LA-UR- 95 - 2308

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BEAM-BREAKUP TRANSIT-TIME OSCILLATORS

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Submitted to:

THE INTERNATIONAL SOCIETY OF OPTICAL ENGINEERING
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Comparison of axial and radial electron beam-breakup transit-time oscillators

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ABSTRACT

Comparison of two configurations of a novel high-power microwave generator is presented in this article. Coupling the beam-breakup instability with the transit-time effect of the electron beam in the cavity, rapid energy exchange between the electrons and cavity modes can occur. The dominant cavity modes in the axial and radial configurations are different but their growth rates are comparable. We found that the radial configuration can have a beam impedance less than 10Ω and therefore more suitable for low-voltage and high power operation. Good agreements have been obtained between linear theory and simulation for both configurations.

keywords: microwave generator; instability; transit-time; cavity modes; growth rates; beam impedance; simulation.

2. INTRODUCTION

Physical processes leading to the excitation of coherent electromagnetic radiation by relativistic electron beams interacting with various passive structures often depend on axial bunching of the electron beam. The bunching mechanism becomes increasingly inefficient as the kinetic energy of the electron beam increases. Typically, the growth rate of the electromagnetic radiation scales like $1/\gamma^3$. With space-charge limiting currents decreasing with lower voltages, high efficiency is difficult to achieve simultaneously with high power (>1 GW). However, if the interaction makes use of beam bunching in the transverse direction the growth rate would instead scale with $1/\gamma$. The conventional transit-time oscillators (TTO),¹ such as the monotron,^{2,3} where a continuous electron beam passing axially through a simple pillbox cavity generates microwaves if the cavity dimensions are chosen correctly. Electrons traversing the cavity either gain or lose energy, depending on their transit time across the cavity and on the phase of the microwave fields. This interaction leads to axial bunching of the electron beam which results in the unfavorable scaling with beam energy. Also, the large value of the interaction Q implies that little power can be extracted from the cavity.

The beam-breakup transit-time oscillator (BTO) does not depend on axial bunching to facilitate energy exchange between the electron beam and electromagnetic modes. We investigated this novel high-power microwave source in two different configurations. The axial version BTO (ABTO) is depicted in Fig. 1 where a solid electron beam is injected into a simple cylindrical cavity with appropriate dimensions. The nonaxisymmetric electromagnetic cavity mode such as TM_{110} or TE_{112} deflects the electron beam sideways by their finite transverse magnetic field. The transverse excursion causes energy exchange between the electron beam and the cavity mode. The cavity mode grows or damps

depending only on the transit time, $T = d/v$, where v is the axial velocity of the electron beam and d is the length of the cavity. This interaction is inherently 3D, even without

Figure 1: Configuration of ABTO and cavity mode TM_{110} structure

Figure 2: Configuration of RBTO used in the simulation

extraction consideration. The beam is deflected sideways and lies in a plane containing the maximum axial electric field. Since the interaction depends on transverse deflections of the beam the growth rate would have the more favorable scaling, $1/\gamma$, with beam energy. Because of the limitation of the space-limiting current in a cylindrical cavity, the axial BTO is constrained to a beam impedance no lower than about 400 Ω . Therefore, a high voltage electron beam is likely required for high power operation.

In order to increase power without going to impractically-high voltages and bulky insulation, a radial configuration of the BTO was studied. The radial beam-breakup transit-time oscillator (RBTO) is shown in Fig. 2 for a radially inward-directed beam. The beam is generated in a coaxial vacuum transmission line, with symmetric power feed to a radial field-emission diode, and it enters the cavity through a screen (indicated by the dashed line in Fig. 2). It is then axially deflected like a rippled drum membrane by the cavity wave. In contrast to the ABTO, the RBTO uses an annular cavity with a ribbon-like beam fanning its way radially across the cavity, with the beam deflected back and forth (along the axis of the annular cavity) uniformly around the annular cavity in the θ direction. Because the motion is uniform in θ , the problem is now 2D. Because an annular cavity can support a much larger space-charge-limited current than a cylindrical cavity, the beam impedance can be greatly reduced. Thus, the RBTO has several advantages: (1) the 2D TEM microwave mode is easy to analyze and compute; (2) the radial beam can carry 50 times more current and power than an axial beam ($< 10 \Omega$ impedance); (3) the high efficiency and growth rate decrease slowly with increasing voltage; and (4) the device is compact with no external magnetic field. Advantages (3) and (4) are shared with the ABTO over many other devices.

3. LINEAR THEORY AND COMPUTER SIMULATION

We have calculated the linear growth rate, nonlinear saturation level, and space-charge-limited current of the BTO in the axial and radial configurations.

3.1 Axial beam-breakup transit-time oscillators

We can express the vector potential of the TM_{110} cavity mode near the axis as

$$A_z = A_0[x\cos(\omega t - \phi) - y\sin(\omega t - \phi)] ,$$

where ω is the mode frequency. A_0 and ϕ are the field amplitude and injection phase angle. The field components are given by

$$E_z = \omega A_0[x\sin(\omega t - \phi) + y\cos(\omega t - \phi)]$$

$$B_x = A_0\sin(\omega t - \phi), \quad B_y = -A_0\cos(\omega t - \phi) .$$

Under these assumptions, the linear growth rate can be derived⁴ to be

$$\omega_i = (v/d)(v/\gamma)(J_1'\chi)^{-2}[2[\cos(\omega d/v) - 1] + (\omega d/v)\sin(\omega d/v)] .$$

Figure 3: Growth rate versus cavity length for ABTO from linear theory and simulation. The dashed line is the space-charge limiting current.

Figure 4: Electron beam exhibits strong deflections and energy loss in an ABTO

The transit-time effects enter through the sine and cosine terms in the equation. The growth rate ω_i is maximized for a transit time near $\omega d/v = (l + 1/2)\pi$, where l is an even nonzero integer. Also, the maximum growth rate is proportional to the frequency ω of the cavity mode. In Fig. 3, we show the growth rate as a function of cavity length. The cavity has a radius $R = 7.5$ cm and the electron beam has a voltage $V = 2.55$ MV and a current $I = 5$ kA. In the evaluation of the growth rate, the velocity v is taken to be the initial electron velocity as an approximation. Furthermore, space charge effects, which would depress the beam velocity, are ignored in the linear theory. For electron beams with current much smaller than the space-charge limiting current, space charge effects are only small corrections. The space charge limiting current of the cylindrical cavity is shown in Fig. 3 as a reference to the approximation. The three-dimensional particle-in-cell code IVORY has been used successfully to model the physics of the BTO interaction in an unloaded cavity. The growth rates obtained from computer simulations are shown in Fig. 3. The comparison with the linear theory is quite good. The slight shift in cavity length is due to the space-charge effects which was not included in the theory.

Dynamics of the electron beam can be visualized by examining the phase-space structure of the electrons at various times during the simulation. Figure 4 shows a snapshot of the electron beam real space [y (i.e., $r\sin\theta$) vs z] and phase space [$mc^2(\gamma - 1)$ vs z]. At this time, the instability has already saturated. The interaction between the cavity mode TM_{110} and the beam electrons causes increasing radial excursion as the electron beam propagates through the cavity. At the same time energy exchange occurs as showed in the phase space diagram. In this simulation, the average kinetic energy loss of the electron beam when it exits the cavity is about 35%.

3.2 Radial beam-breakup transit-time oscillators

The cavity fields for the RBTO are the TEM_{00m} mode given by

$$E_r = (\Phi/r)\sin(m\pi z/h)\sin(\omega t - \phi),$$

$$B_z = (\Phi/cr)\cos(m\pi z/h)\cos(\omega t - \phi),$$

where $\omega = m\pi c/h$, h is the axial length of the cavity, ϕ is an arbitrary phase angle, and Φ is the mode amplitude. The mode number m corresponds to the number of half wavelengths of the mode axially across the cavity. The closest analogue to the ABTO device is the $m=2$ mode. The unperturbed beam travels radially across the cavity with velocity v_r and position $z_0 = h/2$ centered in the cavity. Note that h is the only length which sets the cavity frequency for this mode. Ignoring space-charge and nonlinear effects, or assuming a very relativistic beam (i.e., treating the radial velocity constant), the linear growth rate is given by⁵

$$\omega_i/\omega = (Av/m\pi\gamma)/\ln(b/a),$$

where $A \equiv (f(\Theta_b) - f(\Theta_a)) \sin \Theta_a + (g(\Theta_b) + g(\Theta_a)) (1 - \cos \Theta_a)$ and $\Theta_a \equiv \omega d/\beta c$; $\Theta_b \equiv \omega a/\beta c$; $\Theta_c \equiv \omega b/\beta c$. The dimensions a , b , d , and h are defined in Fig. 2 and $v = I/17kA$ is Budker's parameter. The functions $f(z) \equiv Ci(z)\sin(z) - si(z)\cos(z)$ and $g(z) \equiv Ci(z)\cos(z) - si(z)\sin(z)$ are defined in terms of Sine and Cosine integrals.⁶

An obvious question at this stage is how does the growth rate of the RBTO depend on the direction of the beam? Repeating this calculation with v_r positive and the beam going from a to b , and using the symmetry relations $f(-z)=-f(z)$ and $g(-z)=g(z)$, it is easy to show that the growth rate is independent of the beam direction if all other parameters are the same. However, it has been shown⁵ that the nonlinear saturation is remarkably different that the case with a radially inward-directed beam has a much higher efficiency.

In Fig. 5, we show an example of how the growth rate depends on the annular width $d=b-a$, for the case of an inward-directed beam of voltage 1MV and current 50kA, in a cavity of axial length $h=15\text{cm}$ and fixed outer radius $b=35\text{cm}$, and mode $m=2$. The growth rate shows the expected transit-time resonances with increasing d , as in the ABTO, but here the range of variation of d is limited by the axis so only two growth-rate peaks are possible. We also show the results of several simulations using the MRC code IVORY. As with the ABTO, space-charge depression of the beam velocity shifts the transit-time resonances to smaller values of d than the theory from Eq. 2, which does not contain space-charge corrections.⁴ This effect becomes much more pronounced as the inner radius a approaches the axis, and the second growth-rate peak is almost 180° out of phase with the simple theory. Nevertheless, the magnitudes of the observed growth rates are in good agreement with the simple theory.

Figure 5: Growth rate versus cavity width for RBTO from theory and simulation

Another obvious question is which device, the ABTO or the RBTO, has the higher growth rate? One can take the limiting case of the RBTO with a large radius, holding fixed the beam energy and current (i.e., γ and v) and the cavity frequency and transit-time (i.e., the axial length h and the radial width d). The growth rate is then inversely proportional to radius. In this sense, the RBTO would have a lower growth rate than the ABTO. Fanning the beam out in the θ direction to form a radial beam crossing a large annular cavity reduces the local strength of the cavity fields and their ability to perturb the beam, and it also reduces the local density of the beam and its ability to generate cavity fields. However, the advantage of the RBTO is that one does not have to hold the beam current constant. The annular cavity has a much higher space-charge-limited current, roughly proportional to radius, which makes the growth rate nearly the same if the space-charge limited current is injected into each device. For the ABTO, with a 2.55MV beam having 5kA of current, injected into a cylindrical cavity with length $d=14.5\text{cm}$, radius 7.5cm, and frequency 2.44GHz, simulations showed a growth rate of 0.24/ns. The results correspond to a cavity interaction "Q" of $Q_b=32$. For the RBTO with a similar frequency of 2.0GHz, the simulation in Fig. 5 with inner radius $a=10\text{cm}$ ($d=25\text{cm}$) showed a growth rate of 0.255/ns or $Q_b=25$. Because the relative change in cavity energy per cycle is $\Delta U/U=2\pi/Q$, roughly a quarter of the cavity energy is being regenerated each cycle. It is hard to imagine a device doing much better in this regard than either the ABTO or the RBTO. Alternatively, these devices support and can efficiently use a beam current large enough to drive a cavity with a low quality factor, Q_L . With the optimum growth rate or Q nearly the same for the two devices, this makes the power and cavity energy nearly proportional to radius and the beam impedance inversely proportional to radius for the RBTO.

The space-charge limiting current is an important consideration in high-power operation of any microwave sources. An estimate of the upper bound for the space-charge current of for both the ABTO and RBTO can be obtained. For the ABTO, it is given by⁶,

$$v = (17/4)(r_b/r_w)^2[(2.405)^2 + (\pi r_w/d)^2](\gamma^{2/3} - 1)^{3/2} \text{ kA},$$

where r_b is the radius of the electron beam and r_w and d are the radius and length of the cavity. The numerical evaluation is shown in Fig. 3 and the values are less than 10 kA for the parameters used in the simulation. For the RBTO, it is given by⁵

$$v = 17b\alpha \coth(\alpha h/2)(\gamma^{2/3} - 1)^{3/2} \text{ kA}.$$

This yields an upper bound of space-charge limiting current of 110kA for the simulation parameters: $h = 15 \text{ cm}$, $a = 10 \text{ cm}$, $b = 35 \text{ cm}$, and $V = 1 \text{ MeV}$. The order of magnitude increase in the limiting current makes the RBTO uniquely suitable for extremely high power operation with low impedance electron beams.

4. CONCLUSION

The axial and radial electron beam-breakup transit-time oscillators have been demonstrated in our simulation and analysis to be promising microwave sources. They both are compact and simple, and they do not require an external magnetic field. The frequency of radiation only depends on cavity dimensions. They have favorable efficiency scalings with respect to microwave frequency and electron beam energy. Furthermore, the radial BTO has been shown to be particularly suitable for high-power operation with low voltage and high current electron beams.

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