RIKEN Winter School

STRUCTURE OF HADRONS
—INTRODUCTION TO QCD HARD PROCESSES—

December 9–12, 1998

Organizers
Naohito Saito, Toshi-Aki Shibata and Koichi Yazaki

RIKEN BNL Research Center
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Preface to the Series

The RIKEN BNL Research Center was established this April at Brookhaven National Laboratory. It is funded by the “Rikagaku Kenkyusho” (Institute of Physical and Chemical Research) of Japan. The Center is dedicated to the study of strong interactions, including hard QCD/spin physics, lattice QCD and RHIC physics through nurturing of a new generation of young physicists.

For the first year, the Center will have only a Theory Group, with an Experimental Group to be structured later. The Theory Group will consist of about 12-15 Postdocs and Fellows, and plans to have an active Visiting Scientist program. A 0.6 teraflop parallel processor will be completed at the Center by the end of this year. In addition, the Center organizes workshops centered on specific problems in strong interactions.

Each workshop speaker is encouraged to select a few of the most important transparencies from his or her presentation, accompanied by a page of explanation. This material is collected at the end of the workshop by the organizer to form a proceedings, which can therefore be available within a short time.

T.D. Lee
July 4, 1997

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Introduction

The 9th Riken Winter School on "Structure of Hadrons - Introduction to QCD Hard Processes" was held from December 9th through 12th at Shimoda Tokyu Hotel, Shimoda, Shizuoka, Japan. The school was sponsored by the RIKEN BNL Research Center and the RIKEN Accelerator Research Facility. The purpose of the school was to offer young researchers an opportunity to learn the exciting physics associated with the structure of hadrons probed by QCD hard processes. The subjects discussed in the school cover theoretical and experimental aspects of the hard processes including polarization phenomena and the nucleon structure functions, starting from the basics of the perturbative QCD.

We had 3 main courses each consisting of 4 one-hour lectures, 3 tutorial sessions including Q&A and exercises on the contents of the lectures, 2 evening sessions for discussions on interesting and important subjects and 4 short lectures on specific topics. These were given by 3 main lecturers and 4 tutors. The list of them together with the titles of their lectures are given below.

Main Lecturers
R. Jaffe (MIT) "Spin in Hard QCD Processes"
J. Kodaira (Hiroshima) "Perturbative QCD and Introduction to Structure Functions"
K. Rith (Erlangen) "Quark-Gluon Structure of the Nucleon"

Tutors
N. Hayashi (Riken) "RHIC Spin Program"
Y. Koike (Niigata) "Twist-3 Effects in Hard Processes"
S. Kumano (Saga) "Polarized Proton-Deuteron Drell-Yan Processes"
K. Tanaka (Juntendo) "Hard Exclusive Processes and Vector Meson Wave Functions in QCD"

24 students attended the school and actively participated in the program. The 3 main lecturers gave excellent courses which were both pedagogical and inspiring. They also attended the tutorial and evening sessions and were kind enough to respond to any questions asked either during the sessions or in smaller circles.

The 4 tutors played very important roles by organizing the tutorial sessions, making exercise problems, helping students in understanding the lectures and giving interesting lectures on their own works.

At the end of the school, we asked the students to write their opinions about the school. The responses were all positive, and the students were grateful to the lecturers and the tutors. Many of them seem to have felt that 4 days were a little too short for fully understanding the broad field of QCD hard processes. They also wanted to have this kind of school regularly, every year if possible. We are thus considering the possibility of organizing such a school at least every two years.

We are grateful to Prof. T. D. Lee and Prof. M. Ishihara for their approval, which enabled us to organize this school. We also thank the lecturers, the tutors and the students for making the school fruitful. Special thanks are due to two secretaries, Ms. Yoko Kishino and Ms. Toshiko Nakamura, who did most of the administrative works and took care of drinks and snacks during the coffee breaks and the evening sessions. The hotel is located at a beautiful spot in Izu peninsula and we were lucky about the weather. We thank...
Shimoda Tokyu Hotel for their excellent service and comfortable atmosphere.

Naohito Saito, Toshi-Aki Shibata and Koichi Yazaki
Introduction to Perturbative QCD and Structure Functions

Lectures Presented at the 9th RIKEN Winter School,
Shimoda, December 8-12, 1998

Jiro Kodaira

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Abstract

In this lecture I give a pedagogical introduction to the Perturbative QCD to understand the short-distance dynamics of the strong interaction. Starting with fundamental concepts such as the color degree of freedom of QCD, non-abelian gauge field theory, renormalization group equation etc., I explain a basic idea of the perturbative QCD and apply this idea to the e^+e^- processes and the structure functions. The notion of mass singularity and the necessity of its factorization is discussed in some detail.
INTRODUCTION
TO
PERTURBATIVE QCD
AND
STRUCTURE FUNCTIONS

J.Kodaira (Hiroshima)

December 9-12 '98
at RIKEN WINTER SCHOOL

1. PROLOGUE
   - Colored Quark Model
   - Quantization
   - QCD as Gauge Theory
   - Parton Model

2. PERTURBATIVE QCD
   - Renormalization
   - Basic Idea
   - Asymptotic Freedom

3. INFRARED SAFE OBSERVABLES
   - KLN Theorem
   - Jet
   - $e^+e^-$ Process

4. DEEP INELASTIC SCATTERING
   - Factorization
   - Parton Distribution Function and OPE
1. PROLOGUE

Colored Quark Model

1. History

Sakata Model $\rightarrow$ SU$_f$(3) Classification of Hadrons

$$3 = (p, n, \Lambda)$$

QUARK Model by Gell-Mann and Zweig

$$\bar{3} = (u, d, s)$$

Quark Model tells us:

**Meson** $\leftarrow$ 2-quark Bound State

$$(3 \times \bar{3}^* = 1 + 8)$$

**Baryon** $\leftarrow$ 3-quark Bound State

$$(3 \times 3 \times 3 = 1 + 8 + 8 + 10)$$

e.g.

$$\pi^+ = u\bar{d} \quad \pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \quad \pi^- = \bar{u}d$$

$$p = uud \quad n = udd \quad \Sigma = uus \quad \Omega^- = s^+_s^+_s^+_s^-$$
2. Statistics

QUARKs are Fermions ↔ Pauli Principle

e.g. for $\Omega^-$ → Not S-wave .
but S-wave from Exp. (e.g. Magnetic Moment)

QUARK Should have
an Additional Quantum Number
下达
COLOR

For $SU_c(3)$

$\epsilon_{ijk} q^i q^j q^k$

is Antisymmetric !!

3. $\pi^0 \to 2\gamma$

\[ \pi^0 \rightarrow \begin{array}{c}
\text{q}\text{i} \\
\end{array} \]

if $q$ is Colorless,

$\text{Exp.} = 9 \ (\text{Theor.})_{\text{Colorless}}$

if $q$ is Colored,

$\text{Theor.} = 3^2 \ (\text{Theor.})_{\text{Colorless}} = \text{Exp.}$
4. R - ratio

\[ R(s) \equiv \frac{e^+e^- \rightarrow \text{hadrons}}{e^+e^- \rightarrow \mu^+\mu^-} \]

if \( q \) is Colorless, \( R_{\text{Theor.}} = \Sigma_i Q_i^2 \)
if \( q \) is Colored, \( R_{\text{Theor.}} = 3 \Sigma_i Q_i^2 = R_{\text{Exp.}} \)

5. Append.

QUARKs are Confined
\[ \uparrow \]
Colorless State is only Observable

6. Colored Quark

<table>
<thead>
<tr>
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<th>( \leftarrow ) SU(_c)(3)</th>
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QCD as Gauge Theory

Color $\leftrightarrow$ SU$_c$(3)
$\rightarrow$ SU$_c$(3) Non-Abelian Gauge Theory

Non-Abelian Gauge Field Theory

Start with Free Dirac Theory

$$L_{\text{free}} = \bar{\psi}(i \not\!D - m)\psi$$

For Matter (quark) field $\psi$,

$$\psi \rightarrow \psi' = U\psi = e^{i\theta^a(x)T^a}\psi .$$

where $T^a$ is Representation of Group (SU(3))

$$[T^a, T^b] = if^{abc}T^c ,$$

Introduce Gauge Field $A^a_\mu$

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu - ig T^a A^a_\mu ,$$

and Define its Transformation such that

$$D_\mu \psi \rightarrow (D_\mu \psi)' = UD_\mu \psi ,$$

$$(D_\mu \psi)' = (\partial_\mu - ig T^a A^a_\mu)\psi'$$

$$= U(\partial_\mu + U^{-1}\partial_\mu U - ig U^{-1}T^a A^a_\mu U)\psi ,$$

so

$$T^a A^a_\mu = UT^a A^a_\mu U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1} ,$$

With these Fields, we get Non-Abelian Gauge Field Theory

$$L = \bar{\psi}(i \not\!D - m)\psi - \frac{1}{4}F^{a\mu\nu} F_{a\mu\nu} ,$$

with

$$F^{a\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc} A^b_\mu A^c_\nu .$$
Quantization

1. General Strategy

PERTURBATION THEORY

- Take Interaction Picture

\[ \mathcal{L}(x) \equiv \mathcal{L}_0(x) + \mathcal{L}_I(x) \]

- Quantize Free Part \( \mathcal{L}_0 \)

2. Naive (Canonical Quantization)

\[ \mathcal{L}(x) = \mathcal{L}(\phi(x), \partial_\mu \phi(x)) \]

Canonical Variable = \( \phi(x) \)

Canonical Momentum = \( \pi(x) \equiv \frac{\partial \mathcal{L}}{\partial (\partial_0 \phi)} \)

Commutation Relation:

\[ [\phi(x), \pi(y)]_{x^0 = y^0}^{\pm} = i\delta(\vec{x} - \vec{y}) \]
\[ [\phi(x), \phi(y)]_{x^0 = y^0}^{\pm} = [\pi(x), \pi(y)]_{x^0 = y^0}^{\pm} = 0 \]

3. Complexity for Gauge Field

- \( A_\mu \) vs Physical Degree of Freedom (= 2)

Gauge Fixing

- Non-Abelian

Faddeev-Popov Ghost
Effective Lagrangian

\[
\mathcal{L}_{\text{eff}} = \mathcal{L} + \mathcal{L}_{\text{g.f.}} + \mathcal{L}_{\text{gc}}
\]

\[
\mathcal{L}_{\text{g.f.}} = \frac{1}{2\alpha} (f^a(A))^2 = -\frac{1}{2\alpha} (\partial^\mu A^a_\mu)^2
\]

\[
\mathcal{L}_{\text{gc}} = -\bar{c}_a \frac{\partial f^a_\mu}{\partial A^b_\mu} D^{bd}_\mu c_d = \partial^\mu \bar{c}_a D^{bd}_\mu c_d
\]

4. S-Matrix, Feynman Rule e.t.c.

- Dyson’s S-Matrix

\[
|t = +\infty\rangle \equiv S |t = -\infty\rangle
\]

\[
S = T \left( e^{\int_{-\infty}^{\infty} d^4x \mathcal{L}_I(x)} \right)
\]

- Unitarity

\[
S_{fi} = \langle f | S | i \rangle \equiv \delta_{fi} + i(2\pi)^4 \delta^4(p_f - p_i) T_{fi}
\]

Insert \( SS^\dagger = S^\dagger S = 1 \)

\[
T_{fi} - T_{fi}^\dagger = i(2\pi)^4 \sum_n \delta^4(p_f - p_n) T_{fn} T_{ni}^\dagger
\]

When \( f = i \)

\[
2\text{Im} T_{ii} = (2\pi)^4 \sum_n \delta^4(p_i - p_n) |T_{in}|^2
\]

- Cross section for \( p_1 + p_2 \rightarrow k_1 + k_2 + \cdots + k_n \)

Define

\[
M_{fi} \equiv \prod_l (2\pi)^{3/2} \sqrt{2p_l^0} \prod_j (2\pi)^{3/2} \sqrt{2k_j^0} T_{fi}
\]

with

\[
s_{ij}^\pm = s - (m_i \pm m_j)^2 , \quad s = (p_1 + p_2)^2
\]

\[
d\sigma = \frac{1}{2\sqrt{s_{12} s_{12} s_{12}}} (2\pi)^4 \delta^4 \left( \sum_j k_j - p_1 - p_2 \right) |M_{fi}|^2 \prod_j \frac{d^3 k_j}{(2\pi)^3 2k_j^0}
\]

10
• Feynman Rule

Field $\phi$ can be Expanded into Plane Waves in Int. Picture

\[ \downarrow \]


\[ i \overrightarrow{\delta_{ij}} \]

\[ \frac{i}{k^2} \delta^{ab} \left( g_{\mu\nu} - (1 - \alpha) \frac{k_\mu k_\nu}{k^2} \right) \]

\[ \frac{-i}{k^2} \delta^{ab} \]

\[ i g \gamma^\mu \Gamma_{ij}^a \]

\[ g f^{abc} p^\mu \]

\[ g f^{abc} \{ g_{\mu\nu} (p - q)_\lambda + g_{\nu\lambda} (q - r)_\mu + g_{\lambda\mu} (r - p)_\nu \} \]

External Line

\[ \leftrightarrow \bar{u}(p), u(p), \bar{v}(p), v(p), \epsilon_\mu(k) \]
Parton Model

1. Deep Inelastic Scattering

Lorentz Invariants:

\[ Q^2 = -q^2 = -(k - k')^2 \]
\[ P \cdot q = M \nu \]
\[ x = \frac{1}{\omega} = \frac{Q^2}{2P \cdot q} \]

Cross section:

\[ k'_0 \frac{d\sigma}{d^3 k'} = \frac{\pi}{4k \cdot P} e^4 \sum_X |M_{fi}|^2 \delta^4(P_X - P - q) \]

where

\[ M_{fi} \equiv \langle eX|M|eN \rangle = \langle k'|j_\mu(0)|k, s \rangle \langle X|J_\nu(0)|PS \rangle D^{\mu\nu}(q) \]

Leptonic tensor:

\[ L_{\mu\nu} = \frac{1}{2} \sum_{s'} \langle k, s|j_\mu(0)|k', s' \rangle \langle k', s'|j_\nu(0)|k, s \rangle \]

Hadronic tensor:

\[ W_{\mu\nu} = \frac{1}{2\pi} \sum_X \langle PS|J_\mu(0)|X \rangle \langle X|J_\nu(0)|PS \rangle (2\pi)^4 \delta^4(P_X - P - q) \]
\[ = \frac{1}{2\pi} \int d^4x e^{-iq \cdot z} \langle PS|J_\mu(0)J_\nu(z)|PS \rangle \]
\[ = \frac{1}{2\pi} \int d^4x e^{-iq \cdot z} \langle PS|[J_\mu(0), J_\nu(z)]|PS \rangle \]

so

\[ k'_0 \frac{d\sigma}{d^3 k'} = \frac{1}{k \cdot P} \left( \frac{e^2}{4\pi Q^2} \right)^2 L^{\mu\nu} W_{\mu\nu} \]
In General

\[ W_{\mu\nu} = - \left( g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2} \right) F_1 + \left( P_{\mu} - \frac{P \cdot q}{q^2} q_{\mu} \right) \left( P_{\nu} - \frac{P \cdot q}{q^2} q_{\nu} \right) \frac{2}{P \cdot q} F_2 \]

\[ + 2i \epsilon_{\mu\nu\lambda\sigma} q^\lambda \left\{ \frac{1}{P \cdot q} g_1 + (P \cdot q S^\sigma - \bar{q} \cdot S P^\sigma) \frac{1}{(P \cdot q)^2} g_2 \right\} \]

with

\[ P \cdot S = 0 \quad S^2 = -M^2 \]

2. Bjorken Scaling

How to see "Constituents = Quarks" Directly?

Hit by "Light" which has

Wave Length \( \ll L \)

(Short W.L \( \leftrightarrow \) High Energy)

Bjorken Limit:

\[ x : \text{fixed} \quad Q^2, P \cdot q \to \infty \]

Bjorken Scaling:

Structure Functions \( F_i, (g_i) \)

depend on Two Invariants

\[ F_i = F_i(x, Q^2) = F_i(P \cdot q, Q^2) \]
\[ \to F_i(x) \]
3. Parton Model

Nucleon is Assembly of Free Quarks (Partons) at High (Deep) Energy

Note: At High Energy (Short Distance), We will also See Quantum Effects

So Not Only 3 Quarks but Many

Parton Model (Impulse Approximation)

\[ \uparrow \]

Interaction Time \( \tau \approx \frac{1}{q^0} \ll \text{Life Time of Partons} \ T \)

Old Fashioned P.T. in Infinite Momentum Frame

\( T^{-1} \sim \text{Energy Denominator} \)

\[ \sum_i E_i - E_P = \sum_i \left[ (x_i P)^2 + p_{i\perp}^2 + \mu_i^2 \right] - \sqrt{P^2 + M^2} \]

with \( \sum_i x_i = 1 \)

Take \( P \to \infty \)

\[ \sum_i E_i - E_P \approx \sum_i \left[ x_i P + \frac{p_{i\perp}^2 + \mu_i^2}{2|x_i|P} \right] - P - \frac{M^2}{2P} \]

When all \( x_i > 0 \), \( T \) becomes Max

\[ T \sim \frac{2P}{\sum (p_{i\perp}^2 + \mu_i^2)/x_i - M^2} \sim \frac{2P}{M^2} \quad \text{when} \quad p_{\perp} \sim \mu \sim 0 \]
Take a Lepton-Nucleon CM frame
\[ q = \left( \frac{2P \cdot q - Q^2}{4P}, q_{\perp}, -\frac{2P \cdot q + Q^2}{4P} \right) \]

Then
\[ \tau \sim \frac{1}{q^0} = \frac{4P}{2P \cdot q - Q^2} \]

So,
\[ \tau \ll T \Rightarrow P \cdot q(1 - x) \gg M^2 \]

Structure Function in Parton Model

Introduce "Parton Distribution Function"

\[ q(\xi) d\xi, \quad p_q = \xi P \]

Cross Section:
\[ \sigma(l + N \to l' + X) = \sum_q \int \sigma_{\text{parton}}(l + p_q \to l' + X) q(\xi) d\xi \]

with \( q = u, \bar{u}, d, \bar{d}, \ldots \)

Scaling:
\[ \sigma_{\text{parton}} \propto \delta((\xi P + q)^2 - (\mu_{\text{parton}}^2 \sim 0)) \]

\[ x = \xi \]
Structure Function:
For Helicity state of Nucleon, introduce

\[ q_\uparrow(\xi) \quad \text{and} \quad q_\downarrow(\xi) \]

\[ q(\xi) \equiv q_\uparrow(\xi) + q_\downarrow(\xi) \]
\[ \delta q(\xi) \equiv q_\uparrow(\xi) - q_\downarrow(\xi) \]

Finally

\[ F_1(x) = \frac{1}{x} F_2(x) = \sum_q e_q^2 \, q(x) \]
\[ g_1(x) = \frac{1}{2} \sum_q e_q^2 \, \delta q(x) \]
2. PERTURBATIVE QCD

✧ Renormalization

1. Lagrangian and Physics

Two Interrogation about "Lagrangian"

\[
\mathcal{L} = \bar{\psi}(i \not{D} - m_0)\psi - \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \cdots
\]

\[
= \bar{\psi}(i \not{\partial} - m_0)\psi + g_0 \bar{\psi} \not{A}^a T^a \psi_0 - \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \cdots
\]

- Normalization of \( \mathcal{L} \)? Wrong with \( 2\mathcal{L} \)?
- What are \( m_0 \) and \( g_0 \)?

Both were Fixed at Classical (Tree) Level 0
but

\( m \) and \( g \) are Parameters Determined from Exp.

Exp. Value \( \neq \) Tree Level

e.g.

\[ i g_0 \Gamma^\mu(p, p') T^{a}_{ij} = i g_0 \gamma^\mu T^a_{ij} + i g_0^2 \gamma^\mu f(p, p') T^a_{ij} + \cdots \]
2. Renormalization

One MUST Define Coupling Constant \( g \)
Corresponding to an Experiment, at \( p^2 = p'^2 = \mu^2 \)

\[
g(\mu) \equiv g_0 + g_0^3 f(p^2 = p'^2 = \mu^2) + \cdots
\]
\[
\downarrow
\]
\[g_0 = Z_g(\mu) g(\mu)\]

"Physical" Coupling Const. \( g(\mu) \)
Depend on (arbitrary) Scale \( \mu \)

The Same for \( m \) and Normalization

For Quark (Gluon):

Residue and (Pole Position) of Propagator
Change from Tree Level
\[
\downarrow
\]
Renormalization

\[ \psi_0 = Z_2^{1/2} \psi \ , \ A_0^{a\mu} = Z_3^{1/2} A^{a\mu} \]
\[ g_0 = Z_g g \ , \ m_0 = Z_m m \ , \ \cdots \]

\[ \mathcal{L} = Z_2 \bar{\psi}(i \partial - Z_m m)\psi + Z_g Z_2 Z_3^{1/2} g \bar{\psi} A^a T^a \psi \]
\[ - \frac{Z_3}{4} F^a_{\mu\nu} F^{a\mu\nu} + \cdots \]
\[ = \bar{\psi}(i \partial - m)\psi + g \bar{\psi} A^a T^a \psi - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} \]
\[ + (Z_2 - 1) \bar{\psi}(i \partial - m)\psi + Z_2 (1 - Z_m) m \bar{\psi} \psi \]
\[ + (Z_g Z_2 Z_3^{1/2} - 1) g \bar{\psi} A^a T^a \psi - \frac{1}{4} (Z_3 - 1) F^a_{\mu\nu} F^{a\mu\nu} + \cdots \]

\( Z_i \) should be Determined by some Condition

\[ \| \]

Renormalization Condition

\( Z_i \) depend on \( \mu = \) Renormalization Scale

n.b. \( Z_i \) are in General Utraviolet Divergent

3. Renormalization Group Equation

Physical Quantity e.g. Cross Section

CAN NOT Depend on \( \mu \)

\[ \frac{d}{d\mu} \sigma(p, \mu) = 0 \]
Asymptotic Freedom

1. Anti-Screening

- \( \mu \) Dependence of \( g(\mu) \)

Remember
\[
g_0 = Z_g(\mu)g(\mu)
\]
so
\[
\mu \frac{d}{d\mu} g_0 = \left( \mu \frac{d}{d\mu} Z_g(\mu) \right) g(\mu) + Z_g(\mu) \mu \frac{d}{d\mu} g(\mu) = 0
\]
i.e.
\[
\mu \frac{d}{d\mu} g(\mu) = -g(\mu) \mu \frac{d}{d\mu} \ln Z_g(\mu) \equiv \beta(g)
\]

- Asymptotic Freedom

For QCD
\[
\beta(g) = -\beta_0 g^3 - \beta_1 g^5 - \cdots
\]
\[
= -\frac{g^3}{(4\pi)^2} \frac{1}{3} (33 - 2n_f) - \mathcal{O}(g^5)
\]
This Means

when \( \mu \to \text{Large} \), \( g(\mu) \to \text{Small} \)

if \( \beta_0 > 0 \) i.e. \( 33 - 2n_f > 0 \)

- Physics

QED v.s. QCD

Photon Does NOT have Charge
Gluon DOES have Charge
2. QCD Coupling Constant

\[ \mu \frac{d}{d\mu} g(\mu) = \beta(g) = -\beta_0 g^3 - \beta_1 g^5 - \cdots \]

- 1-loop:

\[ \frac{1}{2} \ln \frac{Q^2}{\mu^2} = \frac{1}{2 \beta_0} \left[ \frac{1}{g^2(Q^2)} - \frac{1}{g^2(\mu^2)} \right] \]

Now Introduce \( \mu \) Independent Parameter \( \Lambda_1 \)

\[ \Lambda_1 = \mu \exp \left( -\frac{1}{2 \beta_0 g^2(\mu^2)} \right) \]

Then

\[ g^2(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda_1^2)} \]

- 2-loop:

Now

\[ \Lambda_2 = \mu \exp \left( -\frac{1}{2 \beta_0 g^2(\mu^2)} \right) (\beta_0 g^2(\mu^2))^{-\beta_1/2\beta_0^2} \]

Then

\[ g^2(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda_2^2)} - \frac{\beta_1 \ln(\ln(Q^2/\Lambda_2^2))}{\beta_0^3 \ln^2(Q^2/\Lambda_2^3)} + \cdots \]

- QCD Parameter

\( \Lambda \) is the Only Parameter of QCD

except Quark Mass

R.G.E. \( \rightarrow \mu \) and \( g(\mu) \) are not Independent
3. Scheme Dependence

- Explicit Value of Parameter $g, m, \cdots$ Depend on Renormalization Condition

Called Scheme Dependence

- Definition of $\Lambda$

There are some arbitrariness in defining $Z_g$

$$g_0 = Z_g g = Z'_g g'$$

Relation between $g$ and $g'$

$$g^2 = (Z'_g/Z_g)^2 g'^2 \equiv z_g^2 g'^2 = (1 + z_2 g'^2 + z_4 g'^4 + \cdots) g'^2$$

This means QCD $\Lambda$ Depend on Scheme

1-loop Example:

$$g^2(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2)} = g'^2(Q^2)(1 + z_2 g'^2(Q^2) + \cdots)$$

$$= \frac{1}{\beta_0 \ln(Q^2/\Lambda'^2)} \left[ 1 + \frac{z_2}{\beta_0 \ln(Q^2/\Lambda'^2)} \right]$$

i.e. $\Lambda = \Lambda' e^{z_2/(2\beta_0)}$

Commonly Used Scheme:

**MS**: $Z_i$ has only $1/\epsilon$ in $D = 4 - 2\epsilon$

**$\overline{\text{MS}}$**: $Z_i$ has $1/\epsilon - \gamma_E + \ln 4\pi$ in $D = 4 - 2\epsilon$

**MOM**: $Z_i$ in Momentum Subtraction Scheme
Basic Idea

1. General Strategy

In QCD When $\mu \to \infty$, $g(\mu) \to 0$

What will Happen?

if we apply Perturbative Expansion

from the Beginning
to Quark and Gluon System

In Real World, Quark and Gluon are Confined

Relation to Real Observable will be considered Later

Consider "Observable" in Quark and Gluon System

$$\mu \frac{d}{d\mu} \sigma \left( \frac{Q^2}{\mu^2}, \frac{p^2}{\mu^2}, \cdots \right) = \left( \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) \sigma = 0$$

$Q = \text{Typical Mom.}$, $p = \text{Mom. of External Particle}$

$\mu = \text{Renormalization Scale}$

Neglect Mass Effects for Simplicity

Formal Answer:

$$\sigma \left( \frac{Q^2}{\mu^2}, \frac{p^2}{\mu^2}, \cdots, g(\mu^2) \right) = \sigma \left( 1, \frac{p^2}{Q^2}, \cdots, g(Q^2) \right)$$

$p^2$ Should be Considered as

$$p^2 = m_{\text{quark}}^2 \simeq (\text{hadron scale})^2 \simeq 0,$$
• Scheme Dependence in Observables

Observable like Cross section
Should NOT depend on Scheme

\[ S(g) = S'(g') \]

Expand in Perturbation Series

\[ S(g) = s_0 + s_2 g^2 + s_4 g^4 + \cdots \]
\[ S'(g') = s_0' + s_2' g'^2 + s_4' g'^4 + \cdots \]

Rewrite \( S(g) \) using Relation \( g \) and \( g' \)

\[ S(g) = s_0 + s_2 g^{r_2} + (s_4 + z_2 s_2) g^{r_4} + \cdots \]

This Means

\[ s_0 = s_0' , \quad s_2 = s_2' \]

\( s_{2n} \ (n \geq 2) \) Depend on Scheme

and

\[ [S'(g')]_{2n} - [S(g)]_{2n} = \mathcal{O}(g^{2n+2}) \]

\[ \downarrow \]

Perturbative Prediction for Observable
Depend on Scheme !!
Now, Perturbatively Expand when $Q^2 \sim \text{Large}$

$$\sigma \left(1, \frac{p^2}{Q^2}, g(Q^2)\right) = 1 + a g^2(Q^2) \ln \left(\frac{Q^2}{p^2}\right) + b g^2(Q^2) + \mathcal{O}(g^4)$$

Problem: Second Term, if

$$\ln \left(\frac{Q^2}{p^2}\right) \gg 1$$

then

$$g^2(Q^2) \ln \left(\frac{Q^2}{p^2}\right) \geq 1$$

Perturbative Expansion Breaks Down

Singularity in $p^2 = m^2 \to 0$ is Called

Mass Singularity

Related to Long Distance (Low Momentum) Behavior

of QCD

Two Cases to Overcome this Problem

- Factorization

$$\sigma = 1 + a g^2 \ln \left(\frac{Q^2}{p^2}\right) + \cdots$$

$$= \left[1 + a g^2 \ln \left(\frac{Q_0^2}{p^2}\right) + \cdots\right] \left[1 + a g^2 \ln \left(\frac{Q^2}{Q_0^2}\right) + \cdots\right]$$

- Singularity Free Case

$$a = 0$$
2. Sketch of Factorization

Consider DIS

Start with Parton Model (Parton Ansatz) $q^0(z)$

$$F(x, Q^2) = \int_x^1 \frac{dz}{z} F^0 \left( \frac{x}{z} \right) q^0(z)$$

$F^0$ is Parton Structure Func.

$$F^0(\xi) = \delta(\xi - 1) + a(\xi) g^2 \ln \left( \frac{Q^2}{p^2} \right) + \cdots ,$$

Take Moment (for Simplicity)

$$\int dx x^{n-1} F(x, Q^2) = \int dzz^{n-1} q^0(z) \int dyy^{n-1} F^0(y, Q^2)$$

$$\equiv M_n \left[ 1 + a_n g^2 \ln \left( \frac{Q^2}{p^2} \right) + \cdots \right]$$

with

$$M_n = \int dzz^{n-1} q^0(z), \quad a_n = \int dyy^{n-1} a(y),$$

$$\downarrow$$

$$\int dx x^{n-1} F(x, Q^2) = M_n(Q_0^2) \left[ 1 + a_n g^2 \ln \left( \frac{Q^2}{Q_0^2} \right) + \cdots \right]$$

where

$$M_n(Q_0^2) \equiv M_n \left[ 1 + a_n g^2 \ln \left( \frac{Q_0^2}{p^2} \right) + \cdots \right]$$
3. Interpretation and Comment

- **Interpretation**
  - 1st Case
    Factorized Second Term Should be a Prediction
  - 2nd Case
    NOT Need Long Distance Information
    So, $\sigma$ itself Can be a Prediction

- **Comment**
  Two (Technically Different) Approach to
  \[
  \text{Large Log } \ln \left( \frac{Q^2}{p^2} \right)
  \]
  - 1st Approach
    \[ p^2 \to 0 \quad \text{with } Q^2 = \text{fixed} \]
    \[ \Downarrow \]
    Factorization of Mass Singularity
  - 2nd Approach
    \[ Q^2 \to \infty \quad \text{with } p^2 = \text{fixed} \]
    \[ \Downarrow \]
    Renormalization of U.V.Singularity
    \[ \Downarrow \]
    Operator Product Expansion
3. INFRARED SAFE OBSERVABLES

♠ KLN Theorem

1. BN (Bloch-Nordsieck) Theorem

Ultraviolet Sing. $\leftrightarrow$ "Unphysical" Remormalization
Infrared Sing. $\leftrightarrow$ "Physical" Massless Particle

e.g. for QED at 1-Loop

\[
\sim e^2 \left\{ 2 \left( \ln \frac{-q^2}{m^2} - 1 \right) \ln \frac{\lambda^2}{m^2} - \ln^2 \frac{-q^2}{m^2} + 3 \ln \frac{-q^2}{m^2} + \cdots \right\}
\]

\[
\sim e^2 \left\{ 2 \left( \ln \frac{-q^2}{m^2} - 1 \right) \ln \frac{4 \omega_{\text{max}}^2}{\lambda^2} - \ln^2 \frac{-q^2}{m^2} + 2 \ln \frac{-q^2}{m^2} + \cdots \right\}
\]

$\lambda = \text{Photon Mass}$ , $\omega_{\text{max}} = \text{Max Energy of Photon}$

When $k \to 0$, Propagator $\longrightarrow$ becomes "On-Shell"
Infrared Sing. Cancel in the Sum
\[ \downarrow \]
Zero Mom. Emission = No Emission
\[ \downarrow \]
Should Sum Degenerate States in Energy

2. KLN (Kinoshita-Lee-Nauenberg) Theorem

When \( m \to 0 \), Singularity in \( m^2 \) Remains !!
\[ \downarrow \]
Mass Singularity (in Narrow Sense)
\[ \uparrow \]
Massless Particle CAN DECAY into TWO On-Shell Particles

\[ p \to k + l \quad \text{with} \quad k \neq l \neq 0 \quad \vec{k}/\vec{l} \]

Infrared Sing. + Mass Singularity (in Narrow Sense)
\[ \parallel \]
Mass Sing.

QED vs QCD

- \( \alpha_{\text{QED}} \sim 1/137 \) vs \( \alpha_{\text{QCD}} \sim 1/10 \)
- Gluons Interact themselves

KLN Theorem

Mass Singularity Cancel when All Degenerate States in Energy are Summed over Initial and Final States
\( e^+e^- \) Process

1. R - ratio

\[
R(s) = \frac{e^+e^- \rightarrow \text{hadrons}}{e^+e^- \rightarrow \mu^+\mu^-}
\]

- No QCD in the Initial State
- Total Cross Section
  i.e. Sum up All States in the Final States
- No Mass Singularity

Tree Level

\[
\sigma_0 = \frac{4\pi\alpha^2}{3s} N_c \sum_i Q_i^2
\]

One-Loop Level

\[
\sigma = \sigma_0 \left( 1 + \frac{\alpha_s}{\pi} \right)
\]

i.e.

\[
R = N_c \sum_i Q_i^2 \left( 1 + \frac{\alpha_s}{\pi} \right)
\]
2. Scheme Dependence and How to Determine $\Lambda$

$$R = N_c \sum_i Q_i^2 \left[ 1 + \frac{\alpha_s(Q^2)}{\pi} + B \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 + \cdots \right]$$

$B$ has been calculated in MS scheme

$$B = \frac{C_F}{4} \left[ \frac{123}{8} N_c \frac{3}{8} C_F - \frac{11}{4} f + (11N_c - 2f) \left\{ -\zeta(3) + \frac{1}{4} (\ln 4\pi - \gamma_E) \right\} \right]$$

where

$$\zeta(3) = 1.202..., \quad \gamma_E = 0.5772...$$

Remove $\ln 4\pi - \gamma_E$ in MS scheme

When $N_c = 3$

$$B = \begin{cases} 7.36 - 0.44f & \text{(MS)} \\ 1.99 - 0.12f & \text{(MS)} \end{cases}$$

Determination of $\Lambda$

Suppose One Determines $\Lambda$ from Exp. at LO

$$R = N_c \sum_i Q_i^2 \left( 1 + \frac{\alpha_s(Q^2/\Lambda_{\text{Exp.}}^2)}{\pi} \right)$$

Scheme Dependence Tells

$$\Lambda_B = \Lambda_{\text{Exp.}} \exp \left( -\frac{B}{8\pi^2 \beta_0} \right),$$

e.g.

$$\Lambda_{\text{MS}} = 0.260 \Lambda_{\text{Exp.}}, \quad \Lambda_{\text{MS}} = 0.696 \Lambda_{\text{Exp.}}$$

↓

Need NLO Results to Determine $\Lambda$!!
Jet

1. Sterman-Weinberg Jet

Jet: Correlated Hadron's Beam in some Direction

Jet Definition by Sterman-Weinberg

\[ \Downarrow \]

Direct Application of KLN Theorem

Definition of 2-Jet Production:

\[ \sigma(\sqrt{s}, \Omega, \kappa, \delta) \]

Cross Section for Energy \( \sqrt{s} \geq E \geq (1 - \kappa)\sqrt{s} \)

Emitted into the Cone with Angle \( \delta \)

[Diagram of e^+ e^- collision showing jets]

Tree Level

\[ \left( \frac{d\sigma}{d\Omega} \right)_0 = \frac{\alpha^2}{4s} N_c \sum_i Q_i^2 (1 + \cos^2 \theta) \]

One-Loop Level

\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_0 \left[ 1 - \frac{\alpha_s}{\pi} C_F \left( 4 \ln \delta \ln(2\kappa) + 3 \ln \delta + \frac{\pi^2}{3} - \frac{5}{2} \right) \right] \]
2. Thrust and Spherocity

Other Physical Quantities which are Free from M.S.

\[ T = 2 \max \left( \frac{\Sigma_i p_i^i}{\Sigma_i |p_i^i|} \right) \]

\[ S = \left( \frac{4}{\pi} \right)^2 \min \left( \frac{\Sigma_i |p_T^i|}{\Sigma_i |p_i^i|} \right)^2 \]

where

\( p^i \) is momentum of \( i \)

Meaning of Sum:

Fix some Axis and take Sum over Produced Hadrons

\[ \downarrow \]

Move Axis such that \( T(S) \) becomes Max (Min)

\[ \downarrow \]

This Axis is Called “Jet Axis”

\( S, T \) take

\[ 1 \geq T \geq \frac{1}{2}, \quad 0 \leq S \leq 1, \]

where

1.h.Limit = 2 - Jet Event, \quad r.h.Limit = Spherical Event

- \( T, S \) DOES NOT Discriminate Degenerate States
- Mass Singularity Free
4. DEEP INELASTIC SCATTERING

• Factorization

1. Strategy and Scenario

\[ F(x, Q^2) = \int_x^1 \frac{dz}{z} F^0 \left( \frac{x}{z}; (Q^2, p^2) \right) q^0(z; (p^2)) \]

\[ \rightarrow \int_x^1 \frac{dz}{z} C \left( \frac{x}{z}; \alpha_s(\mu^2), \frac{Q^2}{\mu^2} \right) q(z, \mu^2) \]

with \( C \) and \( q \) being Finite and \( \mu^2 \gg \Lambda_{QCD}^2 \)

What we Should (Want to) Show

\[ F^0 = C \otimes \Gamma, \quad q = \Gamma \otimes q^0 \]

\[ \otimes = \text{Complete Sum} \]

Mass Singularity is Factorized into \( \Gamma \)

2. Preliminaries

• \( F^0 \) at 1-Loop Example

\[ \text{Mass Sing. occurs when} \]

\[ p^2 \rightarrow 0 \]

\[ \downarrow \]

\[ \theta \rightarrow 0 \Rightarrow k^2 \rightarrow 0 \]
• Technical Points
  Light-Cone Parametrization
  \[ p^\mu \equiv p(1, 0_\perp, 1), \ n^\mu \equiv (1/2p)(1, 0_\perp, -1) \]
  Sudakov Parametrization for \( k^* \)
  \[ k'^\mu = z p^\mu + (k_1^2 + k_\perp^2)/2x, \ n^\mu + k_\perp^\mu \equiv k_\parallel^\mu + \cdots \]
  Regularization of M.S. (Dimensional)
  \[ D = 4 + 2\epsilon, \ \ g^2 \to g^2 \mu^{-2\epsilon} \]
  \[ \ln(Q^2/p^2) \to 1/\epsilon + \ln(Q^2/\mu^2) \text{ when } p^2 \to 0 \]

  Integration Measure
  \[ d^D k = \frac{dz}{2z}dk^2d^{D-2}k_\perp = \frac{dz}{4z}dk^2d\Omega_{2+2\epsilon}dk_\perp^2(k_\perp^2)^\epsilon \]

• Axial Gauge and Important Facts
  Axial gauge
  \[ D_{\mu\nu}(l) = i/l^2 \ [ -g_{\mu\nu} + (l_\mu n_\nu + l_\nu n_\mu)/n \cdot l ] \]

  Facts without Proof
  2-P-I Amplitudes are Finite
  when External Mom. are Kept
  \[ 2PI = \text{Un-Separated when t-channel 2 Lines are Cut} \]

\[ \begin{array}{c}
  k^2 \\
  \hline
  \hline
  = \\
  + \\
  + \\
  + \\
\end{array} \]
3. Factorization

- Generalized Ladder in Axial Gauge

\[ F^0 = [C^0 (1 + K^0 + K^0 K^0 + \cdots)]_{\alpha\alpha'} u^\alpha(p) \bar{u}^{\alpha'}(p) Z_F \]

\[ = \left[ C^0 \frac{1}{1 - K^0} \right] Z_F \]

- Suppress \( \otimes \) (= \( d^D k \), Sum over Spinor indices, Color)
- \( K^0 (C^0) \) Contains Upper lines (Propagators)
- \( A_{\alpha\alpha'}^{**} \hat{p}^{\alpha\alpha'} \equiv A \hat{p} \), \( \hat{p}^{\alpha\alpha'} A_{\alpha\alpha'}^{**} \equiv [ \hat{p} A \]

• Projection of Mass Singularity

We MUST Decouple both in Spin Indices and \( k \) Integral to Factorize M.S. (Pole in \( \epsilon \))

\[ P = P^s \otimes P^\epsilon \]

Consider e.g. the Term

\[ K^0 \hat{p} \] \[ = \frac{1}{(k^2)^2} (\text{dim. 1 func.}) \]

\[ = \frac{1}{(k^2)^2} \left( A k_\parallel + B k_\perp + C \hat{p} k^2 + \cdots \right) \]

with \( A, \cdots \) being Dimensionless \( A = A(k^2/\mu^2, x, \epsilon), \cdots \)

Note: \([m] = M^{-1}\) and \( k_\parallel = x p \)
Integration over $k$:

$$
\int d^D k \ K^0 \phi = \int \frac{dz}{4z} d\Omega_{2+2\epsilon} \times \int_0^{\mathcal{O}(Q^2)} dk^2 \int_0^{-(1-z)k^2} dk_+^2 \left( \frac{k_+^2}{\mu^2} \right)^\epsilon \frac{1}{(k^2)^2} (A \ k_+ + \cdots)
$$

- Upper Limit of $k^2$ Can be Any Scale
- Since M.S. Comes from $k^2 \sim 0$
- Upper Limit of $k_+^2 \leftarrow (p - k)^2 > 0$

Only $A$ Term and Region $k^2 \sim 0$ in Integral Produces Mass Singularity.

\[\Downarrow\]

$$
\mathcal{P} K^0 \phi = \mathcal{P}^s \otimes \mathcal{P}^e K^0 \phi
$$

\[= \int \frac{dz}{2z} [k^2 = 0] \int \frac{dk^2 d^{2+2\epsilon}k_+}{(2\pi)^D} \left[ \frac{\rho}{4k \cdot n} K^0 \phi \right]
\]

\[= \int \frac{dz}{z} [k^2 = 0] \int \frac{d^P k}{(2\pi)^D} \delta(z - k \cdot n) \left[ \frac{\rho}{4k \cdot n} K^0 \phi \right]
\]

Extension to Two Kernels $A, B$ Connected by $k$:

$$
A \mathcal{P} B = A \mathcal{P}^s \otimes \mathcal{P}^e B
\]

\[= A \ [k^2 = 0] \mathcal{P}^e \left[ \frac{\rho}{4k \cdot n} B \right]
\]

- $k^2 = 0 \rightarrow k = zp$
- $\mathcal{P}^e \left[ \frac{\rho}{4k \cdot n} B \right]$ Extracts Poles in $e$ i.e. M.S.
- Def. of $\mathcal{P}^e$ is NOT Unique

Poles + *** $\rightarrow$ Factorization Scheme Dep.
• Factorization

\[ M \equiv C^0(1 + K^0 + K^0 K^0 + \cdots) \]

\[ \bigodot \text{Proof is Iterative} \]

First Factorize Last Kernel

\[
M = C^0 \left[ 1 + \sum_{i=1}^{\infty} (K^0)^{i-1} \mathcal{P} K^0 + \sum_{i=1}^{\infty} (K^0)^{i-1} (1 - \mathcal{P}) K^0 \right]
\]

\[ = C^0 \left[ 1 + \sum_{i=0}^{\infty} (K^0)^i (1 - \mathcal{P}) K^0 \right] + M \mathcal{P} K^0
\]

So

\[ M(1 - \mathcal{P} K^0) = C^0 \left[ 1 + \sum_{i=0}^{\infty} (K^0)^i (1 - \mathcal{P}) K^0 \right]
\]

Next Factorize \( K^0(1 - \mathcal{P}) K^0 \) in r.h.s.

\[
M(1 - \mathcal{P} K^0 - \mathcal{P}(K^0(1 - \mathcal{P}) K^0))
\]

\[ = C^0 \left[ 1 + (1 - \mathcal{P}) K^0 \sum_{i=0}^{\infty} (K^0)^i (1 - \mathcal{P})(K^0(1 - \mathcal{P}) K^0) \right]
\]

Finally

\[ M(1 - \mathcal{P} K) = C^0 \frac{1}{1 - (1 - \mathcal{P}) K^0}
\]

where

\[ K \equiv K^0 \frac{1}{1 - (1 - \mathcal{P}) K^0}
\]

or

\[ M = \left( C^0 \frac{1}{1 - (1 - \mathcal{P}) K^0} \right) \left( \frac{1}{1 - \mathcal{P} K} \right)
\]
Final Result

\[ F^0(x, Q^2, (\alpha_s)_{\text{bare}}, 1/\epsilon) = [M, \mathcal{P}] Z_F \]

\[ = \left[ \left( C^0 \frac{1}{1 - (1 - \mathcal{P}) K^0} \right) \mathcal{P} \right. \]

\[ + \left( C^0 \frac{1}{1 - (1 - \mathcal{P}) K^0} \right) \mathcal{P} K \frac{1}{1 - \mathcal{P} K} \mathcal{P} \right] Z_F \]

\[ = \int \frac{dz}{z} \left[ \left( C^0 \frac{1}{1 - (1 - \mathcal{P}) K^0} \right) K \right] \kappa_{k^2=0} \]

\[ \times Z_F \left\{ \delta(1 - z) \right. \]

\[ + \int \frac{d^D k}{(2\pi)^D} \delta(z - k \cdot n) \left[ \frac{\mathcal{P}}{4 k \cdot n} K \frac{1}{1 - \mathcal{P} K} \mathcal{P} \right] \}

\[ = \int \frac{dz}{z} C \left( \frac{z}{z}; \alpha_s(\mu^2), \frac{Q^2}{\mu^2} \right) \Gamma \left( z, \alpha_s, \frac{1}{\epsilon} \right) \]

4. Renormalization Group Equation

- \( Q^2 (\mu^2) \) Independence of \( \Gamma \)

Possible \( Q^2 \) Dependence

\[ \uparrow \]

\[ \int_{0}^{\mathcal{O}(-Q^2)} dk^2 \]

Note: \( \mathcal{P} \) acts on \( K \)

\[ K = K^0 \left[ 1 + (1 - \mathcal{P}) K^0 + (1 - \mathcal{P})(K^0(1 - \mathcal{P}) K^0 + \cdots) \right] \]

\[ \downarrow \]

Only Last Integral is Divergent
So with Finite Function $\Phi$

\[
\Gamma(z, \alpha_s, \frac{1}{\epsilon})
= \int_0^\infty \frac{d \Phi(k^2)}{k^2} \Phi(\frac{k^2}{\mu^2}, z, \epsilon = 0)
= Z_F \left[ \delta(1 - z) + \frac{1}{\epsilon} \Gamma_1(z, \alpha_s) + \frac{1}{\epsilon^2} \Gamma_2(z, \alpha_s) + \cdots \right]
\]

By Differentiating Both Sides,

\[
\frac{\partial}{\partial Q^2} \Gamma_i(z, \alpha_s) = 0
\]

Namely

\[
\frac{\partial}{\partial Q^2} \Gamma \left( z, \alpha_s, \frac{1}{\epsilon} \right) = 0 = \left[ \frac{\partial}{\partial \mu^2} \Gamma \left( z, \alpha_s, \frac{1}{\epsilon} \right) \right]
\]

- Renormalization Group Equation

\[
F^0 \text{ (Bare) Physical Quantity}
\]

CAN NOT Depends on $\mu$

\[
\mu \frac{d}{d\mu} F^0 = \left( \mu \frac{\partial}{\partial \mu} + \beta(g, \epsilon) \frac{\partial}{\partial g} \right) F^0 = 0
\]

Take Moment of $F^0$ for Simplicity

\[
F^0 \left( N, Q^2, (\alpha_s)_{\text{bare}}, \frac{1}{\epsilon} \right) = C \left( N, \frac{Q^2}{\mu^2}, \alpha_s \right) \Gamma \left( N, \alpha_s, \frac{1}{\epsilon} \right)
\]

with

\[
f(N) \equiv \int_0^1 dx x^{N-1} f(x)
\]
Renormalization Group Equation

\[ 0 = \mu \frac{d}{d\mu} \ln F^0(N) = \mu \frac{d}{d\mu} \ln C(N) + \mu \frac{d}{d\mu} \ln \Gamma(N) \]

i.e.

\[ \left[ \mu \frac{d}{d\mu} - \gamma(N, \alpha_s) \right] C \left( N, \frac{Q^2}{\mu^2}, \alpha_s \right) = 0 \]

with

\[ C \left( N, \frac{Q^2}{\mu^2}, \alpha_s \right) \]

\[ = C \left( N, 1, \alpha_s(Q^2) \right) \exp \left[ - \int_{\alpha_s(\mu^2)}^{\alpha_s(Q^2)} \frac{d\lambda}{\lambda} \frac{\gamma(N, \lambda)}{2\beta(\lambda, \epsilon)/g} \right] \]

where

\[ \gamma(N, \alpha) = -\mu \frac{d}{d\mu} \ln \Gamma(N) = -\beta(g, \epsilon) \frac{\partial}{\partial g} \ln \Gamma(N, \alpha_s, 1/\epsilon) \]

and

\[ \Gamma \left( N, \alpha_s, \frac{1}{\epsilon} \right) = \exp \left[ - \int_{\alpha_s(\mu^2)}^{\alpha_s(Q^2)} \frac{d\lambda}{\lambda} \frac{\gamma(N, \lambda)}{2\beta(\lambda, \epsilon)/g} \right] \]

Note that \( \mu \) can be Any Value

5. Final Step and DGLAP Equation

- Dressed Parton Distribution Function

For the Hadron Target, we prepare

"Bare" Distribution Function inside the Hadron

\[ q^0 \left( z, \frac{1}{\epsilon} \right) = z \int \frac{d^D p}{(2\pi)^D} \delta \left( z - \frac{p \cdot n}{P \cdot n} \right) \left[ \frac{p}{4 p \cdot n} \frac{H}{P} \right] \]
Define "Dressed" Parton Distribution Function
"Renormalized" at $\mu^2$

$$q(N, \mu^2) = \exp \left[ - \int_0^{\alpha_s(\mu^2)} \frac{d\lambda}{\lambda} \frac{\gamma(N, \lambda)}{2\beta(\lambda, \epsilon)/g} \right] q^0 \left( N, \frac{1}{\epsilon} \right)$$

So

$$F'(N, Q^2) = F^0 \left( N, Q^2, (\alpha_s)_{\text{bare}}, \frac{1}{\epsilon} \right) q^0(N)$$

$$= C \left( N, \frac{Q^2}{\mu^2}, \alpha_s \right) q(N, \mu^2)$$

$$= C \left( N, 1, \alpha_s(Q^2) \right) q(N, Q^2)$$

- **DGLAP Equation**

  $Q^2$ Evolution of Dressed Parton Distribution Function

  $$Q \frac{d}{dQ} q(N, Q^2) = -\gamma(N, \alpha_s(Q^2)) q(N, Q^2)$$

  In $x$ Space

  $$Q \frac{d}{dQ} q(x, Q^2) = \int_x^1 \frac{dz}{z} P \left( \frac{x}{z}, \alpha_s(Q^2) \right) q(z, Q^2)$$

  with

  $$\int_0^1 dx x^{N-1} P \left( x, \alpha_s(Q^2) \right) = -\gamma(N, \alpha_s(Q^2))$$

  This is DGLAP Equation !!
Parton Distribution Function and OPE

"Modern Def.” of Parton Distribution Function
Parton Model and Twist and OPE

1. Some Notation and Light Cone Dominance

Bjorken Limit in Hadronic Tensor

$$W_{\mu\nu} = \frac{1}{2\pi} \int d^4x e^{-i\vec{q}\cdot\vec{x}} \langle PS| [J_{\mu}(0), J_{\nu}(z)] |PS \rangle$$

Only Region $0 \leq z^2 \leq 1/Q^2$ is Important
when $Q^2 \to \infty$, $x = Q^2/2(P \cdot q)$; fixed

Take Frame $\vec{P}/\vec{\eta}/\vec{\epsilon}_z$ and Introduce

$$p^\mu \equiv \frac{\mathcal{P}}{\sqrt{2}} (1, 0, 0, 1), \quad n^\mu \equiv \frac{1}{\sqrt{2\mathcal{P}}} (1, 0, 0, -1)$$

with $p^2 = n^2 = 0$, $p \cdot n = 1$

$$P^\mu = p^\mu + \frac{M^2}{2} n^\mu \sim p^\mu$$

$$\lim_{\text{Bj}} q^\mu \sim \left( P \cdot q + \frac{1}{2} M^2 x \right) n^\mu - x p^\mu$$

Writing $z^\mu = \eta p^\mu + \lambda n^\mu + z_\perp^\mu$

$$\lim_{\text{Bj}} q \cdot z \simeq \eta P \cdot q - \lambda x$$

Riemann-Lebesgue Theorem says

$$|q \cdot z| \lesssim 1 \to |\eta| \lesssim 1/(P \cdot q), \quad |\lambda| \lesssim 1/x$$

Namely with Causality

$$0 \leq z^2 = 2\eta\lambda - z_\perp^2 \leq 2|\eta\lambda| \leq \frac{\text{const}}{Q^2}$$
2. Parton Distribution Function

- "Free Field Theory"

Wick Theorem

\[
[J_\mu(0), J_\nu(z)] = - (\partial^\alpha \Delta(z)) [S_{\mu\alpha\nu}\sigma \mathcal{O}_V^{\sigma}(0, z) - i\epsilon_{\mu\alpha\nu}\sigma \mathcal{O}_A^{\sigma}(0, z)]
\]

with

\[
J_\mu = \bar{\psi} \gamma_\mu \psi, \quad \Delta(z) = -(1/2\pi)\epsilon(z^0)\delta(z^2)
\]

\[
S_{\mu\alpha\nu\sigma} \equiv g_{\mu\alpha}g_{\nu\sigma} - g_{\mu\nu}g_{\alpha\sigma} + g_{\mu\sigma}g_{\nu\alpha}
\]

where

\[
\mathcal{O}_V^{\sigma}(0, z) \equiv \bar{\psi}(0)\gamma^\sigma \psi(z) - \bar{\psi}(z)\gamma^\sigma \psi(0)
\]

\[
\mathcal{O}_A^{\sigma}(0, z) \equiv \bar{\psi}(0)\gamma^\sigma \gamma^5 \psi(z) + \bar{\psi}(z)\gamma^\sigma \gamma^5 \psi(0)
\]

Now

\[
\eta \sim z_+ \sim 0
\]

\[
\langle PS|\mathcal{O}_{V,A}^{\sigma}(0, z)|PS \rangle \simeq \langle PS|\mathcal{O}_{V,A}^{\sigma}(0, \lambda n)|PS \rangle
\]

E.g.

\[
\langle PS|\bar{\psi}(0)\gamma^\sigma \psi(\lambda n)|PS \rangle = 2p^\sigma \dot{q}(\lambda) + 2n^\sigma M^2 \dot{f}_4(\lambda)
\]

\[
\simeq 2p^\sigma \dot{q}(\lambda)
\]

Insert \( W_{\mu\nu} \) with \( \Delta(z) \) and

\[
\dot{q}(\lambda) = \int d\xi e^{-i\lambda\xi} q(\xi) = \int d\xi e^{-i\xi p \cdot z} q(\xi)
\]

One gets

\[
F_1(x) = (1/x)F_2(x) = q(x) - q(-x)
\]
with
\[ q(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0) \phi \psi(\lambda n)|PS\rangle \]
\[ -q(-x) \mapsto \text{Anti-Quark} \]

• "Full Theory"

QCD Interaction
\[ \Delta(z) \rightarrow \Delta(z)[0, z] \]
\[ q(x) \rightarrow \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)[0, \lambda n] \phi \psi(\lambda n)|PS\rangle \]

where
\[ [y, z] = P \exp \left( ig \int_0^1 dt (y - z) \mu A^\mu (ty + (1 - t)z) \right) \]

New q(x) is Gauge Invariant !!

\[ \Delta(z) \text{ and } q(z) \text{ Develop Mass Singularity} \]
\[ \Delta(z)[0, z] \leftrightarrow F^0(z, Q^2, 1/\epsilon) \]
\[ q(y) \leftrightarrow q^0(y, 1/\epsilon) \]

We Need Factorization (Renormalization)
\[ \Delta(z) \rightarrow C(y, Q^2/\mu^2, \alpha_s) \]
\[ q(y) \rightarrow q(y, \mu^2) = \Gamma(y/\xi, \alpha_s, 1/\epsilon) \]
\[ \otimes q^0(\xi, 1/\epsilon) \]

Similar for \( g_1(x) \) and \( g_2(x) \)
3. Operator Product Expansion

Light Cone Dominance

↓

Systematic Expansion in $x^2$ of Current Product

↓

Operator Product Expansion (OPE)

Current (Operator) Product can be Expanded as

$$J(x)J(0) \sim \sum_{i,N} C_{i,N}(x^2) x^{\mu_1} x^{\mu_2} \cdots x^{\mu_N} O^{i,N}_{\mu_1 \mu_2 \cdots \mu_N}(0)$$

$C_{i,N}$: c-Number Function (Wilson's Coefficient Function)

Short-Distance Expansion:

What Happens when $x^\mu \to 0$?

$$C_i(x) \sim x^{dO-d_A-d_B} \quad (d: \text{scale dim.})$$

Operators with Small Dimension Dominate

Light-cone Expansion:

What Happens when $x^2 \to 0$ but $x^\mu \neq 0$?

$$C_{i,N}(x^2) \sim (x^2)^{d_O^N-N-d_A-d_B}/2$$

Operators with Low Twist Dominate

$$\tau_N \equiv d_O^N - N$$

Caveat (short distance vs. light cone):

OPE is valid when $x^\mu \sim 0 \iff |q^2| \geq P \cdot q$

Deep inelastic region $x^2 \sim 0 \iff |q^2| \leq P \cdot q$
Moment Sum Rule: (Lorentz Structure Neglected)
Consider "Forward Virtual Compton Scatt"

\[ T = i \int dx \, e^{iQ \cdot x} \langle P|TJ(x)J(0)|P \rangle \]

Valid in \( P \cdot q/|q^2| \leq 1 \)

Relation to structure functions;
- Optical theorem \( W = 1/\pi \text{ Im}T \)
- Crossing symmetry \( W(q, P) = -W(-q, P) \)

Calculation Goes as Follows
Take Fourier Transform

\[ i \int d^4x \, e^{iQ \cdot x} T J(x) J(0) \sim \sum_{i,N} C_{i,N}(Q^2) \left( \frac{2}{Q^2} \right)^N q_{\mu_1} \cdots q_{\mu_N} O_{i,N}^{\mu_1 \cdots \mu_N} \]

Matrix element of \( O \)

\[ \langle P|O_{i,N}^{\mu_1 \cdots \mu_N}(0)|P \rangle \equiv 2 A_i^N P^{\mu_1} \cdots P^{\mu_N} \]

so

\[ T(P \cdot q, q^2) = 2 \sum_{i,N} \omega^N A_i^N C_{i,N}(Q^2) \], \( \omega = \frac{2P \cdot q}{Q^2} = \frac{1}{x} \)

Structure Functions in Physical Region \( \omega \geq 1 \) \((x \leq 1)\)

\[ \frac{1}{2\pi i} \int d\omega \omega^{-N-1} T = \]

\[ = \sum_i A_i^N C_{i,N}(Q^2) = \int_0^1 dx x^{N-1} W(x, Q^2) \]
4. Relation to OPE

OPE

Lowest Twist Operator

\[ O^{\mu_1 \cdots \mu_N} = i^{N-1} \overline{\psi} \gamma^1 \mu_1 D^{\mu_2} \cdots D^{\mu_N} \psi \]

Moment Sum Rule

\[ \int_0^1 dx x^{N-1} F(x, Q^2) = C \left( N, \frac{Q^2}{\mu^2}, \alpha_s \right) A^N(\mu^2) \]

where

\[ \langle P | O^{\mu_1 \cdots \mu_N} | P \rangle = 2A^N(\mu^2) P^{\mu_1} \cdots P^{\mu_N} \]

Namely

\[ A^N(\mu^2) = \frac{1}{2} n_{\mu_1} \cdots n_{\mu_N} \langle P | O^{\mu_1 \cdots \mu_N} | P \rangle \]

Note \( P \cdot n = 1 \)

On the Other Hand

Moment of (Renormalized) \( q(x, \mu^2) \)

\[ q(N, \mu^2) = \int dx x^{N-1} q(x, \mu^2) \]

\[ = \frac{1}{4\pi} \int d\lambda \left\{ \left( -i \frac{\partial}{\partial \lambda} \right)^{N-1} 2\pi \delta(\lambda) \right\} \times \langle P | \overline{\psi}(0)[0, \lambda n] \not{v} \psi(\lambda n) | P \rangle \]

\[ = \frac{1}{2} \langle P | \overline{\psi}(0) \not{v}(i n \cdot D)^{N-1} \psi(0) | P \rangle = A^N(\mu^2) \]

Equivalent to OPE !!
Spin in Hard QCD Processes: An Introduction
Lectures Presented at the 9th RIKEN Winter School,
Shimoda, December 8-12, 1998

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Abstract

The foundations and several applications of inclusive QCD spin physics are presented in four lectures. First, I discuss the properties of relativistic spin-$\frac{1}{2}$ and spin-1 particles and the tensor density describing angular momentum in QCD. Next I introduce the light cone description of parton distribution and fragmentation functions for quarks and gluons in a helicity basis. I discuss the classification of spin dependent effects with respect to helicity, chirality, transversity and twist. Two applications are discussed in some detail: the properties of the transverse spin structure function $g_2(x,Q^2)$, and the parton description of spin and orbital angular momentum.
SPIN IN HARD QCD PROCESSES
AN INTRODUCTION

R L JAFFE
SHIMODA
DECEMBER 1998

* THANK YOU TO RIKEN,
   PROF YAZAKI, SHIBATA
   DR SAITO

* I HOPE YOU ALL HAVE OPPORTUNITY TO VISIT
   RIKEN-BNL RESEARCH CENTER AT BROOKHAVEN

* THANKS TO PROF. KODAIRA FOR HIS LECTURES
   FORGIVE REPEITION, EXCURSIONS BEYOND.
   WANT TO GIVE INTRODUCTION BUT ALSO INSPIRE
   WITH POWERFUL RESULTS

* CHANGES TO HELP WAKEFULNESS
BJORKEN SUM RULE

\[ \int_{0}^{1} dx q_{1}^{p-n}(x, Q^2) = \frac{1}{6} \frac{q_{A}}{q_{V}} \left\{ 1 - \frac{\alpha_{s}}{\pi} - \frac{43}{12} \left( \frac{\alpha_{s}}{\pi} \right)^2 + \frac{20}{27} \left( \frac{\alpha_{s}}{\pi} \right)^3 \right\} \]

\[ + \frac{M^2}{Q^2} \int_{0}^{1} dx x^2 \left\{ \frac{2}{3} q_{2}^{p-n}(x, Q^2) + \frac{1}{6} q_{2}^{p-n}(x, Q^2) \right\} \]

\[ - \frac{1}{27} \frac{1}{Q^2} \mathcal{F}^a_{u-d}(Q^2) \]

\[ q_2^{\mu}(Q^2) S^a = \langle PS | g \bar{u} \gamma^\mu \gamma^5 \chi_u | UPS \rangle \]

- PQCD
- TARGET MASS
- HIGHER TWIST

LOWEST NON-VANISHING MOMENT OF \( q_2(x, Q^2) \)

\[ \int_{0}^{1} dx x^2 q_2(x, Q^2) = - \frac{2}{3} \int_{0}^{1} dx x^2 q_1(x, Q^2) + \frac{1}{6} d_2(Q^2) \]

\[ d_2(Q^2)[2S_{\sigma \rho \pi \rho} - S_{\omega \rho \pi \rho} - S_{\phi \rho \pi \rho}] \]

\[ = \frac{2}{3} \langle PS | g \bar{q} \left( F_{\sigma \mu} \gamma_\nu + i F_{\sigma \nu} \gamma_\mu \right) q | UPS \rangle_{Q^2} \]

A SIAC, HERMES

CEBAF (?) PROJECT
SPIN IN HARD QCD PROCESSES

I. ANGULAR MOMENTUM ≠ RELATIVITY - INTRODUCTION
   SPIN-0
   SPIN-1/2
   SPIN-1

II. KINEMATICS OF SPIN IN DEEP INELASTIC PROCESSES
    DEEP INELASTIC SCATTERING
    DISTRIBUTION & FRAGMENTATION FUNCTIONS
    CHIRALITY, TWIST, TRANSVERSITY...

III. APPLICATIONS/EXAMPLES
    SUM RULES (BRIEF)
    TRANSVERSITY
    g₂
    PARTON DISTR. OF ORBITAL ANGULAR MOMENTUM

THANKS TO
   ANEESH MANOHAR
   XIANGDONG JI
   GARY GOLDSTEIN
   SERGEI BASHINSKY
I. ANGULAR MOMENTUM IN RELATIVISTIC Q.M.

A. DIRAC PARTICLES - SPIN 1/2

- Conservation Laws
- Structure of Dirac Wave Functions
- Free Dirac Spinors
  Spin, helicity, chirality...
- Noether's Theorem
  Angular Momentum Tensor Density

B. GAUGE FIELDS

- Recap a la Spin 1/2
- Free Field Polarization States
- Angular Momentum Tensor Density
  Gauge Dependence

C. QCD

- Angular Momentum Tensor Density (last time!)
- Gauge & Interaction Issues
A. SPIN-1/2

- **DIRAC EQUATION**  \((i\not\partial - m) \Psi = 0 \Rightarrow (\not\partial - m) u = 0\)

- \(\Psi \neq u\) are four component, Dirac Spinors, but Dirac equation describes only 2 degrees of freedom for each \(E \neq \not p\)
  \[\Lambda_{\pm} = \frac{1}{2m} (m \pm \not p)\]

- \(\Lambda_{\pm}(p)\) are projection operators

They project out a 2-dim. subspace. So for each \(E \neq \not p\), there are only 2 modes
  (spin) for relativistic Dirac particle

- **Spin-1/2 ≠ Polarization**

  Any state of spin-1/2 particle is described by 2-component spinor in its rest frame:

  \[u_j \quad j=1,2 \quad u^+ u = 1\]

  Spin density matrix \(M_{ij} = u_i u_j^*\)

  1. \(M^+ = M\)
  2. \(tr M = 1\)
  3. \(M^2 = M\)
Mixed states: spinor formalism difficult

M formalism excellent

Most general polarization state for spin $-\frac{1}{2}$

$M$ with $M^\dagger = M \quad \text{and} \quad N = 1$

Parameterize by $M = \frac{1}{2} \left( 1 + P \vec{\sigma} \cdot \hat{S} \right)$

$P = \text{polarization}$

$\hat{S} = \text{direction of spin}$

All you need to know @ spin half.

Pure state? $M^2 = M \implies P = 1 \quad M = \frac{1}{2} \left( 1 + \vec{\sigma} \cdot \hat{S} \right)$

Suggests ANY PURE SPIN STATE FOR SPIN HALF IS

A (POSITIVE EIGENVALUE) EIGENSTATE

OF $\vec{\sigma} \cdot \hat{S}$ FOR SOME $\hat{S}$

Proof: Let $\hat{S} \equiv U^\dagger \vec{\sigma} U$

Then $\hat{S} \cdot \vec{\sigma} \cdot U = U$ left as exercise

So any dirac particle is an eigenstate of spin in some direction. This is not true for spin $> \frac{1}{2}$ and simple misconceptions arise.
• **Symmetries of Dirac Eq.**

\[ H = \vec{\alpha} \cdot \vec{p} + \beta m + V \]

(Strong or general background)

\[ V = U \gamma^5 \beta, \text{ etc.} \]

\[ \vec{\sigma} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \]

**TEMPORARY DIRAC MATRICES**

\[ \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

\[ \vec{\sigma} = \begin{pmatrix} \sigma_x & i \sigma_y \\ i \sigma_y & \sigma_x \end{pmatrix} \]

\[ \hat{J}^2 \Rightarrow j (j+1) \]

\[ \hat{J}_z \Rightarrow m_j \text{ are good c.n.} \]

\( \hat{L} \neq \hat{\hat{S}} \) are not conserved. They mix in Dirac energy eigenstates

• **TT Parity**

\[ \text{TT} \equiv \hat{P} \beta \text{ WHERE } \hat{P} |\chi\rangle = 1 \chi \langle \chi| \text{ IS NANE PArity} \]

\[ [\text{TT}, H] = 0 \text{ for parity invariant interactions} \]
• Symmetries implemented on states

Standard $\chi$-matrix basis $\chi^0 = \beta = (1^0, 0^-)$

$$\Psi = (\phi \chi) \quad \Pi \Psi(\vec{r}) = \left( \begin{array}{c} +\phi(-\vec{r}) \\ -\chi(-\vec{r}) \end{array} \right)$$

$\gamma_0 \text{ did it}$

$\gamma_{lm}(-\vec{r}) = (-1)^l \gamma_{lm}(\vec{r})$

So $\phi$ and $\chi$ differ in $l$ by 1 unit $l_\phi = l_\chi \pm 1$

Example (important)

• Typically lowest energy state
• Reduces to $1S_{1/2}$ in NR limit
• Upper components $- l = 0$
  Lower components $- l = 1$

$$\Phi = \left( \begin{array}{c} f(r) \\ \text{s-wave} \\ \text{i} \vec{\sigma} \cdot \vec{\gamma} g(r) \uparrow \text{p-wave} \end{array} \right) u_m$$

$$\langle S_3 \rangle = \frac{1}{2} \int_0^\infty r^2 dr \left( f^2 - \frac{1}{3} g^2 \right)$$

$$\langle L_3 \rangle = \frac{2}{3} \int_0^\infty r^2 dr \ g^2$$

NB $\langle S_3 \rangle + \langle L_3 \rangle = \frac{1}{2} \left( \int_0^\infty r^2 dr \ (f^2 + g^2) = 1 \right)$

Moral $\Rightarrow$ Orbital and spin angular momentum cannot be separated in a relativistic theory.

$\Rightarrow$ Even the lowest energy Dirac states carry $\langle L_3 \rangle$ in a relativistic $Q\bar{q}$. 57
• Free Dirac Spinors

\[ u(p, \xi) \quad \overline{u}(p, \xi) \]

(set \( u(p, \xi) \) aside)

\[
\begin{align*}
(p - m) u(p, \xi) &= 0 \\
(m + \xi \gamma_5) u(p, \xi) &= 0
\end{align*}
\]

\[ \text{CONVENTIONAL} \]

\[ \begin{align*} 
\text{in rest frame} & \quad \hat{s} \cdot \hat{s} u(p, \xi) = u(p, \xi) \\
& \quad s^\mu = m (0, \hat{s})
\end{align*} \]

or

\[
\begin{align*}
\overline{u}(p, \xi) \gamma^\mu u(p, \xi) &= 2 p^\mu \\
\overline{u}(p, \xi) \gamma^\mu \gamma_5 u(p, \xi) &= 2 s^\mu
\end{align*}
\]

\[
p^\mu p_\mu = m^2 \quad s^\mu s_\mu = -m^2 \quad p^\mu s_\mu = 0
\]

\( p^\mu \) is a 4-vector \quad \( s^\mu \) is an axial 4-vector

Note: Consider an amplitude with a fermion scattering forward -

Bilinear in \( \overline{u}(p, \xi) \quad u(p, \xi) \quad \overline{u}(p, \xi) \quad \overline{u}(p, \xi) \)

\[ \Rightarrow \text{AT MOST LINEAR IN } s^\mu \]

• Bases

\[ \lambda = \left( \begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix} \right) \]

\[ \lambda = \frac{1}{2} \hat{s} \cdot \hat{p} \]

\[ \lambda = \lambda_1, \lambda_2 \]

\[ [\lambda, \lambda_1] = 0 \quad [\lambda, \lambda_2] = 0 \quad \lambda u(p, \xi) = \pm \frac{1}{2} u(p, \xi) \]
**CHIRALITY**

\[ \gamma_5 \left[ \gamma_5, H \right] = 0 \quad \text{iff} \quad m = 0 \]

Of course we can use a chiral basis whether or not \[ \left[ \gamma_5, H \right] = 0 \].

\[ \gamma_5 \, u_\text{R}(p) = u_\text{R}(p) \]
\[ \gamma_5 \, u_\text{L}(p) = -u_\text{L}(p) \]

**TRANSVERSITY**

\[ \text{Praj \& X. Ji, NP 378 (1992)} \]
\[ p = 327 \]

**Transverse Spin does not commute with \( H \):**

EG. \[ \left[ \sigma_1, H \right] = \left[ \sigma_1, \alpha \cdot \vec{p} \right] = \left[ \sigma_1, \alpha_3 \right] p_3 = -i \alpha_2 p_3 \]

So a Dirac particle with Energy \( E \) & Momentum \( \vec{p} \)

Cannot be labelled by Transverse Spin.

**However** \( \gamma_1 \gamma_5 \) does commute with \( H = \alpha^3 p^3 + \beta m \)

\[ \left[ \gamma_1 \gamma_5, \alpha_3 p_3 \right] = 0 \] so this "Transversity"

Is a good Q.N.

Let \[ \hat{S} = m \hat{e}_\perp \]

\[ \frac{1}{2} \left( m \mp \gamma_5 \gamma \right) \Rightarrow \frac{1}{2} m \left( 1 \mp \gamma_5 \gamma^\perp \right) \]

\[ \gamma_5 \gamma^\perp u_\perp(p) = u_\perp(p) \quad \gamma^\perp = \gamma^1 \text{or} \gamma^2 \]
\[ \gamma_5 \gamma^4 u_\perp(p) = -u_\perp(p) \]

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RELATIONS

- For $\gamma^1 = \gamma^1$
  
  \[ u_\perp (p) = \frac{1}{\sqrt{2}} \left( u(p,+) + u(p,-) \right) \]
  
  \[ u_T (p) = \frac{1}{\sqrt{2}} \left( u(p,+) - u(p,-) \right) \]

  So relation between transversity & helicity is analogous to $\sigma_1$ and $\sigma_3$ in NR QCD.

- Of course chirality & helicity coincide for $m=0$.

Angular Momentum in Q.F. QCD. (Spin $0 \& \frac{1}{2}$)

- Conservation of 4-momentum + Noether's Thm $\Rightarrow T_{\mu\nu}$
  
  \[ \partial^\mu T_{\mu\nu} = 0, \quad T_{\mu\nu} = T_{\nu\mu} \]
  
  \[ P^\nu = \int d^3x \ T^{0\nu}, \quad \frac{\partial}{\partial t} P^\nu = \int d^3x \ (-\partial_i T^{i\nu}) = 0 \]

- Lorentz invariance + Noether's Thm $\Rightarrow \ M_{\mu\nu\lambda}$

  \[ M_{\mu\nu\lambda} = -M_{\mu\lambda\nu} \]
  
  \[ \text{ROTATIONS} \]

  \[ \text{M}_{\mu\nu\kappa} \text{ BOOSTS} \]

"Simple" construction

\[ M_{\mu\nu\lambda} \equiv \chi \cdot T_{\mu\lambda} - \chi \times T_{\mu\nu} \]

(SEE RLJ &)

(ANCHOR)
\[ \delta^\mu M_{\mu \nu \lambda} = \delta^\mu \nu T_{\mu \lambda} - \delta^\mu \lambda T_{\mu \nu} + x \nu \partial^\mu T_{\mu \lambda} - x \lambda \partial^\mu T_{\mu \nu} = 0 \text{ if } T_{\mu \nu} \text{ is symmetric} \neq \text{conserved} \]

\[ \int d^3x \ M^0_{ij} = \epsilon_{ijk} J_k \text{ rotations} \]

\[ \int d^3x \ M^0_{\alpha \epsilon} = B \epsilon \text{ boosts} \]

\text{Note: } M_{\mu \nu \lambda} \text{ obtained from this construction and from Noether's theorem are slightly different.}

\[ M_{\mu \nu \lambda} = M_{\mu \nu \lambda}^{(w)} + \partial^\beta B_{[\mu, \nu, \lambda]} \text{ \quad \text{see RJF} \quad \text{manohar} \quad \text{NP 5337 (1996)}} \]

\^ "superpotential" does not contribute to charges

"What matters is what's observable!"
• Scalar field $\rho$

$$\mathcal{L}^{\mu \nu \lambda} = \partial^{\mu} \phi \left( x^\nu \partial_{\lambda} - x^\lambda \partial_{\nu} \right) \phi - \left( g^{\mu \nu} x^\lambda - g^{\mu \lambda} x^\nu \right) \mathcal{L}$$

Interpretation:

$$J^k = \int d^3x \, \dot{\phi} \left( \hat{x} \times \hat{\nabla} \right)^k \phi$$

Canonical:

$$L^k = \int d^3x \sum_{\lambda} i \Pi^\lambda(x) \left( \hat{x} \times \hat{\nabla} \right)^k \hat{x}^\lambda(x)$$

So $\Pi^k \left( \hat{x} \times \hat{\nabla} \right) \phi^k$ is "convective"

can "orbital" angular momentum.

• Dirac field $\psi$

$$\mathcal{L}^{\mu \nu \lambda}_{(1/2)} = \frac{i}{2} \bar{\psi} \gamma^\mu \left( x^\nu \partial_{\lambda} - x^\lambda \partial_{\nu} \right) \psi + \text{h.c.}$$

$$+ \frac{1}{2} \epsilon^{\mu \nu \lambda} \sigma_{\alpha \beta} \bar{\psi} \gamma^\alpha \gamma_5 \gamma^\beta \psi$$

* Note appearance of spin associated with axial vector operator.

* The fact that spin tensor density reduces to an axial vector is, in fact, a degeneracy associated with spin-1/2 fields and does not hold for higher spins.
B. SPIN 1 (AND GAUGE FIELDS)

• Many aspects $\leftrightarrow$ spin $-\frac{1}{2}$
  * $\vec{l} \neq \vec{3}$ are not separately conserved - only $\vec{j}$
  * Stationary states in external field mix various $\ell$ values to obtain eigenvectors of $\vec{J}^2 \neq$ parity
  * Three propagating degrees of freedom (helicity +1, 0, -1) for each $E \neq \vec{p}$. Gauge invariance $\rightarrow 2$. No helicity 0 for gluons

• Polarization in rest frame (needs specialization for gauge particles)

  $\epsilon_{k} \quad k=1,2,3 \quad \text{POLARIZATION VECTOR}$

  $\star \quad M_1 \quad 3x3 \quad \text{POLARIZATION DENSITY MATRIX}$

  $M_1^\dagger = M_1 \quad \text{tr } M_1 = 1 \quad M_1^2 = M_1$ (for pure state)

  $\Sigma : \quad 3x3 \quad \text{spin matrices} \quad (\Sigma^k)_{ij} = -i \epsilon_{kij}$

  To parameterize $M_1$ requires 8 $3x3$ hermitian matrices (cf. Gell-Mann's $SU(3)$)
\[ \Theta_{ij} = \{\Sigma_i, \Sigma_j\} - \frac{1}{2} \delta_{ij} \quad \text{Tr} \Theta = \Theta_{ij} = 0 \quad 5 \text{ independent } \Theta_{ij} \]

Then \[ M_1 = \frac{1}{3} \mathbb{I} + P \hat{S} \cdot \vec{\Sigma} + Q_{ij} \Theta_{ij} \]

VECTOR POL. TENSOR POL.

So polarization state of spin-1 particle contains both vector (\( \hat{S} \)) and tensor (\( Q \)) information.

* Spin direction \[ \hat{S} = \vec{\varepsilon}^* \vec{\Sigma} \vec{\varepsilon} \quad (\text{cf. } \vec{v}^* \vec{v}) \]

\[ S_k = \varepsilon_i^* (-i \varepsilon_{kij}) \varepsilon_j = -i \vec{\varepsilon}^* \times \vec{\varepsilon} \]

However - an arbitrary \( \vec{\varepsilon} \) is NOT an eigenstate of \( \vec{\Sigma} \)
along some direction \( \hat{S} \). Can prove \( \vec{\varepsilon} \) is eigenstate iff \( \vec{\varepsilon} \cdot \vec{\varepsilon} = 0 \). [Then \( (\vec{\Sigma} \cdot \vec{S}) \vec{\varepsilon} = \vec{\varepsilon} \)]

* Relativistic generalization

\[ \vec{\varepsilon} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \]

\[ \vec{\varepsilon} \to \varepsilon^\mu \] with \( k^\mu \varepsilon_{\mu} = 0 \) (still only 3 dof.)

* SCALAR: \( \varepsilon_{\mu}^* \varepsilon^\mu \)

* VECTOR POLARIZATION: \( S_{\mu} = i \varepsilon_{\mu \nu \alpha \beta} P^\nu \varepsilon_{\alpha \beta}^* \varepsilon^\beta \)

* TENSOR POLARIZATION: \( \gamma_{\mu \nu} = \varepsilon_{\mu}^* \varepsilon^\nu + \varepsilon_{\nu}^* \varepsilon^\mu \]

\[ -\frac{1}{2} g_{\mu \nu} \varepsilon_{\alpha}^* \varepsilon^{\alpha} \]
SCALAR $\varepsilon^* \cdot \varepsilon$

VECTOR $s_{\mu} = i \varepsilon^{\mu\nu\rho\sigma} F_{\nu}^\sigma \varepsilon^* \varepsilon^\rho$

TENSOR $\eta_{\mu\nu} = \varepsilon_{\mu}^* \varepsilon_{\nu} + \varepsilon_{\nu}^* \varepsilon_{\mu} - \frac{1}{2} g_{\mu\nu} \varepsilon^*_\alpha \varepsilon^\alpha$

Spin physics of gluons is richer because they can carry tensor polarization, but poorer (than massive spin-one) because no helicity zero state exists.

- $M_{\mu\nu\lambda}$ for gauge fields

$M_{\mu\nu\lambda}^g = -F_{\mu\nu}^\alpha (x^\nu \partial^\lambda - x^\lambda \partial^\nu) A_\alpha + F_{\mu\nu}^\lambda A_\nu + F_{\nu\mu}^\lambda A_\nu$

$- \frac{1}{4} F^2 (x^\nu g_{\mu\lambda} - x^\lambda g_{\mu\nu})$ (ABELIAN)

* Spin part (no $x^\alpha$, no $\partial^\alpha$) is not related to a vector operator.
  (Note $F_{\mu\nu}^\lambda A_\nu + F_{\nu\mu}^\lambda A_\nu + \overline{F}_{\nu\mu}^\lambda A_\nu = \varepsilon_{\mu\nu\rho\sigma} K_{\rho\sigma}$ but we don't have the third term.)

* $M_{\mu\nu\lambda}^g$ is gauge invariant, but separate spin and orbital pieces are not.
\[ \mathcal{J}^k = \frac{1}{2} \varepsilon^{k \ell m} \int d^3x \, M^{\ell m}(x) \]

\[ \mathcal{J} = \int d^3x \, E^\ell \cdot (x \times \nabla) A^\ell + \int d^3x \, E \times A \]

\[ \mathcal{J} = \int d^3x \, x \times (E \times B) \quad \text{familiar} \]

\[ M^{\mu \nu} = \frac{i}{2} \bar{\Phi} \gamma^\mu (x^\nu \gamma^\lambda - x^\lambda \gamma^\nu) \Phi + \frac{i}{2} \varepsilon^{\mu \nu \lambda \sigma} \bar{\Phi} \gamma^\lambda \gamma^\sigma \Phi \]

\[ -2 \text{Tr} \{ F^{\mu \nu} (x^\lambda \gamma^\sigma - x^\sigma \gamma^\lambda) A_{\sigma} \} + 2 \text{Tr} \{ F^{\mu \lambda} A^\nu + F^{\nu \lambda} A^\mu \} \]

\[ -\frac{1}{2} \text{Tr} F^2 (x^\nu g^{\mu \lambda} - x^\lambda g^{\mu \nu}) \]

* Lack of gauge invariance clouds interpretation of the individual terms. On the other hand, no interaction dependent terms appear (\( M^{\mu \nu} \) is quadratic in canonical variables).

* Ji suggested gauge invariant decomposition by \( \mathcal{J}_\alpha \rightarrow \mathcal{D}_\alpha \) — addition terms in quark part cancel against gluon part.

\[ X. Ji, PRL 78 (1997), 10. \]
\[ M_{\mu \nu}^{\text{QCD}} = \frac{i}{2} \bar{\Psi} \gamma_{\mu} (x^\nu D^\lambda - x^\lambda D^\nu) \Psi + \frac{1}{2} \epsilon_{\mu \nu \lambda \sigma} \bar{\Psi} \gamma_\lambda \gamma_\sigma \Psi \]

- 2 Tr \{ \mathbf{F}^{\mu \nu} (x^\nu D^\lambda - x^\lambda D^\nu) \mathbf{A}_\mu \} + 2 \text{Tr} \{ \mathbf{F}^{\mu \alpha} A_\mu + F^{\nu \beta} A_\nu \}

**Summary**

- **Quark Spin** - separately gauge invariant and directly measurable.
- **Gluon Spin** - not separately gauge invariant but a quantity is measurable that coincides with this in a definite gauge (see last lecture).
- **Quark Orbital** - separately gauge invariant but measurement unknown.
- **Gluon Orbital** - not separately gauge invariant and measurement unknown.

See later lecture for more discussion.

Why is it hard to measure matrix elements of orbital angular momentum?  
R.L.J. MANOHAR

Analog with charge and magnetic moment:

\[ j^\mu = \bar{\Psi} \gamma^\mu \Psi \]

\[ \langle \psi | j^0 | \psi \rangle = 2 P^0 Q \]

But \[ \langle \psi | j^\mu | \psi \rangle = 0 \]
Moment of $j^\mu$ appears in off-forward matrix element

$$\int a^3 x e^{i q x} \langle P \bar{S}^\prime | j^\mu(x) | P S \rangle$$

$$= \bar{u}(P \bar{S}^\prime) \left[ g_\mu F_1(q^2) + \frac{i g_{\mu\nu} q^\nu}{2 M} F_2(q^2) \right] u(P S)$$

Expand about $\vec{q} = 0$. Linear term on RHS

RHS $\propto F_2(0) \vec{q} \times \vec{S} \propto \vec{q} \times \vec{S} \mu$

LHS $\propto \vec{q} \times \int a^3 x \langle P \bar{S}^\prime | \frac{1}{2} \vec{\gamma} \times \vec{j} | P S \rangle$

Lesson: To measure a coordinate space moment of an operator you must measure the operator matrix element at non-zero momentum transfer.

Angular momenta are moments of $T^{\mu\nu}$ ($x^\nu T^{\mu\lambda} - x^\lambda T^{\mu\nu}$). So one must measure $\langle P \bar{S}^\prime | T^{\mu\nu} | P S \rangle$.

Hard QCD processes naturally give forward matrix elements — hard.
But see X. Ji PR DSE (1997) 7114.
II. KINEMATICS OF SPIN IN DEEP INELASTIC PROCESSES

A. DEEP INELASTIC SCATTERING FROM A SPIN-$1/2$ TARGET
   - STRUCTURE FUNCTIONS
   - MEASUREMENT

B. INTRODUCTION TO QUARK & GUYON DISTRIBUTION
   - FRAGMENTATION FUNCTIONS
     - A GRAPHICAL GUIDE
     - FROM GRAPHS TO EQUATIONS
     - GENERALIZATIONS, DIFFICULTIES, F.A.Q.
     - SUMMARY
     - LEARNING FROM THE LIGHT CONE

C. HELICITY BASIS & DENSITY MATRIX NOTATION
• QCD hard processes $\Rightarrow$ forward matrix elements

\[
\frac{q}{P} \rightarrow \frac{q}{P} \quad \Rightarrow \quad \frac{q}{P} \rightarrow \frac{q}{P}
\]

$\langle P | \text{Operators} | P \rangle$ NEVER $\langle P' | \text{Operators} | P \rangle$

• How magnetic moment $\vec{\mu} = \frac{\vec{x} \times \vec{d}}{2}$ can be measured?

\[
\int d^4 x \ e^{i q \cdot x} \langle P S' | \frac{i}{\hbar} \left[ \frac{i}{2} \delta_{k} (x) | P S \rangle = (2\pi)^4 \delta^4 (P+q-P')
\]

\[
\bar{u}(P S') \left[ \delta_{k} F_1 (q^2) + \frac{i}{2M} \sigma_{R u q'} F_2 (q^2) \right] u(P S)
\]

Take $\vec{q} \rightarrow 0 \quad q^0 \approx q^2 / 2M \quad \delta_{k} = \frac{(P+P')}{2M} + \frac{i \sigma_{R u q'}}{2M}

\text{GORDON IDENTITY}

LHS: $\ e^{i q \cdot x} = 1 + i q \cdot x + \ldots$ \quad \text{NOTE: term linear in} \ x \ \text{is also linear in} \ \vec{q}

ALGEBRA (see e.g. JACKSON)

\[
\int d^3 x \ \frac{1}{2} \vec{x} \times \langle P S | \bar{u} (P S') \frac{\vec{q}}{2M} u(P S) | P S \rangle \quad \frac{F_1 (0) + F_2 (0)}{2M}
\]

REQUIRES $\vec{q} \neq 0$ EXPT.
A. DEEP INELASTIC SCATTERING FROM A SPIN-$\frac{1}{2}$ TARGET

\[ P \cdot q = v = M v_{\text{exp}} \]
\[ q^2 = -Q^2 < 0 \]
\[ x = Q^2 / 2v \]
\[ u = v/M \]
\[ c = 1 + 4M^2 x^2 / Q^2 \]
\[ s_1, s_2 = \text{mass} \]

Structure Functions

\[ W_{\mu}(q, \rho, s) = \frac{1}{2\pi} \sum_x (2\pi)^4 \delta(q - P - x) \langle PS | J_{\mu} | x \rangle \langle x | IJ_{\mu} | PS \rangle \]

\[ = \int d^4x \ e^{iq \cdot x} \langle PS | J_{\mu} (x) | IJ_{\mu} (0) | PS \rangle \]

\[ = \frac{1}{2\pi} \text{Im} \ T_{\mu\nu}(q, P, s) \]

- \( q^\mu W_{\mu} = 0 \)
- \( W_{\mu\nu}^* = W_{\mu\nu} \)
- \( \leq \text{linear in } s^\mu \)

\[ W_{\mu\nu} = -g_{\mu\nu} F_1 (x, Q^2) + \frac{P_{\mu} P_{\nu}}{s} F_2 (x, Q^2) \]

\[ + \frac{i\epsilon_{\mu\rho\sigma\nu}}{v} q^\alpha S^\beta g_1 (x, Q^2) \]

\[ + \frac{i\epsilon_{\mu\rho\sigma\nu}}{v^2} (v S^\beta - q \cdot S^\rho) q^\alpha q_2 (x, Q^2) \]
$g_1$ and $g_2$ are the playthings of DIS.

Why are they called longitudinal and transverse?

**Define important light-like basis vectors**

\[ \mathbf{p} \equiv p(1,0,0,1) \quad \mathbf{n} = p^2 = 0 \]

\[ \frac{1}{\sqrt{2}} \mathbf{n} \equiv \frac{1}{p} (1,0,0,-1) \quad \mathbf{n} \cdot \mathbf{p} = 1 \]

**Work in frame where nucleon \& photon**

**Define $Z$-axis (theorists frame changes event by event)**

- **Nucleon at rest**
- **Photon along $Z$-axis**

\[ F^\mu = p^\mu + \frac{M^2}{2} n^\mu \]

\[ q^\mu = (v + \frac{1}{2} M^2 x) n^\mu - x p^\mu \]

\[ S^\mu = S \cdot n p^\mu + S \cdot p n^\mu + S_\perp^\mu \quad S \cdot p \ll S \cdot n \]

**Rest frame** $\mathbf{p} = \frac{\sqrt{s}}{2}$

\[ W_{\mu\nu} = \text{spin independent terms} \quad S \cdot \mathbf{p} = 0 \]

\[ + i \epsilon_{\mu \nu \lambda \sigma} (S \cdot n) n^\lambda p^\sigma g_1(x, \xi^2) \]

\[ + i \epsilon_{\mu \nu \lambda \sigma} n^\lambda S_\perp^\sigma (g_1(x, \xi^2) + g_2(x, \xi^2)) \]

So $q_1 \Leftrightarrow \text{longitudinal}$

\[ q_1 + g_2 \equiv g_T \Leftrightarrow \text{transverse} \]
WHY THE LIGHT CONE?

Bjorken limit: \( P, q \to \infty \quad x = \frac{Q^2}{P \cdot q} \text{ fixed} \)

Rest frame:
\[
P = (M, 0, 0, 0) \\
q = (\frac{Q}{M}, 0, 0, -(\frac{Q^2}{2M^2} - \frac{Q^2}{2}))
\]

\[
\lim_{\text{Bj}} q = (\frac{Q}{M}, 0, 0, -\frac{Q}{M} + Mx) \\
q^\pm = \frac{1}{\sqrt{2}} (q^0 \pm q^3)
\]

\[
q^+ = \frac{Mx}{\sqrt{2}} \quad q^- = \sqrt{2} \frac{\nu}{M} \quad P^+ = P^- = M \sqrt{2}
\]

So Bj-limit is single component of \( q^u \to \infty \) SIMPLE

- \( q^- \to \infty \implies \xi^+ \to 0 \) uncertainty principle
  
  What coordinate is \( \xi^+ \)?
  
  It's a Compton scattering coord!

- \( \xi^2 = 2 \xi^+ \xi^- - \xi_\perp^2 > 0 \) (causality) \([J(\xi), J(\xi)]\)

\[
\xi^- \sim \frac{1}{\nu^2} q^+ \sim \sqrt{2} M x
\]

So \( \xi \to 0 \) IF \( x \to 0 \)

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Measurement

Electron spin (helicity) asymmetry

\[ d\sigma_R = d\bar{\sigma} + d\Delta\sigma \]
\[ d\sigma_L = d\bar{\sigma} - d\Delta\sigma \]
\[ d\bar{\sigma} \Rightarrow F_1 \neq F_2 \text{ (standard)} \]

\[ \frac{d\Delta\sigma}{dxdy\ d\phi} = \frac{e^4}{4\pi^2Q^2} \left\{ \cos\alpha \left[ (1-\frac{y}{2}-\frac{y^2}{4}(k-1))g_1 - \frac{y}{2}(k-1)g_2 \right] - \sin\alpha\cos\phi\sqrt{(k-1)(1-y-\frac{y^2}{4}(k-1)) \left( \frac{y}{2}g_1 + g_2 \right)} \right\} \]

NB \( k-1 \sim \frac{1}{Q^2} \left( = \frac{4\alpha^2x^2}{Q^2} \right)^{1/3} \)

* Complication: electron beam is not parallel to virtual photon.

* At any fixed polarization \( \alpha \neq 90^\circ \), \( g_1 \) dominates and one measures \( \sim (1-y/2)g_1(x,Q^2) \)

* Exactly at \( 90^\circ \) \( g_1 \) term drops out and subdominant \( g_2 \) term is measured

* \( g_2 \) term is multiplied by an extra \( \frac{1}{1-Q^2} \) \( \sim \sqrt{k-1} \) signalling non-leading twist.

* This is the best and most important example where twist-3 dominates because twist-2 is kinematically suppressed.
B. INTRODUCTION TO QUARK & GUYON DISTRIBUTION & FRAGMENTATION FUNCTIONS

A GRAPHICAL GUIDE

Too much to do and too little time to start with first principles and develop everything mathematically. Instead, motivate definitions pictorially.

Deep inelastic processes:
- large four-momentum transfer $Q^2 \gg \Lambda_{QCD}$
- unconstrained development

QCD/FARTON FACTORIZATION

DEEP INELASTIC SCATTERING
$e^+e^- \rightarrow \ell^+\ell^- \rightarrow \ell^+\ell^-X$

$Q^2 \gg \Lambda_{QCD}$

Q$^2$, $P$, $q$ $\rightarrow \infty$

QCD/FARTON FACTORIZATION

ELECTRON-POSITRON ANNIHILATION
$e^+e^- \rightarrow \ell^+\ell^- \rightarrow \ell^+\ell^-X$

$Q^2 \gg \Lambda_{QCD}$

Q$^2$, $P$, $q$ $\rightarrow \infty$

SINGLE PARTICLE INCLUSIVE ANNihilation
$e^+e^- \rightarrow \ell^+\ell^- \rightarrow \ell^+\ell^-X$

$Q^2 \gg \Lambda_{QCD}$

Q$^2$, $P$, $q$ $\rightarrow \infty$
Hore processes (suppress lepton lines →)

**Drell-Yan**

\[ pp \rightarrow \ell\ell X \quad Q^2, P_{T\ell}, P_{Tq} \rightarrow \infty \]

2 JET PRODUCTION

\[ pp \rightarrow \text{jet + jet} + X \]

Experimenters measure cross sections

\[ lp \rightarrow l' X \]

\[ e^+e^- \rightarrow \ell\ell X \]

\[ 2 \rightarrow \]

\[ 2 \rightarrow \]

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Basic building blocks

**DISTRIBUTION FUNCTIONS**

**FRAGMENTATION FUNCTIONS**

Cross sections for deep inelastic processes in QCD are composed of distribution & fragmentation functions $\otimes$ hard parton cross sections $\otimes$ QCD radiative corrections

Example: SINGLE HADRON INCLUSIVE ELECTROPRODUCTION $ep \rightarrow e^{+}H^{X}$

- Building Blocks
- Helicity Density/Matrix Formalism
2. FROM GRAPHS TO EQUATIONS

\[ f(x, p, s, \eta) = \sum_k S(k^3-xP^3) \left| \langle X | \bar{\eta} \psi | P, s \rangle \right|^2 \]

\[ k^3 = xP^3 \]

**SUM ON UNOBSERVED HADRONS**

**QUARK WITH**

**LONGITUDINAL MOMENTUM FRACTION x**

**RJI IN "RELATIVISTIC DYNAMICS & QUARK NUCLEAR PHYSICS" (NILEY, 1986)**

**DIRAC FIELD FOR QUARK**

**ARBITRARY SPINOR TO ABSORB DIRAC INDICES**

**\leftrightarrow** REST OF DIAGRAM

**NOTE:**

- \( P^2 = M^2, \ S^2 = -M^2, \ P \cdot S = 0 \)
- Formulated in INFINITE MOMENTUM FRAME
  \[ P = (\sqrt{P^2+M^2}, 0, 0, P) \Rightarrow P \rightarrow \infty \]
- Only \( k^3 = xP^3 \) of quark is observed
  \( \Sigma \chi \) includes other components of \( \eta^\mu \)
- \( f \) is a probability, \( f \geq 0 \)
  \( \eta \) (a four component Dirac spinor, selects quark spin state)
Toward a frame independent formulation of \( \phi \).

**Aside on light-cone coordinates**

\[ a^\mu = (a^0, a^1, a^2, a^3) \text{ a Lorentz 4-vector} \]

\[ a^+ = \frac{1}{\sqrt{2}} (a^0 + a^3) \quad a^- = \frac{1}{\sqrt{2}} (a^0 - a^3) \]

\[ a_\mu a^\mu = 2 a^+ a^- - a_\perp^2 = (a^0)^2 - a^2 \]

\[ a^+ \perp = (0, a^1, a^2, 0) \]

**Target momentum**

\[ P^\mu \approx (\frac{1}{2} M^2, 2, 0, 0, 0) \]

\[ P^+ = \sqrt{2} P \to \infty \]

\[ P^- \approx \frac{M^2}{2\sqrt{2} P} \approx 0 \]

\( P^\mu \) has only one non-trivial component \( P^+ \)

**Define two "unit" vectors**

\[ \phi^\mu = \frac{1}{\sqrt{2}} (1, 0, 0, 1) \quad \phi^2 = 0 \]

\[ \eta^\mu = \frac{1}{\sqrt{2} P} (1, 0, 0, -1) \quad \eta^2 = 0 \]

\( \phi \cdot \eta = 1 \)
\[ P^\mu = p^\mu + \frac{M^2}{2} n^\mu \]
\[ S^\mu = (S \cdot n) p^\mu + (S \cdot p) n^\mu + S^\mu_1 \]

**Delta function in \( \xi \)**
\[ k^3 = x P^3 \quad (P^3 = p \to \infty) \]
\[ \Rightarrow k \cdot n = x P \cdot n = x \]

\[ \sum X \ S(k \cdot n - x) \langle PS | \bar{\psi} \eta | X \rangle \langle X | \bar{\eta} \psi | PS \rangle \]

\[ \sum_X S(k \cdot n - x) = \sum_X \int \frac{d\lambda}{2\pi} e^{i \lambda x - i \lambda k \cdot n} \]

\[ k \cdot n = (P - P_x) \cdot n \]

\[ e^{i (P_x - P) \cdot n} \langle X | \bar{\eta} \psi | PS \rangle = \langle X | \bar{\eta} \psi(\lambda n) | PS \rangle \]

Perform \( \sum_X \)

\[ f(x, P, S, \eta) = \int \frac{d\lambda}{2\pi} e^{i \lambda x} \langle P, S | \bar{\psi}(\lambda) \eta \bar{\psi}(\lambda n) | PS \rangle \]

**Quark distribution function is Fourier transform of light-cone correlation function**

- Starting point for field-theoretic analysis
- Frame independent! \( p = M \sqrt{2} \to \) rest frame
- Light cone physics controls distribution functions
I. STANDARD PARTON DISTRIBUTIONS

A PARTON DISTRIBUTION

\[ f(x) = \sum_x \delta \left( \frac{P_x^+}{P^+} - (1-x) \right) \left| \left\langle x \left| \bar{\psi}_+(0) \psi_+(x^-) \right| P \right\rangle \right|^2 \]

Light Cone Momentum Distr.
"Good" Component of Quark Field

\[ P^+ \rightarrow \begin{array}{c} \hline \hline \end{array} \]

\[ P^+ \]

\[ (1-x)P^+ \]

\[ P_{x}^+ \]

\[ xP^+ \]

\[ \begin{array}{c} \text{PROBABILTY} \\
\text{SELECTS OUT SPECIAL DIRECTION!} \quad \sqrt{2}P^+ = P^+P^3 \\
\text{DETERMINED BY EXPT.} \\
\text{DISTRIBUTION IN} \quad x = x_0 \end{array} \]

\[ \begin{array}{c} \text{Q}^2 \text{ DEPENDENCE (FROM QCD RADIATIVE CORRECTIONS)} \\
\text{IS SUPPRESSED.} \end{array} \]

IN COORDINATE SPACE

\[ f(x) = \frac{1}{2\pi} \int dx^- e^{iP^+_x x^-} \left\langle P \left| \bar{\psi}_+(0) \psi_+(x^-) \right| P \right\rangle \]

\[ \begin{array}{c} x^+ = 0 \\
x_L = 0 \end{array} \]
\[ f(x) = \frac{1}{2\pi i} \int dx^- e^{i\mathbf{p} \cdot x} \langle \mathcal{P} \mid \psi^+(0) \psi^-(x^-) \mid \mathcal{P} \rangle \]

* Gauge Invariance:

Not manifest. Actually (*) is the form of a manifestly G.I. expression in \( A^+=0 \) gauge

\[ \psi^+_+(0) \psi^-_+(x^-) \rightarrow \psi^+_+(0) \mathcal{P} \text{exp} \left[ \int_{x^-}^{x^+} dy^+ A^+_+(y^-) \psi^+_+(y^-) \right] \]

Nilson Line on l.c.

* Lorentz Invariance

\[ \psi^+_+(0) ... \psi^+_+(x^-) = \overline{\psi}_-(0) \psi^+_+ ... \psi^-_-(x^-) \]

\( x^+ dx^- \neq \text{all else is boost invariant} \)

Along \( x^3 \) direction

\[ f(x) = \frac{1}{2\pi i} \int dx^- e^{i \mathbf{p} \cdot x} \langle \mathcal{P} \mid \overline{\psi}(0) \mathcal{P} e^{i \int_{x^+}^{x^-} A^+ dy} \psi(x^-) \mid \mathcal{P} \rangle \]

NB: Path dependence
\( \int_{-1}^{1} dx \, x^{n-1} \frac{f(x)}{x} = A_n \)

\[ S \langle P, \overline{\psi} \chi^{\mu_1} \cdots iD^{\mu_n} \psi \mid P \rangle = S(P_{\mu_1} \cdots P_{\mu_n})A_n \]

\( S = \text{SYMMETRIZE} \neq \text{REMOVE TRACES} \)

\[ S(P_{\mu_1}P_{\mu_2}) = P_{\mu_1}P_{\mu_2} - g_{\mu_1\mu_2} M^2/4 \]

* OPERATORS ARE G.I., DEFINITE TWIST \((t=2)\), WITH
  * NEU. DEFINED ANOMALOUS DIMENSIONS (HENCE \(Q^2\)-EVILOUTION)

* SUMMARY: AN IDEAL PARTON DISTRIBUTION

  * WEIGHTED LIGHT-CONE MOMENTUM PROBABILITY DISTRIBUTION
    (WEIGHTED BY SPIN, FLAVOR, TRANSVERSITY, ETC.)
  * MANIFESTLY G.I. BI (OR MULTI-) LOCAL LIGHT-CONE FOURIER TRANSFORM
  * TOWER OF LOCAL, GAUGE INVARIANT OPERATORS
    OF DEFINITE TWIST
* Examples of Ideal Parton Distributions

- Momentum ($p^+$) Distribution

$$ q(x) = \int \frac{dx}{4\pi} e^{i\lambda x} \langle P | \Phi(p) \gamma \psi(x) | IP \rangle $$

$$ p = \frac{1}{\sqrt{2}} (p, p^+, p) $$

- Helicity Distribution

$$ \Delta q(x) = \int \frac{dx}{4\pi} e^{i\lambda x} \langle P \parallel \Phi(p) \gamma \psi(x) | IP \rangle $$

$$ n = \frac{1}{\sqrt{2}} \left( \frac{1}{p}, \frac{q^+}{p}, \frac{1}{p} \right) $$

- Transversity Distribution

$$ s_q(x) = \int \frac{dx}{4\pi} e^{i\lambda x} \langle P_\perp \Phi(p) | g_{\perp}(x) \gamma_5 \psi(x) | IP \rangle $$

- Gluon Momentum Distribution

$$ g(x) = \int \frac{dx}{4\pi} e^{i\lambda x} \langle P | n_\mu F_{\mu \nu}^{ax}(0) n_\nu F_{\nu x}^{ax}(\lambda x) | IP \rangle $$

* Not So Ideal Parton Distributions

- Higher Twist Distributions
- Gluon Spin Distribution
- Transverse Momentum & Angular Momentum Distributions
III. GENERALIZATIONS, DIFFICULTIES & FAQ

- Generalization to fragmentation functions

\[ \hat{f}(z,P,S,\eta) = \sum_X S(k - \frac{1}{2} P) \langle 0 | \bar{\psi} \psi | PS_X^0 \rangle \langle PS_X^0 | \bar{\psi} \eta | 0 \rangle \]

After similar algebra

\[ \hat{f}(z,P,S,\eta) = \sum_X \int \frac{d\alpha}{2\pi} e^{-i\lambda z} \langle 0 | \bar{\psi}(\alpha) PS_X^0 \rangle \langle PS_X^0 | \bar{\psi}(\alpha) \eta | 0 \rangle \]

Fragmentation function is also a L-C correlation function, but inherently more complicated because \( \sum_X \) cannot be performed

- Generalization to gluons

\[ G(x,P,S,\eta) = \int \frac{d\alpha}{2\pi} e^{i\lambda x} \langle PS | \gamma^\mu G^\mu_\nu(0) \eta^\nu \gamma^\nu G_{\alpha\beta}(\alpha) | PS \rangle \]
• Gauge invariance

• $\bar{\psi}(0)\eta\bar{\eta}\psi(\lambda n)$ is not gauge invariant.

• Such problem usually means a class of diagrams has been omitted.

\[ \bar{\psi}(0)\eta\bar{\eta}\psi(\lambda n) \Rightarrow \bar{\psi}(0)\eta \mathcal{P}(e^{i\int_{\beta}^{0} n.A(\beta n)} \bar{\eta}\psi(\lambda n) \]

$\mathcal{P}(\ )$ is Wilson link from $\lambda n \to 0$

sums up gluon insertions

• Simplify with gauge choice.

\[ n.A = 0 \iff \text{light-cone gauge} \]

$\mathcal{P}(\ ) \Rightarrow 1$ so gauge invariant result reduces to

\[ \bar{\psi}(0)\eta\bar{\eta}\psi(\lambda n) \]

in light-cone gauge
• "But the intermediate state |X⟩ is colored!"

Soft hadronization physics is assumed not
to spoil dynamical predictions. What actually happens

\[ \text{Hadronization occurs at a much later time scale } \sim \frac{1}{\Lambda_{\text{QCD}}} \text{ after hard scattering.} \text{ "Lead brick effect"} \]

• But \( \bar{\Psi}(0) \gamma_5 \Psi(\lambda n) \) doesn't exist in QCD?!

**e.g. Cheng & Li or other standard text**

In the \( \Sigma x \) is an integral over the other 3 components
of the quark (or gluon) momentum. This integral
generally diverges in renormalizable field theories

\[ f(x,P,S,\eta) = \int d^4k \ 8(k.n-x) \sum_X \langle P S | \bar{\Psi} \gamma_5 X \rangle \langle X | \eta \Psi | P S \rangle \]

Example

\[ f \sim \int \frac{d^2k_1}{k_1^2} \sim \log \Lambda^2 \]

\[ \text{or} \]

\[ \text{or} \]

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Aside

\[ f(x, \ldots) \propto \int \frac{d^4k}{(2\pi)^4} \overline{u}(PS) \gamma^\mu \not{k} \not{\bar{\eta}} \gamma^\nu \not{\bar{\bar{\eta}}} u(PS) \ S(k_n-x) \]

\[ \propto S((P-k)^2) + \text{gauge terms} \]

Brief calculation \Rightarrow \int \frac{dx_1^2}{k_{-1}^2}

Remedy - In physical processes divergence is cut off by physical momentum transfer - e.g. $Q^2$ in deep inelastic electron scattering

\[ \int dx_1^2 \Rightarrow \int dx_1^2 \]

- $f(x, P, S, \eta)$ depends logarithmically on $Q^2$

- $\bar{\Psi}(o) \eta \bar{\eta} \Psi (\lambda n) \Rightarrow \bar{\Psi}(o) \eta \bar{\eta} \Psi (x^2 \approx \frac{1}{Q^2}, x^0 = \lambda)$

Not quite local on light-cone
- Light-cone correlator doesn't actually exist ($x^0 = 0$)
- Problem is logarithmic and well-understood

- $\ln q^2 \leftrightarrow$ QCD radiative corrections

$\bar{\Psi}(o) \frac{1}{Q^2} \not{\bar{\eta}} \Psi (\lambda n)$
SUMMARY — DISTRIBUTION FUNCTIONS

QUARKS

\[ q(x, Q^2) = \frac{1}{\sqrt{2} p^+} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \left( \langle p | \gamma^+_{\perp} p(0, \lambda n) \gamma^+_{\perp} (\lambda n) | p \rangle \right) \mid_{Q^2} \]

\[ \Delta q(x, Q^2) = \frac{1}{\sqrt{2} p^+} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \left( \langle p^+ \gamma^+_{\perp} p(0, \lambda n) \gamma^+_{\perp} (\lambda n) | p \rangle \right) \mid_{Q^2} \]

\[ \Sigma q(x, Q^2) = \frac{1}{\sqrt{2} p^+} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \left( \langle p^+ \gamma^+_{\perp} p(0, \lambda n) \gamma^+_{\perp} (\lambda n) | p \rangle \right) \mid_{Q^2} \]

\[ q(x, Q^2), \Delta q(x, Q^2) \neq \Sigma q(x, Q^2) \text{ describe all quark inclusive hard physics that survives at large } Q^2 \text{ (twist-2)} \]

\[ q(x, Q^2) \leftrightarrow f_1(x, Q^2) \text{ SPIN AVERAGE} \]

\[ \Delta q(x, Q^2) \leftrightarrow g_1(x, Q^2) \text{ LONGITUDINAL SPIN} \]

\[ \Sigma q(x, Q^2) \leftrightarrow h_1(x, Q^2) \text{ TRANSVERSE SPIN} ^* \]

\[ P^\mu = p^\mu + \frac{m^2}{2} \eta^\mu \]

NOTE: NO \( g_2 \) — SUBDOMINANT

\( h_1(x, Q^2) \) DECOUPLES FROM INCLUSIVE D.I.S.

\[ \Theta(0, \lambda n) = P \left( \exp i \int \frac{d\xi}{\lambda n} A^\mu(\xi) \right) \]

\[ \gamma^+ = \Pi^+ \gamma \quad P^\pm = \frac{1}{2} \gamma^+ \gamma^\pm \quad \gamma^\pm = \gamma^0 \pm \gamma^3 \]

VARIATION ON THIS

\[ \text{89} \]
**GUONS**

\[ G(x,Q^2) = \frac{i}{\sqrt{2} \times p^+} \int \frac{d\lambda^+}{2\pi} e^{i\lambda^+ x} \langle P | F^{+\alpha}(0) P(0,\lambda n) F^{+\alpha}(\lambda n) | P \rangle \]

\[ \Delta G(x,Q^2) = \frac{i}{\sqrt{2} \times p^+} \int \frac{d\lambda^+}{2\pi} e^{i\lambda^+ x} \langle PS^+_\perp | F^{+\alpha}(0) P(0,\lambda n) \bar{F}^{+\alpha}(\lambda n) | PS^+_\perp \rangle \]

\[ G(x,Q^2) \leftrightarrow \text{SPIN AVERAGE GUON DISTRIBUTION} \]
\[ \Delta G(x,Q^2) \leftrightarrow \text{LONGITUDINAL SPIN DISTRIBUTION} \]

- **LEARNING FROM THE LIGHT CONE**

All the quantities of interest appear as correlation functions at light-like coordinate separation

Suspect there is something fundamental to learn from formulating QCD on the light cone.

Large subject, long history.
Small piece here.

---

* MORE ABOUT THIS LATER
*If we quantize Dirac field on surfaces of constant $\xi^+$, then correlation function will reduce to a distribution of Fourier components of independent degrees of freedom, just as equal time corr. fn. reduces to a momentum distribution.*

Formal basis for extension of parton model to spin dependent & higher-twist effects.

**Light Cone Quantization**

$$\chi^\pm \equiv \frac{1}{\sqrt{2}} (\gamma^0 \pm \gamma^3) \quad \chi^1 = (\gamma', \gamma^2)$$

**Useful projection operators**

$$P_\pm \equiv \frac{1}{2} \gamma^\pm \gamma^\pm$$

$$P_+ \psi = \psi_+$$

$$P_- \psi = \psi_-$$

$$P_+ P_- = P_- P_+ = 0$$

$$P_\pm^2 = P_\pm$$

$$P_+ + P_- = 1$$

$\psi_+$ & $\psi_-$ are two component spinor fields

$\psi_+ \Leftrightarrow "good"$ components of $\psi$

$\psi_- \Leftrightarrow "bad"$ components of $\psi$

**New notation**

$\psi_+ \Rightarrow \phi$ 

$\psi_- \Rightarrow \chi$
Importance of $\Psi_\pm$: At constant $x^+$, $x^+$ is evolution variable analogous to time in usual formulation.

Using $P^\pm$, Dirac equation

\[
\frac{\partial}{\partial x^+} \Psi_\pm = \cdots
\]

\[
(i \frac{\partial}{\partial x^-} - qA^-) \Psi_\pm = m \Psi_\pm
\]

\[
i \frac{\partial}{\partial x^-} \Psi_- = qA^- \Psi_- + \frac{i}{2} \left[ (i \frac{\partial}{\partial x^-} - qA^\pm) \Psi_+ + m \right] \chi \Psi_+
\]

Gauge $nA = 0 \rightarrow n^-A^- = 0 \Rightarrow A_- = A^+ = 0$

Eqn. for $\Psi_\pm$ does not involve $x^+$-evolution. It is a constraint. Likewise for $A^- = A^+$

\[
i \frac{\partial}{\partial x^-} \Psi_- = F[A^\pm, \Psi_+]
\]

So $\Psi_- \text{ is not an independent field} \rightarrow \text{it is a functional of } \Psi_+ \neq A_+$

- $\Psi_+$ and $A^\pm$ are independent canonical variables
- $\Psi_-$ and $A^\pm$ are dependent and should be eliminated from formulas to display their particle content
ON TO SPIN & RELATED ISSUES

$\Psi_{\pm}$ are still two component spinors. Decompose this spin space. Choose a suitable set of $\gamma$-matrices

\[
\begin{align*}
\gamma^0 & = \rho_1 \sigma_3 = \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix} & \gamma^2 & = i \sigma_2 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & i\sigma_3 \end{pmatrix} \\
\gamma^1 & = i \sigma_1 = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & i\sigma_1 \end{pmatrix} & \gamma^3 & = -i \rho_2 \sigma_3 = \begin{pmatrix} 0 & -\sigma_3 \\ \sigma_3 & 0 \end{pmatrix} \\
\gamma_5 & = i \gamma^0 \gamma^2 \gamma^3 = \rho_3 \sigma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}
\end{align*}
\]

Why? \( P^+ = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad P^- = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix} \)

So \( \Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} \) Upper = good Lower = bad

NOTE: \( \gamma^1, \gamma^2, \gamma_5 \neq \sigma_3 \) ALL COMMUTE \( \Psi \) \( P_{\pm} \)

CHOICES

1. DIAGONALIZE \( \gamma_5 \) (AND \( \sigma_3 \))
   HELICITY OR CHIRALITY BASIS

2. DIAGONALIZE \( \gamma_1 \) OR \( \gamma_2 \)
   TRANSVERSITY BASIS

1. \( \sigma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} \)
2. \( \gamma_5 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix} \)

\[ \Psi = \begin{pmatrix} \phi_+ \\ \phi_- \\ \chi_+ \\ \chi_- \end{pmatrix} \] 
GOOD
\( \pm \) ARE HELICITY LABELS
BAD

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2. **Rotate so $Y'$ is diagonal**

$$Q_I = \frac{1}{2} (1 \pm \gamma^2 \gamma^5)$$

$$\Psi = \begin{pmatrix} \phi_T \\ \phi_\perp \\ \chi_T \\ \chi_\perp \end{pmatrix} \text{ I are Transversity Labels}$$

- $Q_T \phi_T = \phi_T$ **Positive Transversity**
- $Q_\perp \phi_\perp = \phi_\perp$ **Negative Transversity**

Of course, $\phi_T$ and $\phi_\perp$ are linear combinations of $\phi_{\pm}$.
Also $\gamma^2$ could be equally well used to define transversity.

**Summary**

<table>
<thead>
<tr>
<th>Component</th>
<th>Helicity</th>
<th>Chirality</th>
<th>Effective Twist *</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_+$</td>
<td>$\frac{1}{2} \uparrow$</td>
<td>$+R$</td>
<td>1</td>
</tr>
<tr>
<td>$\phi_-$</td>
<td>$-\frac{1}{2} \downarrow$</td>
<td>$-L$</td>
<td>1</td>
</tr>
<tr>
<td>$\chi_+$</td>
<td>$\frac{1}{2} \uparrow$</td>
<td>$-L$</td>
<td>2</td>
</tr>
<tr>
<td>$\chi_-$</td>
<td>$-\frac{1}{2} \downarrow$</td>
<td>$+R$</td>
<td>2</td>
</tr>
</tbody>
</table>

*See subsequent discussion

$\uparrow \downarrow$ Alternative notation for helicity

$L,R$ Alternative notation for chirality.
WHY DO \( \phi_+ \) AND \( \chi_+ \) HAVE SAME HELICITY
BUT OPPOSITE CHIRALITY?

\[
\begin{bmatrix}
\text{Note} \\
\chi_+ = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} \\
\chi_5 = \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix}
\end{bmatrix}
\]

ANSWER: \( \chi_\pm \) ARE ACTUALLY QUARK-GLUON COMPOSITE FIELDS

\[ \chi_+ \propto (\phi A_\perp)^+ \]

\((\phi A_\perp)^+\) IS QUARK+GLUON IN HELICITY +\(\frac{1}{2}\)

STATE:

\[ \begin{array}{c}
\uparrow \\
\downarrow \\
\end{array} \]

\[ +1 - \frac{1}{2} = +\frac{1}{2} \]

SO \( \chi_+ \) REALLY CONTAINS \( \phi \rightarrow \) ODD CHIRALITY.

(IMPORTANT FOR SORTING OUT TWIST)
2. QUARK & GLUON DISTRIBUTION & FRAGMENTATION 
FUNCTIONS IN A HELICITY BASIS

$q, \Delta q, S_\perp$ all seem to be described as helicity projections of quark-hadron scattering

- Quark $h = \pm \frac{1}{2}$ "good" light-cone components
- Gluon $h = \pm 1$ "good" light-cone components
- Nucleon $h = \pm \frac{1}{2}$

\[ A_{H_h H_h'} \]

\[ \begin{array}{c}
H \\
\downarrow h \\
\downarrow h' \\
\uparrow H' \\
\end{array} \quad = \quad \text{disc}_u \quad \begin{array}{c}
H' \\
\downarrow h' \\
\downarrow h \\
\uparrow H \\
\end{array} \]

- **Conservation of $J_3$**: $H + h' = H' + h$

- **Parity** $A_{H_h H_h'} = A_{-H-h',-H'-h}$

- **T-Invariance** $A_{H_h H_h'} = A_{H_h', H_h}$

*Not valid for fragmentation function $\xi$ to $t^*$
• QUARKS IN THE NUCLEON

\[
\begin{align*}
A_{\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}} &= q(x, Q^2) + \Delta q(x, Q^2) \\
A_{\frac{1}{2} \frac{1}{2} \frac{1}{2} -\frac{1}{2}} &= q(x, Q^2) - \Delta q(x, Q^2) \\
A_{\frac{1}{2} -\frac{1}{2} \frac{1}{2} -\frac{1}{2}} &= S_q(x, Q^2)
\end{align*}
\]

• GLUONS IN THE NUCLEON

\[
\begin{align*}
A_{\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}} &= G(x, Q^2) + \Delta G(x, Q^2) \\
A_{\frac{1}{2} -\frac{1}{2} \frac{1}{2} -\frac{1}{2}} &= G(x, Q^2) - \Delta G(x, Q^2)
\end{align*}
\]

• HILBERT DENSITY MATRIX FORMALISM

Gather non-vanishing helicity amplitudes into a matrix rotation.

QUARK-NUCLEON:

\[
A(x, Q^2) = q(x, Q^2) \Pi \otimes \Pi + \Delta q(x, Q^2) \sigma_3 \otimes \sigma_3 \\
+ S_q(x, Q^2) \left( \sigma_1 \otimes \sigma_1 + \sigma_2 \otimes \sigma_2 \right)
\]

GLUON-NUCLEON:

\[
A(x, Q^2) = G(x, Q^2) \Pi \otimes \Pi + \Delta G(x, Q^2) \sigma_3 \otimes \Sigma_3
\]
2. DIRAC COVARIANTS, DIMENSION ≠ TWIST

Return to \( \Phi_i, \Phi_j \) and construct \( \Phi^T \Phi \) for \( T = 1, 8, \ldots \)

### ASIDE ON DIMENSIONS

Dimension means "engineering dimension" or units in [mass] \( ^d \).

- Operator: \( \Theta \), notation \([\Theta] = m^{d_\Theta}\)
- Examples:
  - \( S = \int d^4x \mathcal{L} \Rightarrow [\mathcal{L}] = m^4 \Rightarrow d_\mathcal{L} = 4\)
  - \([\Phi \Phi \Phi] = m^4 \Rightarrow d_\Phi = 3/2\)
  - \([\mathcal{G}_{\mu \nu} \mathcal{G}^{\mu \nu}] = m^4 \Rightarrow d_\mathcal{G} = 2\)
  - \(d_p = 1, \quad d_n = -1\)
  - \(\langle PIP^- \rangle = (2\pi)^3 2\pi S^3 (\mathcal{E}_\mathcal{F} - \mathcal{E}^-)\)
  - \([\langle PIP^- \rangle] = m^{-2}\)

Dimensional consistency requires

\[
[\langle P|\Phi_i(0)\Phi_j(\lambda n)|P\rangle] = m^1
\]

Dimensions can be made up from powers of \( p^\mu, n^\mu, S^\mu \) and target mass.
POWER COUNTING (TWIST)

- Looking for largest contribution as momentum transfers $\to \infty$
- Hard parton cross sections are scale invariant (up to logs)
  \[ \frac{d\hat{\sigma}}{d\theta_{ab\cdots}} \sim \frac{1}{Q^d} \] where $d$ is determined by dimensional analysis
- No masses in denominator
- Any masses introduced in $g$ or $f$ will persist through to end of calculation and suppress contribution by $\Theta(m/Q)^S$
- Leading contribution (no $m/Q$ suppression) is "TWIST-2"
  Each numerator factor of $m$ increases suppression and twist by 1.

EXAMPLE \[ \Gamma \Rightarrow \gamma^\mu \]

\[ \frac{1}{2\pi} \int d\lambda \, e^{i\lambda x} \left\langle \langle PS \bar{\psi}(o) \gamma^\mu \psi(\lambda n) \rangle P \right\rangle \frac{1}{Q^2} \]

\[ = 2 \left\{ g_1(x,Q^2) p^\mu + \bigg( \frac{3}{2} g_4(x,Q^2) n^\mu \bigg) \right\} \]

\[ \frac{g_1}{\text{TWIST-2}} \quad \frac{g_4}{\text{TWIST-4}} \]

$g_1$ will eventually contribute at leading order $\Theta(1)$.
$g_4$ will contribute at order $1/Q^2$. 
DEMONSTRATION (D. I. S.)

\[
W_{\mu \nu} = \frac{1}{4} \text{Tr} \left( \gamma_{\mu} \gamma_{\nu} \gamma_{\sigma} \gamma_{\tau} \right) \frac{(q + x P)^{\rho}}{2 P \cdot q} f^{\sigma}_{\rho} \\
\]

\[
f^{\sigma}_{\rho} = \frac{1}{2\pi} \int d\lambda e^{i\lambda x} \langle P | \bar{\psi} \gamma_{\sigma} \psi (\lambda x) | LP \rangle = 2 \left( f_{\mu \nu} \sigma + M^{\mu} f_{\sigma} \right) \\
\]

\[
P^{\mu} = p^{\mu} + \frac{M^{\mu}}{2} n^{\mu} \quad \quad P^{2} = M^{2} \quad \quad P \cdot q = \gamma \quad \quad q^{2} = -2x\gamma = -Q^{2} \]

\[
\frac{1}{4} \text{Tr} \left( \gamma_{\mu} \gamma_{\nu} \gamma_{\sigma} \gamma_{\tau} \right) = g_{\mu \rho} g_{\nu \sigma} + g_{\mu \sigma} g_{\nu \rho} - g_{\mu \nu} g_{\rho \sigma} \]

\[
W_{\mu \nu} = -g_{\mu \sigma} \frac{q \cdot f}{2q \cdot P} + \left( \frac{(q^{\mu} + x P^{\mu}) f^{\sigma}_{\rho}}{2q \cdot P} + (\mu \leftrightarrow \nu) \right) \]

\[
= -g_{\mu \sigma} f_{1}(x,Q^{2}) + \frac{q^{\mu} P^{\nu} + q^{\nu} P^{\mu}}{2q \cdot P} f_{1}(x,Q^{2}) + x \frac{P^{\mu} P^{\nu}}{q \cdot P} f_{1} \]

\[
\rightarrow \quad + g_{\mu \sigma} \frac{x M^{2}}{q \cdot P} f_{4} + \frac{q^{\mu} q^{\nu} M^{2}}{(q \cdot P)^{2}} f_{4} + \ldots \]

SUPPRESSED
DECOMPOSITION INTO DISTRIBUTION FUNCTIONS

\[ \frac{1}{2\pi} \int d\lambda e^{i\lambda x} \langle \bar{\psi}(1) \gamma_\mu \psi(\lambda n) | PS \rangle = 2 \{ f_2(x) p_\mu + f_4(x) n_\mu M^2 \} \]

\[ \frac{1}{2\pi} \int d\lambda e^{i\lambda x} \langle \bar{\psi}(1) \gamma_\mu \gamma_5 \psi(\lambda n) | PS \rangle = 2 \{ g_2(x) S \cdot n p_\mu + g_4(x) S \cdot n \}
\]

\[ \frac{1}{2\pi} \int d\lambda e^{i\lambda x} \langle \bar{\psi}(1) \psi(\lambda n) | PS \rangle = 2M e^{i\lambda x} \]

\[ \frac{1}{2\pi} \int d\lambda e^{i\lambda x} \langle \bar{\psi}(1) \gamma_\mu i \gamma_5 \psi(\lambda n) | PS \rangle = 2 \{ \mathcal{P}_2(x) (S_{\mu} p_\nu - S_{\nu} p_\mu) / M 
\]

\[ \hspace{0.5cm} + \mathcal{P}_4(x) M (p_\mu n_\nu - p_\nu n_\mu) S \cdot n + \mathcal{P}_6(x) M (S_{\mu} n_\nu - S_{\nu} n_\mu) \} \]

NOTE: IN DIMENSIONAL ANALYSIS \( S \) COUNTS AS POWER OF MASS IN NUMERATOR.

THREE LEADING TWIST DISTRIBUTION FUNCTIONS \( f, g, h \).

THREE SUBDOMINANT \( \Theta(1/Q) \) DISTR. FUNCTIONS \( e, g_T, h_L \).

THREE TWIST-4, \( \Theta(1/Q^2) \) DISTR. FUNCTIONS - INDIFFERENT FROM MULTIPARTICLE CORRELATIONS ARISING ELSEWHERE.
V LEADING TWIST DISTRIBUTION FUNCTIONS

- $f_1(x) = \frac{\alpha}{4\pi} \int e^{i\lambda x} \langle PL \bar{\psi}(0) \not{\gamma} \not{\psi}(\lambda n) | P \rangle$

- $g_1(x) = \frac{\alpha}{4\pi} \int e^{i\lambda x} \langle PS_{II} \bar{\psi}(0) \not{\gamma} \gamma_5 \not{\psi}(\lambda n) | PS_{II} \rangle$

- $h_1(x) = \frac{\alpha}{4\pi} \int e^{i\lambda x} \langle PS_{II} \bar{\psi}(0) \not{\gamma} \gamma_5 \not{\psi}(\lambda n) | PS_{II} \rangle$

Decompose in light-cone chirality basis

\[ f_1(x) = \frac{1}{x} \int d^2k_\perp \langle PL | \Phi_R^+(x, k_\perp) \Phi_R(x, \bar{k}_\perp) + \Phi_L^+(x, k_\perp) \Phi_L(x, \bar{k}_\perp) | P \rangle \]

\[ g_1(x) = \frac{1}{x} \int d^2k_\perp \langle P \hat{\epsilon}_3 | \Phi_R^+(x, k_\perp) \Phi_R(x, \bar{k}_\perp) - \Phi_L^+(x, k_\perp) \Phi_L(x, \bar{k}_\perp) | P \hat{\epsilon}_3 \rangle \]

\[ h_1(x) = \frac{2}{x} \text{Re} \int d^2k_\perp \langle P \hat{\epsilon}_3 | \Phi_L^+(x, k_\perp) \Phi_R(x, \bar{k}_\perp) | P \hat{\epsilon}_3 \rangle \]

or in light-cone transversity basis

\[ g_1(x) = \frac{2}{x} \text{Re} \int d^2k_\perp \langle P \hat{\epsilon}_3 | \Phi_T^+(x, k_\perp) \Phi_T(x, \bar{k}_\perp) | P \hat{\epsilon}_3 \rangle \]

\[ h_1(x) = \frac{1}{x} \int d^2k_\perp \langle P \hat{\epsilon}_1 | \Phi_T^+(x, k_\perp) \Phi_T(x, \bar{k}_\perp) - \Phi_T^-(x, k_\perp) \Phi_T(x, \bar{k}_\perp) | P \hat{\epsilon}_1 \rangle \]
<table>
<thead>
<tr>
<th>LIGHT-CONE</th>
<th>HELICITY</th>
<th>CHIRALITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOOD</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>GOOD</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>BAD</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>BAD</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

\[ \text{TWIST-TWQ} = -(\text{GOOD} - \text{GOOD}) \]

\[ \text{SCALING} \]

\[ \text{HEMICITY AVERAGE} \]
\[ (\uparrow \uparrow \Rightarrow \uparrow \uparrow + \downarrow \downarrow \Rightarrow \downarrow \downarrow) \]
unpolarized target & beam

\[ \text{HEMICITY DIFFERENCE} \]
\[ (\uparrow \uparrow \Rightarrow \uparrow \uparrow - \downarrow \downarrow \Rightarrow \downarrow \downarrow) \]
longitudinal spin asymmetry

\[ \text{HEMICITY FLIP} \]
\[ (\uparrow \downarrow \Rightarrow \downarrow \uparrow) \]
transverse spin asymmetry

chiral even

chiral even

chiral odd
TWIST-3 EXAMPLE: \( e(x) \)

- SPIN AVERAGE
- CHIRAL ODD

*Which Dirac structure?*

\[
2Me(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle P | \overline{\phi}(0) \psi(\lambda n) | P \right\rangle
\]

- CLEARLY SPIN-AVERAGE
- CLEARLY CHIRAL-ODD
- TWIST-3 \( \Theta(M/Q) \) BY DIMENSIONAL ANALYSIS

*Light-cone Helicity Analysis*

\[
e(x) = \frac{1}{x} \int d^3k_\perp \left\langle P | \phi_R^+(x\mu, k_\perp) \chi_R(x\mu, k_\perp) + \phi_R^+(xR, k_\perp) \chi_R(xR, k_\perp) \right\rangle
\]

- REDUCTION TO INDEPENDENT FIELDS \( \chi = F[\phi, A] \)

\[
e(x) = -\frac{i}{4Mx} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle P | \overline{\phi}(0) i\gamma^\mu D_\mu(\lambda n) \phi(\lambda n) | P \right\rangle + H.C.
\]
• **NO SIMPLE FOCK SPACE INTERPRETATION OF** \( \rho(x) \)

• **GRAPHICALLY**

\[ \begin{array}{c}
\text{\( x\rho \)} \\
\longrightarrow \\
\text{\( \text{\( (x+y)\rho \)} \)
\end{array} \]

• **GENERALIZATIONS**

Each unit of twist adds independent fields:

\[ \begin{align*}
\phi^\dagger \phi & \leftrightarrow \text{TWIST 2} \\
\phi^\dagger \chi & \leftrightarrow \text{TWIST 3} \\
\chi^\dagger \chi & \leftrightarrow \text{TWIST 4}
\end{align*} \]

• Twist-3 is (barely) tractible, twist-4 is not yet.
III. SPIN DYNAMICS  SHORT EXAMPLES

A. SUM RULES

B. TRANSVERSE SPIN EFFECTS

C. $q_2 \neq$ TWIST-3

D. A LOCAL OPERATOR FOR GLUON SPIN
A. **SUM RULES**

* GENERIC *

\[ f(x) = \frac{1}{2\pi} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle \frac{1}{\mu^{\nu}} \frac{1}{P} P (0, \omega) \phi (\omega) | PS \right\rangle \]

\[ \int_{-\infty}^{\infty} dx \, x^n f(x) = \ldots \int dx \int \frac{d\lambda}{2\pi} x^n e^{i\lambda x} \ldots \]

\[ = \ldots \int dx \int \frac{d\lambda}{2\pi} \left( i \frac{\partial}{\partial \lambda} \right)^n e^{i\lambda x} \ldots \]

\[ = \ldots \int dx \int \frac{dx}{2\pi} e^{i\lambda x} \left( i \frac{\partial}{\partial \lambda} \right)^n \ldots \]

\[ \left( \int \frac{dx}{2\pi} e^{i\lambda x} = \delta (\lambda) \right) \]

\[ = \ldots \left( i \frac{\partial}{\partial \lambda} \right)^n \left\langle PS \phi (0, \omega) \phi (\omega) | PS \right\rangle \]

\[ \int_{-\infty}^{\infty} dx x^n f(x, Q^2) = \ldots \left\langle PS \phi (0) \phi^\dagger (0, \lambda) \frac{1}{P} (u D^\nu) \phi (0) | PS \right\rangle \]

QUARKS \[ \rightarrow \bar{f}(x, Q^2) = -\bar{f}(-x, Q^2) \]

GLUONS \[ \rightarrow G(x, Q^2) = -G(-x, Q^2) \]

have various signs

* EXAMPLES (STRIPPED OF QCD RADIATIVE CORRECTIONS) *

• **BJORKEN** (1972)

\[ \bar{\psi} \gamma^+ \gamma_5 \tau_3 \gamma \psi \]

\[ \int d [g_{1}^{ep}(x) - g_{1}^{en}(x)] = \frac{1}{6} g_\Lambda \frac{g_\Lambda}{Q} \Delta u - \Delta d \]

**QCD**
• ELLIS-REI

\[
\int_0^1 \frac{dx}{x} q_1^{ep}(x) = \frac{4}{3} \Delta u + \frac{1}{3} \Delta d + \frac{1}{3} \Delta s \propto \langle \bar{\psi} \gamma^5 \gamma_5 \gamma^2 \psi \rangle
\]

\[
\alpha = \frac{1}{2} F + \frac{1}{18} D + \frac{1}{3} \Sigma \quad \text{SU(3) SYMMETRY}
\]

\[
F \neq D \Leftrightarrow \text{BARYON } B-\text{DECAY AXIAL CHARGES}
\]

\[
\Sigma \leftrightarrow \langle \psi 1 \Phi \gamma^\mu \gamma_5 \gamma_\mu \psi 1 \psi 3 \rangle = \text{SPIN FRACTION}
\]

• BURKHARDT-COTTINGHAM SR

\[
\int_0^1 \frac{dx}{x} g_2(x) = 0
\]

QCD

\[\text{ABSENCE OF } J=0 \text{ FIXED POLE} \]

THE IS NO OPERATOR
(SPIN-0, DIMENSION-3)
FOR R.H.S.

B. TRANSVERSE SPIN EFFECTS

\[ S_0 \text{ DECOUPLES FROM INCLUSIVE D.I.S.} \]

\[ S_0 \propto A \begin{array}{c}
\frac{1}{2} - \frac{5}{2} - \frac{1}{2} \frac{1}{2} \\
\text{QUARK} \\
\text{HELIUMITY} \\
\text{FLIP}
\end{array} \]

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$S_q$ does not mix with gluons.

There is no twist-2 gluon helicity flip:

\[
\begin{array}{c}
\frac{1}{2} - 1 \\
\uparrow \\
\Delta H = -1 \\
\hline
\frac{1}{2} + 1 \\
\uparrow \\
\Delta H = +2
\end{array}
\]

$S_q$ dominates transverse spin effects in hard processes with two active hadrons.

Ex. $e\bar{p} \rightarrow e\bar{H}X$

\[
\begin{array}{c}
\text{\textbf{\large $S_q$}} \\
\text{\textbf{\large $S_q$}}
\end{array}
\]

$A_{T\gamma}/A_{all} \ll 1$ in $\bar{p}p \rightarrow 2\text{jets}$

- 2-jet production is dominated by $gg$ & $gq$, but there is no gluon transversity.
C. $g_2$ IS HIGHER TWIST & HAS NO SIMPLE PARTON INTERPRETATION

$$g_2 \propto \bar{\psi} \gamma^+ \gamma_5 \psi \rightarrow \psi^+ \gamma^+ \gamma_5 \psi + \psi^- \gamma^- \gamma_5 \psi.$$  
BAD L.C. COMPONENTS  
$\Rightarrow$ HIGHER TWIST.

$$g_1 \propto \bar{\psi} \gamma^+ \gamma_5 \psi$$

GUONS WILL APPEAR:

$$\int_0^1 dx \, x^2 \, g_2(x,Q^2) \propto \langle \text{PS} \mid \bar{q} \gamma^+ \bar{F}_\mathbf{q} \gamma^+ \psi \mid \text{PS} \rangle$$

[SHURYAK-VAINShteIN S.R.]

- BAD NEWS: Nearly infinite number of papers claiming to "understand" $g_2$ in parton model are flawed (or at least very model dependent).
- GOOD NEWS: $g_2$ probes quark gluon correlations

D. A LOCAL OPERATOR FOR GUON SPIN FRACTION

$$\Delta G(x,Q^2) = G^+(x,Q^2) - G^-(x,Q^2)$$

$$\int_0^1 dx \, \Delta G(x,Q^2) = \Delta G$$

- WHAT IS RELATION TO GENERATOR OF GUON SPIN ROTATIONS IN $M^{\mu\nu\lambda}$?

$$\Delta G(x,Q^2)$$

- RHS SHOULD BE SPIN-1, DIM-3
- BUT THERE IS NO LOCAL
- GAUGE INvariant, $s=1, d=3$ GUON OP.?
I. Why is $q_2(x, Q^2)$ interesting?

$q_2(x, Q^2)$ is not the quark transverse spin distribution.

\[ q_1(x) = \sum_a \left[ q^+_a(x) - q^+_a(x) \right] e_a^2 \]

where $q^+_a (q^-_a)$ is the probability to find a quark with flavor $a$, momentum fraction $x \neq$ helicity $\uparrow$ or $\downarrow$ relative to the target nucleon.

III. Using operator product expansion, $q_2(x, Q^2)$ can be related to quark-gluon operators:

\[
\begin{align*}
\int_0^1 dx \ x^n q_1(x, Q^2) &= \frac{1}{4} a_n(Q^2) \\
\int_0^1 dx \ x^n q_2(x, Q^2) &= \frac{1}{4} \frac{n}{n+1} (d_n(q^2) - a_n(q^2)) \\
\Rightarrow q_2 &= \bar{g}_2 + g_{2WW} \\
g_{2WW} &= -q_1 + \int_x^1 \frac{dy}{y} q_1(y, Q^2)
\end{align*}
\]
\[ \int_0^q dx \ x^n \ \overline{q}_2(x, Q^2) = \frac{1}{4} \ \frac{n!}{n+1} \ \Delta_n(Q^2) \]

\[ \langle PS | \Theta \left[ \sigma_5 \left\{ \mu_1 \right\} \mu_2 ... \mu_n \right] \overline{Q}_2^n \rangle = \frac{d_n(Q^2)}{(n+1)} \left[ (S_{\mu_1} P_{\mu_1} - S_{\mu_1} P_0) P_{\mu_2} ... P_{\mu_n} \right.
+ (S_{\mu_2} P_{\mu_2} - S_{\mu_2} P_0) P_{\mu_3} ... P_{\mu_n} + ... \left. \right] \]

- traces

\[ \Theta \left[ \sigma_5 \left\{ \mu_1 \right\} \mu_2 ... \mu_n \right] = \mathcal{J}_n \left\{ \frac{g}{8} \sum_{l=0}^{n-2} \overline{q} D_{\mu_1} ... D_{\mu_l} F_{\overline{P}_{\mu_1} \mu_2} D_{\mu_3} ... D_{\mu_{n-1}} q \right\}
+ \frac{g}{8} \sum_{l=0}^{n-3} \overline{q} D_{\mu_1} ... D_{\mu_l} \left( D_{\mu_{l+1} ... \mu_{n-1}} \right) D_{\mu_n} \gamma_5 q \]

- traces

\[ g \ (QCD \ COUPLING) = 0 \ \Rightarrow \ \overline{q}_2 = 0 \]

\( \overline{q}_2 \) IS INTERACTION DEPENDENT

\[ \int_0^1 dx \ \overline{q}_2(x, Q^2) = 0 \quad \text{B.C.S.R.} \]

SIMPLEST NON-TRIVIAL CASE — \( n = 2 \)

\[ \Theta \left[ \sigma_5 \left\{ \mu \right\} \mu \right] = \frac{g}{8} \ \overline{q} \left( \gamma_\nu \gamma_5 \gamma_\mu + \gamma_\mu \gamma_5 \gamma_\nu \right) q \]
.2 Parton Structure

Generic Twist-Two Structure Function

\[ p(x, q^2) \]

Generic Twist-Three Structure Function

\[ g(x_1, x_2, q^2) \]

Then \( q_T(x) \) is obtained from \( G(x_1, x_2) \)

\[ q_T(x) = \frac{1}{2x} \int dy \left\{ G(x,y) - G(y,x) + \tilde{G}(x,y) - \tilde{G}(y,x) \right\} \]

\( G(x,y) \) is a "natural" light-cone distribution function

\( q_T(x) \) has information integrated out (Compressed)

Note: The process \( G(x_1, x_2) \Rightarrow q_T(x) \) is not reversible
where \( x \) denote other tensor structures of higher twist (\( > 3 \))

\[
\ldots \left( x, y \right) \mathcal{O} + 
\sum_{\mathcal{O}} \left( x, y \right) \mathcal{O} + 
\sum_{\mathcal{O}} \left( x, y \right) \mathcal{O} + 
\ldots
\]

\[
\text{LICHT CONE GEOMETRIE, } \text{N. A. = O}
\]

\[
P = \mathbf{r} \times \mathbf{r} + \mathbf{r} \times \mathbf{r} + \mathbf{r} \times \mathbf{r} + \mathbf{r} \times \mathbf{r}
\]

\[
P \cdot n = 0
\]

\[
P \cdot n = 0
\]

\[
x \neq y, \quad \text{KINETIKS: LICHTFÖRMIGE IERARIKES}
\]

\[
\text{III.3. THEORETISCHE INTERWÜMDE: EXPRESSIONS FOR (X, Y) = (x, y)}
\]
AN EQUIVALENT — AND DECEPTIVE — EXPRESSION
FOR $g_2(x, Q^2)$

$$\frac{d\alpha}{2\pi} e^{ixx} \langle PS \mid q(x) \gamma^\mu \gamma^5 q(y) \mid PS \rangle = 2 \left\{ g_1(x, Q^2) p^\mu s \cdot n \\
+ g_T(x, Q^2) S_\mu \\
+ g_3(x, Q^2) S \cdot n n^\perp \right\}$$

Looks simple: $g_1$ and $g_T$ both related to quark distribution alone.

But: $q^+_\perp Y^\perp \gamma_5 q^+$ is independent light cone distribution whereas

$q^+_\perp Y^\perp \gamma_5 q^-$ is dependent

Many misguided interpretations of $g_2$ begin with this eqn.

QCD EVOLUTION OF $g_2(x, Q^2)$

A complex story with a hopeful ending

General situation summarized diagrammatically
\[ G(x_1, x_2, Q^2) \rightarrow \bar{q}_2(x, Q^2) \]

EXPERIMENTAL MEASUREMENT OF \( q_2(x, Q^2) \) DOES NOT GIVE ENOUGH INFORMATION \((G(x_1, x_2, Q^2))\) TO EVOLVE.

TECHNICAL HISTORY

* AHMED & ROSS discovered \( q_2(x, Q^2) \) and calculated anomalous dimensions \( \{ \gamma_n \} \) leading to straightforward evolution

* SHURYAK & VAINSHTEYN showed \( q_2(x, Q^2) \) to be related by eqns of motion to tower of gluon-quark operators. And that \((n-1)\) gluon-quark operators mix with \( q_2 \) under renormalization
- Lipatov et al. \{ Calculated anomalous dimension matrix for each moment n \( (\gamma_n)_{ij} \) \\
Ratcliff

\textbf{MATRIX MIXING \Rightarrow UNDERLYING TWO VARIABLE DISTRIBUTION EVOLVING IN "STANDARD" FASHION}

\[
\frac{d}{d\ln Q^2} G(x_1, x_2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int \frac{dy_1}{x_1} \frac{dy_2}{x_2} \mathcal{P}(x_1 y_1, x_2 y_2) G(y_1, y_2, Q^2)
\]

* SCHEMATIC

- Ali, Braun, Hiller. Showed that evolution simplifies in \( N_c \to \infty \) limit.

\( \bar{q}_z(x, Q^2) \) evolves in "standard" way with non-standard anomalous dimension for \( N_c \to \infty \)

\[
(\gamma_n^\star)^{\text{NS}} = 2N_c \left( 4(1/n+1) + \gamma_E - \frac{1}{4} + \frac{1}{2(n+1)} \right) + \mathcal{O}(1/N_c)
\]

Although approximate it may be good enough to transfer model predictions from model scales \( (Q^2 = \mu^2_{\text{MODEL}}) \) to experiment \( Q^2 = 10 \text{ GeV}^2 \).

\textbf{END OF THEORETICAL INTERLUDE}
III.4 BURKHARDT-COTTINGHAM SUM RULE

\[ \int \frac{dx}{2\pi} e^{i\lambda x} \langle PS | \bar{q}(0) \gamma^\mu \gamma_5 q(\lambda x) | PS \rangle = 2 \{ g_1(x) s \cdot n p^\mu + q_T(x) S_1 + \ldots \} \]

I. Contract with \( n \mu \) \( (n \cdot p = 1, n^2 = 0, n \cdot S_1 = 0) \)

\[ 2 s \cdot n g_1(x) = \int \frac{dx}{2\pi} e^{i\lambda x} \langle PS | \bar{q}(0) \gamma_5 q(\lambda x) | PS \rangle \]

- Integrate over \( x \) (restore \( Q^2 \)-label)

\[ 2 s \cdot n \int dx \ g_1(x, Q^2) = \langle PS | \bar{q}(0) \gamma_5 q(0) | PS \rangle \bigg|_{Q^2} \]

- In target rest frame choose \( z = \hat{z} \)

\[ \int dx \ g_1(x, \hat{z}^2) = \langle P \hat{e}_3 | \bar{q}(0) \gamma_5 q(0) | P \hat{e}_3 \rangle \bigg|_{Q^2} \]

II. Let \( s \cdot n = 0 \), by choosing \( z = \hat{z} = \hat{e}_1 \)

\[ \int dx \ g_T(x, Q^2) = \langle P \hat{e}_1 | \bar{q}(0) \gamma^\mu \gamma_5 q(0) | P \hat{e}_1 \rangle \bigg|_{Q^2} \]

III. Compare underlined equations \( \Rightarrow \int dx \ g_T = \int dx \ g_1 \)

Or \( \int dx g_2(x, Q^2) = 0 \) \underline{APARENTLY A CONSEQUENCE OF ROTATION INVARIANCE}

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A POSSIBLE HITCH IN THE B.C. SUM RULE

\[ \int_{-1}^{1} dx \ q_2(x, Q^2) = 0 \quad \text{is gold plated} \]

BUT \( q_2(x, Q^2) \) IS A DISTRIBUTION IN \( x \), IE A GENERALIZED FUNCTION \( \neq \) CAN HAVE \( \delta(x) \)

\[ q_2(x, Q^2) = q_2^{\text{OBSERVABLE}}(x, Q^2) + C \delta(x) \quad ? \]

WHENCE \[ \int_{-1}^{1} dx \ q_2^{\text{OBSERVABLE}}(x, Q^2) = - \frac{1}{2} C \]

NOTE: SUCH A PATHOLOGY IS NOT SO ARBITRARY AS IT LOOKS

- IT IS AN EXAMPLE OF A TIME HONORED DISEASE KNOWN AS A "J=0 FIXED POLE WITH NON-POLYNOMIAL RESIDUE." FIRST STUDIED IN REGGE THEORY, EMERGES NATURALLY IN DERIVATION OF B.C. AS SUPERCONVERGENCE RELN.

- TWO SIGNIFICANT INCIDENCES OF J=0 F.P. RUINING SUM RULES ARE
  1. BURKHARDT COTTINGHAM
  2. DREU-HEARN-GERASIMOV

BOTH TESTIBLE IN NEAR FUTURE!
EVIDENCE ON B.C. SUM RULE

Several other similar sum rules are violated perturbatively in QCD.

Burkhardt Cottingham does not appear to be violated in pQCD at 1-loop, but non-perturbative effects & higher orders remain unstudied.

$g_2(x)$

CASE I OR II?

OR PERHAPS MORE NODES?

LOOK AT MODELS
5 The shape of $q_2(x, Q^2)$

An experimental agenda.

1. Is $\int_0^1 dx \ q_2(x, Q^2) = 0$ ?

2. If yes, does $q_2(x, Q^2)$ have just one (non-trivial) zero ?

3. If yes, what is the sign of

$$\int_0^1 dx \ x^2 q_2(x, Q^2) = M_2[q_2]$$

- $M_2[q_2] < 0 \Rightarrow \begin{array}{c}\Downarrow \text{sign} \end{array}$
- $M_2[q_2] > 0 \Rightarrow \begin{array}{c}\Downarrow \text{sign} \end{array}$

4. Can a signal of twist-3 be seen ?

$$\overline{q}_2(x, Q^2) = q_2(x, Q^2) - q^{WW}_2(x, Q^2) \neq 0$$
RECALL
\[
\Delta G(x, Q^2) = \frac{i}{\sqrt{2} P^+ x} \int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle PS_z | F^{+\alpha}(\lambda n) P(\lambda\eta, 0) F^{\alpha+}(0) | PS_z \rangle |_{Q^2}.
\]

\[
\int_{-\infty}^{\infty} dx \Delta G(x, Q^2) \Rightarrow \int \frac{d\lambda}{\lambda} e^{i\lambda x} \quad \text{A LOCAL SUM RULE}
\]

\[
\int \frac{d\lambda}{\lambda} e^{i\lambda x} \Rightarrow \epsilon(\lambda) \quad \text{for} \quad \lambda > 0 \quad \text{and} \quad \lambda < 0.
\]

\[
\Delta G(Q^2) = \int_{-\infty}^{\infty} d\lambda \epsilon(\lambda) \langle PS_z | F^{+\alpha}(\lambda n) P(\lambda\eta, 0) F^{\alpha+}(0) | PS_z \rangle |_{Q^2}
\]

GAUGE INVARIANT BUT NOT LOCAL!

CHOOSE $A^+ = 0$ GAUGE (light-cone gauge, often used in parton model)

$F^{+\alpha} = \partial^+ A^\alpha - \partial^\alpha A^+ + [A^+, A^\alpha] \Rightarrow \partial^+ A^\alpha \Rightarrow \partial^+ A^\alpha$

$P(\lambda\eta, 0) \Rightarrow 1$

\[
\Delta G(Q^2) = \int_{-\infty}^{\infty} d\lambda \epsilon(\lambda) \frac{\partial}{\partial\lambda} \langle \ldots \rangle = 2 \langle \ldots \rangle \bigg|_{\lambda=0}
\]

(Surface terms at $\lambda = \pm \infty$ can be dropped.)

\[
\Delta G(Q^2) = \frac{i}{\sqrt{2} M} \langle PS_z | 2 \text{Tr} (A^F F^{+2} - A^2 F^{+1}) | PS_z \rangle |_{Q^2}.
\]

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THE DILEMMA OF ORBITAL ANGULAR MOMENTUM

R.L. JAFFE

\[ \Delta \Sigma \approx 0.2 - 0.3 \]

IN SOME SENSE

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_Q + L_G \]

AFTER CONSIDERABLE STRUGGLE WE KNOW

* WHAT \( \Delta G \) IS
* HOW TO MEASURE IT

\( L_Q \neq L_G \) ARE POORLY UNDERSTOOD

* DEFINITION?
* GAUGE INVARIANCE, UNIQUENESS?
* DISTRIBUTION IN \( x_Bj \)?
* MEASURABLE?

ALL PROBLEMATIC, SUBJECT OF THIS TALK

HOPE: STIMULATE WORK.
THANKS TO
SERGEI BASHINSKY
XIANGDONG JI
ANEESH MANOHAR

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OUTLINE

I. Standard Parton Distributions

II. Momentum & Angular Momentum Tensor Densities in QCD

III. Operator Expressions for Integrated Quark & Gluon Contributions to the Ang. Momentum

IV. Local Distributions in $x_{bj}$

V. The Present Dilemma.
Essentially all workers agree that

\[ \overline{Q} \gamma_5 q \]

is quark spin operator. [And is measurable in polarized D.I.S. — modulo renormalization scheme controversies.]

- Gluon spin: appears gauge dependent. However, it is related to correct operator description of gluon spin (see below).

- Quark & gluon orbital angular momentum look physical, but gauge dependent.

- Note: \( M_{\mu\nu} \), in total, is gauge invariant \( RJJAM \).

- "Angular momentum sum rule"?

In some sense, \( M_{\mu\nu} \) is a sum of 4 terms. How does it imply

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_Q + L_G \]
**Gauge Invariant Decomposition of** $M_{\mu\nu}$

**REPLACE** \( j^\nu \rightarrow j^\nu - igA^\nu \) **IN QUARK**

**ORBITAL TERM** USE EQ'S OF MOTION TO MASSAGE INTO TERM THAT SENDS \( j^\nu \rightarrow j^\nu - igA^\nu \) **IN GWON**

**ORBITAL TERM**

\[
M_{J1}^{\mu\nu} = \frac{i}{2} \bar{\psi} \gamma^\mu \left( x^\nu \frac{\partial}{\partial x^\nu} - x^\nu \right) \psi + \frac{i}{2} \epsilon^{\mu\nu\alpha\beta} \bar{\psi} x^\alpha x^\beta \psi \\
- 2 \text{Tr} \left\{ F^{\mu\nu} \left( x^\nu \frac{\partial}{\partial x^\nu} - x^\nu \right) A_\alpha \right\} \\
+ 2 \text{Tr} \left\{ F^{\mu\nu} A^\nu - F^{\mu\nu} A^\nu \right\} - \frac{1}{2} \text{Tr} F^2 (x^\nu q^\mu - x^\nu q^\mu)
\]

**Observations (Ji)**

- Quark orbital term is now separately GI
- Gluon: only total spin + orbit is GI
- Quark orbital term is measurable (in principle) thru DCS.

**Questions**

- What is physical significance of \( j \rightarrow j - igA \)?
  Relation to rotations etc.?
- What is meaning of gluon spin that experimenters measure?
- What can be said about \( x E_j \) distributions?
III. INTEGRATE PIECES OF THE ANGULAR MOMENTUM

Deep inelastic processes probe light-cone momentum distributions. The lowest moments of these may be local symmetry generators that appear in sum rules. Before studying light cone momentum distributions for spin and orbital angular momentum, we look here at:

WHAT CAN BE DETERMINED @ EXPECTATION VALUES OF PIECES OF SYMMETRY GENERATORS?

\( \Phi \gamma_5 \bar{\Psi} \equiv \Sigma_{\mu} \)

"FORM FACTOR" FOR \( \Sigma_{\mu} \): Imagine an external probe that couples to \( \Sigma_{\mu} \)

\[ \Sigma_{\mu} (p, q, s) \equiv \int dx \langle p's | \Sigma_{\mu} (x) | ps \rangle e^{iq \cdot x} \]

General covariant parameterization

\[ \langle p's | \Sigma_{\mu} (s) | ps \rangle = S_{\mu} A(q^2) + q_{\mu} s \cdot q B(q^2) + ... \]

\[ \Sigma_{\mu} (p, q, s) = (2\pi)^4 \delta^4 (p' + q - p) S_{\mu} A(q^2) + ... \]
\[ \mathcal{A}_\mu(p,0,s) = \int dx^0 \int d^3x \left\langle \psi_s \Sigma_{\mu}(x) \right\rangle \psi_s \]

\[ = (2\pi)^4 \delta(0) \left\langle \psi_s \int d^3x \Sigma_{\mu}(x) \right\rangle \psi_s \]

Use \[ \left\langle \psi_s \psi_s \right\rangle = (2\pi)^3 2E^3 \delta^3(p-p') \] to obtain

\[ A(0) S = \frac{\left\langle \psi_s \int d^3x \Sigma(x) \right\rangle \psi_s}{\left\langle \psi_s \psi_s \right\rangle} \]

measured by

\[ \text{forward scattering} \]

**Comments**

* of course no current couples directly to \( \Sigma_{\mu} \), but flavor generalizations \( q \gamma_{\mu} q \frac{1}{2} q \gamma \) are measured this way (in \( \beta \)-decay)

* why did i belabor this obvious point? because it's not so obvious for operators with explicit \( x^\mu \).

* *quark orbital angular momentum*  

first do pedagogical example of magnetic moment.
MAGNETIC MOMENT

\[ \vec{\mu}(x) = \frac{1}{2} \times \vec{j}(x) \]

More generally

\[ \frac{1}{2} \left[ X_{\mu}^{\dagger} \beta(x) - X_{\mu} \beta(x) \right] \]

\[ \mathcal{F}_{\mu}^{\dagger}(p, q, s) = \frac{1}{2} \int d^4x \ e^{i p' \cdot x} \left( \langle p' s' | (X_{\mu}^{\dagger} \beta(x) - X_{\mu} \beta(x)) | p s \rangle \right) \]

General covariant parameterization

\[ P^\alpha = \frac{1}{2} (p + p')^\alpha \]

\[ \langle p' s' | j_{\mu}(0) | p s \rangle = 2 P_{\mu} F_1(q^2) + 2 i e_{\mu \rho \sigma \chi} P^\rho q^\sigma s^\chi F_2(q^2) \]

Use

\[ x_{\mu} = -i \partial q_{\mu} e^{i q \cdot x} \]

\[ \mathcal{F}_{\mu}(p, q, s) = -i \frac{\partial}{\partial q_{\mu}} \left\{ (2\pi)^{4-4} (p' + q - p) \langle p' s' | j_{\mu}(0) | p s \rangle \right\} - (\mu \leftrightarrow \rho) \]

\[ = -i \frac{\partial}{\partial q_{\mu}} \left( (2\pi)^{4-4} (p' + q - p) \left[ 2 P_{\mu} F_1 + 2 i e_{\mu \rho \sigma \chi} P^\rho q^\sigma s^\chi \right] \right) \]

\[ - (\mu \leftrightarrow \rho) \]

Take \( q^\mu \to 0 \)

And compare:

\[ \Xi_{\mu}(p, q, s) = -i (2\pi)^{4} \left\{ 2 \partial_{\rho} \delta^{4}(0) P_{\mu} - 2 \partial_{\rho} s^{4}(0) P_{\mu} \right\} \]

\[ + 4 (2\pi)^{4} \delta^{4}(0) e_{\chi \alpha \beta \gamma} P_{\alpha} s_{\beta} F_{2}(0) \]

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IN THE END: \[ F_2(0) = \frac{\langle PS \int d^3x \frac{1}{2} x \cdot \bar{x}(x) 1P\rangle}{\langle PS 1P \rangle} \]

MEASURED BY OFF-FRONTWARD SCATTERING

INTERPRETABLE BY THEORY

MORAL: WHEN \( x^\lambda \) APPEARS IN AN OPERATOR, IT IS NECESSARY TO MEASURE OFF-FRONTWARD MATRIX ELEMENTS

\( \neq \) LEAVE THE USUAL WORLD OF "HARD QCD"

HENCE DVCS.

JI'S PROPOSAL FOR MEASURING

\[ x^\lambda T^\mu_\nu - x^\nu T^\mu_\lambda \leftrightarrow \frac{1}{2} \Delta \Sigma + L_Q \]

\[ x^\lambda T^\mu_\nu - x^\nu T^\mu_\lambda \leftrightarrow \Delta G + L_G \]

\( Q^2 \to \infty \) \( \Rightarrow \) \( P' - P = q \)

\( q^2 = t \approx 0 \) OFF-FRONTWARD TO PICK UP MOMENT OF \( T^\mu_\nu \)

USE PHOTON KINEMATICS TO INTEGRATE SECOND MOMENT OF \( \bar{G} x^\lambda \Psi \) ON LIGHT CONE
TAKE STOCK

- $\Delta \Sigma$ Measure thru sum rules for polarized DIS

- $L_\xi$ Measure $\frac{1}{2} \Delta \Sigma + L_\xi$ in DVCS where $L_\xi$ here is Ji’s covariant version

- $L_G + \Delta G$ In principle measure this sum via DVCS.

PROBLEMS

- No place for $\Delta G$. No local operator (GI)

- No physical interpretation for covariant $L_\xi$ vis a vis naive $L_\xi$.

- No generalization of $L_\xi$ (or $L_G$) to $L_\xi(x)$. Is a Bjorken-$x$ distribution of orbital angular momentum meaningful?

- No general analysis of what quantities may (or may not) appear as parton distributions.

- No simple connection to partons in $A^1 = 0$ gauge (as exists for $T^\mu$—see pg 10).

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IV. **Local Distributions in $x_B$**

This section is based entirely on work by **Sergei Bashinsky**

SB & RJ  REP-PH 9804377

- Anticipate significance of space direction $x^3 \rightarrow x^+$, which defines parton model.
- Define fields corresponding to notion of quark or gluon parton with "momentum" $q^+ = xp^+$.
- Study transformations of those fields corresponding to space-time transforms.
- Parton distributions are neither charges corresponding to those transformations.

- Keep close contact with $A^+ = 0_3$ but maintain gauge inv.
- Associate parton distributions with field transformations.
- Don’t worry too much about experimental measurement.

**Result**

- Partonic definitions of all four terms in angular momentum S.R. as functions of $x_B$
- Understanding of $\Delta g$ as local operator.
1. **WHAT IS A PARTON?** (ILLUSTRATE FOR QUARK)

A QUANTUM WITH DEFINITE $P^+$ IN $A^+=0$ GAUGE

$$T^q_-(a^-) \psi(k^+, x) = e^{-ik^+a^-} \psi(k^+, x)$$

$T^q_-$ — TRANSLATION ALONG $x^-$ BY $a^-$

$\psi$ — QUARK FIELD IN L.C. GAUGE $\chi = (x^+, x^1, x^2)$

SO QUARK PARTON IS DEFINED AS EIGENSTATE OF TRANSLATION ALONG $x^-$ IN $A^+=0$ GAUGE

SUPERPOSE PARTONS TO MAKE QUARK FIELD IN $A^+=0$ GAUGE

$$\psi(x) = \int \frac{dk^+}{2\pi} e^{-ik^+x^-} \psi(k^+, x)$$

$$T^q_-(a^-) \psi(x) = \psi(x+a^-)$$

NAIVE TRANSLATION IN $A^+=0$ GAUGE

GENERALIZE TO ARBITRARY GAUGE

$$\Psi(x) = e^{i\alpha(x)} \psi(x)$$
In arbitrary gauge, partons are defined as eigenstates of the gauge covariant translation

\[ T^q_\alpha(a^-) \psi(x) = U(x, x+a^-) \psi(x+a^-) \]

\[ U(x, x+a^-) = P \exp \left( ig \int_{x}^{x+a^-} d\xi A^+ \right) \]

Generalize to gluons (\( D_\lambda(x) \) is matrix in fundamental representation)

\[ T^g_\alpha(a^-) \Rightarrow U(x, x+a^-)D_\lambda(x+a^-)U(x+a^-) \]

In infinitesimal:

\[ \delta^g\alpha \Rightarrow a^- F^- \lambda \]

\[ T^g_\alpha(a^-) A^\lambda_\alpha(k^+, \xi) = e^{-ik^+a^-} A^\lambda_\alpha(k^+, \xi) \]

In \( A^+ = 0 \) gauge

\[ \delta^g\alpha \Rightarrow a^- F^- \lambda \]

\[ T^g_\alpha(a^-) \Rightarrow U(x, x+a^-)D_\lambda(x+a^-)U(x+a^-) \]

\[ \Rightarrow e^{i\alpha(x)} \left( A_\alpha^\lambda(x) + \frac{i}{g} \partial_\lambda \right) e^{-i\alpha(x)} \]

Properties of covariant translation

Independent

\[ \left[ T^q_\alpha(a^-), T^g_\alpha(b^-) \right] = 0 \]

Leave gauge unchanged

\[ T^{q, g}_\alpha(a^-) : \alpha(x) \rightarrow \alpha(x) \]
SEPARATE $T^{-9}$ AND $T^{-g}$ ARE NOT SYMMETRIES

But

$$T^{-g}(a^{-}) = T^{-9}(a^{-}) + T^{-g}(a^{-})$$

IS SYMMETRY OF

GAUGE INVARIANT LAGRANGIAN

WHOOPS! RESIDUAL GAUGE SYMMETRY

Speaking of $\gamma$ and $A^\lambda$ as physical was premature.

If $\alpha = \alpha(x)$ only then

$$\psi(k^+,x) \rightarrow e^{i\alpha(x)} \psi(k^+,x)$$

$$A^\lambda(k^+,x) \rightarrow e^{i\alpha(x)} (A^\lambda(k^+,x) + \frac{2\pi i \delta(k^+)}{g} \partial_\lambda) e^{-i\alpha(x)}$$

OBSERVABLES MUST BE IN Variant UNDER RESIDUAL G.I.

BREAK OUT "RESIDUAL GAUGE FIELD"

$$A^\lambda(k^+,x) \equiv 2\pi \delta(k^+) a^\lambda(x) + G^\lambda(k^+,x)$$

with

$$G^\lambda(k^+,x) \bigg|_{k^+=0} = 0 \quad \therefore \partial^\lambda \rightarrow e^{i\alpha(x)} \partial^\lambda e^{-i\alpha(x)}$$

$$a^\lambda \rightarrow e^{i\alpha(x)} (a^\lambda + \frac{i}{g} \partial^\lambda) e^{-i\alpha(x)}$$
2. WHAT ARE PARTON ATTRIBUTES?

NOETHER CHARGES OF TRANSFORMATIONS THAT COMMUTE
WITH $T_9 \neq T^3$

- DEFINE A FIELD TRANSFORMATION $T$ (NOT
  NECESSARILY A SYMMETRY OF QCD)
- ONLY ACCEPT $[T, T_9] = [T, T^3] = 0$
  SO $T$ IS DIAGONAL IN PARTON BASIS.
- THEN $T$ CAN BE DEFINED INDEPENDENTLY
  IN EACH $k^+$ SUBSPACE. ASSOCIATED
  NOETHER CHARGE CARRIES $k^+$ LABEL.

IN $A^+ = 0$ GAUGE LET $\delta^{(q^+)}_{T^9}$ BE TRANSFORMATION
ACTING ON $k^+ = q^+$ SUBSPACE:

$$\delta^{(q^+)}_{T^9} \varphi(k^+) = i e T^9 \varphi(q^+) \delta(q^+ - k^+)$$

$\vec{x}$ DEPENDENCE SUPPRESSED

SUPERPOSE $k^+$ SUBSPACES TO FIND FIELD TRANSFORMATION
IN $A^+ = 0$ GAUGE

$$\delta^{(q^+)}_{T^9} \varphi(x) = i e \int \frac{d\xi^-}{2\pi} e^{i q^+ \xi^-} T^9 \varphi(x + \xi^-)$$
AND FOR SU(3)

\[ \delta^{(q^+)} \rho = 0. \]

\[ \mathcal{S}_{q^+} \rho(x) = \int \frac{d\xi^-}{2\pi} e^{iq^+\xi^-} \left( \mathcal{A}^\rho(x+\xi^-) - \mathcal{A}^\rho(x) \right). \]

THESE TRANSFORMATIONS ARE GENERATED BY NOETHER CHARGES, WITH ASSOCIATED CURRENTS

\[ J^\mu_{q^+}(x,x_{\rho}) = \frac{p^+}{2\pi} \int d\xi^- e^{ix_{\rho}P^+\xi^-} \bar{\mathcal{A}}^\rho(x) x^\mu T^q \mathcal{A}^\rho(x+\xi^-). \]

1. WE HAVE SEEN \( q^+ \equiv x^\rho p^+ \)

2. \( J^\mu_{T^q} = -i \frac{\partial}{\partial (x^\rho)} T^q \Phi \) A LA NOETHER

3. \( J^+_{T^q} \) IS GENERATOR OF \( T^q \) TRANSFORMATION BY COMMUTATION AT EQUAL \( x^+ \)

SO \( \int d^2x_ \perp dx^- J^+_{T^q}(x,x_{\rho}) \) IS OBSERVABLE

CANONICALLY ASSOCIATED WITH \( T^q \)

PARTON DISTRIBUTION OF \( T^q \)

\[ f_{T^q}(x_{\rho}) = \frac{\langle P|Q^{(q^+)}_{T^q}(x_{\rho})|P\rangle}{\langle P|P\rangle} \]

where \( Q^{(q^+)}_{T^q} = \int dx^- d^2x_ \perp J^+_{T^q}(x,x_{\rho}) \)
TIME FOR EXAMPLES  IDEAL PARTON DISTRIBUTIONS

1. $T^g = 1$  \[ S^{(q^+)}(q^+) S^{(q^-)}(q^- - k^-) \]
   \[ \Rightarrow \text{NUMBER OPERATOR} \]
   \[ \mathcal{f}_1(x_{B_1}) \equiv q_1(x_{B_1}) = \frac{1}{i} \int \frac{d^2 \xi^-}{2\pi} e \cdot \frac{i x_{B_1} \xi^- P^+}{2} \left< P \right| \Psi_{q^+}^{+}(0) | \Psi_{q^+}^{-(\xi^-)} | P \left> \right. \]
   \[ \text{STANDARD UNPOLARIZED QUARK DISTRIBUTION} \]

2. $T^q = x^5$  \[ \text{QUARK SPIN ROTATION} \]
   \[ \mathcal{f}_{x^5}(x_{B_1}) \equiv \Delta q_1(x_{B_1}) = \frac{1}{i} \int \frac{d^2 \xi^-}{2\pi} e \cdot \frac{i x_{B_1} \xi^- P^+}{2} \left< P \right| \Psi_{q^+}^{+}(0) | x^5 \Psi_{q^+}^{-(\xi^-)} | P \left> \right. \]
   \[ \text{STANDARD QUARK HELICITY DISTRIBUTION} \]

   BOTH ARE WRITTEN IN $A^+ = 0$ GAUGE, BUT INTRODUCTION OF WILSON LINES PUTS THEM INTO ARBITRARY GAUGE.

3. $T^g = \delta_{AB}^g$  \[ \text{GLUON NUMBER OPERATOR} \]
   \[ \mathcal{f}_{\gamma^g}(x_{B_1}) = \frac{i}{4\pi} \int d^2 \xi e \cdot \frac{i x_{B_1} P^+ \xi^-}{2} \left< P \right| F_{+A}^{x^g}(0) | \left. \left. A_{+A}^{-(\xi^-)} \right| P \right> \]
   \[ \text{WRITTEN IN AN (ARBITRARY) } A^+ = 0 \text{ GAUGE.} \]

NOTE: The reason it doesn't look GT is that it is written in a particular gauge.

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RESTORE MANIFEST GI.

\[ e^{i x_{B_j} P^+ \xi^-} = \frac{1}{i x_{B_j} P^+} \frac{\partial}{\partial \xi^-} e^{i P^+ \xi^- x_{B_j}} \]

- INTEGRATE BY PARTS

- USE \( \Theta^- A_{\lambda} = - F_{\lambda}^+ \)

\[ G(x_{B_j}) = \frac{i}{4\pi i x_{B_j} P^+} \int d\xi^- e^{i x_{B_j} P^+ \xi^-} \langle P| F^{+\lambda}(0) F_{\lambda}^+(\xi^-)|P \rangle \]

+ surface term that can be dropped

**Finally restore MANIFEST GI by Wilson line.**

\[ G(x_{B_j}) = \frac{i}{4\pi i P^+ x_{B_j}} \int d\xi^- e^{i x_{B_j} P^+ \xi^-} \langle P| F^{+\lambda}(0) U(0, \xi^-) F_{\lambda}^+(\xi^-)|P \rangle \]
ROTATIONS ABOUT $\hat{e}_3$ COMMUTE WITH $T_-$

$$\delta_j \psi = -\epsilon \left[ -\frac{i}{2} \sigma^3 + (x'^{\nu} \partial_{\nu} - x'^{\rho} \partial_{\rho}) \right] \psi$$

$$\delta_j G_{\lambda} = -\epsilon \left[ - (\delta^i_{\lambda} \delta^p_{\sigma} - \delta^p_{\lambda} \delta^i_{\sigma}) + (x'^{\nu} \partial_{\nu} - x'^{\rho} \partial_{\rho}) \right] G_{\rho}$$

**Consider each transformation independently**

**Terms containing $\partial^j$ are not invariant under residual gauge group. Restore $g_i$ to each term (sum is G.I. regardless) by**

$$\delta_j \rightarrow \delta_j - igA_j = \partial_j \quad \text{(Russian $\partial$)}$$

**$\Delta \Sigma$**

$$\delta_\Sigma \psi = i \epsilon \sigma^{12} \psi \quad \quad T_\Sigma = \gamma^3 \gamma^5$$

**$L_\theta$**

$$\delta_{L_\theta} \psi = -\epsilon (x'^{\nu} \partial_{\nu} - x'^{\rho} \partial_{\rho}) \psi \quad \quad T_{L_\theta} = (x'^{\nu} \partial_{\nu} - x'^{\rho} \partial_{\rho})$$

**$\Delta G$**

$$\delta_{\Delta G} G_{\lambda} = \epsilon (\delta^i_{\lambda} \delta^p_{\sigma} - \delta^p_{\lambda} \delta^i_{\sigma}) G_{\rho} \quad \left( T_{\Delta G} \right)_{\lambda}^\rho = -i \epsilon^{\text{+ }-} \lambda \rho$$

$$\delta_{L_G} G_{\lambda} = -\epsilon (x'^{\nu} \partial_{\nu} - x'^{\rho} \partial_{\rho}) G_{\lambda} \quad \quad \left( T_{L_{G}} \right)_{\lambda}^\rho = (x'^{\nu} \partial_{\nu} - x'^{\rho} \partial_{\rho}) \delta^i_{\lambda}$$

**Note:** $[T_i, T_j] = 0$ for all $T_i$'s. However, had we used "naive" covariantization $\delta_j \rightarrow \delta_j - igA_j$ then $[T_i, T_j] \neq 0$ and parton interpretation would be ruined.
\[ \Delta G(x_{B_1}) = \frac{1}{4\pi} \int d^2 \xi^- \ e^{i x_{B_1} P^+ \xi^-} \langle P | F^{+\lambda}(0) \varepsilon^+ \lambda^\beta A_\beta(\xi^-) | P \rangle \]

- Restore manifest GI

\[ \Delta G(x_{B_1}) = \frac{1}{4\pi x_{B_1} P^+} \int d^2 \xi^- \ e^{i x_{B_1} P^+ \xi^-} \langle P | F^{+\lambda}(0) U(0, \xi^-) F^+_{\lambda}(\xi^-) | P \rangle \]

- This result agrees with standard parton model expression


- Lowest moment, \( \Delta G \), is non-local in arbitrary gauge

\[ \Delta G = \frac{1}{2P^+} \int d^2 \xi^- \langle P | F^{+\lambda}(0) U(0, \xi^-) \tilde{F}^+_{\lambda}(\xi^-) | P \rangle \varepsilon(\xi^-) \]

But in \( A^+ = 0 \) gauges

\[ \Delta G = \frac{1}{2P^+} \langle P | 2(A^1 F^{+2} - A^2 F^{+1}) | P \rangle \]

- LOCAL
- IDENTICAL TO GENERATOR \( \int d^3 x M^{+12} \) FROM SYMMETRY ANALYSIS.

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CONCLUSION ON QUARK SPIN

- 3. A GI PARTON DISTRIBUTION IN $x_{B_i}$, $A_G(x)$
- THE LOWEST MOMENT OF $A_G$ MEASURES QUARK SPIN IN NUCLEON.
- OPERATOR FOR $A_G$ IS (IN GENERAL GAUGE) NON-LOCAL.
- IN $A^+ = 0$ GAUGES IT IS LOCAL & CORRESPONDS TO TRADITIONAL QUARK SPIN GENERATOR IN $\gamma^\mu$.
- SEE RLJ - PLB 356 (1995) 359 FOR EARLIER.
  DEMONSTRATION FROM LESS FORMAL BEGINNINGS.

2) APPLY TO QUARK & GLUON ORBITAL ANGULAR MOMENTUM.

$$L_q(x_{B_j}) = \frac{1}{2\pi i} \int d^2 x_1 e^{i\mathbf{x}_B \mathbf{P} \cdot \mathbf{x}_1} \langle P | \int d^2 x_1$$

$$\otimes \phi_+(x_1) \left(x_1^i D_i - x_1^2 \sigma_i \right) U(x_1, x_1 + \frac{\mathbf{z}}{2}) \phi_+(x_1 + \frac{\mathbf{z}}{2}) | P \rangle$$

$$\otimes x_{\zeta} = \partial_i - i g A_i \quad \text{(IN $A^+ = 0$ GAUGE)}$$

$$A_i = \frac{\int dx^- A_i(x, x^-)}{\int dx^-} \quad \text{SO $A_i = 0$ IN ANY GAUGE WHERE $A_i \to 0$ AS $x^\mu \to \infty$}$$

IN SUCH WELL-BEHAVED GAUGES $\otimes x_{\zeta} \to \partial_i$.
**NOTE** \[ \mathcal{U}(x_-, x_+ + \varepsilon^-) \left| \Theta_2 \right| \] is 0, so there is no ambiguity in placement of \( \mathcal{U} \).

**CONTRAST FULL COVARIANT DERIVATIVE:**

\[ \psi^+(x) \iff D_j \psi(x) \]

Actually involves 3 light-cone points

\[ D_\xi = \partial_\xi - ig A_\xi \left( \eta^- \right) \]

\[ \begin{cases} x^- = 0 \\ x^- = \eta^- \\ x^- = \frac{\eta^-}{2} \end{cases} \]

\( D_j \) has parton content, does not commute w/ T- interpretation as quark orbital angular momentum is lost: \( D_j \) (as part of \( x^i D_2 - x^2 D_i \)) generates no recognizable transformation of \( \psi \).

**LOWEST MOMENT**

\[ L_Q = \int dx_+ L_Q(x) = \frac{1}{\sqrt{2p^+}} \int d^2x_\perp \langle P | \int d^2x_\perp \psi^+_\perp(x_\perp) (x^i \Theta_1 - x_1^2 \Theta_1) \psi_\perp(x_\perp) | P \rangle \]

Again connects to piece of \( M^{+12} \) in \( A^+ = 0 \) gauge.
\[ G(x_{L}) = \frac{1}{4\pi x_{L} P+\int d^{2}x_{\perp}} \int d^{2}x_{\perp} e^{-i x_{L} P^{+} \frac{y}{2} - i x_{\perp} P^{+} \frac{\alpha}{2}} \langle P | \int d^{2}x_{\perp} \]

* \( F^{+\lambda}(x_{L}) (x_{L}^{i} \partial_{2} - x_{L}^{2} \partial_{1}) \bar{J}(x_{L}, x_{L}+\xi) F_{\lambda}^{+}(x_{L}+\xi) | P \rangle \)

**LOWEST MOMENT**

\[ L_{G} = \frac{i}{2 P+\int d^{2}x_{\perp}} \langle P | \int d^{2}x_{\perp} F^{+\lambda}(x_{L}) (x_{L}^{i} \partial_{2} - x_{L}^{2} \partial_{1}) A_{\lambda}(x_{L}) | P \rangle \]

**SUMMARY**

- **SIMPLE & ELEGANT WAY TO DEFINE A PARTON DISTRIBUTION FUNCTION WITH A FIELD TRANSFORMATION**
- **UNAMBIGUOUSLY DEFINES QUARK & GLUON SPIN & ORBITAL ANGULAR MOMENTUM IN GAUGE INVARIANT WAY THATobeys**
  \[ \frac{1}{2} = \frac{1}{2} \Delta Z + \Delta G + L_{Q} + L_{G} \]
- **GAVE SAME RELATION AMONG TENSOR DENSITY  \[ ^{\mu\nu} \]
  - \( A^{+} = 0 \) GAUGES & \( \times \) PARTON DISTRIBUTIONS, AS WE
  FOUND AMONG \( ^{T}_{\mu\nu} \), \( A^{+} = 0 \) & \( \times \) SPIN AVERAGE PARTON DISTRIBUTIONS IN CLASSICAL ENERGY-MOMENTUM ANALYSIS.
I. THE PRESENT DILEMMA

EASY TO STATE BUT HARD TO RESOLVE

REGARDING ORBITAL ANGULAR MOMENTUM

(a) WHAT CAN BE INTERPRETED CANNOT BE MEASURED
(b) WHAT CAN BE MEASURED CANNOT BE INTERPRETED

(YET)

DILEMMA BEST SET OUT BY HOOBHOUR, WHO STUDY HIGHER GENERALIZATIONS OF $X^\lambda T^\mu - X^\nu T^\mu$

NAMELY:

$M_{\alpha \beta \mu_1 \ldots \mu_n} = X^{\alpha \beta} T_{\mu_1 \ldots \mu_n} - X^{\beta} T_{\alpha \mu_1 \ldots \mu_n}$

WHERE

$T_{\alpha \mu_1 \ldots \mu_n} = S \left( \Phi \gamma_\alpha \gamma_\beta D^{\mu_1} \gamma_\delta \gamma_\epsilon D^{\mu_2} \ldots \gamma_\chi D^{\mu_n} \Phi \right)$

$T_{\alpha \beta \mu_1 \ldots \mu_n}$ IS THE STANDARD LEADING TWIST OPERATOR THAT ARISES IN $J_\alpha(x) J_\mu(0)$ AND CAN BE MEASURED OFF-FORWARD IN DVCS.
So the invariants associated with $M^{\mu_1\mu_2...\mu_n}$ can be measured but they don't correspond to angular momentum.

$$\int d^3x\ N_i^{12++...} = S^{++...} + L_q^{++...} + \Delta L_q^{++...}$$

$$S^{++...} = \frac{2}{n+1} \int d^3x \bar{\psi} \chi^{+\frac{3^3}{2}} i\Sigma...iD^+\psi$$

$$L_q^{++...} = \frac{1}{n} \int d^3x \bar{\psi} \chi^{+} (x^iD^2-x^2iD' \Sigma...iD^+\psi$$

$$\Delta L_q^{++...} = \frac{1}{n(n+1)} \int d^3x \left\{ \bar{\psi} \chi^{+} (x^iD^2-x^2iD') igF_{\mu\nu}^+ \psi \right\}$$

Note: $L_q^{++...}$ is not the orbital angular momentum.

$\Delta L_q^{++...}$ is not interpretable!

So—At least at this time—the measurable quantity has no direct interpretation.
ON THE OTHER HAND

BASHINSKY'S METHOD YIELDS UNAMBIGUOUS PARTON DISTRIBUTIONS ASSOCIATED WITH ANY PARTONIC \( (T', T_q, (a^+)) = 0 \) TRANSFORMATION OF FIELDS.

CERTAINLY YIELDS DEFINITIONS OF QUARK \( q \) GLUON DISTRIBUTIONS \( l_q \neq l_G \) IN \( x_{B_j} \)

IN MOST GAUGES THEY COINCIDE W7 THE NAME EXPRESSIONS USED BY HAGLER \& SCHAFFER AND BY HARINDRANATH \& KUNDU

HOWEVER, THESE DISTRIBUTIONS HAVE NOT (YET) BEEN IDENTIFIED W7 EXPERIMENT.

FINALLY NOTE SOME EXPERIMENTS CAN BE DONE ON COMPUTERS! WE HAVE GOOD (i.e. "IDEAL") PARTON DISTRIBUTIONS FOR \( \Delta \Sigma, \Delta G, \frac{l_q \neq l_G}{l_q} \) SUCH THAT

\[
\frac{1}{Z} \int dx_j \left\{ \frac{1}{2} \Delta \Sigma (x_{B_j}) + \Delta G (x_{B_j}) + l_q (x_{B_j}) + l_G (x_{B_j}) \right\} = \frac{1}{Z}
\]
Quark and Gluon Structure of the Nucleon

Lectures Presented at the 9th RIKEN Winter School,
Shimoda, December 8-12, 1998

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Abstract

Recent results on unpolarised and polarised deep inelastic lepton nucleon scattering (DIS) and other hard QCD processes are presented. The first part of the lectures covers following topics:
- the definition of structure functions, early experimental determination of quark distributions;
- the flavour asymmetry of the light quark sea and its recent determination from the Drell-Yan process and from pion production in semi-inclusive DIS, both on hydrogen and deuterium targets;
- the Q2 dependence of parton distributions and its description by perturbative QCD;
- recent results from HERA, especially the behaviour of the proton structure function F2 at very low values of x and moderate Q2 and at large x and very high Q2;
- the determination of the gluon distribution from the Q2 dependence of structure functions, from charm production and from direct photons at high transverse momenta in nucleon nucleon scattering.
The second part of the lectures summarizes the status of polarised DIS. It covers:
- the different experiments and their technological achievements;
- recent results on the asymmetry $A_1$ and the structure function $g_1$ for proton, deuteron and neutron;
- sum rules and their implications for the decomposition of the nucleon spin into its contributions from quarks, gluons and orbital angular momenta;
- determination of the individual polarised parton distributions from semi-inclusive DIS;
- measurements of the second structure function $g_2$ and the first attempts to derive the twist-3 matrix element $d_2$.

The lectures close with an outlook on future measurements at HERMES, SLAC, COMPASS, RHIC and HERA with polarised protons.
Structure of the Nucleon

K. Rith

Selected results from unpolarized and polarized deep inelastic $eN$ scattering
Outline

1. Deep inelastic lepton nucleon scattering (DIS)
   Structure functions
   parton distributions

2. DIS and perturbative QCD
   scale breaking, $Q_s^2 (Q^2)$

3. DIS at very high $Q^2$ — HERA

4. Polaris ed DIS
   Spin of the nucleon — HERMES at
\[ M_{if} = \frac{e}{q^+} L_{\mu\nu} W^{\mu\nu} \]

unpolarized case

\[ L_{\mu\nu} = \text{leptonic tensor} = L^{(S)}_{\mu\nu} + L^{(RS)}_{\mu\nu} \]

\[ = 2 (k_\mu k_\nu + k_\nu k_\mu + (m_e^2 - q_{\mu\nu} k k') \]

\[ - i \varepsilon_{\mu\nu\lambda\beta} q^\lambda s^\beta \]

\[ W^{\mu\nu} = \text{hadronic tensor} = W^{\mu\nu}_{(S)} + W^{\mu\nu}_{(RS)} \]

\[ = W_1 (\gamma, Q^2) (-q^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2}) + W_2 (\gamma, Q^2) \cdot \frac{1}{M_a} (p^\mu - p q \frac{q^\nu}{Q^2} q^\nu) (p^\nu - p q \frac{q^\mu}{Q^2} q^\mu) \]

\[ + G_1 (\gamma, Q^2) \cdot M \cdot i \varepsilon^{\mu\nu\lambda\sigma} q_{\lambda} s^h_{\sigma} + G_2 (\gamma, Q^2) \cdot \frac{1}{M} \cdot i \varepsilon^{\mu\nu\lambda\sigma} q_{\lambda} (p \cdot q \cdot s^h_{\sigma} - s^h_{\sigma} \cdot p q) \]

\[ s^h = \text{Polarization vector of } \phi N \]

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$W_1, W_2, G_1, G_2 : \text{Structure functions}$

\[
\begin{align*}
M W_1 (\gamma, Q^2) &= F_1 (x, Q^2) \quad \text{unpolarized} \\
\gamma W_2 (\gamma, Q^2) &= F_2 (x, Q^2) \\
M^2 \gamma G_1 (\gamma, Q^2) &= q_1 (x, Q^2) \quad \text{polarized \rightarrow later} \\
M \gamma^2 G_2 (\gamma, Q^2) &= q_2 (x, Q^2)
\end{align*}
\]

\[
\frac{d^3 \sigma}{d \Omega dE} = \frac{4 \pi^2 E^2}{Q^4} \cos^2 \theta \left[ W_2 (\gamma, Q^2) + 2 W_4 (\gamma, Q^2) t_0^2 \theta \right]
\]

\text{Jacobian:}

\[
\frac{d^3 \sigma}{d \Omega dE} = \frac{EE'}{\pi} \frac{x}{Y} \frac{d^3 \sigma}{dQ^2 dx} = \frac{EE'}{\pi} \frac{d^3 \sigma}{dQ^2 dy} = \frac{1}{2M \gamma} \frac{E'}{\pi} \frac{d^3 \sigma}{dx dy} \quad \text{with} \quad Y = \frac{\gamma}{E}
\]

\[
\frac{d^3 \sigma}{dQ^2 dx} = \frac{4 \pi^2 L^2}{Q^4} \left[ (1 - Y - \frac{M x y}{2 E}) \frac{F_2 (x, Q^2)}{x} + y^2 F_1 (x, Q^2) \right]
\]

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### Examples

<table>
<thead>
<tr>
<th>Elastic, Nucleus change distr. $g(\tau)$ Spin $= 0$</th>
<th>$W_1(\gamma, Q^2)$</th>
<th>$W_2(\gamma, Q^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$F_\ell^2(Q^2) \sigma(\frac{Q^2}{2M^2} - \gamma)$</td>
<td></td>
</tr>
</tbody>
</table>

| Elastic, Nucleon $g(\tau)$, Spin $= \frac{1}{2}$ | $\tau \frac{Q^2}{4M^2} \sigma(\frac{Q^2}{2M^2} - \gamma)$ | $\frac{G_E(Q^2) + \tau G_M(Q^2)}{1 + \tau} \sigma(\frac{Q^2}{2M^2} - \gamma)$ |

| Elastic, Muon pointlike, Spin $= \frac{1}{2}$ | $\frac{Q^2}{4M^2} \mu_1 \sigma(\frac{Q^2}{2M^2} - \gamma)$ | $1 \sigma(\frac{Q^2}{2M^2} - \gamma)$ |

| Elastic, Quark pointlike, Spin $= \frac{1}{2}$ | $e_q^2 \frac{Q^2}{4M_x^2} \sigma(\frac{Q^2}{2M_x^2} - \gamma)$ | $e_q^2 \sigma(\frac{Q^2}{2M_x^2} - \gamma)$ |

Mass $M_q = M$, charge $e_Q$.

$\Rightarrow 2M W_1 = \frac{1}{x} \gamma W_2$, $2x F_1 = F_2$

$$F_\ell^2(Q^2) = \text{Formfactor} = \int_0^\infty e^{i \frac{Q^2}{2M^2} \tau} f(\tau) d\tau \bigg|_{g(\tau) = 2e_q f(\tau)}$$

$$= 1 - \frac{1}{6} q^2 \langle \tau^2 \rangle + \cdots$$

$$\left. \frac{dF(q^2)}{dq^2} \right|_{q^2, 0} = -\frac{1}{6} \langle \tau^2 \rangle$$
Elastic e-Nucleus scattering

\[ S(\tau) = \frac{S(0)}{1 + e^{(\tau - c)/a}} \]

\[ c = 1.07 \text{ fm } A^{1/3} \]

\[ a = 0.54 \text{ fm} \]

\[ q(1st \ min): R = 4.5 \]

\[ R_{\text{gCa}} > R_{\text{roCa}} \]

\[ q = 2E \sin \frac{\Theta}{2} \]
Fig. 4.5. Elastic scattering cross section of deuterium from $^{40}Ca$ from experiments performed at Stanford and Saclay, France. (Kennedy L. Sci. Phys. Lett. 245, 345 (1979).)
Elastic $e$-Nucleon scattering

\[ G_E^p (Q^2) = \left( 1 + \frac{Q^2}{0.71 \text{GeV}^2} \right)^{-2} \quad (\text{Dipole}) \]

\[ g(\tau) = g(0) e^{-\alpha \tau} \quad \alpha = 4.27 \text{ fm}^{-1} \]

\[ \sqrt{\langle \tau_p^2 \rangle} = 0.862 \text{ fm} \]
Fig. 4.25. Values of the proton magnetic form factor $G_M$ normalized by division with the proton magnetic moment, plotted against the square of the momentum transfer $Q^2$. An empirical fit to the data is shown as a solid line. The inset shows $Q^2G_M(Q^2)$ as a function of $Q^2$, where $Q^2 = -p^2$, together with some theoretical curves. [From S. Araki et al., Phys. Rev. Lett. 57, 174 (1986)].

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Inelastic e-Nucleon scattering

\[ W^2 = (p+q)^2 = M^2 + 2M \gamma - Q^2 \]
Deep inelastic e-Nucleon scattering

$F_2(x, Q^2) \neq f(Q^2)$

$x$ scaling

$\rightarrow$ nucleon has pointlike substructure

(Partons, Quarks)

$2F_1(x) \approx F_2(x)$

$\rightarrow$ spin $\frac{1}{2}$
Deep inelastic $e$-$N$-scattering

is incoherent sum of elastic scattering off pointlike spin-$\frac{1}{2}$ constituents.

Light cone:

$$X = \frac{p_0^2 + p_2^q}{p_0^2 + p_2^N} = \frac{\sqrt{m^2 + p_2^{\text{lab}}^2 + p_2^q^2}}{\sqrt{M^2 + p_2^N^{\text{lab}}^2 + p_2^N}}$$

\[ \to \frac{m^q}{M} \quad \text{(nucleon at rest)} \]

\[ \to \frac{p_2^q}{p_2^N} \quad \text{(infinite momentum far)} \]

$q_f(x) dx = \text{probability to find quark with flavor } f \text{ in } [x, x+dx]$  

$q_f(x) = \text{quark distribution} = \text{quark number density}$
\[ F_2(x) = \sum_f e_f^2 x [q_f(x) + \bar{q}_f(x)] \]

**Proton**  
\[ |p\rangle = |u_u_d_u\rangle + \text{sea} \]

\[
\frac{1}{x} F_2^p(x) = \frac{4}{3} (u(x) + \bar{u}(x)) + \frac{1}{3} (d + \bar{d}) + \frac{1}{3} (s + \bar{s}) + \frac{4}{3} (c + \bar{c})
\]

\[
u(x) = U(x) - \bar{U}(x) \quad \bar{u}_s(x) = \bar{u}(x)
\]

\[
\int_0^1 (u - \bar{u}) \, dx = \int_0^1 u_v \, dx = 2
\]

\[
\int_0^1 (d - \bar{d}) \, dx = \int_0^1 d_v \, dx = 1
\]

\[
\int_0^1 (s - \bar{s}) \, dx = \int_0^1 (d_s - \bar{d}) \, dx = \int_0^1 (c_s - \bar{c}) \, dx = 0
\]

**Neutron**  
\[ |n\rangle = |d_d_u_u\rangle + \text{sea} \]

**Isospin symmetry**  
\[ u^p = d^n = u \]
\[ d^p = u^n = d \]

\[
\frac{1}{x} F_2^n(x) = \frac{1}{3} (u + \bar{u}) + \frac{4}{3} (d + \bar{d}) + \frac{1}{3} (s + \bar{s}) + \frac{4}{3} (c + \bar{c})
\]
\[ F_2^n(x) / F_2^p(x) \]

**assumption:** \( \bar{u} = \bar{d} = \bar{s} = \bar{c} = S \)

\[ \Rightarrow \frac{F_2^n(x)}{F_2^p(x)} = \frac{4d_v + u_v + 20S}{4u_v + d_v + 20S} \]

<table>
<thead>
<tr>
<th>( S=0, u_v = 2d_v )</th>
<th>( \frac{F_2^n}{F_2^p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/5</td>
<td>1/4</td>
</tr>
</tbody>
</table>

\( S=0, u_v = 0 \)

\( S=0, d_v = 0 \)

\( u_v, d_v \ll S \)

At large \( x \): quark which defines quantum numbers dominates, i.e. \( u, \bar{u}, p, \bar{p}, d, \bar{d} \) in \( n \)

At small \( x \): seaquarks dominate
NMC spectrometer

replaced driftchambers by proportional chambers
added many wire chambers
optimised for ratios
\textbf{Shadowing} in the deuteron?  

\textbf{But } \lambda = \frac{\hbar c}{l_0^2} \approx 6 \text{ fm} \gg R_d
QCD analysis

NMC

BCDMS/SLAC
Quark carry fractional charges

\[ Q = \frac{2}{3}, \quad \frac{1}{3}, \quad \frac{1}{3} \]

Extrapolation \( x \to 0 \)

\[ S_c = 0.28 \pm 2 \]

\[ \rho \neq 0 \quad \therefore \quad \frac{3}{L} = \int \left[ x \rho \left( \frac{E}{E} - (x) \frac{E}{E} \right) \right] \quad 2 \int \rho = \kappa \]

Gottefield Sum Rule:

\[ (p - n) \frac{3}{L} + (\bar{p} - \bar{n}) \frac{3}{L} = \]

\[ L \rho + \bar{p} - \bar{n} + n + \bar{n} \]
\[ S_G = \int_0^1 \frac{1}{x} \left( F_2^+(x) - F_2^-(x) \right) \, dx \]

\[ = \frac{1}{3} \int \left( u_\nu(x) - d_\nu(x) \right) \, dx + \frac{2}{3} \int \left( \bar{u}(x) - \bar{d}(x) \right) \, dx \]

\[ = \frac{1}{3} \quad \text{if} \quad \bar{u} \equiv \bar{d} \]

NMC. P.R. D50 (1994), R1

\[ S_G \left( 0.004 - 0.8 \right) = 0.221 \pm 0.008 \text{(stat)} \pm 0.013 \text{(sys)} \]

\[ S_G \left( Q^2 = 4 \text{GeV}^2 \right) = 0.235 \pm 0.026 \]

\[ \int_0^1 \, dx \left( \bar{u}(x) - \bar{d}(x) \right) = -0.147 \pm 0.038 \]

\[ u_\nu > d_\nu \]

\[ \bar{d} > \bar{u} \]
Why $\bar{u} + d$?

Feynman + Field (1977)

$$x \cdot \bar{u} = 0.17 (1-x)^{10}$$

$$x = 0.17 (1-x)^7 \Rightarrow \int \frac{dx}{x} (F_u^p - F_u^n) = 0.2$$

Reason: Pauli blocking

$$(uud)(u\bar{u}) < (uud)(dd)$$

$$\Rightarrow 4 \text{ more } u\bar{u} \quad 5 \text{ more } d\bar{d}$$

equivalent to:

$$p \leftrightarrow p + \pi^0 \quad \pi^0 = (u\bar{u} + dd)$$

$$p \leftrightarrow n + \pi^+ \quad \pi^+ = (u\bar{d})$$

$$p \leftrightarrow \Delta^{++}, \pi^- \quad \pi^- = (\bar{u}d) \text{ suppressed}$$

More $\bar{d}$ than $\bar{u}$ in proton

correspondingly $$n \leftrightarrow p + \pi^-$$

$$n \leftrightarrow \Delta^- + \pi^+$$

Sea not only perturbative

[Box: 171]
Antiquark distributions
\( \gamma, \bar{\gamma} - N \) scattering, charged currents

\[
\begin{align*}
(\gamma_e)_{L} & \quad (\gamma_{\mu})_{L} & \quad (\gamma_{\tau})_{L} & \quad 0 \\
(e^{-})_{L} & \quad (\mu^{-})_{L} & \quad (\tau^{-})_{L} & \quad 1 \\
(u)_{L} & \quad (c)_{L} & \quad (t)_{L} & \quad \frac{2}{3} \\
(d)_{L} & \quad (s)_{L} & \quad (b)_{L} & \quad -\frac{1}{3}
\end{align*}
\]

Angular momentum conservation:

\[
S_3 = 0 \quad \rightarrow \quad \gamma_L \quad q_L \quad S_3 =
\]

\[
\gamma_L \quad 180^\circ \quad \gamma_L \quad \bar{q}_R \quad S_3 =
\]

\[
S_3 = 0 \quad \rightarrow \quad \gamma_L \quad \bar{q}_R \quad S_3 =
\]

\[
\bar{\gamma}_R \quad q_L \quad \sim (1 - \gamma)^2
\]

\[
\gamma = \frac{E_\gamma - E_{\gamma'}}{E_\gamma}
\]
Isoscalar target (Fe, C, ...)

\[
P = \frac{M_\omega^2}{M_\omega^2 + Q^2}
\]

\[
\frac{d^3\sigma}{dx dy} = \frac{G_F^2 \cdot P^2}{\pi} \cdot \frac{M \cdot E_y \cdot 2 \times \left[ \left( u + d + 2s \right) + (1 - \gamma)^2 \left( \bar{u} + \bar{d} + 2\bar{s} \right) \right]}{S_q}
\]

\[
\frac{d^3\sigma}{dx dy} = \left[ \left( u + d + 2s \right) + (1 - \gamma)^2 \left( u + d + 2s \right) \right]
\]

\[
\begin{align*}
\bar{u} + \bar{d} + 2\bar{s} \quad \{ & -x(x) \quad \bar{u} + \bar{d} + 2\bar{s} \\
\bar{u} + d + 2s \quad \{ & -x(x) \quad \bar{u} + d + 2s \\
\end{align*}
\]

CCFR, S.R. Nishina et al., P.R.L. 68 (32) 3493

\[
xq(x)
\]

\[
xQ(x) \sim (1-x)^7
\]
\[ F_2^{\gamma N} = x \left[ u(x) + \bar{u}(x) + d(x) + \bar{d}(x) + s(x) + \bar{s}(x) + c(x) + \bar{c}(x) \right], \quad N = \frac{1}{3}(p^+) \]

\[ F_2^{\mu N} = \frac{5}{18} x \left[ u(x) + \bar{u}(x) + d(x) + \bar{d}(x) + s(x) + \bar{s}(x) + c(x) + \bar{c}(x) \right] + \frac{1}{6} \left[ c(x) + \bar{c}(x) - s(x) - \bar{s}(x) \right] \]

\[ \approx \frac{5}{18} F_2^{\gamma N} \]

\[ F_2 \]

**Expectation:**

\[ \int_0^1 F_2^{\gamma N}(x) \, dx = \sum_f \int_0^1 x \left[ q_f(x) + \bar{q}_f(x) \right] \, dx = 1 \]

**Experiment:**

\[ \approx 0.48 \]

**Quarks carry only 48% of nucleon's momentum → Rest: Gluons**
Total $\gamma, \bar{\gamma} - N$ cross sections

$$V = \int (u_N(x) + d_N(x)) d\chi \cdot \int (1 - y)^{1/3} dy = \frac{1}{3}$$

$$Q_s = \frac{1}{\sqrt{2}} \sum_i \bar{q}_i(x) dx$$

$$G = \int x Q_s(x) dx \quad \int x \bar{Q}_i(x) dx = V + Q_s + \bar{Q} = 0.41$$

$$\frac{\sigma_T(\gamma)}{\sigma_T(\bar{\gamma})} = \frac{\bar{Q} + \frac{1}{3} (V + Q_s)}{(V + Q_s) + \frac{1}{3} \bar{Q}} = \frac{1 + 4 \bar{Q}/V}{3 + 4 \bar{Q}/V} = 0.67$$

$$\Rightarrow V = 0.31 \quad S = Q_s + \bar{Q} = 0.17 \quad G = 0.52$$

CCFR:

$$\frac{2 \int x \bar{s}(x) dx}{\int x (\bar{u}(x) + \bar{d}(x)) dx} = 0.477 \pm 0.051 \pm 0.017$$

$$- 0.050 - 0.036$$

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'Slow Rescaling': \[ \frac{d^2 \sigma}{dy \, dE_y} (\gamma N \rightarrow \mu^+ \mu^- X) = \ldots \left[ 1 - \frac{m_c^2}{2M E_N} \right] \delta(2) \]

- \( m_c = 1.31 \pm 0.236 \)
- \( B_c = 0.105 \pm 0.01 \)
- \( \xi = x \left( 1 + \frac{m_c^2}{Q^2} \right) \)
- \( x S(x) \sim (1-x)^3 \)
- \( Q^2 = 22.2 \, g_{\mu \nu} \)
The Drell-Yan process

\[ \frac{d^2 \sigma}{dx_1 dx_2} = \sigma_0 \times \sum_f e_f^2 [\overline{q}_f(x_1)q_f(x_2) + q_f(x_1)\overline{q}_f(x_2)] \]

\[ \sigma_0 = \frac{4\pi \alpha^2}{3s} \cdot \frac{1}{x_1 x_2} \cdot \frac{3}{g} \cdot \frac{3}{q_{q\bar{q}} \text{ colour}} \]

Select \( x_1 > 0.4 \) \( \Rightarrow \overline{q}(x_1) = 0 \)

\( \Rightarrow \) Measure \( \overline{q}(x_2) \)

p, n target \( \Rightarrow \overline{d}(x)/\overline{u}(x) \)
\[ \frac{\theta_{pd}^{pp}}{2 \theta_{pp}} \bigg|_{x_1 \gg x_2} = \frac{1}{2} \left( \frac{1 + \frac{1}{4} \frac{d_s}{u_1}}{1 + \frac{1}{4} \frac{d_s}{u_1} + \frac{d_s}{u_1}} \right) \cdot \left( 1 + \frac{\bar{d}_s}{\bar{u}_1} \right) \] (28)

140,000 events: with $M_{\mu^+} = 5$.

\[ \int (\bar{d} - \bar{u}) \, dx = 0.06 \pm 0.3 \, (0.04 \pm 0.08 \, \text{(sys)}) \]
Deep inelastic lepton-nucleon scattering and perturbative QCD

- Q.P.M.: Scaling for $Q^2, \gamma \to \infty$
  \[ F_2(x, Q^2) \text{ for fixed } x = \frac{Q^2}{2 \pi \gamma} \]
  Data: Scale breaking

- Quarks carry only ~50% of nucleons momentum
  \[ \sum_f \int_0^1 x (q(x) + \bar{q}(x)) dx \approx 0.5 \]

  \[ \text{strong interaction} \]

  \[ \begin{array}{c}
    q \\
    \text{q} \\
    \text{q} \\
    \text{q}
  \end{array} \quad \begin{array}{c}
    \text{q} \\
    \text{q} \\
    \text{q} \\
    \text{q}
  \end{array} \quad \begin{array}{c}
    \text{q} \\
    \text{q} \\
    \text{q} \\
    \text{q}
  \end{array} \quad \begin{array}{c}
    \text{Y} \\
    \text{Y} \\
    \text{Y} \\
    \text{Y}
  \end{array} \]

- Asymptotic freedom:
  \[ L_s (Q^2) \equiv \frac{12 \pi}{(33 - 2n_f) \ln (Q^2 / \Lambda^2)} \to 0 \quad \text{for } Q^2 \to \infty \]
QCD evolution of parton distribution

\[ x q^v(x) \quad x \bar{q}(x) \quad x g(x) \]

\[ Q^2 > Q^2_0 \]

R.P.L.G. equations

\[ \frac{d}{d \ln Q^2} \left[ \frac{Q^2(x, Q^2)}{Q^2(x, Q^2_0)} \right] = \frac{\alpha_s}{2 \pi} \left[ \frac{P_{qg}^2 - (\bar{P}_{qg})^2}{P_{qg}^2} \right] \]

From pattern of scale breaking

\[ \chi_s, \bar{Q}(x) \]
DGLAP equations

1) Non-Singlet (Valence like: $F_2^p - F_2^n, xF_3$)

\[ \frac{dQ_v(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s}{2\pi} \int_0^1 \frac{dy}{y} Q_v(y, Q^2) P_{qq}(\frac{x}{y}) \]

\[ = \frac{\alpha_s}{2\pi} P_{qq} \otimes Q_v \]

2) Singlet ($F_1^{\mu, \nu-N}$)

\[ \frac{d}{d \ln Q^2} \left[ Q_s(x, Q^2) \right] = \frac{\alpha_s}{2\pi} \left[ \begin{array}{cc} P_{gg} & 2n_f P_{qg} \\ P_{qg} & P_{gg} \end{array} \right] \otimes \left( Q_s^{\ast} \right) \]
Higher order corrections

Expressions for 'Splittingfunctions' $P_{ij}, x$
are more complicated

$$\mathcal{L}^{nlo}_S = \frac{4\pi}{\beta_0 \beta (Q^2/\Lambda^2)} \left( 1 - \frac{\beta_1}{\beta_0} \frac{\beta u \beta u (Q^2/\Lambda^2)}{\beta u (Q^2/\Lambda^2)} \right)$$

$$\beta_0 = 11 - \frac{2}{3} n_f \quad \beta_1 = 102 - \frac{38}{3} n_f$$

$$\mathcal{L}^{nlo}_S = \mathcal{L}^{lo}_S (1 - 0.2) \quad \text{at} \quad Q^2 = 100 \frac{\text{GeV}^2}{\text{uf}}$$

\[ \Lambda \text{ depends on renormalisation-scheme} \]

usually: $\Lambda_{\overline{MS}}$

(\footnote{Roberts, Ch.5})
NLO perturbative QCD fits to NMC

Fitted are:

1. $x Q^N_S \times Q^S_N \times Q^d \times Q^d$ at $Q^2 = 7 \text{ GeV}^2$

   $x Q^N_S = x \{ (u + \bar{u}) - (d + \bar{d}) \}$

   $x Q^d = x Q^N_S - \frac{2}{3} \times \tilde{Q}^N_S$

   $x Q^S = x \{ (u + \bar{u}) + (d + \bar{d}) + (s + \bar{s}) + (c + \bar{c}) \}$

   $x \tilde{Q}^N_S = x \{ (s + \bar{s}) - (c + \bar{c}) \}$ (From KMR3-86)

QPM:

- $F^p_2 = \frac{5}{18} \times Q^d + \frac{3}{18} \times Q^N_S$
- $F^d_2 = \frac{5}{18} \times Q^d$
- $F^p_2 - F^d_2 = \frac{1}{3} \times Q^N_S$

NLO $Q^2$ evolution program (Vivchaux, Gouganon, Botje

- Fix $\Lambda_{HS}^{(4)} = 263 \pm 42 \text{ MeV}$
- $Q^2_{min} = 1 \text{ GeV}^2$
- Impose momentum S.R. $\int dx \,(x Q^S_N \times Q^d) > 1$

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QCD fit to NMC data
(A fixed)

\[ \frac{1 - u F_2}{1 - u Q^2} \]

Preliminary

- NMC
- SLAC/BGDMS

\[ \Lambda^{(4)}_{\pi\pi} = 262 \text{ MeV} \]
X and $Q^2$ evolution of parton distributions

Fit at $Q^2 > 7$ GeV$^2$

A.P. evolution equations
\[ \Lambda_{RS} : \Delta_S (Q^2) \]

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Experiment</th>
<th>[ \Lambda_{RS}^{(1)} \text{ (MeV)} ]</th>
<th>[ \Delta_S (Q^2 = M^2) ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( xF_3^{uN} )</td>
<td>CCFR</td>
<td>173 ( \pm 36 \pm 41 )</td>
<td>0.117 ( \pm 0.008 ) ( \pm 0.004 ) ( \pm 0.004 )</td>
</tr>
<tr>
<td>( xF_3^{uN} ) ( \times F_2^{uN} \text{ (x&gt;0.25)} )</td>
<td>CCFR</td>
<td>210 ( \pm 28 \pm 41 )</td>
<td>0.118 ( \pm 0.003 ) ( \pm 0.003 ) ( \pm 0.004 )</td>
</tr>
<tr>
<td>( F_2^{eN} )</td>
<td>SLAC-BONNS</td>
<td>263 ( \pm 42 \pm 55 )</td>
<td>0.118 ( \pm 0.004 ) ( \pm 0.004 ) ( \pm 0.004 )</td>
</tr>
<tr>
<td>( F_2^{eN} )</td>
<td>ENC</td>
<td>211 ( \pm 80 \pm 80 )</td>
<td>0.118 ( \pm 0.004 ) ( \pm 0.004 ) ( \pm 0.004 )</td>
</tr>
<tr>
<td>( F_2^{eN} )</td>
<td>NMC</td>
<td>211 ( \pm 80 \pm 80 )</td>
<td>0.118 ( \pm 0.004 ) ( \pm 0.004 ) ( \pm 0.004 )</td>
</tr>
<tr>
<td>SF</td>
<td>world average</td>
<td>234 ( \pm 26 \pm 50 )</td>
<td>0.117 ( \pm 0.003 ) ( \pm 0.003 ) ( \pm 0.004 )</td>
</tr>
<tr>
<td>( e^+ e^- )</td>
<td>world average</td>
<td>234 ( \pm 26 \pm 50 )</td>
<td>0.121 ( \pm 0.006 ) ( \pm 0.006 ) ( \pm 0.006 )</td>
</tr>
</tbody>
</table>

Rule of the thumb: \( \frac{\Lambda}{\pi} \approx 200 \text{ MeV fm} \)

\( R_N = 0.86 \text{ fm} \)

\[ \Lambda \approx \frac{\frac{\Lambda}{\pi}}{R_N} \]
Figure 25.2: Summary of the values of \( \alpha_s(\mu) \) at the values of \( \mu \) where they are measured. The lines show the central values and the \( \pm 1\sigma \) limits of our average. The figure clearly shows the decrease in \( \alpha_s(\mu) \) with increasing \( \mu \).

**Rule of the thumb:**

\[
L_S (Q = 5 \text{ GeV}) = \frac{1}{5}
\]

\[
L_S (Q = 30 \text{ GeV}) \approx \frac{1}{3}
\]
\[ R(x,Q^2) \]

\[
\frac{d^2\sigma}{dQ^2 dx} = \frac{4\pi\alpha^2}{xQ^4} F_2(x,Q^2) \left\{ 1 - y \frac{Q^2}{4E^2} + \frac{y^2 + Q^2/E^2}{2(1 + R(x,Q^2))} \right\}
\]

\[ R(x,Q^2) = \frac{F_L(x,Q^2)}{2xF_s(x,Q^2)} \]

\[
F_L(x,Q^2) = \frac{2as(Q^2)}{2\pi} \int \frac{dy}{y^3} \int \left[ \frac{g}{3} F_2(y,Q^2) + \frac{4}{3}(1 - \frac{y}{2}) \right] G(y,Q^2)
\]

- Knowledge of \( R(x,Q^2) \) needed for determination of \( F_2 \) from \( \frac{d^2\sigma}{dQ^2 dx} \)
  - Large contribution to system error of \( F_2 \)

- Dito: polarised s.f. \( g_1(x,Q^2) \sim \frac{R_s F_2}{2x(1 + R_l)} \)

- In principle: tool for det. of \( g_2(x,Q^2) \)

- In reality: data 'compatible' with QCD
d

\( \text{Substantial improvement urgently needed!} \)

(Several beam energies, systematics!)
\[ F_L(x, Q^2) = \frac{e^2}{2\pi} x^2 \int \frac{dy}{y^3} \left\{ g \frac{\delta}{\delta x} F_L(x, Q^2) + \frac{40}{3} (1 - x) y G(y, Q^2) \right\} \]

different from p.n.d.? 

\[ \frac{Q^2}{F_p} = \frac{F_{d}}{F_{p}} \quad \text{if} \quad R_d = R_p \]
New data from SLAC-E143 for $x < 0.1$

--- NNLO pQCD

--- R1998

--- V. Abe et al., hep-ex/980828
**DIS at very high $Q^2$ - HERA**

Very high $Q^2, W^2$ only possible with colliding beams

### Fixed target:

- $\mathbf{r} = (E_\gamma, \vec{r})$, $\mathbf{p} = (M, 0)$
- $s = (\mathbf{r} + \mathbf{p})^2 = 2E_eM$
- FNAL-E665, $E_\gamma \sim 5506$
- $\sqrt{s} = 32$ GeV
- $Q^2 \lesssim 500$ GeV$^2$

### Collider:

- $\mathbf{r} = (E_e, \vec{r})$, $\mathbf{p} = (E_p, \vec{p})$
- $s = (\mathbf{r} + \mathbf{p})^2 = 2E_eE_p$

**HERA 1984 - 1987**:

- $E_e = 27.5$ GeV; $E_p = 820$ GeV $\rightarrow \sqrt{s} = 300$ GeV
- $Q^2 = s \cdot x \cdot y \quad x \approx 10^{-5}$
- $\lesssim 30000$ GeV$^2$
Study electromagnetic (γ) strong (q) weak (W^+ Z^0) interactions
Kinematic reach

Several upgrades

$10^{-6} < x < 0.7$

$10^{-1}$ GeV$^2 \lesssim Q^2 \lesssim 30 000$ GeV$^2$
HERA luminosity 1992-97

Mar Apr May Jun Jul Aug Sep Oct Nov

Integrated Luminosity (pb⁻¹)
\[ Q^2 = 22.860 \text{ GeV}^2 \]
Reduced cross section

\[ \sigma := \frac{xQ^4}{2\pi\alpha^2Y_+} \frac{d^2\sigma^{NC}}{dx dQ^2} = F_2 - \frac{y^2}{Y_+} F_L - \frac{Y_+}{Y_+} x F_3 \]

\[ Y_\pm = 1 \pm (1 - y)^2 \]

Steep rise at low x
ZEUS Preliminary 1996-97

$P_{2m}$ vs $Q^2$

$Q^2 = 800$  $Q^2 = 1200$  $Q^2 = 1500$

$Q^2 = 2000$  $Q^2 = 3000$  $Q^2 = 5000$

$Q^2 = 8000$  $Q^2 = 12000$  $Q^2 = 20000$

$10^{-3}$  $10^{-2}$  $10^{-1}$  $10^{0}$  $10^{1}$  $10^{2}$  $10^{3}$

$x$
Steep rise of $F_2$ at low $x$

⇒ strong increase of parton densities in the proton at low $x$
low $x$: $F_2(x) = \frac{5}{18} x \cdot q(x)$  

@ $q(x=25\cdot10^{-3})=230c$

@ rather crowded.
Reduced Cross-section at High $x$

- Difference visible in the QCD fit when the high $Q^2$ data is or is not included.
- High $Q^2$ HERA data now also have an influence at high $x$. 
Gluon distribution $g(x)$

1) From scale-breaking
   - 2 coupled Altarelli-Parisi equations
     (Problem $g(x) \leftrightarrow A$ correlated)

2) Direct photons at high $p_t$
   (QCD - Compton)

3) $J/\Psi$ - Production
   Photon - Gluon - Fusion

4) ($F_L(x, Q^2) \leftrightarrow R(x, Q^2) = \frac{g_\gamma}{g_\gamma}$)
\[
\frac{dF_2}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \left[ \int_x^1 \frac{dz}{z} \left( \frac{x}{z} \right) P_{qq}(\frac{x}{z}) F_2(x, Q^2) \right] + \int_x^1 \frac{1}{x} \sum_q e_q^2 \frac{dz}{z} \left( \frac{x}{z} \right) P_{qg}(\frac{x}{z}) g(x, Q^2)
\]

NLO QCD fit \( Q^2 = 20 \text{ GeV}^2 \)

\begin{itemize}
  \item H1 1995+96 \((0.118\pm0.005)\) (preliminary) \(m_\nu=1.3 - 1.8 \text{ GeV}\)
  \item ZEUS 1994 \((0.113\pm0.005)\) (preliminary) \(m_\nu=1.3 - 1.5 \text{ GeV}\)
  \item NMC \((0.113)\)
  \item MRSRI \((0.113)\)
  \item CTEQ4M \((0.116)\)
  \item GRV94-HO \((0.116)\)
\end{itemize}
ZEUS 1995 Preliminary

$Q^2 = 20 \text{ GeV}^2$

$Q^2 = 7 \text{ GeV}^2$

$Q^2 = 1 \text{ GeV}^2$
$J/\Psi$ production from $H, D, Sn, C$

Photon-gluon-fusion model

$\sigma_{\text{inelastic}} \sim q(x)$

$\text{inelastic: } z = \frac{E_{J/\Psi}}{y} < 0.3$

$P_t^2 > 0.2 \text{ GeV}^2$

$x = \frac{S_{x^a g^a}}{S_{x^a N}} = \frac{1}{W^2} \left[ \frac{M_{J/\Psi}^2}{z} + \frac{P_t^2}{z(1-z)} \right]$
$E_\mu = 280$ GeV

$I = (3.05 \pm 0.06) \times 10^{12} \mu \nu \nu L = 125 \text{ pb}^{-1}$

$$\frac{\sigma^{3/4}(H)}{\sigma^{3/4}(D)} = 0.96 \pm 0.08$$

---

Thesis: M. de Jong

C. Mariotti

CERN-EP/80-178

P.L. 258B (1951) 493.

---

$537 \pm 25$  \hspace{2cm} $D_2$

$238 \pm 17$  \hspace{2cm} $H_2$

$\gamma 3/4 = 3.095 \pm 0.003 \mu^{-} \mu^{-} \text{ Mass (GeV/c}^2) \rightarrow$

FIG. 2  \hspace{1cm} $m_{3/4} = 3.093 \pm 0.003$
Inelastic J/ψ Production

\[ Z = \frac{E_{\text{miss}}}{\gamma} \leq 0.8; \quad P_t^2 > 0.1 \text{ GeV}^2 \]

(278 ± 25 events)

\[ \gamma \times G(x) \sim \frac{\eta^4}{2} (1 - x)^\eta; \quad \eta = 5.1 \pm 0.3 \]

P.L. B258 (1991) 498
NH; CERN-PPE/90-178

- This Experiment
- EMC NH; J/ψ [20]
$312 \rightarrow \mu^+ \mu^-$
J/ψ and γ production at HERA

ZEUS 95-97 Preliminary

γ cross section larger than expected
$g(x)$ from open charm

$\gamma^* g \rightarrow c\bar{c} \rightarrow D^* X$

![Graph showing $xg(x)$ vs. log $x$ with data points for $D^*$ (DIS) and $D^*$ (p+p) and a QCD fit to $F_2$ with $\mu^2 = 25 \text{ GeV}^2/c^2$.](image)
$F_2$ - charm

NLO QCD: $g(x,Q^2), P_g \rightarrow F_2^{cc}$

Use $D^*$ events

(Transition probabilities $c \rightarrow D^*, \bar{c} \rightarrow \bar{D}^*$ and $D^*, \bar{D}^*$ decays known from LEP)

Measurement of $F_2^{charm}$ consistent with NLO prediction

At low $x$ contribution to $F_2$ 20-25%
Higher Twist Effects

\[ F_2(x, Q^2) = \begin{cases} F_2(x, Q^2)^{LT} \left(1 + \frac{C_i(x)}{Q^2}\right) \\ F_2(x, Q^2)^{LT} + \frac{C'(x)}{Q^2} + \ldots \end{cases} \]

- Measure of quark-quark correlations
  (Less clean than quark-gluon corr. in \( g_2(x, Q^2) \) - Twist-3)

- Effects largest at high \( x \)

- H.T. effects different for bound and free quarks?

  Nuclear binding: Correlations between quarks of different nucleons.

Fits: \( C_p(x) \approx C_d(x) \)

\[ \frac{F_2^n}{F_2^p}(Q^2) \Rightarrow C^n(x) > C^p(x) \]

- Small effect
- Similar to EMC effect

- Need high lumi
$100 < \Lambda_{\overline{MS}} < 300$ MeV

**Structure function $F_2$ of proton**

- **SLAC** ➔ **EMC**
- **QCD** ($\Lambda_{\overline{MS}} = 90$ MeV)
- $x = 0.45$
- **Higher twist**
- $F_2^{HT} \sim \frac{1}{Q^n}$

**Have to increase $Q^2$ range to test QCD!**
**One measurement!**
\[ d\sigma^{NC}/dx \text{ für } Q^2 > 1000 \text{ und } > 10000 \text{ GeV}^2 \]

- Gute Beschreibung der Daten durch das Standardmodell mit
destruktiver \( \gamma-Z^0 \)-Interferenz für \( e^+p \).

D. Pitzl, DPG Freiburg 24.3.1998
HERA e\(^+\)p DIS cross section 94 – 97

\(\frac{d\sigma}{dQ^2} (\text{pb/GeV}^2)\)

Preliminary

- Neutral current
- Charged current

- ZEUS
- H1

\(y < 0.9\)

\(Q^2 (\text{GeV}^2)\)
4 Polarisied DIS
Spin of the nucleon
\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_2^q + \Delta G + L_2^q \]

Quarks \quad Gluons
orbital angular momenti

\[ \Delta \Sigma = \Delta u_v + \Delta d_v + 2 (\Delta \bar{u} + \Delta \bar{d} + \Delta \bar{s}) \]
\[ = 1 \quad \text{QPM} \]
\[ \approx 0.6 \quad \text{rel. QPM} \]

\[ \Delta \Sigma \approx 0.2 \quad ? \]
\[ \Delta s \approx -0.1 \quad ? \]

\{ EMC/SLAC: Spin 'Puzzle' \}
Quark-Modell (1961) - SU(6)

\[ p = |uud\rangle \quad e_u = \frac{2}{3} \text{e} \]
\[ n = |d\,d\,u\rangle \quad e_d = -\frac{1}{3} \text{e} \]

\[ p^+ = \frac{1}{\sqrt{18}} \left\{ 2|uud\rangle - |uud\rangle - |uud\rangle \right\} + \ldots \]

\[ u^+ = \frac{5}{3} \quad u^- = \frac{1}{3} \quad \Delta u = u^+ - u^- = \frac{4}{3} \]
\[ d^+ = \frac{1}{3} \quad d^- = \frac{2}{3} \quad \Delta d = d^+ - d^- = -\frac{1}{3} \]

\[ \langle \mu^0 \rangle = \frac{1}{3} \left\{ 4 \langle \mu^+ \rangle - \langle \mu^- \rangle \right\} \]
\[ \langle \mu^n \rangle = \frac{1}{3} \left\{ 4 \langle \mu^+ \rangle - \langle \mu^- \rangle \right\} \]

\[ m_u = m_d = m_q \]

\[ \frac{\langle \mu^n \rangle}{\langle \mu^0 \rangle} = -\frac{2}{3} = -\frac{1.81}{2.73} \quad \Rightarrow \quad m_q \approx \frac{M_p}{2.73} \]
Polarised deep inelastic lepton-nucleon scattering

Measure probability for quark-spin and nucleon-spin $\uparrow\uparrow$ or $\downarrow\downarrow$

Spin-Structure Function $g_1^{p,n}(x, Q^2)$

In addition:
- $g_2^{p,n}(x, Q^2),$ $b^d,$ $\Delta^d,$ $h_1,$
- $\Delta V,$ $\Delta S,$ $\Delta G,$ $L_\perp$
Polarised lepton nucleon scattering

\[ \frac{d^3 \sigma(\alpha)}{dx \, dy \, d\phi} \bigg| \text{ (e)} = \frac{d^3 \sigma}{dx \, dy \, d\phi} + \frac{d^3 \sigma^*}{dx \, dy \, d\phi} \]

\[ \sim f(F_1, F_2) \quad \sim f(g_1, g_2) \]

\[ \frac{d^3 \sigma^*(\alpha)}{dx \, dy \, d\phi} = \frac{e^4}{4 \pi^2 Q^2} \left\{ \cos \alpha \cdot \left[ a \cdot g_1(x) + b \cdot g_2(x) \right] \right. \]

\[ \left. - \cos \phi \sin \alpha \cdot c \left( \frac{Y}{2} g_1(x) + g_2(x) \right) \right\} \]

\( a \gg b \quad (a = 1 - \frac{Y}{2} - \frac{Y^2}{4} (\kappa - 1); \quad b = \frac{Y}{2} (\kappa - 1), c = \sqrt{(\kappa - 1)a} \)

\( \alpha = 0^\circ : \text{ measure dominantly } g_1(x) \)

\( \alpha = 90^\circ : \text{ both } g_1(x), g_2(x) \)

Asymmetries:

\[ R = \frac{\sigma(\alpha + \pi) - \sigma(\alpha)}{\sigma(\alpha + \pi) + \sigma(\alpha)} \]
\[ \sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow} = \frac{1}{2} \sum_f e_f^2 \{ q_f^+(x, Q^2) - q_f^-(x, Q^2) \} \]

\[ \sigma_{\uparrow\uparrow} + \sigma_{\uparrow\downarrow} \sim F_1(x) = \cdots \]

\[ \frac{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\uparrow\downarrow}} \sim \frac{R_1(x)}{F_1(x)} \approx \frac{g_1(x)}{F_1(x)} \]

\[ \Delta q_f = \int_0^1 \{ q_f^+(x) - q_f^-(x) \} \, dx \]

\[ \Delta q_f = \langle \bar{q}_f \gamma_\mu \gamma_5 q_f | P, S \rangle \frac{1}{2M} \]
Integrals

a) unpolarised

\[ F_2(x) = \sum_f e_f^2 x (q_f(x) + \bar{q}_f(x)) \]

\[ I_2 = \int F_2(x) \, dx \]

- \[ I_2^p + I_2^n \sim \text{quark-momenta} \]
- \[ I_2^p - I_2^n \sim 0.24 \] (\( \approx 0.24 \))
- \( \text{quark charges:} \bar{u} + \bar{d} \)

b) polarised

\[ q_1(x) = \frac{1}{2} \sum_f e_f^2 \Delta q_f(x) \]

\[ I_1 = \int q_1(x) \, dx \]

- \( I_1^p + I_1^n \sim \text{quark-spins} \)
- \( \bar{q} \text{quarks contribute little to N-spin} \)
- \( I_1^p - I_1^n \) (BJ, SR)
- \( \text{axial charge:} q_a \)

But: Only global informations

Detailed test of QCD from

\[ F_2(x, Q^2) \]

(last \( \approx 25 \) years)
**Sum Rules:**

1. **Bjorken (66)** Fundamentally:

\[ \int_0^1 dx \left( g_{1}^p(x) - g_{1}^n(x) \right) = \frac{1}{6} \left| \frac{g_R}{g_V} \right|_{np}^{\text{QCD Kott.}} \]

\[ = 0.191 \pm 0.003 \]

\( g_R, g_V \): weak coupling constant. 

from Gamow-Teller decay

---

2. **Ellis-Jaffe** - SU(3)

\[ \int_0^1 dx \, q_{1}^{D,(n)}(x) = \frac{1}{12} \left| \frac{g_R}{g_V} \right|_{np}^{\text{QCD Kow.}} \left\{ 1 + 5 \cdot \frac{3F-D}{F+D} \right\} \]

\[ \downarrow \Delta S = 0, \ F/D = 0.579 \]

\[ = 0.175 \pm 0.007 \]

\[ = -0.016 \pm \ldots \]

F,D: SU(3) coupling constant; baryon decay:

np: F+D; \( \Lambda p: F + \frac{1}{3} D \);

\( \Sigma \Lambda: F - \frac{1}{3} D \)

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**Sum Rules**

\[ \Gamma_{1}^{p.n}(Q_{0}^{2}) \equiv \int_{0}^{1} g_{1}(x,Q_{0}^{2}) \, dx \]

(via QCD fit)

**Expt:**
- Evolve data from $Q_{m}^{2}(x) \rightarrow Q_{o}^{2}$
- Extrapolations:
  \[ \int_{x_{\text{min}}}^{1} \int_{0}^{\text{big?}} \int_{\text{small}}^{x_{\text{max}}} \]

**Theorie:**

\[ \Delta_{5}(Q^{2}) - \text{Corrections:} \]


(Kataev, Stavshenkov, Mod. Phys. Let.

Known to $O(\Delta_{5}^{3})$, estimates $O(\Delta_{5}^{4})$

\[ \Gamma_{1}^{p.n}(Q^{2}) = \frac{1}{12} \left( \frac{1}{4} a_{3} + \frac{1}{3} a_{g} \right) C^{NS}(Q^{2}) + \frac{1}{3} a_{0} C^{S}(Q^{2}) \]

\[ a_{3} = \frac{g_{R}}{g_{v}} = F+D = (\Delta u - \Delta d) \quad \text{n-decay} \]
\[ a_{g} = \frac{3}{2} (3F-D) = (\Delta u + \Delta d - 2\Delta s) \quad \text{hyperon-decay} \]
\[ a_{0} = \Delta \Sigma = (\Delta u + \Delta d + \Delta s) \quad \text{hyperon-decay} \]

\[ g_{R}/g_{v} = 1.257 \pm 0.0028 \]
\[ 2F-\Lambda = 0.579 \pm 0.025 \]
\[ \alpha_s(Q^2) \text{ CORRECTIONS} \]


→ Bjorken Sum Rule

\[
\Gamma_1^2 - \Gamma_1^0 = \frac{1}{6} \left[ 1 - \frac{\alpha_s(Q^2)}{\pi} - 3.58 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - 20.22 \left( \frac{\alpha_s(Q^2)}{\pi} \right) \right]
\]

→ Ellis-Jaffe Sum Rules

\[
\Gamma_1^{p(n)} = \Gamma_1^{NS} + \Gamma_1^S = \frac{1}{12} \left[ \frac{(-) a_3 + \frac{a_8}{\sqrt{3}}}{1 - \frac{\alpha_s}{\pi} - 3.58 \left( \frac{\alpha_s}{\pi} \right)^2 - 20.21 \left( \frac{\alpha_s}{\pi} \right)^2} \right] \Delta \Sigma \\
\]

\[ a_3 = \frac{g_a}{g_v}, \quad F + D = (\Delta u - \Delta d) \quad \text{neutron decay} \]

\[ a_8 = \frac{3F - D}{\sqrt{3}} = (\Delta u + \Delta d - 2\Delta s) \quad \text{hyperon decay} \]

\[ \Delta \Sigma = \Delta u + \Delta d + \Delta s \]

+ higher twist corrections
\[ \int_0^1 g^p_1(x) \, dx = 0.126 \pm 0.010 \pm 0.015 \]

\[ \mathcal{J} \mathcal{P}_x = \frac{1}{N_{N}^{+} N_{N}^{-}} \left( f_N \cdot P^B \cdot P^T \cdot D \right)^{-1} \]

\[ f(NH_3) = \frac{3}{17} \]
\[ f(C+H_3O\bar{H}) = \frac{10}{74} \]

But:
\[ f(H) = f(D) = 1 \]

\[ \mathcal{J} g_1 \sim \mathcal{J} R_{x^*} F_1 \sim \mathcal{J} R_{x^*} F_2 / x \]

---

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Consequences from $\Gamma_1' = 0.126 \pm 0.013$

\begin{align*}
\text{(LO - QCD)} \\
\Gamma_1^{\text{exp}} &= \Gamma_1^{\text{E3}} + \frac{1}{3} c_2 \Delta S \\
0.126 &= 0.175 + \frac{1}{3} \cdot 0.393 \Delta S \\
\Delta S &= -0.147 \\
\end{align*}

QPM

\begin{align*}
1) \Delta u - \Delta d &= 1.2572 \pm \ldots \\
2) \Delta u + \Delta d - 2\Delta S &= 0.5796 \pm \ldots \\
3) \Delta S &= -0.147 \pm \ldots \\
\end{align*}

\begin{align*}
\Delta u &= 0.771 \pm \ldots \\
\Delta d &= -0.486 \pm \ldots \\
\Delta S = \Delta u + \Delta d + 2\Delta S &= 0.138 \pm 0.170 \\
\end{align*}

'Spin puzzle'
- Very difficult experiments, tiny asymmetry; tremendous effort

- Phantastic progress in very different new technologies:
  - Polarized (strained GaAs) e\(^{-}\) sources with \(p_e \approx 85\%\) (E-143, E-154)
  - Long. beam polarization (Sovolov-Ternov) in HERA with \(p_e \approx 70\%\)
  - Cryogenic polarized targets (\(\text{NH}_3, \text{C}_6\text{H}_5\text{OH}, \text{h}\) (SMC, E143, TJNAL, ELSA, ...) highly developed
  - Polarized \(^3\text{He}\) gas targets \(+ \text{Li}^+\text{D}^+\)
    - Spin exchange with optically pumped \(\text{Rb}^\uparrow\)
    - \(\oplus\) 10 adm (E142, E-154)

- Metastability exchange (HERMES)

- Internal storage cell target works perfectly (HERMES)

- Atomic Beam Source for \(\text{H}^+\text{D}^+\) h.d. (HERMES)

- Laser Driven Source for \(\text{H}^+\text{D}^+\) coming

- Polarimetry
Spin experiments

<table>
<thead>
<tr>
<th>SMC/CERN</th>
<th>$P_T$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu^+ \to P_B = 0.8$</td>
<td>$I \sim \frac{1}{2} \text{ pA}$</td>
<td>$\sim 4 \times 10^{25} \text{ cm}^{-2}$</td>
</tr>
</tbody>
</table>

$E = 180 \text{ GeV}$

$E_{142} / E_{154} / SLAC$

$^3\text{He}^+ (10 \text{ atm})$ ~ $0.33$ ~ $1$

$^3\text{He}^+$ (optically pumped Rb) ~ $0.35$ ~ $1$

$Ga-As \uparrow e$ $P_B \sim 0.4, I \sim 0.1 \mu A$

$P_B \sim 0.8, I \sim 10 \mu A$

$E = 26 / 48 \text{ GeV}$ $T \sim 3 \times 10^{22} \text{ cm}^{-2}$

$E_{143} / E_{155} / SLAC$

$^{15}\text{NH}_3$ $< 0.8$ ~ $1$

$^{18}\text{ND}_3$ $< 0.3$ ~ $1$

$\text{LiD}$ $\sim 0.5$ ~ $1$

$E = 29 / 48 \text{ GeV}$ $T \sim 10^{24} \text{ cm}^{-2}$

HERMES / DESY

$e^+ e^-$ $P_B = 0.5 - 0.7$

$I \sim 20 \text{ mA}$

$E = 28 \text{ GeV}$ $T = (1 \to 10) \times 10^{14} \text{ cm}^{-2}$

HERA

$e^+ (\text{Sokolov-Ternov})$

$P_B = 0.5 - 0.7$

$I \sim 20 \text{ mA}$

$E = 28 \text{ GeV}$ $T = (1 \to 10) \times 10^{14} \text{ cm}^{-2}$
E154 $^3$He Target Assembly & NMR

- 4 Ti:Sapphire Laser + 3 Laser Diode Arrays,
- 30 cm long target vessel, 50 µm windows,
- ~10 Amagat $^3$He density.
The HERMES target

- Internal storage cell:

- undiluted target atoms: $\overline{H}$, $\overline{D}$, $^{3}\overline{He}$

- Gas targets:
  - Laser driven source: $^{3}\overline{He}$
    $$\rho = 1 \cdot 10^{15} \text{ N/cm}^2 \quad \Delta t_{flip} \sim 10 \text{ min} \quad \overline{P}_T = 46 \%$$
  - Atomic beam source: $\overline{H}$
    $$\rho = 7 \cdot 10^{13} \text{ N/cm}^2 \quad \Delta t_{flip} \sim 1 \text{ min} \quad \overline{P}_T = 92 \%$$
  - Unpolarized gases: $^1H$, $^2D$, $^3He$ and $^{14}N$. 
Several upgrades

$10^{-6} < x < 0.7$

$10^{-1} \text{ GeV}^2 \lesssim Q^2 \lesssim 30\ 000 \text{ GeV}^2$
\[
\Delta Q^2 = 22.860 \text{ GeV}^2
\]
Reduced cross section

\[ \sigma := \frac{xQ^4}{2\pi\alpha^2Y_+} \frac{d^2\sigma^{NC}}{dx dQ^2} = F_2 - \frac{y^2}{Y_+} F_L - \frac{Y_-}{Y_+} x F_3 \]

\[ Y_\pm = 1 \pm (1 - y)^2 \]

Steep rise at low x
ZEUS Preliminary 1996-97

$F_2$ vs. $x$ for different $Q^2$ values:

- $Q^2 = 800$
- $Q^2 = 1200$
- $Q^2 = 1500$
- $Q^2 = 2000$
- $Q^2 = 3000$
- $Q^2 = 5000$
- $Q^2 = 8000$
- $Q^2 = 12000$
- $Q^2 = 20000$
Steep rise of $F_2$ at low $x$

strong increase of parton densities in the proton at low $x$
Low $x$: $F_2(x) = \frac{5}{18} x \cdot q(x)$ and $q(x=2.5 \times 10^{-3}) = 230c$

A rather crowded.
ZEUS 1995 Preliminary

\[ \langle Q^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} Q^2_i \]

ZEUS DATA

GRV94

DL

\[ \frac{d^2F}{d\ln Q^2} \]

\[ x \]

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Reduced Cross-section at High $x$

- Difference visible in the QCD fit when the high $Q^2$ data is or is not included.
- High $Q^2$ HERA data now also have an influence at high $x$.
1) From scale-breaking
   2 coupled Altarelli-Parisi equations
   (Problem $g(x) \leftrightarrow A$ correlated)

2) Direct photons at high $p_t$
   (QCD - Compton)

3) $J/\psi$ Production
   Photon - Gluon - Fusion

4) $(F_L(x, Q^2) \leftrightarrow R(x, Q^2) = \frac{g_\gamma}{g_\gamma})$
$g(x)$ from scaling violations

$$\frac{dF_2}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \left[ \frac{1}{x} \int \frac{dz}{z} \left( \frac{x}{z} \right) P_{qq}(\frac{x}{z}) F_2(x, Q^2) \right. $$

$$+ \left. \frac{1}{x} \sum q e_q^2 \frac{dz}{z} \left( \frac{x}{z} \right) P_{qg}(\frac{x}{z}) g(x, Q^2) \right]$$

NLO QCD fit $Q^2 = 20$ GeV$^2$

$x$ vs $g(x)$

- H1 1995+96 (preliminary) $m_c = 1.3 - 1.6$ GeV
- ZEUS 1994 (preliminary) $m_c = 1.3 - 1.5$ GeV
- NMC
- MRST1
- CTEQ4M
- GRV94-HO

$\alpha_s(M_Z^2)$

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ZEUS 1995 Preliminary

$Q^2 = 20 \text{ GeV}^2$

$Q^2 = 7 \text{ GeV}^2$

$Q^2 = 1 \text{ GeV}^2$
$J/\Psi$ production from $H, D, Sn, C$

**Photon-gluon-fusion model**

\[ 0 \rightarrow J/\Psi \sim g(x) \]

**Inelastic**

\[ z = \frac{E_{J/\Psi}}{\gamma} < 0.9 \]

\[ p_t^2 \gtrsim 0.2 \text{ GeV}^2 \]

\[ x = \frac{S_x g}{S_{x^* N}} = \frac{1}{W^2} \left[ \frac{M_{J/\Psi}^2}{z^2} + \frac{p_t^2}{z (1-z)} \right] \]

\[ M_{J/\Psi} = \frac{W^2}{2 \rightarrow 1} \]
$E_{\mu} = 280 \text{ GeV}$

$I = (3.05 \pm 0.06) \cdot 10^{12} \mu \tau \cdot L = 125 \text{ pb}^{-1}$

\[ \frac{\sigma^{3/4}(D)}{\sigma^{3/4}(H)} = 0.96 \pm 0.08 \]

---

**Thesis:** M. de Jong

C. Mariani

**NMGCERN-PPE/80-178**


\[ m_{3/4} = 3.085 \pm 0.002 \text{ GeV/c}^2 \]

**Fig. 2** \[ m_{3/4} = 3.083 \pm 0.003 \]
**Inelastic \( J/\psi \)-Production**

\[
Z = \frac{E_{3/4}}{\gamma} \leq 0.8; \quad P_t^2 > 0.1 \text{ GeV}^2
\]

\[
(278 \pm 25 \text{ events})
\]

\[
\gamma \times g(x) = \eta^+ \left(1 - x\right)^{\gamma}; \quad \eta = 5.1 \pm 0.3
\]


NMC; CERN-PPE/90-178
$3174 \rightarrow \mu^+\mu^-$
J/ψ and γ production at HERA

ZEUS 95-97 Preliminary

γ cross section larger than expected
$g(x)$ from open charm

$\chi^* g (\rightarrow c\bar{c}) \rightarrow D^* X$

H1 preliminary
- $D^*$ (DIS)
- $D^*$ ($p\bar{p}$)
- QCD fit to $F_2$

$\mu^2 = 25 \text{ GeV}^2/c^2$

CTEQ4F3
$F_2^{\text{charm}}$

NLO QCD: $g(x, Q^2), P_{g \to c\bar{c}} \rightarrow F_2^{c\bar{c}}$

Use $D^*$ events

(Transition probabilities $c \rightarrow D^*, \bar{c} \rightarrow \bar{D}^*$ and $D^*, \bar{D}^*$ decays known from LEP)

Measurement of $F_2^{\text{charm}}$ consistent with NLO prediction

At low $x$ contribution to $F_2$ 20-25%
Higher Twist Effects

\[ F_2(x, Q^2) = \begin{cases} 
F_2(x, Q^2)^{LT} & (1 + \frac{C_i(x)}{Q^2}) \\
F_2(x, Q^2)^{LT} + \frac{C'_i(x)}{Q^2} & + \ldots
\end{cases} \]

- Measure of quark-quark-correlations (Less clean than quark-gluon-corr. in $g_\perp(x, Q^2)$ - Twist-3)
- Effects largest at high $x$
- H.T. - effects different for bound and free quarks?
- Nuclear binding: Correlations between quarks of different nucleons?

Fits: $C^p(x) \approx C^d(x)$

\[ \frac{F_2^n(Q^2)}{F_2^p(Q^2)} \Rightarrow C^n(x) > C^p(x) \]

Small effect

Similar to EMC effect

\[ \# \text{ Need high lumi} \]
$100 < \Lambda_{\text{MS}} < 300 \text{ MeV}$

**Structure function $F_2$ of proton**

Have to increase $Q^2$ range to test QCD!

One measurement!
\[ \frac{d\sigma^{NC}}{dx} \text{ für } Q^2 > 1000 \text{ und } > 10000 \text{ GeV}^2 \]

- Gute Beschreibung der Daten durch das Standardmodell mit destruktiver \(\gamma-Z^0\)-Interferenz für \(e^+p\).

D. Pitzl, DPG Freiburg 24.3.1998
4. Polarised DIS

Spin of the nucleon
\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_2^q + \Delta G + L_2^c \]

Quarks | Gluons
orbital angular moment.

\[ \Delta \Sigma = \Delta u + \Delta d + 2(\Delta \bar{u} + \Delta \bar{d} + \Delta \bar{s}) \]
\[ = 1 \quad \text{QPM} \]
\[ \approx 0.6 \quad \text{rel. QPM} \]

\[ \Delta \Sigma = 0.2 \ ? \]
\[ \Delta S \approx -0.1 \ ? \]  
EMC/SLAC: Spin 'Puzzle'

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Quark-Modell (1961) - SU(6)

\[ P = |uud\rangle \quad e_u = \frac{2}{3} \text{ ele} \]
\[ n = |d\bar{d}u\rangle \quad e_d = -\frac{1}{3} \text{ ele} \]

\[ P^\uparrow = \frac{1}{\sqrt{18}} \left\{ 2|u^\uparrow ud\rangle - |u^\uparrow ud\rangle - |u^\uparrow ud\rangle + \ldots \right\} \]

\[ \Delta u = u^+ - \bar{u} = \frac{4}{3} \]
\[ \Delta d = d^+ - \bar{d} = -\frac{1}{3} \]

\[ <\mu^e> = \frac{1}{3} \left\{ 4 <\mu^e> - <\mu^d> \right\} \]
\[ <\mu^n> = \frac{1}{3} \left\{ 4 <\mu^d> - <\mu^u> \right\} \]

\[ m_u = m_d = m_\sigma \]

\[ \frac{<\mu^n>}{<\mu^e>} = -\frac{2}{3} \approx -1.81 \text{ \( \frac{1}{2} \)}; \quad m_\sigma \approx \frac{M_p}{2.73} \]
Measure probability for quark-spin and nucleon-spin \( \uparrow \uparrow \) or \( \downarrow \downarrow \)

**Spin-Structure Function** \( g_1(n)(x,Q^2) \):

In addition:

\( g_2(n)(x,Q^2) \), \( b_1 \), \( \Delta^d \), \( h_1 \),

\( \Delta V, \Delta S, \Delta G, L_2 \)
Polarised lepton nucleon scattering

\[ \frac{d^3\sigma(\alpha)}{dx
dy
d\phi} = \frac{1}{dx
dy
d\phi} \frac{d^3\sigma}{dx
dy
d\phi} + \frac{d^3\sigma^*}{dx
dy
d\phi} \]

\[ \sim f(F_1,F_2) \sim f(g_1,g_2) \]

\[ \frac{d^3\sigma^*}{dx
dy
d\phi} = \frac{e^4}{4\pi^2Q^2} \left\{ \cos \alpha \cdot \left[ a \cdot g_1(x) + bg_2(x) \right] - \cos \phi \sin \alpha \cdot c \left[ \frac{1}{2} g_1(x) + g_2(x) \right] \right\} \]

\[ a >> b \quad (a = 1 - \frac{1}{2} - \frac{1}{4} (k-1) ; b = \frac{1}{2} (k-1), c = \sqrt{(k-1)a} \]

\[ \alpha = 0^\circ: \text{measure dominantly } g_1(x) \]

\[ \alpha = 90^\circ: \text{both } g_1(x), g_2(x) \]

Asymmetries: \[ R = \frac{\sigma(\alpha,\pi) - \sigma(\alpha)}{\sigma(\alpha,\pi) + \sigma(\alpha)} \]

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Integrals

a) un-polarised

\[ F_2(x) = \sum_f e_f^2 x (q_f(x) - \bar{q}_f(x)) \]
\[ I_2 = \int_0^1 F_2(x) \, dx \]

- \( I_2^p + I_2^n \) quark-momenta
- only ~50% of N-momentum
- \( I_2^p - I_2^n = \frac{1}{3} \) (= 0.24)
- quark charges: \( u \dagger d \)

b) polarised

\[ q_1(x) = \frac{1}{2} \sum_f e_f^2 \Delta q_f(x) \]
\[ I_1 = \int_0^1 q_1(x) \, dx \]

- \( I_1^p + I_1^n \) quark-spins
- quarks contribute little to N-spin
- \( I_1^p - I_1^n \) (B_3 - SR)
- axial charge: \( g_\alpha \)

But: Only global informations

Detailed test of QCD from

\[ F_2(x, Q^2) \]
(last ~25 years)
**Sum Rules:**

1. **Bjorken (66)** Fundamental

\[ \int_0^1 dx \left( g_R^p(x) - g_R^n(x) \right) = \frac{1}{6} \left| \frac{g_R}{g_V} \right|_{V,F} QCD \text{ Kott.} \]

\[ = 0.191 \pm 0.003 \]

\[ g_R, g_V: \text{ weak coupl. const.} \]

From Gamow-Teller \( \beta^- \text{-decay} \)

2. **Ellis-Jaffe** \(-\text{SU}(3)\)

\[ \int_0^1 dx \left( g_R^p(x) \right) = \frac{1}{12} \left| \frac{g_R}{g_V} \right|_{V,F} \left\{ \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \right\} \text{QCD Korn.} \]

\[ \Delta S = 0, F/D = 0.579 \]

\[ = 0.175 \pm 0.007 \]

\[ (n) = -0.016 \pm \ldots \]

\( F, D: \text{SU}(3) \) coupling const.; baryon decays:

\( np: F + D \);

\( \Lambda p: F + \frac{1}{3} D \);

\( \Sigma \Lambda: F - \frac{1}{3} D \)
**Sum Rules**

\[ \Gamma_{p,n}^{p,n}(Q^2) \equiv \int_{Q} q_n(x,Q^2) \, dx \]  
(via QCD fit)

**Expt:**
- Evolve data from \( Q_m(x) \Rightarrow Q_o \)
- Extrapolations:
  \[ \int_{x_{\text{min}}}^{x_{\text{max}}} \int_{Q}^{Q} \]
  \[ \uparrow \quad \uparrow \]
  \[ \text{big?} \quad \text{small} \]

**Theorie:**

\( \mathcal{L}_s(Q^2) \) - Corrections:

( Larin, Phys. Lett. B334 (1994) 1

Known to \( O(\mathcal{L}_s^3) \), estimates \( O(\mathcal{L}_s^4) \)

\[ \Gamma_{p,n}^{p,n}(Q^2) = \frac{1}{12} \left( + \frac{1}{3} a_3 + \frac{1}{3} a_8 \right) C^{NS}(Q^2) \]

\[ + \frac{1}{3} a_6 C^S(Q^2) \]

\( a_3 = q_R/q_v = F+D = (\Delta u - \Delta d) \) \( \text{r-decay} \)

\( a_8 = (3F-D) = (\Delta u + \Delta d - 2\Delta s) \) \( \text{hyperon-decay} \)

\( a_6 = \Delta \Sigma = (\Delta u + \Delta d + \Delta s) \)

\[ q_R/q_v = 1.2573 \pm 0.0028 \]

\[ 2F-D = 0.579 \pm 0.025 \]
\[ \alpha_s(Q^2) \text{ CORRECTIONS} \]

\[ \text{Bjorken Sum Rule} \]

\[ \Gamma_1^p - \Gamma_1^n = \frac{1}{6} g_s \left[ \frac{1}{ \pi} - \frac{\alpha_s(Q^2)}{\pi} - 3.58 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - 20.22 \left( \frac{\alpha_s(Q^2)}{\pi} \right) \right] \]

\[ \text{Ellis-Jaffe Sum Rules} \]

\[ \Gamma_1^{p(n)} = \Gamma_1^{NS} + \Gamma_1^S \]

\[ = \frac{1}{12} \left( \frac{\alpha_s}{\sqrt{3}} \right) \left[ \frac{1}{ \pi} - \frac{\alpha_s}{\pi} - 3.58 \left( \frac{\alpha_s}{\pi} \right)^2 - 20.21 \left( \frac{\alpha_s}{\pi} \right)^3 \right] \]

\[ + \frac{1}{9} \left[ 1 - \frac{\alpha_s}{\pi} - 0.55 \left( \frac{\alpha_s}{\pi} \right)^2 \right] \Delta \Sigma \]

\[ \alpha_3 = \frac{g_3}{g_s} = F + D = (\Delta u - \Delta d) \quad \text{neutron decay} \]

\[ \alpha_8 = \frac{\sqrt{3} F - D}{\sqrt{3}} = (\Delta u + \Delta d - 2\Delta s) \quad \text{hyperon decay} \]

\[ \Delta \Sigma = \Delta u + \Delta d + \Delta s \]

+ higher twist corrections

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\[ \int_{0}^{1} g_{1}(x) \, dx = 0.126 \pm 0.010 \pm 0.015 \]

\[ \mathcal{R}_{x^*} = \frac{1}{N_{P}^{M} N_{P}^{T}} \left( f_{N} \cdot P \cdot P^{T} \cdot D \right)^{-1} \]

\[ f(NH_{3}) = \frac{9}{7} \; ; \quad \text{But:} \quad f(H) = f(D) = 1 \]

\[ \mathcal{S}_{R_{1}} \sim \mathcal{S}_{R_{x^*} F_{1}} \sim \mathcal{R}_{x^*} F_{2} / x \]
Consequences from $\Gamma_1' = 0.126 \pm 0.018$

\[ \Gamma_{1\exp}^p = \Gamma_{1\text{ E3}}^p + \frac{1}{3} c_2 \Delta S \]

\[ 0.126 = 0.175 + \frac{1}{3} \cdot 0.997 \Delta S \]

$\Delta S = -0.147$

1) $\Delta u - \Delta d = 1.2572 \pm \ldots$
2) $\Delta u + \Delta d - 2\Delta S = 0.5796 \pm \ldots$
3) $\Delta S = -0.147 \pm$

$\Delta u = 0.771 \pm \ldots$

$\Delta d = -0.486 \pm \ldots$

$\Delta \Sigma = \Delta u + \Delta d + \Delta S = 0.138 \pm 0.170$

'Spin puzzle'
- Very difficult experiments, tiny asymmetry, tremendous effort

- Phantastic progress in very different new technologies:

  - Polarized (strained GaP) $e^-$ sources with $p_e \sim 85\%$ (E-143, E-154)

  - Long. beam polarization (Sovolov-Ternov) in HERA with $p_e \sim 70\%$

  - Cryogenic polarized targets (NH$_3$, C$_6$H$_5$OH, $\ldots$) SMC, E143, FNAL, ELSA, $\ldots$) highly developed

  - Polarized $^3$He gas targets + Li$^+D^+$

  - Spin exchange with optically pumped Rb$^+$

  - 10 cm (E142, E-154)

  - High power laser diode

  - Metastability exchange (HERMES)

- Internal storage cell target works perfectly (HERMES)

- Atomic Beam Source for H$^+, D^+$ h. d. (HERMES)

- Laser Driven Source for H$^+, D^+$ coming

- Polarimetry
<table>
<thead>
<tr>
<th>Spin experiments</th>
<th>$P_T$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMC/CERN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu^+ \rightarrow P_B \sim 0.8$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I \sim \frac{1}{2}$ mA</td>
<td>$T \sim 4 \cdot 10^{25}$ cm$^{-2}$</td>
<td></td>
</tr>
<tr>
<td>$E = 180$ GeV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_4H_9OH^+$</td>
<td>$\sim 0.85$</td>
<td>$\sim 1/1$</td>
</tr>
<tr>
<td>$C_4D_9OD^+$</td>
<td>$&lt; 0.5$</td>
<td>$\sim 1/1$</td>
</tr>
<tr>
<td>$NH_3^+$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.83$</td>
<td>$\sim 1/1$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>E-142 / E-154 / SLAC</th>
<th>$^3$He$^+$ (10 atm)</th>
<th>$\sim 0.33$</th>
<th>$\sim 1/1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(optically pumped Rb)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_{\text{Ga-As}}$</td>
<td>$P_B \sim 0.8$, $I \sim 10$ mA</td>
<td>$\sim 0.38$</td>
<td>$\sim 1/1$</td>
</tr>
<tr>
<td>$P_B \sim 0.8$, $I \sim 100$ nA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E = 2648$ GeV</td>
<td>$T \sim 3 \cdot 10^{22}$ cm$^{-2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>E-143 / E-155 / SLAC</th>
<th>$^{15}$NH$_3$</th>
<th>$&lt; 0.8$</th>
<th>$\sim 1/1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}$ND$_3$</td>
<td>$&lt; 0.3$</td>
<td>$\sim 1/1$</td>
<td></td>
</tr>
<tr>
<td>LiD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sim 0.5$</td>
<td>$\sim 1/1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E = 2848$ GeV</td>
<td>$T \sim 10^{24}$ cm$^{-2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>HERMES / DESY</th>
<th>$^3$He$^+$</th>
<th>$\sim 0.9$</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+$ HERA</td>
<td>$P_B = 0.5 - 0.7$</td>
<td>$\sim 0.5$</td>
<td>$\sim 1/1$</td>
</tr>
<tr>
<td>$I \sim 20$ mA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E = 28$ GeV</td>
<td>$T = (1 \rightarrow 10) \times 10^{16}$ cm$^{-2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
E154 $^3$He Target Assembly & NMR

- 4 Ti:Sapphire Laser + 3 Laser Diode Arrays,
- 30 cm long target vessel, 50 μm windows,
- ~10 Amagat $^3$He density.

SLAC E154 APP (Adiabatic Fast Passage) NMR Signals

The HERMES target

- Internal storage cell:

- undiluted target atoms: $\overline{H}$, $\overline{D}$, $\overline{3He}$

- Gas targets:
  - Laser driven source: $\overline{3He}$
    $\rho = 1 \cdot 10^{15}$ N/cm$^2$  $\Delta t_{flip} \sim 10$ min  $\overline{P_T} = 46\%$
  - Atomic beam source: $\overline{H}$
    $\rho = 7 \cdot 10^{13}$ N/cm$^2$  $\Delta t_{flip} \sim 1$ min  $\overline{P_T} = 92\%$
  - Unpolarized gases: $^1H$, $^2D$, $^3He$ and $^{14}N$. 

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HI:

ABS sextupole  RFT

B

chopper detector

TGA

magnet coils

iron yoke

e^+ beam

sextupole

chopper detector

BRP

storage cell
HERMES Hydrogen target performance, 1996–97

\[ \alpha = \text{degree of dissociation} = \frac{N_H}{N_H + 2N_{H_2}} \]
E155 Spectrometers
E155 kinematic coverage by spectrometer
transverse and longitudinal vertex distribution:

no background from wall scattering!
Beam polarisation and polarimetry

Transverse polarisation: through emission of synchrotron radiation in the curved sections ('Sokolov-Ternov effect'):

\[ P(t) = P_{\text{max}} \cdot (1 - e^{-t/\tau}) \]

Longitudinal polarisation: achieved by a 90° spin rotation in the region of the HERMES experiment:

annual mean: \( \overline{P_B} = .55 \)
\( \sqrt{P_B} = 5.4\% \) ('95)
\( \sqrt{P_B} = 4.1\% \) ('96,'97)
SMC

- Final data sets - CERN-EP/88-85
- Improved beam polarisation
  \[ A_{\chi, \text{new}} = 0.965 \ A_{\chi, \text{old}} \]
- Hadron tagging
  - Eliminate elastic contribution to DIS rate
  - Reduced radiative corrections
  - Smaller dilution factor at low \( x \)
  - Extend low \( x \), low \( Q^2 \) range

- Hadrons: flavour decomposition \( \Delta \xi \)
- QCD analysis CERN-EP/88-86
SMC - updated, final results

* Normalisation change ($\rho^3$): -3.5%

* low x, only events with hadrons $\rightarrow$ reduced rad. corr.
  $\rightarrow$ smaller dilution
High statistics (200 M events) inclusive results from \( \text{NH}_3 \), \( \text{Li} \bar{D} \) targets

\[ P_{1,2}(x), \quad Q_{1,2}(x) \]

3 spectrometers \((2.75^\circ, 5.5^\circ, 10.5^\circ)\)

\( \text{Li} \bar{D} \) target

\[ f = 0.5 \]

\( p^T = 0.2 - 0.25 \)
Very impressive statistical accuracy

Also results for $g_1^d/F_1^d$ from LiD target
HERMES

- \( A_1^p, g_1^p \) from '87 \( \bar{H} \) data

- Hadron asymmetries from
  '85 \(^3\text{He}\) data
  '96 \( \bar{H} \)

\[ \Rightarrow \Delta u_\nu(x), \Delta d_\nu(x), \Delta \bar{q}(x) \]
'97 proton results

\[ p^B = 0.55 \pm 0.02 \]

\[ p^T = 0.88 \pm 0.04 \]

\[ Q^2 > 0.8 \text{ GeV}^2 \]

\[ W^2 > 3.24 \text{ GeV}^2 \]

\[ 0.04 \text{ rad} < \theta < 0.22 \text{ rad} \]

\[ 0.15 \gamma < 0.8 \]

\[ \Rightarrow 1.73 \times 10^6 \text{ events} \]

\[ 0.8 \times 10^6 \text{ events} \]

\[ 0.15 \gamma \leq 0.8 \]

\[ \frac{g_1}{F_1} = \frac{1}{1 + y^2} \left( \frac{P^y}{D} - (\gamma - \chi) F_2 \right) \]
Neutron results from $^3\text{He}$
\[ A(x, Q^2) \rightarrow g_1(x, Q^2) \]

- \[ A(x, Q^2) = \frac{\sigma^{\uparrow\downarrow}}{\sigma^{\uparrow\uparrow} + \sigma^{\downarrow\downarrow}} = D(A_1 + \gamma A_2) \] (small correction)

- \[ g_1(x, Q^2) \approx A_1(x, Q^2) \cdot \frac{F_2(x, Q^2)}{2x \left( 1 + R(x, Q^2) \right)} \]

- \( F_2(x, Q^2) \): NMC - parametrisation

- \( R(x, Q^2) \): SLAC - parametrisation: \( R_{143} \) (Recently new one from E143 for 0.03, hep-ex/9808028)

- \( Q^2 \) measured - different for each \( x \)-bin

Integrals: require data at fixed \( Q_0^2 \)

* Assumption: \( A_1(x, Q^2) \neq f(Q^2) \) \( QCD \): small \( Q^2 \)-dependence data: ok within e

* Extrapolation via

\( Q^2 \) dependence of \( F_2(x, Q^2), R(x, Q^2) \)

or

* Use results of NLO QCD analysis

(However: many assumptions, e.g. \( Q_0^2(x, Q^2) \)
SMC final results

(a) proton

- SMC $Q^2 > 0.2 \text{ GeV}^2$
- SMC $Q^2 > 1.0 \text{ GeV}^2$

(b) deuteron

- SMC $Q^2 > 0.2 \text{ GeV}^2$
- SMC $Q^2 > 1.0 \text{ GeV}^2$

$g_1$ vs $x$ for proton and deuteron.
Li D - target; nuclear effects understood.

July 22, 1998

improvements highly desirable

- E155 (preliminary)
- E143
- SMC 92+94+95
at $Q^2=5 \text{ GeV}^2$

E155 systematic error
July 22, 1998

- E155 (preliminary)
  - E143
  - SMC 93+96 at $Q^2 = 5$ GeV$^2$

Preliminary
3 He results

HERMES: $W^2 > 4 \text{(GeV/c)}^2$
$Q^2 > 1 \text{(GeV/c)}^2$
$\gamma < 0.85$

$X.~Reutterstaff~et~al.,~P.L.B.404(87)38$
E155 (preliminary)

O E154

at $Q^2 = 5 \text{ GeV}^2$

E155 systematic error
$g_1^n(x)$ strongly decreasing for $x \rightarrow 0$?

low $x$ extrapolation?

New SMC results much flatter!!
The 'small x problem':

SLAC - E154 result:

- measured region:
  \[ \int_{0.014}^{0.3} g_1^n(x) \, dx = 0.036 \pm 0.004 \pm 0.005 \]

- power law fit: \( g_1^n(x) = -0.02 x^{-0.8} \)
  \[ \int_{0}^{0.014} g_1^n(x) \, dx = -0.041 \]

\[ 100\% \text{ error from low x extrapolation?} \]

**But:** Maybe 'problem' is over-emphasized by logarithmic presentation of data

- Need more precise data \( \forall \)very low \( x \)
  - HERA \((e\rightarrow\mu)\) (unfolding \( D \) very small)
  - RHIC
  - TeV - Linear - Collider
\( Q^2 = 5 \text{ GeV}^2 \)

\( (x^2)_{LJ} \)

\( (x)_{LJ} \)

Based on BCDS data

Based on EMC data

New parametrisation

H1, ZEUS, NMC
Table 4. Integrals over the polarised structure function $g_1$ from recent experiments.
Results for integrals over $Q^2 < 5\text{GeV}^2$

World data (without HERMES $\pi^0 \rightarrow e^+\nu\rightarrow e^-\nu\rightarrow e^-\gamma\rightarrow e^-\gamma$

(Recent SMC analysis)

<table>
<thead>
<tr>
<th>Particle</th>
<th>$\Gamma_1$</th>
<th>(Stat.)</th>
<th>(Syst.)</th>
<th>(theor.)</th>
<th>Extrapol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>proton</td>
<td>0.121 ± 0.003 ± 0.005 ± 0.026</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>neutron</td>
<td>-0.068 ± 0.007 ± 0.005 ± 0.023</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>deuteron</td>
<td>0.021 ± 0.004 ± 0.003 ± 0.023</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The singlet axial matrix element $a_0(1)$

\[ \bar{\text{MS}} \text{- scheme: } a_0(Q^2) = \Delta \Sigma(Q^2) = 0.19 \pm 0.05 \]

\[ \text{PB - scheme: } a_0(Q^2) = \Delta \Sigma(Q^2) - n_f \frac{\Sigma(Q^2)}{2\pi} \Delta g \]

\[ 0.38 \pm 0.03 \]  \[ = 0.24 \pm 0.07 \]
Bjorken Sum Rule satisfied
Ellis-Jaffe S.R. violated by 2-3 $\sigma$
Ellis–Jaffe Sum Rule

- Ellis–Jaffe Sum Rule is violated
- Deuteron data most significant
  \(4.6\sigma\) (E-143, 3 GeV\(^2\)) and \(3.5\sigma\) (SMC, 10 GeV\(^2\))
anomaly: q gluon contribution

Adler-Bardeen fact. scheme:

\[
\begin{align*}
\alpha_o(Q^2) &= \Delta \Sigma - n_f \frac{\Delta s(Q^2)}{2\pi} \Delta g(Q^2) \\
\alpha_{o,f}(Q^2) &= \Delta q_f - \frac{\Delta s(Q^2)}{2\pi} \Delta g(Q^2)
\end{align*}
\]
QCD Fits

Results are very scheme dependent

Comparison \( \overline{\text{MS}} \) and Adler Bardeen sch.

\[
\int_0^1 \Delta g(x) \, dx = 0.99 \pm 0.31 - 0.22 \pm 0.4,
\]

\( \overline{\text{MS}}: \quad \Delta g = 0.3 \pm \cdots \)

Gluon polarization badly known
Spin Puzzle

\[ \Delta \Sigma \approx 0.2 - 0.4 \]

confirmed by recent inclusive data!

What is the origin of the nucleon Spin?

Many models and theor. explanations

Need additional experimental info

\[ \Delta u_v(x) \quad ? \]
\[ \Delta \bar{u}(x) \quad ? \]
\[ \Delta d_v(x) \quad ? \]
\[ \Delta \bar{d}(x) \quad ? \]
\[ \Delta s(x) \quad ? \]
\[ \Delta q(x, Q^2) \quad ? \]
\[ L_\perp^2 (Q^2) \quad ? \]

Semi-inclusive data

Domain of HERMES (good PID)

(Compass → 2001)
Orbital angular momenta $L_z^q, L_z^g$

$2^2g^o \Delta \Sigma + 2 L_z^q = \frac{3 n_f}{16 + 3 n_f} = 2 \gamma^q = 0.43$

$2 \Delta g^q + 2 L_z^g = \frac{16}{16 + 3 n_f} = 2 \gamma^g = 0.57$

$\Delta L_z^q (q^2 = 3 - 106 \text{eV}^2) \approx 0.075$

Experimentally very difficult to attack (DIVC deeply virtual Compton scat.)
Semi-Inclusive DIS

\[ \frac{1}{\sigma_{tot}} \frac{d\sigma^h(x, Q^2, z)}{dz} = \frac{\sum_q e_q^2 q(x, Q^2) D^h_q(z)}{\sum_q e_q^2 q(x, Q^2)} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expression</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Four-momentum Transfer</td>
<td>( Q^2 )</td>
<td></td>
</tr>
<tr>
<td>Bjorken's scaling variable</td>
<td>( x = Q^2 / 2M \nu )</td>
<td>incl.</td>
</tr>
<tr>
<td>Parton density function</td>
<td>( q_f(x, Q^2) )</td>
<td>incl.</td>
</tr>
<tr>
<td>Energy fraction of hadron</td>
<td>( z_{lab} = E_h / \nu )</td>
<td>semi-incl.</td>
</tr>
<tr>
<td>Fragmentation function</td>
<td>( D^h_o(z) )</td>
<td>semi-incl.</td>
</tr>
</tbody>
</table>
- Proton - neutron pion multiplicity difference:

\[ r(x, z) = \frac{N_p^\pi^- - N_n^\pi^-}{N_p^\pi^+ - N_n^\pi^+} \]

- Extract flavour asymmetry of the light sea:

\[
\begin{align*}
\bar{d}(x) - \bar{u}(x) &= \frac{(1+D^-/D^+)[1 - r(x, z)] - [1 + r(x, z)]}{u(x) - d(x)} \\
&= \frac{(1+D^-/D^+)[1 - r(x, z)] + [1 + r(x, z)]}{u(x) - d(x)}
\end{align*}
\]

- Ratio of fragmentation functions \( D^-/D^+ \):

![Graph showing the ratio of fragmentation functions \( D^-/D^+ \)]

HERMES D/D' (x, P>4 GeV, sea corrected)
Assumption of factorisation a.k.
\[ u_v(x) > d_v(x) \]

\[ d\bar{d} - \text{sea} > u\bar{u} - \text{sea} \]
Particle Asymmetries

- Photon-Nucleon Asymmetry:

\[
A_1^h(x, z) = \frac{\sigma_h^+ - \sigma_h^-}{\sigma_h^+ + \sigma_h^-} = \frac{\sum_q e_q^2 (q^- - q^-) D_h^2(z)}{\sum_q e_q^2 (q^+ + q^-) D_h^2(z)}
\]

- Measured Asymmetry:

\[
A_{||}^h(x, z) = \frac{\sigma_h^{\perp} - \sigma_h^{\parallel}}{\sigma_h^{\perp} + \sigma_h^{\parallel}} = A_1^h(x, z) \cdot D,
\]

with photon depolarization factor

\[
D(x, Q^2) = \frac{y(2 - y)}{y^2 + 2(1 - y)(1 + R(x, Q^2))}.
\]

- Here we assumed the naive QPM.
- Also possible in improved QPM, factors of \((1 + R(x, Q^2))\) and \((1 + Q^2/\nu^2)\) have to be added.
Photon-Nucleon Asymmetry:

\[ A_1^h(x, z) = \sum_q \frac{e^2 q(x) D^q_{z^-} z^+}{\sum_{q'} e^2 q'(x) D^{q'}_{z^-} z^+} \cdot \frac{\Delta q(x)}{q(x)} \]

\[ = \sum_q P_q^h(x, z) \cdot \frac{\Delta q(x)}{q(x)} \]

- The hadron quark-purity \( P_q^h(x, z) \) is the probability that a quark \( q \) was struck in an event \( l + N \rightarrow l' + h + X \).

- Purities are spin-independent (unpolarized) quantities!

- Define

\[
\vec{A} = \begin{pmatrix}
A_h^1(x) \\
\ldots \\
A_h^{1m}(x)
\end{pmatrix}, \quad \vec{Q} = \begin{pmatrix}
\Delta q_1(x)/q_1(x) \\
\ldots \\
\Delta q_n(x)/q_n(x)
\end{pmatrix}, \quad P = \begin{bmatrix}
P_q^h(x) \end{bmatrix}.
\]

To measure quark polarizations invert:

\[ \vec{A} = P \vec{Q} \]
Purities for proton target

dominated by $u(x), \bar{u}(x)$

dependence very small
Polarised quark distributions ('35+’)

$Q^2 > 1 \text{ GeV}^2$

$w^2 > 4 \text{ GeV}^2$

$0.023 < x < 0.6$

HERMES preliminary $(\Delta u_{\text{sea}} / u_{\text{sea}} = \Delta d_{\text{sea}} / d_{\text{sea}} = \Delta s / s)$

Similar results for

* assumption $\Delta u = \Delta d = \Delta s$

* different choice of $D_n (x)$
Errors dominated by correlations!

**statistical errors only**

- HERMES preliminary ($\Delta u_{sea}/u_{sea} = \Delta d_{sea}/d_{sea} = \Delta s/s$)
- SMC 97 ($\Delta u_{sea} = \Delta d_{sea} = \Delta s$)

* Reasonable agreement with SMC
* '97 data: 3-times higher statistics for $p$
* Need high-quality deuteron data → '98
HERMES preliminary ($\Delta u_{sea}/u_{sea} = \Delta d_{sea}/d_{sea} = \Delta s/s$)

\[ \frac{\Delta u + \Delta u}{(u + \bar{u})} \]

\[ \frac{\Delta d + \Delta d}{(d + \bar{d})} \]

\[ \frac{\Delta q_{sea}/q_{sea}}{X_{BJ}} \]

$10^{-1}$
\[ \Delta g (x, Q^2) \] rather weakly constrained by present data

- Study processes where \( \Delta g (x) \) occurs directly already in L.O. \( \gamma^*(x)\Delta g \rightarrow c \bar{c} \)

- open charm production
  \( D \rightarrow K^+ \tau \), \( D \rightarrow K^+ \mu \nu \)

- high \( p_T \) pions, kaons

\[ \tilde{p}p \rightarrow \gamma + \text{jet} ; \tilde{p}p \rightarrow X_2 (3550) \text{ RHIC} \]

\[ \rightarrow \gamma + X \]

\[ \rightarrow \text{jet} + X \]

\[ \rightarrow \text{jet} + \text{jet} \]

HERA \( \tilde{p}p \) fixed
The COMPASS Spectrometer

- Spectrometer Components
  Tracking detectors, Rich Detectors, electromagnetic and hadronic Calorimeters, Magnets, Muon filters (iron)
Upgrade I: Dual radiator RICH

- Clear silica aerogel \( n = 1.03; \gamma_n = 4.2 \)
- \( \text{C}_4 \text{F}_{10} \) \( n = 1.0005; \gamma_n = 32 \)

<table>
<thead>
<tr>
<th></th>
<th>Aerogel</th>
<th>Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold (( \pi, K, p )) GeV/c</td>
<td>0.6, 2, 1.3, 3</td>
<td>4, 15.8, 30</td>
</tr>
<tr>
<td>Separation (( \pi/K, K/p )) GeV/c</td>
<td>1-9; 2-15.5</td>
<td>4-23.5; 10 &gt; 25</td>
</tr>
</tbody>
</table>

\[ \text{Aerogel (5cm)} \]

\[ \text{C}_4 \text{F}_{10} \]

\[ \theta_{\text{threshold}} \text{ vs Momentum} \]
\[ g_2(x) = -g_1(x) + \int \frac{d^2}{x^2} g_4(x) + \bar{g}_2(x) \]

Quark-Gluon-Correlation (Twist-3 Operator)

Long. pol. beam

Long. + transverse pol. target

Interesting Sum Rules:

1. \[ \int g_2(x) \, dx = 0 \quad (\text{Buranov-Cottingham}) \]

2. \[ \int x^n g_2(x) \, dx = \frac{1}{4} \frac{n}{n+1} (-a_n + d_n) \quad n = 2, 4, \ldots \]

3. \[ \int x^n g_4(x) \, dx = \frac{a_n}{4} \quad \text{Twist-3} \quad n = 0, 2, 4, \ldots \]

Precise measurement of \( g_2(x) \) very helpful in determining the nucleon wave function
\( P^2, \text{ very small!} \)
$g_2(x)$ small

Data compatible with twist-2 ($W\bar{W}$)

Twist-3 very small
I
\{0.02 \sim Proton
\}

E - \sim___ ___ ___ ___ _
\simNeutron
_— _ ___ ___ 
F

-Lj..-

\sim-0.02
\sim_{/- ~–– –__ _}

\sim$\sim1$

\[PI_white_{CTIONS and DATA}

\simQCD Sum Rules

\simBag Model

\simLattice

\simNeutron

\sim0.00
d_2

\sim0.02

\sim0.00
d_2

\sim0.02

\sim0.00
d_2

\sim0.02

\sim0.00

\simPwhite_{CTIONS and DATA
FESpectives

HERMES

1998/99: \( \overline{D} \) (+ RICH, \( \mu \)-wall, FQS, \( \Lambda \)-wheel)

- \( g_1^d, g_1^n \), Bjorken S.R
- \( \Delta u, \Delta d, \Delta \bar{u}, \Delta \bar{d} \)
- \( \Delta g \) open charm \( D \rightarrow K\pi, K\mu\nu \) \( \langle x_g \rangle \sim 0.3 \)
- high \( \rho_t \) \( \pi \) \( \langle x_g \rangle \sim 0.15 \)
- \( \delta \left( \frac{\Delta g}{g} \right) = 0.45 \pm 0.2 \)
- \( \Lambda \) - polarisation \( J \)-transfer

- nuclear (unpol.) targets \( (D, ^3He, N, K^+) \)
  - hadronisation in nuclei
  - \( g, \phi \) - production \( (\text{ang}, \text{disly, nucl. transp.}) \)

---------------

2000 \rightarrow 2002

- \( H \) (+ RICH \ldots )

- transverse target pol.
  - \( g_2 \), \( b_1(x) \), \( \Delta(x) \), \( h_1(x) \) \( \ldots \)

SLAC - E15X

1999: transverse target pol.: \( g_2 \), \( d_2 \)

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COMPASS/CERN 2000/1

- 2-stage spectrometer (ext. of SMC) hadron- ID with 2 RICH,
  high luminosity, \( E_\mu = 100 - 200 \text{ GeV} \)
  \( ^t\text{LiD} \), \( ^t\text{NH}_3 \)-targets

- Main aim:
  \[ \Delta Q_8 \]
  open charm \( 0.07 < x_8 < 0.4 \) \( \sigma(\frac{\Delta Q_8}{Q}(0.14)) = 0.1 \)
  high-\( p_t \) \( K^+, H^+ (0.04 < x_q < 0.2) \) \( \sigma(\frac{\Delta Q_8}{Q}(0.1)) = 0.05 \)

- \( g_2, h_1, \ldots \)

\[ \text{RHIC} \] 2000

- \( \vec{p} \vec{p} \); \( \sqrt{s} = 50 - 500 \text{ GeV} \); PHENIX, STAR

- \[ \Delta Q_8 \]
  \( \vec{p} \vec{p} \rightarrow \pi^+ \text{jet} \ \bar{\chi} \bar{q} + q \rightarrow \pi^+ q \)
  200 GeV: \( 0.07 < x_q < 0.2 \) \( \sigma(\frac{\Delta Q_8}{Q}) = 0.05 \rightarrow 0.4 \)
  500 GeV: \( 0.03 < x_q < 0.1 \) \( \sigma = 0.05 \rightarrow 0.2 \)

- \[ \Delta u/u \] \[ \Delta d/d \]
  \( A_L \) \( x > 0.05 \)

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\[ \text{HERA} - \vec{p} \quad 200 \rightarrow \]

\[ \bar{e} \rightarrow \vec{p} \quad \text{\textit{p}} \text{S} = 300 \text{ GeV} \]

\[ 27.5 \text{ GeV} \quad 820 \text{ GeV} \]

\[ g_\pi(x) \times \sqrt{5 \cdot 10^{-5}} \]

\textit{(but: depd factor D large \rightarrow error)}

\[ \Delta g \quad \times \text{NLO-fits} \]

\[ \times 2 \cdot \text{jet events} \times \sqrt{5 \cdot 10^{-4}} \]
Conclusions

- Many new and interesting results.

- Still plenty of open questions:
  * $\Delta G(x)$
  * $L^q$
  * $\Delta q_f(x)$
  * Low-$x$ extrapolation
  * $q^2(x)$, $\rightarrow d_3$
  * $Q^2$-dependence of polarised quark distribution

- Plenty of exciting results to be expected in the future
The RHIC spin program is a new type of the experiments to study the spin-dependent structure function of the nucleon and probe possible new phenomena.

The RHIC (Relativistic Heavy Ion Collider) is now under constructing at BNL will be the first polarized proton-proton collider. Previously it was not possible to keep the polarization inside the synchrotron, but special sets of magnets, Siberian snake, make this type of experiment possible. The polarized proton acceleration study at AGS, which will be used as an injector, has made a big progress and now it has more than 0.50 polarization at over 20 GeV/c. The proton beam polarimetry is also important and many activities are going on.

The quark spin contributions to the nucleon spin have been studied extensively in polarized deep inelastic scattering (pDIS) experiments. However, it was found that the quark contribution is very small and the gluon spin contribution has a very weak constraint. The gluon contribution can be studied with prompt photon which mostly comes from quark-gluon compton reaction at RHIC energy. Fine granular PHENIX EM Calorimeter is suitable to suppress a large background and large acceptance of STAR EMCal allows to detect aside jet together with photon. Moreover, pDIS inclusive reaction alone is not possible to separate individual quark or anti-quark flavor contribution. The flavor decomposition can be done with W-boson production at RHIC and high transverse momentum lepton is a W decay signature.

In addition to these important subjects, this program can measure new structure function, the transversity distribution, through the transversely polarized proton collision. Studying parity violation may give us a hint for a beyond standard model physics. The experiments contain those programs will start taking data in the year 2000.
Introduction

- Polarized Nucleon Structure Function

  - Proton Spin Crisis:

    \[ \Delta \Sigma \approx 0, \Delta s \approx -0.2 \]

  - More pol-DIS experiments — Update on \( \Delta \Sigma \):

    (S.M.C. SLAC HERMES 1992~)

    \[ \Delta \Sigma \approx 0.3 \]

  - We know

    \[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_z \]

  - Rest of the nucleon spin in \( \Delta G \)?
    QCD NLO analysis \( \Rightarrow \Delta G \approx 0 \sim 2 \pm 2 \)
    * Current knowledge on \( \Delta G \) is not satisfactory

  - Some information about flavor decomposition

    (\( \Delta u, \Delta d, \Delta s \)). But always \( \Delta \bar{u}(x) = \Delta \bar{d}(x) \)?

    C.Bourrely et al., hep-ph/9803229

    Unpolarized case \( \Rightarrow \bar{u}(x) \neq \bar{d}(x) \) (N.M.C. NACHT, ESGT)

  - Polarized RHIC (PHENIX, STAR) will measure

    \( \Delta G(x), \Delta \bar{q}(x) \ldots \)

  - Transversity Distribution
Relativistic Heavy Ion Collider (RHIC)

as The First Polarized Proton-Proton Collider

RIKEN-BNL Collaboration

Since 1995

\[ \sqrt{s} = 50 \sim 500 \text{ GeV} \]

- Beam polarization: \( P_B \simeq 0.7 \)

- Luminosity:
  \[ \mathcal{L} = 8 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1} \text{ at } \sqrt{s} = 200 \text{ GeV} \]
  \[ \mathcal{L} = 2 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1} \text{ at } \sqrt{s} = 500 \text{ GeV} \]

- Integrated Luminosity:
  - 10 weeks, 70\% efficiency with full luminosity
  \[ \int \mathcal{L} dt = 320 \text{ pb}^{-1} \text{ at } \sqrt{s} = 200 \text{ GeV} \]
  \[ \int \mathcal{L} dt = 800 \text{ pb}^{-1} \text{ at } \sqrt{s} = 500 \text{ GeV} \]
Todays Status (Nov '98)

Current PHENIX Experimental Hall picture


STAR construction picture of the day
**Hadron-Hadron Reaction**

\[ A + B \rightarrow C + X \]

- Parton distributions with \( x_a, x_b \):
  \[ f_{a/A}(x_a), f_{b/B}(x_b) \]

- Parton reaction:
  \[ \frac{d\sigma_{ab}}{dt} \]

- Fragmentation:
  \[ D_{C/c}(z) \]

\[
E \frac{d^3\sigma}{dp^3} \sim \sum_{abcd} f_{a/A}(x_a) \otimes f_{b/B}(x_b) \otimes \frac{d\sigma_{cd}}{dt} \otimes D_{C/c}(z)
\]

**Asymmetry for \( A + B \rightarrow C + X \)**

\[ A = E \frac{d^3\Delta\sigma}{dp^3} / E \frac{d^3\sigma}{dp^3} \approx \frac{\Delta f_{a/A}(x_a)}{f_{a/A}(x_a)} \otimes \frac{\Delta f_{b/B}(x_b)}{f_{b/B}(x_b)} \otimes \hat{a}_{LL}(ab \rightarrow cd) \]

- \( A \): Experimentally measured asymmetry

- Polarized parton distributions with \( x_a, x_b \):
  - \( \Delta f_{a/A}(x_a), \Delta f_{b/B}(x_b) \)

- Parton level asymmetry:
  \( \hat{a}_{LL}(ab \rightarrow cd) \)
  - calculated in pQCD

- Fragmentation:
  - Not relevant for \( \gamma, W \)
  - Cancel for asymmetry in principle
**Prompt Photon**

\[ p\bar{p} \rightarrow \gamma + X \]

- Two processes to produce \( \gamma \)
  
  (a) Gluon Compton
  
  \[ g + q \rightarrow \gamma + q \]
  
  80 \( \sim \) 90 \%
  
  Dominant

  (b) Annihilation
  
  \[ q + \bar{q} \rightarrow \gamma + g \]
  
  10 \( \sim \) 20 \%

\[ A_{LL}(p_T) = \frac{\Delta G(x)}{G(x)} \frac{g_1(x)}{F_1(x)} A_{LL}(g + q \rightarrow \gamma + q) \]

![Graph showing \( A_1 \) vs. \( x_{\text{quark}} \) for DIS and pDIS experiments.]

- Theoretically rather clear process

N. Hayashi
Prompt Photon Asymmetry and Sensitivity

- $\Delta G = 1.71$
- $\Delta G = 1.63$
- $\Delta G = 1.02$

5-30% at $\sqrt{s} = 200$ GeV

5-20% at $\sqrt{s} = 500$ GeV
Anti-quark polarization measured with $A_L^W$

More systematic studies are underway

- background from $\pi/K$ decays
- background from $Z^0$ decays
**Summary**

- Significant Progress on Polarized proton acceleration study
- Many Polarimetry works going on
- Detectors installation progress
- Spin Structure Function Studies
  - $\Delta G(x)$ measurements:
    * prompt $\gamma^* 5\text{--}30\%$ at $\sqrt{s} = 200$ GeV,
      $5\text{--}20\%$ at $\sqrt{s} = 500$ GeV (PHENIX)
    * $\gamma + jet$, di-jets (STAR)
    * $\pi^0$ inclusive, open heavy quark, quarkonium production
  - $\Delta q(x)$ measurements:
    * 2\% error through $A_L^W$
- further systematic studies are underway ...
- New measurements
  - Transversity
  - Single spin asymmetry $A_N$
  - Search for new parity violated interaction
**RHIC/Spin Plan**

- RHIC turn on (Engineering Run), January to July 1999
- Siberian Snakes installation (1999 summer)
- RHIC Physics Run, October 1999!
  - Polarized proton commissioning
  - $1 \sim 10\%$ luminosity (?)
- Spin rotators installation (2000 summer)
- First polarized proton-proton physics run (2000-01)
  Heavy Ion $pp$ comparison run
- Full capabilities: $\sqrt{s} = 500$ GeV (2002-?)
Twist-3 Effects in Hard Processes

Yuji Koike

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Abstract

The quark and gluon distribution functions in the nucleon can be classified by the "twist". Twist-2 distributions have a simple parton model interpretation and contribute to the cross sections with no power suppression with respect to the the hard momentum. On the other hand, higher twist distributions represent complicated quark-gluon correlation and appear in the cross section as a power suppressed contribution. In general, it is difficult to measure the higher twist contributions in hard processes, since they are hidden behind the leading twist-2 contributions. However, when the leading twist-2 contribution is absent due to a kinematics, one can measure some twist-3 contributions. As an example for this type of interesting "twist-3 processes", We discuss (1) Longitudinal-transverse spin-asymmetry in the nucleon-nucleon polarized Drell-Yan process, and (2) Semi-inclusive production of the polarized spin-1/2 baryon in the deep-inelastic scatterings.

In the first part of this talk, we present a first estimate for the longitudinal-transverse spin asymmetry \( A_{LT} \) in the nucleon-nucleon polarized Drell-Yan process at RHIC and HERA-\( \bar{N} \) energies in comparison with \( A_{LL} \) and \( A_{TT} \). \( A_{LT} \) receives contribution from \( g_1 \), the transversity distribution \( h_1 \), and the twist-3 distributions \( g_T \) and \( h_L \). For the twist-3 contribution we use the bag model prediction evolved to a high energy scale by the large-\( N_c \) evolution equation. We found that \( A_{LT} \) (normalized by the asymmetry in the parton level) is much smaller than the corresponding \( A_{TT} \). Twist-3 contribution given by the bag model also turned out to be negligible.

In the second part of this talk, we analyze the semi-inclusive deep-inelastic scattering from the nucleon detecting a polarized spin-1/2 baryon in the final state. It turns out that with the unpolarized electron beam and the transversely (longitudinally) polarized nucleon target, detection of the longitudinally (transversely) polarized spin-1/2 baryon gives rise to a leading twist-3 contribution to the spin-asymmetries.
Twist Expansion for Inclusive Cross Sections

\[ \sigma(Q^2) \sim \sigma_0(\ln Q^2) + \frac{\Lambda_{QCD}}{Q} \sigma_1(\ln Q^2) + \frac{\Lambda_{QCD}^2}{Q^2} \sigma_2(\ln Q^2) + \ldots \]

- Twist-2
  - "Parton model"
  - Incoherent scatt. from each parton

Twist-3
  - Quark-Gluon correlation
  - Peculiar to spin-dependent processes

Twist-4

Interesting "Twist-3" processes

\[ \sigma(Q^2) \sim \sigma_0(\ln Q^2) + \frac{\Lambda_{QCD}}{Q} \sigma_1(\ln Q^2) + \frac{\Lambda_{QCD}^2}{Q^2} \sigma_2(\ln Q^2) + \ldots \]

- Kinematic suppression
- New test of p-QCD beyond twist-2

- $e^+p \rightarrow e' + X : g_T(x, Q^2)$
- $\bar{p} + p \rightarrow e^+e^- + X : h_L(x, Q^2), g_T(x, Q^2)$

semi-inclusive DIS

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**Polarized N-N Drell-Yan**

\[
P_A \cdot S_A \\
\begin{array}{c}
\rightarrow X \\
\times \rho_A \\
y \rho_B \\
\end{array} \\
Q \\
\rightarrow X \\
P_B \cdot S_B
\]

\[S = (P_A + P_B)^2 \quad (= (CM \text{ energy})^2)\]

\[Q^2 = (x \rho_A + y \rho_B)^2 \approx xy S \quad (M_N^2 \ll S)\]

*Double spin asymmetry*

\[
A_{SA SB} = \frac{\sigma(S_A S_B) - \sigma(S_A, -S_B)}{\sigma(S_A S_B) + \sigma(S_A, -S_B)}
\]

\[S_A, S_B = +, - : \text{Longitudinal pol.}\]

\[\uparrow \downarrow : \text{Transverse pol.}\]
\[ A_{LL} = \frac{\sigma^{(+)} - \sigma^{(-)}}{\sigma^{(+)} + \sigma^{(-)}} = \frac{\sum_a e_a^2 g_1(x, Q^2) g_{1a}(y, Q^2)}{\sum_a e_a^2 f_i^a(x, Q^2) f_i^{\bar{a}}(y, Q^2)} \]

\[ A_{TT} = \frac{\sigma^{(\perp)} - \sigma^{(\perp)}}{\sigma^{(\perp)} + \sigma^{(\perp)}} = \frac{\sin^2 \theta \cos 2\phi}{1 + \cos^2 \theta} \frac{\sum_a e_a^2 h_1^a(x, Q^2) h_{1a}(y, Q^2)}{\sum_a e_a^2 f_i^a(x, Q^2) f_i^{\bar{a}}(y, Q^2)} \]

\[ A_{LT} = \frac{\sigma^{(\pm)} - \sigma^{(\mp)}}{\sigma^{(\pm)} + \sigma^{(\mp)}} = \frac{2 \sin2\theta \cos \phi}{1 + \cos^2 \theta} \frac{M}{Q} \frac{\sum_a e_a^2 [g_1^a(x, Q^2) y g_{1a}(y, Q^2) + x h_1^a(x, Q^2) h_{1a}(y, Q^2)]}{\sum_a e_a^2 f_i^a(x, Q^2) f_i^{\bar{a}}(y, Q^2)} \]

---

Quark distribution functions (Jaffe-Ji, '92)

<table>
<thead>
<tr>
<th>Spin twist</th>
<th>Average</th>
<th>Longitudinal</th>
<th>Transverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>twist-2</td>
<td>( f_i(x, \mu^2) )</td>
<td>( g_1(x, \mu^2) )</td>
<td>( h_1(x, \mu^2) )</td>
</tr>
<tr>
<td>twist-3</td>
<td>( e(x, \mu^2) )</td>
<td>( h_L(x, \mu^2) )</td>
<td>( g_T(x, \mu^2) )</td>
</tr>
</tbody>
</table>
Distribution functions in QCD

= Light-cone correlation functions in a Nucleon

Assume $\hat{\Phi} \parallel \hat{e}_3$

Put $\phi(z) = \psi_+(z)$ : Light-cone "good" component

\[
\left( \begin{array}{c}
z^+ = z = 0 \\
z^z = 0 \\
z^\perp = \frac{1}{2}(z^0 \pm z^3)
\end{array} \right)
\]

Dynamically independent variables.

\[
\psi_{\perp} \equiv P^+ \psi, \quad P^+ \equiv \frac{1}{2}(1 \mp \gamma^+) \rho_{\perp}
\]

$\star$ Twist-2

\[
g_1(x, \mu^2) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^-z^-} \left< \Phi(0) \gamma^+ \gamma_5 \phi(z) \right|_{\mu^2}^{P_{\perp}} \left| \Phi(0) \right>
\]

\[
h_1(x, \mu^2) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^-z^-} \left< \Phi(0) \gamma^+ \gamma_5 \phi(z) \right|_{\mu^2}^{P_{\perp}} \left| \Phi(0) \right>
\]

$\star$ Twist-3 $\star$ Twist-2

\[
g_T(x) = \int_{-x}^{1} \frac{g_1(y)}{y} \, dy + \tilde{g}_T(x) \equiv g_T^{uw}(x) + \tilde{g}_T(x)
\]

\[
h_L(x) = 2x \int_{x}^{1} \frac{h_1(y)}{y^2} \, dy + \tilde{h}_L(x) \equiv h_L^{uw}(x) + \tilde{h}_L(x)
\]

Wandzura-Wilczek part purely twist-3 part

\[
\tilde{h}_L(x, \mu^2) = \frac{ip^+}{M} \int_{-\infty}^{\infty} \frac{dz^-}{2\pi} e^{-2iP^-z^-} \int_{0}^{1} \frac{d\bar{u}}{u} \frac{dz^2}{2\pi} \int_{-\infty}^{\infty} \frac{dt}{t^2} \left< \Phi(uz) \gamma_5 \gamma^+ g_{T+}(t\vec{z}) \phi(-uz) \right|_{\mu^2}^{P_{\perp}} \left| \Phi(0) \right>
\]

quark-glueon correlation

(Similarly for $\tilde{g}_T(x, \mu^2)$)
Estimate $A_{LT}$ using Bag model and available parametrization for distribution functions:

- $f_1$ (GRSV standard distribution) 
  - Glück et al., Phys. Rev. D53, 4775 (96)
- $g_1$ (GS (A) distribution) 
  - Gehrmann-Striling, Phys. Rev. D53, 6100 (96)

$h_1$: Assume at low $Q^2$

$$h_1(x, Q^2) = g_1(x, Q^2)$$

- GRSV @ $Q^2 = 0.23 \text{ GeV}^2$
- GS @ $Q^2 = 1 \text{ GeV}^2$

- This is suggested by a low energy effective model for the nucleon

- LO Evolution equation

$$h_1(x, Q^2) @ \text{high } Q^2$$
Twist-3 distributions

\[ g_T(x, q^2) = g_T^{\text{hw}}(x, q^2) + \tilde{g}_T(x, q^2) \]
\[ h_L(x, q^2) = h_L^{\text{hw}}(x, q^2) + \tilde{h}_L(x, q^2) \]

Evolution, separately

\[ \tilde{g}_T(x, q^2), \tilde{h}_L(x, q^2) \]

Low scale \[ \text{MIT bag model} \]

\[ \mu^2_{\text{bag}} = 0.081 \text{ GeV}^2 \]

\[ \mu^2_{\text{bag}} = 0.25 \text{ GeV}^2 \]

\[ \text{Large-}N_c \text{ evolution} \]

(Ali et al., P.L. B266, 117 (91))
(Balitsky et al., P.R.L. 77, 3078 (96))

- As simple as twist-2 evolution
- Accurate to \(O(1/N_c^2) \sim 10\%\)

High scale

(Kanazawa-Koike, P.L. B403, 357 (97))

NB. Ignore mixing with gluon distribution for \(\tilde{g}_T(x, \mu^2)\)
Results for the asymmetries

- $-A_{LT}/a_{LT}$ in comparison with $-A_{LL}, -A_{TT}/a_{TT}$

- Kinematic Variables

  $S$, $Q^2$ and $x_F = x - y$

  $Q^2 = x y S \implies x = \frac{1}{2} \left[ x_F + \sqrt{x_F^2 + \frac{4Q^2}{S}} \right]$

  $y = \frac{1}{2} \left[ -x_F + \sqrt{x_F^2 + \frac{4Q^2}{S}} \right]$

- CM frame

\[
\begin{array}{c}
\bullet \\
+ \rightarrow \rightarrow \leftarrow \leftarrow \\
-0.5 & 0 & 0.5 \\
\end{array}
\]

$\sqrt{s} = 50, 200 \text{ GeV}$

(cf. RHIC, $50 < \sqrt{s} < 500 \text{ GeV}$

HERA-\overrightarrow{N}, $\sqrt{s} = 40 \text{ GeV}$)
Figure 2: Double spin asymmetries, $\tilde{A}_{LL}$, $\tilde{A}_{TT}$, $\tilde{A}_{LT}$, for the polarized Drell-Yan using the GRSV parton distribution and the bag model at $Q = 8, 10 \text{ GeV}$ and $\sqrt{s} = 50, 200 \text{ GeV}$. The solid line denotes $\tilde{A}_{LT}$ with only the Wandzura-Wilczek contributions in $g_T$ and $h_L$. The short dash-dot line denotes $\tilde{A}_{LT}$ with the bag scale $\mu_{bag}^2 = 0.25 \text{ GeV}^2$, and the dotted line denotes $\tilde{A}_{LT}$ with the bag scale $\mu_{bag}^2 = 0.081 \text{ GeV}^2$. The long dashed line corresponds to $\tilde{A}_{LL}$, and the long dash-dot line corresponds to $\tilde{A}_{TT}$. 
Summary

- Estimate of $-AL_T/a_{LT}$, $-AL_L$, $-AT_T/a_{TT}$ using GRSV & GS parton distributions and Bag model.

- Asymmetries $\uparrow$ as $Q^2/s \uparrow$.

- $|AL_T/a_{LT}| < |AL_L|$, $|AT_T/a_{TT}|$

- Bag model input for the purely twist-3 pants $\tilde{h}_L(x, Q^2)$, $\tilde{g}_T(x, Q^2)$

  $\rightarrow$ Small

  Because of strong $Q^2$-dependence of those pieces.

$\Rightarrow$ WAIT FOR EXP. AT RHIC & HERA-$\overline{\text{N}}$. 

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Polarized proton-deuteron Drell-Yan processes

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ABSTRACT

In order to investigate spin structure of spin-1 hadrons, we study general formalism of proton-deuteron Drell-Yan processes [1]. Because of the spin-1 nature of the deuteron, there are new structure functions which cannot be measured in the proton-proton reactions. Imposing Hermiticity, parity conservation, and time-reversal invariance, we find that 108 structure functions exist in the Drell-Yan processes. However, the number reduces to 22 after integrating the cross section over the virtual-photon transverse momentum $Q_T$ or after taking the limit $Q_T \to 0$. There are 11 new structure functions in addition to the 11 ones in the proton-proton Drell-Yan processes. The additional structure functions are associated with the tensor structure of the deuteron, and they could be measured by quadrupole spin asymmetries. We show a number of spin asymmetries for extracting the polarized structure functions.

Next, we analyze the polarized proton-deuteron Drell-Yan processes in a naive parton model [2]. Quark and antiquark correlation functions are expressed in terms of possible combinations of Lorentz vectors and pseudovectors with the constraints of Hermiticity, parity conservation, and time-reversal invariance. Then, we find tensor polarized distributions for the deuteron. There are only four finite structure functions and the others vanish after integrating the cross section over the virtual-photon transverse momentum $Q_T$ or after taking the limit $Q_T \to 0$. One of the finite structure functions is related to the tensor structure function $b_1$, and it does not exist in the proton-proton reactions. The vanishing structure functions should be associated with higher-twist physics. The tensor distributions, particularly the tensor-polarized antiquark distributions, can be measured by the quadrupole polarization measurements. The proton-deuteron reactions may be realized in the RHIC-Spin project and other future ones.

References

Research purpose

description of proton-deuteron Drell-Yan

• Spin-1/2 hadrons (proton)

  Proton spin structure:
  well investigated theoretically and experimentally

• Spin-1 hadrons
  – tensor spin structure as a new ingredient
  – ed scattering: some theoretical studies
  – pd Drell-Yan: no paper
    → investigate polarized pd Drell-Yan process

Purposes

1) test our knowledge of spin physics
   by another observable

2) understand tensor structure in a parton model

3) establish theoretical formalism
   for the next-generation RHIC-Spin project
\[
\int dx \ b_1^p(x) = \frac{5}{6} \left[ \Gamma_{0,0} - \frac{1}{2} (\Gamma_{1,1} + \Gamma_{-1,-1}) \right] + \frac{1}{9} (\delta Q + \delta \overline{Q})_{\text{sea}}
\]

macroscopically

\[
\Gamma_{0,0} = \lim_{t \to 0} \left[ F_c(t) - \frac{t}{3 M^2} F_Q(t) \right]
\]

\[
\Gamma_{+1,+1} = \Gamma_{-1,-1} = \lim_{t \to 0} \left[ F_c(t) + \frac{t}{6 M^2} F_Q(t) \right]
\]

F.E. Close & SK, 

\[
\int dx \ b_1^D(x) = \lim_{t \to 0} - \frac{5}{12} \frac{t}{M^2} F_Q(t) + \frac{1}{9} (\delta Q + \delta \overline{Q})_{\text{sea}}
\]

\[
\Rightarrow \lim_{t \to 0} - \frac{5}{12} \frac{t}{M^2} F_Q(t)
\]

Gottfried sum rule

\[
\frac{d}{dx} [F_2^p(x) - F_2^n(x)] = \frac{1}{3} \int dx [u_v - d_v] + \frac{2}{3} \int dx [\bar{u} - \bar{d}]
\]

Experimental possibilities

HERA, ELFE, RCNP ?
\[
\frac{d\sigma}{d^4Q\,d\Omega_0} = \frac{1}{4\pi} \sum_{L,M} f(L) \, D^L_{M0} (\phi_0, \theta, 0) \sum_{\lambda, \lambda'} \sum_{X} \delta^4 (P_A + P_B - Q - P_X) \times <1\lambda : LM|1\lambda'> \sum_{\mu, \mu'; \nu, \nu'} \rho(AB)_{\mu\nu; \mu'; \nu'} F^{\dagger}_{\lambda'\mu'; \nu'} F_{\lambda\mu\nu}
\]

\[
F_{\lambda\mu\nu} = \sqrt{\frac{\pi \alpha^2}{3 s k^2}} \langle X | \tilde{\varepsilon}^*(\lambda) \cdot \tilde{J}(0) | A(\mu) B(\nu) \rangle
\]

Spin density matrix:

\[
\rho(AB)_{\mu\nu; \mu'; \nu'} = \rho(A)_{\mu\mu'} \rho(B)_{\nu\nu'}
\]

\[
\rho = \frac{1}{2S + 1} \sum_{K, N} <\tau_{KN}(S) > \tau_{KN}^\dagger(S)
\]

\[
\tau_{KN}(S) = \sqrt{2S + 1} \sum_{\mu, \mu'} (-1)^{S-\mu} < S\mu' : S - \mu | KN > | S\mu' > < S\mu |
\]

\[
\rho_{\mu\mu'} = < S\mu | \rho | S\mu' >
\]

\[
= \sum_{K, N} \frac{\sqrt{2K + 1}}{2S + 1} < S\mu' : KN | S\mu > < \tau_{KN}^\dagger(S) >
\]

\[
< \tau_{00}^\dagger(S) > = 1 , \quad < \tau_{1x}^\dagger(1/2) > = | \bar{S}_A | D_{N0}^1 (\alpha_A - \Phi, \beta_A, 0)
\]

\[
< \tau_{1x}^\dagger(1) > = \sqrt{\frac{3}{2}} | \bar{S}_B | | \bar{S}_A |\sum_{N_2} R_{L00}^{M0N_2} D_{N20}^1 (\alpha_B + \Phi, \beta_B, 0), \quad < \tau_{2x}^\dagger(1) > = \sqrt{\frac{3}{2}} \bar{S}_B^2 D_{N0}^2 (\alpha_B + \Phi, \beta_B, 0)
\]
Structure functions

\[ R_{L K_1 K_2}^{M N_1 N_2} = \frac{1}{6} \sum_{\lambda, \mu, \nu} \sum_X \delta^4(P_A + P_B - Q - P_X) < 1 \lambda : LM|1\lambda' > \times F_{\lambda^* \mu^* \nu} \cdot F_{\lambda \mu \nu} < \frac{1}{2} \mu' : K_1 N_1 |\frac{1}{2} \mu > < 1 \nu' : K_2 N_2 |1 \nu > \]

\[ R_{L K_1 K_2}^{M N_1 N_2} = (-1)^{M+L+M_1+N_2} \left( R_{L K_1 K_2}^{M-M_1-N_2} \right)^* \text{ (Hermiticity)} \]

\[ R_{L K_1 K_2}^{M N_1 N_2} = \begin{cases} 
\text{real} & L + K_1 + K_2 = \text{even} \\
\text{imaginary} & L + K_1 + K_2 = \text{odd} \\
0 & L + K_1 + K_2 = \text{odd and } M + N_1 + N_2 = 0 
\end{cases} \]

Imposing these constraints, we obtain 108 independent structure functions.

Integration over \( \Phi \) (azimuthal angle of \( \vec{Q}_T \)) : 22

\[ 2 \int d\Phi \frac{d\sigma}{dQ d\Omega} = f(0) \left[ R_{000}^{000} + \frac{3\sqrt{5}}{2} |S_A| |S_B| \left\{ \cos \beta_A \cos \beta_B R_{000}^{000} + \sin \beta_A \sin \beta_B \cos(\alpha_A + \alpha_B) R_{011}^{011} \right\} \right. \]
\[ + \frac{\sqrt{5}}{2} |S_B|^2 (3 \cos^2 \beta_B - 1) R_{200}^{200} - 3\sqrt{10} |S_A| |S_B| \sin \beta_A \sin \beta_B \cos \beta_A \sin \beta_B \sin(\alpha_A + \alpha_B) i R_{011}^{011} \]
\[ + f(2) (3 \cos^2 \theta - 1) \left[ \frac{1}{2} R_{200}^{000} + \frac{3\sqrt{5}}{2} |S_A| |S_B| \left\{ \cos \beta_A \cos \beta_B R_{211}^{211} + \sin \beta_A \sin \beta_B \cos(\alpha_A + \alpha_B) R_{211}^{211} \right\} \right. \]
\[ + \frac{\sqrt{5}}{2} |S_B|^2 (3 \cos^2 \beta_B - 1) R_{200}^{200} + \frac{5\sqrt{5}}{\sqrt{2}} |S_A| |S_B| \sin \beta_A \sin \beta_B \cos \beta_A \sin \beta_B \sin(\alpha_A + \alpha_B) i R_{211}^{211} \]
\[ + f(2) \sin \theta \cos \theta \left[ \sin(\phi - \alpha_A) \left\{ 3 |S_A| \sin \beta_A i R_{110}^{110} + \frac{3\sqrt{5}}{\sqrt{2}} |S_A| |S_B| \sin \beta_A (3 \cos^2 \beta_A - 1) i R_{112}^{112} \right\} \right. \]
\[ - \sin(\phi + \alpha_B) \left\{ \frac{3\sqrt{5}}{\sqrt{2}} |S_A| \sin \beta_B : R_{101}^{101} + 3\sqrt{10} |S_A| |S_B| \cos \beta_A \sin \beta_B \cos \beta_B \sin(\alpha_A + \alpha_B) i R_{112}^{112} \right\} \]
\[ - \cos(\phi - \alpha_A) \left\{ \frac{9}{2} |S_A| |S_B| \sin \beta_A \cos \beta_B R_{110}^{110} \right\} \]
\[ + \cos(\phi + \alpha_B) \left\{ 3 \sqrt{10} |S_B|^2 \sin \beta_B \cos \beta_B R_{211}^{211} + \frac{9}{\sqrt{2}} |S_A| |S_B| \cos \beta_A \sin \beta_B R_{211}^{211} \right\} \]
\[ + f(2) \sin^2 \theta \left[ \cos(2\phi + \alpha_B) \frac{3\sqrt{5}}{2 \sqrt{2}} |S_B|^2 \sin \beta_B R_{200}^{200} - \cos(2\phi - \alpha_A + \alpha_B) \frac{9}{4} |S_A| |S_B| \sin \beta_A \sin \beta_B R_{211}^{211} \right. \]
\[ + \sin(2\phi - \alpha_A + \alpha_B) \frac{3\sqrt{15}}{2} |S_A| |S_B| \sin \beta_B \sin \beta_B \cos \beta_B i R_{211}^{211} \]
\[ - \sin(2\phi + \alpha_B) \left\{ \frac{3\sqrt{15}}{2 \sqrt{2}} |S_A| |S_B| \cos \beta_A \sin \beta_B R_{211}^{211} \right\} \right] \]

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Hadron tensor and polarization states of proton and deuteron

\[ W^{\mu\nu} \equiv \int \frac{d^4 \xi}{(2\pi)^4} e^{i Q \cdot \xi} < P_A S_A P_B S_B | J^\mu(0) J^\nu(\xi) | P_A S_A P_B S_B > \]

- \( X^\mu = P_A^\mu Q^2 Z \cdot P_B - P_B^\mu Q^2 Z \cdot P_A + Q^\mu (Q \cdot P_B Z \cdot P_A - Q \cdot P_A Z \cdot P_B) \)
- \( Y^\mu = \epsilon^{\mu\alpha\beta\gamma} P_A^\alpha P_B^\beta Q_\gamma \)
- \( Z^\mu = P_B^\mu Q \cdot P_B - P_B^\mu Q \cdot P_A \)

- \( S_A^\mu = \lambda_A P_A^\mu / M_A + S_A^\mu - \delta_- (\lambda_A M_A / P_A^+) \)
- \( S_B^\mu = \lambda_B P_B^\mu / M_B + S_B^\mu - \delta_+ (\lambda_B M_B / P_B^-) \)
- \( T^\mu = \epsilon^{\mu\alpha\beta\gamma} S_\alpha Z_\beta Q_\gamma \)

- \([W^{\mu\nu}(Q; P_A S_A; P_B S_B)]^* = W^{\mu\nu}(Q; P_A S_A; P_B S_B) \) (Hermiticity)
- \( W_{\mu\nu}(\tilde{Q}; \tilde{P}_A - \tilde{S}_A; \tilde{P}_B - \tilde{S}_B) = W^{\mu\nu}(Q; P_A S_A; P_B S_B) \) (Parity)
- \( W_{\mu\nu}(\tilde{Q}; \tilde{P}_A \tilde{S}_A; \tilde{P}_B \tilde{S}_B) = [W^{\mu\nu}(Q; P_A S_A; P_B S_B)]^* \) (Time reversal)

\[ \tilde{P}^\mu \equiv (P^0, -\vec{P}) \]

\[ (W^{\mu\nu})_{Q=0} = -g^{\mu\nu} A - \frac{Z^{\mu} Z^{\nu}}{Z^2} B' + Z^{\mu T^{\nu}} C + Z^{\mu T^{\nu}} D \]

\[ + Z^{\mu S^{\nu}} E + Z^{\mu S^{\nu}} F - S^{\mu}_{BT} S^{\nu}_{BT} G' - S^{\mu}_{AT} S^{\nu}_{BT} H' \]

\[ + T^{\mu S^{\nu}} I' + S^{\mu}_{BT T^{\nu}} J + Q^\nu Q' K + Q^{\mu Z^{\nu}} L \]

\[ + Q^{\mu S^{\nu}} M + Q^{\mu S^{\nu}} N + Q^{\mu T^{\nu}} O + Q^{\mu T^{\nu}} P \]

where \( Q^{\mu Z^{\nu}} \equiv Q^\mu Z^{\nu} + Q^\nu Z^{\mu} \)

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\[ A_1 = W_{0,0}, \quad A_2 = V^{LL}_{0,0}, \quad A_3 = V^{TT}_{0,0}, \quad A_4 = U^{Q_0}_0, \quad A_5 = V^{TQ_1}_{0,0}, \]
\[ B_1 = W_{2,0}, \quad B_2 = V^{LL}_{2,0}, \quad B_3 = V^{TT}_{2,0}, \quad B_4 = U^{Q_0}_2, \quad B_5 = V^{TQ_1}_{2,0}, \]
\[ C_1 = U^{TU}_{2,1}, \quad C_2 = U^{TQ_0}_{2,1}, \quad D_1 = U^{UT}_{2,1}, \quad D_2 = U^{LQ_1}_{2,1}, \quad D_3 = U^{TQ_2}_{2,1}, \]
\[ E_1 = U^{TL}_{2,1}, \quad F_1 = U^{LT}_{2,1}, \quad F_2 = U^{UQ_1}_{2,1}, \]
\[ H_1 = U^{TT}_{2,2}, \quad G_1 = U^{UQ_2}_{2,2}, \quad I_1 = U^{TQ_1}_{2,2}, \quad J_1 = U^{LQ_2}_{2,2}. \]

\[ W^{\mu\nu} = \left[ g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right] \left\{ W_{0,0} + \frac{M_A M_B}{S Z^2} Z \cdot S_A Z \cdot S_B V^{LL}_{0,0} - S_A T \cdot S_B V^{TT}_{0,0} \right. \]
\[ - \left. \left( \frac{8 M_B^2 (Z \cdot S_B)^2}{S^2 (Q \cdot P_B)^2} + \frac{4}{3} S_B^2 \right) V^{Q_0}_0 + \frac{M_B}{Z^2 Q \cdot P_B} Z \cdot S_B T_A \cdot S_B V^{TQ_1}_{0,0} \right\} \]
\[ - \left[ \frac{Z^\mu Z^\nu}{Z^2} \right] - \frac{1}{3} \left( g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) \left\{ W_{2,0} + \frac{M_A M_B}{S Z^2} Z \cdot S_A Z \cdot S_B V^{LL}_{2,0} - S_A T \cdot S_B V^{TT}_{2,0} \right. \]
\[ - \left. \left( \frac{8 M_B^2 (Z \cdot S_B)^2}{S^2 (Q \cdot P_B)^2} + \frac{4}{3} S_B^2 \right) V^{Q_0}_0 + \frac{M_B}{Z^2 Q \cdot P_B} Z \cdot S_B T_A \cdot S_B V^{TQ_1}_{2,0} \right\} \]
\[ - Z^{(\mu T_{\nu A}^A)} \frac{1}{\sqrt{Q^2 Z^2}} \left\{ U^{TU}_{2,1} \left( \frac{8 M_B^2 (Z \cdot S_B)^2}{S^2 (Q \cdot P_B)^2} + \frac{4}{3} S_B^2 \right) U^{TQ_0}_{2,1} \right\} \]
\[ - Z^{(\mu T_{\nu B}^B)} \frac{1}{\sqrt{Q^2 Z^2}} \left\{ U^{TU}_{2,1} + \frac{M_A M_B}{S Z^2} Z \cdot S_A Z \cdot S_B U^{LQ_1}_{2,1} + S_A T \cdot S_B U^{TQ_2}_{2,1} \right\} \]
\[ + Z^{(\mu S_{\nu A}^A)} \frac{\sqrt{Q^2 M_B}}{Z^2 Q \cdot P_B} Z \cdot S_B U^{LT}_{2,1} \]
\[ + Z^{(\mu S_{\nu B}^B)} \left\{ - \frac{\sqrt{Q^2 M_A}}{Z^2 Q \cdot P_A} Z \cdot S_A U^{LT}_{2,1} + \frac{\sqrt{Q^2 M_B}}{Z^2 Q \cdot P_B} Z \cdot S_B U^{LQ_1}_{2,1} - \frac{1}{\sqrt{Q^2 Z^2}} T_A \cdot S_B U^{TQ_2}_{2,1} \right\} \]
\[ - 2 S^{(\mu S_{\nu A}^A)} \left\{ \frac{Q^\mu Q^\nu}{Q^2} - \frac{Z^\mu Z^\nu}{Z^2} \right\} U^{Q_1}_{2,2} \]
\[ - S^{(\mu S_{\nu A}^A)} - S_{\nu A} - S_B \left\{ \frac{Q^\mu Q^\nu}{Q^2} - \frac{Z^\mu Z^\nu}{Z^2} \right\} U^{TT}_{2,2} \]
\[ - T_{\mu A}^{(\mu S_{\nu B}^B)} - T_A \cdot S_B \left\{ \frac{Q^\mu Q^\nu}{Q^2} - \frac{Z^\mu Z^\nu}{Z^2} \right\} U^{LQ_1}_{2,2} \]
\[ + S_{\nu B}^{(\mu T_{\nu B}^B)} \frac{M_A}{Z^2 Q \cdot P_A} Z \cdot S_A U^{LQ_1}_{2,2} \]

\[ Q_0 \text{ for the term } 3 \cos^2 \beta_B - 1 \sim Y_{20} \]
\[ Q_1 \text{ for } \sin(2 \beta_B) \sim Y_{21} \]
\[ Q_2 \text{ for } \sin^2(\beta_B) \sim Y_{22} \]
\[
\frac{d\sigma}{d^4Q \, d\Omega} = \frac{\alpha^2}{2 s \, Q^2} \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) W_{ij}
\]
\[
= \frac{\alpha^2}{2 s \, Q^2} \left\{ 2 \left[ W_{0,0} + \frac{1}{4} \lambda_A \lambda_B \, V_{0,0}^{LL} + |\bar{S}_{AT}| \, |\bar{S}_{BT}| \cos(\phi_A - \phi_B) \, V_{0,0}^{TT} \right. \\
+ \left. \frac{2}{3} \left( 2 |\bar{S}_{BT}|^2 - \lambda_B^2 \right) \, V_{0,0}^{UQ_0} + |\bar{S}_{AT}| \, \lambda_B \, |\bar{S}_{BT}| \sin(\phi_A - \phi_B) \, V_{0,0}^{TQ_1} \right] \\
+ \left( \frac{1}{3} - \cos^2 \theta \right) \left[ W_{2,0} + \frac{1}{4} \lambda_A \lambda_B \, V_{2,0}^{LL} + |\bar{S}_{AT}| \, |\bar{S}_{BT}| \cos(\phi_A - \phi_B) \, V_{2,0}^{TT} \right. \\
+ \left. \frac{2}{3} \left( 2 |\bar{S}_{BT}|^2 - \lambda_B^2 \right) \, V_{2,0}^{UQ_0} + |\bar{S}_{AT}| \, \lambda_B \, |\bar{S}_{BT}| \sin(\phi_A - \phi_B) \, V_{2,0}^{TQ_1} \right] \\
+ 2 \sin \theta \cos \theta \left[ \sin(\phi - \phi_A) \, |\bar{S}_{AT}| \left( U_{2,1}^{TU} + \frac{2}{3} \left( 2 |\bar{S}_{BT}|^2 - \lambda_B^2 \right) \, U_{2,1}^{TQ_0} \right) \\
+ \sin(\phi - \phi_B) \, |\bar{S}_{BT}| \left( U_{2,1}^{UT} + \frac{1}{4} \lambda_A \lambda_B \, U_{2,1}^{LQ_1} \right) \\
+ \sin(\phi + \phi_A - 2 \phi_B) \, |\bar{S}_{AT}| \, |\bar{S}_{BT}|^2 \, U_{2,1}^{TQ_2} \\
+ \cos(\phi - \phi_A) \, |\bar{S}_{AT}| \, \lambda_B \, U_{2,1}^{TL} \\
+ \cos(\phi - \phi_B) \, |\bar{S}_{BT}| \left( \lambda_A \, U_{2,1}^{LT} + \lambda_B \, U_{2,1}^{UQ_1} \right) \right] \\
+ \sin^2 \theta \left[ \cos(2\phi - 2\phi_B) \, |\bar{S}_{BT}|^2 \, U_{2,2}^{UQ_2} \\
+ \cos(2\phi - \phi_A - \phi_B) \, |\bar{S}_{AT}| \, |\bar{S}_{BT}| \, U_{2,2}^{TT} \\
+ \sin(2\phi - \phi_A - \phi_B) \, |\bar{S}_{AT}| \, \lambda_B \, |\bar{S}_{BT}| \, U_{2,2}^{TQ_1} \\
+ \sin(2\phi - 2\phi_B) \, \lambda_A \, |\bar{S}_{BT}|^2 \, U_{2,2}^{LQ_2} \right] \right\} 
\]
Possible spin asymmetries

\[ \text{pp Drell-Yan: } \langle \sigma \rangle, A_{LL}, A_{TT}, A_{LT}, A_T \]

\[ \downarrow \]

\text{pd Drell-Yan:}

\[ \langle \sigma \rangle, A_{LL}, A_{TT}, A_{LT}, A_{TL}, A_{UT}, A_{TU}, A_{UQ_0}, A_{TQ_0}, A_{UQ_1}, A_{LQ_1}, A_{TQ_1}, A_{UQ_2}, A_{LQ_2}, A_{TQ_2} \]

The quadrupole spin asymmetries are new ones in spin-1 hadron reactions.
Quadrupole \( Q_0 \) spin asymmetries

\[
A_{UQ_0} = \frac{1}{2} <\sigma> \left[ \sigma(\bullet,0_L) - \frac{\sigma(\bullet,+1_L) + \sigma(\bullet,-1_L)}{2} \right]
\]

\[
= \frac{2V_{0,0}^U Q_0 + \left(\frac{1}{3} - \cos^2\theta\right)V_{2,0}^U Q_0}{2W_{0,0} + \left(\frac{1}{3} - \cos^2\theta\right)W_{2,0}}
\]

\[
A_{TQ_0} = \frac{1}{2} <\sigma> \left[ \begin{array}{c}
\{ \sigma(\phi_A = 0,0_L) - \sigma(\phi_A = \pi,0_L) \} / 2 \\
- \{ \sigma(\phi_A = 0,+1_L) + \sigma(\phi_A = 0,-1_L) \\
- \sigma(\phi_A = \pi,+1_L) - \sigma(\phi_A = \pi,-1_L) \} / 4
\end{array} \right]
\]

\[
= \frac{2\sin\theta \cos\theta \sin\phi U_{2,1}^{TQ_0}}{2W_{0,0} + \left(\frac{1}{3} - \cos^2\theta\right)W_{2,0}}
\]
Spin asymmetries in the parton model (\( \int d\bar{Q}_T \) case)

\[ f_1 \rightarrow q_a, \quad g_{1L} \rightarrow \Delta q_a, \quad h_1 \rightarrow \Delta T q_a, \quad b_1 \rightarrow \delta q_a \]

Unpolarized cross section

\[
\left( \frac{d\sigma}{d x_A d x_B d\Omega} \right) = \frac{\alpha^2}{4 Q^2} (1 + \cos^2 \theta) \bar{W}_T \\
= \frac{\alpha^2}{4 Q^2} (1 + \cos^2 \theta) \frac{1}{3} \sum_a e_a^2 \left[ q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B) \right]
\]

Spin asymmetries

\[
A_{LL} = -\frac{\bar{V}_T^{LL}}{4 \bar{W}_T} = \frac{\sum_a e_a^2 \left[ \Delta q_a(x_A) \Delta \bar{q}_a(x_B) + \Delta \bar{q}_a(x_A) \Delta q_a(x_B) \right]}{\sum_a e_a^2 \left[ q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B) \right]}
\]

\[
A_{(T)T} = A_{T(T)} = A_{TT}^\parallel = \frac{\sin^2 \theta \cos 2\phi}{1 + \cos^2 \theta} \frac{\bar{U}_{2,2}^{TT}}{\bar{W}_T} \\
= \frac{\sin^2 \theta \cos(2\phi)}{1 + \cos^2 \theta} \frac{\sum_a e_a^2 \left[ \Delta T q_a(x_A) \Delta T \bar{q}_a(x_B) + \Delta T \bar{q}_a(x_A) \Delta T q_a(x_B) \right]}{\sum_a e_a^2 \left[ q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B) \right]}
\]

\[
A_{TT}^\perp = \tan(2\phi) A_{TT}^\parallel \\
A_{UQ_0} = \frac{\bar{V}_T^{UQ_0}}{\bar{W}_T} = \frac{\sum_a e_a^2 \left[ q_a(x_A) \delta \bar{q}_a(x_B) + \bar{q}_a(x_A) \delta q_a(x_B) \right]}{\sum_a e_a^2 \left[ q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B) \right]}
\]

\[
A_{LT} = A_{TL} = A_{UT} = A_{TU} = A_{TQ_0} = A_{UQ_1} = A_{LQ_1} \\
= A_{TQ_1} = A_{UQ_2} = A_{LQ_2} = A_{TQ_2} = 0
\]

Advantage of the hadron reaction (\( \delta \bar{q} \) measurements)

\[
A_{UQ_0}(\text{large} \ x_F) \approx \frac{\sum_a e_a^2 q_a(x_A) \delta \bar{q}_a(x_B)}{\sum_a e_a^2 q_a(x_A) \bar{q}_a(x_B)}
\]
Spin asymmetries in the parton model \((\int d\vec{Q}_T \text{ case})\)

\[ f_1 \rightarrow q_a, \quad g_{1L} \rightarrow \Delta q_a, \quad h_1 \rightarrow \Delta T q_a, \quad b_1 \rightarrow \delta q_a \]

Unpolarized cross section

\[
\left\langle \frac{d\sigma}{dx_A dx_B d\Omega} \right\rangle = \frac{\alpha^2}{4 Q^2} (1 + \cos^2 \theta) \overline{W}_T
\]

\[
= \frac{\alpha^2}{4 Q^2} (1 + \cos^2 \theta) \frac{1}{3} \sum_a e_a^2 \left[ q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B) \right]
\]

Spin asymmetries

\[
A_{LL} = -\frac{V_T^{LL}}{4 \overline{W}_T} = \frac{\sum_a e_a^2 \left[ \Delta q_a(x_A) \Delta \bar{q}_a(x_B) + \Delta \bar{q}_a(x_A) \Delta q_a(x_B) \right]}{\sum_a e_a^2 \left[ q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B) \right]}
\]

\[
A_{TT} = A_{T(T)} = A_{TT}^{\parallel} = \frac{\sin^2 \theta \cos 2\phi \overline{U}_{2,2}^{TT}}{1 + \cos^2 \theta} \overline{W}_T
\]

\[
= \frac{\sin^2 \theta \cos(2\phi)}{1 + \cos^2 \theta} \sum_a e_a^2 \left[ \Delta_T q_a(x_A) \Delta_T \bar{q}_a(x_B) + \Delta_T \bar{q}_a(x_A) \Delta_T q_a(x_B) \right]
\]

\[
A_{TT}^{\perp} = \tan(2\phi) A_{TT}^{\parallel}
\]

\[
A_{UQ_0} = \frac{\overline{V}_T^{UQ_0}}{\overline{W}_T} = \frac{\sum_a e_a^2 \left[ q_a(x_A) \delta \bar{q}_a(x_B) + \bar{q}_a(x_A) \delta q_a(x_B) \right]}{\sum_a e_a^2 \left[ q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B) \right]}
\]

\[
A_{LT} = A_{TL} = A_{UT} = A_{TU} = A_{TQ_0} = A_{UQ_1} = A_{LQ_1}
\]

\[
= A_{TQ_1} = A_{UQ_2} = A_{LQ_2} = A_{TQ_2} = 0
\]

Advantage of the hadron reaction (\(\delta \bar{Q}\) measurements)

\[
A_{UQ_0} \text{(large } x_F) \approx \frac{\sum_a e_a^2 q_a(x_A) \delta \bar{q}_a(x_B)}{\sum_a e_a^2 q_a(x_A) \bar{q}_a(x_B)}
\]

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Hard Exclusive Processes and Vector Meson Wave Functions in QCD

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Department of Physics, Juntendo University, Inba-gun, Chiba 270-1606, Japan

The light-cone wave functions (distribution amplitudes) of vector mesons describe a "long distance part" for hard exclusive processes involving vector meson in the final state[1], such as exclusive semileptonic and radiative $B$ decays, hard electroproduction of vector mesons, etc. In particular, the higher twist distribution amplitudes are relevant for understanding preasymptotic corrections to the hard exclusive amplitudes.

We discuss a systematic study of light-cone distribution amplitudes of vector mesons in QCD[2]. We give operator definitions of quark-antiquark distribution amplitudes up to twist-4 and quark-antiquark-gluon distribution amplitudes of twist-3, based on matrix elements of nonlocal light-cone operators between the vacuum and the vector meson state. A detailed operator expansion analysis is performed for the twist-3 distribution amplitudes, and the constraints among them from the QCD equations of motion are derived and solved. The solutions give explicit relations between the quark-antiquark and the quark-antiquark-gluon distribution amplitudes.

Based on (approximate) conformal invariance in massless QCD, we introduce conformal partial wave expansion as a powerful framework for systematic treatment of these solutions. A complete set of twist-3 distribution amplitudes is constructed, which satisfies all (exact) equations of motion and constraints from conformal expansion. The renormalization scale dependence of the distribution amplitudes is also worked out in the leading logarithmic approximation, utilizing conformal symmetry.

Taking into account a few low order terms in the conformal expansion, we obtain a consistent set of distribution amplitudes which involve a minimum number of independent nonperturbative parameters as expansion coefficients. These nonperturbative parameters are calculated from QCD sum rules. We also take into account the SU(3) flavor violation effects induced by quark masses, and construct models for the distribution amplitudes of $\rho$, $\omega$, $K^*$, and $\phi$ mesons up to twist-3, which satisfy all QCD constraints.

References

Hard processes in QCD
light-cone nonlocal operators

$$\bar{q}^{(z)} \Gamma z \gamma^a q^{(-z)}$$

$$\text{P} e^{ig \int_{-z}^{z} dx \mu A_{\mu}}$$

$$z^2 = 0; \ z \mu = (0, z^-, 0)$$

$$\lambda^a: \text{flavor matrix}$$

$$\Gamma = \gamma^\mu, \gamma^\mu \gamma^5, \sigma^{\mu \nu}, 1, \gamma^5$$

Inclusive

DIS

Structure fn.

$$\langle P | \bar{q}^{(z)} \Gamma z \gamma^a q^{(-z)} | P \rangle$$

Exclusive

Heavy meson (semileptonic) decay

$$B \rightarrow \ell e \nu \bar{\nu}$$

Radiative

$$B \rightarrow J + \gamma$$

$$\text{LC distribution amplitudes}$$

$$\langle 0 | \bar{u}(z) \Gamma d(-z) | P^- \rangle$$

electroproduction of vector meson
Higher twist distribution amplitudes
preasymptotic corrections \( \frac{1}{Q^2} \)
new information on QCD and hadron structure

\[ \langle p^e | \bar{u}(z) \gamma^\mu d(-z) | p^e \rangle \]

**Jaffe-Ji (92)**

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<th>( S_{\parallel} )</th>
<th>( S_{\perp} )</th>
<th>LC-proj.</th>
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\[ \langle p^e | \bar{u}(z) \gamma^\mu d(-z) | p^e \rangle \]

**Ball, Braun, Koike & Tanaka (98)**

<table>
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<td>( g_{\perp}, g_{\perp}^{(a)} )</td>
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<td>4 ( O(\frac{1}{6^2}) )</td>
<td>( g_2 )</td>
<td>( h_3 )</td>
<td>++</td>
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</table>

\( LD = 4I + 4I \)
Classification of chiral-odd DA's

\[ P_\mu = p_\mu + \frac{1}{2} m_\perp^2 \frac{z_\mu}{(p \cdot z)} ; \quad p_\mu = (p^+, 0, 0, \epsilon_\perp) \]

\[ z_\mu = \left( \frac{E \cdot z}{(p \cdot z)} \right) p_\mu + \left( \frac{E \cdot p}{(p \cdot z)} \right) z_\mu + \epsilon_\perp^\mu ; \quad \epsilon_\perp^\mu \epsilon_\perp^\nu = -1 \]

\[ z = u - (1-u) \]

\[ = 2u - 1 \]

\[ \langle 0 | \bar{u}(z) \gamma_\mu i \gamma_5 d(-z) | f^- (p, \epsilon) \rangle = \zeta_f^T \phi(u) \]

\[ = i \int_p^T \left[ \left( \frac{E_\perp^\mu}{(p \cdot z)} \right) p_\mu - \frac{E_\perp^\mu}{(p \cdot z)} m_\perp^2 \right] \frac{1}{(p \cdot z)^2} \int_0^1 dz e^{i \frac{z}{p \cdot z}} \frac{m_\perp^2}{(p \cdot z)} \phi(u) \]

\[ + \frac{1}{z} \left( \frac{E_\perp^\mu}{(p \cdot z)} \right) m_\perp^2 \int_0^1 dz e^{i \frac{z}{p \cdot z}} \frac{m_\perp^2}{(p \cdot z)} \phi(u) \]

\[ + \frac{1}{z} \left( \frac{E_\perp^\mu}{(p \cdot z)} \right) m_\perp^2 \int_0^1 dz e^{i \frac{z}{p \cdot z}} \phi(u) \]

\[ = 2 \left[ (S_\perp^\mu p_\mu - S_\perp^\mu p_\mu) \right] \frac{1}{(p \cdot z)} \int_1^0 dx \epsilon^{2 i z p \cdot z} h_1(x) \]

\[ + \frac{m_\perp^2}{(p \cdot z)^2} \int_1^0 dx \epsilon^{2 i z p \cdot z} h_2(x) \]

\[ + \frac{1}{(p \cdot z)^2} \int_1^0 dx \epsilon^{2 i z p \cdot z} h_3(x) \]

\[ \langle 0 | \bar{u}(0) \gamma_\mu d(0) | f^- (p, \epsilon) \rangle = i \zeta_f^T (\epsilon_\perp p_\perp - \epsilon_\parallel p_\perp) \]
\[ \langle 0 | \bar{u}(z) d(-z) | \Psi^-(p, \epsilon) \rangle = -i \left( f_\mu - f_\rho \frac{m_\mu + m_d}{m_\rho} \right) (\epsilon \cdot z) \int_0^1 du \ e^{i z p \cdot z} \ \rho_{\mu}(u) \] 

\[ \langle \Psi^+ | \bar{u}(z) \gamma_\mu d(-z) | \Psi^- \rangle = 2 M \int_0^1 dx \ e^{2 i x p \cdot z} e(x) \]

\[ \langle 0 | \bar{u}(0) \gamma_\mu d(0) | \Psi^- (p, \epsilon) \rangle = f_\mu m_\rho \epsilon \mu \]

\[ \int_0^1 du \ h_{\mu \mu}(u) = 1 \]

- Particle DA's
Constraints on twist-3 DA's

Using QCD-EOM \((i\hat{D} - m_u)U = 0\) etc.

\[
\frac{\partial}{\partial x_\mu} \left[ \overline{U}(x) \sigma_{\mu\nu} x_\nu \sigma^{\nu\sigma} d(-x) \right] = i \int_{-1}^{1} dv \int_{-1}^{1} du \overline{U}(x) \sigma_{\alpha\beta} g G_{\mu\nu}(vx) x_\alpha x_\mu d(-x) \\
- i x_\alpha \partial_\alpha \left[ \overline{U}(x) d(-x) \right] \\
- (m_u - m_d) \overline{U}(x) \gamma_\mu x_\mu d(-x)
\]

\[
\overline{U}(x) d(-x) - \overline{U}(0) d(0) = \int_{0}^{1} dt \int_{-1}^{1} dv \overline{U}(tx) \sigma_{\alpha\beta} g G_{\mu\nu}(vx) x_\alpha x_\mu d(-tx) \\
+ i \int_{0}^{1} dt \partial_\alpha \left[ \overline{U}(tx) \sigma_{\alpha\beta} x_\beta d(-tx) \right] \\
+ i \int_{0}^{1} dt (m_u + m_d) \overline{U}(tx) \gamma_\mu x_\mu d(-tx)
\]

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Solution via the conformal expansion

\[ h_{\parallel}(u) = h_{\parallel}(u)^{\text{WW}} + h_{\parallel}(u)^{g} \]
\[ = \sum_{k,l=0}^{\infty} \frac{J(\alpha_{d},\alpha_{u})}{k!l!} \alpha_{d}^{k} \alpha_{u}^{l} \]
\[ \phi_{\parallel}(u) = 6u(1-u) \sum_{n=0}^{\infty} A_{n} C_{n}^{3/2}(2u-1) \]

\[ i_{n} \sim \langle 0 | \Sigma_{n,0}(p,\epsilon_{l}) | g^{C}(P,\epsilon_{l}) \rangle, \quad W_{k,l} \sim \langle 0 | H_{k,l}^{C}(0) | g^{C}(P,\epsilon_{l}) \rangle \]
\[ i_{n}^{C}(0) = \overline{u}(0) \sigma_{LV} z_{L} C_{n}^{3/2}(z \cdot \vec{D}) d(0) \]
\[ \xi_{k,l}^{C}(0) = \overline{u}(0) (i \vec{D} \cdot z) e_{k} \sigma_{\mu \nu} g \zeta_{\nu} z_{L} \zeta_{\nu} \zeta_{L} (z \cdot \vec{D}) d(0) \]
\[ + (\text{total derivatives}) \]

\[ \rho(t) \]
\[ h_{\parallel}(u) = \sum_{n=0}^{\infty} (H_{n} - H_{n-1}) \frac{1}{2} C_{n}^{\frac{1}{2}}(2u-1) \]
\[ \rho(s) \]
\[ = 4u(1-u) \sum_{n=0}^{\infty} \frac{H_{n} - H_{n+1}}{(n+1)(n+2)} C_{n}^{3/2}(2u-1) \]
\[ H_{n} = H_{n}^{\text{WW}} + \sum_{\infty} A_{n} \sigma_{\mu} A_{n}^{\nu} \]
\[ j = n + 3/2 \]
Models for $f(\omega)$, $K^*$, $\phi$ DA's

Truncation of conformal expansion: $j \leq \frac{3}{2}$

$$ P_{\perp}(u) = 6u(1-u) \sum_{n=0}^{\phi^2} \alpha_n \frac{\phi^2}{k+l \leq 1} C_n^{3/2}(2u-1) $$

$$ P_{\parallel}(u) = -\sum_{n=0}^{\phi^2} \left( H_n - H_{n-1} \right) C_n^{3/2}(2u-1) $$

$$ P_{\parallel}^{(s)}(u) = 4u(1-u) \sum_{n=0}^{\phi^2} \frac{H_n - H_{n+1}}{(n+1)(n+2)} C_n^{3/2}(2u-1) $$

\[ \text{QCD-SR} \]

\[ \text{onperturbative matrix elements} \]

\[ \tilde{a}_n \sim \langle 0 | \bar{u}(0) \sigma_{\mu\nu} z_\mu C_n^{3/2}(z \cdot \hat{D}) d(0) | \Phi(P, \epsilon_1) \rangle \]

\[ \tilde{u}_{10} \sim \langle 0 | \bar{u}(0) \sigma_{\mu\nu} z_\mu z_\nu \left[ \mathcal{G}_{v}(0)(z \cdot \hat{D}) - (z \cdot \hat{D}) \mathcal{G}_{v}(0) \right] d(0) | \Phi(P, \epsilon_1) \rangle \]

Hirai-even

$$ \phi_{\parallel} \leftrightarrow \phi_{\perp} $$

$$ g_{\parallel}, g_{\perp} \leftrightarrow h_{\parallel}^{(t)}, h_{\parallel}^{(s)} $$

\[ \text{Renormalization:} \]

Keiichi, Nishikawa, Tanaka
P wave: DA's
(twist - 3)

μ = 1 GeV
Summary

Light-cone DA's of vector mesons $\rho, \omega, K^*, \phi$

complete classification of 2-particle DA's

3-particle DA's: chiral-odd & chiral-even

Systematic analysis of twist-3 DA's

QCD-EOM & conformal partial wave expansion

$$h_{\gamma}(t, s) = (WW) + (3\text{-part.}) + O(m_q)$$

Resolution order by order in conformal spin $j$

$j$: good quantum number up to $O(\alpha^2)$

Renormalization based on conformal operator basis

- all anomalous dimensions are determined!
- 3-particle DA's: complicated mixing of many $\bar{u}Gd$ ops.

$$Nc \to \infty \text{ or } n \to \infty: \text{no complicated mixing!}$$

$$\tilde{\gamma}_n \sim \text{Im} \gamma_n \quad (n \to \infty)$$

Explicit models of DA's for $\rho, \omega, K^*, \phi$

- truncation of conformal expansion
- estimates of nonperturbative matrix elements by QCD SR
- SU(3) flavor violation effects due to quark masses: asymmetry

Powerful framework for hard exclusive processes:

All info. predictable based on QCD

Model building consistent with all QCD constraints
Problems for J. Kodaira's Lectures

1. Understand the following facts:

(a) A spin-1/2 fermion field $\psi$ is decomposed into two components due to the eigenvalues of $\gamma_5$ ($\equiv \pm 1$, called "chirality") as

$$
\psi_R = (1 + \gamma_5)\psi/2 \quad \text{and} \quad \psi_L = (1 - \gamma_5)\psi/2.
$$

Show that $\bar{\psi}\Gamma\psi = \bar{\psi}_L\Gamma\psi_L + \bar{\psi}_R\Gamma\psi_R$ for $\Gamma = \gamma_\mu, \gamma_\mu\gamma_5$ and $\bar{\psi}\Gamma\psi = \bar{\psi}_R\Gamma\psi_L + \bar{\psi}_L\Gamma\psi_R$ for $\Gamma = 1, \gamma_5, \sigma_{\mu\nu}$. From this we understand that the standard model lagrangian preserves chirality of the quarks except quark mass effect.

(b) For massless fermions, chirality is identical to helicity. (For anti-fermions, they are opposite.)

(c) In the high-energy $e^+e^-$ collision, the positive-helicity (right-handed) electron can annihilate only in the collision with the negative helicity (left-handed) positron, and accordingly the spin of the virtual photon is transverse.

2. Consider the nucleon structure function in the deep-inelastic scattering

$$
W_{\mu\nu}(q, P, S) = \frac{1}{4\pi} \int d\xi e^{iq\cdot\xi}(PS) \left[ J_\mu(\xi), J_\nu(0) \right] |PS\rangle. \tag{1}
$$

(a) Show $W_{\mu\nu}(q, P, S) = W_{\nu\mu}(q, P, S)$. (Hint: Take complex conjugate of the above equation.)

(b) Show that the parity invariance requires $W_{\mu\nu}(q, P, S) = W^{\mu\nu}(q, \bar{P}, -\bar{S})$ where $\bar{P}$ denotes $\bar{P} = (P^0, -\vec{P})$ for $P = (P^0, \vec{P})$ etc.

(c) Show that the time reversal invariance requires $W_{\mu\nu}(q, P, S) = W_{\nu\mu}(q, \bar{P}, \bar{S})$.

(d) For the unpolarized case (i.e. $|PS\rangle = |P\rangle$), the most general decomposition of $W_{\mu\nu}$ can be written as

$$
W_{\mu\nu}(x, Q^2) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1 + \frac{1}{M^2} \left(P_\mu - \frac{P \cdot q}{q^2} q_\mu\right) \left(P_\nu - \frac{P \cdot q}{q^2} q_\nu\right) W_2
$$

$$
+ \frac{i}{2M^2} \epsilon_{\mu\nu\lambda\sigma} P^\lambda q^\sigma W_3 + \frac{1}{M^2} q_\mu q_\nu W_4 + \frac{1}{2M^2}(P_\mu q_\nu + P_\nu q_\mu) W_5
$$

$$
- \frac{i}{2M^2}(P_\mu q_\nu - P_\nu q_\mu) W_6. \tag{2}
$$

Show that (i) $W_{1-6}$ are real, (ii) Parity invariance kills $W_3$, (iii) T-invariance kills $W_6$, (iv) The current conservation $q^\mu W_{\mu\nu} = 0$ kills $W_{4-6}$.
3. Define the scaling functions as $F_1(x, Q^2) = W_1(x, Q^2)$ and $F_2(x, Q^2) = \nu W_2(x, Q^2)/M^2$ ($\nu = P \cdot q, x = Q^2/2\nu$). Prove that the Callan-Gross relation, $F_2(x) = 2xF_1(x)$, is a direct consequence of the fact that the parton is a spin-1/2 particle.

4. Solve the renormalization group (RG) equation for the coupling constant

$$
\frac{d}{d\mu} g(\mu) = \beta(g) = -\frac{g^3}{16\pi^2} b_0 - \frac{g^5}{(16\pi^2)^2} b_1 - \cdots,
$$

(3)

to obtain

$$
g^2(Q^2) = \frac{1}{b_0 \ln(Q^2/\Lambda^2)} \left[ 1 - \frac{b_1 \ln\ln(Q^2/\Lambda^2)}{b_0^2 \ln(Q^2/\Lambda^2)} + \cdots \right],
$$

(4)

where $b_0 = 11 - \frac{2}{3}N_f$ and $b_1 = 102 - \frac{38}{3}N_f$. The RG equation for an operator $O_n(\mu)$ is given as

$$
\frac{d}{d\mu} O_n(\mu) + \gamma_n(g(\mu))O_n(\mu) = 0.
$$

(5)

Solve this equation when the anomalous dimension $\gamma_n(g)$ is given as

$$
\gamma_n(g) = \frac{g^2}{16\pi^2} \gamma_n^{(0)} + \frac{g^4}{(16\pi^2)^2} \gamma_n^{(1)} + \cdots.
$$

(6)

Note that in the MS (\overline{MS}) scheme $O_n(\mu)$ depends on $\mu$ only through $g(\mu)$. The answer is

$$
O_n(Q^2) = \left( \frac{g^2(Q^2)}{g^2(\mu^2)} \right)^{\gamma_n^{(0)}/2b_0} \left[ 1 + \frac{g^2(Q^2) - g^2(\mu^2)}{16\pi^2} \left( \frac{\gamma_n^{(1)}}{2b_1} - \frac{\gamma_n^{(0)}}{2b_0} \right) \right] O_n(\mu^2).
$$

(7)

(Suggestion: Try to solve the leading order result first by putting $b_1 = \gamma_n^{(1)} = 0$.)

5. The parton distribution functions are defined as

$$
\Phi(x) = \int \frac{d\lambda}{2\pi} e^{ix\lambda}(PS|\bar{\psi}(0)\Gamma\psi(\lambda n_i)|PS).
$$

(8)

Answer to the following questions.

(a) The anti-quark distribution $\overline{\Phi}(x)$ is defined by the replacement $\psi \rightarrow C\bar{\psi}^T$, where $C = i\gamma^5 \gamma^0$ is a charge conjugation matrix. Show that $\Phi(-x) = -\overline{\Phi}(x)$ for $\Gamma = \gamma_\mu, \sigma_{\mu\nu}$ and $\Phi(-x) = \overline{\Phi}(x)$ for $\Gamma = \gamma_\mu\gamma_5, 1, i\gamma_5$. (Hint: Use the following relations; $C = -C^T, \overline{C}C^{-1} = 1, C\gamma_\mu C^{-1} = -\gamma_\mu^T, C\gamma_5 C^{-1} = \gamma_5^T, C^T\gamma_\mu C^{-1} = -\sigma_{\mu\nu}$.)
(b) (Similar to the problem 2. So you may skip.) Show that the time-reversal invariance prohibits the presence of the following $e_L(x)$, $h(x)$ and $f_T(x)$ although they are allowed by parity:

$$\int \frac{d\lambda}{2\pi} e^{i\lambda z} \langle PS | \bar{\psi}(0) i \gamma_5 \psi(\lambda n) | PS \rangle = M(S \cdot n) e_L(x),$$  \hfill (9)

$$\int \frac{d\lambda}{2\pi} e^{i\lambda z} \langle PS | \bar{\psi}(0) i \gamma_5 \sigma_{\mu\nu} \psi(\lambda n) | PS \rangle = \cdots + M \epsilon_{\mu\nu\lambda\sigma} p^\lambda n^\sigma h(x) + \cdots,$$  \hfill (10)

$$\int \frac{d\lambda}{2\pi} e^{i\lambda z} \langle PS | \bar{\psi}(0) \gamma_\mu \psi(\lambda n) | PS \rangle = \cdots + \epsilon_{\mu\nu\lambda\sigma} p^\nu n^\lambda S^\sigma f_T(x) + \cdots.$$  \hfill (11)

The complete set of the quark distribution functions defined from (8) which respect time-reversal and parity invariance is listed in Jaffe & Ji, Nucl. Phys. B375 (92) 527.

6. (Not easy) Understand the origin of the Gauge-link operators in the quark distribution functions. (Answer: As an example, take the deep inelastic scattering. Consider the diagrams in which the usual quark-handbag diagram has additional gluon lines connecting short-distance and long-distance parts. Decompose the gluon fields $A_\mu$ as $A_\mu = \omega^{\mu\lambda} A_\lambda + p^\mu (n \cdot A)$ where $\omega^{\mu\lambda} = g^{\mu\lambda} - p^\mu n^\lambda$. Then the $p^\mu (n \cdot A)$ piece ("scalar polarized gluon") contributes to the process in the same order with respect to $1/Q$ as the simplest handbag diagram. Using the technique of Ward identity, one can show that this scalar polarized gluon contributions can be recast into the form of the gauge-link operator. This gauge-link operator is reduced to unit operator in $n \cdot A = 0$-gauge (light-cone gauge).
Problem set for R. L. Jaffe's lectures  
(except for the last part)

[1] Spin, orbital, and total angular momenta in the Dirac theory.

Prove that the orbital and spin angular momenta are not separately conserved in the Dirac theory although the total angular momentum is conserved:

\[[ \bar{L}, H ] \neq 0, \quad [ \bar{S}, H ] \neq 0, \quad [ \bar{J}, H ] = 0, \]

where

\[
H = \vec{\alpha} \cdot \vec{p} + \beta m, \quad \vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]


(2.1) Consider the translation \( x \rightarrow x + a \) in the Lagrangian of scalar field theory:

\[
\mathcal{L}(x + a) = \mathcal{L}[\phi(x + a), \partial_\mu \phi(x + a)]
\]

Then, the variations of \( \phi \) and \( \partial_\mu \phi \) are given by

\[
\delta \phi = \delta \phi^\nu(x) \partial_\nu \phi(x),
\]

\[
\delta \partial_\mu \phi(x) = \delta \phi^{\sigma}(x) \partial_\sigma \partial_\mu \phi(x) + \partial_\mu [\delta \phi^{\nu}(x)] \partial_\nu \phi(x),
\]

for an infinitesimal \( x \)-dependent transformation. Suppose that the variation of the action is invariant under the transformation. Prove the following conservation law for the energy-momentum tensor:

\[
\partial_\mu T^{\mu \nu} = 0, \quad T^{\mu \nu} = \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} \partial_\nu \phi - g^{\mu \nu} \mathcal{L}
\]

(2.2) Next, suppose that the action is invariant under the infinitesimal Lorentz transformation \( \delta x^\mu = \delta \omega^{\mu \nu} x_\nu \), where \( \delta \omega^{\mu \nu} \) is the antisymmetric (\( \delta \omega^{\mu \nu} = -\delta \omega^{\nu \mu} \)). Show the following conservation law for the angular-momentum tensor:

\[
\partial_\mu M^{\mu \nu \lambda} = 0, \quad M^{\mu \nu \lambda} = x^\nu T^{\mu \lambda} - x^\lambda T^{\mu \nu},
\]

\[
= \partial^\mu \phi (x^\nu \partial^\lambda - x^\lambda \partial^\nu) \phi - (g^{\mu \lambda} x^\nu - g^{\mu \nu} x^\lambda) \mathcal{L},
\]

where the Lagrangian is given by \( \mathcal{L} = \partial_\mu \phi \partial^\mu \phi/2 - m^2 \phi^2/2 - V(\phi) \).
Gauge invariance of the angular momentum tensor in Abelian gauge theory.

(3.1) For the electromagnetic field with
\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}, \]
show that the angular-momentum tensor is given by
\[ M^{\mu\nu\lambda} = - F^{\mu\rho} (\partial^{\nu} \partial^{\lambda} - \partial^{\lambda} \partial^{\nu}) A_{\rho}^{\ast} \text{ orbital} \]
\[ + F^{\mu\lambda} A^{\nu} + F^{\nu\mu} A^{\lambda} \text{ spin} \]
\[ - \frac{1}{4} F^{2} (\partial^{\nu} g^{\mu\lambda} - \partial^{\lambda} g^{\mu\nu}), \]
where \( F^{2} = F_{\mu\nu} F^{\mu\nu} \).

(3.2) Then, prove that the orbital and spin terms are not separately gauge invariant although the whole angular momentum tensor is invariant under the local gauge transformation \( A_{\mu} \rightarrow A_{\mu} + \partial_{\mu} \alpha(x) \).


From the antisymmetric part of the hadron tensor
\[ W_{\mu\nu}^{A} = i \varepsilon_{\mu\nu\lambda\sigma} q^{\lambda} S^{\sigma} g_{1} + i \varepsilon_{\mu\nu\lambda\sigma} \frac{1}{\nu} q^{\lambda} (\nu S^{\sigma} - q \cdot S \cdot P^{\sigma}) g_{2}, \]
show
\[ W_{\mu\nu}^{A} = i \varepsilon_{\mu\nu\lambda\sigma} (n \cdot S) n^{\lambda} p^{\sigma} g_{1} + i \varepsilon_{\mu\nu\lambda\sigma} n^{\lambda} S_{\mu\nu}^{\sigma} (g_{1} + g_{2}), \]
so that \( g_{1} \) and \( g_{T} = g_{1} + g_{2} \) are interpreted as the longitudinal and transverse structure functions, respectively.

[5] "Good" and "bad" components.

From the Dirac equation \( (i \partial - g A) \psi = m \psi \), show that \( i \frac{\partial}{\partial x^{-}} \psi_{-} \) is expressed by \( A_{\perp} \) and \( \psi_{+} \):
\[ i \frac{\partial}{\partial x^{-}} \psi_{-} = \frac{1}{\sqrt{2}} [ (i \partial_{\perp} - g A_{\perp}) \gamma_{\perp}^{\ast} + m ] \gamma^{0} \psi_{+}, \]
so that \( \psi_{-} \) is not independent of \( A_{\perp} \) and \( \psi_{+} \).
Problems for K. Rith's Lectures


\( k \)

\( k' \)

\( P^2 = M^2 \)

\( \theta \)

\( X \)

1. Write the expressions in the laboratory system for the frequently used Lorentz invariants:

\[ \nu = \frac{\vec{q} \cdot \vec{P}}{M}, \]

\[ Q^2 = -\frac{\vec{q}^2}{\nu}, \]

\[ y = \frac{\vec{q} \cdot \vec{P}}{k \cdot P}. \]

The lepton 4-momenta in the lab. system are given by

\[ k^\mu = (E, \vec{k}), \quad k'^\mu = (E', \vec{k}'), \quad \theta : \vec{k} \wedge \vec{k}'. \]

and the lepton mass is assumed to be negligible.

2. Express the invariant mass squared of the final state hadrons

\[ W^2 = (P + q)^2 \]

by the invariants given in (1) and show that the Bjorken variable, \( x = Q^2 / 2 \nu P \), satisfies \( 0 \leq x \leq 1 \). Also, confirm that \( 0 \leq y \leq 1 \).

3. Show

\[ S = (k + P)^2 = Q^2 y + M^2. \]

2. Use the previous results to answer the following.

1. Consider a fixed target deep inelastic scattering experiment \((e, e')\) on proton at \( E = 30 \text{ GeV}\). Obtain the minimum accessible \( x \) for \( Q^2 \geq (1 \text{ GeV/c})^2 \). Assume for simplicity that the proton mass is approximately 1 GeV. What is the electron scattering angle \( \theta \) at \( y = 0.9 \)?

2. Repeat the same calculation for \( E = 200 \text{ GeV} \).

3. In the case of a HERA type experiment \((E_p = 820 \text{ GeV}, E_e = 30 \text{ GeV})\), obtain the total energy in the center-of-mass system. Also obtain \( x_{\text{min}} \) for \( Q^2 \geq (1 \text{ GeV/c})^2 \).

3. Calculate the lepton tensor, \( L^{\mu \nu} \), defined by

\[ L^{\mu \nu} = \frac{i}{2} \sum_{s, s'} \left< k, s | j^{\mu}(o) | k', s' \right> \left< k', s'| j^{\nu}(o) | k, s \right>, \]

\[ = \frac{i}{2} \sum_{s} \overline{U}(k, s) Y^{\mu} \gamma^{\nu} U(k, s) \overline{U}(k', s') Y^{\nu} U(k, s'). \]

The 4-vector, \( s'^\mu \) is given by

\[ \overline{U}(k, s) Y^{\mu} \gamma^{\nu} U(k, s) = 2 s'^\mu, \]

and the lepton mass is again assumed to be negligible.

4. An experiment is performed at the luminosity \( L = 10^{32} \text{ cm}^{-2} \text{ s}^{-1} \) for 100 days. Obtain the integrated luminosity \( S \text{ L} \text{ at } (\rho \text{ pb}) \).

Obtain the expected number of events for the reaction with the cross section \( \sigma = 1(\rho \text{ pb}) \).

What is the statistical error in this case?
The RIKEN Winter School

Structure of Hadrons — Introduction to QCD Hard Processes —

Organizers: N. Saito, RIKEN
            T.-A. Shibata, Tokyo Inst. Tech./RIKEN
            K. Yazaki, Univ. Tokyo/RIKEN

Lecturers: R. Jaffe, MIT
           J. Kodaira, Hiroshima Univ.
           K. Rith, Erlangen-Nuernberg Univ.

Tutors:    N. Hayashi, RIKEN
           Y. Koike, Niigata Univ.
           S. Kumano, Saga Univ.
           K. Tanaka, Juntendo Univ.
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# Program of the School

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<th>14:15–15:15</th>
<th>15:45–17:45</th>
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<td>Kodaira</td>
<td>Kodaira</td>
<td>Jaffe</td>
<td>Rith</td>
<td>Jaffe</td>
<td>** evening session **</td>
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<td>Dec. 10</td>
<td>Rith</td>
<td>Rith</td>
<td>lunch</td>
<td>Koike</td>
<td>Rith</td>
<td>(Koike (Hayashi (Kumano</td>
<td></td>
</tr>
<tr>
<td>Dec. 11</td>
<td>Jaffe</td>
<td>Kodaira</td>
<td>Koike</td>
<td>Hayashi</td>
<td>(Kumano Tanaka)</td>
<td>** tutorial session **</td>
<td></td>
</tr>
<tr>
<td>Dec. 12</td>
<td>Jaffe</td>
<td>Rith</td>
<td>Rith</td>
<td>Tanaka</td>
<td>Rith</td>
<td>Tanaka</td>
<td>banquet</td>
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</table>

** dinner       ** dinner     ** banquet **