] is $\theta_p = 2\pi/l$. On the other hand, for the data point at radius r_i , the azimuthal integrals I_i are discretized with an angular increment $\delta\theta \simeq \sqrt{(dx^2 + dy^2)}/2/r_i$. Thus, if $n_r \geq 2$ summed points are required in an angular period θ_p of the discretized integral, then it follows that $n_{\text{gmin}} = 1 - i + \text{Int}\{(n_r - 1)l/2\pi\}$. To insure that the resolution criteria will be well satisfied for all data points, we fix n_{gmin} with the data point of smallest radius (r_1) corresponding to $i = 1$, obtaining $n_{\text{gmin}} = \text{Int}\{(n_r - 1)l/2\pi\}$. For the case of $l = 0$ or l small, it is also not desirable to let n_{gmin} become too small or too few steps will be employed in the discretized azimuthal integral. A suitable minimum value is $n_{\text{gmin}} = 2$. Therefore, we take

$$n_{\text{gmin}} = \text{Max} \left\{ \begin{array}{c} \text{Int}\{(n_r - 1)l/2\pi\}, \\ 2, \end{array} \right. \quad (\text{B7})$$

where, Max denotes the maximum of the list of arguments. For a typical requirement of $n_r = 9$ points of resolution over the azimuthal angular period of interest and a higher-order moment with $l = 6$, Eq. (B7) yields $n_{\text{gmin}} = 7$. This choice should also allow a reasonable number of radial data points (n) to be used for typical transverse grid resolutions.

With the radial data points chosen and the integrals approximated as outlined above, Eq. (B2) can be numerically solved for the vector of multipole moments $(m_{l,1}, m_{l,1+2}, \dots, m_{l,k_{\text{max}}})$ using standard linear least squares methods. The procedure is carried out for $i_z = 0, 1, \dots, n_z$ to obtain the z -dependence of the vector of multipole moments, and can be repeated for each azimuthal mode number l of interest to numerically calculate the three dimensional multipole structure of the lens. To apply the least squares method, the number of data points n should be chosen such that $n > (k_{\text{max}} - l)/2$ is well-satisfied so that the results will not be sensitive to numerical noise, but not so large that the radial cutoff becomes a poor approximation at the maximum radial data point where $r = r_n$ [i.e., for a good cutoff $(r_n/a)^{k_{\text{max}}} \ll 1$]. For a high degree of noise immunity, it is appropriate to implement the linear least squares solution of Eq. (B2) using singular value decompositions. To explain this method we write Eq. (B2) in matrix form as

$$[r/a] \cdot (m) = (I). \quad (\text{B8})$$

Here, we have denoted the $n \times (k_{\text{max}} - l)/2$ matrix of powers of r/a as $[r/a]$, the $(k - l)/2$ length vector of multipole moments $m_{l,k}$ as (m) , and the n length vector of integrals I_i as (I) . The best solution for (m) in a least squares sense will minimize the square of the error vector $(E) = [r/a] \cdot (m) - (I)$. The solution to this problem is given simply as

$$(m) = \sum_{i=1}^{(k_{\text{max}}-l)/2} \{(U_i) \cdot (I)\}(V_i), \quad (\text{B9})$$

where (U_i) and (V_i) are vectors of length n and $(k_{\text{max}} - l)/2$ formed from the columns of the matrices $[U]$ and $[V]$ defined in terms of the usual singular-value decomposition of the matrix $[r/a]$ as

$$[r/a] = [U][W][V]^T. \quad (\text{B10})$$

Here, $[U]$ is an $n \times (k_{\max} - l)/2$ column-orthogonal matrix, $[W]$ is a $(k_{\max} - l)/2 \times (k_{\max} - l)/2$ diagonal matrix with positive or zero elements, and $[V]^T$ is the transpose of an $(k_{\max} - l)/2 \times (k_{\max} - l)/2$ orthogonal matrix $[V]$. Routines to accomplish this matrix decomposition are standard in many numerical math libraries. Sample singular-value decomposition routines along with more detailed explanations of linear least squares methods are presented by W.H. Press *et. al* in *Numerical Recipes, The Art of Scientific Computing* (Cambridge University Press, New York, 1986).

Fig. 1. Diagram of the small recirculator matching section.

Fig. 2. Transverse cross section of a dipole biased electrostatic quadrupole.

Fig. 3. Dipole biased electrostatic quadrupole as a linear superposition of three simpler quadrupoles, two purely dipole biased along the x - and y -axes, and the third a conventional quadrupole configuration.

Fig. 4. Contours of the electrostatic potential ϕ in the axial midplane of a dipole biased quadrupole with $V_q = 0$, $V_x = 1$, and $V_y = 0$.

Fig. 5(a). Multipole moment $m_{1,1}$ (dipole moment) versus the axial coordinate z for a dipole biased quadrupole centered at $z = 0.254$ meters with $V_q = 0$, $V_x = 1$, and $V_y = 0$.

Fig. 5(b). Multipole moment $m_{1,3}$ versus the axial coordinate z for a dipole biased quadrupole centered at $z = 0.254$ meters with $V_q = 0$, $V_x = 1$, and $V_y = 0$.

Fig. 5(c). Multipole moment $m_{1,5}$ versus the axial coordinate z for a dipole biased quadrupole centered at $z = 0.254$ meters with $V_q = 0$, $V_x = 1$, and $V_y = 0$.

Fig. 5(d). Multipole moment $m_{1,7}$ versus the axial coordinate z for a dipole biased quadrupole centered at $z = 0.254$ meters with $V_q = 0$, $V_x = 1$, and $V_y = 0$.

Fig. 5(e). Multipole moment $m_{3,3}$ (sextupole moment) versus the axial coordinate z for a dipole biased quadrupole centered at $z = 0.254$ meters with $V_q = 0$, $V_x = 1$, and $V_y = 0$.

Fig. 5(f). Multipole moment $m_{3,5}$ versus the axial coordinate z for a dipole biased quadrupole centered at $z = 0.254$ meters with $V_q = 0$, $V_x = 1$, and $V_y = 0$.

Fig. 5(g). Multipole moment $m_{3,7}$ versus the axial coordinate z for a dipole biased quadrupole centered at $z = 0.254$ meters with $V_q = 0$, $V_x = 1$, and $V_y = 0$.

Fig. 5(h). Multipole moment $m_{5,5}$ (decapole moment) versus the axial coordinate z for a dipole biased quadrupole centered at $z = 0.254$ meters with $V_q = 0$, $V_x = 1$, and $V_y = 0$.

Fig. 5(i). Multipole moment $m_{5,7}$ versus the axial coordinate z for a dipole biased quadrupole centered at $z = 0.254$ meters with $V_q = 0$, $V_x = 1$, and $V_y = 0$.

Fig. 6. Geometry of two dipole biased quadrupole lenses for beam steering.

Fig. 7. Transverse cross section of a dipole biased electrostatic quadrupole with asymmetric dipole biases.

Fig. 8. Dipole biased electrostatic quadrupole with asymmetric dipole biases as a linear superposition of an ideal dipole biased quadrupole, a pure quadrupole, and a distorted quadrupole.

Fig. 9. Contours of the electrostatic potential ϕ in the axial midplane of a quadrupole configuration with conductors along the x -axis biased to unit potential and conductors along the y -axis grounded.

Fig. 10(a). Multipole moment $d_{0,0}$ (scaled on-axis potential) versus the axial coordinate z for a quadrupole configuration centered at $z = 0.254$ meters and with conductors along the x -axis biased to unit potential and conductors along the y -axis grounded.

Fig. 10(b). Multipole moment $d_{0,2}$ verses the axial coordinate z for a quadrupole configuration centered at $z = 0.254$ meters and with conductors along the x -axis biased to unit potential and conductors along the y -axis grounded.

Fig. 10(c). Multipole moment $d_{0,4}$ verses the axial coordinate z for a quadrupole configuration centered at $z = 0.254$ meters and with conductors along the x -axis biased to unit potential and conductors along the y -axis grounded.

Fig. 10(d). Multipole moment $d_{0,6}$ verses the axial coordinate z for a quadrupole configuration centered at $z = 0.254$ meters and with conductors along the x -axis biased to unit potential and conductors along the y -axis grounded.

Fig. 10(e). Multipole moment $d_{2,2}$ (quadrupole moment) verses the axial coordinate z for a quadrupole configuration centered at $z = 0.254$ meters and with conductors along the x -axis biased to unit potential and conductors along the y -axis grounded.

Fig. 10(f). Multipole moment $d_{2,4}$ verses the axial coordinate z for a quadrupole configuration centered at $z = 0.254$ meters and with conductors along the x -axis biased to unit potential and conductors along the y -axis grounded.

Fig. 10(g). Multipole moment $d_{2,6}$ verses the axial coordinate z for a quadrupole configuration centered at $z = 0.254$ meters and with conductors along the x -axis biased to unit potential and conductors along the y -axis grounded.

Fig. 10(h). Multipole moment $d_{4,4}$ (octupole moment) verses the axial coordinate z for a quadrupole configuration centered at $z = 0.254$ meters and with conductors along the x -axis biased to unit potential and conductors along the y -axis grounded.

Fig. 10(i). Multipole moment $d_{4,6}$ verses the axial coordinate z for a quadrupole configuration centered at $z = 0.254$ meters and with conductors along the x -axis biased to unit potential and conductors along the y -axis grounded.

Fig. 11(a). Position errors in the final centroid coordinate ($\sqrt{x^2 + y^2}$) as a function of the an initial centroid error x_0 with $y_0 = x'_0 = y'_0 = 0$ (solid curve) and another initial error y_0 with $x_0 = x'_0 = y'_0 = 0$ (dashed curve).

Fig. 11(b). Divergence errors in the final centroid coordinate ($\sqrt{x'^2 + y'^2}$) as a function of the an initial centroid error x_0 with $y_0 = x'_0 = y'_0 = 0$ (solid curve) and another initial error y_0 with $x_0 = x'_0 = y'_0 = 0$ (dashed curve).

Fig. 11(c). Position errors in the final centroid coordinate ($\sqrt{x^2 + y^2}$) as a function of the an initial centroid error x'_0 with $x_0 = y_0 = y'_0 = 0$ (solid curve) and another initial error y'_0 with $x_0 = y_0 = x'_0 = 0$ (dashed curve).

Fig. 11(d). Divergence errors in the final centroid coordinate ($\sqrt{x'^2 + y'^2}$) as a function of the an initial centroid error x'_0 with $x_0 = y_0 = y'_0 = 0$ (solid curve) and another initial error y'_0 with $x_0 = y_0 = x'_0 = 0$ (dashed curve).

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Subject: Estimate of emittance growth from rotated quadrupoles...

Estimate of emittance growth from rotated quadrupoles with application to the Small Recirculator matching section

I. Introduction

When the alignment of the electrostatic quadrupoles in the recirculator matching section was examined by EG&G it was found that, although the displacement of the quads from the centerline was small, the quadrupoles were rotated about the longitudinal (z) axis by as much as 8 mrad. Rotated *dipoles* will cause the beam centroid to wander off axis, (since they give the beam an impulse in the vertical direction), but rotated quads will not cause an initially aligned beam centroid to become misaligned. Quad rotations do however, create a coupling between the two transverse directions: x (in the bend plane) and y (in the vertical direction). Since this coupling enters linearly in the equations of motion, we felt that it was prudent to estimate the effects on the emittance of the quad rotations that were measured by EG&G. In order to make the estimate, we derived a set of moment equations, which serve as a generalization to the conventional envelope equations. We shall show that even under linear equations of motion, the beam emittance in the laboratory frame is not conserved, and that the beam acquires a finite angular momentum, which is also not conserved during the passage of the beam through the rotated quadrupoles.

II. Equations of Motion

To obtain an estimate, we assume that the force on an ion comes from two sources only: The external focusing from a purely quadrupolar field, and the space charge of the beam (image forces have been neglected). For the purposes of this calculation we assume that the space charge is distributed in a uniform density ellipse, but we allow the semi-axes of the ellipse and the rotation angle of the ellipse to evolve as a function of the axial coordinate z . We assume that the quadrupoles are rotated by an angle θ from the x -axis, and that the beam is rotated by an angle α from the x -axis. The relation between the coordinates in the quadrupole frame (indicated by subscript 0) and the lab frame (no subscript) are given by:

$$x = x_0 \cos \theta - y_0 \sin \theta; \quad y = y_0 \cos \theta + x_0 \sin \theta \quad (1)$$

Similarly, the relation between the coordinates in the rotated beam frame (in which the beam semi-axes are parallel to the coordinate axes and are indicated by subscript b) and the lab frame are given by:

$$x - \langle x \rangle = x_b \cos \alpha - y_b \sin \alpha; \quad y - \langle y \rangle = y_b \cos \alpha + x_b \sin \alpha \quad (2)$$

Here $\langle \cdot \rangle$ indicates a statistical average over the distribution function. For a non-relativistic beam moving at constant velocity βc , the paraxial equations of motion can then be written as:

$$x'' = K_{qxx}x + K_{qxy}y + K_{sxx}(x - \langle x \rangle) + K_{sxy}(y - \langle y \rangle) \quad (3)$$

$$y'' = K_{qyy}y + K_{qyx}x + K_{syy}(y - \langle y \rangle) + K_{syx}(x - \langle x \rangle) \quad (4)$$

Here prime (') indicates derivative with respect to z and

$$\begin{aligned} K_{qxx} &\equiv K_{qx0} \cos^2 \theta + K_{qy0} \sin^2 \theta = K_{qx0} \cos 2\theta \\ K_{qxy} &\equiv (K_{qx0} - K_{qy0})(\sin \theta \cos \theta) = K_{qx0} \sin 2\theta \\ K_{qyx} &\equiv (K_{qx0} - K_{qy0})(\sin \theta \cos \theta) = K_{qxy} \\ K_{qyy} &\equiv K_{qy0} \cos^2 \theta + K_{qx0} \sin^2 \theta = -K_{qx0} \cos 2\theta \\ K_{sxx} &\equiv K_{sxb} \cos^2 \alpha + K_{syb} \sin^2 \alpha \\ K_{sxy} &\equiv (K_{sxb} - K_{syb})(\sin \alpha \cos \alpha) \\ K_{syx} &\equiv (K_{sxb} - K_{syb})(\sin \alpha \cos \alpha) = K_{sxy} \\ K_{syy} &\equiv K_{syb} \cos^2 \alpha + K_{sxb} \sin^2 \alpha \end{aligned}$$

and where

$$K_{qx0} \equiv \pm \frac{1}{r_p^2} \frac{q e V_q}{\frac{1}{2} A m_p \beta^2 c^2}; \quad K_{qy0} \equiv -K_{qx0} \quad (5)$$

$$K_{sxb} \equiv \frac{K}{2(\Delta x_b^2 + (\Delta x_b^2 \Delta y_b^2)^{1/2})}; \quad K_{syb} \equiv \frac{K}{2(\Delta y_b^2 + (\Delta x_b^2 \Delta y_b^2)^{1/2})} \quad (6)$$

and Δx_b^2 and Δy_b^2 are the moments in the rotated beam frame:

$$\Delta x_b^2 = \Delta x^2 \cos^2 \alpha + \Delta y^2 \sin^2 \alpha + 2\Delta xy \cos \alpha \sin \alpha \quad (7)$$

$$\Delta y_b^2 = \Delta y^2 \cos^2 \alpha + \Delta x^2 \sin^2 \alpha - 2\Delta xy \cos \alpha \sin \alpha \quad (8)$$

Here, $K \equiv 2qI/(\beta^3 A I_0)$ is the perveance, q is the charge state of the ions, A is the atomic mass of the ions, β is the velocity of the ions in units of c , $I_0 \equiv 4\pi\epsilon_0 m_p c^3/e$ is the proton Alfven current ($\cong 31$ MA), m_p is the proton mass, I is the ion beam current, V_q is the voltage on a quad (with centerline at zero voltage), r_p is half the aperture distance, and the operator Δ is defined (as in ref. 1) by $\Delta ab = \langle ab \rangle - \langle a \rangle \langle b \rangle$ (e.g. $\Delta x^2 \equiv \langle x^2 \rangle - \langle x \rangle^2$), where $\langle \cdot \rangle$ indicates average over particles.

Note that the space charge force, is just the force obtained from the potential of a uniform density ellipse (ref. 2), but where the semi-axes a and b have been replaced by $2(\Delta x_b^2)^{1/2}$ and $2(\Delta y_b^2)^{1/2}$, respectively, and where the location of the centroid determines the zero point of the space charge force.

The beam rotation angle α may be expressed in terms of second order moments. From eq. (2), $\Delta x^2 - \Delta y^2 = (\Delta x_b^2 - \Delta y_b^2) \cos 2\alpha$ and $\Delta xy = (1/2)(\Delta x_b^2 - \Delta y_b^2) \sin 2\alpha$, so that

$$\tan 2\alpha = \frac{2\Delta xy}{\Delta x^2 - \Delta y^2} \quad (9)$$

In deriving eq. 9 we have made use of the fact that $\Delta xy_b = 0$.

III. Moment Equations

Let the distribution function f , be the number of particles dN per unit transverse phase space volume,

$$f(x, x', y, y', z) = \frac{dN}{dx dx' dy dy'}$$

The evolution of f is described by the Vlasov/Boltzmann Equation:

$$\frac{\partial f}{\partial z} + x' \frac{\partial f}{\partial x} + x'' \frac{\partial f}{\partial x'} + y' \frac{\partial f}{\partial y} + y'' \frac{\partial f}{\partial y'} = 0. \quad (10)$$

The equations of motion for x'' and y'' may be substituted into eq. 10.

The average of a variable ξ over the continuous distribution is given by:

$$\langle \xi \rangle(z) \equiv \int \int \int \int \xi f(x, x', y, y', z) dx dx' dy dy'.$$

Using integration by parts, it is then straightforward to calculate the evolution of the following second order moments:

$$\begin{aligned} \frac{d\Delta x^2}{dz} &= 2\Delta x x' \\ \frac{d\Delta x x'}{dz} &= \Delta x'^2 + K_{xx}\Delta x^2 + K_{xy}\Delta x y \\ \frac{d\Delta x'^2}{dz} &= 2K_{xx}\Delta x x' + 2K_{xy}\Delta x' y \\ \frac{d\Delta y^2}{dz} &= 2\Delta y y' \\ \frac{d\Delta y y'}{dz} &= \Delta y'^2 + K_{yy}\Delta y^2 + K_{yx}\Delta x y \\ \frac{d\Delta y'^2}{dz} &= 2K_{yy}\Delta y y' + 2K_{yx}\Delta x y' \\ \frac{d\Delta x y}{dz} &= \Delta x' y + \Delta x y' \\ \frac{d\Delta x' y}{dz} &= \Delta x' y' + K_{xx}\Delta x y + K_{xy}\Delta y^2 \\ \frac{d\Delta x y'}{dz} &= \Delta x' y' + K_{yy}\Delta x y + K_{yx}\Delta x^2 \\ \frac{d\Delta x' y'}{dz} &= K_{xx}\Delta x y' + K_{xy}\Delta y y' + K_{yy}\Delta x' y + K_{yx}\Delta x x' \end{aligned} \quad (11)$$

Similarly the evolution of the first order moments is given by:

$$\begin{aligned} \frac{d\langle x \rangle}{dz} &= \langle x' \rangle \\ \frac{d\langle y \rangle}{dz} &= \langle y' \rangle \\ \frac{d\langle x' \rangle}{dz} &= K_{qxx}\langle x \rangle + K_{qxy}\langle y \rangle \\ \frac{d\langle y' \rangle}{dz} &= K_{qyy}\langle y \rangle + K_{qyx}\langle x \rangle \end{aligned} \quad (12)$$

In eq. 11,

$$\begin{aligned} K_{xx} &\equiv K_{qxx} + K_{sxx} \\ K_{yy} &\equiv K_{qyy} + K_{syy} \end{aligned}$$

$$\begin{aligned} K_{xy} &\equiv K_{qxy} + K_{sxy} \\ K_{yx} &\equiv K_{qyx} + K_{syx} = K_{xy}. \end{aligned}$$

Note that the first order moment equations depend only on first order moments, and the second order moment equations depend only on second order moments, so that the ten equations for the second order moments form a closed set and the four equations for the first order moments form another closed set. The centroid motion is thus decoupled from the envelope motion calculated with respect to the centroid.

The normalized rms emittance defined along the x and y lab frame axes are defined by:

$$\begin{aligned} \epsilon_{nx} &\equiv 4\beta (\Delta x^2 \Delta x'^2 - (\Delta x x')^2)^{1/2} \\ \epsilon_{ny} &\equiv 4\beta (\Delta y^2 \Delta y'^2 - (\Delta y y')^2)^{1/2} \end{aligned}$$

Using eqs. (11) we can calculate the derivatives of ϵ_{nx}^2 and ϵ_{ny}^2 (again assuming constant β):

$$\begin{aligned} \frac{d\epsilon_{nx}^2}{dz} &= 32\beta^2 K_{xy} (\Delta x^2 \Delta x' y - \Delta x y \Delta x x') \\ \frac{d\epsilon_{ny}^2}{dz} &= 32\beta^2 K_{yx} (\Delta y^2 \Delta x y' - \Delta x y \Delta y y') \end{aligned} \quad (13)$$

Since the rotated quads will induce finite correlations between x and y the normalized rms emittance is not conserved.

We may also define a quantity $l \equiv \Delta x y' - \Delta x' y$, which is proportional to the z component of the angular momentum. Again using eqs. (11) and some manipulations using the definitions, eqs. (4), (5) and (9), we find,

$$\frac{dl}{dz} = (K_{qyy} - K_{qxx}) \Delta x y + K_{qxy} (\Delta x^2 - \Delta y^2) \quad (14)$$

As can be seen from eq. 14, the angular momentum is not necessarily conserved when the quads are rotated. Physically, after a beam has passed through a quad the beam will in general be elliptical. On passing through a quad rotated relative to the first, the principal axis of the elliptical beam will not align with the quadrupole axes and a torque will be applied to the beam, causing a rotation of the beam. (Note also that eq. 14, does not depend on the self space-charge forces of the beam, as expected).

Because the focusing strength is a function of z , the effective external potential well within which the beam travels is z dependent, and so the transverse beam energy H is also not a constant in z . However, in the hard-edge model, within each quadrupole and drift section the focusing strength is assumed constant, and therefore the transverse energy is constant. We may use the result of ref. [1], adding the kinetic and potential energy terms to obtain a total transverse energy. To obtain the potential energy of the beam in the external quad field, we transform Δx_0^2 and Δy_0^2 to the lab frame. The result is

$$2H = \Delta x'^2 + \Delta y'^2 - K_{qx0} ((\Delta x^2 - \Delta y^2) \cos 2\theta + 2\Delta x y \sin 2\theta) - K \ln ((\Delta x_b^2)^{1/2} + (\Delta y_b^2)^{1/2}) \quad (15)$$

Here Δx_b^2 and Δy_b^2 may be expressed in laboratory quantities using eq. (8).

A Mathematica code was written to integrate the eqs. (11). The code uses a Runge-Kutta routine from ref. [3], which allows a fixed step size, in the integration routine. The results are plotted in figures 1 and 2. A plot of H vs. z (figure (e)) shows the constancy of the energy in each drift or quad segment, and a plot of l vs. z shows that the angular momentum is conserved in each drift section, both of which help to give some confidence in the code results.

IV. Application to the Matching Section

Measurements of the matching section alignment that were made by Don Sharver of EG&G are presented in table 1. x_1 is the position in x of the middle of the upper electrode relative to the middle of the channel as defined by the point halfway between the left and right electrodes, and x_2 is the position in x of the lower electrode. Similarly, y_1 is the position in y of the left electrode, relative to the midway point between the upper and lower electrodes, and y_2 is this position measured for the right electrode. All of these measurements are in units of thousandths of an inch (mils). δx is the difference between x_2 and x_1 , and similarly $\delta y = y_2 - y_1$. We may define rotation angles $\delta\theta_x = \delta x/2r_p$ and $\delta\theta_y = \delta y/2r_p$, where r_p is the half aperture (1 inch). All but three of the measurements indicate that $|\delta x - \delta y| < 2$ mils, indicating that the quad misalignments were generally consistent with a pure rotation. The rotation angles found were less than 8 mrad. We have used the maximum (in absolute value) of the two measurements (θ_{max}) for the values of θ that appear in eq. 11. We have used only those quads designated by asterisks in table 1, since these are quads that will remain after the SBTE transport section is converted to the small recirculator matching section. Two cases are presented in figures 1 and 2. The first figure shows the integration of eqs. 11, with zero quad rotation errors. The parameters were chosen to be the same as those used by Steve Lund in ref. 4. The equations (when the rotation errors are identically zero) are then exactly equivalent to the conventional envelope equations, and reproduce the results of ref. 4 to three significant figures. In these equations the emittance is not assumed to be constant, and there was only a slight numerical increase of emittance of .0002%, giving confidence in the code results. When the maximum angles of the EG&G measurements were used for the rotation angles of the quadrupoles, the final rms emittance shows an increase of 4% (having reached a maximum increase of 6%). This is an acceptable growth of emittance, since a factor of 2 increase in normalized emittance is allotted from injector to insertion section. The scaling of the emittance growth is such that if all of the rotation angles θ are multiplied by a constant factor μ , then $\epsilon_{nx}^2(z_{final}) - \epsilon_{nx}^2(z_{initial}) \propto \mu^2$.

V. Is this emittance growth 'real'?

Since the equations of motions are linear, it may be of some surprise that the emittance changes at all. In order to examine whether there is a frame where the emittance has not grown, at each point in z we may calculate the emittance in a frame rotated by an angle ψ , and *rotating* with instantaneous rotation rate ψ' . The emittance in such a frame is given by:

$$\begin{aligned}
\epsilon_{nz}(\psi, \psi', z) = 4\beta & \left((\Delta x^2 \cos^2 \psi + \Delta y^2 \sin^2 \psi + 2\Delta xy \cos \psi \sin \psi) \right. \\
& \times (\Delta x'^2 \cos^2 \psi + \Delta y'^2 \sin^2 \psi + 2\Delta x'y' \cos \psi \sin \psi) \\
& + 2\psi' (-\Delta xx' \sin \psi \cos \psi + \Delta yy' \sin \psi \cos \psi - \Delta xy' \sin^2 \psi + \Delta x'y \cos^2 \psi) \\
& + \psi'^2 (\Delta x^2 \sin^2 \psi + \Delta y^2 \cos^2 \psi - 2\Delta xy \cos \psi \sin \psi) \\
& - (\Delta xx' \cos^2 \psi + \Delta xy' \cos \psi \sin \psi + \Delta x'y \cos \psi \sin \psi + \Delta yy' \sin^2 \psi + \\
& \left. \psi' (-\Delta x^2 \cos \psi \sin \psi + \Delta y^2 \cos \psi \sin \psi - \Delta xy \sin^2 \psi + \Delta xy \cos^2 \psi) \right)^{1/2} \quad (16)
\end{aligned}$$

We then used Mathematica to find the minimum emittance as a function of ψ and ψ' . The value of the minimized normalized emittance is plotted in figure 4, together with the values of the x and y normalized emittance in the lab frame. As can be seen, the minimum emittance remains very nearly equal to the original emittance, throughout the entire length of the matching section. (There are small ($< 0.1\%$) spikes which occur when the beam is nearly round. At these points ψ is rapidly varying, and so numerical inaccuracies may be a cause of these spikes. However, the spikes scale with rotation angle error, which may not be consistent with numerical errors). A comparison of the plot of the angle ψ_{min} at which the emittance is minimized, and the rotation angle α of the beam indicates that, except at the points where α undergoes rapid change (again when the beam is nearly round), the curves are very nearly equal. However, a comparison of the rate of change of α with the ψ' which minimizes the emittance (ψ'_{min}) shows that $\psi'_{min} \neq \alpha'$ indicating that the rotating beam frame is not the frame which minimizes the emittance.

Assuming that in some rotated (and rotating) beam frame the normalized emittance is conserved, is the emittance growth real? The results of a two slit scanner experiment would, in an ideal experiment, show an emittance given by figure 2 (e) and so we would conclude that there was growth. All of the second order moments would also be calculable, though, and the minimum emittance could then also be calculated. If the beam rotation rate were large, it would, in principal, be possible to correct for the rotation of the beam, by some set of rotated quads, but it may be impractical. If not corrected, the beam with finite angular momentum, which translates into a larger emittance in the laboratory frame, will presumably focus down to a larger final spot than a non-rotating beam.

VI. Conclusion

We have estimated the emittance growth from rotated quads in the matching section, and have found the emittance growth to be at an acceptable level. We have used a formulation, in which the major assumption is that the space charge force can be calculated by assuming that the beam remains a uniform density ellipse with a shape that evolves in z . Under this assumption we have derived a set of moment equations which generalize the conventional envelope equations. We have found the misalignments cause the beam to acquire an overall angular momentum, and an increase in emittance measured relative to

fixed laboratory axes. Particle-in-cell results, however, are required to validate this method of estimating emittance growth from rotated quads. Finally, we point out that this method may also be applied to estimate rotation alignment tolerances in the ring itself.

VII. Acknowledgements

I wish to acknowledge Don Sharver, Chuck Ward, and Dave Longinotti, for providing and explaining the measurements of quad alignment; Steve Lund for useful discussions and suggestions regarding the constancy of the emittance in a rotated frame, and for critically reading a draft version of this note, and additionally for pointing out the Mathematica Runge-Kutta algorithm in ref. 3; Fred Deadrick, Tom Fessenden, and Steve Lund for asking illuminating questions about quad misalignments during our visit to EG&G; and Alex Friedman and Bill Sharp for additional useful comments.

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- [3]. Roman Maeder "Programming in Mathematica," [Addison-Wesley, Redwood City, CA], Second Edition, 1991, p. 259.
- [4]. Steve Lund, Recirculator Note 31, "Envelope solution for the recirculator matching section," July 11, 1994.

Table 1

Quad No.	x_1 (mil)	x_2 (mil)	δx (mil)	θ_x (mrad)	y_1 (mil)	y_2 (mil)	δy (mil)	θ_y (mrad)	θ_{max} (mrad)
1*	-2.6	-4.1	-1.5	-0.8	-2.2	-1.3	-0.9	-0.5	-0.8
2†*	2.6	-7.4	-10.0	-5.0	-1.5	-2.3	-0.8	-0.4	-5.0
3*	1.2	-1.5	-2.7	-1.4	0.0	-1.7	-1.7	-0.9	-1.4
4	2.8	-1.1	-3.9	-2.0	0.7	-2.2	-2.9	-1.5	-2.0
5*	1.0	2.4	1.4	-0.7	0.7	-2.1	-2.8	-1.4	-1.4
6	2.7	0.0	-2.7	-1.4	2.1	-2.1	-4.3	-2.2	-2.2
7†*	5.4	-4.0	-9.4	-4.7	7.2	-6.1	-13.3	-6.7	-6.7
8	7.0	-6.1	-13.1	-6.6	5.4	-9.0	-14.4	-7.2	-7.2
9*	4.7	-4.5	-9.2	-4.6	4.6	-4.5	-9.1	-4.6	-4.6
10†	5.1	-7.2	-12.3	-6.2	6.4	-9.7	-16.1	-8.1	-8.1
11*	3.5	-8.1	-11.6	-5.8	3.6	-7.0	-11.6	-5.8	-5.8
12	7.4	-4.3	-11.7	-5.9	5.4	-6.4	-11.8	-5.9	-5.9

* indicates quadrupoles which will remain intact for the recirculator matching section.

† indicates quads for which $|\delta x - \delta y| > 2$ mil, and so the assumption of a pure rotation is a poor approximation.

Figure Captions

1. Second order moment evolution with NO quad rotation errors. (a). Conventional envelope parameters, and quad focusing strength. (b). The six second order moments which do not involve x - y coupling. (c). The x and y normalized emittance, and the four second order moments which are coupled in x and y . (d). θ vs. z , α vs. z , and α' vs. z . (e). The normalized emittance on an expanded scale (upper plot) showing that apart from small numerical errors is constant, and the transverse energy (middle plot) showing that it is piecewise constant. The lower plot shows l (proportional to the z component of the angular momentum), showing vs. z , which in the case of no rotational errors is identically zero.

2(a)-(e). Same as figure 1, except the quad rotation errors that were used are the θ_{max} for the quads which will remain in the matching section, as measured at EG&G. In figure (e), note that the emittance is no longer conserved, and that the transverse energy remains piecewise constant. Also note that the rotational misalignments have induced a finite beam angular momentum and that the angular momentum is constant within drift sections.

3. Minimum normalized emittance plotted along with the laboratory normalized emittance; a zoomed in minimum normalized emittance; the beam rotation angle at minimum normalized emittance; and z derivative of beam rotation angle at minimum normalized emittance.

Figure 1 (a): No quad rotation errors

Figure 1 (b): No quad rotation errors
(mks units)

Figure 1 (c): No quad rotation errors
(mks units)

Figure 1 (d): No quad rotation errors

Figure 1 (e): No quad rotation errors

Output from 'momentrunge.m':

Input Parameters:

zvalues(m)={0., 0.1016, 0.2032, 0.254, 0.3556, 0.4064, 0.508, 0.7112,
> 0.8128, 1.016, 1.1176, 1.3208, 1.4224, 1.6256, 1.7272, 1.9812}
Quad voltage values (kV)={0., 2., 0., -4.1, 0., 3.5, 0., -2.184, 0., 2.135,
> 0., -2.096, 0., 1.959, 0.}
Quad rotation angles (theta) (radians)=
> {0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.}
Energy (eV)=80000
Current=0.002

Initial normalized x emittance (pi meter-rad)=5. 10⁻⁸
Initial normalized y emittance (pi meter-rad)=5. 10⁻⁸
perv=0.000358418
rpipe=0.0254
chargestate q =1.
ion mass (amu)=39.
rigidityo=0.255229
thetarms (rad) (avg of (delta theta)^2)^(1/2)=0.
dthetarms (rad) (avg of (delta theta-thetaavg)^2)^(1/2)=0.
thetaavg (avg of theta) (rad)=0.
nmax (number of points plotted per z interval)=40
plotting and integration step dz (m)=0.00651282

Derived Parameters (mks units):

a0=0.005 b0=0.005
amax=0.0152952 bmax=0.014619
amin=0.00500002 bmin=0.00499997
amax-amin=0.0102952 bmax-bmin=0.00961905
afinal=0.00911828 bfina1=0.014619
ap0=0. bp0=0.
apmax=0.0476525 bpmax=0.041328
apmin=-0.0258154 bpmin=-0.0385775
apmax-apmin=0.0734678 bpmax-bpmin=0.0799055
apfinal=-0.0179289 bpfinal=0.0259404

Emittance values (pi meter-rad):

emitnx (initial)=5. 10⁻⁸ emitny (initial)=5. 10⁻⁸
emitnx (max)=5.00001 10⁻⁸ emitny (max)=5.00001 10⁻⁸
emitnx (min)=5. 10⁻⁸ emitny (min)=5. 10⁻⁸
emitnx (final)=5.00001 10⁻⁸ emitny (final)=5.00001 10⁻⁸

Time used=1875.7 sec.

Thu Sep 8 16:10:29 PDT 1994

Input and output parameters for plots in figure 1:
Quads with no quad rotation errors

Figure 2 (a): Quads with finite rotation errors

Figure 2 (b): Quads with finite rotation errors
(mks units)

Figure 2 (c): Quads with finite rotation errors
(in units)

Figure 2 (d): Quads with finite rotation errors

Figure 2 (e): Quads with finite rotation errors

Output from 'momentrunge.m':

Input Parameters:

zvalues(m)={0., 0.1016, 0.2032, 0.254, 0.3556, 0.4064, 0.508, 0.7112,

> 0.8128, 1.016, 1.1176, 1.3208, 1.4224, 1.6256, 1.7272, 1.9812}

Quad voltage values (kV)={0., 2., 0., -4.1, 0., 3.5, 0., -2.184, 0., 2.135,

> 0., -2.096, 0., 1.959, 0.}

Quad rotation angles (theta) (radians)=

> {0., -0.0008, 0., -0.005, 0., -0.0014, 0., -0.0014, 0., -0.0067, 0.,

> -0.0046, 0., -0.0058, 0.}

Energy (eV)=80000

Current=0.002

Initial normalized x emittance (pi meter-rad)=5. 10⁻⁸

Initial normalized y emittance (pi meter-rad)=5. 10⁻⁸

perv=0.000358418

rpipe=0.0254

chargestate q =1.

ion mass (amu)=39.

rigidityo=0.255229

thetarms (rad) (avg of (delta theta)^2)^(1/2)=0.00429701

dthetarms (rad) (avg of (delta theta-thetaavg)^2)^(1/2)=0.00223269

thetaavg (avg of theta) (rad)=-0.00367143

nmax (number of points plotted per z interval)=80

plotting and integration step dz (m)=0.00321519

Derived Parameters (mks units):

a0=0.005 b0=0.005

amax=0.015293 bmax=0.0146196

amin=0.00500002 bmin=0.00499997

amax-amin=0.010293 bmax-bmin=0.0096196

afinal=0.00912148 bfinal=0.0146196

ap0=0. bp0=0.

apmax=0.0476559 bpmax=0.041328

apmin=-0.0258002 bpmin=-0.0385725

apmax-apmin=0.0734562 bpmax-bpmin=0.0799005

apfinal=-0.0179036 bpfinal=0.0259563

Emittance values (pi meter-rad):

emitnx (initial)=5. 10⁻⁸ emitny (initial)=5. 10⁻⁸

emitnx (max)=5.29538 10⁻⁸ emitny (max)=5.29538 10⁻⁸

emitnx (min)=5. 10⁻⁸ emitny (min)=5. 10⁻⁸

emitnx (final)=5.20426 10⁻⁸ emitny (final)=5.20426 10⁻⁸

Time used=3852.9 sec.

Thu Sep 8 20:07:10 PDT 1994

Input and output parameters for plots in figure 2:
Quads with finite rotation errors

Figure 3.

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Introduction

Perhaps the most important diagnostic planned for the small recirculator experiment is the non-intercepting capacitive pick-up probe. We are relying on this type of probe to determine the beam intensity and position within the pipe during acceleration. Fred Deadrick proposed this diagnostic in Recirculator Note #10. This last summer Joshua Rintamaki studied the operation of a C-probe (HIF Note # 94-2) in a small bench set up using a metal rod to simulate the beam. He showed that the diagnostic gives an output signal that is reasonably linear with beam position. However, his description of the physics of the device was a bit brief and we felt that a more complete description was needed. Moreover, with only a slight increase in complexity, we realized that it should be possible to obtain both the x and y position of the beam within the pipe as well as the total line charge density at each C-probe location. Therefore, with considerable assistance from *Mathematica*, we have developed a detailed solution for the case of a line charge of λ Coulombs/meter within the circular beam tube.

Physics Model

Consider the C-probe sketched in Fig 1. Here a beam of charge/meter λ is a normalized distance ρ off center at an angle ϕ to the horizontal. As a result of the presence of the beam, charges Q_1 , Q_2 , Q_3 , and Q_4 are induced on the four 90° sections of the conducting surface that surrounds the beam. The problem then becomes one of finding an expression for the beam position and intensity in terms of the four charges Q_1 , Q_2 , Q_3 , and Q_4 .

Fig. 1 Sketch of the C-probe problem and geometry

There are several methods of solving this field problem. Perhaps the most customary way is by the method of images wherein an image line-charge of value $-\lambda$ at an angle ϕ and position R/ρ from the origin is assumed. It can be shown that the circle of radius R is an equipotential in the field of the two line charges. The field component normal to the conduction surface gives the distribution of charge around its circumference. From this distribution the total charge on each 90° section can be calculated and the voltage on each section determined. Note we have assumed that the gaps between plates do not significantly change the distribution of charges. A somewhat more elegant solution that is described in detail in the Appendix can be obtained using complex variable techniques. One finds the charge on the circumference to be given by

$$\sigma(\vartheta) = \frac{\lambda}{2\pi R} \left[\frac{1 - \rho^2}{1 + \rho^2 - 2\rho \cos(\vartheta - \phi)} \right] \quad (1)$$

where $\sigma(\vartheta)$ is the surface charge distribution on the conducting surfaces. Upon integrating this expression over the conducting surface at R (easier using complex variable techniques) one obtains

the sum of the four charges $Q_1 + Q_2 + Q_3 + Q_4 = \lambda L$ as expected. Here L is the axial length of the C-probe. The total charge on the first sector is then given by

$$Q_1/L = \frac{\lambda}{2\pi} \int_0^{\pi/2} \frac{1-\rho^2}{1+\rho^2-2\rho\cos(\vartheta-\varphi)} d\vartheta \quad (2)$$

Similar expressions exist for the other three sectors. Consider the following definitions:

$$X \equiv \frac{(Q_1 + Q_4 - Q_2 - Q_3)}{(Q_1 + Q_2 + Q_3 + Q_4)}; \quad Y \equiv \frac{(Q_1 + Q_2 - Q_3 - Q_4)}{(Q_1 + Q_2 + Q_3 + Q_4)} \quad (3)$$

Using the expression for the charge around the circumference given in Eq. (1) and integrating from 0 to π as in Eq. 2, one finds an expression for the variable X to be given by

$$X = \frac{1}{2\pi} \left\{ 4 \tan^{-1} \left[\frac{1+\rho}{1-\rho} \tan(\pi/4 + \varphi/2) \right] + 4 \tan^{-1} \left[\frac{1+\rho}{1-\rho} \tan(\pi/4 - \varphi/2) \right] - 2\pi \right\}. \quad (4)$$

In terms of the Cartesian variables one finds the rather formidable

$$X = \frac{1}{2\pi} \left\{ 4 \tan^{-1} \left[\frac{1+\sqrt{x^2+y^2}}{1-\sqrt{x^2+y^2}} \tan \left(\pi/4 + \frac{\tan^{-1}(y/x)}{2} \right) \right] + 4 \tan^{-1} \left[\frac{1+\sqrt{x^2+y^2}}{1-\sqrt{x^2+y^2}} \tan \left(\pi/4 - \frac{\tan^{-1}(y/x)}{2} \right) \right] - 2\pi \right\}. \quad (5)$$

Here x and y are the spatial coordinates divided by the pipe radius R . A similar expression can be found for Y . One can show that X and Y are zero at the origin. For small values of x and y

$$X \approx 4/\pi x; \quad Y \approx 4/\pi y \quad (x, y \ll 1) \quad (6)$$

or

$$x \approx \pi/4 X; \quad y \approx \pi/4 Y. \quad (7)$$

Within the circle $\sqrt{x^2+y^2} = 0.5$, the approximate expressions given in Eq. 7 are accurate to better than 7%. Within $\sqrt{x^2+y^2} = 0.25$ the error becomes less than 2%. For most situations these should be adequate for determining the beam position within the diagnostic. Needless-to-say, one must multiply the variables x and y by the pipe radius R to obtain the actual offset in centimeters or inches.

Using *Mathematica* we investigated the function given in Eqs (4) and (5) in detail. Fig 2 is a three dimensional plot of Eq. (5) over the interior of the C-probe. The plot for *Y* is identical except rotated counter-clockwise by 90°.

Fig. 2 Three dimensional plot of $X(x,y)$.

A plot of X versus x for parametric y is given in Fig. 3.

Fig. 3 Plot of $X(x,y)$ versus x for parametric y values from 0 to 0.9 in steps of 0.1

Perhaps the most useful plot is contained in Fig 4. This is a contour plot of both functions X and Y for values from -0.9 to 0.9 in steps of 0.1. Thus for a particular experimental situation, if the values of X and Y are measured, the position of the beam within the pipe can be determined.

Fig. 4 Contour Plots of X and Y. The solid lines are contours of constant X in increments of 0.1. The dotted lines are corresponding contours of constant Y

Electrical Considerations

Now that we know what it is we have to measure we need only figure out a way to do it. Actually, Fred Deadrick and Josh Rintamaki concentrated on this aspect of the C-probe system in their earlier notes. Since they have essentially covered this subject, we will add only a few new thoughts.

A circuit of the type sketched in Fig. 5 should produce the desired signal voltages. Here the charge on the four sectors is converted to a voltage signal by an external capacitor C_{ext} . The voltage is then amplified and buffered by an operational amplifier that drives the signal into the control room. This circuit must be located as close as possible to the C-probe.

Fig 5 Sketch of an analog C-probe circuit

The total charge induced on the probe sectors is simply the beam line-charge-density times the axial length of the sector L . Assuming a sector length of 0.1 m and a 2 mA potassium beam at 80 kV, we find that approximately 0.32 nC total or 0.08 nC per sector will appear on the plates. Using an external capacitor (C_{ext}) of 0.1 nF as suggested by Deadrick in Note #10 will produce a voltage level near 0.8 volts on each of the sectors. Note that it is important to keep the effective capacitance of each of the four circuits extremely well balanced. A difference of 5% will cause an on-axis beam to appear $5\pi/4$ % off axis. The capacitance of the plates will be near 0.05 nF (50 pf). Stray lead capacitance (≈ 30 pf/foot of cable) will also contribute to the capacitance. These effects can be minimized by using a larger external capacitor at the expense of signal strength. An external capacitance near 1 nF (1000 pf) producing signals near 32 mV might prove to be a better compromise.

If the series resistance of the operational amplifier R_s is 100 k we find that the signal will droop with a time constant of 3 μ sec for a 0.1 nF capacitor and 30 μ sec if 1.0 nF is used. This can be compensated by adjusting the time constant of feedback resistor and capacitor to equal the input droop time. The operational amplifier can also be used to increase the signal level.

Appendix I

Solution for the charge distribution around the pipe circumference

The field problem is solved using conformal mapping techniques. We look for the analytic function $\Phi(z)$ of the complex variable z generated by a singularity at point z_0 within the unit circle. The function $\Phi(z)$ is sometimes called the complex potential. The real part of $\Phi(z)$ is the familiar solution to Laplace's equation. We look for a mapping into the ζ plane that takes the point z_0 to the origin and the circle $|z| = 1$ to $|\zeta| = 1$. Consider the sketch shown in Fig. A1

Fig A1

Mapping of the complex z plane to the ζ plane

The point z_0 maps to the origin of the ζ plane

Once the mapping is found the problem is essentially finished. The mapping we are looking for is

$$\zeta = \frac{z - z_0}{z z_0^* - 1}, \quad z = \frac{\zeta - z_0}{\zeta z_0^* - 1} \quad (\text{A1})$$

We next solve for the potential generated by a line charge of strength λ located at z_0 in the plane and at the origin in the ζ plane. The solution in the ζ plane is found by inspection to be simply

$$\Phi(z) = \frac{\lambda}{2\pi\epsilon_0 R} \log \zeta \quad (\text{A2})$$

The solution for the complex potential in the z plane is then found by substituting the expression in equation A1 into Eq. A2. The charge distribution $\sigma(\vartheta)$ around the boundary $|z| = 1$ is found by evaluating

$$\sigma(\vartheta) = \epsilon_0 \left. \frac{\partial \Phi(z)}{\partial r} \right|_{r=1}; \quad \text{where } z = r e^{i\vartheta}. \quad (\text{A3})$$

We next proceed by chain differentiating according to

$$\left. \frac{\partial \Phi(z)}{\partial r} \right|_{r=1} = \left. \frac{\partial \Phi(\zeta)}{\partial \zeta} \frac{\partial \zeta}{\partial z} \frac{\partial z}{\partial r} \right|_{r=1} \quad (\text{A4})$$

Using $z_0 = \rho e^{i\varphi}$ and cranking through yields the desired result

$$\sigma(\vartheta) = \frac{\lambda}{2\pi R} \left[\frac{1 - \rho^2}{1 + \rho^2 - 2\rho \cos(\vartheta - \varphi)} \right] \quad (\text{A5})$$

This explanation is rather brief. If you really want to understand this derivation study chapter 4 of **Functions of a Complex Variable**, G.F. Carrier, M. Krook, and C.E. Pearson.

Appendix 2. Alternative capacitive probe orientations

Although we have focused on a capacitive probe in which four gaps lie at $\vartheta = 0, 90, 180$, and 270° (our favored orientation), other possibilities exist. We have also looked at detail at the configuration where the gaps between plates lie at $45, 135, 225$, and 315° (see figure A2).

Fig. A2 An alternate orientation of the capacitive probe

We again integrated the charge density on each plate, and formed alternative expressions to measure the position of the beam:

$$X_1 \equiv \frac{(Q_1 - Q_3)}{(Q_1 + Q_2 + Q_3 + Q_4)}; \quad Y_1 \equiv \frac{(Q_2 - Q_4)}{(Q_1 + Q_2 + Q_3 + Q_4)} \quad (\text{A6})$$

As in our favored orientation, the expressions become linear at small x and y :

$$X_1 \equiv 2 \frac{\sqrt{2}}{\pi} x \quad Y_1 \equiv 2 \frac{\sqrt{2}}{\pi} y \quad (x, y \ll 1) \quad (\text{A7})$$

This configuration has one slight advantage, in that X_1 and Y_1 can be formed by subtracting only two voltages, whereas in the previous configuration X and Y are formed by subtracting sums of two voltages each. This was not deemed an important advantage, since the electrical circuit in figure 5, is still straightforward. This configuration has the drawback that the signal strength is weaker by the ratio of the proportionality constants in eqs. (6) and (A7) (a 29 % decrease in signal strength). Figure A3 shows the contour plot of X_1 and Y_1 in the x, y plane analogous to figure 4. A second disadvantage is apparent when the beam is at a large distance from the origin. At these positions, the determination of the beam location would be problematic.

As can be seen from the plot, if, for example $\pi/4 < \varphi < 3\pi/4$ and ρ is near unity, all of the induced charge would collect on the top plate, so that X_1 would be near zero, independent of the actual value of x . Nevertheless, in the regime that we hope to operate in, this configuration would be satisfactory, in that the accuracy of eq. (A7) is comparable to that of eq. (6).

A larger number of plates may also have advantages, since in principle this would yield more detailed information about the charge density around the circumference of the pipe, which may yield more information about the structure of the beam. We have not yet examined in detail the use of capacitive probes for measurement of higher order moments of the beam.

Fig A3
Contour plots for X_1 and Y_1 for the probe orientation of Fig. A2

To: HIF Note Mailing List
From: Dave Grote
Subject: Analysis of the dipole plate shape and location

I. Introduction

The shape and location of the electric dipole plates for the small recirculator have a significant effect on the behavior of the beam. The fringe fields out of the plane of the bend have a focusing effect on the beam. They also have a significant sextupole component ($\Phi \sim r^3 \cos(3\phi)$) which can distort the beam's transverse profile and lead to nonlinear beam focusing with associated emittance growth. The axial fringe fields also focus the beam and have a significant sextupole component as well. The energy effect in the dipole also focuses the beam, with a strength that varies with the location of the potential zero. Also, the energy effect can give a net timing advance or lag to the beam.

The goal of the design is to minimize the detrimental effects on the beam from the plates. This is accomplished by making the focusing in the two transverse planes the same, by minimizing the timing errors, and by minimizing the nonlinear (sextupole) fields. Two plate shapes are examined, flat plates and sector plates. Ridges on the top and bottom of the plates are used to minimize the nonlinear sextupole field.

Figure 1 shows the geometry of the flat dipole plates and figure 2 shows the sector plates.

II. Ridges to reduce the sextupole component of the potential

The fringe fields (both out of the bend plane and axially) produce a sextupole component which has a significant effect on the beam. Figure 3 shows what the beam looks like after four laps with no ridges on the plates. It's not very pleasant.

An acceptable approach in designing dipole plates is to add enough ridges so that the sextupole in the center of the dipole is small. Typically, this means adding ridges with a height that is about 10% of the distance from the plate to the center. Unfortunately, in our case there is also significant sextupole contributions from the axial fringe fields. Cancelling the sextupole only in the center of the dipole is not enough. The approach I have taken is to add the appropriate ridges which make the average of the sextupole component over the full dipole field region (including fringe fields) small.

I first looked at flat plates of equal size that were shifted outward in x 2.3 mm from the centerline. In WARP3d, the dipole plates are made up of grid cells in a Cartesian mesh where the potential is held at the appropriate value. To make the ridges, I added grid cells to the ends of the plates until I could not further reduce the average of the sextupole component. The reduction was limited by the resolution of the computational mesh. The procedure I followed was to stack (in x) grid cells on the furthest ends of the plates (at fixed y), which reduced the average sextupole, until the sign of the average changed. I then removed that last cell and started stacking cells one grid spacing further in, in y , until the sign again changed. I continued until I could not add another grid cell without changing the sign of the average.

Figure 4 shows what the ridge looks like after the above procedure. Each box represents a

grid cell, whose physical size is 1.15 by 1.15 mm. Also shown in the figure is a hand drawn guess of what the ridges could look like. Figure 5 shows the sextupole component with and without the ridges for flat plates centered about $z = -15.6$ cm. Without the ridges, the average sextupole was about 27 volts at $2/3$ the pipe radius (2.46 mm). With the ridges, the average sextupole was reduced to under 1 volt. Figure 6 shows the potential contours at the axial midplane of the dipole. About the same reduction in the average sextupole was obtained for sector plates by using the same ridges.

The ridge described above is not the only possible ridge shape which would make the average of the sextupole small. Triangular ridges were also examined. Some results are shown in table 1. The three triangular ridges each reduce the average to nearly the value obtained with the more carefully designed ridges above. As above, the reduction is limited by the grid resolution. Interpolating between the first two rows to find the triangle which makes the average vanish gives a triangle with a height and width of 12.84 mm. The ridges examined had little effect on the phase advance.

III. Flat versus sector plates

Tables 2 and 3 give the undepressed phase advances and timing changes for flat plates, without and with the ridges described above, as a function of the location of the plates. Table 4 gives the same data for sector plates with ridges. Radial means in the x direction. The phase advances were calculated by following a single particle that started out off axis (5 mm in either x or y) for 80 lattice periods. The number of betatron oscillations made by the particle times the ratio $360/80$ gives the phase advance. This method averages the focusing over the transverse range of the betatron oscillation of the particle. With the initial offset given, that range covered most of the beam size. That method was used since the transfer matrix method is only valid for linear fields. The transfer matrix method was tried anyway and the results it gave were dependent on the starting positions of the particles.

The timing change results from the axial variation of the potential through the dipole. As a particle traverses a dipole, its curved path takes it off the centerline through the dipole making its axial velocity either higher or lower than its nominal axial velocity. That change in velocity causes the particle to get ahead of or behind a (fictitious) particle which would maintain its nominal velocity. The beam, on average, will experience a timing advance or lag depending on the location of the potential zero of the dipole relative to the beam center. Moving the plates out radially puts the beam closer to the inner plate which is at a negative potential. More of the beam then has a higher velocity in the dipole, giving a larger net timing advance to the beam. To avoid having the complication of including these timing changes in the timing of the acceleration and axially confining fields, the plates should be located so that the timing change is zero.

The change is calculated by advancing a single particle, starting on axis, as many time steps as would be needed to traverse a full period at the nominal axial velocity and noting where it actually ended up. If the particle travelled further than one lattice period, it got a positive timing advance. The advance is calculated by dividing the distance travelled past the one period by the nominal velocity.

In all cases, as the plates were shifted, the voltage on the plate was adjusted to maintain the correct bending strength. The correct strength was found by sending a single particle that started on axis in the center of a quadrupole through one full lattice period and adjusting the voltage until the particle also exited on the axis.

In both cases with ridges, the location where the x and y focusing are equal is not the same

place where the timing change is zero. For the flat plates, interpolation gives a radial shift of 2.6 mm to have no timing change. There, the focusing in x is significantly higher than in y . In order to meet both conditions, plate shaping is necessary to have the x and y focusing match. One of two plate shapings can be done. The first is curving the plates in y , the second is to make the plates different sizes. In Recirculator Note 23, Dave Judd theoretically showed that spherical sector plates could result in equal x - and y -focusing with zero timing error. However, this analysis did not include fringe field, and WARP3d is currently unable to easily model plates which are curved in y . Simulation of plates curved in y is therefore left for future work.

The focusing in x from the sector plates is higher than that of the flat plates. In the simulations, this has proved advantageous as the additional focusing of the sector plates keeps the beam smaller as the beam is accelerated. This reduces the emittance growth. The emittance histories of two runs, with flat plates and with sector plates, are shown in figure 7. It needs to be noted that these runs were with a time step that other simulations showed was too big (i.e. the emittance growth is partly of numerical origin). I believe that with a smaller time step, there still will be a difference in the emittance growth of the two cases.

IV. Smaller inner plate

As flat plates are easier to build and align, I examined whether the focusing in x can be increased by shortening the inner plate. Table 5 shows the effect of such shortening. The ridges maintained the same shape, but were shortened appropriately on the inner plate. As above, the voltage was adjusted to maintain the correct bending strength. To obtain the same increase in x -focusing as from sector plates, the length of the inner plate needs to be shortened to about 9.5 cm. The current design of the plates does include a differential in the inner and outer plate lengths. The lengths are the same as in the sector plates but without the curvature. That is close to row 3 of table 5.

To try to match the focusing in x and y , the inner plate was also made narrower in the vertical y -plane. The first try I made did not work well. The data is shown in table 6. The only significant effect narrowing the inner plate had was to increase the sextupole component. The phase advance in y was essentially unaltered. When the inner plate was narrowed, all other parameters, including the shape of the ridges were kept fixed. Perhaps if the shape of the ridges were also adjusted, the focusing in y could be increased.

V. Conclusions

Several adjustments to the shape and location of the electric dipole plates were investigated to improve the beam optics. For flat plates, the radial location of the plates needs to be shifted out about 2 mm from the centerline of the system so that the timing errors from the energy effect are small. Ridges should also be put on the top and bottoms of the plate to reduce the average sextupole component in the dipole. Several possible shapes for these ridges were found. Increasing the x -focusing of the dipoles was shown to be advantageous for minimizing the beam size and thereby reducing the emittance growth. This can be accomplished by either employing sector dipoles or by making the inner plate of a planar dipole shorter. With flat plates, making the inner plates roughly 9.5 cm long gives the same x -focusing as sector dipoles.

No satisfactory case was found in this report for matching the x - and y -focusing. Further studies will be done to find optimal shapes, including study of plates curved in y as well as further study of changing the size of the flat plates and the ridges. One more option is to shift the plates

out far enough so that the x - and y -focusing do match and including the resulting timing advance in the timing of the accelerator gaps.

One last note is that each of the adjustments has a small effect on the others. Fully optimized plate design would require making the adjustments consistent. Whether or not such fine tuning is needed is not known but seems unlikely. For example, shortening the length of the inner plate did effect the average sextupole field component, but the change in the average was small compared to the initial reduction resulting from the ridges.

Table 1: Triangular shaped ridges. Height is in x direction, width is in y.

height (mm)	width (mm)	average sextupole (volts)
12.7	12.7	0.5
13.8	13.8	-3.5
8.1	24.2	1.1
5.8	34.5	-1.2

Table 2: Flat plates without ridges

radial shift (mm)	σ_{ox}	σ_{oy}	time advance (ns/period)	voltage (volts)
0.	83.7°	71.3°	-1.38	7591
2.3	81.0°	74.3°	-0.16	7612
4.6	78.7°	77.4°	1.06	7621
6.9	75.6°	80.1°	2.28	7616

Table 3: Flat plates with ridges

radial shift (mm)	σ_{ox}	σ_{oy}	time advance (ns/period)	voltage (volts)
0.	83.3°	72.9°	-1.4	7485
2.3	81.0°	74.2°	-0.2	7505
4.6	81.0°	74.3°	1.9	7520
6.9	79.9°	76.5°	2.2	7528
9.2	78.7°	76.5°	3.4	7530

Table 4: Sector plates with ridges

radial shift (mm)	σ_{ox}	σ_{oy}	time advance (ns/period)	voltage (volts)
-2.3	85.5°	69.8°	-1.18	7342
0.0	83.7°	70.7°	0.02	7364
2.3	83.3°	72.0°	1.24	7378
4.6	82.8°	72.5°	2.46	7388
6.9	81.5°	74.3°	3.66	7390

Table 5: Flat plates with ridges and a radial shift of 2.3 mm, shortening the inner plate

inner plate length (cm)	σ_{ox}	σ_{oy}	time advance (ns/period)	voltage (volts)	average sextupole (volts)
10.4	81.0°	74.2°	-0.2	7505	-0.4
9.8	83.2°	72.9°	-0.5	7631	-0.8
9.3	83.7°	72.0°	-0.8	7770	-1.0

Table 6: Flat plates with ridges, a radial shift of 2.3 mm, and 9.3 cm long, narrowing the inner plate

inner plate width (cm)	σ_{ox}	σ_{oy}	time advance (ns/period)	voltage (volts)	average sextupole (volts)
11.06	83.7	72.0	-0.8	7770	-1.0
10.8	84.4	72.0	-0.8	7765	-1.8
10.6	84.4	72.0	-0.8	7760	-2.7

Figure 1: Geometry of flat plates

Figure 2: Geometry of sector plates

Figure 3: Transverse profile of the beam after four laps with no shaping of the dipole plates to reduce sextupole.

Figure 4: Structure of the dipole plate with ridges. a) grid cells which make up the ridge. Each square is 1.15 mm on a side. b) approximate idea of what the ridge could look like. This was drawn with guess work. The 73° angle was given by the dimensions of the ridges. It is about 13 grids high by 4 wide. $\text{atan}(13/4) \sim 73^\circ$. This would be a wedged shaped in cross section.

Figure 5: Sextupole component over one full lattice period: 1) without ridges, 2) with ridges. With the ridges, the average sextupole is reduced from 27 volts to under one volt. The dipole is centered about $z = -0.156$ m.

Figure 6: Profile of the dipole plates and the potential contours.

Figure 7: Emittance histories of a constant length beam (with the final design parameters) over 15 laps with flat plates and sector plates. The larger emittance growth occurs with the flat plates. It needs to be noted these runs were with a time step that other simulations showed was too big. I believe that with a smaller time step, there still will be a difference in the emittance growth of the two cases.

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To: HIF Note Distribution

From: Fred Deadrick

Subject: Deflection Dipole Waveform Generation for the
Small Recirculator

Introduction

The problem of generating the precision high voltage electrical waveforms used to drive the deflection dipoles around the small recirculator presents, at first look, some rather formidable electrical engineering challenge. The voltage on the plates must increase from a few kilovolts to nearly 30 kV in approximately 200 μ s (A dV/dt on the order of $1.5E+8$ volts/sec.). This is a fairly high rate of change in voltage. If there is significant amount of distributed capacitance in the circuit, these rates result in large peak charging currents. Additionally, one would like the dipole voltage to ramp up some specified non-linear fashion. The direct approach would be to have an arbitrary waveform generator to synthesize the waveform, and a high power/high voltage linear amplifier to generate the required high voltage waveform. I'm not sure such amplifiers exist at the 30kV level, and if they do, I'm sure they are not cheap. Switched circuits rather than linear circuits are generally easier to implement, and it is usually easier to discharge capacitors fast than it is to charge them.

I think that I have come up with a relatively simple scheme to do the job. The idea is to take the difference of two voltages: one a constant voltage, and the other a time decreasing voltage. See the circuit shown in figure 1. The net voltage difference can generate a positive going waveform. If we first charge a fixed capacitor to some initial voltage before pulsing the recirculator, and then rely on discharging this capacitor in a controlled manner rather than a charging it while the beam circulates, the voltage difference can approximate the required dipole waveform. By time varying the resistive load used in the discharge of the capacitor a piece-wise fit to the required dipole plate waveform can be realized

Figure 1. Simplified circuit used to generate dipole plate voltages.

Note: It should be noted that by using this technique, the voltage across the plates will not be symmetric with respect to ground potential. This may result in undesirable perturbations to the beam trajectory.

Waveforms and Circuits

To evaluate this idea, I first took numbers from Tom Fessenden's calculations of the dipole voltage on gap number 5 (Reference: Recirculator Note #27) as a starting point. (Note that similar voltages are used on all the plates, and in fact it is planned that all the plates can be wired in parallel for the whole recirculator, however in this case the initial and final voltages will be slightly different from those given here for plate 5 only.) The dipole voltage as a function of time is plotted by the squares in figure 2 below. If one does a curve fit to this data, a second order polynomial in time fits the data very well (the fit error is less than 1%), as indicated by the solid line plot and the coefficients shown in the plot. For purposes of this analysis I will use the polynomial fit to the data as my reference waveform to generate.

Figure 2. Dipole voltage on gap number 5 as a function of time for 15 laps of the recirculator.

The waveform shown in figure 2, however, is not characterized by what one would expect from a discharge circuit--i.e. the voltage is increasing, not decreasing. However, by subtracted figure 2 from a constant voltage, say 30 kV, we get a waveform like that shown in figure 3. This corresponds to the required capacitor voltage.

Figure 3. 30 kV minus the dipole voltage waveform shown as a function of time

Figure 3's waveform could in principle be approximated by resistively discharging a capacitor, however, the shape of the decay is not that of a simple $e^{-t/\tau}$ capacitor discharge, and thus a time-varying τ would be needed.

In figure 1 V_1 is fixed at a constant high voltage, and capacitor C would be charged to an initial voltage such that the difference between the constant DC supply voltage V_1 and the initial charge voltage V_2 , would be the initial dipole plate voltage at time $t=0$. Then, as time increases and the ion beam gains energy as it circulates around the loop, the capacitor C could be discharged such that the difference voltage, V_{out} , approximates the required dipole waveform, figure 2. The problem then is how well can a decaying exponential be made to fit the waveform.

$V_2 = V_0 * e^{-t/\tau}$, where V_0 is the capacitor voltage at $t=0$, and τ is a function of time such that:

$$0 \leq t < t_1 \quad \tau = \tau_1$$

$$t_1 \leq t < t_2 \quad \tau = \tau_2$$

$$t_2 \leq t < t_3 \quad \tau = \tau_3$$

$$t_3 \leq t < t_4 \quad \tau = \tau_4$$

•
•
•

$$t_{n-1} \leq t < t_n \quad \tau = \tau_n$$

To implement this in hardware is simple, the circuit would look like that shown in figure 4. The waveform we want to generate is such that the time constant, $\tau = R_{eff} * C$, decreases with time, so in the circuit, we could sequentially close $S_1, S_2 \dots S_n$ and place the resistors in parallel to decrease the effective value of R in the time constant, i.e. $\tau_1 = R_1 * C$, $\tau_1 = (R_1 \parallel R_2) C$, etc. The sequence of closing of the switches

could be synched, for example to the cycles around the recirculator. Closing the switches, and the number of switched terms are really only dependent on how well one wants the exponential terms to fit the desired waveform.

Figure 4 Implementation of multi-term exponential fit.

Model Results

In order to see how well a series of exponential terms fit the required waveform, I used a spread-sheet calculation of the polynomial fit to the dipole waveform as the reference, and did an approximate fit to the waveform by adjusting the value of τ at the end of each lap. Figure 5 shows the error in approximating the dipole polynomial waveform by a series of 14 corrections to a decaying exponential terms, and the τ values used are listed in Table 1.

Figure 5 Error in exponential fit to dipole waveform. 14 terms used.

My choices for the values of the decay time constant term τ were only approximate, and the mathematics for an optimal least squares fit to the data must be performed. However, as can be seen the fit is quite good, and the errors in the fit are generally better than $\pm 0.5\%$.

Tau1	3.50E-04
Tau2	3.06E-04
Tau3	2.52E-04
Tau4	2.200E-04
Tau5	1.925E-04
Tau6	1.650E-04
Tau7	1.460E-04
Tau8	1.250E-04
Tau9	9.800E-05
Tau10	9.000E-05
Tau11	7.000E-05
Tau12	5.660E-05
Tau13	4.300E-05
Tau14	3.100E-05

Table 1 Time constant terms used in exponential curve fit.

Implementation

The hardware implementation shown in figure 4 would require a array of high voltage switches such as thyratrons. Tom Fessenden suggested an alternate implementation of the form shown in figure 6. Here, for example, the voltage across each resistive load could be limited to some fraction of the total voltage, and it may be practical to consider solid state switches such as high voltage MOSFETs or SCRs. A system of isolated switch control signals would also be needed for such a circuit. The values of the time constants are all well within practical physical space, and with this scheme or the one shown in figure 4, one does not need to be particularly concerned about minimizing the capacitance of the deflection elements and the associated voltage distribution.

Figure 6. Alternate scheme for implementing variable time constant decay circuit.

While the values switched into this circuit were chosen to match a particular discharge rate of the capacitor C, it is possible in principle to implement a binary encoded array of values which could be computer controlled to provide an arbitrary discharge rate. Such a scheme could possibly control the deflection plate voltages in real time in a closed loop feedback system which monitors beam energy. Further thought needs to be devoted to this scheme.

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Subject: Measurement Report from Ron Holsinger
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DATE: Oct. 13, 1994

TO: Dr. Tom Fessenden
M/C L-440

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FROM: Ron Holsinger

MESSAGE:

Attached is a summary of the JB'dz measurements for the 15 PMQ's, which hopefully arrived in Livermore yesterday. Did you receive the envelope with the detailed measurement reports that was included with the shipment? Please get in touch if you have any questions.

Yours,

Ron H.

Strength

70°F	=	21.1°C		80°F	=	26.7°C	
B'L(T)		From Mean	From Spec	B'L(T)		From Mean	From Spec
0.9478		-0.15%	1.91%	0.94249232		-0.15%	1.34%
0.9492		-0.01%	2.06%	0.94388448		-0.01%	1.49%
0.9527		0.36%	2.44%	0.94736488		0.36%	1.87%
0.9512		0.20%	2.28%	0.94587328		0.20%	1.71%
0.9478		-0.15%	1.91%	0.94249232		-0.15%	1.34%
0.9474		-0.20%	1.87%	0.94209456		-0.20%	1.30%
0.9474		-0.20%	1.87%	0.94209456		-0.20%	1.30%
0.9473		-0.21%	1.86%	0.94199512		-0.21%	1.29%
0.9499		0.07%	2.14%	0.94458056		0.07%	1.57%
0.9479		-0.14%	1.92%	0.94259176		-0.14%	1.35%
0.951		0.18%	2.26%	0.9456744		0.18%	1.69%
0.9486		-0.07%	2.00%	0.94328784		-0.07%	1.43%
0.9526		0.35%	2.43%	0.94726544		0.35%	1.86%
0.9496		0.04%	2.11%	0.94428224		0.04%	1.54%
0.9486		-0.07%	2.00%	0.94328784		-0.07%	1.43%
0.94926667				0.94395077			
Mean				Mean

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---- QUADRUPOLE HARMONIC ANALYSIS REPORT ----

----- HAL VERSION 2.1 22-MAR-94 -----

RAW data file NAME: QUAD001.RAW OPERATER: CJM
 MEASUREMENT DATE : 27-SEP-1994 TIME: 17:40:13
 MAGNET PART NUMBER: 101167 MAGNET SERIAL NUMBER: 001
 COIL PART NUMBER : 101410 CONDUCTOR SPACING (MM): .160
 R4 (MM) : 33.020 NO. OF TURNS ON OUTER : 10
 R2 (MM) : -16.510 NO. OF LAYERS ON OUTER: 2
 R3 (MM) : 20.650 NO. OF TURNS ON INNER : 20
 R1 (MM) : -4.140 NO. OF LAYERS ON INNER: 4
 LOW (UNBUCKED) GAIN: 6.640 HIGH (BUCKED) GAIN : 9.970
 MEASUREMENT TYPE : UNBUCKED AND BUCKED
 COMMENTS :

B*XL(EFF) = .9478 TESLA AT 87.32 DEGREES
 MAGNETIC CENTER: .05 MM AT 14.03 DEGREES

BUCKED RESULTS:

BUCKING RATIO : 67.7
 NORMALIZATION RADIUS (MM) : 25.40

N	SIGNAL (UV-SEC)	FIELD (TESLA-METER)	B(N)/B(2) (%)	abs-PHASE (DEGREES)	REL-PHASE (DEGREES)
3	.089	0.105E-04	.044	25.0	123.0
4	.174	0.274E-04	.114	27.4	38.1
5	.054	0.106E-04	.044	-24.5	-101.2
6	.074	0.174E-04	.072	165.8	1.9
7	.026	0.712E-05	.030	-54.0	54.7
8	.035	0.111E-04	.046	25.3	46.7
9	.231	0.820E-04	.341	-47.5	-113.4
10	1.041	0.410E-03	1.702	151.0	-2.3
11	.012	0.508E-05	.021	96.7	-143.9
12	.002	0.778E-06	.003	101.9	134.0
13	.001	0.377E-06	.002	-68.1	-123.3
14	.011	0.583E-05	.024	143.3	.8
15	0.000	0.281E-06	.001	89.6	-140.3
16	.001	0.473E-06	.002	-146.1	-103.3
17	.003	0.178E-05	.007	117.5	73.0
18	.006	0.398E-05	.017	-50.1	178.1

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---- QUADRUPOLE HARMONIC ANALYSIS REPORT ----

----- HAL VERSION 2.1 22-MAR-94 -----

RAW data file NAME: QUAD002.RAW OPERATER: CJM
 MEASUREMENT DATE : 28-SEP-1994 TIME: 18:59:15
 MAGNET PART NUMBER: 101167 MAGNET SERIAL NUMBER: *
 COIL PART NUMBER : 101410 CONDUCTOR SPACING (MM): .160
 R4 (MM) : 33.020 NO. OF TURNS ON OUTER : 10
 R2 (MM) : -16.510 NO. OF LAYERS ON OUTER: 2
 R3 (MM) : 20.650 NO. OF TURNS ON INNER : 20
 R1 (MM) : -4.140 NO. OF LAYERS ON INNER: 4
 LOW (UNBUCKED) GAIN: 6.640 HIGH (BUCKED) GAIN : 9.970
 MEASUREMENT TYPE : UNBUCKED AND BUCKED
 COMMENTS :

B*XL(EFF) = .9492 TESLA AT 87.57 DEGREES
 MAGNETIC CENTER: .05 MM AT -17.41 DEGREES

BUCKED RESULTS:

BUCKING RATIO : 69.0
 NORMALIZATION RADIUS (MM) : 25.40

N	SIGNAL (UV-SEC)	FIELD (TESLA-METER)	B(N)/B(2) (%)	abs-PHASE (DEGREES)	REL-PHASE (DEGREES)
3	.171	0.202E-04	.083	-125.1	-27.8
4	.126	0.199E-04	.082	57.4	67.1
5	.084	0.166E-04	.068	-73.2	-151.1
6	.115	0.271E-04	.111	165.2	-.3
7	.072	0.200E-04	.082	-85.0	22.0
8	.022	0.678E-05	.028	86.1	105.5
9	.007	0.233E-05	.010	99.2	31.0
10	1.022	0.403E-03	1.652	153.0	-2.8
11	.014	0.601E-05	.025	98.3	-145.0
12	.010	0.465E-05	.019	132.3	161.4
13	.002	0.111E-05	.005	-178.0	123.6
14	.010	0.570E-05	.023	134.9	-11.2
15	.001	0.845E-06	.003	174.7	-58.9
16	.001	0.351E-06	.001	-158.0	-119.2
17	0.000	0.111E-06	0.000	68.5	19.7
18	.006	0.409E-05	.017	-48.5	175.1

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---- QUADRUPOLE HARMONIC ANALYSIS REPORT ----

----- HAL VERSION 2.1 22-MAR-94 -----

RAW data file NAME: QUAD003.RAW OPERATER: CJM
 MEASUREMENT DATE : 29-SEP-1994 TIME: 13:10:53
 MAGNET PART NUMBER: 101167 MAGNET SERIAL NUMBER: 003
 COIL PART NUMBER : 101410 CONDUCTOR SPACING (MM): .160
 R4 (MM) : 33.020 NO. OF TURNS ON OUTER : 10
 R2 (MM) : -16.510 NO. OF LAYERS ON OUTER: 2
 R3 (MM) : 20.650 NO. OF TURNS ON INNER : 20
 R1 (MM) : -4.140 NO. OF LAYERS ON INNER: 4
 LOW (UNBUCKED) GAIN: 6.640 HIGH (BUCKED) GAIN : 9.970
 MEASUREMENT TYPE : UNBUCKED AND BUCKED
 COMMENTS :

B*XL(EFF) = .9527 TESLA AT 87.43 DEGREES
 MAGNETIC CENTER: .05 MM AT -1.02 DEGREES

BUCKED RESULTS:

BUCKING RATIO : 69.1
 NORMALIZATION RADIUS (MM) : 25.40

N	SIGNAL (UV-SEC)	FIELD (TESLA-METER)	B(N)/B(2) (%)	abs-PHASE (DEGREES)	REL-PHASE (DEGREES)
3	.314	0.370E-04	.151	123.5	-138.8
4	.237	0.373E-04	.152	-10.2	.1
5	.102	0.201E-04	.082	133.1	56.0
6	.117	0.275E-04	.113	99.3	-65.3
7	.043	0.119E-04	.048	-41.8	66.2
8	.019	0.606E-05	.025	-41.1	-20.5
9	.015	0.535E-05	.022	-91.6	-158.5
10	1.029	0.405E-03	1.657	152.2	-2.0
11	.018	0.789E-05	.032	88.8	-152.9
12	.004	0.175E-05	.007	132.1	162.9
13	.002	0.127E-05	.005	144.6	88.0
14	.010	0.538E-05	.022	142.5	-1.5
15	.001	0.867E-06	.004	146.4	-85.0
16	0.000	0.233E-06	.001	162.4	-156.4
17	.001	0.346E-06	.001	169.8	123.5
18	.006	0.403E-05	.016	-50.4	175.9

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---- QUADRUPOLE HARMONIC ANALYSIS REPORT ----

----- HAL VERSION 2.1 22-MAR-94 -----

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RAW data file NAME: QUAD004.RAW      OPERATER: CJM
MEASUREMENT DATE  : 29-SEP-1994      TIME: 13:28:38
MAGNET PART NUMBER: 101167           MAGNET SERIAL NUMBER: 004
COIL PART NUMBER  : 101410           CONDUCTOR SPACING (MM): .160
R4 (MM)           : .33.020          NO. OF TURNS ON OUTER : 10
R2 (MM)           : -16.510          NO. OF LAYERS ON OUTER: 2
R3 (MM)           : 20.650          NO. OF TURNS ON INNER : 20
R1 (MM)           : -4.140          NO. OF LAYERS ON INNER: 4
LOW (UNBUCKED) GAIN: 6.640          HIGH (BUCKED) GAIN : 9.970
MEASUREMENT TYPE  : UNBUCKED AND BUCKED
COMMENTS :

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B*XL(EFF) = .9512 TESLA AT 87.52 DEGREES

MAGNETIC CENTER: .04 MM AT -55.51 DEGREES

BUCKED RESULTS:

BUCKING RATIO : 69.1

NORMALIZATION RADIUS (MM) : 25.40

N	SIGNAL (UV-SEC)	FIELD (TESLA-METER)	B(N)/B(2) (%)	abs-PHASE (DEGREES)	REL-PHASE (DEGREES)
3	.354	0.418E-04	.171	-103.3	-5.8
4	.254	0.400E-04	.164	45.0	54.9
5	.228	0.449E-04	.184	-93.6	-171.2
6	.051	0.119E-04	.049	156.6	-8.5
7	.033	0.904E-05	.037	43.4	150.7
8	.003	0.952E-06	.004	-68.8	-49.0
9	.006	0.219E-05	.009	20.2	-47.5
10	1.019	0.401E-03	1.643	151.8	-3.4
11	.008	0.340E-05	.014	127.0	-115.7
12	.008	0.382E-05	.016	158.1	-172.1
13	.003	0.139E-05	.006	169.4	111.6
14	.010	0.558E-05	.023	155.8	10.5
15	.001	0.778E-06	.003	103.8	-129.0
16	.001	0.338E-06	.001	165.0	-155.3
17	0.000	0.951E-07	0.000	106.0	58.2
18	.006	0.419E-05	.017	-47.7	177.0

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---- QUADRUPOLE HARMONIC ANALYSIS REPORT ----
----- HAL VERSION 2.1 22-MAR-94 -----

RAW data file NAME: QUAD005.RAW OPERATER: CJM
MEASUREMENT DATE : 30-SEP-1994 TIME: 17:53:41
MAGNET PART NUMBER: 101167 MAGNET SERIAL NUMBER: 005H
COIL PART NUMBER : 101410 CONDUCTOR SPACING (MM): .160
R4 (MM) : 33.020 NO. OF TURNS ON OUTER : 10
R2 (MM) : -16.510 NO. OF LAYERS ON OUTER: 2
R3 (MM) : 20.650 NO. OF TURNS ON INNER : 20
R1 (MM) : -4.140 NO. OF LAYERS ON INNER: 4
LOW (UNBUCKED) GAIN: 6.640 HIGH (BUCKED) GAIN : 9.970
MEASUREMENT TYPE : UNBUCKED AND BUCKED
COMMENTS :

B'XL(EFF) = .9478 TESLA AT 88.19 DEGREES
MAGNETIC CENTER: .01 MM AT -78.08 DEGREES

BUCKED RESULTS:

BUCKING RATIO : 68.3
NORMALIZATION RADIUS (MM) : 25.40

N	SIGNAL (UV-SEC)	FIELD (TESLA-METER)	B(N)/B(2) (%)	abs-PHASE (DEGREES)	REL-PHASE (DEGREES)
3	.279	0.329E-04	.137	132.9	-131.7
4	.048	0.758E-05	.031	-81.5	-74.2
5	.094	0.185E-04	.077	52.8	-28.2
6	.040	0.949E-05	.039	-70.4	120.4
7	.096	0.266E-04	.110	16.5	119.1
8	.059	0.186E-04	.077	61.5	75.9
9	.063	0.223E-04	.093	49.0	-24.7
10	.985	0.388E-03	1.611	158.3	-3.6
11	.020	0.854E-05	.035	120.1	-130.0
12	.005	0.237E-05	.010	100.2	121.9
13	.004	0.191E-05	.008	158.5	92.0
14	.010	0.562E-05	.023	149.4	-5.3
15	.001	0.603E-06	.003	170.9	-72.0
16	.001	0.374E-06	.002	-147.8	-118.8
17	0.000	0.884E-07	0.000	-123.8	177.0
18	.006	0.398E-05	.017	-41.2	171.4

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----- QUADRUPOLE HARMONIC ANALYSIS REPORT -----
 ----- HAL VERSION 2.1 22-MAR-94 -----

RAW data file NAME: QUAD006.RAW OPERATER: CJM
 MEASUREMENT DATE : 30-SEP-1994 TIME: 10:38:38
 MAGNET PART NUMBER: 101167 MAGNET SERIAL NUMBER: 006B
 COIL PART NUMBER : 101410 CONDUCTOR SPACING (MM): .160
 R4 (MM) : 33.020 NO. OF TURNS ON OUTER : 10
 R2 (MM) : -16.510 NO. OF LAYERS ON OUTER: 2
 R3 (MM) : 20.650 NO. OF TURNS ON INNER : 20
 R1 (MM) : -4.140 NO. OF LAYERS ON INNER: 4
 LOW (UNBUCKED) GAIN: 6.640 HIGH (BUCKED) GAIN : 9.970
 MEASUREMENT TYPE : UNBUCKED AND BUCKED
 COMMENTS :

B*XL(EFF) = .9474 TESLA AT 88.23 DEGREES
 MAGNETIC CENTER: .03 MM AT-110.50 DEGREES

BUCKED RESULTS:

BUCKING RATIO : 68.7
 NORMALIZATION RADIUS (MM) : 25.40

N	SIGNAL (UV-SEC)	FIELD (TESLA-METER)	B(N)/B(2) (%)	abs-PHASE (DEGREES)	REL-PHASE (DEGREES)
3	.277	0.327E-04	.136	-17.6	77.7
4	.203	0.320E-04	.133	79.8	86.8
5	.066	0.129E-04	.054	133.6	52.5
6	.061	0.144E-04	.060	132.8	-36.6
7	.049	0.135E-04	.056	28.5	130.9
8	.044	0.139E-04	.058	141.9	156.0
9	.061	0.215E-04	.089	79.6	5.5
10	.997	0.393E-03	1.632	158.6	-3.7
11	.017	0.718E-05	.030	138.4	-112.2
12	.001	0.337E-06	.001	88.4	109.6
13	.004	0.193E-05	.008	111.4	44.4
14	.011	0.589E-05	.024	158.0	2.8
15	.001	0.512E-06	.002	146.6	-96.8
16	0.000	0.162E-06	.001	132.3	160.6
17	0.000	0.149E-06	.001	4.3	-55.7
18	.005	0.357E-05	.015	-33.8	178.0

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----- QUADRUPOLE HARMONIC ANALYSIS REPORT -----
----- HAL VERSION 2.1 22-MAR-94 -----

RAW data file NAME: QUAD007.RAW OPERATER: CJM
MEASUREMENT DATE : 3-OCT-1994 TIME: 11:35:51
MAGNET PART NUMBER: 101167 MAGNET SERIAL NUMBER: 007
COIL PART NUMBER : 101410 CONDUCTOR SPACING (MM): .160
R4 (MM) : 33.020 NO. OF TURNS ON OUTER : 10
R2 (MM) : -16.510 NO. OF LAYERS ON OUTER: 2
R3 (MM) : 20.650 NO. OF TURNS ON INNER : 20
R1 (MM) : -4.140 NO. OF LAYERS ON INNER: 4
LOW (UNBUCKED) GAIN: 6.640 HIGH (BUCKED) GAIN : 9.970
MEASUREMENT TYPE : UNBUCKED AND BUCKED
COMMENTS :

B*XL(EFF) = .9474 TESLA AT 88.32 DEGREES
MAGNETIC CENTER: .02 MM AT-100.63 DEGREES

BUCKED RESULTS:

BUCKING RATIO : 68.9
NORMALIZATION RADIUS (MM) : 25.40

N	SIGNAL (UV-SEC)	FIELD (TESLA-METER)	B(N)/B(2) (%)	abs-PHASE (DEGREES)	REL-PHASE (DEGREES)
3	.143	0.169E-04	.070	-19.6	75.4
4	.185	0.291E-04	.121	-74.9	-68.1
5	.175	0.344E-04	.143	7.6	-74.0
6	.088	0.208E-04	.086	106.7	-63.2
7	.083	0.228E-04	.095	-126.7	-24.9
8	.014	0.444E-05	.018	59.8	73.3
9	.029	0.104E-04	.043	86.8	11.9
10	.994	0.391E-03	1.626	159.6	-3.6
11	.006	0.278E-05	.012	149.2	-102.3
12	.007	0.325E-05	.013	-170.3	-150.1
13	.001	0.729E-06	.003	66.0	-2.2
14	.009	0.515E-05	.021	151.1	-5.4
15	.001	0.769E-06	.003	-155.6	-40.4
16	0.000	0.222E-06	.001	116.8	143.7
17	0.000	0.188E-06	.001	-47.3	-108.7
18	.005	0.375E-05	.016	-34.6	175.7

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---- QUADRUPOLE HARMONIC ANALYSIS REPORT ----

----- HAL VERSION 2.1 22-MAR-94 -----

RAW data file NAME: QUAD008.RAW OPERATER: CJM
 MEASUREMENT DATE : 3-OCT-1994 TIME: 13:39:18
 MAGNET PART NUMBER: 101167 MAGNET SERIAL NUMBER: 008B
 COIL PART NUMBER : 101410 CONDUCTOR SPACING (MM): .160
 R4 (MM) : 33.020 NO. OF TURNS ON OUTER : 10
 R2 (MM) : -16.510 NO. OF LAYERS ON OUTER: 2
 R3 (MM) : 20.650 NO. OF TURNS ON INNER : 20
 R1 (MM) : -4.140 NO. OF LAYERS ON INNER: 4
 LOW (UNBUCKED) GAIN: 6.640 HIGH (BUCKED) GAIN : 9.970
 MEASUREMENT TYPE : UNBUCKED AND BUCKED
 COMMENTS :

B*XL(EFF) = .9473 TESLA AT 87.54 DEGREES
 MAGNETIC CENTER: .05 MM AT -2.35 DEGREES

BUCKED RESULTS:

BUCKING RATIO : 68.4
 NORMALIZATION RADIUS (MM) : 25.40

N	SIGNAL (UV-SEC)	FIELD (TESLA-METER)	B(N)/B(2) (%)	abs-PHASE (DEGREES)	REL-PHASE (DEGREES)
3	.326	0.385E-04	.160	-113.9	-16.5
4	.308	0.485E-04	.201	-39.6	-29.8
5	.146	0.287E-04	.119	-156.6	125.6
6	.207	0.489E-04	.203	129.3	-35.9
7	.023	0.641E-05	.027	10.1	117.3
8	.027	0.860E-05	.036	-165.5	-145.9
9	.007	0.264E-05	.011	126.8	58.9
10	.992	0.390E-03	1.623	153.0	-2.5
11	.011	0.465E-05	.019	128.1	-114.9
12	.001	0.354E-06	.001	-29.4	.1
13	.004	0.209E-05	.009	-169.0	132.9
14	.011	0.597E-05	.025	135.2	-10.4
15	.001	0.840E-06	.003	137.1	-96.1
16	.001	0.669E-06	.003	134.3	173.6
17	.001	0.345E-06	.001	96.5	48.3
18	.005	0.387E-05	.016	-52.4	171.8

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---- QUADRUPOLE HARMONIC ANALYSIS REPORT ----
----- HAL VERSION 2.1 22-MAR-94 -----

RAW data file NAME: QUAD009.RAW OPERATER: CJM
MEASUREMENT DATE : 3-OCT-1994 TIME: 14:32:41
MAGNET PART NUMBER: 101167 MAGNET SERIAL NUMBER: 009D
COIL PART NUMBER : 101410 CONDUCTOR SPACING (MM): .160
R4 (MM) : 33.020 NO. OF TURNS ON OUTER : 10
R2 (MM) : -16.510 NO. OF LAYERS ON OUTER: 2
R3 (MM) : 20.650 NO. OF TURNS ON INNER : 20
R1 (MM) : -4.140 NO. OF LAYERS ON INNER: 4
LOW (UNBUCKED) GAIN: 6.640 HIGH (BUCKED) GAIN : 9.970
MEASUREMENT TYPE : UNBUCKED AND BUCKED
COMMENTS :

B'XL(EFF) = .9499 TESLA AT 88.16 DEGREES
MAGNETIC CENTER: .03 MM AT -64.85 DEGREES

BUCKED RESULTS:

BUCKING RATIO : 68.7
NORMALIZATION RADIUS (MM) : 25.40

N	SIGNAL (UV-SEC)	FIELD (TESLA-METER)	B(N)/B(2) (%)	abs-PHASE (DEGREES)	REL-PHASE (DEGREES)
3	.108	0.128E-04	.053	141.0	-123.5
4	.201	0.317E-04	.131	2.9	10.3
5	.132	0.260E-04	.108	36.7	-44.1
6	.199	0.470E-04	.195	111.6	-57.3
7	.027	0.731E-05	.030	102.5	-154.6
8	.042	0.131E-04	.054	58.0	72.7
9	.046	0.162E-04	.067	57.3	-16.1
10	.999	0.393E-03	1.629	158.2	-3.4
11	.012	0.502E-05	.021	164.9	-84.8
12	.002	0.844E-06	.003	153.2	175.4
13	.001	0.527E-06	.002	131.2	65.1
14	.010	0.554E-05	.023	154.3	.1
15	.002	0.923E-06	.004	162.7	-79.6
16	0.000	0.308E-06	.001	-168.6	-139.1
17	.001	0.427E-06	.002	104.6	45.9
18	.005	0.366E-05	.015	-36.7	176.5

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----- QUADRUPOLE HARMONIC ANALYSIS REPORT -----
 ----- HAL VERSION 2.1 22-MAR-94 -----

RAW data file NAME: QUAD010.RAW OPERATER: CJM
 MEASUREMENT DATE : 4-OCT-1994 TIME: 10: 4:55
 MAGNET PART NUMBER: 101167 MAGNET SERIAL NUMBER: 0101
 COIL PART NUMBER : 101410 CONDUCTOR SPACING (MM): .160
 R4 (MM) : 33.020 NO. OF TURNS ON OUTER : 10
 R2 (MM) : -16.510 NO. OF LAYERS ON OUTER: 2
 R3 (MM) : 20.650 NO. OF TURNS ON INNER : 20
 R1 (MM) : -4.140 NO. OF LAYERS ON INNER: 4
 LOW (UNBUCKED) GAIN: 6.640 HIGH (BUCKED) GAIN : 9.970
 MEASUREMENT TYPE : UNBUCKED AND BUCKED
 COMMENTS :

B*XL(EFF) = .9479 TESLA AT 88.83 DEGREES
 MAGNETIC CENTER: .04 MM AT -16.86 DEGREES

BUCKED RESULTS:

BUCKING RATIO : 68.7
 NORMALIZATION RADIUS (MM) : 25.40

N	SIGNAL (UV-SEC)	FIELD (TESLA-METER)	B(N)/B(2) (%)	abs-PHASE (DEGREES)	REL-PHASE (DEGREES)
3	.195	0.230E-04	.095	-110.6	-17.1
4	.097	0.152E-04	.063	34.3	39.0
5	.088	0.172E-04	.072	-60.4	-144.5
6	.194	0.457E-04	.190	155.5	-17.5
7	.078	0.214E-04	.089	-74.3	23.9
8	.032	0.996E-05	.041	-127.4	-118.0
9	.032	0.112E-04	.046	59.5	-20.0
10	.998	0.393E-03	1.633	165.4	-2.9
11	.015	0.647E-05	.027	142.0	-115.2
12	.007	0.320E-05	.013	-171.0	-157.0
13	.003	0.174E-05	.007	-177.0	108.2
14	.012	0.666E-05	.028	157.5	-6.2
15	.001	0.765E-06	.003	-176.7	-69.2
16	.001	0.435E-06	.002	117.4	136.1
17	0.000	0.184E-06	.001	157.5	87.4
18	.006	0.396E-05	.016	-27.1	173.9

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---- QUADRUPOLE HARMONIC ANALYSIS REPORT ----
----- HAL VERSION 2.1 22-MAR-94 -----

RAW data file NAME: QUAD011.RAW OPERATER: CJM
MEASUREMENT DATE : 3-OCT-1994 TIME: 12:55:26
MAGNET PART NUMBER: 101167 MAGNET SERIAL NUMBER: 011
COIL PART NUMBER : 101410 CONDUCTOR SPACING (MM): .160
R4 (MM) : 33.020 NO. OF TURNS ON OUTER : 10
R2 (MM) : -16.510 NO. OF LAYERS ON OUTER: 2
R3 (MM) : 20.650 NO. OF TURNS ON INNER : 20
R1 (MM) : -4.140 NO. OF LAYERS ON INNER: 4
LOW (UNBUCKED) GAIN: 6.640 HIGH (BUCKED) GAIN : 9.970
MEASUREMENT TYPE : UNBUCKED AND BUCKED
COMMENTS :

B'XL(EFF) = .9510 TESLA AT 86.75 DEGREES
MAGNETIC CENTER: .04 MM AT 13.35 DEGREES

BUCKED RESULTS:

BUCKING RATIO : 68.8
NORMALIZATION RADIUS (MM) : 25.40

N	SIGNAL (UV-SEC)	FIELD (TESLA-METER)	B(N)/B(2) (%)	abs-PHASE (DEGREES)	REL-PHASE (DEGREES)
3	.377	0.446E-04	.185	-93.7	6.1
4	.025	0.391E-05	.016	-146.1	-133.1
5	.055	0.109E-04	.045	-102.5	-176.3
6	.008	0.187E-05	.008	5.9	-154.5
7	.050	0.139E-04	.058	-93.8	19.0
8	.022	0.699E-05	.029	30.2	56.2
9	.031	0.112E-04	.046	-47.9	-108.6
10	1.001	0.394E-03	1.632	145.6	-1.9
11	.016	0.674E-05	.028	167.6	-66.6
12	.005	0.247E-05	.010	143.0	-177.9
13	.004	0.199E-05	.008	125.3	77.6
14	.010	0.554E-05	.023	128.5	-6.0
15	.001	0.512E-06	.002	76.1	-145.1
16	.001	0.450E-06	.002	130.7	-177.2
17	.001	0.663E-06	.003	97.5	62.9
18	.005	0.375E-05	.016	-65.3	173.3

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----- QUADRUPOLE HARMONIC ANALYSIS REPORT -----
 ----- HAL VERSION 2.1 22-MAR-94 -----

RAW data file NAME: QUAD012.RAW OPERATER: CJM
 MEASUREMENT DATE : 5-OCT-1994 TIME: 15:23:44
 MAGNET PART NUMBER: 101167 MAGNET SERIAL NUMBER: 012A
 COIL PART NUMBER : 101410 CONDUCTOR SPACING (MM): .160
 R4 (MM) : 33.020 NO. OF TURNS ON OUTER : 10
 R2 (MM) : -16.510 NO. OF LAYERS ON OUTER: 2
 R3 (MM) : 20.650 NO. OF TURNS ON INNER : 20
 R1 (MM) : -4.140 NO. OF LAYERS ON INNER: 4
 LOW (UNBUCKED) GAIN: 6.640 HIGH (BUCKED) GAIN : 9.970
 MEASUREMENT TYPE : UNBUCKED AND BUCKED
 COMMENTS :

B*XL(EFF) = .9486 TESLA AT 87.88 DEGREES
 MAGNETIC CENTER: .03 MM AT-109.81 DEGREES

BUCKED RESULTS:

BUCKING RATIO : 68.7
 NORMALIZATION RADIUS (MM) : 25.40

N	SIGNAL (UV-SEC)	FIELD (TESLA-METER)	B(N)/B(2) (%)	abs-PHASE (DEGREES)	REL-PHASE (DEGREES)
3	.067	0.788E-05	.033	-151.0	-54.7
4	.257	0.405E-04	.168	7.1	15.5
5	.050	0.981E-05	.041	19.8	-59.6
6	.078	0.185E-04	.077	95.9	-71.4
7	.030	0.835E-05	.035	174.7	-80.5
8	.053	0.167E-04	.069	160.3	177.3
9	.030	0.107E-04	.044	124.2	53.3
10	.979	0.385E-03	1.600	155.3	-3.5
11	.010	0.430E-05	.018	118.1	-128.6
12	.004	0.204E-05	.008	109.0	134.4
13	.001	0.515E-06	.002	-161.7	135.8
14	.009	0.507E-05	.021	149.4	-1.0
15	.002	0.123E-05	.005	130.0	-108.3
16	.001	0.410E-06	.002	166.4	-159.7
17	.001	0.403E-06	.002	131.4	77.3
18	.006	0.390E-05	.016	-40.9	177.2

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---- QUADRUPOLE HARMONIC ANALYSIS REPORT ----

----- HAL VERSION 2.1 22-MAR-94 -----

RAW data file NAME: QUAD013.RAW OPERATER: CJM
 MEASUREMENT DATE : 6-OCT-1994 TIME: 8:52:29
 MAGNET PART NUMBER: 101167 MAGNET SERIAL NUMBER: 013A
 COIL PART NUMBER : 101410 CONDUCTOR SPACING (MM): .160
 R4 (MM) : 33.020 NO. OF TURNS ON OUTER : 10
 R2 (MM) : -16.510 NO. OF LAYERS ON OUTER: 2
 R3 (MM) : 20.650 NO. OF TURNS ON INNER : 20
 R1 (MM) : -4.140 NO. OF LAYERS ON INNER: 4
 LOW (UNBUCKED) GAIN: 6.640 HIGH (BUCKED) GAIN : 9.970
 MEASUREMENT TYPE : UNBUCKED AND BUCKED
 COMMENTS :

B'XL(EFF) = .9526 TESLA AT 88.19 DEGREES

MAGNETIC CENTER: .04 MM AT -56.63 DEGREES

BUCKED RESULTS:

BUCKING RATIO : 68.8

NORMALIZATION RADIUS (MM) : 25.40

N	SIGNAL (UV-SEC)	FIELD (TESLA-METER)	B(N)/B(2) (%)	abs-PHASE (DEGREES)	REL-PHASE (DEGREES)
3	.172	0.203E-04	.084	-126.8	-31.4
4	.145	0.228E-04	.094	106.0	113.2
5	.137	0.270E-04	.111	-141.1	137.9
6	.051	0.121E-04	.050	-167.8	23.0
7	.107	0.295E-04	.122	-45.9	56.8
8	.053	0.167E-04	.069	112.9	127.3
9	.009	0.325E-05	.013	31.3	-42.5
10	1.000	0.394E-03	1.628	158.1	-3.8
11	.013	0.543E-05	.022	143.6	-106.5
12	.006	0.307E-05	.013	160.6	-177.7
13	.004	0.204E-05	.008	146.6	80.1
14	.009	0.501E-05	.021	158.6	3.9
15	.001	0.356E-06	.001	150.5	-92.3
16	.001	0.457E-06	.002	159.6	-171.5
17	0.000	0.159E-06	.001	102.9	43.6
18	.005	0.380E-05	.016	-37.9	174.6

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 ----- HAL VERSION 2.1 22-MAR-94 -----

RAW data file NAME: QUAD014.RAW OPERATER: CJM
 MEASUREMENT DATE : 6-OCT-1994 TIME: 10:53:34
 MAGNET PART NUMBER: 101167 MAGNET SERIAL NUMBER: 014A
 COIL PART NUMBER : 101410 CONDUCTOR SPACING (MM): .160
 R4 (MM) : 33.020 NO. OF TURNS ON OUTER : 10
 R2 (MM) : -16.510 NO. OF LAYERS ON OUTER: 2
 R3 (MM) : 20.650 NO. OF TURNS ON INNER : 20
 R1 (MM) : -4.140 NO. OF LAYERS ON INNER: 4
 LOW (UNBUCKED) GAIN: 6.640 HIGH (BUCKED) GAIN : 9.970
 MEASUREMENT TYPE : UNBUCKED AND BUCKED
 COMMENTS :

B'XL(EFF) = .9496 TESLA AT 86.99 DEGREES
 MAGNETIC CENTER: .05 MM AT -26.47 DEGREES

BUCKED RESULTS:

BUCKING RATIO : 68.7
 NORMALIZATION RADIUS (MM) : 25.40

N	SIGNAL (UV-SEC)	FIELD (TESLA-METER)	B(N)/B(2) (%)	abs-PHASE (DEGREES)	REL-PHASE (DEGREES)
3	.409	0.483E-04	.200	51.7	150.7
4	.060	0.945E-05	.039	-55.7	-43.7
5	.157	0.310E-04	.128	177.2	102.3
6	.061	0.145E-04	.060	89.0	-73.0
7	.055	0.152E-04	.063	-110.2	.9
8	.028	0.894E-05	.037	78.3	102.4
9	.023	0.812E-05	.034	49.9	-13.1
10	1.007	0.397E-03	1.644	146.0	-3.9
11	.017	0.755E-05	.031	92.3	-144.6
12	.001	0.311E-06	.001	.7	36.8
13	.004	0.180E-05	.007	132.2	81.3
14	.010	0.557E-05	.023	132.9	-5.0
15	.002	0.144E-05	.006	155.1	-69.8
16	.001	0.495E-06	.002	148.0	-163.9
17	0.000	0.165E-06	.001	72.2	33.4
18	.005	0.365E-05	.015	-59.9	174.2

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----- HAL VERSION 2.1 22-MAR-94 -----

RAW data file NAME: QUAD015.RAW OPERATER: CJM
MEASUREMENT DATE : 6-OCT-1994 TIME: 10: 1:15
MAGNET PART NUMBER: 101167 MAGNET SERIAL NUMBER: 015
COIL PART NUMBER : 101410 CONDUCTOR SPACING (MM): .160
R4 (MM) : 33.020 NO. OF TURNS ON OUTER : 10
R2 (MM) : -16.510 NO. OF LAYERS ON OUTER: 2
R3 (MM) : 20.650 NO. OF TURNS ON INNER : 20
R1 (MM) : -4.140 NO. OF LAYERS ON INNER: 4
LOW (UNBUCKED) GAIN: 6.640 HIGH (BUCKED) GAIN : 9.970
MEASUREMENT TYPE : UNBUCKED AND BUCKED
COMMENTS :

B*XL(EFF) = .9486 TESLA AT 86.49 DEGREES
MAGNETIC CENTER: .01 MM AT -85.17 DEGREES

BUCKED RESULTS:

BUCKING RATIO : 69.0
NORMALIZATION RADIUS (MM) : 25.40

N	SIGNAL (UV-SEC)	FIELD (TESLA-METER)	B(N)/B(2) (%)	abs-PHASE (DEGREES)	REL-PHASE (DEGREES)
3	.323	0.382E-04	.159	-81.4	19.2
4	.039	0.618E-05	.026	12.7	26.7
5	.069	0.137E-04	.057	62.7	-9.7
6	.152	0.360E-04	.149	-151.3	49.7
7	.070	0.192E-04	.080	-43.3	71.3
8	.021	0.655E-05	.027	130.7	158.8
9	.016	0.567E-05	.024	-84.0	-142.4
10	.998	0.393E-03	1.631	141.6	-3.3
11	.011	0.473E-05	.020	141.6	-89.8
12	.003	0.141E-05	.006	150.9	-167.0
13	.003	0.148E-05	.006	90.5	46.1
14	.011	0.580E-05	.024	133.7	2.8
15	.001	0.615E-06	.003	55.8	-161.6
16	.001	0.560E-06	.002	165.4	-138.5
17	.001	0.501E-06	.002	161.5	131.2
18	.005	0.379E-05	.016	-68.8	174.4

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To: HIF Note Mailing List

From: Steven Lund

Subject: Generation of Electric Dipole Bending Waveforms

GENERATION OF ELECTRIC DIPOLE BENDING WAVEFORMS FOR THE SMALL RECIRCULATOR

Recently, there has been concern over the electrical engineering requirements for the generation of high-voltage waveforms to apply to the dipole deflection plates for beam bending in our small recirculator experiment. Likely voltage waveforms are characterized in Recirculator Note 26 (John Barnard) and Recirculator Notes 27 and 32 (Tom Fessenden). In the continuous approximation where the beam uniformly gains energy linearly in distance propagated, the dipole bending waveforms can be thought of as being initial ± 6.8 KV signals simultaneously applied to all dipole plates that increase in time (with a component linear and quadratic in time) to ± 27 KV over the $230 \mu\text{s}$ residence time of the beam in the ring. In practice, a fair degree of flexibility in the waveform shape is desired to accommodate various acceleration schedules, and it is also possible that the waveforms will need to be "tweaked" (i.e., slightly modified in shape and amplitude) at each individual dipole in the ring (or perhaps groups of dipoles) to correct for the head-to-tail variation in beam energy, realistic (discrete) acceleration schedule, waveform synthesis errors, etc. These tweaker signals could also be critical for beam steering to correct for the in-plane trajectory errors in bending the beam as a result of alignment errors, waveform errors, etc. The tweaker signals will likely be on the order of ± 150 V or less, and will need to be timed correctly with flexible waveform shape and duration ($1-4 \mu\text{s}$). Current estimates dictate that the rms voltage error over the pulse must be less than about 1% for a high degree of confidence that the errors will not have a significant detrimental influence on beam dynamics (Bill Sharp, HIF Group Meeting, 10-18-94). In certain situations it is also likely that the amplitude of modifications involved in the possible tweaking of the dipole signals could be on the order of this 1% tolerance. The tweaking amplitude will be maximum if two waveform generators are used for all dipole plates (one for the + plate and one for the - plate) and of lesser magnitude if a number of waveform generators are used and appropriately distributed and synchronized around the ring. In the limit where a separate (smooth) waveform generator is used for each dipole plate, no tweaking signals would be needed in the absence of waveform synthesis and alignment errors. However, in the real machine, it is likely that at least some minimal number of tweaked dipoles set by beam steering requirements will prove necessary to compensate for in-plane bending errors due to waveform synthesis and alignment errors. This could be true even if the tweaking components needed to correct for the head-to-tail variation in beam energy, discrete acceleration schedule, etc., are below the needed 1% tolerance and have negligible influence on the beam dynamics.

The purpose of this note is to distribute several potential solutions for the dipole waveform synthesis problem that were communicated to us by Ed Bowles of General Atomics. Fred Deadrick presented an interesting and potentially simple solution to the waveform generation problem in HIF Note 94-7. In this solution, he proposed charging a capacitor with a high voltage DC source and then discharging this capacitor through a time-varying resistor and obtaining the desired dipole plate voltage as the difference voltage between this DC source and across the capacitor. The desired difference voltage waveform for beam bending would be achieved by tailoring the time-varying resistor appropriately such that the difference voltage fits the desired dipole waveform. In practice this resistor could be implemented by a circuit that rapidly (and perhaps flexibly) switches in appropriate resistance values. The waveform obtained in each switching cycle can be easily calculated using linear circuit analysis. This solution could be characterized as a passive solution. In contrast, the waveform synthesis options presented in this note are active solutions in the sense that a modulator or pulse transformer directly drives the capacitive load of the dipole plates. Several of these active solutions may be more complicated and/or potentially more expensive than Fred Deadrick's passive solution presented in HIF Note 94-7. Nevertheless, certain active options may offer advantages including, smooth waveform structures that would be more compatible with any tweaker signals and possibly lower noise, potentially greater flexibility, and their existence in proven working designs operating in similar regimes to what we need. Several options could possibly be purchased to specification through General Atomics or implemented using borrowed hardware currently at Livermore. In that regard they could provide attractive fallback options in the event we decide to contract out the waveform synthesis problem to Russian research institutes or if we have a design or fabrication failure.

The three active waveform synthesis options presented here are a pulse transformer method, a planar triode modulator, and a solid state switched modulator. We will discuss each of these options in turn. It should be stressed that the example designs presented are due to Ed Bowles of General Atomics, and not the author of this note. The purpose of this note is only to communicate these options and a few relevant characteristics to the group. More detailed information on the electrical engineering implementation of the example designs can be obtained from Ed Bowles.

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Several estimates and arguments are also provided in this note by the author in an effort

to illustrate and compliment the communicated material. In these estimates, material characteristics and possible approaches were obtained from conversations with Ed Bowles, Andy Faltens, Lou Reginato, Fred Deadrick, and Tom Fessenden. However, any errors in these estimates are due entirely to the incompetence of the author in this subject matter.

Before overviewing the design options, a summary of a relevant system parameters that impact the designs is appropriate. First, the capacitance of the deflector plates can be estimated from a simple parallel plate capacitor model as

$$C_d = \epsilon_0 \ell h / 2r_p \sim 1.6 \text{ pF},$$

where $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N meter}^2$ is the permittivity of free-space, $r_p \sim 3.5 \text{ cm}$ is the pipe radius, $\ell = 3.7r_p$ is the effective axial length of the dipole, and $h \sim 10 \text{ cm}$ is the vertical height of the dipole plates. Here, and in all following estimates, ring parameters are taken from Recirculator Note 26 by John Barnard. In certain configurations, it is likely that the waveforms will be guided to the dipoles by coaxial cables. For typical 50Ω impedance cables of length ℓ_{cab} , the cable will have an characteristic capacitance of

$$C_{\text{cab}} = 101 \ell_{\text{cab}} \text{ pF},$$

with ℓ_{cab} measured in meters. To the driving circuit, this cable capacitance will appear in parallel with the capacitance of the deflecting dipole to the driving circuit. The same lead lengths can be used for all dipoles to simplify timing and analysis. For the purposes of this note, we will assume that to be the case. However, it is possible that the waveform output by a single circuit that is applied to a group of dipoles will need to be slightly attenuated between individual dipoles in that group. Such an attenuation could account for the head-to-tail energy variation in the beam, but would also result in a more complicated load structure than outlined above. For a common waveform source with no attenuation, the minimum cable lengths would be the recirculator radius (waveform source in center of the ring), corresponding to $\ell_{\text{cab}} = 2.3$ meters, and the likely maximal cable length would be half the recirculator circumference of 14.4 meters plus some stand-off distance (waveform source outside the ring). For a 2 meter standoff distance, the cables will be $\ell_{\text{cab}} = 9.2$ meters long, with a corresponding capacitance of $C_{\text{cab}} = 930 \text{ pF}$. For a rough estimates of energy and current required to charge the dipoles and cables, we take the voltage across the bending dipole to be given by

$$V_d(t) = 2V_i \left[1 + \left(\sqrt{\frac{V_f}{V_i}} - 1 \right) \left(\frac{t}{\tau_p} \right) \right]^2.$$

Here, t is the time, $\tau_p = 230 \mu\text{s}$ is the pulse duration, and $V_i = 6.8 \text{ KV}$ and $V_f = 27 \text{ KV}$ are the initial ($t = 0$) and final ($t = \tau_p$) pulse voltages applied to a dipole plate. This waveform corresponds to the approximation where the beam is accelerated uniformly in time from 80 KeV to 320 KeV over τ_p seconds. Using this bending waveform, the electrostatic field energy $\{ \mathcal{E} = \frac{1}{2} C V_d^2 \sim 2 C V_i^2 [1 + (\sqrt{V_f/V_i} - 1)(t/\tau_p)]^4 \}$ energy stored in *each* of the 40 beam bending dipoles will increase from 0.15 to 2.3 mJ over the

duration of the pulse. This will correspond to an instantaneous power $\{ P = d\mathcal{E}/dt \sim (8CV_i^2/\tau_p)(\sqrt{V_f/V_i}-1)[1+(\sqrt{V_f/V_i}-1)(t/\tau_p)]^3 \}$ flowing into the dipole that increases from 2.6 to 20 W over the duration of the pulse. Also, the current to charge each dipole will ramp linearly in time from 0.19 to 0.37 mA over the duration of the pulse. On the other hand, the field energy stored in and power flowing into *each* cable will increase from 9.3 to 150 mJ/meter and 0.16 to 1.3 KW/meter, respectively, and the current to charge each cable will linearly ramp from 12 to 24 mA/meter over the duration of the pulse. For the maximal cable length of $\ell_{cab} = 9.2$ meters, these values are 580 times those of the bending dipoles. From these estimates it is clear that the cable lengths and impedances (along with any stray impedances) will need to be carefully controlled to limit the power requirements and concomitant cost of the circuitry.

Another point that should be brought out is that any viable waveform generation scheme must ultimately be compatible with insertion and extraction of the beam into and out of the ring. This will require certain dipoles in the ring to be biased with opposite polarity on the final lap to help kick the beam out of the ring. This procedure is illustrated in detail in Recirculator Note 26 by John Barnard. One solution would be to drive these dipoles by a separate waveform generator. An alternative solution that might be cheaper and more simple would be to use a fast high voltage switch to disconnect these dipoles from the normal orbit waveform after the beam passes them on the final normal lap, and then connect these dipoles to static bias voltage set appropriately for beam extraction. At the final designed beam energy, there would be approximately 10 μ s for the switching and for the switching noise to decay before the head of the beam transverses the ring and arrives back at the dipoles. This should be acceptable, and of course, extraction under this scheme would become easier at lower beam energies and the static biases would need to be preset at the correct values for the lap on which the beam will be extracted.

Pulse Transformer:

The pulse transformer option is attractive because it is conceptually simple and flexible. The basic idea is illustrated on the figure at the end of this section. A low voltage control waveform is fed into a step-up transformer with a ferrite or Metglas core and transformed to the desired level. The pulse transformer shown is a 600 V to ± 30 KV transformer with a $4\mu\text{s}$ pulse duration recently constructed by General Atomics. A photograph of this oil-packed transformer is attached. Provided a pulse transformer is operated far from saturation, the response is very repeatable. Pulse transformers are also straightforward to construct, and a 1% tolerance should be achievable in a careful design.

A circuit diagram of the pulse transformer option is given below.

Here, C is the capacitive load of the deflecting dipoles, cables, feedthroughs, stray capacitances, etc., $V_d(t)$ is the desired dipole voltage waveform, $V_a(t)$ is the required voltage waveform to be applied to the pulse transformer and $I_a(t)$ is the consistent current associated with the applied voltage pulse. If the magnetic material is operated in a linear region and can be described by a permeability μ , the fields can be approximated by those of an ideal solenoid of cross-sectional area A and length ℓ ($B = \mu N I / \ell$), and all flux is linked, then we have

$$V_a(t) = \frac{N_1}{N_2} V_d(t),$$

$$I_a(t) = \frac{N_2}{N_1} C \left[\frac{dV_d(t)}{dt} - \frac{dV_d(t=0)}{dt} \right] + \frac{\ell}{\mu N_1 N_2 f A} \int_0^t dt' V_d(t').$$

Here, N_1 and N_2 are the number of windings in the primary and secondary coil, and f is the packing fraction of the magnetic core material. An arbitrary additive constant in the applied current I_a has been set to zero since it would require that additional energy be supplied by the applied waveform generation circuitry (static magnetic field energy stored in the inductor). From Faraday's law, it can be estimated that the core will undergo a flux swing ΔB over the pulse duration that satisfies

$$2(V_f - V_i)\tau_p = N_2 f A \Delta B.$$

The cross-sectional area A of the core and the number of secondary windings N_2 should be chosen large enough to keep the flux swing ΔB from being driven into or near saturation.

To estimate the core area needed, some ferromagnetic material characteristics must first be discussed. For iron alloys, the saturation field is roughly proportional to the fraction of iron contained times 2 Tesla, the saturation field of pure iron. Using this rule, "Metglas" (80% iron) and silicon steel (97% iron) have saturation fields $B_{\text{sat}} = 1.6$ Tesla and $B_{\text{sat}} = 1.97$ Tesla, respectively. Also, the packing fractions of Metglas and silicon steel are $f = 0.7$ and $f = 0.9$, respectively. Ferrites typically have a lower saturation field of $B_{\text{sat}} = 0.7$ Tesla and a packing fraction of unity ($f = 1$). Provided the core is reset between shots with a pulse to drive the field to negative saturation, the flux swing during the pulse can be double the saturated value. In practice the core should only be driven a fraction of this swing, say for estimation purposes $\Delta B \sim (3/2)B_{\text{sat}}$. Then the product $N_2 A$ should be chosen such that

$$N_2 A > \frac{4}{3f} \frac{(V_f - V_i)\tau_p}{B_{\text{sat}}}$$

For Metglas and ferrite, this corresponds to $N_2 A > 5.5$ meters² and $N_2 A > 8.9$ meters², respectively. The number of secondary turns N_2 should be chosen large enough to limit the core to a practical (economical) size, but not so large that leakage inductances and capacitances between the secondary windings become too large. Estimations of these leakages and their influence, along with breakdown considerations, will become important in a detailed design. Several electrical engineers have expressed the opinion that a pulse transformer with a large number of secondary windings N_2 could prove practical for our particular problem.

As a practical example, consider a ferrite core of circular cross-section with a 4 cm radius. Then the number of secondary windings should be chosen such that $N_2 > 1770$. To allow the use of an inexpensive, low voltage solid-state arbitrary waveform generator and solid-state amplifier combination to drive the transformer primary, the winding ratio N_1/N_2 should be chosen such that $2V_f N_1/N_2 < 250$ V. Consistent with these restrictions, we choose $N_1 = 10$ and $N_2 = 2000$. For these parameters, the the applied voltage waveform should increase from 34 to 135 V over the 230 μ s pulse duration with the same time-dependence as the desired dipole voltage. To estimate the consistent current in the transformer primary, we employ the previous formulas with the ideal waveform corresponding to uniform acceleration to obtain

$$I_a(t) \sim 4 \frac{N_2 C V_i}{N_1 \tau_p} \left(\sqrt{\frac{V_f}{V_i}} - 1 \right)^2 \left(\frac{t}{\tau_p} \right) + \frac{2\ell V_i \tau_p}{3\mu N_1 N_2 f A} \frac{[1 + (\sqrt{V_f/V_i} - 1)(t/\tau_p)]^3 - 1}{(\sqrt{V_f/V_i} - 1)}$$

The primary current will be maximum at the end of the pulse ($t = \tau_p$). To apply this formula, we must first fix the length of core material ℓ , and estimate the magnetic permeability μ and capacitive load C . The core should be chosen sufficiently long such that the outer layers of the secondary windings are not far from the core material. Breakdown between winding layers can be prevented with mylar insulating layers. Generally the detailed geometry and the lead positions will most strongly influence breakdown considerations. And, of course, the transformer can be oil-packed to simplify breakdown problems.

A $\ell = 10$ cm core length will probably meet these practical requirements. As a crude order of magnitude estimate, we take the permeability of ferrite to be $\mu \sim 2500\mu_0$, where $\mu_0 = 4\pi \times 10^{-7}$ H/meter is the vacuum permeability. The actual value of μ will depend both on the detailed material composition, the regime of operation, and the prior magnetization history. This value represents a typical maximum permeability, which should not be too different from the effective linear value in the regime of operation proposed. Finally, we estimate capacitive load to be $C \sim (1.6n + 101n\ell_{cab})$ pF, where n is the number of dipoles being driven by the pulse transformer and ℓ_{cab} is the dipole cable length, in meters. A single pulse transformer driving a single ($n = 1$) dipole with a zero cable length ($\ell_{cab} = 0$), the peak primary current will be $I_a \sim 2.3$ A. On the other hand, for a single pulse transformer driving all 40 dipoles with maximal cable lengths of $\ell_{cab} = 9.2$ meters, the peak primary current will be $I_a \sim 870$ A. If a typical arbitrary waveform generator with a high impedance output is used to generate the waveform, then in all realistic cases it will need to be amplified with a fast, high power amplifier to drive the pulse transformer primary. It is evident from this calculation that the driving circuitry may be much smaller and cheaper if a separate pulse transformer is used for and mounted near each dipole. Finally, it should be stressed that to achieve the 1% tolerances desired, it is likely that a careful design including stray impedances, detailed material characteristics, and readily available component parameters will be necessary. In particular, models for the stray inductances and capacitances of the secondary windings will need to be established to determine the load and actual waveform shape more precisely. Also, a pulse transformer may prove difficult to operate with feedback correction circuitry. However, provided leakage inductances are adequately controlled, the input waveform can be tailored appropriately such that the desired waveform is obtained at the dipoles.

PULSE TRANSFORMER METHOD

$4 \mu \text{ s}$ flat top $\pm 30 \text{ kV}$

Planar Triode Modulator:

Another possible option is a planar triode modulator that would function as a high voltage and power amplifier that would amplify a low voltage waveform to directly drive the capacitive load of the bending dipoles and cables. The basic geometry would be similar to the schematic on the following page. This schematic represents the modulator used to drive the ATA beam correction coils (the load represents an impedance consisting of cables, a splitter/inverter, beam coils, and a damping resistor). Two similar circuits could be used to drive the beam deflector plates with the required $\pm$ biases. The modulators constructed for the ATA correction coils could function for this purpose with little or no modification. They have a designed $0 \rightarrow 25$ KV output with a bandwidth of 50 MHz and a 5 ns slew rate. It should also be possible to use a 30 KV power supply with the current tubes to obtain the needed peak 27 KV signal output. The modulator input is $0 - 5$ V and is designed to be driven by an arbitrary waveform generator. This input drives a fast, buffered operational amplifier, which drives a power MosFET, which in turn drives the planar triode. The circuit is air-cooled and an array of planar triodes were used to obtain a 180 A current capacity at 25 KV. For a single modulator driving all 40 dipoles with maximal 9.2 meter cable lengths, a 8.7 A peak instantaneous current (or 470 KW peak power) would be expected, well within the 180 A capacity of the modulator. Also, a 1% output tolerance should be readily achievable and the system should be well suited for use with feedback corrections. An advantage of this modulator over switched solid state modulators is that it has a smooth output and would likely have lower noise. For our low-power application, only a single tube should be necessary. A 65 KV tube rated 2 MW is available and should function well for this purpose. This tube would allow a substantial margin in the voltage output should increased bending strengths be necessary. Also, depending on the cable arrangement and the safety margin desired in the peak output voltage, the 2 MW capacity of this tube should be adequate. Several tubes and tube arrays are illustrated on the page following the circuit schematic.

Modulators of this type are currently being produced by General Atomics for Physics International. They cost on the order of 75 thousand dollars each. However, it is possible that they could be manufactured for our application somewhat cheaper, since the tubes are the dominant cost in the manufacture and our low-power application would require only a single tube as opposed to an array of tubes. Even in this case, it seems likely that these modulators would be too expensive to be used to drive each dipole individually. Two modulators would most likely be used to drive all dipoles simultaneously. In this case, any fine-scale tweaking of the main signal at the individual dipoles would likely require considerable additional waveform generation circuitry, further driving up the cost and complication of the system. On the other hand, it is also possible that the needed number of ATA modulators could be borrowed for our experiment making this an attractive option if tweaking proves to be unnecessary. Some number of the original ATA modulators are still at Livermore, although Steve Sampayan is currently using some fraction of those in his experiments. For reference purposes, a portion of a research memo "Specifications and Original Baseline Description of the Fast Coil System" is included at the end of this note. This portion of the memo contains further descriptions of the modulator.

MODULATOR USED FOR ATA BEAM CORRECTION

(50 MHz, 180 A at 25 kV)

(50 MHz, 180 °A at 25 kV)

(A)

(B)

(C)

(D)

- A) 32 MW, 50 MHz PLANAR TRIODE TUBE ARRAY
(7 ns RISETIME, $>0.6 \text{ GW/m}^3$)
- B) 32 MW TUBE ARRAY ON ETA-II INDUCTION CELL
(TESTS COMPLETED OCTOBER 89)
- C) MODIFIED CIRCUIT FOR 65 kV PLANAR TETRODE
(3 IN. DIA, 2 MW EACH)
- D) 175 kV PLANAR TETRODE, EIMAC YU-146
(6 IN. DIA, 5 MW EACH)

Solid State Switched Modulator:

Another option is a solid state switched modulator. This circuit would also function as a high voltage and power amplifier that would amplify a low voltage waveform to directly drive the capacitive load of the bending dipoles and cables. A practical implementation of such a modulator is illustrated on the following page. In this circuit a low-voltage waveform from an arbitrary waveform generator would drive a fast, high impedance input analog to digital converter which would in turn drive the gates of a switched power supply constructed with IGBTs FETs or some other fast, high voltage solid-state switch. The circuit sketched is a 20 KV supply with an 8 bit analog to digital converter operating the gates of IGBTs. The 8 bit resolution corresponds to a 78 V resolution in the least significant bit. This circuit was used in a long pulse application for which a pulse transformer would not have been practical. Two similar circuits could be used to drive the beam deflector plates with the required $\pm$ biases. Such a circuit should be fast and be well suited for operation with feedback corrections. For our application, a circuit similar to the one illustrated with a maximum output of 40 KV and a 10 or 12 bit digital to analog converter would have 39 V and 9.8 V resolutions on the least significant bit, which are well within the required 1% tolerance at the initial 6.8 KV waveform amplitude. The the switches for the most significant bits would also need to be constructed from a chain of FETs or IGBTs in series to avoid voltage breakdown. Power requirements of the load along with the power capacity of the circuit will dictate if several modulators will need to be ganged in parallel. Implications of noise generated by the switching on the dynamics of the beam may also need to be considered. Cost estimates will also need to be performed to determine if the circuitry would be cheap enough to allow separate modulators to drive each individual dipole and eliminate the possible need for any additional tweaking circuitry.

SOLID STATE, 20 kV FERROELECTRIC TUNER BIAS SUPPLY WITH EIGHT BITS

Beam Research Program

Lawrence Livermore National Laboratory

Title: Specifications and Original Baseline Description of
the Fast Coil System

RM 89-14

Date 1/11/89

Author(s): K. Whitham, G. Caporaso, R. Thomas, G. Vogtlin,
J. Zentler, F. Coffield, T. Orzechowski, E. Bowles

No. pages 23

Introduction

The fast correction coil system is a set of coils that provide transverse magnetic fields to electron beams at wide bandwidth to correct beam position and angle during the beam pulse. Beam position and angle are derived from beam position monitors along the beam line. Error signals are produced and fed into function generators which drive amplifiers which drive coils to produce the correcting fields. A basic assumption is that these corrections are repetitive from pulse to pulse and that changes in the beam occur slowly. Bandwidths of 30 MHz minimum is required. The maximum repetition rate is 800 Hz. Individual pulses are 70 ns FWHM with 10 ns rise time required.

The approach is to use a wideband amplifier based on planar triodes. These tubes can drive up to 15 amperes apiece at voltages up to 15 kV with bandwidths limited by the circuit application. They can be operated in parallel and provide a source of multimegawatt power as amplifiers, albeit unipolar and nonlinear.

3. Amplifier Circuit Design

The amplifier circuit used to drive the fast correction coils will be designed to meet the following requirement:

1. Rise time < 10 ns.
2. Supply a maximum of 120 amps to opposing sets of steering coils with a positive pulse to one coil and a negative pulse to the other.
3. Drive a total cable impedance of 100 ohms.
4. Accommodate (attenuate) the reflections caused by one end of the coils being grounded.

The complexity of the steering system, especially the control subsystem, will be reduced by minimizing the number of steering bars to four. The maximum coil length has been chosen to minimize the amount of current needed from the driver circuits to achieve a steering field of ± 300 G-cm. The 15 cm beampipe I.D. for the steering coil matches that of the rest of the transport system, although this might be increased in later designs to lower the driver current requirement. A clear space diameter of 7.5 cm was found to give suitable steering field uniformity in the beam region. M , the per length mutual inductance between the beam current and each bar, is 92 nH/m.

The impedance of each 1.6 cm steering bar, referenced to the beampipe wall, is 100 Ω . Lower impedances cause a dramatic increase in the driver current requirement. Each pair of steering bars in the baseline coil design produces the equivalent of a 1 Gauss field per 9.2 A current. Since the driver circuits have a unipolar output, the steering bars must work against an externally generated DC bias field to allow beam deflection in any direction. To produce the specified ± 300 G-cm steering effort, each pair of bars must generate 0-600 G-cm steering effort, requiring a maximum of 92 A per bar. Assuming separate drive circuits are used for each bar, the total current needed is 340 A, well under the 480 A constraint. The magnitude of the DC bias field is 7.1 Gauss.

3. Amplifier Circuit Design

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FAST CORRECTION COIL CIRCUIT
Figure 4

5. The circuit must not be damaged by the beam induced voltage.
6. Successfully operate in the EMI environment of the PCA catwalk area.
7. The package shall be air insulated and cooled and in a configuration compatible with the PCA catwalk blumlein storage area.

The amplifier system to drive the coils will utilize the circuit designed by Ed Bowles for the injector energy regulator. This circuit utilizes two parallel Eimac YU114 planar triodes driven in cascode by a power MosFET. The MosFET is driven by a buffered high speed operational amplifier. An amplifier gain control is provided to aid in adjusting system current sharing. See Fig. 4 for details. The circuit is on a 45 degree pie shaped PC card that is intended to be mounted in a cylindrical package of 8 cards.

The fast correction coil amplifier system will utilize four of the amplifier modules packaged in a cylinder that is approximately 14" in diameter. The unused four amplifier positions will be blanked off with copper dummy cards. The height is not critical, and can be made adequate to comfortably house the circuits, power supplies, and other components. A fan mounted on the outside of the cylinder will blow air across the tubes to remove the 113 watts of filament power. The present plan is to power the tube filaments from an external 15 VDC power supply. Module-to-module filament isolation will be provided with ferrite bead inductors on the leads. Other necessary voltages will be provided with DC-DC converters powered from the filament supply.

Refer to the system block diagram Fig. 5. The four amplifier modules are capacitively coupled to a ferrite isolated splitter/inverter arrangement of two 50 ohm coaxial cables driven in series. 14" of high frequency ferrite will provide about 25 uh inductance on the outside of the coax shield to isolate the pulse from ground. The 1K resistor to ground is needed to damp out the reflections from the shorted cable. Tests of this arrangement indicate that the beam induced voltage is effectively cancelled. Our circuit

Fast Correction Coil Amplifier System

has enough voltage margin to accommodate any resulting beam induced voltage from the beam being off center. The design of the coil system in the beam pipe should eliminate the possibility of a beam strike on the coils, however, a protective circuit will be provided to protect the circuit during tune-up operations when off center beams are likely.

4. Feed Throughs and Interconnections

The feed throughs and interconnecting cable will be designed for a 75 kV peak voltage. Voltages in excess of 80 kV will cause a gas breakdown in the feed through connector. Fig. 6 shows the expected stresses with 30 kV applied. At the amplifier end the voltage induced by the beam will be reduced due to ferrite inversion and bucking of the two pulses. This circuit is shown in Fig. 7.

The cable discussed at the design review of 11 Jan. had a 100 ohm impedance. This cable would have a dielectric diameter of at least one inch.

Alternatives discussed have been 50 ohms and 68 ohms. These impedances will cause a mismatch that may have an interaction with the beam. Another option considered would be to short one end of the steering bar. This also would result in mismatches but would have the following benefits:

1. Minimum electrostatic steering
2. Double current over 100 Ω case
3. Provide dump path for beam
4. Reduce normal feed through stress from 50 kV to 35 kV.

5. Controls System and Controls Algorithm

Requirements - The trajectory errors in the beam are expected to vary from pulse to pulse within a burst but to be relatively constant from burst to burst for a given pulse. This means that a unique correction waveform is

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