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3-D Target Location from Stereoscopic SAR Images

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3-D Target Location from Stereoscopic SAR Images

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ABSTRACT

SAR range-Doppler images are inherently 2-dimensional. Targets with a height offset lay over onto offset range and azimuth locations. Just which image locations are laid upon depends on the imaging geometry, including depression angle, squint angle, and target bearing. This is the well known layover phenomenon.

Images formed with different aperture geometries will exhibit different layover characteristics. These differences can be exploited to ascertain target height information, in a stereoscopic manner. Depending on the imaging geometries, height accuracy can be on the order of horizontal position accuracies, thereby rivaling the best IFSAR capabilities in fine resolution SAR images.

All that is required for this to work are two distinct passes with suitably different geometries from any plain old SAR.

Details follow.

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1. Introduction

A number of techniques exist to extract a 3-Dimensional position fix using Synthetic Aperture Radar (SAR). These include 3-D SAR, Interferometric SAR (IFSAR), and stereo SAR. All are related to each other in the sense that they use the equivalent of two (or more) 2-Dimensional images gathered in such a way to force differences between them that depend on target height.¹

IFSAR, in particular, uses phase differences to infer height, and is widely studied and reported in the literature.

This report deals with stereo SAR, which uses amplitude displacements between image pairs to extract information about the third dimension. For this to happen requires that targets at other than the nominal height be displaced somewhat between the two images. The degree of displacement depends on target height above the nominal focal plane. The direction of displacement depends on the imaging geometry. Any two images made from geometries that cause different layover directions can be used to ascertain target height.² We note that some geometries are better than others for particular target characteristics, like distributed targets that exhibit speckle.³

This report deals with the mathematics of finding target height from two images from arbitrary (but straight) apertures (flight paths). While stereo SAR is not a new idea,^{4,5,6,7} and using it for this purpose is not a new idea either,⁸ what has not yet been observed in the literature is a sufficiently detailed presentation of optimal flight geometries for height estimation using stereo techniques.

Note: This report is a somewhat expanded version of an earlier report that was originally distributed as a memo, Doerry, A. W., "3-D Target Location from Stereoscopic SAR images", memo to Distribution, August 10, 1999.

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1. Jakowatz, C. V. Jr., Daniel E. Wahl, Paul H. Eichel, Dennis C. Ghiglia, Paul A. Thompson, "Spotlight-Mode Synthetic Aperture Radar: A Signal Processing Approach", ISBN 0-7923-9677-4, Kluwer Academic Publishers, 1996.
 2. Doerry, Armin W., "SAR in a 3-D Target Space for Precision Guided Munitions", memo to J. R. Fellerhof, 15 February 1993.
 3. Jakowatz, Charles V. Jr., Daniel E. Wahl, Paul A. Thompson, "Three-Dimensional SAR Imaging Using Cross-Track Coherent Stereo Collections", Conf. Record of the 31st Asilomar Conf. on Signals, Systems & Computers, Vol. 2, 1998.
 4. Desai, Mita D., "Spotlight Mode SAR Stereo Technique for Height Computation", IEEE Transactions on Image Processing, Vol. 6, No. 10, October 1997.
 5. Leberl, Franz W., "Radargrammetric Image Processing", ISBN 0-89006-273-0, Artech House, Inc., 1990.
 6. Hensley, Scott, Scott Shaffer, "Automatic DEM Generation Using Magellan Stereo Data", IGARSS '94, Vol. 3, 1994.
 7. Dowman, I., "The geometry of SAR images for geocoding and stereo applications", Int. J. Remote Sensing, Vol. 13, No. 9, 1992.
 8. Abbot, Anthony S., Douglas E. Atkinson, "B-2 GPS Aided Munition and Targeting System", Proceedings of the Inst. of Navigation National Technical Meeting, Santa Monica, CA, Jan. 14-16, 1997.

2. Basic Geometry

Consider a SAR where geometry is defined as in figure 1. We identify

$$\begin{aligned} \text{MCP} &= \text{Motion Compensation Point, or the focal point of the image, and} \\ \text{APC}_0 &= \text{Antenna Phase Center location at the aperture center.} \end{aligned} \quad (1)$$

In this image we identify

$$\begin{aligned} \mathbf{r}_{c0} &= \text{vector from the MCP to the APC}_0, \\ s &= \text{vector from to MCP to the target location,} \\ \hat{\mathbf{n}} &= \text{unit vector normal to the image focal plane, to determine its orientation,} \\ \mathbf{m}_{\text{MCP}} &= \text{vector from the universal coordinate frame center to the MCP.} \end{aligned} \quad (2)$$

We can further identify

$$\begin{aligned} \hat{\mathbf{n}}_{c0} &= \mathbf{r}_{c0} / |\mathbf{r}_{c0}| = \text{unit vector in the direction of the aperture center,} \\ \psi_{c0} &= \text{elevation (grazing) angle such that } \sin \psi_{c0} = \hat{\mathbf{n}}_{c0} \cdot \hat{\mathbf{n}}, \\ \hat{\mathbf{n}}_{d0} &= \frac{\hat{\mathbf{n}}_{c0} - \hat{\mathbf{n}}_{c0} \cdot \hat{\mathbf{n}}}{\cos \psi_{c0}} = \text{unit vector in direction of } \mathbf{r}_{c0} \text{ projected into focal plane,} \\ \mathbf{v}_{c0} &= \left. \frac{d\mathbf{r}_c}{dt} \right|_{\mathbf{r}_c = \mathbf{r}_{c0}} = \text{velocity vector at the aperture center,} \end{aligned}$$

Figure 1. Geometric Definitions

$\hat{n}_{v0} = \mathbf{v}_{c0}/|\mathbf{v}_{c0}|$ = unit vector in direction of aircraft velocity,

ψ_{c0} = pitch angle of velocity vector at aperture center such that $\sin \psi_{c0} = \hat{n}_{v0} \cdot \hat{n}$,
and

Φ_0 = squint angle in the ground plane such that

$$\cos \Phi_0 = \frac{\hat{n}_{v0} \cdot (-\hat{n}_{d0})}{\cos \psi_{c0}} \text{ and } \sin \Phi_0 = \frac{(\hat{n}_{v0} \times \hat{n}_{d0}) \cdot \hat{n}}{\cos \psi_{c0}}. \quad (3)$$

With this geometry we can define the image coordinate frame (icf) with the following orthonormal basis vectors

$$\begin{aligned} \hat{u}_a &= \frac{\hat{n}_{c0} \times \hat{n}}{\cos \psi_{c0}} \\ \hat{u}_r &= \hat{n} \times \hat{u}_a, \text{ and} \\ \hat{u}_z &= \hat{n}. \end{aligned} \quad (4)$$

Consequently, the target vector is identified in the image coordinate frame as

$$\mathbf{s} = s_a \hat{u}_a + s_r \hat{u}_r + s_z \hat{u}_z. \quad (5)$$

The problem is that the SAR image formation process will project a target at this actual location to a different apparent location given by

$$\mathbf{s}_{\text{apparent}} = s_a' \hat{u}_a + s_r' \hat{u}_r = \mathbf{s} + s_z \mathbf{q}, \quad (6)$$

where the vector $\mathbf{q}$ identifies the 3-D displacement of the apparent target location from the true target location, but normalized to a unit target height. That is,

$$\mathbf{q} = \frac{(s_{\text{apparent}} - s)}{s_z}. \quad (7)$$

These relationships are illustrated in figure 2.

For not-too-tall targets especially near the scene center, the vector $\mathbf{q}$ will be independent of target coordinates, and hence a constant. Consequently the vector $\mathbf{q}$ also defines a direction or line in 3-D space along which all targets will project to the same image point. This presumption is, however, less accurate for especially tall targets distant from the SAR focal point, due primarily to wavefront and iso-Doppler cone curvature.

For straight and level flight, near the image center we calculate⁹

$$\begin{aligned} s_a' &\approx s_a - s_z \left(\frac{\tan \psi_{c0}}{\tan \Phi_0} \right), \text{ and} \\ s_r' &\approx s_r - s_z \tan \psi_{c0}. \end{aligned} \quad (8)$$

9. Doerry, A. W., "Squint Mode SAR in 3-D", memo to Distribution, September 18, 1997.

In matrix/vector notation we can state that in the image coordinate frame

$$s_{icf} = \begin{bmatrix} s_a \\ s_r \\ s_z \end{bmatrix} \approx \begin{bmatrix} s_a' \\ s_r' \\ 0 \end{bmatrix} + \begin{bmatrix} \tan \psi_{c0} / \tan \Phi_0 \\ \tan \psi_{c0} \\ 1 \end{bmatrix} s_z = \begin{bmatrix} s_a' \\ s_r' \\ 0 \end{bmatrix} - q_{icf} s_z. \quad (9)$$

More generally, aircraft may not always fly straight and level, but it does not seem too unreasonable to expect them to fly pretty straight and pretty level most of the time. Consequently, this approximation is usable most of the time. Nevertheless, we will develop a more general expression.

Suppose now that flight is straight but not necessarily level.

For a short target at the MCP, we can relate

$$\frac{q}{|q|} = \frac{v_{c0} \times (-r_{c0})}{|v_{c0} \times (-r_{c0})|} \text{sgn}(\Phi_0). \quad (10)$$

The signum function $\text{sgn}(\Phi_0)$ forces positive heights to project downwards.

Furthermore, we can calculate the incidence angle of the projection vector as

$$\cos \phi_q = \frac{q}{|q|} \cdot (-\hat{n}), \quad (11)$$

Figure 2. Target geometry, true vs. apparent position.

but we also know that since $\mathbf{q}$ is always downward,

$$\cos\phi_q = \frac{1}{|\mathbf{q}|}, \quad (12)$$

which allows us to combine these bits of knowledge to form the expression

$$\mathbf{q} = \frac{\mathbf{q}}{|\mathbf{q}|}|\mathbf{q}| = \frac{\mathbf{q}}{|\mathbf{q}|} \left(\frac{1}{\frac{\mathbf{q}}{|\mathbf{q}|} \cdot (-\hat{n})} \right) = \left(\frac{\mathbf{v}_{c0} \times (-\mathbf{r}_{c0})}{|\mathbf{v}_{c0} \times (-\mathbf{r}_{c0})|} \right) \left(\frac{1}{\left(\frac{\mathbf{v}_{c0} \times (-\mathbf{r}_{c0})}{|\mathbf{v}_{c0} \times (-\mathbf{r}_{c0})|} \right) \cdot (-\hat{n})} \right). \quad (13)$$

Simplifying a bit yields-

$$\mathbf{q} = \left(\frac{\mathbf{v}_{c0} \times (-\mathbf{r}_{c0})}{(\mathbf{v}_{c0} \times (-\mathbf{r}_{c0})) \cdot (-\hat{n})} \right). \quad (14)$$

With the geometry previously identified, we observe in the icf frame

$$\mathbf{v}_{c0} = |\mathbf{v}_{c0}| \begin{pmatrix} -\cos\upsilon_{c0} \sin\Phi_0 \hat{u}_a \\ + \cos\upsilon_{c0} \cos\Phi_0 \hat{u}_r \\ + \sin\upsilon_{c0} \hat{u}_z \end{pmatrix} = |\mathbf{v}_{c0}| \begin{bmatrix} -\cos\upsilon_{c0} \sin\Phi_0 \\ \cos\upsilon_{c0} \cos\Phi_0 \\ \sin\upsilon_{c0} \end{bmatrix}, \text{ and} \quad (15)$$

$$\mathbf{r}_{c0} = |\mathbf{r}_{c0}| \begin{pmatrix} -\cos\psi_{c0} \hat{u}_r \\ + \sin\psi_{c0} \hat{u}_z \end{pmatrix} = |\mathbf{r}_{c0}| \begin{bmatrix} 0 \\ -\cos\psi_{c0} \\ \sin\psi_{c0} \end{bmatrix}. \quad (16)$$

Consequently,

$$\mathbf{v}_{c0} \times (-\mathbf{r}_{c0}) = |\mathbf{v}_{c0}| |\mathbf{r}_{c0}| \begin{bmatrix} (\cos\upsilon_{c0} \cos\Phi_0 \sin\psi_{c0} + \cos\psi_{c0} \sin\upsilon_{c0}) \\ \cos\upsilon_{c0} \sin\Phi_0 \sin\psi_{c0} \\ \cos\upsilon_{c0} \sin\Phi_0 \cos\psi_{c0} \end{bmatrix}, \quad (17)$$

and

$$\mathbf{q}_{\text{icf}} = \begin{bmatrix} -\left(\frac{\tan\psi_{c0}}{\tan\Phi_0} + \frac{\tan\upsilon_{c0}}{\sin\Phi_0} \right) \\ -\tan\psi_{c0} \\ -1 \end{bmatrix}. \quad (18)$$

Now for some observations.

- Projection in the range dimension ($\hat{u}_r$) depends only on the grazing angle.
- Projection in the azimuth dimension ($\hat{u}_a$) depends on all angles, that is, grazing, squint and pitch.

Of course we can rotate this vector to any other coordinate frame by multiplying with some rotation matrix. For example, to achieve an east-north-up (enu) frame, and assuming that up in the icf frame is the same up as in the enu frame, we identify the rotation matrix as

$$R_{\phi, \text{enu}} = \begin{bmatrix} \cos \phi_b & \sin \phi_b & 0 \\ -\sin \phi_b & \cos \phi_b & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (19)$$

where ϕ_b = the ground bearing from APC_0 to the MCP. In this case

$$s_{\text{enu}} = R_{\phi, \text{enu}} \begin{bmatrix} s_a \\ s_r \\ s_z \end{bmatrix} = R_{\phi, \text{enu}} \begin{bmatrix} s_a \\ s_r \\ 0 \end{bmatrix} - R_{\phi, \text{enu}} q_{\text{icf}} s_z. \quad (20)$$

Furthermore, we identify the rotated projection vector in the enu frame as

$$q_{\text{enu}} = R_{\phi, \text{enu}} q_{\text{icf}}. \quad (21)$$

The task at hand is to find s_z , and ultimately the true s_a and s_r .

From the universal coordinate frame origin, the target is located at a position

$$p = m_{\text{MCP}} + s. \quad (22)$$

This holds for all coordinate frames. For example

$$p_{\text{enu}} = m_{\text{MCP}, \text{enu}} + s_{\text{enu}}. \quad (23)$$

One final point is that the unit vector $q/|q|$, along with the MCP, defines a unique ‘slant plane’. This slant plane is often used to identify an equivalent plane in Fourier space to describe the nature of the collected phase-history set.¹⁰

10. Jakowatz, C. V. Jr., Daniel E. Wahl, Paul H. Eichel, Dennis C. Ghiglia, Paul A. Thompson, “Spotlight-Mode Synthetic Aperture Radar: A Signal Processing Approach”, ISBN 0-7923-9677-4, Kluwer Academic Publishers, 1996.

3. Multiple Coordinate Sets

Consider two distinct SAR images that contain the same target point. We identify the various coordinates and frames with different subscript indices.

For image number 1,

$$\begin{aligned}
 p_{\text{enu}, 1} &= m_{\text{MCP}, \text{enu}, 1} + R_{\phi, \text{enu}, 1} \begin{bmatrix} s_{a, 1} \\ s_{r, 1} \\ s_{z, 1} \end{bmatrix} \\
 &= m_{\text{MCP}, \text{enu}, 1} + R_{\phi, \text{enu}, 1} \begin{bmatrix} s_{a, 1} \\ s_{r, 1} \\ 0 \end{bmatrix} - R_{\phi, \text{enu}, 1} q_{\text{icf}, 1} s_{z, 1} \cdot
 \end{aligned} \tag{24}$$

For image number 2,

$$\begin{aligned}
 p_{\text{enu}, 2} &= m_{\text{MCP}, \text{enu}, 2} + R_{\phi, \text{enu}, 2} \begin{bmatrix} s_{a, 2} \\ s_{r, 2} \\ s_{z, 2} \end{bmatrix} \\
 &= m_{\text{MCP}, \text{enu}, 2} + R_{\phi, \text{enu}, 2} \begin{bmatrix} s_{a, 2} \\ s_{r, 2} \\ 0 \end{bmatrix} - R_{\phi, \text{enu}, 2} q_{\text{icf}, 2} s_{z, 2} \cdot
 \end{aligned} \tag{25}$$

But, the target is stationary, which implies

$$p_{\text{enu}, 1} = p_{\text{enu}, 2} \tag{26}$$

Consequently,

$$\begin{aligned}
 m_{\text{MCP}, \text{enu}, 2} - m_{\text{MCP}, \text{enu}, 1} + R_{\phi, \text{enu}, 2} \begin{bmatrix} s_{a, 2} \\ s_{r, 2} \\ 0 \end{bmatrix} - R_{\phi, \text{enu}, 1} \begin{bmatrix} s_{a, 1} \\ s_{r, 1} \\ 0 \end{bmatrix} \\
 = R_{\phi, \text{enu}, 2} q_{\text{icf}, 2} s_{z, 2} - R_{\phi, \text{enu}, 1} q_{\text{icf}, 1} s_{z, 1} \cdot
 \end{aligned} \tag{27}$$

This can be rewritten as the overconstrained set of linear equations

$$Ax = b \tag{28}$$

where

$$A = \begin{bmatrix} -(R_{\phi, \text{enu}, 1} q_{\text{icf}, 1}) & (R_{\phi, \text{enu}, 2} q_{\text{icf}, 2}) \end{bmatrix} = \begin{bmatrix} -q_{\text{enu}, 1} & q_{\text{enu}, 2} \end{bmatrix}, \tag{29}$$

$$\mathbf{x} = \begin{bmatrix} s_{z,1} \\ s_{z,2} \end{bmatrix}, \text{ and} \quad (30)$$

$$\mathbf{b} = \mathbf{m}_{\text{MCP, enu, 2}} - \mathbf{m}_{\text{MCP, enu, 1}} + \mathbf{R}_{\phi, \text{enu, 2}} \begin{bmatrix} s_{a,2} \\ s_{r,2} \\ 0 \end{bmatrix} - \mathbf{R}_{\phi, \text{enu, 1}} \begin{bmatrix} s_{a,1} \\ s_{r,1} \\ 0 \end{bmatrix}. \quad (31)$$

Solving for target height by solving for $\mathbf{x}$ is accomplished by calculating

$$\begin{bmatrix} s_{z,1} \\ s_{z,2} \end{bmatrix} = \mathbf{x} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}. \quad (32)$$

This is a least-squares fit to the given data. Since the problem is overconstrained, an exact answer may not always exist due to measurement errors, and the like. Furthermore, equal weighting is given to all measurements. While one might envision an algorithm that places more weight on range measurements than perhaps inherently less accurate azimuth measurements, this report will not do this, and treat all measurements as equally valid.

Of course, once these height parameters are known, true positions with respect to the MCPs can be found via

$$\mathbf{s}_{\text{icf, 1}} = \begin{bmatrix} s_{a,1} \\ s_{r,1} \\ s_{z,1} \end{bmatrix} = \begin{bmatrix} s_{a,1} \\ s_{r,1} \\ 0 \end{bmatrix} - \mathbf{q}_{\text{icf, 1}} s_{z,1}, \quad (33)$$

and

$$\mathbf{s}_{\text{icf, 2}} = \begin{bmatrix} s_{a,2} \\ s_{r,2} \\ s_{z,2} \end{bmatrix} = \begin{bmatrix} s_{a,2} \\ s_{r,2} \\ 0 \end{bmatrix} - \mathbf{q}_{\text{icf, 2}} s_{z,2}. \quad (34)$$

Relative Height Measurements

We note that by choosing a common reference point in the two images, and measuring azimuth and range offsets with respect to it, this is tantamount to shifting the MCPs for the two images to a common point (as far as target location is concerned), such that

$\mathbf{m}_{\text{MCP, enu, 2}} = \mathbf{m}_{\text{MCP, enu, 1}}$. Consequently, the target height estimates $s_{z,1}$ and $s_{z,2}$ should be identical and yield the relative height with respect to the common reference point.

We also note that more than two images might be employed to further overconstrain the problem, but nevertheless to perhaps find a better ‘average’ height solution. Details for this are omitted in this report.

4. Limitations and Errors

A reasonable question might be “How well should we expect this to work in real life?” With a little more thought, we might modify our question to “What do we have to do to make this technique work well?”

4.1. Flight path limitations

The first limitation has already been discussed, that is, we presume straight, though not necessarily level, flight during an aperture for any particular SAR image.

However, additional limitations exist on the relationship between the apertures.

To investigate this we return to the calculation of the target height solution

$$\begin{bmatrix} s_{z,1} \\ s_{z,2} \end{bmatrix} = \mathbf{x} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}. \quad (35)$$

We note that a matrix inversion is required. That is, it is presumed that the entity $(\mathbf{A}^T \mathbf{A})^{-1}$ exists. For this to be true, $(\mathbf{A}^T \mathbf{A})$ must be of full rank. Let us examine this further.

Recall that the matrix $\mathbf{A}$ is of the form

$$\mathbf{A} = \begin{bmatrix} -\mathbf{q}_{\text{enu},1} & \mathbf{q}_{\text{enu},2} \end{bmatrix} \quad (36)$$

where the interior column vectors are defined as

$$\begin{aligned} \mathbf{q}_{\text{enu},1} &= \mathbf{R}_{\phi, \text{enu},1} \begin{bmatrix} -\left(\frac{\tan \psi_{c0,1}}{\tan \Phi_{0,1}} + \frac{\tan \nu_{c0,1}}{\sin \Phi_{0,1}} \right) \\ -\tan \psi_{c0,1} \\ -1 \end{bmatrix}, \text{ and} \\ \mathbf{q}_{\text{enu},2} &= \mathbf{R}_{\phi, \text{enu},2} \begin{bmatrix} -\left(\frac{\tan \psi_{c0,2}}{\tan \Phi_{0,2}} + \frac{\tan \nu_{c0,2}}{\sin \Phi_{0,2}} \right) \\ -\tan \psi_{c0,2} \\ -1 \end{bmatrix}. \end{aligned} \quad (37)$$

Restating from before, these vectors represent the scale factors for $s_{z,1}$ and $s_{z,2}$ respectively for layover in the enu coordinate frame. That is, this is how targets with height lay over in the reference enu frame. These vectors also represents a line in 3-D space indicating the projection direction in 3-D space, or line of projection, not to be confused with the layover direction in the SAR image itself, which is itself a projection of this line. Confusing, eh? See figure 3.

We may then calculate

$$(A^T A) = \begin{bmatrix} (q_{\text{enu},1}^T q_{\text{enu},1}) & -(q_{\text{enu},1}^T q_{\text{enu},2}) \\ -(q_{\text{enu},2}^T q_{\text{enu},1}) & (q_{\text{enu},2}^T q_{\text{enu},2}) \end{bmatrix}, \text{ and} \quad (38)$$

$$(A^T A)^{-1} = \frac{\begin{bmatrix} (q_{\text{enu},2}^T q_{\text{enu},2}) & (q_{\text{enu},1}^T q_{\text{enu},2}) \\ (q_{\text{enu},2}^T q_{\text{enu},1}) & (q_{\text{enu},1}^T q_{\text{enu},1}) \end{bmatrix}}{\begin{pmatrix} (q_{\text{enu},1}^T q_{\text{enu},1})(q_{\text{enu},2}^T q_{\text{enu},2}) \\ -(q_{\text{enu},2}^T q_{\text{enu},1})(q_{\text{enu},1}^T q_{\text{enu},2}) \end{pmatrix}} = \frac{\begin{bmatrix} \|q_{\text{enu},2}\|^2 & q_{\text{enu},1}^T q_{\text{enu},2} \\ q_{\text{enu},1}^T q_{\text{enu},2} & \|q_{\text{enu},1}\|^2 \end{bmatrix}}{\begin{pmatrix} \|q_{\text{enu},1}\|^2 \|q_{\text{enu},2}\|^2 \\ -(q_{\text{enu},1}^T q_{\text{enu},2})^2 \end{pmatrix}}. \quad (39)$$

Clearly, one obvious condition is that we must insist

$$q_{\text{enu},1} \neq \pm q_{\text{enu},2} \quad (40)$$

which means that the directions of projection can not be colinear. This in turn implies that the flight paths can not be located in the same slant plane. For example, subsequent apertures from the same straight and level flight path will not work. This is because the lines of projection would also be colinear, rendering no real projection differences between the two images. This is bad. This could, however, be mitigated by incorporating an appropriate turn between two subsequent apertures.

Figure 3. Projection directions illustrated.

If we did have the condition $q_{\text{enu},1} \approx q_{\text{enu},2}$ (note that $q_{\text{enu},1} = -q_{\text{enu},2}$ isn't really viable), then $(A^T A)$ would be ill-conditioned, implying that small errors in vector b would yield perhaps large errors in solution vector x . We don't like that. In fact, we want the solution vector x to be as insensitive to errors in vector b as possible. That is we want $(A^T A)$ as un-ill-conditioned as possible.

A reasonable inference is then that we want vectors $q_{\text{enu},1}$ and $q_{\text{enu},2}$ to be as different as possible. In the images, this means that we want the apparent positions due to layover in the reference enu frame to be as different as possible, but in a 3-D projection sense. We note that any two coplanar (same slant plane) apertures will cause layover to the exact same apparent position in the reference enu frame.¹¹ We don't want this.

4.2. Effects due to position measurement errors

The question now is "How sensitive are we to position measurement errors?"

All position measurements are wrapped up in vector b , which we recall is

$$b = m_{\text{MCP}, \text{enu}, 2} - m_{\text{MCP}, \text{enu}, 1} + R_{\phi, \text{enu}, 2} \begin{bmatrix} s_{a, 2} \\ s_{r, 2} \\ 0 \end{bmatrix} - R_{\phi, \text{enu}, 1} \begin{bmatrix} s_{a, 1} \\ s_{r, 1} \\ 0 \end{bmatrix}. \quad (41)$$

Matrix A contains strictly angle data, which are generally pretty accurate. More will be said on this later.

Note that b is essentially a position-difference vector, with units of distance (such as meters). Consequently errors in position, whether the presumed MCP vectors or the apparent target position coordinates, at most get rotated but not scaled (by R_{ϕ}).

The effect of a position error is to yield an error in the height estimates, as follows

$$\begin{bmatrix} \varepsilon_{z, 1} \\ \varepsilon_{z, 2} \end{bmatrix} = \Delta x = [(A^T A)^{-1} A^T] \Delta b \quad (42)$$

where

$\varepsilon_{z, 1}$ = the error in $s_{z, 1}$,

$\varepsilon_{z, 2}$ = the error in $s_{z, 2}$,

Δx = the error in x , and

Δb = the error in b .

(43)

So, the thing that takes a Δb and turns it into a Δx is the transformation matrix

11. Doerry, A. W., "SAR in 3-D Target Space for Precision Guided Munitions", memo to J. R. Fellerhof, February 15, 1993.

$$[(A^T A)^{-1} A^T] = \begin{bmatrix} \frac{(-\|q_{\text{enu}, 2}\|^2 q_{\text{enu}, 1}^T + (q_{\text{enu}, 1}^T q_{\text{enu}, 2}) q_{\text{enu}, 2}^T)}{(\|q_{\text{enu}, 1}\|^2 \|q_{\text{enu}, 2}\|^2 - (q_{\text{enu}, 1}^T q_{\text{enu}, 2})^2)} \\ \frac{(\|q_{\text{enu}, 1}\|^2 q_{\text{enu}, 2}^T - (q_{\text{enu}, 1}^T q_{\text{enu}, 2}) q_{\text{enu}, 1}^T)}{(\|q_{\text{enu}, 1}\|^2 \|q_{\text{enu}, 2}\|^2 - (q_{\text{enu}, 1}^T q_{\text{enu}, 2})^2)} \end{bmatrix}. \quad (44)$$

The point is that this transformation both rotates and scales $\Delta \mathbf{b}$ into perhaps a much larger $\Delta \mathbf{x}$. We really don't want a big $\Delta \mathbf{x}$, so we prefer transformations that make as small a $\Delta \mathbf{x}$ as possible. The appropriate constraints on vectors $q_{\text{enu}, 1}$ and $q_{\text{enu}, 2}$ that do this also constrain flight geometries.

Sources for position errors might include inaccurate target location within any SAR image (peak interpolation/location), inadequate orthorectification, and unaccounted for residual geometric errors such as from residual wavefront curvature within the image.

The transformation matrix can be manipulated to the form

$$[(A^T A)^{-1} A^T] = \begin{bmatrix} \frac{1}{\|q_{\text{enu}, 1}\|} \left(\frac{-q_{\text{enu}, 1}^T}{\|q_{\text{enu}, 1}\|} + \left(\frac{q_{\text{enu}, 1}^T q_{\text{enu}, 2}}{\|q_{\text{enu}, 1}\| \|q_{\text{enu}, 2}\|} \right) \frac{q_{\text{enu}, 2}^T}{\|q_{\text{enu}, 2}\|} \right) \\ \frac{1}{\|q_{\text{enu}, 2}\|} \left(\frac{q_{\text{enu}, 2}^T}{\|q_{\text{enu}, 2}\|} - \left(\frac{q_{\text{enu}, 1}^T q_{\text{enu}, 2}}{\|q_{\text{enu}, 1}\| \|q_{\text{enu}, 2}\|} \right) \frac{q_{\text{enu}, 1}^T}{\|q_{\text{enu}, 1}\|} \right) \\ \frac{1}{\|q_{\text{enu}, 1}\|} \left(\frac{q_{\text{enu}, 2}^T}{\|q_{\text{enu}, 1}\|} - \left(\frac{q_{\text{enu}, 1}^T q_{\text{enu}, 2}}{\|q_{\text{enu}, 1}\| \|q_{\text{enu}, 2}\|} \right) \frac{q_{\text{enu}, 1}^T}{\|q_{\text{enu}, 1}\|} \right) \\ \frac{1}{\|q_{\text{enu}, 2}\|} \left(\frac{-q_{\text{enu}, 1}^T}{\|q_{\text{enu}, 2}\|} + \left(\frac{q_{\text{enu}, 1}^T q_{\text{enu}, 2}}{\|q_{\text{enu}, 1}\| \|q_{\text{enu}, 2}\|} \right) \frac{q_{\text{enu}, 2}^T}{\|q_{\text{enu}, 2}\|} \right) \end{bmatrix}. \quad (45)$$

Since everything is written in terms of normalized vectors, and acknowledging the constraints on the magnitudes for vectors $q_{\text{enu}, 1}$ and $q_{\text{enu}, 2}$, we can bound the magnitudes of the row vectors to

$$\left\| \frac{1}{\|q_{\text{enu}, 1}\|} \left(\frac{-q_{\text{enu}, 1}^T}{\|q_{\text{enu}, 1}\|} + \left(\frac{q_{\text{enu}, 1}^T q_{\text{enu}, 2}}{\|q_{\text{enu}, 1}\| \|q_{\text{enu}, 2}\|} \right) \frac{q_{\text{enu}, 2}^T}{\|q_{\text{enu}, 2}\|} \right) \right\| \leq \left(\frac{2}{1 - \left(\frac{q_{\text{enu}, 1}^T q_{\text{enu}, 2}}{\|q_{\text{enu}, 1}\| \|q_{\text{enu}, 2}\|} \right)^2} \right), \quad (46)$$

and

$$\left\| \frac{1}{\|q_{\text{enu}, 2}\|} \left(\frac{q_{\text{enu}, 2}^T}{\|q_{\text{enu}, 2}\|} - \left(\frac{q_{\text{enu}, 1}^T q_{\text{enu}, 2}}{\|q_{\text{enu}, 1}\| \|q_{\text{enu}, 2}\|} \right) \frac{q_{\text{enu}, 1}^T}{\|q_{\text{enu}, 1}\|} \right) \right\| \leq \left(\frac{2}{1 - \left(\frac{q_{\text{enu}, 1}^T q_{\text{enu}, 2}}{\|q_{\text{enu}, 1}\| \|q_{\text{enu}, 2}\|} \right)^2} \right). \quad (47)$$

These are, of course, minimized when $q_{\text{enu}, 1}^T q_{\text{enu}, 2} = 0$. What this does is place the direction of uncertainty from the first aperture orthogonal to the direction of uncertainty from the second aperture, that is, each aperture maximally addresses the uncertainty of the other.¹² This is desirable.

4.3. Angle measurement errors

For a straight-line flight path, the velocity vector of the aircraft (along with the aperture center) determines the bearing, squint angle, and pitch angle of the data collection. These angles fully determine the A matrix, and hence the quantity $[(A^T A)^{-1} A^T]$ which rotates and scales image coordinates to find target height.

Errors in these angle measures lead to errors in the rotation and scaling of image coordinates, thereby assigning wrong height values. These height errors tend to scale with target distance from the reference point, which for a collection of target points manifests itself as a tilt in the image. Consequently, wrong angle measures will tilt any reconstructed topography of a scene.

Generally, for Sandia SAR systems, instantaneous measures of these angles will be very accurate, to sub-degree precision.

4.4. Non-straight-line flight paths

The presumption heretofore has been a straight-line aperture or flight path, yielding unambiguous angle measures to be incorporated into the A matrix. To the extent that a flight path isn't straight, there exists variation in these angles (notably squint and pitch) over the length of an aperture. Consequently, some sort of average (perhaps a weighted average) of the squint and pitch angles may be more appropriate than any single instantaneous measure such as at the center of an aperture, particularly if significant non-linear motion exists.

In practice, if the vehicle tries to fly straight, the instantaneous values for these angles sampled at the center of the aperture is suitably representative, and often good enough.

12. Abbott, Anthony, Neeraj Putra, Peter Howe, "GPS Exploitation for Precision Targeting - Observability Using Synthetic Aperture Radar", Proceedings of the 49th Annual Meeting on Future Global Navigation and Guidance, Cambridge, Massachusetts, June 21-23, 1993.

5. Analysis of Flight Geometries

5.1. Case: Coplanar flight paths revisited

As previously discussed, if $\mathbf{q}_{\text{enu},1} = \mathbf{q}_{\text{enu},2}$, then the denominators of the transformation matrix go to zero and all terms of the transformation matrix blow up to infinity. Consequently, errors in $\Delta \mathbf{x}$ will be infinite, implying no real solution. This is bad. We don't want this.

5.2. Case: Orthogonal projection directions

For a given magnitude $\|\mathbf{q}_{\text{enu},1}\|^2$ and $\|\mathbf{q}_{\text{enu},2}\|^2$, we can maximize the transformation matrix denominators by forcing $\mathbf{q}_{\text{enu},1}^T \mathbf{q}_{\text{enu},2} = 0$, that is, making the 3-D projection directions orthogonal. The transformation matrix then becomes

$$[(\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T] = \begin{bmatrix} -(\mathbf{q}_{\text{enu},1}^T / \|\mathbf{q}_{\text{enu},1}\|^2) \\ (\mathbf{q}_{\text{enu},2}^T / \|\mathbf{q}_{\text{enu},2}\|^2) \end{bmatrix}. \quad (48)$$

We note that we can compute the magnitude of $\mathbf{q}_{\text{enu},1}$ and $\mathbf{q}_{\text{enu},2}$ respectively as

$$\begin{aligned} \|\mathbf{q}_{\text{enu},1}\| &= \sqrt{\left(\frac{\tan \psi_{c0,1}}{\tan \Phi_{0,1}} + \frac{\tan \nu_{c0,1}}{\sin \Phi_{0,1}}\right)^2 + (\tan \psi_{c0,1})^2 + 1}, \text{ and} \\ \|\mathbf{q}_{\text{enu},2}\| &= \sqrt{\left(\frac{\tan \psi_{c0,2}}{\tan \Phi_{0,2}} + \frac{\tan \nu_{c0,2}}{\sin \Phi_{0,2}}\right)^2 + (\tan \psi_{c0,2})^2 + 1}. \end{aligned} \quad (49)$$

For typically geometries where $\nu_{c0,1} = 0$, $0 < \psi_{c0,1} \leq 45^\circ$, and $45^\circ < \Phi_{0,1} \leq 135^\circ$, and similarly for $\nu_{c0,2}$, $\psi_{c0,2}$ and $\Phi_{0,2}$, the magnitudes are always less than $\sqrt{3} \approx 1.732$, and usually in the neighborhood of 1. Consequently, since error scaling is inversely proportional to this magnitude, a relative position error of 1 m from whatever source might cause us to expect no more than a 1 m error in our height estimate. This is a very important observation.

An illustration of orthogonal projection directions is given in figure 4.

So, one might ask "What flight geometries does it take to make the lines of projection orthogonal?"

Recall that by definition we are considering

$$\mathbf{q}_{\text{enu},1}^T \mathbf{q}_{\text{enu},2} = (\mathbf{R}_{\phi,\text{enu},1} \mathbf{q}_{\text{icf},1})^T (\mathbf{R}_{\phi,\text{enu},2} \mathbf{q}_{\text{icf},2}) = 0, \quad (50)$$

or, expanding a bit

$$\begin{bmatrix} \frac{\tan \psi_{c0,1} + \frac{\tan v_{c0,1}}{\sin \Phi_{0,1}}}{\tan \Phi_{0,1}} \\ \tan \psi_{c0,1} \\ 1 \end{bmatrix}^T R_{\phi, \text{enu}, 1}^T R_{\phi, \text{enu}, 2} \begin{bmatrix} \frac{\tan \psi_{c0,2} + \frac{\tan v_{c0,2}}{\sin \Phi_{0,2}}}{\tan \Phi_{0,2}} \\ \tan \psi_{c0,2} \\ 1 \end{bmatrix} = 0, \quad (51)$$

Now let us consider some level flight scenarios, where $v_{c0,1} = v_{c0,2} = 0$

Similar bearings and broadside squint

If the two apertures are made with identical bearings to target scene, then $R_{\phi, \text{enu}, 1}^T R_{\phi, \text{enu}, 2}$ degenerates to the identity matrix, and we are left with

$$\begin{bmatrix} \tan \psi_{c0,1} \\ \tan \Phi_{0,1} \\ \tan \psi_{c0,1} \\ 1 \end{bmatrix}^T \begin{bmatrix} \tan \psi_{c0,2} \\ \tan \Phi_{0,2} \\ \tan \psi_{c0,2} \\ 1 \end{bmatrix} = \tan \psi_{c0,1} \tan \psi_{c0,2} \left(1 + \frac{1}{\tan \Phi_{0,1} \tan \Phi_{0,2}} \right) + 1 = 0. \quad (52)$$

Figure 4. Orthogonal projection directions illustrated.

For a broadside squint angle, $\tan \Phi_{0,1} \tan \Phi_{0,2} \rightarrow \pm\infty$, depending on which side of the aircraft. This leaves us with

$$\tan \psi_{c0,1} \tan \psi_{c0,2} = -1 \quad (53)$$

which implies a 90° difference in the depression angles.

Similar bearings and depression angles

This leaves us with the condition on squint angles of

$$\tan \Phi_{0,1} \tan \Phi_{0,2} = \frac{-\tan \psi_{c0,1} \tan \psi_{c0,2}}{1 + \tan \psi_{c0,1} \tan \psi_{c0,2}} = \frac{-(\tan \psi_{c0,1})^2}{1 + (\tan \psi_{c0,1})^2} = -(\sin \psi_{c0,1})^2. \quad (54)$$

For example, with $\psi_{c0,1} = 45^\circ$, we require $\tan \Phi_{0,1} \tan \Phi_{0,2} = -0.5$, which would be satisfied by squint angles $\Phi_{0,1} = 35.3^\circ$ and $\Phi_{0,2} = 144.7^\circ$.

Note that the actual flight paths represented by their squint angles are not orthogonal, and in fact neither are the layover directions in the SAR images themselves. Nevertheless, the 3-D directions of projection are orthogonal in 3-D space.

This case amounts to crossed flight paths thru the same point in 3-D space.¹³

Broadside squint and similar depression angles

In this case, the bearings will be different. Recall that for straight and level flight we desire

$$\begin{bmatrix} \tan \psi_{c0,1} \\ \tan \Phi_{0,1} \\ \tan \psi_{c0,1} \\ 1 \end{bmatrix}^T \mathbf{R}_{\phi, \text{enu}, 1}^T \mathbf{R}_{\phi, \text{enu}, 2} \begin{bmatrix} \tan \psi_{c0,2} \\ \tan \Phi_{0,2} \\ \tan \psi_{c0,2} \\ 1 \end{bmatrix} = 0. \quad (55)$$

Note that the entity $\mathbf{R}_{\phi, \text{enu}, 1}^T \mathbf{R}_{\phi, \text{enu}, 2}$ is a relative rotation matrix, so without loss of generality we may presume that $\mathbf{R}_{\phi, \text{enu}, 1} = \mathbf{I}$ = identity matrix. This is tantamount to presuming that $\phi_{b,1} = 0^\circ$. Consequently,

13. Jakowatz, Charles V. Jr., Daniel E. Wahl, Paul A. Thompson, "Three-Dimensional SAR Imaging Using Cross-Track Coherent Stereo Collections", Conf. Record of the 31st Asilomar Conf. on Signals, Systems & Computers, Vol. 2, 1998.

$$\mathbf{R}_{\phi, \text{enu}, 1}^T \mathbf{R}_{\phi, \text{enu}, 2} = \begin{bmatrix} \cos \phi_{b, 2} & \sin \phi_{b, 2} & 0 \\ -\sin \phi_{b, 2} & \cos \phi_{b, 2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (56)$$

and

$$\begin{aligned} \mathbf{q}_{\text{enu}, 1}^T \mathbf{q}_{\text{enu}, 2} &= \begin{pmatrix} \cos \phi_{b, 2} \frac{\tan \psi_{c0, 1} \tan \psi_{c0, 2}}{\tan \Phi_{0, 1} \tan \Phi_{0, 2}} + \cos \phi_{b, 2} \tan \psi_{c0, 1} \tan \psi_{c0, 2} \\ + \sin \phi_{b, 2} \frac{\tan \psi_{c0, 1} \tan \psi_{c0, 2}}{\tan \Phi_{0, 1}} - \sin \phi_{b, 2} \frac{\tan \psi_{c0, 1} \tan \psi_{c0, 2}}{\tan \Phi_{0, 2}} \\ + 1 \end{pmatrix} \\ &= \begin{pmatrix} \cos \phi_{b, 2} \tan \psi_{c0, 1} \tan \psi_{c0, 2} \left(\frac{1}{\tan \Phi_{0, 1} \tan \Phi_{0, 2}} + 1 \right) \\ + \sin \phi_{b, 2} \tan \psi_{c0, 1} \tan \psi_{c0, 2} \left(\frac{1}{\tan \Phi_{0, 1}} - \frac{1}{\tan \Phi_{0, 2}} \right) \\ + 1 \end{pmatrix} = 0. \quad (57) \end{aligned}$$

If we assume broadside squint angles such that $\Phi_{0, 1} = \Phi_{0, 2} = 90^\circ$, then this reduces to

$$\cos \phi_{b, 2} = \frac{-1}{\tan \psi_{c0, 1} \tan \psi_{c0, 2}}. \quad (58)$$

For $\psi_{c0, 1} = \psi_{c0, 2} = 45^\circ$, then $\phi_{b, 2} = 180^\circ$, which is directly across the target from the first bearing.

Note again that although the layover in the images are 180° different, the 3-D directions of projection are only 90° different.

While only level scenarios were presented here, we note that climbing and descending apertures are just as easily exploited.

6. Example - Contrived Geometries

Consider the following example.

The geometry illustrated in figure 5, which is defined in a universal east-north-up (enu) coordinate frame by the vectors

$$\begin{aligned}
 \mathbf{m}_{\text{MCP}, \text{enu}, 1} &= [-10 \ 20 \ 0]^T \text{ m}, \\
 \mathbf{r}_{c0, \text{enu}, 1} &= [10 \ 200 \ 50]^T \text{ m}, \\
 \hat{\mathbf{n}}_{v0, \text{enu}, 1} &= [0.8944 \ -0.4472 \ 0]^T, \\
 \mathbf{m}_{\text{MCP}, \text{enu}, 2} &= [40 \ -30 \ 15]^T \text{ m}, \\
 \mathbf{r}_{c0, \text{enu}, 2} &= [300 \ -30 \ 70]^T \text{ m, and} \\
 \hat{\mathbf{n}}_{v0, \text{enu}, 1} &= [-0.1961 \ -0.9806 \ 0]^T.
 \end{aligned} \tag{59}$$

The target truth position is stated in the enu frame as

$$(\mathbf{m}_{\text{MCP}, \text{enu}, 1} + \mathbf{s}_{\text{enu}, 1}) = (\mathbf{m}_{\text{MCP}, \text{enu}, 2} + \mathbf{s}_{\text{enu}, 2}) = [20 \ 40 \ 50]^T \text{ m}. \tag{60}$$

Figure 5. Geometry and ground truth.

From these we calculate the relevant angles

$$\begin{aligned}
 \psi_{c0,1} &= 14.0194^\circ, \\
 \Phi_{0,1} &= 66.2974^\circ, \\
 \phi_{b,1} &= -177.1376^\circ, \\
 v_{c0,1} &= 0, \\
 \\
 \psi_{c0,2} &= 13.0711^\circ, \\
 \Phi_{0,2} &= 84.4007^\circ, \\
 \phi_{b,2} &= -84.2894^\circ, \text{ and} \\
 v_{c0,2} &= 0,
 \end{aligned} \tag{61}$$

In an ideal range-Doppler SAR image, the target would appear at the following image coordinate frame locations, illustrated in figure 6.

$$\begin{aligned}
 s_{icf,1} &= [-34.4448 \ -33.9576 \ 0]^T \text{ m, and} \\
 s_{icf,2} &= [66.8658 \ 18.7399 \ 0]^T \text{ m.}
 \end{aligned} \tag{62}$$

Now using just these angles and image frame coordinates, we identify

Figure 6. Apparent target locations in image coordinate frames.

$$\mathbf{A} = \begin{bmatrix} -0.1220 & 0.2288 \\ -0.2439 & -0.0458 \\ 1 & -1 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{bmatrix} 1.9090 \\ -13.7964 \\ 15 \end{bmatrix}. \quad (63)$$

We therewith calculate

$$[(\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T] = \begin{bmatrix} 1.0497 & -3.0654 & 0.3804 \\ 1.2292 & -2.9992 & -0.5816 \end{bmatrix} \quad (64)$$

which allows us to calculate height estimates

$$\begin{bmatrix} s_{z,1} & s_{z,2} \end{bmatrix}^T = \begin{bmatrix} 50 & 35 \end{bmatrix}^T \text{ m}, \quad (65)$$

so that proper locations in the respective image coordinate frames are

$$\begin{aligned} s_{\text{icf},1} &= \begin{bmatrix} -28.9638 & -21.4732 & 50 \end{bmatrix}^T \text{ m, and} \\ s_{\text{icf},2} &= \begin{bmatrix} 67.6625 & 26.8660 & 35 \end{bmatrix}^T \text{ m.} \end{aligned} \quad (66)$$

To close the loop, we calculate that back in the enu frame

$$\begin{aligned} (\mathbf{m}_{\text{MCP}, \text{enu}, 1} + s_{\text{enu},1}) &= (\mathbf{m}_{\text{MCP}, \text{enu}, 1} + \mathbf{R}_{\phi, \text{enu}, 1} s_{\text{icf},1}) = \begin{bmatrix} 20 & 40 & 50 \end{bmatrix}^T \text{ m, and} \\ (\mathbf{m}_{\text{MCP}, \text{enu}, 2} + s_{\text{enu},2}) &= (\mathbf{m}_{\text{MCP}, \text{enu}, 2} + \mathbf{R}_{\phi, \text{enu}, 2} s_{\text{icf},2}) = \begin{bmatrix} 20 & 40 & 50 \end{bmatrix}^T \text{ m,} \end{aligned} \quad (67)$$

which matches our original ground truth. Yippee!

Basically, the math works.

7. Example - Real Images

Ok, so the math works, but what about real images?

We present in figures 7 and 8 two SAR images taken of the Sandia department 2343 antenna-range Vee by the Lynx SAR flying on the DeHavilland DH-6 Twin-Otter on February 12, 1999. One corner reflector on the Vee was chosen as the reference target, and others were chosen as targets for which we desire relative heights. Note that the two images have not only very different aspect angles, but are at different resolutions, as well. The azimuth and range offsets were eye-balled from the real-time images using cross-hair position readings with no additional image interpolation. The necessary geometry parameters (sampled at the aperture centers) are summarized in table 1.

Table 1: Geometric parameter comparison

	<i>Parameter</i>	<i>View 1</i>	<i>View 2</i>
Bearing to scene center	ϕ_b	90.5753°	175.6082°
Depression angle	Ψ_{c0}	34.2013°	4.3839°
Squint angle	Φ_0	-75.2096°	74.6482°
Velocity pitch angle	ν_{c0}	0.2146°	-0.4938°

Two other corner reflectors on the Vee are located 40 m on either side of the reference. Since the Vee slopes at a pitch of 1.4°, these corners are respectively 0.98 m above and below the reference in height.

The first corner is analyzed in table 2.

Table 2: Target #1 parameter comparison

	<i>Parameter</i>	<i>View 1</i>	<i>View 2</i>
Azimuth offset	s_a'	-9.7878 m	-39.6075 m
Range offset	s_r'	38.2506 m	-6.4096 m
Calculated relative target height	s_z	0.8348 m	0.8218 m

The second corner is analyzed in table 2.

Table 3: Target #2 parameter comparison

	<i>Parameter</i>	<i>View 1</i>	<i>View 2</i>
Azimuth offset	s_a'	9.8701 m	39.3695 m
Range offset	s_r'	-37.8735 m	6.6995 m
Calculated relative target height	s_z	-1.0634 m	-1.0593 m

file: /home/awdoerr/projects/stereo/matlab/f1_494668_220.limg

Figure 7. View 1 of the antenna range Vee.

file: /home/awdoerr/projects/stereo/matlab/f1_501468_888.limg

Figure 8. View 2 of the antenna range Vee.

Since we are looking for relative height from a common reference corner reflector, the two solution heights for either of the corners should be the same, and they nearly are (to within 2 cm). Consequently we can reasonably take an average and claim that the two target corners are one 0.8283 m above and the other 1.0613 m below the reference corner. These represent errors of 14.9 cm and 8.4 cm, respectively, or less than half the coarsest resolution cell size in the images used.

The errors are attributed to two main sources. The first is that the corner reflectors did not sit directly on the crown of the Vee, but rather on top of some sandbag cushions on the crown in order to stabilize their mounting, which undoubtedly varied in thickness from corner to corner. The second main error source is simply the eyeballing of the corner distances in the images themselves. No peak-finding algorithm was used beyond dead reckoning.

Nevertheless, this is pretty darned good.

Further Analysis

We identify the transformation matrix as

$$[(A^T A)^{-1} A^T] = \begin{bmatrix} 1.2637 & 0.4388 & 0.0649 \\ 1.2419 & 0.5011 & -0.9307 \end{bmatrix}. \quad (68)$$

From this we see that we are most sensitive to errors in the east-west direction, but that in any case error scaling isn't too bad, and we may expect height errors to be comparable to horizontal position measurement errors, and hence on the order of the coarsest resolution cell size.

8. Forming Topographic Maps

To find the height of a single target point does not require forming a topographic map of an entire scene, but rather just processing the necessary pixels of interest.

Nevertheless, finding relative height for all points in a scene will yield a topographic map of the scene. This is what stereo SAR imaging is typically all about. Points for which height information is desired must be identifiable in both SAR images. This may constrain flight geometries somewhat for non-isotropic scatterers. For example, distributed targets exhibiting speckle need to have identical speckle patterns, requiring collocated aperture centers for both images.¹⁴ The bottom line is that target points have to be registrable between the two images.

In the previous example, we label some of the common targets in figure 10, and illustrate their relative topography in figure 11.

The ground truth for targets not on the Vee itself is not well known, and certainly not as well known as the slope of the crown of the Vee, but do seem to match the stereoscopically derived heights fairly well. For example, USGS topographical maps of the area do indicate a west-facing gentle slope to the area, and a visual inspection of the area reveals no dramatic elevation changes for the corner reflectors indicated.

Figure 9. Optical photograph of antenna range Vee (looking east).

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14. Jakowatz, Charles V. Jr., Daniel E. Wahl, Paul A. Thompson, "Three-Dimensional SAR Imaging Using Cross-Track Coherent Stereo Collections", Conf. Record of the 31st Asilomar Conf. on Signals, Systems & Computers, Vol. 2, 1998.

Figure 10. Selected targets from view 1 of the antenna range Vee.

Figure 11. Topography of selected targets.

9. Conclusions

We make the following observations.

- Layover differences from two distinct SAR images can be effectively used to ascertain target height information. Images generally need not even be at the same resolution. The equations for this are straightforward and given in this report.
- The accuracy and precision of this technique depends on the relative geometries of the two apertures used. Some geometries work better than others. Geometric limitations and error sensitivities are given in this report.
- A limitation is that target points need to be identifiable in both SAR images. Non-isotropic scatterers (such as distributed targets exhibiting speckle) place further limits on aperture geometries.
- To derive the topography of an entire scene optimally requires an automated registration operation. Issues and details of image registration were not addressed in this report.

For example, an image registration algorithm might need to operate with two available images with each at different resolutions.

- With this technique, it is possible to determine height accuracy to on the same order as the horizontal position accuracy.

Relative height accuracy within either image (with respect to another target in the images) depends on the measurement accuracy of relative positions within the images.

Relative height accuracy with respect to the aircraft also depends on the relative accuracy of the aperture centers.

Absolute height accuracy additionally yet depends on the absolute height accuracy of the aperture centers.

In any case, stereoscopically-derived DTED Level 5 spot maps would seem to be not particularly troublesome for a Lynx-class SAR.

- This is not IFSAR, since it deals with amplitude responses in the detected images. Phase is irrelevant. Complex images are not required. Multiple phase centers on a single aircraft are not required. Monopulse is not required. This is, in fact, a two-pass stereo technique.
- Any plain old SAR can do this. An example with Lynx SAR detected images is presented.
- While it has been suggested by a number of others, it still seems worth repeating here, that stereoscopically derived height information might be useful in resolving ambiguities and other errors in IFSAR generated height maps.

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