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Abstract— A model-based approach is developed to solve an adaptive ocean acoustic signal processing problem. Here we investigate the design of a model-based identifier (MBID) for a normal-mode model developed from a shallow water ocean experiment and then apply it to a set of experimental data demonstrating the feasibility of this approach. In this problem we show how the processor can be structured to estimate the horizontal wave numbers directly from measured pressure-field and sound speed thereby *eliminating* the need for synthetic aperture processing or a propagation model solution.

I. INTRODUCTION

Ocean acoustic signal processing has made great strides over the past decade necessitated by the development of quieter submarines and the recent proliferation of diesel powered vessels. These improvements have been achieved by developing processors that incorporate knowledge of the surrounding ocean environment and noise into their processing schemes [1-4]. However, it is well-known that if the incorporated model is inaccurate either parametrically or incorrect from the basic principles, then the processor can actually perform worse in the sense that the predicted error variance is greater than that of the raw measurements [5,6]. In fact, one way to choose the "best" model or processor is based on comparing predicted error variances — the processor achieving the smallest wins! In practice, the usual procedure to check for model adequacy is to analyze the statistical properties of the resulting residual or innovations sequence, that is, the difference between the measured and predicted measurements. Here again the principle of minimum (residual) variance is applied to decide on the best processor or equivalently the best embedded model [7]. Other sophisticated statistical tests have been developed for certain classes of models with high success to make this decision [8-10]. In any case the major problem with *model-based* signal processing (MBP) schemes is assuring that the model incorporated in the algorithm is ad-

equated for the proposed application, that is, it can faithfully represent the on-going phenomenology. Therefore, it is necessary, as part of the MBP design procedure, to estimate/update the model parameters either through separate experiments or jointly (adaptively) while performing the required processing [11,12]. The introduction of a recursive, on-line MBP can offer a dramatic detection improvement in a tactical passive or active sonar-type system especially when a rapid environmental assessment is required [12,13]. In this paper, we discuss the development of a processor capable of adapting to the ever changing ocean environment and providing the required signal enhancement for detection and localization.

Here we investigate the development of a "model-based identifier," (MBID) that is, a identifier that incorporates an initial mathematical representation of the ocean acoustic propagation model into its framework and adapts, on-line, its parameters as the ocean changes environmentally. Here we are interested primarily in a shallow water environment characterized by a normal-mode model and therefore, our development will concentrate on adaptively adjusting parameters of the normal-mode propagation model to "fit" the ocean surrounding our sensor array. In fact, one way to think about this processor is that it passively listens to the ocean environment and "learns" or adapts to its changes. It is clear that the resulting processor will be much more sensitive to changes than one that does not, thereby, providing current information and processing. The algorithm uses the incoming data to adaptively update the parameter set, jointly, with the acoustic signal processing. In the following, we define the MBID as a Kalman filter whose estimated states are the modal functions $\hat{\phi}(z_\ell)$ and states representing the estimated ocean acoustic parameters $\hat{\theta}(z_\ell)$ that have been augmented into the processor. The basic processor is shown in Figure 1. The inputs to the MBID are raw data $\{p(z_\ell)\}, \{c(z_\ell)\}$

Figure 1: Model-Based Ocean Acoustic Identification: the Basic Processor.

and its outputs are $\theta(z_t)$ the set of parameters of interest.

The application of this adaptive approach to other related problems of interest is apparent. For *signal enhancement*, the adaptive MBP or MBID can provide enhanced signal estimates of modal functions (modal filtering), pressure-field estimates (measurement filtering), and parameters of interest (parameter estimation) such as wave numbers, range-depth functions, sound speed, etc. [11,12]. For *model monitoring* and *source detection* purposes, the MBID provides estimates of the residuals or innovations sequence, which can be statistically tested for adequacy [8] or used to calculate a decision function [11]. For *localization*, the MBID provides estimates of the enhanced range-depth and modal (modal filter) functions used for model-based localization as discussed in a recent paper [14]. In fact for rapid assessment of the ocean environment — a definite requirement in a tactical situation — it is possible to perform *model-based inversion*, on-line, using the MBID scheme to adaptively estimate the changing parameters characterizing the sound speed profile [12,13]. Thus, the MBID provides a technique capable of “listening and learning”.

II. ADAPTIVE WAVE NUMBER ESTIMATION

In this section we develop a model-based identifier for use with an ocean acoustic propagation model. System identification is typically concerned with the estimation of a model and its associated parameters from noisy measurement data. Usually the model structure is pre-defined (as in our case) and then a parameter estimator is developed to “fit” parameters according to some error criterion. After completion or during this estima-

tion, the quality of the estimates must be evaluated to decide if the processor performance is satisfactory or equivalently the model adequately represents the data. There are various types (criteria) of identifiers employing many different model (usually linear) structures [8-10]. Since our efforts are primarily aimed ocean acoustics in which the models and parameters are usually nonlinear, we will concentrate on developing a parameter estimator capable of on-line (shipboard) operations and nonlinear dynamics.

From our previous work, it is clear the the extended Kalman filter (EKF) identifier will satisfy these constraints nicely. The general nonlinear identifier or equivalently parameter estimator structure can be derived directly from the EKF algorithm (see Refr. [16] for details). We note that this algorithm is *not* implemented in this fashion, it is implemented in the numerically stable UD-factorized form as in SSPACK_PC [18].

For propagation in a shallow water environment we choose the normal-mode model which can easily be placed in state-space form (see Refr. 11 for details). We choose the “depth only” structure and assume a vertical array which yields a linear space-varying formulation, then we develop the identifier — a nonlinear processor.

Assuming a horizontally-stratified ocean of depth h with a *known* source position (x, y) and also that the acoustic energy from a point source can be modeled as a trapped wave governed by the wave equation. We follow the approach of Clay [15] using the separation of variables technique leading to a set of ordinary differential equations. The resulting depth relation is an eigenvalue equation in z with

$$\frac{d^2}{dz^2} \phi_m(z) + \kappa_z^2(m) \phi_m(z) = 0, \quad m = 1, \dots, M \quad (1)$$

whose eigensolutions $\{\phi_m(z)\}$ are the so called *modal functions* and κ_z is the wavenumber in the z -direction. These solutions depend on the sound speed profile, $c(z)$, and the boundary conditions at the surface and bottom as well as the corresponding *dispersion* relation given by

$$\kappa^2 = \frac{\omega^2}{c^2(z)} = \kappa_r^2(m) + \kappa_z^2(m), \quad m = 1, \dots, M \quad (2)$$

where κ_r, κ_z are the respective wave numbers in the r and z directions with c the depth-dependent sound speed profile and ω the harmonic source frequency.

For our purpose we are concerned with the estimation of the pressure field, therefore we remove

the time dependence, normalize units, and obtain the *acoustic pressure propagation model*,

$$p(r, z) = \sum_{m=1}^M q \phi_m(z_s) \phi_m(z) \frac{e^{-\alpha_r(m)r}}{\sqrt{\kappa_r(m)r}} e^{j\kappa_r(m)r} \quad (3)$$

where p is the acoustic pressure; q is the source amplitude; ϕ_m is the m^{th} modal function at z and z_s ; α is the modal attenuation; $\kappa_r(m)$ is the horizontal wavenumber associated with the m^{th} mode; and r is the horizontal range.

The normal-mode solutions can easily be placed in state-space form and we refer the interested reader to Refs. 12 and 13 for the detailed theory. This approach leads to a Gauss-Markov representation which includes the second order statistics. The measurement noise can represent the near-field acoustic noise field, flow noise on the hydrophone and electronic noise. The modal noise can represent sound speed errors, distant shipping noise, errors in the boundary conditions, sea state effects and ocean inhomogeneities. By assuming that the horizontal range of the source r_s is known *a-priori*, we can use the Hankel function $H_0(\kappa_r r_s)$ which is the source range solution; therefore, we reduce the state-space model to that of "depth only" and the Gauss-Markov model for this constrained problem is given by

$$\frac{d}{dz} \phi(z) = A(z) \phi(z) + B(z) u(z) + w_\phi(z) \quad (4)$$

where

$$A_m(z) = \begin{bmatrix} 0 & 1 \\ -\kappa_z^2(m) & 0 \end{bmatrix} \quad (5)$$

with the pressure field measurement model given by

$$p(r_s, z) = C^T(r_s, z) \phi(z) + v(z) \quad (6)$$

where

$$C^T(r_s, z) = [\beta_1(r_s, z_s) \ 0 | \beta_2(r_s, z_s) \ 0 | \dots | \beta_M(r_s, z_s) \ 0] \quad (7)$$

with $\beta_m(r_s, z_s) = \frac{q \phi_m(z_s)}{\int_0^h \phi_m^2(z) dz} H_0(\kappa_r(m) r_s)$. The random noise vectors w_ϕ and v are assumed gaussian, zero-mean with respective covariance matrices, $R_{w_\phi w_\phi}$ and R_{vv} .

Since our array spatially samples the pressure-field, we choose to discretize the differential state equations using a finite (first) difference approach, that is, for the m^{th} -mode we use

$$\frac{d}{dz} \phi_m(z_\ell) \approx \frac{\phi_m(z_{\ell+1}) - \phi_m(z_\ell)}{\Delta z_\ell} \quad (8)$$

where $\Delta z_\ell = z_{\ell+1} - z_\ell$. Therefore, substituting into the differential equation, we obtain

$$\phi_m(z_{\ell+1}) = [I + \Delta z_\ell A_m(z_\ell)] \phi_m(z_\ell) + \Delta z_\ell B(z_\ell) u(z_\ell) + \Delta z_\ell w_\phi(z_\ell) \quad (9)$$

Since we have a vertical line sensor array to measure the pressure-field, the measurement model for the m^{th} mode becomes

$$p_m(r_s, z_\ell) = C_m^T(r_s, z_\ell) \phi_m(z_\ell) + v_m(z_\ell) = \sum_{m=1}^M \beta_m(r_s, z_s) \phi_{m1}(z_\ell) + v_m(z_\ell) \quad (10)$$

It is this model that we employ in our model-based identifier. Next suppose we assume that the horizontal wave numbers, $\{\kappa_r(m)\}$, are unknown and we would like to estimate them *directly* from the pressure-field measurements. Note that the horizontal wave numbers are *not* a function of depth — they are constant or invariant over depth. This is a crucial first step for any ocean acoustic processing requirement. By estimating the wave numbers directly from the noisy array measurements, not only do we obtain the parameters required for our subsequent model-based processor designs, but also we replace the need to perform the synthetic aperture processing using a towed array which is the usual approach in obtaining the horizontal wave numbers. Once estimated, the horizontal wave numbers along with known sound speed can be used to determine the vertical wave numbers directly from the dispersion relation. Note also that this implies that by identifying the wave numbers directly from the sensor measurements we are essentially *eliminating* the need to solve the boundary value problem using a numerical eigenvalue solver. This is possible because it is known that the wave numbers carry all of the essential information (boundary, temporal frequency etc.) directly — a powerful set of parameters! Thus, we define the basic *wave number identification problem* as GIVEN a set of noisy pressure-field and sound speed measurements, $\{p(r_s, z_\ell)\}, \{c(z_\ell)\}$, FIND the "best" (minimum error variance) estimate of the horizontal wave numbers, that is, $\{\hat{\kappa}_r(m)\}, m = 1, \dots, M$.

The basic form of the coupled modal equations follow from Eq. 4 with $\kappa_r \rightarrow \theta$ and $m = 1, \dots, M$

$$\begin{aligned} \phi_{m1}(z_\ell) &= \phi_{m1}(z_{\ell-1}) + \Delta z_\ell \phi_{m2}(z_{\ell-1}) \\ \phi_{m2}(z_\ell) &= -\Delta z_\ell \left(\frac{\omega^2}{c^2(z_{\ell-1})} - \theta_m^2(z_{\ell-1}) \right) \phi_{m1}(z_{\ell-1}) \\ &\quad + \phi_{m2}(z_{\ell-1}) \\ \theta_m(z_\ell) &= \theta_m(z_{\ell-1}) \end{aligned} \quad (11)$$

and with corresponding measurement model

$$p_m(r_s, z_\ell) = \sum_{m=1}^M \beta_m(r_s, z_s) \phi_{m1}(z_\ell) \quad (12)$$

The MBID requires the following jacobian matrices for the wave number identification problem [17]

$$A[\phi, \theta] = \begin{bmatrix} A_\phi[\phi, \theta] & | & A_\theta[\phi, \theta] \\ \hline O & | & I_M \end{bmatrix} \quad (13)$$

The associated wave number (parameter jacobian) is with $A_\phi \in R^{2M \times 2M}$, and $A_\theta \in R^{2M \times M}$ and the overall jacobian matrix given by $A[\phi, \theta] \in R^{3M \times 3M}$ with

$$A_\phi[\phi, \theta] = A(z_\ell, \theta) \quad (14)$$

For the measurement system, we have that

$$c_m[\phi, \theta] = \frac{q\phi_m(z_s)}{\int_0^h \phi_m^2(z) dz} H_0(k_r(m)r_s) \phi_{m1}(z_\ell) \quad (15)$$

or substituting for the known and unknown parameters and using the well-known approximation to the Hankel function for $\kappa_r r_s \gg 1$, we obtain

$$c_m[\phi, \theta] = \gamma_m(z_s) \sqrt{\frac{2}{\pi \theta_m(z_\ell) r_s}} \cos(\theta_m(z_\ell) r_s - \pi/4) \phi_{m1}(z_\ell) \quad (16)$$

where $\gamma_m(z_s) := \frac{q\phi_m(z_s)}{\int_0^h \phi_m^2(z) dz}$ and $H_0(\kappa_r(m)r_s) \approx (\sqrt{\frac{2}{\pi \theta_m(z_\ell) r_s}} \cos(\theta_m(z_\ell) r_s - \pi/4))$. Summing over the total number of modes we have

$$c[\phi, \theta] = \sum_{m=1}^M \gamma_m(z_s) \sqrt{\frac{2}{\pi \theta_m(z_\ell) r_s}} \cos(\theta_m(z_\ell) r_s) \phi_{m1}(z_\ell) \quad (17)$$

The $1 \times 3M$ measurement jacobian vector for this problem is given by

$$C[\phi, \theta] = [C_\phi[\phi, \theta] \mid C_\theta[\phi, \theta]] \quad (18)$$

Using this information we can easily construct the overall parameter estimator. The *prediction equations* for the m^{th} mode and wave number are

$$\begin{aligned} \hat{\phi}_{m1}(z_\ell|z_{\ell-1}) &= \hat{\phi}_{m1}(z_{\ell-1}|z_{\ell-1}) + \Delta z_\ell \hat{\phi}_{m2}(z_{\ell-1}|z_{\ell-1}) \\ \hat{\phi}_{m2}(z_\ell|z_{\ell-1}) &= -\Delta z_\ell \left(\frac{\omega^2}{c^2(z_\ell)} - \hat{\theta}_m^2(z_{\ell-1}) \right) \\ &\quad \hat{\phi}_{m1}(z_{\ell-1}|z_{\ell-1}) + \hat{\phi}_{m2}(z_{\ell-1}|z_{\ell-1}) \\ \hat{\theta}_m(z_\ell|z_{\ell-1}) &= \hat{\theta}_m(z_{\ell-1}|z_{\ell-1}) \end{aligned}$$

and the corresponding innovations (parameterized by θ) are given by

$$e(z_\ell, \theta) = p(r_s, z_\ell) - \sum_{m=1}^M c_m[\phi, \theta] \hat{\phi}_{m1}(z_\ell|z_{\ell-1}, \theta) \quad (20)$$

with the *correction equations*

$$\begin{aligned} \hat{\phi}(z_\ell|z_\ell) &= \hat{\phi}(z_\ell|z_{\ell-1}) + K_\phi(z_\ell) e(z_\ell, \theta) \\ \hat{\theta}_m(z_\ell|z_\ell) &= \hat{\theta}_m(z_\ell|z_{\ell-1}) + K_\theta(z_\ell) e(z_\ell, \theta) \end{aligned} \quad (21)$$

This completes the basic implementation of the MBID, next we discuss its application to a set of noisy experimental measurements. We utilize the experiment at the Hudson Canyon located off of the New Jersey coast, a well-known shallow water (73m) ocean environment [19]. Here a 23-element vertical array is deployed from the bottom with 2.5m separation to measure the pressure-field. Note that the subsequent plots show the function over the aperture length (57.5m) not its actual position in the water column. We use a 1.25m spacing using the following average horizontal wave numbers: $\{0.208, 0.199, 0.183, 0.175, 0.142\} m^{-1}$ for the 5 modes supporting the water column from a 36m deep, 50Hz source at 0.5Km range (see Refs. 8 and 33 for more details). We investigate the results of the MBID design using the actual experimental hydrophone measurements from the Hudson Canyon. Here we have the 23 element array and initialize the MBID with the average set of horizontal wave numbers from [18]. The resulting estimates are quite reasonable as shown in Figure 2. Note that although there is a little difficulty tracking the first couple of modes, the results actually appear better than those reported previously for this data set (see Refr. 7). The results for the higher order modes follow those predicted by the model as observed in the figure and corresponding estimation errors. From the figure we see the reconstructed pressure-field and innovations are also quite reasonable as shown in Figure 3 and indicates a "tuned" processor with its zero-mean ($1.9 \times 10^{-3} < 6.7 \times 10^{-3}$) and white. Recall that it is necessary for the innovations sequence to be zero mean and white for the processor to be deemed as "tracking" for the modes and associated parameters. Thus, the processor is successfully tracking and the model is *valid* for this data set. The final parameter estimates are shown in Figure 4 with the predicted error statistics for this data which are also included in Table I for comparison to the simulated. We note that the parameter estimates continue to adapt to the changing ocean environment based on the pressure-field measurements. We initially start the wave numbers at their averages and then allow them to adapt to the measured sensor data. The first wave number estimate appears to converge (approximately) to the average with a slight bias but the others

adapt to other values due to changes in the data. We see that the MBID appears to perform better than the fixed MBP (see Table II) with the augmented identifier simply because the horizontal wave numbers are "adaptively" estimated, on-line providing a superior fit to the raw data. Thus, we see that the use of the MBID in conjunction with vertical array measurements enables us to circumvent the need for a propagation modal solver as long as we have reasonable estimates to initialize the processor. So in this particular application we see how the model-based identifier can be employed to estimate the wave numbers (horizontal) from noisy pressure-field and sound speed measurements evolving using a vertical array or hydrophones. This completes the section on applying the identifier to a critical ocean acoustic estimation problem.

Table I. MBID: Wave Number Estimation

<i>Hudson Canyon Experiment Wave Numbers</i>		
Model	Simulation	Experiment
0.2079	0.2105 ± 0.0035	0.2076 ± 0.0043
0.1991	0.1993 ± 0.0052	0.1978 ± 0.0036
0.1827	0.1846 ± 0.0359	0.1817 ± 0.0251
0.1746	0.1770 ± 0.0149	0.1746 ± 0.0098
0.1423	0.1466 ± 0.0385	0.1479 ± 0.0288

Table II. MBID: Modal Estimation

<i>Hudson Canyon Experiment: Modeling Error</i>		
Mode	Fixed MBP	Adaptive
1	1.8×10^{-3}	1.2×10^{-3}
3	1.9×10^{-4}	3.0×10^{-4}
4	5.8×10^{-4}	3.2×10^{-4}
5	5.4×10^{-4}	6.7×10^{-4}

III. DISCUSSION

In this paper we have developed an on-line, adaptive, model-based solution to the ocean acoustic signal processing problem based on coupling the normal-mode propagation model to a vertical sensor array. The algorithm employed was the nonlinear extended Kalman filter identifier/parameter estimator. It was shown that the model-based identifier (MBID) could be designed to estimate the set of horizontal wave numbers from noisy Hudson Canyon experimental data yielding results better than those reported previously [8] in the sense that the estimated modal functions "track" those predicted by propagation models more closely (smaller variances, etc.). It appears that the application of the adaptive MBID scheme yields superior performance then a

Figure 2: Model-Based Identification of Hudson Canyon Experiment (0.5Km): (a) Mode 1 and error (91% out). (b) Mode 2 and error (83% out). (c) Mode 3 and error (0% out). (d) Mode 4 and error (0% out).

Figure 3: Model-Based Identification of Hudson Canyon Experiment (0.5Km): (a) Mode 5 and error (0% out). (b) Pressure-field and innovation (2% - 0% out). (c) Whiteness test and WSSR (0% out).

Figure 4: Model-Based Identification of Hudson Canyon Experiment (0.5Km): (a) Parameter 1 and error (8.7% out). (b) Parameter 2 and error (4.4% out). (c) Parameter 3 and error (0% out). (d) Parameter 4 and error (0% out).

fixed processor enabling it to passively "listen" to the everchanging ocean environment and "learn" about its changes in the true spirit of a well-informed array.

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