**A COMPTON SCATTER CAMERA
FOR SPECTRAL IMAGING
OF 0.5 TO 3.0 MeV GAMMA RAYS**

by

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ABSTRACT

A COMPTON SCATTER CAMERA FOR SPECTRAL IMAGING OF 0.5 TO 3.0 MeV GAMMA RAYS

by

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Chairman: Glenn F. Knoll

A prototype Compton scatter camera for imaging gamma rays has been built and tested. This camera addresses unique aspects of gamma-ray imaging at nuclear industrial sites, including gamma-ray energies in the 0.5 to 3.0 MeV range and polychromatic fields.

Analytic models of camera efficiency, resolution and contaminating events are developed. The response of the camera bears strong similarity to emission computed tomography devices used in nuclear medicine. A direct Fourier based algorithm is developed to reconstruct two-dimensional images of measured gamma-ray fields. Iterative ART and MLE algorithms are also investigated.

The point response of the camera to gamma rays of energies from 0.5 to 2.8 MeV is measured and compared to the analytic models. The direct reconstruction algorithm is at least ten times more efficient than the iterative algorithms and produces images that are, in general, of the same quality.

Measured images of several phantoms are shown. Important results include angular resolutions as low as 4.4° , reproduction of phantom size and position within 7%, and contrast recovery of 84% or better. Spectral imaging is demonstrated with independent images from a multi-energy phantom consisting of two sources imaged simultaneously.

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CHAPTER I

INTRODUCTION

Gamma-ray imaging is not widely used in the nuclear industry today, but a well designed imaging device could speed nuclear power plant downtime procedures, act as a diagnostic tool during power plant operation and quickly survey large numbers of radioactive waste storage containers, along with many other applications. If such a device were mounted on a remotely controlled robot, it could also survey high radiation areas, reducing radiation exposure to nuclear industry workers. Although gamma-ray imaging is widely used in the disciplines of nuclear medicine and astronomy, the nuclear industry has not benefited from advances in these disciplines, because the gamma-ray fields encountered at nuclear industrial sites are very different from those imaged in either nuclear medicine or astronomy. Among the most important differences are:

1. Energies of interest range from about 0.5 MeV to 3.0 MeV; this includes important sources of electromagnetic radiation such as positron annihilation (0.511 MeV), and gamma rays emitted during the decay of ^{137}Cs (0.662 MeV), ^{60}Co (1.17 and 1.33 MeV) and ^{24}Na (1.37 and 2.75 MeV)
2. Radiation fields are often polychromatic with several different gamma-ray lines important enough to warrant imaging
3. Sources of gamma rays are spatially extended
4. Plant-wide average gamma-ray exposure rates are as high as 1 R/h in operating nuclear reactors [Ott89].

In addition to the difficult imaging environment, an industrial imaging device must be portable to be useful.

Gamma-ray imaging devices commonly used in nuclear medicine and gamma-ray astronomy are described in the following section. Reasons why these devices are not good candidates for direct application in the nuclear industry are discussed.

1.1 Gamma-Ray Imaging Devices Used in Nuclear Medicine and Astronomy

Perhaps the oldest and simplest of gamma-ray imaging devices used in nuclear medicine is the rectilinear scanner. Scanners consist of an active gamma-ray detector, normally a scintillation crystal, and a mechanical collimator. The collimator acts to "focus" incident gamma rays by shielding the detector from gamma rays incident from all but a small range of directions. A single-bore collimator restricts incident gamma rays to a narrow range of directions, while the more common multi-bore collimator efficiently accepts gamma rays from a point at a fixed depth from the detector [Bar81]. As implied by the name, the collimated detector is scanned over a patient who has been injected with a radio-pharmaceutical to generate a two-dimensional (2D) image of the gamma radiation emerging from the patient. Gamma rays emitted from the focusing depth are imaged sharply, while those from other depths are recorded as a blurred image.

Rectilinear scanners are used to image singly emitted gamma rays at low energies of 0.140 MeV to 0.511 MeV [Hin74]. Imaging more penetrating, high energy gamma rays requires increased collimator thickness, thus increasing device weight and limiting portability. In addition, the scanner must dwell long enough at each pixel in the source space to acquire sufficient gamma-ray counts to generate a statistically accurate image. Slow image formation is the principle reason that scanners are not widely used in the field of nuclear medicine today.

Scanner-based imaging systems are also employed in gamma-ray astronomy. An example is the Oriented Scintillation Spectrometry Experiment (OSSE) currently in

service on the Compton Gamma Ray Observatory (CGRO). OSSE has a single-bore tungsten collimator to limit its field of view (FOV) [Cam92].

A raster-scanning camera designed for imaging gamma-ray fields in nuclear power stations has been built and tested [Dev90]. This single-bore camera successfully generated images with good resolution, but suffered from long acquisition times and high background penetration through its collimating shield.

The most common device in nuclear medical imaging is the scintillation or Anger camera [Ang58, Ang68]. The gamma-ray detector in this camera is typically a rectangular slab scintillator of thallium-activated sodium iodide, NaI(Tl), usually about 1 cm thick and up to 50 cm in width and/or length [Sor87]. The crystal is viewed with an array of photomultiplier tubes (PMTs); the scintillation light from an interaction is split among several PMTs. Signals from the PMTs are combined to give signals that represent the interaction location and total deposited energy.

As with the scanners, the direction of incident gamma rays is restricted by mechanical collimation. One type of collimator has a single pinhole aperture that projects an inverted view of the source distribution onto the detector, much like a "box camera". Pinhole collimators magnify sources by different factors depend on the source depth of field, and are normally used to image small organs. Parallel hole collimators are more commonly used with Anger cameras than pinhole collimators. The thousands of parallel holes restrict the gamma rays passing through it to directions nearly perpendicular to the detector, with each hole functioning like one single-bore collimator in a scanner. Thickness of the solid septa between the collimator holes is set to minimize the number of gamma rays that can penetrate through the collimator from one hole to another, while maintaining a reasonable efficiency. Converging hole collimators are also used in Anger cameras when the subject source distribution needs to be magnified, while diverging hole collimators are used to reduce or *minify* the source projection on the detector [Sor87].

For imaging the 0.140 MeV gamma-rays from ^{99m}Tc , a parallel hole collimator made of lead typically has a length of 2.5 cm with a septal thickness less than 0.08 cm [Sor87, Sin83a]. When the energy is increased to 0.4 MeV, the septal thickness must be increased by a factor of ten, due to the decreased attenuation coefficient of lead, with a resulting factor of three decrease in efficiency. For application in industrial gamma-ray fields, the collimator length and septal thickness would have to be increased even further to image the penetrating gamma rays with energies between 0.5 and 3.0 MeV. The resulting collimator would be heavy, have poor spatial resolution and be very inefficient. Low collection efficiency due to the collimator would be compounded by low interaction efficiency of the thin scintillator. Scintillator thickness can be increased but with a penalty of larger mass and poorer resolution. In most Anger cameras, the system spatial resolution is dominated by the *collimator resolution*, rather than the *intrinsic resolution* of the detector. However, as their thickness is increased the intrinsic resolution of these detectors is significantly degraded by scattering of the gamma rays in the detector itself [Ang64, Kno69]. Also in thicker crystals, the light will be more widely spread by the time it reaches the PMTs, further degrading the intrinsic resolution. Thus, there are several reasons why Anger cameras with pinhole or parallel hole collimation are poor choices for industrial gamma-ray imaging.

To increase the efficiency of Anger cameras in nuclear medicine, coded apertures have been used to replace the pinhole and parallel-hole collimators. Coded apertures are more efficient than the other collimators because gamma rays from a point source are spread over many pixels of the camera detector; Barrett refers to this as *multiplexing* [Bar81]. One of the first coded apertures investigated was the Fresnel zone plate, which spatially modulates the gamma rays with a pattern nearly identical to the hologram of a point source of coherent light [Bar72, Rog72a]. The position of the center of the shadow cast by the coded aperture on the detector indicates the direction of the source, and depth of field information can be determined from the size of the shadow [Rog72b]. Before its

application in nuclear medicine the Fresnel zone plate was investigated as a coded aperture for imaging X-ray stars [Mer61]. On-axis Fresnel zone plate apertures were found to work well for dilute sources such as stars; but not for spatially extended sources of nuclear medicine. The multiplexing that increases the efficiency also reduces the utility of coded apertures for extended sources, because each pixel receives counts from many overlapping aperture shadows, decreasing the signal-to-noise ratio (SNR) in the image after it is decoded [Bar81]. Off-axis zone plates were also used but were found to require recording devices with higher resolution than provided by Anger cameras.

Other coded apertures have been studied, including random pinhole arrays [Dic68], non-redundant pinhole arrays [Wou73, Cha74], an annular coded aperture [Sim78] (which is described in Section 3.2) and uniformly redundant arrays [Fen78]. None of these coded apertures have found widespread use in nuclear medicine. In addition, extending this technique to high energy gamma-rays requires increased aperture thickness [Joh80], which may make its application to the fields found at industrial sites impractical.

1.2 Compton Scatter Imaging in Nuclear Medicine and Astronomy

None of the mechanically collimated gamma-ray imaging devices used in nuclear medicine or astronomy appears to offer an attractive solution for industrial applications. An alternative concept exploits Compton scattering between two position sensitive detectors. When an incident gamma-ray Compton scatters in one detector and the scattered gamma-ray escapes to interact in the second detector, its direction may be localized to the surface of a cone. The physics of Compton scatter imaging is fully described in Section 2.1.

The Compton scatter technique was first employed in telescopes for gamma-ray astronomy. Schonfelder, Horner and Schneider [Sch73, Sch82]; Herzo, et al. [Her75, Ait90], and Lockwood, et al. [Loc79] have successfully deployed balloon-born Compton

telescopes. Currently the Compton telescope, COMPTEL, is in orbit on the CGRO [Sch84]. Compton telescope designs must be efficient due to the faint gamma-ray sources with energies between 0.5 and 30 MeV that they image. As a result, telescopes are large, consisting of two planar detectors each having areas in excess of 1 m^2 and separated by a distance of 1.3 m or more.

Compton imaging was first applied to nuclear medicine by Singh, who replaced the mechanical collimator of an Anger camera with a planar high purity germanium (HPGe) detector array [Sin83a, Sin83b, Sin85]. A prototype *electronically collimated gamma camera* (ECC) has been used to measure single photon emission computed tomography (SPECT) images of $^{99\text{m}}\text{Tc}$ and ^{137}Cs source phantoms at energies of 0.140 and 0.662 MeV, respectively [Sin90]. Comparison of mechanically collimated SPECT measurements with prototype ECC measurements extrapolated to a full system indicate that the ECC has an order of magnitude higher sensitivity at 0.140 MeV for equal noise and resolution. The sensitivity gain increases to nearly three orders of magnitude at 0.662 MeV. Results from the ECC demonstrate that Compton scatter imaging is possible for single photons in the energy range of interest for nuclear industrial applications. In addition, the size of the ECC, with 0.5 cm x 0.5 cm x 0.6 cm elements in the HPGe array and 40 cm diameter Anger camera, fills the portability criteria more readily than the bulky Compton telescopes.

Others have suggested Compton scatter cameras for nuclear medicine that employ two planar position-sensitive detectors like the ECC but with either a silicon detector [Sol88] or a gas scintillation proportional counter [Fuj87] replacing the HPGe array. However, no other medical imaging device has been built to date. A recent review of mechanically collimated and Compton scatter gamma-ray imaging devices is presented by Dogan [Dgn93a].

1.3 Concept of the Ring Compton Camera

In this work the Compton scatter technique is being investigated for industrial applications because it offers solutions for each of the unique aspects of the industrial gamma-ray fields identified at the beginning of this chapter.

1. For materials commonly used in radiation detectors (such as HPGe and NaI), over the 0.5 to 3.0 MeV energy range, Compton scattering is the dominant interaction mechanism, and the scattering cross section varies by less than a factor of three. These physical characteristics suggest that Compton scattering is a promising choice for imaging gamma rays at these energies.
2. Polychromatic fields can be imaged by selecting events based on the summed energy deposited in the two detectors.
3. Spatially extended sources, which subtend large fractions of the system FOV have been successfully imaged [Sin90].
4. Random background count rates and dead times that may be caused by the high exposure rates can be minimized by using multi-element detectors with good timing properties.

The resolution of a Compton scatter camera is dependent on the energy resolution of the first detector and the size of its active elements. To minimize impact of element size on resolution, they must be kept small. (The elements cannot be made arbitrarily small, however, because they must be able to absorb recoil electrons generated when gamma rays Compton scatter.) The mass of such a camera will be less than a comparable mechanically collimated device which relies on shielding for image formation. The angular resolution is improved by increasing the separation distance between the two detectors, as a result, Compton scatter cameras tend to be relatively large.

While the ECC quite successfully imaged single energy photons from medically significant extended sources, its geometry of two parallel planar position sensitive

detectors is not optimal for industrial gamma-ray fields. In Compton cameras with planar second detectors, small angle Compton scattering events are most probable, particularly for incident gamma-ray energies above 0.5 MeV. As will be explained in Chapter II, these events are poorly resolved. Additionally a scattered gamma-ray retains most of the energy of the incident gamma-ray after a small angle scatter in the first detector. The peak-to-total ratio for these high energy scattered gamma rays is low in the second detector, especially if it is a thin Anger camera detector. Since events are to be selected based on their summed energy, most of the small scattering angle events will only contribute to the dead time and random background in the camera. Additionally, if a particular gamma-ray line is to be imaged in the presence of a higher energy line, higher energy gamma-ray events depositing only part of their energy may be confused with lower energy gamma-ray events depositing their full energy and possibly contaminate the image of the lower energy gamma-rays.

The number of small angle scattered events can be reduced by using a ring shaped second detector. The fraction of events which are poorly resolved or only contribute to dead time and random background is reduced in a ring shaped second detector compared to a planar second detector. With the ring shaped second detector, the average scattering angle will be increased, resulting in lower scattered gamma-ray energies and enhanced peak-to-total ratios in the second detector. The summed energy spectrum in the camera will have enhanced peak-to-total ratio which is desirable for any radiation spectroscopy system [Kno89].

Images from the ECC have been reconstructed using iterative algorithms [Sin90, Bre90]. While these algorithms produce high quality images, they are computationally intensive. With the ring shaped second detector, it is possible to adapt a more efficient Fourier transform based reconstruction algorithm from emission computed tomography. This algorithm is described in Chapter III. While efficient transform based algorithms

may be possible for the ECC and other Compton devices with planar second detectors, no examples have been found to date.

Some characteristics of a demonstration Compton scatter camera with a ring shaped second detector have been reported [Mrn93]. In this dissertation, an improved version of that camera, now called the Ring Compton Camera is investigated; it is designed to generate 2D images of polychromatic gamma-ray radiation. The Ring Compton Camera is intended to serve as a prototype for imaging devices to be used in nuclear industrial applications.

1.4 Organization of the Dissertation

In Chapter I, the motivation for this work was presented. Some relevant gamma-ray imaging techniques were reviewed, and the primary concepts behind the Ring Compton Camera were explained. Chapter II is a development of analytic mathematical models for efficiency and resolution of a Compton camera. Also, analytic models for estimating rates of contaminating events are presented there. Techniques for reconstructing 2D images from a Compton scatter camera are developed in Chapter III. In that chapter, parallels between tomographic imaging systems and Compton scatter systems are described. The hardware, electronics and software of the Ring Compton Camera are described in Chapter IV, along with an evaluation of basic system parameters. The system response to point sources is evaluated in Chapter V. Specifically, procedures used to measure point spread functions and comparisons between predictions from the analytic model to measurements are given. Chapter VI details the imaging capabilities of the Ring Compton Camera, including images of polychromatic fields, signal to noise ratio and contrast in spatially extended sources. Conclusions and suggestions for further work are given in Chapter VII.

CHAPTER II

COMPTON CAMERA ANALYTIC MODELS

For the Ring Compton Camera (RCC) to successfully image an incident gamma ray, that photon must Compton scatter in the first detector array, and the scattered photon must escape that detector and deposit its full energy in the second detector array. The geometry of such an interaction is illustrated in Figure 2-1, showing a single element first detector and a ring second detector. To gain insight into how the RCC functions, analytic mathematical models of its efficiency and resolution to point sources on the ring axis are developed in this chapter. Possible contributions from undesired, image contaminating events are also modeled. A brief review of the physics of Compton scattering introduces concepts used to develop these models.

Figure 2-1. Localization of a point source on the ring axis of the RCC.

2.1 Physics of Compton Scattering

The underlying principle of Compton scatter imaging is that the energy of a gamma ray scattered by an electron E_γ' can be related to the incident gamma ray energy E_γ as a function of the scattering angle θ . A.H. Compton first developed this relationship using relativistic momentum and mass-energy conservation, and assuming that the gamma rays were discrete quanta. Compton expressed the relationship in terms of the wavelengths (λ and λ') of the photons [Com23].

$$\lambda = \lambda' + \frac{h}{m_0 c^2} [1 - \cos(\theta)] \quad (2-1)$$

where, h is Planck's constant and $m_0 c^2$ is the rest mass energy of the electron. Equation (2-1) can be recast in terms of the energies using $E_\gamma = hc/\lambda$, which upon rearranging gives

$$E_\gamma' = \frac{E_\gamma}{1 + \alpha [1 - \cos(\theta)]} \quad (2-2)$$

$$\alpha \equiv \frac{E_\gamma}{m_0 c^2} \quad (2-3)$$

The energy passed to the recoil electron E_{re} can be computed from Equation (2-2) and energy conservation.

$$E_{re} = E_\gamma - E_\gamma' = E_\gamma \left\{ \frac{\alpha [1 - \cos(\theta)]}{1 + \alpha [1 - \cos(\theta)]} \right\} \quad (2-4)$$

Like E_γ' and E_{re} , the angle of the recoil electron ϕ can be expressed in terms of E_γ and θ .

$$\tan(\phi) = \frac{1}{(1 + \alpha) \tan(\theta/2)} \quad (2-5)$$

Compton's derivation assumed that the electron from which the gamma ray scatters is free; however, in solid detector crystals, electrons are usually bound. Recently, it has been

shown that even tightly bound k-shell electrons may be treated as though they are free when incident gamma-ray energies exceed 0.3 MeV and the scattering angles are large [Sur92]. The RCC is designed to image gamma rays at energies above 0.5 MeV, and Compton's relation can safely be assumed to hold true.

The direction of a gamma ray incident on a Compton camera can be localized to the surface of a cone when it scatters in the first detector, and the scattered gamma ray deposits its full energy in the second detector. The cone apex is the point of interaction in the first detector, and its axis is a backprojection of the scattered photon direction. Such an interaction sequence is illustrated in Figure 2-1. The cone angle θ can be determined from E_{re} for a known E_γ . By rearranging Equation (2-4), θ can be expressed in terms of E_γ and E_{re} .

$$\cos(\theta) = 1 - \left[\frac{E_{re}}{\alpha(E_\gamma - E_{re})} \right] \quad (2-6)$$

In addition to localizing the incident radiation, an imaging system must quantify the number of photons emitted from the region being imaged. The probability that a photon will interact in a material is specified by its total interaction coefficient μ in terms of the probability of interaction per distance in the material. The probability that a photon undergoes a particular type of interaction is specified by its partial interaction coefficient. Figure 2-2 shows μ and partial interaction coefficients for photoelectric absorption τ , Compton scattering σ , and pair production κ , for gamma rays in germanium as a function of their incident energy. The total coefficient is simply the sum of all the partial coefficients.

Interaction coefficients shown in Figure 2-2 have some important implications for using Compton scattering to image gamma rays in the 0.5 to 3 MeV energy range. First, Compton scattering is the dominant interaction type over the specified range. This is desirable because the fraction of gamma rays undergoing photoelectric and pair

Figure 2-2. Gamma-ray interaction coefficients in germanium. Total attenuation, μ ; photoelectric, τ ; Compton scattering, σ , and pair production κ .

production processes in a first detector made of germanium will be significantly smaller than those undergoing Compton scattering. Secondly, unlike the two partial interaction coefficients, σ is fairly constant over the energy range of interest, varying by less than a factor of 3. The Compton scatter camera can then be expected to have a relatively uniform efficiency across the energy range.

Once a gamma ray Compton scatters in the first detector it must be emitted at an angle subtended by the second detector. The probability of a scattered gamma ray being emitted at a particular θ is expressed by the Klein-Nishina relation as a differential cross section with solid angle [Kle42].

$$\frac{d\sigma}{d\Omega} = Zr_0^2 \left\{ \frac{1}{1 + \alpha[1 - \cos(\theta)]} \right\}^2 \left\{ \frac{1 + \cos^2(\theta)}{2} \right\} \left\{ 1 + \frac{\alpha^2 [1 - \cos(\theta)]^2}{\{1 + \cos^2(\theta)\} \{1 + \alpha[1 - \cos(\theta)]\}} \right\} \quad (2-7)$$

Figure 2-3. Differential Compton scattering cross section, $(d\sigma/d\Omega)/Zr_0^2$, with scattering angle for a range of incident gamma-ray energies.

In Equation (2-7), r_0 is the classical electron radius and Z the atomic number of the target nucleus. For gamma ray energies above the rest mass energy of the electron, the probability of scattering becomes highly anisotropic, with forward scattering preferred. This anisotropic behavior is illustrated in Figure 2-3, which shows the Klein-Nishina relation for a series of different incident gamma-ray energies.

2.2 Efficiency Model

Before the efficiency of the RCC can be modeled, there must be a definition of what constitutes a useful imaging event. This definition was given early in this chapter, but warrants repeating here. An incident gamma ray must Compton scatter in the first detector array, and the scattered photon must escape that detector and deposit its full energy in the second detector array. Useful events are selected when the sum of the energy deposited in the two detector arrays falls in an energy window set about the

incident gamma-ray energy. This also makes it necessary that the full energy of the recoil electron be deposited in a single element of the first detector. These requirements are somewhat stringent, but are necessary to successfully image individual gamma-ray lines in a polychromatic field. Despite these requirements, there are several possible gamma-ray interactions which introduce contaminating events in the camera. Image contaminating interactions are discussed in detail in Section 2.4.

For simplicity, only the response of the detector to a point source on the RCC ring axis has been modeled. The fraction of gamma rays emitted by a source which lead to valid events in the camera is the absolute efficiency ϵ_{acc} . In Equation (2-8), ϵ_{acc} is the product of four terms, each representing an independent random process.

$$\epsilon_{acc} = (\Omega_1/4\pi)P_c f_2 \epsilon_{ip2} \quad (2-8)$$

The first term in parenthesis reflects the probability that a gamma ray emitted by the source will fall in the solid angle subtended by the first detector array Ω_1 .

In Equation (2-8) P_c is the probability that a gamma ray incident on the first detector will Compton scatter and escape into the angular range subtended by the second detector. A relation for this process was proposed by Llacer and Cho [Lla73] and first used to model a Compton camera by Singh [Sin83a]. That relation is developed here. First, the probability that a gamma ray undergoes a scattering event in a differential thickness dx at depth x is given by

$$p_{cs}(x)dx = \sigma e^{-\mu x} dx \quad (2-9)$$

where σ and μ are the partial Compton and total interaction coefficients in the first detector, as illustrated in Figure 2-2. Given that a Compton scattering event has occurred, the probability that the scattered gamma ray is emitted into a differential solid angle $d\Omega$ about Ω is $(1/\sigma)(d\sigma/d\Omega)d\Omega$, where $d\sigma/d\Omega$ is from Equation (2-7). By expressing the differential solid angle in polar coordinates and using the fact that scattered photons

from unpolarized incident gamma rays are uniformly distributed in the azimuthal angle, [Eva55] the differential probability is

$$p_{\theta}(\theta)d\theta = \frac{2\pi}{\sigma} \left(\frac{d\sigma}{d\Omega} \right) \sin(\theta)d\theta \quad (2-10)$$

Recognizing that σ is equal to $d\sigma/d\Omega$ integrated over all space, Equation (2-10) can now be rewritten as

$$p_{\theta}(\theta)d\theta = \frac{2\pi \left(\frac{d\sigma}{d\Omega} \right) \sin(\theta)d\theta}{2\pi \int_0^{\pi} \left(\frac{d\sigma}{d\Omega} \right) \sin(\theta)d\theta} = \frac{P(\theta)d\theta}{\int_0^{\pi} P(\theta)d\theta} \quad (2-11)$$

where,

$$P(\theta) = \frac{\sin(\theta)(d\sigma/d\Omega)}{Zr_0^2}$$

After a scattered photon is emitted with a scattering angle θ , the probability that the photon escapes the detector element p_{es} is a function of x and θ

$$p_{es}(x, \theta) = e^{-\mu'(\theta)S'(x, \theta)} \quad (2-12)$$

where,

$\mu'(\theta)$ = total interaction coefficient at the scattered photon energy E_{γ}' ,

$S'(x, \theta)$ = path length traversed by scattered gamma ray to escape detector.

Now, the probability of a gamma ray Compton scattering in the first detector element and escaping at an angle subtended by the second detector is found by integrating the product of the probabilities in Equations (2-9), (2-11) and (2-12) over the of the first-detector length ($2L_1$) and the range of scattering angles ($\theta_{\min} < \theta < \theta_{\max}$) subtended by the second detector.

Figure 2-4. Geometric factors used to model Compton camera efficiency and angular resolution.

$$P_c = \int_{\theta_{\min}}^{\theta_{\max}} \int_0^{2L_1} p_{cs}(x) p_{\theta}(\theta) p_{es}(x, \theta) dx d\theta$$

$$= \frac{\int_{\theta_{\min}}^{\theta_{\max}} P(\theta) \left\{ \int_0^{2L_1} \sigma e^{-\mu x} e^{-\mu'(\theta) S'(x, \theta)} dx \right\} d\theta}{\int_0^{\pi} P(\theta) d\theta} \quad (2-13)$$

The limits of these integrals are illustrated in Figure 2-4, showing a single element from both of the RCC detectors.

When the probability of the incident gamma-ray interacting in the crystal is small ($\mu 2L_1 \ll 1$), interactions should be uniformly distributed through the detector element, and $S'(x, \theta)$ can be approximated by half of the average thickness of the crystal when viewed from an angle θ , $\tilde{S}(\theta)$. (See Appendix A for a derivation of $\tilde{S}(\theta)$.) With this approximation, Equation (2-13) can be integrated over x to yield

$$P_c = \frac{\frac{\sigma}{\mu} (1 - e^{-\mu 2L_1}) \int_{\theta_{\min}}^{\theta_{\max}} P(\theta) e^{-\mu'(\theta) \bar{S}(\theta)/2} d\theta}{\int_0^{\pi} P(\theta) d\theta} \quad (2-14)$$

Although it is not explicitly shown in Equation (2-14), P_c is a function of E_γ because, μ , σ and $P(\theta)$ all vary with gamma-ray energy.

Once a scattered gamma ray escapes the first detector in the proper angular range, the probability that it will fall on the second detector is the ratio of the area of the detector to the area of the angular scattering range, A_{scat} . In the RCC, the second detector consists of several (N) discrete right circular cylinders, and this ratio f_2 is given by

$$f_2 = N \left\{ \frac{A_2}{A_{\text{scat}}} \right\} = N \left\{ \frac{\pi(R_2)^2}{2\pi(d - L_2/2)^2 [\cos(\theta_{\min}) - \cos(\theta_{\max})]} \right\} \quad (2-15)$$

where the area of each element is $A_2 = \pi(R_2)^2$.

The final factor in Equation (2-8) is the intrinsic peak efficiency ϵ_{ip2} representing the fraction of gamma rays incident on the second detector which deposit their full energy [Kno89]. This efficiency can be represented as the product of the intrinsic total efficiency ϵ_{it2} and the peak to total ratio r_{pt} .

$$\epsilon_{\text{ip2}} = r_{\text{pt}} \epsilon_{\text{it2}} = r_{\text{pt}} \left[1 - e^{-\mu_2(\theta_c) \bar{C}_2} \right] \quad (2-16)$$

The intrinsic total efficiency depends on the mean chord length through the second detector $\bar{C}_2$ and the total interaction coefficient in the second detector material μ_2 at the energy of a gamma ray scattered at the central angle θ_c . Equations for $\bar{C}_2$ and a parametric model of r_{pt} are presented in Appendix A, which also includes polynomial fits to the energy dependence of the various interaction coefficients.

This model does not yet account for recoil electron penetration of the first detector element. Recoil electron penetration will only be significant when the electron range is a large fraction of the dimension of the detector, and hence the recoil electron energy E_{re} is high. This problem is rather complex, not lending itself to analytic modeling, and so a very simple approximation was used: the length of the first detector crystal was reduced by the range of the recoil electron. The approximation is based on an assumption that the recoil electrons are forward directed. This assumption is justified from Equation (2-4); the largest values of E_{re} (> 0.9 MeV) occur when E_γ is high (> 1.5 MeV) and θ large ($> 60^\circ$), where Equation (2-5) shows that the recoil electron angle ϕ to be small ($< 24^\circ$).

In Chapter V, predictions from this model are compared to measured efficiencies. In addition to magnitude comparisons, variations with several important parameters are illustrated, and the applicability of this model to off-axis point sources is examined.

2.3 Resolution Model

A model of angular resolution of the RCC to a point source on its ring axis is developed in this section. The model is an adaptation of one proposed for a Compton camera with a planar second detector [Sin83a]. In addition to that model, others have proposed angular resolution models for Compton cameras [Sol88] and Compton telescopes for gamma-ray astronomy [Sch82]. Each of the models breaks the total angular resolution $\Delta\theta_t$ into independent geometric ($\Delta\theta_g$) and energy ($\Delta\theta_e$) components, and the following relation is taken from Singh [Sin83a].

$$\tan^2(\Delta\theta_t) = \tan^2(\Delta\theta_g) + \tan^2(\Delta\theta_e) \quad (2-17)$$

In Section 2.1, localization of incident gamma rays with a Compton camera was described as a cone whose axis is the backprojection of the scattered gamma-ray direction and whose angle is the Compton scattering angle. Angular uncertainty in the direction of the cone axis is given by $\Delta\theta_g$, while $\Delta\theta_e$ reflects the uncertainty in the cone angle.

The finite size (or position resolution) of the two Compton camera detectors are the cause of geometric uncertainty in the angular resolution. In determining the direction of the scattered gamma ray, the position in each element is quantized. In other words, some point in each element must be chosen to represent all interactions in that element. Here, the volume centroid of each element is chosen as its quantization position. (In Appendix A, using a weighted mean interaction location as the quantization position is investigated, and found to be nearly identical to the volume centroid for all cases of interest.) Any event scattering from the first into the second detector will have some associated *quantization error* [Lim90].

Quantization errors due to the half-width of a first detector element W_1 , the half-length of a first detector element L_1 , and the radius of a cylindrical second element R_2 (which are shown in Figure 2-4) contribute to $\Delta\theta_g$.

$$\tan^2(\Delta\theta_g) = 4 \left[\tan^2(\Delta\theta_{W_1}) + \tan^2(\Delta\theta_{L_1}) + \tan^2(\Delta\theta_{R_2}) \right] \quad (2-18)$$

The sum of the squares of the quantization error components is multiplied by 4 because each component gives only the half-width at half-maximum (HWHM), and $\Delta\theta_g$ represents the full-width at half-maximum (FWHM) angular resolution

In Figure 2-5a, the error due to W_1 is illustrated. A gamma ray incident from the ring axis, traveling at an angle Φ relative to the axis strikes the upper edge of the first detector, and scatters with angle θ into the second detector. This path is indicated with solid vectors. To backproject this event (indicated with the dotted vectors), the interaction is assumed to have taken place at the centers of both detector elements, but the measured scattering angle θ is used, resulting in an error of angle $\Delta\theta_{W_1}$. An equation for $\Delta\theta_{W_1}$ can be derived from the projection of these vectors onto a line perpendicular to the ring axis that passes through the center of the second detector element. Figure 2-5a shows that the perpendicular distance from the ring axis to the center of the second detector is equal to

Figure 2-5. Diagrams illustrating geometric angular resolution components: a) $\Delta\theta_{w1}$, b) $\Delta\theta_{L1}$ and c) $\Delta\theta_{R2}$.

$W_1 + d' \tan(\theta + \Phi)$ and it is also equivalent to $d' \tan(\theta + \Delta\theta_{W_1})$; thus $\Delta\theta_{W_1}$ can be found from known geometry parameters of the RCC.

$$d' \tan(\theta + \Delta\theta_{W_1}) = W_1 + d' \tan(\theta + \Phi) \quad (2-19)$$

Figure 2-5b illustrates the angular uncertainty due to the half-width of the first detector. As before, the equation for $\Delta\theta_{L_1}$ can be derived by examining the orthogonal distance from the ring axis to the middle of the second detector, yielding:

$$d' \tan(\theta + \Delta\theta_{L_1}) = (d' + L_1) \tan(\theta) \quad (2-20)$$

The third uncertainty term in Equation (2-18), is caused by the finite radius of the second detector. As seen in Figure 2-5c, this component can best be visualized as the angle that a scattered gamma ray striking the second detector's outer radius makes with the line connecting the centers of the two elements, and can be written as

$$\tan(\Delta\theta_{R_2}) = R_2 / d \quad (2-21)$$

The energy resolution component $\Delta\theta_e$ can be derived from the expression for θ in terms of E_γ and E_{re} given in Equation (2-6). When E_γ is known a priori, only the uncertainty in measuring E_{re} will affect $\Delta\theta_e$.

$$(\Delta\theta_e)^2 = \left(\frac{\partial\theta}{\partial E_{re}} \right)^2 (\Delta E_{re})^2 \quad (2-22)$$

ΔE_{re} is the FWHM energy resolution of the first detector. Taking the derivative of θ in Equation (2-6) with respect to E_{re} yields

$$\frac{\partial\theta}{\partial E_{re}} = \frac{m_0 c^2}{\sin(\theta)} \left[\frac{1}{(E_\gamma - E_{re})^2} \right] \quad (2-23)$$

In Figure 2-6 the variation of $\partial\theta/\partial E_{re}$ with θ is plotted for several energies over the range of interest. It shows that the sine term in the denominator of Equation (2-23) causes the derivative to become infinite at scattering angles of 0° and 180° , which is a primary reason

Figure 2-6. Variation of $\partial\theta/\partial E_{re}$ with Compton scattering angle, θ .

for designing the RCC to exclude small scattering angle events. At the minimum of $\partial\theta/\partial E_{re}$, the energy contribution to the angular resolution will be lowest for a fixed energy resolution. The figure also shows the minimum in the derivative to be near 45° for lower incident energies, moving toward lower angles with increasing energy.

While E_γ is always known for the work reported here, there are two other cases that should be considered. In the first case, E_γ is unknown but can be determined a posteriori from the summed energy $E_s (= E_\gamma' + E_{re})$ of many interactions. If for example, E_s is found from the centroid of a peak in the summed energy spectrum and taken to be equal to E_γ , then a new formulation for the energy contribution to the angular resolution is needed. It is designated $\Delta\theta_e'$ and is expressed as

$$(\Delta\theta_e')^2 = \left(\frac{\partial\theta}{\partial E_{re}} \right)^2 (\Delta E_{re})^2 + \left(\frac{\partial\theta}{\partial E_\gamma} \right)^2 (\Delta E_\gamma)^2 \quad (2-24)$$

where $\partial\theta/\partial E_{re}$ is given in Equation (2-23) and $\partial\theta/\partial E_\gamma$ is

$$\frac{\partial\theta}{\partial E_\gamma} = -\frac{m_0 c^2}{\sin(\theta)} \left[\frac{1}{(E_\gamma - E_{re})^2} - \frac{1}{E_\gamma^2} \right] \quad (2-25)$$

In this example, the value of ΔE_γ is the uncertainty in determining the centroid of the summed energy peak. Note that there is some nonzero covariance between the two terms in Equation (2-24) that has been neglected. The covariance is small because the summed energy resolution of the RCC is dominated by the scintillator used as the second detector.

In the second case, the summed energy from *each event* is taken to be equal to E_γ , so only measured values of E_γ' and E_{re} are used to determine θ . The energy resolution component is designated $\Delta\theta_e''$ for this case.

$$(\Delta\theta_e'')^2 = \left[\left(\frac{\partial\theta}{\partial E_{re}} \right)'' \right]^2 (\Delta E_{re})^2 + \left[\left(\frac{\partial\theta}{\partial E_\gamma'} \right)'' \right]^2 (\Delta E_\gamma')^2 \quad (2-26)$$

The double primed connotation is used on the derivatives to avoid confusion with derivatives that apply to the earlier cases. To derive an expression for $\Delta\theta_e''$, Equation (2-6) is rewritten in terms of E_γ' and E_{re} .

$$\cos(\theta) = 1 - m_0 c^2 \left[\frac{E_{re}}{(E_\gamma' + E_{re}) E_\gamma'} \right] \quad (2-27)$$

Evaluating the partial derivative with respect to E_{re} ,

$$\left(\frac{\partial\theta}{\partial E_{re}} \right)'' = \frac{m_0 c^2}{\sin(\theta)} \left[\frac{1}{(E_\gamma' + E_{re})^2} \right] \quad (2-28)$$

and with respect to E_γ' ,

$$\left(\frac{\partial\theta}{\partial E_\gamma'}\right)'' = \frac{m_0 c^2}{\sin(\theta)} \left[\frac{1}{(E_\gamma' + E_{re})^2} - \frac{1}{(E_\gamma')^2} \right] \quad (2-29)$$

For the RCC, with a first detector of high purity germanium (HPGe) and a second detector made of NaI(Tl) scintillator, the last case results in significantly worse resolution than the other two. As an example, consider a 0.662 MeV ^{137}Cs gamma ray scattering through an angle of 60° . Then, ΔE_{re} is about 1.5 keV; $\Delta E_\gamma'$ is about 48 keV, and the error associated with finding the peak centroid, ΔE_γ , can be taken as 6 keV. The energy component of the angular resolution can be described for the three cases described above. Using the a priori information of the incident gamma-ray energy gives $\Delta\theta_e = 0.31^\circ$. Using the a posteriori measurement of the summed energy peak centroid yields $\Delta\theta_e' = 0.85^\circ$, and using only the measured quantities gives $\Delta\theta_e'' = 6.4^\circ$.

2.4 Contaminating Events

In Section 2.2, a model describing a Compton camera's efficiency for measuring useful imaging events from a point source ϵ_{acc} has been developed. Apart from the useful events, erroneous direction information will be drawn from several possible interactions in the Compton camera, and these interactions may contaminate the image generated from the Compton camera data. Here equations describing efficiencies from an on-axis point source or count rates for contaminating events are developed. The events can be divided into two broad classifications: those caused by a single photon emitted from the source, and those caused by two photons emitted from the source within $2\tau_r$, the resolving time of the camera.

A contaminating event due by a single photon occurs when the photon Compton scatters two or more times in the first detector; then the scattered photon escapes and is subsequently absorbed in the second detector. The probability of such events occurring

has been modeled by Singh, who found that the ratio of double to single Compton interaction probabilities ($P_{cc}:P_c$) in 0.6 cm thick germanium is 0.10 at 0.14 MeV and approximately 0.077 at 0.50 MeV [Sin83a]. Decrease in this ratio with increasing energy results from the decrease in interaction probabilities with energy. (See Figure 2-2.) Since the interaction coefficients continue to monotonically decrease with energy over the 0.5 to 3.0 MeV energy range, the double scattered events will never exceed 8% of the single scattered events. When they occur, double scattered events will deposit a larger fraction of their energy in the first detector than single scattered events. Equation (2-6) shows that when double scatter events are mistakenly treated as single scattered events, they will be assigned a scattering angle that is too large.

If a Compton scatter in the first detector is preceded or followed by a coherent scattering event, the direction information will be in error, because the energy of a photon is not changed by the coherent scatter, only its direction is. The coherent scattering cross section is small for energies of interest with values equal to or less than 3% of the Compton cross section. As a result coherent scattering interactions are neglected in this study.

Another contaminating event caused by a single photon occurs when a gamma ray first strikes the second detector, undergoes a Compton scatter, and escapes the second detector to deposit its full energy in the first detector. This event is termed a "reverse scattering event", and the absolute efficiency with which these events occur can be written as:

$$\epsilon_{arev} = (\Omega_2/4\pi)P_c f_1 \epsilon_{ip1} \quad (2-30)$$

The terms in Equation (2-30) have the same meaning as those in Equation (2-8), except that the roles of the two detectors are reversed. For each of the remaining single photon contaminating events investigated in this section, only sequences beginning in the first detector are discussed. However, sequences beginning in the second detector are possible

for each of them. Efficiency equations for events initiated in the second detector can be derived as has been done for the reverse scattering, by exchanging the roles of the two detectors.

When incident gamma-ray energies exceed $2m_0c^2$, pair production events are possible, but from Figure 2-2, the pair production interaction coefficients κ only makes significant contributions to μ at energies above 2 MeV. It is possible to get a contaminating coincidence in a Compton camera from a pair production interaction in either detector when only one annihilation photon escapes the initial element and it escapes to deposit its entire energy in an element of the other detector. This sequence is referred to as a "single escape coincidence event". When the pair production occurs in an element of the first detector, the absolute efficiency these events can be written as:

$$\epsilon_{ase1} = (\Omega_1/4\pi)P_{pp}2P_{aab}P_{aes}(\tilde{\Omega}_2/4\pi)\epsilon_{ip2} \quad (2-31)$$

The order of the terms in this equation reflects the order of interactions in the event. The first term is the fraction of the solid angle covered by the first detector relative to the source. The product of the three probability terms gives the probability that an incident gamma ray results in a single escape. The second solid angle term $\tilde{\Omega}_2/4\pi$ is the fraction of the solid angle subtended by the second detector relative to the first, and ϵ_{ip2} is the intrinsic peak efficiency of the second detector evaluated at 0.511 MeV. The terms P_{pp} , P_{aab} and P_{aes} in Equation (2-31) are the probabilities of the following three events: an incident gamma ray undergoes pair production in the first detector element; a particular annihilation photon is absorbed in that element, and the other photon escapes the element. The factor of two accounts for the fact that the interactions of two annihilation photons can be interchanged. The three probability terms can be determined from the following equations:

$$P_{pp} = (\kappa/\mu)(1 - e^{-\mu 2L_1}) \quad (2-32)$$

$$P_{aab} = 1 - e^{-\mu'(\theta)\tilde{S}(\theta)/2} \quad (2-33)$$

$$P_{aes} = e^{-\mu'(\theta)\tilde{S}(\theta)/2} \quad (2-34)$$

Interaction coefficients are evaluated at the incident gamma ray energy in Equation (2-32), and at 0.511 MeV in Equations (2-33) and (2-34). The absorption path length is estimated to be the average thickness of the first detector element when viewed from the angle of the center of the second detector $\tilde{S}(\theta_c)$, and because the photons are emitted back-to-back, the same average thickness applies to the escape path.

A final type of single photon contaminating event occurs when a gamma ray line is imaged in the presence of one or more higher energy lines. Consider the situation where only two gamma-ray lines are present. Full energy events from the lower line of energy E_γ^a are selected by setting a window about the desired energy in the summed energy spectrum ($E_\ell^a < E_s < E_h^a$). If a gamma ray at the higher energy E_γ^b underwent the desired scatter and escape in the first detector but then scattered and escaped from the second detector, the energy deposited could fall between E_ℓ^a and E_h^a , contaminating the lower energy image. The probability that a higher energy gamma ray emitted by a point source results in such a "partial energy deposition event" is ε_{aped} .

$$\varepsilon_{aped} = (\Omega_1/4\pi)P_c f_2 \left(1 - e^{-\mu' \tilde{C}_2}\right) \int_{E_\ell^a - E_{re}^b}^{E_h^a - E_{re}^b} \eta_2(E_2, E_\gamma^{b'}) dE_2 \quad (2-35)$$

This equation is identical to Equation (2-8), except that peak-to-total ratio r_{pt} has been replaced by an integral over the normalized response function of the second detector $\eta_2(E_2, E_\gamma^{b'})$. The normalized response function is the probability of measuring an energy E_2 in the second detector, given an interaction by an incident gamma ray of energy $E_\gamma^{b'}$. Limits on this integral give the range of energies that, when summed with the energy E_{re}^b deposited in the first detector for a scattering angle of θ_c , will cause a count in the summed

energy window. As with double Compton events in the first detector, scattering angles calculated from partial energy deposition events will be larger than the true angle.

The following sources of contaminating events result from two or more gamma rays interacting in a camera within the resolving time $2\tau_r$. Consider random coincidences which occur at a first order rate equal to $2\tau_r r_1 r_2$, where r_1 and r_2 are the single count rates in the first and second detector, respectively [Kno89]. These contaminating events occur when the summed energy deposited in the two detectors falls in the energy window $E_l < E_s < E_h$. The rate of contaminating random events r_{rc} for a Compton camera in a single energy gamma-ray field of energy E_γ is

$$r_{rc} = 2\tau_r \int_{E_{1min}}^{E_h - E_{2min}} \eta_1 \eta_1(E_1, E_\gamma) \left\{ \int_{E_l - E_1}^{E_h - E_1} \eta_2 \eta_2(E_2, E_\gamma) dE_2 \right\} dE_1 \quad (2-36)$$

Limits of the integration over E_1 include the lower level thresholds in the two detectors, E_{1min} and E_{2min} . In theory these thresholds can go to zero, and the first integral would range over all energies up to E_h . Limits on the integral over E_2 restrict the summed energy to values falling in the window, and η_1 and η_2 are the response functions of the first and second detectors, as introduced in Equation (2-35).

A final contaminating event type results from two gamma rays emitted in cascade. Common isotopes that decay by this mechanism include ^{24}Na and ^{60}Co . Compared to the resolving time of a Compton camera (~ 10 ns), the lifetime of the intermediate nucleus that exists between the two gamma ray emissions is generally very short [Eva55]. When each of the cascade gamma rays strikes one of the Compton camera detectors, the event is recorded as a coincidence. Hence, such events have been dubbed "cascade coincidence events". For two cascade photons of energies E_γ^a and E_γ^b emitted in a single decay, the probability that they interact in the two detectors and randomly deposit enough energy to fall in a summed energy window of interest is:

$$\begin{aligned}
\varepsilon_{acce} = & \bar{\omega}(\xi) \varepsilon_{at1}^a \varepsilon_{at2}^b \left\{ \int_{E_{1min}}^{E_h - E_{2min}} \eta_1(E_1, E_\gamma^a) \left[\int_{E_\ell - E_1}^{E_h - E_1} \eta_2(E_2, E_\gamma^b) dE_2 \right] dE_1 \right\} \\
& + \bar{\omega}(\xi) \varepsilon_{at1}^b \varepsilon_{at2}^a \left\{ \int_{E_{1min}}^{E_h - E_{2min}} \eta_1(E_1, E_\gamma^b) \left[\int_{E_\ell - E_1}^{E_h - E_1} \eta_2(E_2, E_\gamma^a) dE_2 \right] dE_1 \right\} \quad (2-37)
\end{aligned}$$

The term $\bar{\omega}(\xi)$ is the average, normalized angular correlation for the cascade gamma rays at the angle between the two detectors from the source position, and typically is within 10% of unity. The absolute efficiencies for interaction of the cascade gamma rays in the first detector are ε_{at1}^a and ε_{at1}^b , and in efficiencies in the second detector are ε_{at2}^a and ε_{at2}^b .

To evaluate the integrals over the response functions in Equations (2-35), (2-36) and (2-37), the response functions have been represented by simple geometric shapes after Gardner [Gar76]. The Compton continuum in either detector can be adequately modeled with a step function; the full energy peak of the NaI(Tl) detector can be modeled with a Gaussian function, and the full energy peak of the HPGe detector can be modeled with a delta function.

CHAPTER III

IMAGE RECONSTRUCTION

In Chapter II, the response of the RCC to point sources was modeled. Here the process of reconstructing images from the measured response of the camera to spatially extended gamma-ray sources (as well as point sources) is studied. Due to a number of constraints, a plane orthogonal to the RCC ring axis is used as the source space for reconstructing images of data measured in this work.

Consider a pair of detector elements, one from the first detector of the RCC and one from the second. The direction $\hat{u}$, from the second detector element to the first, forms the axis for a family of localization cones. By varying the cone angle θ from 0° to 180° all space is covered by these cones. (An equation for the RCC localization cones is given in Section 3.3 as Equation (3-30).) For Compton scattering Equation (2-6) relates θ to the recoil electron energy deposited in the first detector E_{re} , given an incident gamma ray energy E_γ . Any gamma rays emitted by sources lying on one particular localization cone will deposit the same E_{re} in the first-detector element when the scattered gamma ray is absorbed in the second-detector element and will thus be indistinguishable. From this argument it is apparent that the RCC integrates the intensity of all sources lying on one localization cone (though there is a weighting factor dependent on source-detector separation).

Intersecting the localization (or integration) cones with the source plane yields conic sections, which represent integration paths. Equation (3-31) describes the integration paths in a plane given by $z = z_s$. The conic sections that are possible for the RCC integration paths include ellipses, parabolas and hyperbolas, and can cover the entire source plane through variation in θ . Figure 3-1 illustrates a family of conic integration paths from a single detector pair with θ varying in 10° increments. The central scattering

Figure 3-1. A planar source space showing typical integration paths for the RCC when $\theta_c = 60^\circ$.

angle of the RCC θ_c , which is also the angle between the source plane normal line and $\hat{u}$, is 60° in that figure. These integration paths are discussed more fully at the end of Section 3.3 after the appropriate equations have been developed.

Now consider a continuous ring-shaped second detector that contains the position v_2 of the second-detector element described above, with the first-detector element residing on the ring axis. Moving around the ring from v_2 , yields integration paths identical to those of Figure 3-1 except that they are rotated azimuthally. Figure 3-2a shows how the hyperbolic integration path at $\theta = 60^\circ$ in Figure 3-1 appears for 8 equally spaced azimuthal positions around the ring. Figure 3-2a also represents integration paths of a ring Compton camera with 8 second-detector elements intercepting a point source positioned on the ring axis.

Radiation intensity integration paths of the RCC are analogous to the line integrals of "straight-ray" transaxial tomography. In Figure 3-1 the integration paths from one detector pair is shown to cover the entire source plane, much like a single viewing angle covers the entire field of view in tomography, and Figure 3-2a shows that

Figure 3-2. Paths of integration from a ring Compton camera with 8 second-detector elements equally spaced in azimuthal angle and with $\theta_c = 60^\circ$ for point sources a) on the ring axis and b) 30° off of the ring axis. The path for the element at $\phi = 90^\circ$ is indicated with a heavier line in both parts.

the RCC records each set of integration paths at equally spaced azimuthal angles, like the different viewing angles used in tomography. The integration paths for the on-axis point source in Figure 3-2a thus share with the line integrals of straight-ray tomography the attributes of being symmetric and equally distributed in azimuth. In addition these paths are nearly straight. However, Figure 3-2b shows that the symmetry is lost for an off-axis point source, because the second-detector elements accept gamma rays scattered at a range of angles. This off-axis asymmetry is a significant variation from straight-ray tomography, but the paths in Figure 3-2b do retain some of the features of the on-axis point: they are nearly straight at the intersection point, and they are well distributed azimuthally, although the distribution is not uniform. The asymmetry in the integration paths will be studied further to determine its impact on the quality of RCC images.

Another significant difference between integrating over curved paths instead of straight rays is that curved paths of integrations do not directly relate to the two-dimensional (2D) Fourier transform of the source. In straight-ray tomography, this

relation is known as the Central Slice Theorem; it is developed in the following section. Nevertheless, others have had success applying techniques from straight-ray transaxial tomography to systems that integrate over curved paths. In Section 3.2, two such systems are reviewed, where they are compared to the RCC and straight-ray tomographs. A common feature between straight-ray and curved path systems, which makes it possible to apply the straight-ray techniques to the other systems, is described. To serve as a basis, techniques developed for reconstructing images in straight-ray transaxial tomography, such as X-ray computed tomography (X-ray CT), single photon emission computed tomography (SPECT), and positron emission tomography (PET), are reviewed.

3.1 Computed Tomography Image Reconstruction

In straight-ray transaxial tomography, data are collected by measuring line integrals through the region to be imaged. In a common and easily analyzed case, parallel line integrals are measured from a series of different views around the imaged region [Bar81, Kak88, Mac83]. Figure 3-3 shows $\lambda_\phi(x_r)$, which is the projection of straight line integrals through an object whose intensity is $f(x,y)$. y_r is the axis parallel to the integration direction in a Cartesian coordinate system (given by x_r, y_r) rotated at an angle ϕ from a fixed reference system (given by x,y). The coordinate systems are related by

$$\begin{aligned} x_r &= x \cos \phi + y \sin \phi \\ y_r &= -x \sin \phi + y \cos \phi \end{aligned} \tag{3-1}$$

$\lambda_\phi(x_r)$ is related to the intensity function represented in the rotated frame of reference $f(x_r, y_r)$ by line integrals over y_r .

$$\lambda_\phi(x_r) = \int_{-\infty}^{\infty} f(x_r, y_r) dy_r \tag{3-2}$$

Figure 3-3. Projections of intensity function $f(x,y)$ by straight line integrals onto an axis rotated at angle, ϕ .

The one-dimensional (1D) Fourier transform of the projection $\Lambda_\phi(\xi_r) = \mathfrak{F}\{\lambda_\phi(x_r)\}$ is equal to one slice through the 2D Fourier transform of the object $F(\xi_r, \eta_r) = \mathfrak{F}\{f(x_r, y_r)\}$ [Bar81].

$$\begin{aligned}
 \Lambda_\phi(\xi_r) &= \int_{-\infty}^{\infty} \lambda_\phi(x_r) e^{-j2\pi\xi_r x_r} dx_r \\
 &= \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} f(x_r, y_r) dy_r \right\} e^{-j2\pi\xi_r x_r} dx_r \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x_r, y_r) e^{-j2\pi\xi_r x_r} e^{-j2\pi\eta_r y_r} dy_r dx_r \Big|_{\eta_r=0}
 \end{aligned} \tag{3-3}$$

In Equation (3-3), the integrand has been multiplied by the factor $e^{-j2\pi\eta_r y_r}$, which is unity when $\eta_r=0$. This trivial multiplication casts the integral as a 2D Fourier transform over the Cartesian space represented by (x_r, y_r) .

Figure 3-4. Central slice of a two-dimensional Fourier transform computed from line integral projections measured at angle ϕ .

$$\Lambda_{\phi}(\xi_r) = F(\xi_r, \eta_r) \Big|_{\eta_r=0} \quad (3-4)$$

This result is the Central Slice Theorem mentioned earlier in this chapter; it is the basis for applying Fourier transform methods to computed tomography. To recap this theorem in words, the 1D Fourier transform of an object's projection is equal to a central slice (i.e. when $\eta_r=0$) through its 2D transform at the angle of the projection, ϕ . Figure 3-4 shows that portion of $F(\xi_r, \eta_r)$ determined from the projection illustrated in Figure 3-3. Since the angle of the slice in Fourier space is equal to ϕ , $F(\xi, \eta)$ can be completely specified by measuring projected line integrals at all rotation angles, $0 < \phi < \pi$.

A very crude estimate of the source intensity function $f(x, y)$ can be formed by backprojecting and summing the data from every measured angle ϕ . The backprojection of a single line integral $g_{\phi}(x_r, y_r)$ can be written as

$$g_{\phi}(x_r, y_r) = \lambda_{\phi}(x_r) \quad (3-5)$$

This process is equivalent to smearing the integral back over the path of integration. To generalize the backprojection to the unrotated frame, a delta function $\delta(x_r - (x \cos \phi + y \sin \phi))$ is used to represent the backprojection path.

$$g_\phi(x, y) = \int_{-\infty}^{\infty} \lambda_\phi(x_r) \delta(x_r - (x \cos \phi + y \sin \phi)) dx_r \quad (3-6)$$

The backprojections are summed over all ϕ , which in the limit of continuous measurements requires an integration over ϕ . The resulting image $\hat{f}_b(x, y)$ is referred to as a *laminogram*.

$$\hat{f}_b(x, y) = \int_0^\pi \int_{-\infty}^{\infty} \lambda_\phi(x_r) \delta(x_r - (x \cos \phi + y \sin \phi)) dx_r d\phi \quad (3-7)$$

In polar coordinates (r, θ) , the line x_r is equal to $r \cos(\phi - \theta)$, and $\hat{f}_b(r, \theta)$ can be written as

$$\hat{f}_b(r, \theta) = \int_0^\pi \int_{-\infty}^{\infty} \lambda_\phi(x_r) \delta(x_r - r \cos(\phi - \theta)) dx_r d\phi \quad (3-8)$$

To characterize the laminogram, the system response from a delta function point source at the origin $f(x_r, y_r) = \delta(x_r) \delta(y_r)$ is determined. This is known as the system point spread function or PSF. Combining Equations (3-2) and (3-8) yields the PSF of the laminogram p_b .

$$\begin{aligned} p_b(r, \theta) &= \int_0^\pi \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} \delta(x_r) \delta(y_r) dy_r \right\} \delta(x_r - r \cos(\phi - \theta)) dx_r d\phi \\ &= \int_0^\pi \int_{-\infty}^{\infty} \delta(x_r) \delta(x_r - r \cos(\phi - \theta)) dx_r d\phi \\ &= \int_0^\pi \delta(r \cos(\phi - \theta)) d\phi \end{aligned} \quad (3-9)$$

To evaluate (3-9) the following relationship for a delta function must be used [Bar81]

$$\delta[f(x)] = \sum \left. \frac{\delta(x - x_i)}{|df(x)/dx|} \right|_{x=x_i} \quad (3-10)$$

where $f(x_i)=0$. The only root in $r \cos(\phi - \theta)$ between 0 and π occurs at $\phi - \theta = \pi/2$.

$$\begin{aligned} p_b(r, \theta) &= \int_0^\pi \left. \frac{\delta(\phi - \theta - \pi/2)}{|r \sin(\phi - \theta)|} d\phi \right|_{\phi - \theta = \pi/2} \\ &= 1/r \end{aligned} \quad (3-11)$$

Since the PSF is $1/r$, the laminogram for any general source intensity $f_b(x, y)$ is the 2D spatial convolution of $f(x, y)$ with $1/r$.

$$f_b(x, y) = f(x, y) ** (1/r) \quad (3-12)$$

The $1/r$ PSF of the laminogram is undesirable because it causes severe blurring of the original $f(x, y)$. To remove the blurring, the Fourier transform of f_b can be inverse filtered. Inverse filtering is a common image processing technique, where the Fourier transform of the blurred image is divided by the transform of the blurring function, and the image is recovered by inverse transforming the results [Lim90]. The estimated image from the inverse filtering process is $\hat{f}$.

$$\hat{f} = \mathfrak{F}^{-1} \left\{ \frac{F_b}{\mathfrak{F}(p_b)} \right\} \quad (3-13)$$

From Barrett $\mathfrak{F}(1/r) = 1/\rho$ [Bar81], and the inverse filter is simply equal to ρ , where ρ is the Fourier space complement to r . A standard measure for the frequency response of an imaging system is the modulation transfer function (MTF). For a system with a circularly symmetric PSF, the MTF can be defined as

$$\text{MTF}(\rho) = |P(\rho)| / |P(0)| \quad (3-14)$$

where $P(\rho) = \mathfrak{F}\{p(r)\}$ is the system transfer function and $P(0)$ is its amplitude at DC. Thus, the MTF of the laminogram is equal to $1/\rho$, and after filtering, the MTF of $\hat{f}$ is unity at all frequencies, which is the Fourier transform of the desired spatial delta function.

While inverse filtering is, theoretically, a very elegant solution to the blurring problem, increasing the weight of higher frequencies leads to noise amplification in measured images. In a real system line integrals are drawn from recorded counts which are Poisson distributed random variables. Statistical variation of the line integrals leads to point-to-point variations in the image called *spatial noise*. Barrett treats the random line integrals $\lambda_\phi(x_r)$ as the sum of a mean $\bar{\lambda}_\phi(x_r)$ and a randomly-distributed, zero-mean variable $\Delta\lambda_\phi(x_r)$ [Bar81]. He proceeds to show that for the special case of the laminogram from a large, uniform object that the MTF of the noise is proportional to $1/\sqrt{\rho}$. Thus, the signal amplitude of such an object falls more rapidly with frequency than the noise amplitude, and the signal will be dominated by the noise when the frequency becomes sufficiently large. In Section 5.5, the laminogram of a point source measured with the RCC is also shown to be dominated by noise at high frequencies. From these two limiting cases, it is reasonable to assume that the laminogram of any object is dominated by noise at high frequencies. To compensate, ρ is multiplied by a heuristic apodizing function, $A(\rho)$, which de-emphasizes the high frequencies where noise dominates. In Cartesian coordinates, where $\rho = \sqrt{\xi^2 + \eta^2}$, the estimated image takes the following form

$$\hat{f}(x,y) = \mathfrak{F}^{-1} \left\{ \sqrt{\xi^2 + \eta^2} A(\sqrt{\xi^2 + \eta^2}) F_b(\xi, \eta) \right\} \quad (3-15)$$

The backprojection; summation and filtering steps described above are all linear operations, so their order can be changed. In a common technique, projection data are Fourier transformed first to get $\Lambda_\phi(\xi_r)$ and a one dimensional (1D) filter is applied. Filtered projections are then inverse transformed and backprojected over the source space. Mathematically this process can be represented as

$$\hat{f}(x,y) = \int_0^\pi \mathfrak{F}^{-1}\{\Lambda_\phi(\xi_r)|\xi_r|A(\xi_r)\}\delta(x_r - (x\cos\phi + y\sin\phi))d\phi \quad (3-16)$$

Barrett and Swindell [Bar81], Kak and Slaney [Kak88] and Macovski [Mac83] all present derivations of expressions equivalent to Equation (3-16).

The process of removing the $1/r$ blur in tomographic laminograms is variably referred to as *ramp-filtering* (because of the shape of the frequency filter) or *ρ -filtering*. An undesirable effect of ramp-filtering is a result of setting the DC component to zero. After ramp-filtering, the average intensity in $\hat{f}(x,y)$ must then be zero, and slowly varying regions of the laminogram are driven negative. Solutions to this problem are discussed in Section 3.4, where an implementation of the ramp-filtered laminogram technique is developed.

It is important to note that ideal reconstructions are only achieved with the ramp-filtering technique if the system PSF is spatially invariant. In practice many of the SPECT [Bud79, Bar81] and PET [Bud79, Lec84, Der88, Lec90, Evn91] systems in use have variations in the point response across the imaging region.

3.2 Correspondence Between Tomographic and Compton Camera Line Integral Projections

To employ straight-ray tomographic reconstruction techniques for images from a Compton camera, a correspondence between the two systems must be developed. Others have studied this problem for systems that integrate over curved paths in the source region [Nor79, Sim78].

3.2.1 Ultrasound Reflectivity Tomography

Norton and Linzer describe ultrasound reflectivity tomography; this system consists of a ring array of point transducers that form an image of the transaxial slice

inside the ring [Nor79]. In the most basic ultrasound imaging systems, ultrasound pulses are transmitted and received by the same transducer. When the pulse is shorter than the transit time for the pulse to reach and reflect back from the nearest point of interest, the transducer is switched to receive mode immediately after the pulse, and reflections are received as a function of time. This is referred to as an *A-scan* in the ultrasound literature [Mac83]. Knowing the ultrasound velocity c , the round trip transit time t can be related to the distance between the transducer and the reflector r .

$$t = 2r/c \quad (3-17)$$

If, as in Norton and Linzer's system, the transducer is a point, the ultrasound wave will diverge as a circle in the transverse plane, and signals reflected at the same radial distance from a transmitter will be received at the transducer simultaneously. The signal received at any t is then an integral of the reflectivity over a circle of radius $r (=ct/2)$ centered on the transducer.

An alternative mode for ultrasound reflectivity tomography uses separate transducers to transmit and receive the ultrasound waves. All signals measured at a given t will have traveled the same distance from the emitting transducer to the reflector, then back to the receiving transducer. Hence, the measured signal is integrated over elliptical paths, with the two transducers serving as the foci of the ellipses.

For this ultrasound tomograph, data are taken with a complete ring of transducers surrounding the imaging plane. Signals from each transducer are then backprojected over the circular (or elliptical) paths and summed in a process identical to the one represented by Equation (3-7) except for the backprojection path.

The PSF from backprojection $p_u(r, \theta)$ has been computed and then simplified using a power series expansion about the location of the point source (r_0, θ_0)

$$p_u(r, \theta) = p_0(X) + \varepsilon_0(r, \theta, r_0, \theta_0) \quad (3-18)$$

where, $X = [r^2 + r_0^2 - 2r_0 \cos(\theta - \theta_0)]^{1/2}$. The first term of the expansion $p_0(X)$ is spatially invariant; $\varepsilon_0(r, \theta, r_0, \theta_0)$ is a spatially variant error term, and X is the distance from (r_0, θ_0) to (r, θ) . When there is no preshaping of the pulse or filtering applied before backprojection, which is analogous to the formation of the laminogram, $p_0(X)$ is proportional to $1/X$. As a result, p_0 is equal to p_b , which is the PSF derived in Equation (3-11) for straight-ray tomography, and the $1/r$ blurring from backprojections over circular and elliptical paths is significantly reduced by ramp-filtering.

To examine the relative contribution of both p_0 and ε_0 to the system PSF, Norton and Linzer [Nor79] give expressions for p_0 and ε_0 after ramp-filtering, with a *rect* function defined below as the apodizing function.

$$\text{rect}(\rho/\rho_{\max}) = \begin{cases} 1 & \text{if } |\rho|/\rho_{\max} < 1 \\ 0 & \text{otherwise} \end{cases} \quad (3-19)$$

The value of the maximum frequency $\rho_{\max}$ is chosen, such that $1/\rho_{\max}$ is on the order of the system resolution. The post-filtered expressions p_0' and ε_0' are both proportional to $X^{-3/2}$, when $X \gg 1/\rho_{\max}$. The ratio ε_0' / p_0' can readily be computed along the radius on which the point source lies, that is where $\theta = \theta_0$, to illustrate the relative contributions of the two PSF components.

$$\zeta(r, \theta_0) = \frac{\varepsilon_0'(r, \theta_0, r_0, \theta_0)}{p_0'(X)} \leq \frac{\pi^2}{2} \left[\frac{r^2 - r_0^2}{R/\rho_{\max}} \right]^2 \quad (3-20)$$

where R is the radius of the ring of transducers. As expected from a series expansion, $\varepsilon_0' = 0$, when $r_0 = r$ and $\theta_0 = \theta$, and Equation (3-20) shows that $\zeta(r, \theta_0)$ is less than unity when $r^2 - r_0^2 < R\sqrt{2}/\pi\rho_{\max}$. Thus, the PSF is dominated by ε_0' only when $X \gg 1/\rho_{\max}$ (assuming that $R \gg 1/\rho_{\max}$). Because ε_0' falls as $X^{-3/2}$, its contribution to the total PSF is very small when it is the dominant component. This behavior illustrates why Norton and Linzer were successful in ramp-filtering backprojections.

3.2.2 Annular Coded Aperture Gamma-Ray Camera

Another imaging system that integrates data over a curved path is an annular coded aperture gamma-ray imaging camera developed by Simpson [Sim78]. This work is thoroughly reviewed by Barrett and Swindell [Bar81]. The annular coded aperture camera consists of a position sensitive gamma-ray detector, such as an Anger camera, with an opaque shield interposed between the detector and the source to be imaged. The shield has a transmissive annulus (or ring) of mean radius $\bar{r}'$ and width $\Delta r'$. A sketch is shown in Figure 3-5.

This coded aperture system does not image transaxial tomographic slices, but simply measures a two dimensional view of the incident photons. In this way, it is similar to the RCC. Images are reconstructed (or decoded) by mathematically correlating the coded image $h(\mathbf{r}'')$ with an annular function $q_{\text{ann}}(\mathbf{r}'')$,

$$h^+(\mathbf{r}'') = h(\mathbf{r}'') \otimes q_{\text{ann}}(\mathbf{r}'') \quad (3-21)$$

where $h^+(\mathbf{r}'')$ is the filtered version of $h(\mathbf{r}'')$, and $q_{\text{ann}}(\mathbf{r}'')$ is a function describing the annular aperture, scaled to the detector plane. (Bold characters represent vector

Figure 3-5. Sketch of annular coded aperture gamma-ray camera.

quantities.) The correlation decoding performs an integration of all counts received at the detector falling within the annular function.

The coded image of a point source is equal to the same $q_{\text{ann}}(\mathbf{r}'')$ function used in decoding. After decoding, the PSF is equal to the autocorrelation of $q_{\text{ann}}(\mathbf{r}'')$, scaled to the source plane. For a point source on the central axis of the annular aperture ($\delta(x)\delta(y)$) and in the limit that $\bar{r}'$ is infinitesimally small, this autocorrelation results in the following PSF [Bar81]

$$P_{\text{ann}}(r, \theta) = \frac{4\bar{r}^2}{r(4\bar{r}^2 - r^2)^{1/2}} \quad (3-22)$$

where $\bar{r}$ is the radius of the aperture $\bar{r}'$, scaled to the source plane. When $r < \bar{r}/2$ it is apparent from Equation (3-22) that the PSF exhibits a $1/r$ fall-off much like the PSF from straight-ray tomography. Note that it is the decoding process which leads to the $1/r$ blurring in the annular coded aperture. In contrast, integration in the RCC, ultrasound tomograph and straight-ray tomographs result from the data collection process, and the $1/r$ blurring results from back projection.

The PSF begins to deviate from the $1/r$ behavior as r becomes greater than $\bar{r}/2$ until it becomes infinite at $r = 2\bar{r}$. This singularity results when two annuli being correlated are separated by $2\bar{r}$ and overlap tangentially, and it is not found in ultrasound tomography, because within the region bounded by the ring of transducers, only arcs of the circular and elliptical integration paths occur, never a complete circle or ellipse. The possibility that two paths of integration from spatially separated source positions overlap tangentially is eliminated. To avoid effects from this singularity in the annular coded aperture, Simpson limited its field of view to a circular region of radius slightly less than $2\bar{r}$, effectively making its field of view equivalent to that of ultrasound tomography.

For the annular aperture, the system transfer function P is proportional to the square of a Bessel function and thus has circular symmetry [Sim78]

Figure 3-6. MTF of the annular coded aperture compared with a $1/\rho$ envelope. The frequency is in units of $b/\bar{r}'$.

$$P_{\text{ann}}(\rho) \propto [J_0(2\pi\rho\bar{r}'/b)]^2 \quad (3-23)$$

where $b = s_2/(s_1 + s_2)$, as defined in Figure 3-5.

In Figure 3-6, the MTF of the annular coded aperture computed from Equations (3-23) and (3-14) is shown. Like the PSF and system transfer function, it is circularly symmetric. A line that falls off as $1/\rho$ is included in this figure to illustrate that the frequency response of the annular coded aperture exhibits this characteristic of straight-ray tomography. The zeros in this MTF occur at intervals of $b/2\bar{r}'$ and are related to the discontinuity at $r = 2\bar{r}$ in the PSF.

Even though its PSF does not fall off precisely as $1/r$ and the MTF contains the zeros illustrated in Figure 3-6, Simpson greatly reduced blurring by applying a 2D ramp-filter to the correlation decoded images. The ramp-filtering process he used is the one described by Equation (3-15).

3.2.3 Ring Compton Camera

The simple, spatially-invariant annulus used to decode images from the annular coded aperture yielded a simple close formed approximation of its PSF and MTF. The PSF of ultrasound tomography, with more complex spatially-variant integration paths, had to be modeled with a series expansion. This series expansion does not provide much insight into the system PSF and MTF. Ultimately, Norton and Linzer used simulations to study the PSF [Nor79]. The paths of integration in the RCC are more complex than ultrasound tomography, including spatially-variant parabolic and hyperbolic paths shown in Figures 3-1 and 3-2 on top of the elliptical paths. As a result of this increased complexity, a simulation is used to investigate the PSF and MTF of the RCC. In this simulation, interaction probabilities for each pair of RCC detector elements are computed due to gamma rays emitted from a discretized source plane, and a simulated laminogram is generated from these probabilities. A complete description of the simulation is delayed until Section 5.5, where simulated and measured system parameters are compared.

Figure 3-7 shows the simulated response of the RCC to both an on-axis and an off-axis point source. The on-axis source fills a pixel nearest the center of a 128 by 128 pixel source plane of 0.2 cm by 0.2 cm wide pixels. The source plane is centered on the ring axis and is perpendicular to that axis at a distance of 10 cm from the first-detector array. Figure 3-7a shows horizontal and vertical contours through the peak of the simulated laminogram of the on-axis source. The two contours overlay exactly because the geometry of the on-axis source is identical in these two directions. The magnitude of the simulated PSF smoothly falls as approximately $1/r$, where r is the distance from the source position. This PSF does not exhibit a singularity like the annular coded aperture because the geometry of the RCC and the source plane in the simulation does not allow for complete circular or elliptical integration paths in the source plane. Horizontal and vertical

Figure 3-7. RCC system parameters from simulated laminograms of point sources. a) Contours through the simulated PSF of an on-axis point source, and b) contours through the MTF computed from that PSF. c) Contours through the simulated PSF of a source displaced 5.8 cm horizontally from the ring axis, and d) contours through the MTF computed from that off-axis PSF. The RCC simulation includes a 16 element first detector and a 32 element second detector with $\theta_c = 60^\circ$ and $d = 30$ cm. Source plane parameters are described in the text.

contours through the on-axis MTF are shown in Figure 3-7b, where the contours are compared to an envelope that falls as $1/\rho$: the MTF closely follows this envelope.

Figures 3-7c and 3-7d show the PSF and MTF for an off-axis point source. The source plane is identical to that used to generate the on-axis case with the source now displaced horizontally by 5.8 cm from the source plane center. A line from the center of the first detector through the off-axis source makes a 30° angle with the ring axis. As with

the on-axis case, the PSF falls as $1/r$. The vertical contour through the PSF maximum is the same width as the on-axis PSF contour, while the horizontal contour is 27% wider. The vertical contour also has oscillations in the tails that are about 1% of the peak magnitude. The MTF of the vertical contour is very similar to the on-axis case, but the horizontal contour has significant oscillations. If the simulated RCC PSF is replaced with a spatially-invariant $1/r$ PSF the MTF is relatively smooth for an on-axis point, like the RCC MTF in Figure 3-7b. If the $1/r$ PSF is shifted 5.8 cm off-axis horizontally, a horizontal contour through the MTF shows oscillations while the vertical is smooth, similar to Figure 3-7d. Thus, ringing in the off-axis MTF is not unique to the RCC and is due to the difference in the off-axis PSF magnitude at the left and right image boundary. (The discrete Fourier transform, used to compute the MTFs in Figure 3-7, treats the image as one period of an infinite sequence. This topic is discussed further in Section 3.4.)

Simulations show that the RCC system response to an on-axis point source closely resembles that of a straight-ray tomograph. For a source 30° off the ring axis, the system response diverges somewhat from this ideal. However, both ultrasound tomography and the annular coded aperture have system responses which diverge from the ideal $1/r$ PSF. In the case of the annular coded aperture, with the singularity in its PSF and zeros in its MTF, the divergence is more severe than that of the RCC. Consequently, the ramp-filtering reconstruction technique is expected to yield good quality images from RCC data as it did when applied to these other imaging systems with curved paths of integration.

3.3 Backprojecting and Summing Compton Camera Line Integral Projections to Form Laminograms

In Figure 3-7, contours through a PSF are shown. This PSF is a backprojection (without filtering) and summation of integration paths onto a plane orthogonal to the RCC ring axis. For simplicity, the term laminogram has been borrowed from tomography to describe unfiltered backprojection-summation images computed from the RCC integration

paths. The laminogram serves two purposes: most importantly, it is the starting point for reconstruction techniques described later in this chapter, and it is a crude initial reconstruction that can provide a rough estimate of the source intensity.

A starting point for developing a backprojection algorithm is established by considering the first detector to be a single element. Cones backprojected from one element of the second detector through the first form a family of conic sections as shown in Figure 3-1. The second detector consists of N_2 elements with an angular separation Φ_2 ($= 2\pi/N_2$) between each element. With the first detector centered on the ring axis, the backprojected integration paths from an element at angle ϕ are equal to those of the neighboring element at $\phi - \Phi_2$ rotated by Φ_2 about the ring axis. Figure 3-2a illustrates backprojections of a single scattering angle from an eight element second detector, where $\Phi_2 = 45^\circ$. The rotated conic integration paths in that figure are analogous to the projections recorded at different viewing angles in straight-ray tomography for a single x_r .

Before backprojecting, RCC line integral data are formulated as a function of scattering angle θ for the second-detector elements at every ϕ , $\lambda_\phi(\theta)$. This process is described in detail in the data analysis section of Chapter V. $\lambda_\phi(\theta)$ is a differential distribution of counts per scattering angle. Working backward from $\lambda_\phi(\theta)$, the number of gamma rays incident on the first detector needed to cause $\lambda_\phi(\theta)d\theta$ counts (on average) is $L_\phi(\theta)d\Omega$.

$$L_\phi(\theta)d\Omega = \frac{\lambda_\phi(\theta)d\theta}{\epsilon_{icc}} \quad (3-24)$$

The differential solid angle of the cone of width $d\theta$ about θ is given by $d\Omega$. θ is measured relative to the direction from the second detector through the center of the first, $\hat{u}$. The intrinsic Compton camera efficiency ϵ_{icc} is computed from the absolute efficiency ϵ_{acc} in Equation (2-8). The intrinsic and absolute efficiencies are related by

$$\varepsilon_{icc} = \frac{\varepsilon_{acc}}{(\Omega_1/4\pi)} \quad (3-25)$$

Here, ε_{acc} is evaluated for the particular θ of interest, except that the escape path length, $\tilde{S}(\theta)$ is computed using the central scattering angle of the camera θ_c .

If at this point, $L_\phi(\theta)$ were backprojected onto a spherical surface of radius R centered on the first detector, the backprojection would be spatially invariant. To illustrate, consider the intersection of a spherical surface with a cone of angle θ_o projected from any second-detector element. The intersection will form a circle of diameter $R\cos(\theta_o)$ for every second-detector element position. While projecting on a sphere offers a spatially invariant response, it is difficult to combine the information from the planar multi-element first detector of the RCC, without mapping the results onto a planar surface. Here, the spherical surface projection is skipped, and the backprojection is performed directly on a planar source space.

To facilitate computation, $L_\phi(\theta)$ is discretized into M_θ bins of equal θ , yielding $L_{\phi k}$:

$$L_{\phi k} \Delta\Omega_k = \int_{\Omega_{k-1}}^{\Omega_k} L_\phi(\theta) d\Omega = \int_{\theta_{k-1}}^{\theta_k} \frac{\lambda_\phi(\theta)}{\varepsilon_{icc}(\theta)} d\theta \quad (3-26)$$

where,

$$\begin{aligned} \Delta\Omega_k &= \Omega_{k-1} - \Omega_k \\ \Omega_k &= 2\pi(1 - \cos\theta_k) \\ k &= 1, 2, \dots, M_\theta \end{aligned} \quad (3-27)$$

$\varepsilon_{icc}(\theta)$ is a smoothly varying function of θ , and the angular bin width (given by $\theta_k - \theta_{k-1}$) is necessarily small so that it does not impact the resolution. As a result, $\varepsilon_{icc}(\theta)$ can be evaluated at the midpoint angle $\bar{\theta}_k = (\theta_k + \theta_{k-1})/2$. Rewriting (3-26),

$$L_{\phi k} = \frac{\lambda_{\phi k}}{\Delta\Omega_k \varepsilon_{icc}(\bar{\theta}_k)} \quad (3-28)$$

where,

$$\lambda_{\phi k} = \int_{\theta_{k-1}}^{\theta_k} \lambda_{\phi}(\theta) d\theta \quad (3-29)$$

Discrete versions of the integration paths such as $\lambda_{\phi k}$ are commonly referred to as *ray-sums* in the straight-ray tomography literature, and this terminology will be adopted here.

To backproject the angular binned data onto the source plane, the intersection of the plane and the localization cones must be computed. The cone equation can be expressed as a dot product between the unit direction from the second-detector element to the first $\hat{u}$, which defines the cone axis, with the vectors $\mathbf{v}$ that define the cone.

$$\cos(\theta) = \frac{\mathbf{v}}{|\mathbf{v}|} \cdot \hat{u} \quad (3-30)$$

Defining a Cartesian coordinate system, where the origin is centered on the first detector and the ring axis is the z direction, the source plane is given by $z = z_s$. Therefore, the intersection is

$$\cos(\theta) = \frac{xu_x + yu_y + z_s u_z}{\sqrt{x^2 + y^2 + z_s^2}} \quad (3-31)$$

where,

$$\mathbf{v} = x\hat{i} + y\hat{j} + z_s\hat{k} \quad (3-32)$$

As with the angular bins, the source plane is discretized to make the problem computationally tractable. The source plane, centered on the z axis, is segmented into N_1 by N_2 square pixels. To backproject $L_{\phi k}$, the area of the intersection of the k^{th} angular projection bin with each pixel is computed, and the solid angle of this area relative to the origin is determined. The solid angle resulting from the intersection of the j^{th} pixel ($j = 1, 2, \dots, N$, $N = N_1 N_2$) with the k^{th} angular bin is Ω_{jk} . Computing the intersection area of

the angular bins with each pixel directly would involve performing numerical integrations for each bin-pixel pair. Instead a more efficient algorithm, in which each pixel is divided into subpixels, has been devised. In this algorithm, the solid angles of all bin-pixel intersection areas are computed for a given second-detector element before moving to the next element. Considering the j^{th} pixel, the angle θ_{jm} between the direction to the middle of the pixel $\mathbf{v}_{jm}$ (or $x_{jm}\hat{\mathbf{i}} + y_{jm}\hat{\mathbf{j}} + z_s\hat{\mathbf{k}}$) and $\hat{\mathbf{u}}$ is computed from Equation (3-31). The solid angle subtended by a subpixel centered at the middle of the pixel Ω_{jm} is computed relative to the origin. The angle of the center of each subpixel (x_{js}, y_{js}) is determined using a first order Taylor series expansion of (3-31) about the pixel midpoint (x_{jm}, y_{jm}) .

$$\theta_{js} = \theta_{jm} - \frac{1}{\sqrt{1-c_{jm}^2}} \left\{ \left[\frac{u_x |\mathbf{v}_{jm}| - c_{jm} x_{jm}}{|\mathbf{v}_{jm}|^2} \right] (x_{js} - x_{jm}) + \left[\frac{u_y |\mathbf{v}_{jm}| - c_{jm} y_{jm}}{|\mathbf{v}_{jm}|^2} \right] (y_{js} - y_{jm}) \right\} \quad (3-33)$$

where,

$$c_{jm} = \cos(\theta_{jm}) \quad (3-34)$$

The subpixel angles θ_{js} are sorted according to the scattering angle bins to form a histogram, where the number of subpixels falling in the k^{th} bin (i.e. $\theta_k < \theta_{js} \leq \theta_{k+1}$) is s_{kj} .

From this histogram, the solid angle of the bin intersections for this pixel can be approximated by:

$$\Omega_{kj} \cong s_{kj} \Omega_{jm} \quad (3-35)$$

Solid angles computed with this algorithm were found to be within 0.7% of the actual solid angle using a source space and angular bins representative of those used to generate laminograms presented throughout this work.

The product $L_{\phi k} \Omega_{kj}$ represents the number of gamma rays emitted from the j^{th} pixel which actually strike the first detector. To estimate the total number of gamma rays emitted by that pixel, $L_{\phi k} \Omega_{kj}$ must be corrected for $1/r^2$ attenuation. Dividing by $\tilde{\Omega}_{1j}/4\pi$, the fractional solid angle of the first detector relative to pixel j , accomplishes this correction. The solid angle can be estimated as $\tilde{\Omega}_{1j} \cong A_1 \cos(\alpha)/|\mathbf{v}_{jm}|^2$, where α is the angle between the pixel direction and the normal to the first detector. Backprojection from one element of the second detector is similar to the continuous backprojection of straight-ray tomography in Equation (3-6). Here the operation is discrete and the backprojection onto one pixel $g_{\phi j}$ is computed from a summation over all angular bins.

$$g_{\phi j} = \sum_{k=1}^{M_\theta} L_{\phi k} \Omega_{kj} 4\pi |\mathbf{v}_{jm}|^2 / A_1 \cos(\alpha) \quad (3-36)$$

A laminogram $\hat{f}_b$ is formed by summing the backprojections from all the second-detector elements. The estimated gamma-ray intensity in the j^{th} pixel $\hat{f}_{bj}$ is

$$\hat{f}_{bj} = \sum_{\phi=1}^{M_\phi} g_{\phi j} = (4\pi/A_1 \cos(\alpha)) \sum_{\phi=1}^{M_\phi} \sum_{k=1}^{M_\theta} L_{\phi k} \Omega_{\phi kj} |\mathbf{v}_{jm}|^2 \quad (3-37)$$

The subscript ϕ has been added to $\Omega_{\phi kj}$ in order to distinguish the solid angles of different second-detector backprojections.

Finally, the single element first detector is replaced by one consisting of M_1 elements. The laminogram must now include a summation over the backprojections of these elements.

$$\hat{f}_{bj} = (4\pi/A_1 \cos(\alpha)) \sum_{\ell=1}^{M_1} \sum_{\phi=1}^{M_\phi} \sum_{k=1}^{M_\theta} L_{\ell \phi k} \Omega_{\ell \phi kj} |\mathbf{v}_{jm}|^2 \quad (3-38)$$

Substituting the relation for L from Equation (3-28) into Equation (3-38) yields

$$\begin{aligned}
\hat{f}_{bj} &= (4\pi/A_1 \cos(\alpha)) \sum_{\ell=1}^{M_1} \sum_{\phi=1}^{M_\phi} \sum_{k=1}^{M_\theta} \frac{\lambda_{\ell\phi k} \Omega_{\ell\phi k} |\mathbf{v}_{jm}|^2}{\Delta\Omega_k \varepsilon_{icc}(\bar{\theta}_k)} \\
&= \sum_{\ell=1}^{M_1} \sum_{\phi=1}^{M_\phi} \sum_{k=1}^{M_\theta} \lambda_{\ell\phi k} \omega_{\ell\phi k j} \\
&= \sum_{i=1}^M \lambda_i \omega_{ij}
\end{aligned} \tag{3-39}$$

The backprojection weights ω_{ij} have been introduced to represent the number of gamma rays backprojected into the j^{th} pixel for each count in the i^{th} angular projection bin; as well as the index i to simplify the equation, where $i = \ell M_\phi M_\theta + \phi M_\theta + k$, $i = 1..M$, and $M = M_1 M_\phi M_\theta$.

Note that in Equation (3-39), if the i^{th} projection bin does not fall completely within the source plane, the solid angle of the bin-pixel intersections summed over all pixels is less than the total solid angle of the bin.

$$\sum_{j=1}^N \Omega_{ij} < \Delta\Omega_k \tag{3-40}$$

As a result, the counts collected in λ_i will not be conserved under backprojection with ω_{ij} .

$$\omega_{ij} = \omega_{\ell\phi k j} = \frac{\Omega_{\ell\phi k j} 4\pi |\mathbf{v}_{jm}|^2}{A_1 \cos(\alpha) \varepsilon_{icc}(\bar{\theta}_k) \Delta\Omega_k} \tag{3-41}$$

When an accurate approximation of the total gamma-ray intensity in the source plane is needed, an alternative to ω_{ij} must be used. To conserve counts, the total solid angle of the bin $\Delta\Omega_k$ must then be replaced by the solid angle of the bin-source plane intersection. This backprojection weighting is designated $\bar{\omega}_{ij}$.

$$\tilde{\omega}_{ij} = \tilde{\omega}_{\ell\phi kj} = \frac{\Omega_{\ell\phi kj} 4\pi |\mathbf{v}_{jm}|^2}{A_1 \cos(\alpha) \varepsilon_{icc}(\bar{\theta}_k) \sum_{j=1}^N \Omega_{\ell\phi kj}} \quad (3-42)$$

While conserving counts and providing an approximation of the source intensity, backprojecting with $\tilde{\omega}_{ij}$ leads to noisy laminograms. Counts from angular projection bins that intersect a small number of pixels are completely distributed in those pixels. A few counts due to background in such an angular bin will be amplified relative to signals from angular bins intersecting a larger portion of the source plane.

Finding the backprojection weights requires a large number of computations. The RCC consists of 16 elements in both detectors. When 64 angular bins are backprojected onto a 64 x 64 pixel source space and 4 x 4 subpixels are used per pixel, the total number of angles computed in the algorithm described above is 6.7×10^7 . However, this number can be reduced eight times by exploiting symmetries in the RCC and source plane. Figure 3-8 shows a view of the system looking from the ring axis back through the source plane to the camera. This figure illustrates that the system is symmetric for reflections about horizontal, vertical and diagonal planes passing through the ring axis, indicated by the dashed lines. Thus the number of weights that must be computed are reduced by a factor of eight, and the remainder are found by performing the appropriate reflections.

In the opening paragraphs of this chapter, the shape of the RCC integration paths are identified as conic sections. These integration paths are given by Equation (3-31) and are either elliptical, parabolic or hyperbolic in shape. To illustrate, consider the detector element pair where $u_y = 0$, and thus $u_x = \sin(\theta_c)$ and $u_z = \cos(\theta_c)$. Integration paths for all other pairs can be computed by simple translations and rotations in the source plane. Substituting for u_x , u_y and u_z in Equation (3-31), squaring and rearranging yields

Figure 3-8. View of RCC showing planes of symmetry.

$$y^2 + \left(1 - \frac{\sin^2 \theta_c}{\cos^2 \theta}\right) x^2 + 2 \left(\frac{\sin \theta_c \cos \theta_c}{\cos^2 \theta}\right) x + \left(1 - \frac{\cos^2 \theta_c}{\cos^2 \theta}\right) z_s^2 = 0 \quad (3-43)$$

If $\theta = 90^\circ - \theta_c$, the x^2 coefficient becomes zero, and the resulting integration path is a parabola. On the other hand, if $\theta < 90^\circ - \theta_c$, the x^2 coefficient becomes positive thus having the same sign as the y^2 term, and the integration path is an ellipse. Finally, if $\theta > 90^\circ - \theta_c$, the x^2 coefficient becomes negative, and the path is hyperbolic [Swo79]. The hyperbolic shape is maintained for $\theta > 90^\circ - \theta_c$ until $\theta > 90^\circ + \theta_c$ at which point the source plane no longer intersects the single-napped localization cone.

3.4 Direct Image Reconstruction: Ramp-Filtered Laminogram

In earlier sections, laminograms were shown to suffer a $1/r$ blurring, and here an algorithm is developed for removing that $1/r$ blurring by ramp filtering. This algorithm is

a discrete version of Equation (3-15), where the ramp filter is applied in the Fourier domain.

3.4.1 Algorithm Description

To begin the ramp-filtered laminogram (RFL) algorithm, Equation (3-15) is rewritten to emphasize that frequency domain filtering is equivalent to convolution in the spatial domain.

$$\hat{f}(x,y) = \mathcal{F}^{-1}\{H(\xi,\eta)F_b(\xi,\eta)\} \quad (3-44)$$

where,

$$H(\xi,\eta) = A\left(\sqrt{\xi^2 + \eta^2}\right)\sqrt{\xi^2 + \eta^2} \quad (3-45)$$

Defining $h(x,y)$ to be the inverse transform of $H(\xi,\eta)$, $\hat{f}(x,y)$ is a linear convolution of the laminogram $\hat{f}_b(x,y)$ with the spatial filter $h(x,y)$.

$$\hat{f}(x,y) = h(x,y) ** \hat{f}_b(x,y) \quad (3-46)$$

Discrete representations of continuous functions are defined by sampling the continuous variables. For example, $h(n_1,n_2)$ equals $h(x,y)$, where $x = n_1\Delta x$ and $y = n_2\Delta y$, and Δx and Δy are sampling intervals in the x and y directions, respectively. A convolution of the sampled spatial filter $h(n_1,n_2)$ and laminogram $\hat{f}_b(n_1,n_2)$ can be performed using the discrete Fourier transform (DFT) and its inverse (IDFT); but, the discrete operation cannot be performed directly on these functions. In using the DFT and IDFT, a discrete sequence of finite extent (i.e. $0 \leq n_1 \leq N_1 - 1$ and $0 \leq n_2 < N_2 - 1$) is treated as if it were one period of an infinitely repeating sequence. As a consequence, convolving two sequences by multiplying their DFTs in the Fourier domain and applying an IDFT to the product results in a circular convolution [Lim90]. However, the desired result is a linear convolution of the two discrete functions, as in the continuous case. The circular

convolution can be used to give the same result as the linear convolution simply by increasing the periodicity of the sequences so that two periods of one sequence do not overlap a single period of the other. Commonly the period of each sequence is extended by adding zeros in a process known as zero-padding [Str88]. The sequences of interest here have the same size ($N_1 \times N_2$) and hence the same periodicity; therefore, to properly convolve them, their periods must be doubled. However, increasing the periodicity of the laminogram by zero-padding creates sharp discontinuities at its boundaries where the laminogram is still nonzero due to the $1/r$ PSF. The discontinuities are amplified by ramp filtering and leave artifacts near the boundaries.

Rowland [Row79] has shown that better results can be achieved by forming the laminogram on an extended source space that is twice as large as the original region of interest (i.e. $2N_1 \times 2N_2$ pixels) with the original region centered in the extended space. Convolution of the extended laminogram $\hat{f}_{be}(n_1, n_2)$ with a ramp filter $h_e(n_1, n_2)$, also of size $2N_1 \times 2N_2$, yields an estimate of the image $\hat{f}_e(n_1, n_2)$ on the extended source space.

$$\begin{aligned} \hat{f}_e(n_1, n_2) &= \text{IDFT}\{H_e(k_1, k_2)F_{be}(k_1, k_2)\} \\ &= \frac{1}{(2N_1)(2N_2)} \sum_{\substack{k_1=-N_1 \\ k_2=-N_2}}^{N_1} \left[\sum_{\substack{k_1=-N_1 \\ k_2=-N_2}}^{N_2} H_e(k_1, k_2)F_{be}(k_1, k_2)W_{2N_1}^{-k_1 n_1} W_{2N_2}^{-k_2 n_2} \right] \end{aligned} \quad (3-47)$$

where,

$$W_{2N} = e^{-j(2\pi/2N)} \quad (3-48)$$

An original size estimate of the image $\hat{f}(n_1, n_2)$ is recovered by simply extracting a region of size $N_1 \times N_2$ from the center of $\hat{f}_e(n_1, n_2)$.

The discrete Fourier space representation of the laminogram, $F_{be}(k_1, k_2)$, is equal to the DFT of $\hat{f}_{be}(n_1, n_2)$.

$$F_{be}(k_1, k_2) = \text{DFT}\{\hat{f}_{be}(n_1, n_2)\} = \sum_{n_1=0}^{2N_1-1} \left[\sum_{n_2=0}^{2N_2-1} \hat{f}_{be}(n_1, n_2) W_{2N_1}^{k_1 n_1} W_{2N_2}^{k_2 n_2} \right] \quad (3-49)$$

A vector representation of the extended laminogram $\hat{f}_{bej'}$ can be computed from Equation (3-39) and can be reconstituted into a 2D image $\hat{f}_{be}(n_1, n_2)$ using $j' = n_2(2N_1) + n_1$.

The discrete filter $H_e(k_1, k_2)$ of size $2N_1 \times 2N_2$ is computed from a discrete version of Equation (3-45).

$$H_e(k_1, k_2) = A \left[\sqrt{\left(\frac{k_1}{2N_1}\right)^2 + \left(\frac{k_2}{2N_2}\right)^2} \right] C \sqrt{\left(\frac{k_1}{2N_1}\right)^2 + \left(\frac{k_2}{2N_2}\right)^2} \quad (3-50)$$

$$\left\{ \begin{array}{l} -(N_1 - 1) \leq k_1 \leq N_1 \\ -(N_2 - 1) \leq k_2 \leq N_2 \end{array} \right\}$$

Discrete frequency variables k_1 and k_2 are related to the continuous variables by $k_1 = 2N_1(\xi/\xi_{\max})$ and $k_2 = 2N_2(\eta/\eta_{\max})$. The maximum frequencies must be determined for the camera geometry with consideration of aliasing and sampling, which are discussed later in this section. Because neither of the basis directions is imaged better than the other in the RCC, $\xi_{\max}$ is equal to $\eta_{\max}$, and both are defined equal to $\rho_{\max}$. Further, the extended source plane is a square grid with $2N_2=2N_1$, and $H_e(k_1, k_2)$ is circularly symmetric about the DC frequency $k_1=k_2=0$. By substitution, the constant C in Equation (3-50) has the value $\rho_{\max}$.

3.4.2 Apodizing Functions

By defining $v = \sqrt{k_1^2 + k_2^2} / 2N_1$, the apodizing function in Equation (3-50), which is also circularly symmetric about DC, can now be written as $A(v)$. It is used to limit the frequencies that are boosted by the ramp filter. The simplest apodizing function is the $\text{rect}(v/v_{\max})$ given in Equation (3-19), where $v_{\max}$ is less than or equal to 0.5 to be consistent with Equation (3-50). Unfortunately, the sharp frequency cutoff of the rect

function results in ringing in the spatial domain, an effect known as Gibb's phenomenon. To limit this ringing a number of other apodizing (or windowing) functions have been developed. Each transitions smoothly from a maximum of one to zero (or in some cases, a small fraction). The following five windowing functions are common alternatives to the rect function [Bud79, Tod86]:

1) Shepp and Logan,

$$w(v) = \text{sinc}(|v|/v_{\max}) \quad (3-51)$$

where,

$$\text{sinc}(x) = \sin(\pi x) / (\pi x) \quad (3-52)$$

2) Hann,

$$w(v) = \begin{cases} 0.5 + 0.5 \cos(\pi v/v_{\max}) & \text{if } |v| < v_{\max} \\ 0 & \text{if } |v| > v_{\max} \end{cases} \quad (3-53)$$

3) Hamming,

$$w(v) = \begin{cases} 0.54 + 0.46 \cos(\pi v/v_{\max}) & \text{if } |v| < v_{\max} \\ 0 & \text{if } |v| > v_{\max} \end{cases} \quad (3-54)$$

4) Butterworth,

$$w(v) = \left[\frac{1}{(1 + |v|/v_c)^{2n}} \right]^{1/2} \quad (3-55)$$

where n is the order of the Butterworth function,

5) Parzen,

$$w(v) = \begin{cases} 1 - 6(v/v_{\max})^2(1 - |v|/v_{\max}) & \text{if } |v| \leq v_{\max}/2 \\ 2(1 - |v|/v_{\max})^3 & \text{if } v_{\max}/2 < |v| \leq v_{\max} \\ 0 & \text{if } |v| > v_{\max} \end{cases} \quad (3-56)$$

Figure 3-9. Examples of the apodizing functions. a) Plots of the functions given in Equations (3-19) and Equations (3-51) through (3-56) and b) of the ramp filter apodized by these functions.

In Figure 3-9a, each of the above windowing functions is plotted. For the Butterworth function the critical frequency ν_c is 0.33 and the order n is 12, and for all of the other functions the maximum frequency $\nu_{\max}$ is 0.5. Figure 3-9b shows the ramp filter apodized by these same functions. To simplify discussion of the RFL algorithm, filters resulting from apodizing the ramp filter with the above windowing functions are named, following Budinger [Bud79]. The rect apodized ramp filter is labeled RECT, and the other five will be identified as SHEP, HANN, HAMM, BUTR and PARZ, henceforth.

As noted in Section 3.1, a consequence of the ramp-filtering is that the DC term is set to zero; and the intensity information of the laminogram is lost. When the reconstruction is done on the double size source space, the lowest intensity regions of the image, which are driven negative by setting the DC component to zero, are discarded when the space is truncated to its original size. This improves the estimate of the source intensity somewhat; however, to find an accurate estimate of the source intensity, the extracted image $\hat{f}(n_1, n_2)$ must be normalized. To normalize the extracted image it would first be forward projected to compute the ray-sums that would result from this estimated

source strength. All measured ray-sums are summed, as are the estimated ray-sums. Each pixel of $\hat{f}(n_1, n_2)$ is then multiplied by the ratio of the measured to estimated sums. Equation (3-71) in Section 3.5 provides an algorithm for this normalization procedure, but it has not been implemented in this work.

3.4.3 Example Reconstructed Point Sources

To illustrate the RFL algorithm, two reconstructed point sources are shown in Figure 3-10. Laminograms of these same simulated point sources have been shown in Figure 3-7 to demonstrate the $1/r$ character of the RCC laminogram. A BUTR filter with $v_c = 0.33$ and $n = 12$ is used to reconstruct both the on-axis and off-axis point sources. The angular bins used to form the simulated ray-sums are 0.87° wide, corresponding to a spatial width of 0.15 cm at the ring axis. The spatial width is chosen to be less than the 0.2 cm pixel width in order to minimize the impact of angular bin width on resolution. Remaining system parameters are described in Figure 3-7. Figure 3-10a is a surface plot of the reconstructed on-axis source. Horizontal and vertical contours through its maximum are shown in part b. The image is approximately circularly symmetric with radial oscillations that damp out at a distance of 3 cm from the center pixel. The largest amplitude of the undershoot is 7.0% of the maximum and the FWHM is 0.43 cm in both contours.

Simulation of a centered point source imaged with a straight-ray tomograph yielded an undershoot equal to 6% of the maximum and a FWHM 1.5 times the pixel width [Row79]. That straight-ray simulation used 30 angular views and apodized the ramp filter with a rect function ($v_{\max} = 0.5$). When the simulated RCC on-axis source is also reconstructed with a rect apodizing function, it has an undershoot of 3.8% and a FWHM of 0.28 cm or 1.4 pixel widths. In addition to the quantitative agreement between the simulated RCC and straight-ray tomography reconstructions, both simulations share several important features. The straight-ray tomography reconstruction shows

Figure 3-10. Surface plots of simulated point sources reconstructed with the ramp-filtered laminogram technique for a) a point on the RCC ring axis and c) a point displaced 5.8 cm from the ring axis. Horizontal and vertical contours through the peak of these surface plots are shown in b) for the on-axis point and d) for the off-axis point. Laminograms of the point sources were generated on a source plane of 128 by 128 pixels each 0.2 cm on a side, and reconstructed using a BUTR filter with $v_c = 0.33$ and $n=12$. The displayed results represent the center 68 by 68 pixel region of that reconstruction.

approximately circularly symmetric radial oscillations, which damp out away from the point, and azimuthal oscillations which appear at the larger radial distances. These similarities indicate that the ramp-filtered laminogram is properly implemented and that the artifacts in the PSF are not unique to the curved-path algorithm used here.

Parts c and d of Figure 3-10 show a surface plot of, and contours through, the maximum of a reconstructed point source displaced 5.8 cm (or 30°) horizontally from the

ring axis. The undershoot grows to 9.6% and the FWHM of the horizontal (or radial) contour increased to 0.47 cm, while the vertical (or tangential) FWHM decreases to 0.40 cm. Differences in the radial and tangential resolutions are also noted by Rowland in the straight-ray reconstruction of a small, simulated disk source positioned away from the center of the source space, but no quantitative measure of the asymmetry is offered [Row79]. It is evident that azimuthal oscillations in the off-axis PSF of Figure 3-10c are larger than their counterparts in the on-axis source. The oscillation amplitudes are 4.1% of the maximum amplitude in the off-axis PSF versus 1.3% for the on-axis PSF. Increased azimuthal oscillations are also present in Rowland's off-center source, and these oscillations begin at smaller radial distances than in the on-axis PSF, much like Figure 3-10c. The cause of these enhanced artifacts in the off-axis source is shown to be undersampling along the radius (as opposed to the azimuth).

The radial oscillations present in the simulated PSFs of Figure 3-10 will cause distortions in the images of extended sources. These oscillations can be reduced by replacing the high-order BUTR filter with a filter, such as the PARZ, which transitions to zero more gradually. Reconstructing the on-axis source with a PARZ filter ($v_{\max} = 0.5$) reduces the largest undershoot to 0.7% of the maximum amplitude, and the undershoot in the off-axis source is reduced to 3.3%. This improvement in the smoothness of the PSFs comes at the expense of resolution: the FWHM of contours through these PSFs is 34% to 38% wider than reported for the BUTR filtered PSFs above.

3.4.4 Aliasing and Sampling

A result of using discretized images is that spatial variations on the same scale as the sampling interval (Δx) will be aliased. Aliasing occurs because frequencies above $1/(2\Delta x)$ from one period of the DFT of the image will overlap the next period. These high frequency components then look like lower frequencies in the particular period of interest; hence, the term aliasing. Artifacts due to aliasing (i.e. undersampling) have been noted in

the simulated PSF reconstructions of Figure 3-10. One way to eliminate aliasing is to apply a low-pass filter to the data before it is recorded. With the RCC, as well as straight-ray tomography systems, low-pass filtering is provided by the finite spatial resolution of the system. The resolution of a straight-ray tomograph is limited by the size of its detectors δ , and from the Nyquist sampling criteria, the sampling interval should be $\delta/2$ [Bar81]. The RCC resolution is normally characterized by the angular FWHM of its PSF $\Delta\theta_t$. The spatial resolution will be a minimum at the ring axis where $\delta = 2z_s \tan(\Delta\theta_t/2)$, and if $\Delta\theta_t$ is less than 10° , $\delta \approx z_s \Delta\theta_t$ is an excellent approximation. To adequately sample a circular imaging region of radius a in the radial direction, M_θ angular bins are required, where

$$M_\theta = \frac{2a}{(\delta/2)} \approx \frac{4a}{z_s \Delta\theta_t} \quad (3-57)$$

To determine the number of second-detector elements M_ϕ needed to properly sample the same circular source space of radius a , the sampling interval around the source space circumference should be the same as the sampling along the radial direction. This azimuthal sampling requirement is represented by

$$(\delta/2)M_\phi = 2\pi a \quad (3-58)$$

The spatial width δ of an angular bin changes from the ring axis to the radial distance a , and is given by:

$$\delta = z_s [\tan(\alpha + \Delta\theta_t/2) - \tan(\alpha - \Delta\theta_t/2)] \quad (3-59)$$

where, $\alpha = \arctan(a/z_s)$. Using the approximation that $\Delta\theta_t$ is small and applying some trigonometric manipulation to Equation (3-59), δ can be expressed as

$$\delta \approx z_s (\Delta\theta_t) \left[1 + (a/z_s)^2 \right] \quad (3-60)$$

By substituting Equation (3-60) into (3-58) and rearranging, the required number of second-detector elements can be estimated.

$$M_{\phi} \equiv \frac{4\pi a}{(\Delta\theta_t)z_s \left[1 + (a/z_s)^2 \right]} \quad (3-61)$$

Equation (3-57) provides a conservative estimate of the required number of radial samples, because it is derived assuming only one element in the first detector. The 16 elements of the first detector cover a square area, 2 cm on a side, with sampling every 0.5 cm. For source planes near the RCC, where the spatial resolution is on the order of this 0.5 cm interval, fewer samples are necessary than predicted by this equation. This effect is analogous to the translation or *wobbling* motion used in many PET systems to improve sampling [Phe75, Boh77, Evn91]. Likewise, azimuthal sampling will be improved by having 16 elements instead of the assumed one, because the azimuthal angle from a given source pixel to one second-detector element varies depending on the position of the first-detector element involved in the interaction.

Improved sampling due to the finite spatial extent of the first detector is demonstrated in Figure 3-10. The surface plot of the on-axis PSF only has significant azimuthal oscillations at radial distances beyond 3 cm. The 32 second-detector elements of the simulation thus provide adequate sampling to $a = 3$ cm. However, by assuming that $\Delta\theta_t$ is twice the angular bin width of 0.87° , Equation (3-61) requires 114 elements to properly sample to the same radius.

3.5 Iterative Image Reconstruction Techniques

Iterative techniques have traditionally been used to reconstruct images from Compton scatter cameras. In contrast, the direct RFL algorithm presented in Section 3.4 has not previously been used. Iterative techniques are more computationally intensive

than direct filtering; nevertheless, three iterative techniques are considered here to provide comparisons by which the performance of the direct technique can be gauged.

An algebraic reconstruction technique (ART) algorithm is presented first. ART has been a standard technique in tomographic image reconstruction for years and has been reviewed by many authors [Gor70, Gor74, Bar81, Tod86]. Kak and Slaney present a particularly thorough review [Kak88]. Singh and coworkers have successfully used ART to reconstruct images from measured Compton camera projection data [Sin90, Bre90]. A second iterative technique is the Simultaneous Iterative Reconstruction Technique (SIRT), a variation of the standard ART technique. The third technique considered here is a maximum likelihood estimator (MLE) algorithm, first applied to emission computed tomography by Shepp and Vardi [She82]. MLE algorithms have also been used by Singh and coworkers to reconstruct images for a Compton camera [Bre90, Heb87]. An MLE algorithm has been developed specifically for reconstructing images from projection data measured with the RCC [Dgn93a].

All of these iterative techniques treat the $N (=N_1N_2)$ pixels as a basis set for the M ray-sums λ_i ,

$$\lambda_i = \sum_{j=1}^N w_{ij} f_j \quad i = 1, 2, \dots, M \quad (3-62)$$

where w_{ij} is the probability that a gamma ray emitted from the j^{th} pixel falls in the i^{th} ray-sum. An advantage of these methods is that no assumptions or approximations regarding similarities between the data from the RCC and straight-ray tomographs are necessary. On the other hand, iterative techniques require more computations and tend to be noisier than direct techniques like filtered backprojection. Also, they are sensitive to errors in the weights, w_{ij} .

3.5.1 Algebraic Reconstruction Technique

To begin the ART algorithm, note that the N-dimensional pixel basis provides a simple relationship between ray-sums and the estimated source images. The source images are points in this space, and ray-sums represent *hyperplanes* in this space, as illustrated by Equations (3-62). When a unique solution to these equations exists, all of the hyperplanes intersect at a single point. Equations (3-62) can be rewritten as a dot product of N-dimensional vectors.

$$\lambda_i = \mathbf{w}_i \cdot \mathbf{f} \quad i = 1 \dots M \quad (3-63)$$

If the ray-sums are treated as an M-dimensional vector, Equation (3-63) can be rewritten as a single matrix equation

$$\lambda = \mathbf{W}\mathbf{f} \quad (3-64)$$

where, $\mathbf{W}$ is the $N \times M$ transition probability matrix. Casting the relation between the source image and the ray-sums as a matrix equation makes it apparent that solving for a source image $\mathbf{f}$ from its ray-sums and $\mathbf{W}$ is indeed a matrix inversion problem. Each iterative technique is simply a matrix inversion algorithm that suits the type of transition probability matrix encountered in emission computed tomography.

With ART, an initial estimate of the image $\hat{\mathbf{f}}^{(0)}$ is made, and this vector is successively projected onto each of the M hyperplanes. An iteration is completed when all M of these projections are completed. Figure 3-11 illustrates this idea in a 2D space.

The first projection consists of calculating the orthogonal distance $\Delta \hat{\mathbf{f}}^{(1)}$ between the initial estimate $\hat{\mathbf{f}}^{(0)}$ and the first ray-sum hyperplane $\lambda_1 = \mathbf{w}_1 \cdot \mathbf{f}$, then subtracting it from $\hat{\mathbf{f}}^{(0)}$.

For the k^{th} estimate $\hat{\mathbf{f}}_j^{(k)}$, the updated value of pixel j , is given by

$$\hat{\mathbf{f}}_j^{(k)} = \hat{\mathbf{f}}_j^{(k-1)} - \Delta \hat{\mathbf{f}}_j^{(k)} \quad j = 1 \dots N \quad (3-65)$$

Figure 3-11. Two-dimensional illustration of projections on hyperplanes used in the ART algorithm.

The correction term $\hat{\Delta f}_j^{(k)}$ is computed from the difference between the measured ray-sum λ_j and the estimated ray-sum q_j from the previous source estimate.

$$\Delta f_j^{(k)} = \frac{\lambda_j - q_j}{\sum_{\ell=1}^N w_{i\ell}^2} w_{ij} \quad j=1 \dots N \quad (3-66)$$

where,

$$q_j = \sum_{\ell=1}^N f_{\ell}^{(k-1)} w_{i\ell} \quad (3-67)$$

Equations (3-65) through (3-67) are the basic formulation for the ART algorithm.

However, when reconstructing from measured ray-sums, statistical errors in the measured values result in images with salt-and-pepper noise. To reduce this effect, a damping factor κ is introduced [Kak88]. It is also recognized that $\hat{f}_j^{(k)}$ must be greater

than or equal to zero. Incorporating the damping factor and this non-negativity constraint, Equation (3-65) can now be written as

$$\hat{f}_j^{(k)} = \max \left[0, \hat{f}_j^{(k-1)} - \kappa \Delta \hat{f}_j^{(k)} \right] \quad j = 1 \dots N \quad (3-68)$$

The damping factor is set between 0 and 1 depending on the desired rate of convergence and smoothness in the final image. Brechner and Singh [Bre90] used the following factor, which is a function of the ray-sum intensity.

$$\kappa_i = \kappa_1 \sqrt{\frac{\lambda_i}{\max(\lambda)}} + \kappa_2 \quad i = 1 \dots M \quad (3-69)$$

This damping factor assigns the largest weights to those ray-sums with the largest values and hence the smallest relative uncertainty. (The counts in the ray-sums are Poisson distributed with relative variances equal to $1/\sqrt{\lambda_i}$.) κ_1 and κ_2 are each chosen to be less than 0.5

Elements of the transition probability matrix w_{ij} are determined using the efficiency models developed in Chapter II for on-axis point sources. These weights are used to forward project the source estimate into the ray-sums, and as a result are inversely proportional to the backprojection weights ω_{ij} computed from Equation (3-41),

$$w_{ij} = \left[\frac{\tilde{\Omega}_{\ell j}}{4\pi} \right] \left[\frac{s_{kj}}{N_s} \right] \varepsilon_{icc}(\bar{\theta}_k) \quad i = 1 \dots M, \quad j = 1 \dots N \quad (3-70)$$

where $\tilde{\Omega}_{\ell j}$ is the solid angle subtended by element ℓ of the first detector relative to the center of pixel j , and $\bar{\theta}_k$ is the midpoint of the scattering angle bin. The ratio s_{kj}/N_s is the fraction of subpixels in j , which fall in ray-sum i . Recall that $i = \ell M_\phi M_\theta + \phi M_\theta + k$ and $M = M_1 M_\phi M_\theta$, where M_1 is the number of first-detector elements, M_ϕ is the number of second-detector elements and M_θ is the number of scattering angle bins.

The forward probability weights are computed using the same software routine as the backprojection probabilities, and are stored for later use with the ART algorithm. To

minimize storage space only non-zero pixel numbers and weights for each ray-sum are actually stored. Data storage requirements are reduced an additional factor of eight by exploiting the RCC symmetry. The eight-fold symmetry of the system, described in Section 3.3, is also used to reduce memory requirements in the ART algorithm itself.

The initial estimate for the ART algorithm $\hat{f}^{(0)}$ is simply the laminogram $\hat{f}_b$ normalized to have the same number of total counts in the forward projected ray-sums q_{bi} as the measured ray-sums λ_i .

$$\hat{f}_j^{(0)} = \hat{f}_{bj} \frac{\sum_{i=1}^M \lambda_i}{\sum_{i=1}^M q_{bi}} \quad j = 1 \dots N \quad (3-71)$$

Forward projections of $\hat{f}_b$ to determine q_{bi} are computed as shown in Equation (3-67).

Convergence of the ART algorithm is monitored with the root mean square (RMS) error. After iteration k , this error is found by

$$e_{rms}^{(k)} = \left[\frac{\sum_{i=1}^M (\lambda_i - q_i^{(k)})^2}{M} \right]^{1/2} \quad (3-72)$$

The algorithm is stopped when the relative difference between two iterations is less than a predetermined threshold.

Because all pixels making up a ray-sum are updated immediately after the correction factor for that ray-sum is computed, the order of stepping through the M ray-sums can impact the result, particularly in the rate of convergence. To illustrate, consider the lines (hyperplanes) in Figure 3-11 for λ_1 and λ_2 . If these lines were orthogonal, the solution would be reached after only two projections, for any initial guess. In the general N -dimensional case, convergence is reached more quickly if the hyperplanes representing successive ray-sums are nearly orthogonal, than if they are parallel. Thus, to facilitate

convergence, successive ray-sums are chosen from different second-detector elements separated by approximately $\phi = 90^\circ$, minimizing the number of source pixels shared between them. Since the pixels form the basis for the space, hyperplanes containing the fewest number of common pixels will be most nearly orthogonal.

3.5.2 Simultaneous Iterative Reconstruction Technique

The SIRT algorithm is closely related to the ART algorithm; its formulation is identical to ART, except that updates are delayed until the end of each iteration, and then are applied simultaneously [Kak88]. SIRT removes the dependence on the ray-sum order and generally results in a smoother image than ART; however, it converges more slowly.

3.5.3 Maximum Likelihood Estimator Algorithm

An MLE iterative reconstruction technique incorporates the random nature of the gamma rays emitted from each source pixel, by treating each f_j as a Poisson distributed variable [She82, She84, Heb87, Dgn93]. The joint probability or likelihood that a particular source estimate $\hat{\mathbf{f}}$ results in the measured ray-sums λ is $P(\lambda | \hat{\mathbf{f}})$.

$$P(\lambda | \hat{\mathbf{f}}) = \prod_{i=1}^M \left\{ \frac{\left[\sum_{j=1}^N \hat{f}_j w_{ij} \right]^{\lambda_i} \exp \left[- \sum_{j=1}^N \hat{f}_j w_{ij} \right]}{\lambda_i!} \right\} \quad (3-73)$$

Taking the logarithm of Equation (3-73) yields a linear, log-likelihood function, which can readily be maximized. Since the logarithm of $P(\lambda | \hat{\mathbf{f}})$ is a monotonically increasing function of $P(\lambda | \hat{\mathbf{f}})$, maximizing the logarithm also maximizes $P(\lambda | \hat{\mathbf{f}})$. Hebert, et al., derive an algorithm that iteratively modifies the source estimate to maximize this log-likelihood function [Heb87]. This algorithm is summarized by the following equation

$$\hat{f}_j^{(k)} = \frac{\hat{f}_j^{(k-1)}}{\sum_{i'=1}^M w_{i'j}} \sum_{i=1}^M \left[\frac{\lambda_i w_{ij}}{q_i^{(k-1)}} \right] \quad (3-74)$$

The forward projections q_i are computed using Equation (3-67), and the initial source estimate $\hat{f}^{(0)}$ must be strictly positive to assure convergence.

CHAPTER IV

RCC SYSTEM DESCRIPTION

In this chapter, the physical characteristics and performance of the RCC are detailed. The two detector arrays are described individually. Mechanical support and alignment systems are described next, along with estimated uncertainties in alignment. Then, signal processing and data acquisition electronics are reviewed. After the electronics, the data acquisition software is described. The chapter is concluded with a summary of important performance parameters for the RCC.

4.1 First-Detector Array

The first detector is a 4 by 4 planar array of high-purity germanium detectors (HPGe). The array was fabricated by segmenting the contacts of a 6 mm thick, planar HPGe detector into 5 mm by 5 mm elements using a gold-mask technique. A photograph of the array is shown in a paper by Luke [Luk84], where the fabrication process is described.

The HPGe array and its front-end feed back electronics are mounted inside an evacuated cylindrical cryostat, which is cooled with liquid nitrogen. The cryostat and front-end electronics were designed to minimize attenuation of gamma rays entering and leaving the HPGe array [Sin85]. Signals from the front-end electronics were fed through the vacuum seal to individual preamplifiers mounted just outside the cryostat. Pehl, et al. describe the cryostat, front-end electronics and preamplifiers [Peh85]. Vacuum is maintained in the system with a continuously operating ion pump. Figure 4-1 is a photograph of the cryostat. The position of the elements is etched on the cryostat and the preamplifiers are visible around its outside edge. Negative high voltage is applied through an SHV connector on the top of the cryostat.

Figure 4-1. Photograph of first-detector cryostat which holds the HPGe array.

The preamplifiers consist of two feedback stages: the first stage is charge sensitive, and the second inverts the signal and amplifies it with a gain of 5. The second stage also provides the low output impedance necessary to drive loads presented by the shielded output ribbon cable and subsequent signal processing electronics. Two lines of the coaxial cable are used to provide $\pm 24\text{V}$ power to the preamplifiers [Peh85].

Knowledge of the HPGe energy resolution is needed to determine the summed energy resolution of the RCC as well as its angular resolution. The energy resolution of each element of the HPGe array has been measured at energies of 0.122 MeV and 0.662 MeV. These measurements are tabulated in Table B-1 of Appendix B. The FWHM resolution of a full energy peak in a germanium detector is usually expressed as the quadrature sum of three terms: statistical spread in number of charge carriers, electronic noise, and incomplete charge collection [Kno89]. Since the elements of this HPGe array are only 6 mm thick and the applied bias is relatively high at -800V, there is little effect from incomplete charge collection. Thus, the FWHM energy resolution ΔE_1 can be

represented by a term that is independent of the deposited energy and a statistical term that is proportional to the deposited energy

$$(\Delta E_1)^2 = (a_1 + a_2 E) \quad (4-1)$$

A linear least squares fit of $(\Delta E_1)^2$ versus E was performed to characterize the variation of ΔE_1 with energy deposited in any element. For a deposited energy E in MeV, the following equation gives ΔE_1 (also in MeV).

$$\Delta E_1 = 10^{-3} \sqrt{1.068E + 1.067} \quad (4-2)$$

Because the only interactions of interest in the first detector are Compton scattering events, the full energy peak efficiencies of the HPGe elements are of little consequence. However, the probability of Compton scattering is incorporated in the absolute Compton camera efficiency ϵ_{acc} given in Equation (2-8), and measurements of ϵ_{acc} are described in Chapter V. The relative efficiency of these elements was found to vary by as much as $\pm 12\%$ from the mean. The relative coincident event rates between each of the first-detector elements and all elements of the second detector for an on-axis ^{137}Cs point source are illustrated in Figure 4-2a. The diagram represents the HPGe array with the relative count rate of each element written in its array position. The relative efficiencies were experimentally corroborated, because a number of factors could cause relative errors in the coincident counts of the HPGe elements, such as misadjustment or drift in the timing. Pulse height spectra of the array were recorded for the same ^{137}Cs point source on axis but with the coincidence requirement removed. The counts falling between 0.290 and 0.400 MeV from each element were recorded. This energy range represents a minimum in the Compton continuum where calibration errors would have the least impact. Relative counts of the HPGe elements from this measurement are displayed in Figure 4-2b. Both measurements were recorded from an uncollimated point source position 10 cm from the first detector so the solid angle of the elements varied by less than 1%, and they are in good agreement with one another. Note that in both Figure

1.08	1.03	0.97	0.94
1.09	0.95	1.01	0.94
1.11	1.00	0.92	0.92
1.12	1.00	1.02	0.93

Uncertainty: ± 0.04
 Scattering Angle = 45°
 $E_{re} = 0.182$ MeV

a)

1.12	1.02	0.95	0.94
1.12	0.91	0.96	0.93
1.09	0.99	0.95	0.90
1.15	1.03	1.01	0.94

Uncertainty: ± 0.02
 Energy Range: 0.290 - 0.400 MeV

b)

Figure 4-2. Sketches of HPGe detector array showing relative element count rates from an on-axis ^{137}Cs point source a) in coincidence with the second detector at summed energies between 0.600 and 0.720 MeV, and b) without coincidence requirement for energies between 0.290 and 0.400 MeV.

4-2a and 4-2b the high relative efficiency in the left column of elements steadily decreases to the lowest relative efficiencies in the right column. To limit artifacts in the reconstructed images, the counts from each HPGe element were divided by the relative efficiency from Figure 4-2b before backprojection.

4.2 Second-Detector Array

The RCC second detector consist of 16 thallium-activated sodium iodide (NaI(Tl)) scintillating elements. The active volume of each NaI element is a crystal in the shape of a right circular cylinder, 19.1 mm in diameter by 50.8 mm in length. The NaI crystals are optically coupled to individual, 38 mm diameter RCA 6199 photomultiplier tubes (PMTs). These elements were originally used in the SPRINT circular ring tomograph [Wil79].

The 16 second-detector elements are arrayed in a ring, equally spaced in azimuthal angle. The small circular face of each element is perpendicular to a line from its center to the center of the first-detector array. The geometry is illustrated in Figures 2-4 and 3-7. Each element has the same central scattering angle θ_c and center-to-center distance d also illustrated in Figure 2-4.

Positive high voltage is supplied to all of the PMTs through a ribbon cable from a single voltage divider circuit. The divider uses an active network of high voltage transistors to maintain a constant potential on the anode and all dynodes of each PMT at high count rates. A diagram of this circuit is included in Appendix B as Figure B-1.

Due to their inherently poorer energy resolution, the NaI array elements make a much larger contribution to the RCC summed energy resolution than the HPGe array elements, so knowledge of their resolution is more important. The energy resolution of all 16 elements was measured at six energies ranging from 0.511 to 1.691 MeV. The measured FWHM energy resolution of all 16 elements, at each energy, is tabulated in Appendix B as Table B-2. To represent the resolution of the second detector as a whole, R_2 , the average of the 16 measured resolutions at each energy, has been fit to an equation of the form

$$R_2 = \frac{\sqrt{b_1 + b_2 E}}{E} \quad (4-3)$$

as suggested by Knoll [Kno89], where R_2 is the ratio of the pulse height peak FWHM to its amplitude. Values of $b_1=8.861 \times 10^{-4}$ and $b_2=5.496 \times 10^{-3}$ were found by performing a linear least squares fit on a graph of $(R_2 E)^2$ versus E where E is in MeV. Results of this fit are shown in Figure 4-3. Resolution in the individual elements ranged from 8.9% to 11.7% at 0.662 MeV.

The intrinsic efficiency of the NaI elements ϵ_{ip2} is an important parameter in the efficiency of the Compton camera. In Chapter II, a parametric model of ϵ_{ip2} is developed.

Figure 4-3. Graph of $(R_2 E)^2$ versus E , in MeV, to determine fit of average energy resolution, R_2 , of the 16 NaI elements as a function of deposited energy, E .

Measurements of ϵ_{ip2} for a single NaI crystal at several energies are shown in Appendix A along with a fit of the parametric model to the measured data.

4.3 Support and Alignment System

In this section the system used to support and align the two detectors is described, and estimates are made of the accuracy with which the backprojection direction $\hat{u}$ can be determined. A pair of carts are used to support the two detectors, align them relative to one another and align them to the source to be imaged. Figure 4-4 is a photograph showing the RCC with the HPGe array on its cart positioned in front of the NaI ring array, also mounted on its cart. The alignment and support system is designed to allow adjustment of a broad range of many camera parameters, so that many camera geometries can be investigated.

4.3.1 First-Detector Array

The cart for the first detector (i.e. the HPGe array) is made of steel beams in the shape of a cube, 91 cm on a side, mounted on casters. A photograph of the first detector mounted on its cart is shown as Figure 4-5. The detector's liquid nitrogen dewar rests on a leveling table, which in turn is placed on a shelf extending from the bottom of the cube-shaped cart. The leveling table is used to adjust the array to be plumb and level. The cart top is covered with a 1.0 cm thick aluminum plate. The plate is a 91 cm by 91 cm square with a 46 cm wide by 13 cm deep tab, extending from one side of the square. A semicircle with a radius of 1.6 cm is cut in the tab on the center line of the plate; the detector cryostat neck fits tightly in this semicircle to hold the detector in the proper position relative to the cart.

The HPGe array is positioned so that it is parallel to the front edge of the aluminum plate. The uncertainty in this positioning is approximately ± 0.15 cm over 25 cm for an angular uncertainty $\Delta\theta_y$ of $\pm 0.35^\circ$. The angular accuracy in setting the HPGe array to be plumb $\Delta\theta_{x1}$ is $\pm 0.4^\circ$. These angular uncertainties can be thought of as the difference in direction between a vector normal to the face of the HPGe array and the normal vector of a vertical plane passing through the front edge of the first-detector cart plate. Figure 4-6 is a sketch illustrating this geometry. The origin of a Cartesian coordinate system is defined so that the vertical plane passing through the front edge of the cart is at $z = 0$. The line defining $x = 0$, $y = 0$ passes orthogonally through the center of the HPGe array.

The angular uncertainty term $\Delta\theta_{y1}$ represents rotations about the y axis, and $\Delta\theta_{x1}$ is a rotation about the front edge of the cart top, parallel to the x axis. The vertical distance from the cart top to the center of the HPGe array y_c is 23.5 cm. The top edge of the array can be leveled to an accuracy of $\pm 0.4^\circ$. This angular accuracy $\Delta\theta_{z1}$ is the difference between direction of the normal to the top face of the array and the y axis, and represents a rotation about a line parallel to the z axis with $x = 0$ and $y = -y_c$.

Figure 4-4. Photograph of the RCC viewed from front.

Figure 4-5. Close up photograph of RCC first detector and cart.

Once the first detector is aligned with the cart, the assumed position of the center of a given element in the array is (x_{i1}, y_{i1}, z_{i1}) , and its position uncertainty can be written as a vector e_{i1} with components, Δx_{i1} , Δy_{i1} and Δz_{i1} . These uncertainty components can be computed from the angular uncertainties described above. As will be shown, similar angular alignment uncertainties also occur with the second detector. Here, a general relation between the position uncertainties and angular uncertainties is derived. In each position uncertainty component, there are cross terms between the angular uncertainties; however, the cross terms are of order $\sin^2(\Delta\theta)$, which for the small angular uncertainties can be neglected relative to the other terms of order $\sin(\Delta\theta)$. By neglecting the cross terms, each position uncertainty component can be separated into independent pieces corresponding to the different angular uncertainties. Taking the general x coordinate uncertainty Δx_i for example, it can be separated into uncertainties, $\Delta x_i^{(x)}$, $\Delta x_i^{(y)}$, and $\Delta x_i^{(z)}$, due to $\Delta\theta_x$, $\Delta\theta_y$ and $\Delta\theta_z$, respectively.

$$\begin{aligned}\Delta x_i &\equiv \Delta x_i^{(x)} + \Delta x_i^{(y)} + \Delta x_i^{(z)} \\ &\equiv (x_i^{(x)} - x_i) + (x_i^{(y)} - x_i) + (x_i^{(z)} - x_i)\end{aligned}\quad (4-4)$$

The same relation can be used for position uncertainty components Δy_i and Δz_i . In Equation (4-4), the variables $x_i^{(x)}$, $x_i^{(y)}$ and $x_i^{(z)}$ are the new positions of x_i after rotation by $\Delta\theta_x$, $\Delta\theta_y$ or $\Delta\theta_z$.

Rotating the point (x_i, y_i, z_i) about any line parallel to the x axis, where $y = y_0^{(x)}$ and $z = z_0^{(x)}$, by the angle $\Delta\theta_x$ results in the new position $(x_i^{(x)}, y_i^{(x)}, z_i^{(x)})$.

$$\begin{aligned}x_i^{(x)} &= x_i \\ y_i^{(x)} - y_0^{(x)} &= (y_i - y_0^{(x)}) \cos(\Delta\theta_x) - (z_i - z_0^{(x)}) \sin(\Delta\theta_x) \\ z_i^{(x)} - z_0^{(x)} &= (y_i - y_0^{(x)}) \sin(\Delta\theta_x) + (z_i - z_0^{(x)}) \cos(\Delta\theta_x)\end{aligned}\quad (4-5)$$

Rotating (x_i, y_i, z_i) about any line parallel to the y axis, where $x = x_0^{(y)}$ and $z = z_0^{(y)}$, by the angle $\Delta\theta_y$ results in the new position $(x_i^{(y)}, y_i^{(y)}, z_i^{(y)})$.

Figure 4-6. Sketch illustrating alignment of RCC relative to a vertical plane passing through the front edge of the first-detector cart plate.

$$\begin{aligned}
 x_i^{(y)} - x_0^{(y)} &= (z_i - z_0^{(y)}) \sin(\Delta\theta_y) + (x_i - x_0^{(y)}) \cos(\Delta\theta_y) \\
 y_i^{(y)} &= y_i \\
 z_i^{(y)} - z_0^{(y)} &= (z_i - z_0^{(y)}) \cos(\Delta\theta_y) - (x_i - x_0^{(y)}) \sin(\Delta\theta_y)
 \end{aligned} \tag{4-6}$$

The last rotation to consider is about a line parallel to the z axis by an angle $\Delta\theta_z$. The resulting position is $(x_i^{(z)}, y_i^{(z)}, z_i^{(z)})$.

$$\begin{aligned}
 x_i^{(z)} - x_0^{(z)} &= (x_i - x_0^{(z)}) \cos(\Delta\theta_z) - (y_i - y_0^{(z)}) \sin(\Delta\theta_z) \\
 y_i^{(z)} - y_0^{(z)} &= (x_i - x_0^{(z)}) \sin(\Delta\theta_z) + (y_i - y_0^{(z)}) \cos(\Delta\theta_z) \\
 z_i^{(z)} &= z_i
 \end{aligned} \tag{4-7}$$

Equations (4-4) to (4-7) can be combined to give general position uncertainties due to the general angular uncertainties.

$$\begin{aligned}
\Delta x_i &\equiv -(x_i - x_0^{(y)})[1 - \cos(\Delta\theta_y)] - (x_i - x_0^{(z)})[1 - \cos(\Delta\theta_z)] \\
&\quad - (y_i - y_0^{(z)})\sin(\Delta\theta_z) + (z_i - z_0^{(y)})\sin(\Delta\theta_y) \\
\Delta y_i &\equiv (x_i - x_0^{(z)})\sin(\Delta\theta_z) - (y_i - y_0^{(x)})[1 - \cos(\Delta\theta_x)] \\
&\quad - (y_i - y_0^{(z)})[1 - \cos(\Delta\theta_z)] - (z_i - z_0^{(x)})\sin(\Delta\theta_x) \\
\Delta z_i &\equiv -(x_i - x_0^{(y)})\sin(\Delta\theta_y) + (y_i - y_0^{(x)})\sin(\Delta\theta_x) \\
&\quad - (z_i - z_0^{(x)})[1 - \cos(\Delta\theta_x)] - (z_i - z_0^{(y)})[1 - \cos(\Delta\theta_y)]
\end{aligned} \tag{4-8}$$

At small angles, $1 - \cos(\theta) \approx \sin^2(\theta)/2$, so the terms multiplied by $1 - \cos(\Delta\theta)$ in Equation (4-8) may also be dropped to give

$$\begin{aligned}
\Delta x_i &\equiv -(y_i - y_0^{(z)})\sin(\Delta\theta_z) + (z_i - z_0^{(y)})\sin(\Delta\theta_y) \\
\Delta y_i &\equiv (x_i - x_0^{(z)})\sin(\Delta\theta_z) - (z_i - z_0^{(x)})\sin(\Delta\theta_x) \\
\Delta z_i &\equiv -(x_i - x_0^{(y)})\sin(\Delta\theta_y) + (y_i - y_0^{(x)})\sin(\Delta\theta_x)
\end{aligned} \tag{4-9}$$

To determine position uncertainties in the first-detector array, the following substitutions are made in Equation (4-9): $\Delta\theta_x = \Delta\theta_{x1}$, $y_0^{(x)} = -y_c$, $z_0^{(x)} = 0$, $\Delta\theta_y = \Delta\theta_{y1}$, $x_0^{(y)} = 0$, $z_0^{(y)} = 0$, $\Delta\theta_z = \Delta\theta_{z1}$, $x_0^{(z)} = 0$, $y_0^{(z)} = -y_c$. The largest uncertainty vector e_{i1} occurs for an element furthest from the array center and the cart top. The assumed position of this element is (0.75, 0.75, 0) cm. Taking the positive angular uncertainties, $\Delta\theta_{x1} = 0.4^\circ$, $\Delta\theta_{y1} = 0.35^\circ$ and $\Delta\theta_{z1} = 0.4^\circ$, gives the result $e_{i1} = -0.17\hat{i} + 0.01\hat{j} + 0.17\hat{k}$ cm. The magnitude of e_{i1} in this example is 0.24 cm. The magnitude of the uncertainty vector for the other HPGe elements will be less than or equal to this.

To position sources relative to the HPGe array, the first-detector plate is marked in polar coordinate grid with the center of the cryostat neck hole as its origin. When properly aligned the origin of the polar grid is coincident with the origin of the x-z plane defined in Figure 4-6. To locate a desired source position, a plumb bob is placed over the appropriate polar grid coordinates, and the source is placed against the plumb line at the

proper height above the plate. The uncertainty in positioning the source is approximately ± 0.15 cm in each direction.

4.3.2 Second-Detector Array

Figure 4-7 is a photograph of the NaI array, along with its cart and other mechanical support equipment. Each NaI element and an associated pulse processing circuit board are mounted on a "sled". (The pulse processing electronics are described in Section 4.4.) Each sled is mounted on a rail, which is in turn mounted on a 46 cm radius by 0.6 cm thick disk. These rails lie along radial lines from the center of the disk. Drawings of this sled and rail are included in Appendix B. (Sleds, rails and disk are all made from aluminum.) At its center, the disk is attached to a 1.3 cm diameter steel axle, and the axle is mounted to two horizontal beams via sleeve bearings. The bearings allow the axle and disk to be rotated azimuthally. A particular azimuthal position can be held using friction brakes mounted to a horizontal beam on either side of the disk.

Figure 4-7. Photograph of RCC second detector and cart.

The second-detector cart is a parallelepiped, 183 cm high by 130 cm wide by 69 cm deep, made of steel beams and mounted on casters. The height of the disk axle is adjusted by moving the beams on which it rests. Thus, the center of the axle can be moved to roughly the same height as the center of the first-detector array.

The position of the NaI ring array as a whole is set by the position of the second-detector disk. The orientation of this disk is defined relative to the same vertical plane (at $z = 0$) used to define the orientation of the HPGe array. The desired alignment has both detectors parallel to the $z = 0$ plane. Also, for the case where the azimuthal angle between the x axis and NaI element 0 (ϕ_0) is equal to zero, a radial line passing through the middle of the uppermost element is parallel to the y axis when aligned. This radial line is then parallel with the normal to the top edge of the HPGe array.

The horizontal alignment of the second-detector disk with the first-detector cart is accurate to ± 0.3 cm in 75 cm for an angular uncertainty, $\Delta\theta_{y2}$, of $\pm 0.25^\circ$. $\Delta\theta_{y2}$ is measured about the y -axis shown in Figure 4-6. The accuracy for setting the disk to be plumb is $\pm 0.4^\circ$, which is the angular uncertainty $\Delta\theta_{x2}$ for rotation about the x axis. The accuracy for setting the azimuthal rotation angle of the disk about a line parallel to the z axis passing through the center of the ring $\Delta\theta_{z2}$ is $\pm 0.25^\circ$ (or ± 0.2 cm at the edge of the 46 cm radius disk).

Position uncertainty of the NaI elements relative to the center of the ring due to the angular alignment uncertainties described above can be computed. The desired distance between the center of the HPGe array and the center of each NaI element is d , and the desired central scattering angle is θ_c . (See Figure 2-4.) Thus, the desired position of the ring center in the Cartesian coordinate system of Figure 4-6 is $(0, 0, -d\cos(\theta_c))$, and the ring radius is $d\sin(\theta_c)$. For an element at azimuthal angle ϕ , the desired position is $(d\sin(\theta_c)\cos(\phi), d\sin(\theta_c)\sin(\phi), -d\cos(\theta_c))$. The components of a position uncertainty vector, $\mathbf{e}_{i2}$, for this element are Δx_{i2} , Δy_{i2} and Δz_{i2} . These components can be computed for angular uncertainties about the center of the ring array using Equation (4-9) and making

the following substitutions: $\Delta\theta_x = \Delta\theta_{x2}$, $y_0^{(x)} = 0$, $z_0^{(x)} = -d \cos(\theta_c)$, $\Delta\theta_y = \Delta\theta_{y2}$, $x_0^{(y)} = 0$, $z_0^{(y)} = -d \cos(\theta_c)$, $\Delta\theta_z = \Delta\theta_{z1}$, $x_0^{(z)} = 0$, $y_0^{(z)} = 0$. For a numerical example, consider an NaI element at $d = 30\text{cm}$, $\theta_c = 60^\circ$, and $\phi = 0^\circ$. The value of e_{12} is then, $0\hat{i} + 0.07\hat{j} - 0.07\hat{k}$ cm.

Uncertainties in the angular orientation of both detectors relative to a vertical plane through the front edge of the first-detector cart have been quantified, and related to position uncertainties in the individual elements. Uncertainty in the position of the second-detector center relative to the origin of the coordinate system must also be considered. The desired position of the center of the ring is $(0, 0, -d\cos(\theta_c))$, and the uncertainty in measuring the position components are $\Delta x_{c2} = \pm 0.25$ cm, $\Delta y_{c2} = \pm 0.25$ cm and $\Delta z_{c2} = \pm 0.3$ cm. As with the other position uncertainties, an uncertainty vector e_{c2} is defined from these three components.

Total uncertainty in the relative position of an element in the second detector to an element in the first detector $e_{1:2}$ is a function of the three uncertainty vectors e_{i1} , e_{i2} and e_{c2} . To set an upper bound on $e_{1:2}$, the vector $e_{\max}$ is defined by summing the absolute values of e_{i1} , e_{i2} and e_{c2} component by component.

$$e_{\max} = (|\Delta x_{i1}| + |\Delta x_{i2}| + |\Delta x_{c2}|)\hat{i} + (|\Delta y_{i1}| + |\Delta y_{i2}| + |\Delta y_{c2}|)\hat{j} + (|\Delta z_{i1}| + |\Delta z_{i2}| + |\Delta z_{c2}|)\hat{k} \quad (4-10)$$

The uncertainty vector $e_{1:2}$ contributes to angular uncertainty in the backprojected direction $\hat{u}$. The angular uncertainty in $\hat{u}$ is equal to the angular uncertainty in the vector from the center of the first-detector element to the center of the second-detector element u_f where $\hat{u} = -u_f/|u_f|$. The contribution of $e_{1:2}$ to angular uncertainty in u_f is proportional to the dot product between the assumed direction u_f and the unknown true direction $u_f + e_{1:2}$. This angular uncertainty $\Delta\alpha_{1:2}$ can be computed from:

$$\cos(\Delta\alpha_{1:2}) = \frac{\mathbf{u}_f \cdot (\mathbf{u}_f + \mathbf{e}_{1:2})}{|\mathbf{u}_f| |\mathbf{u}_f + \mathbf{e}_{1:2}|} \quad (4-11)$$

If $e_{1:2} \ll u_f$ an upper bound on $\Delta\alpha_{1:2}$ can be found by assuming that $e_{1:2}$ is orthogonal to u_f and setting $|e_{1:2}| = |e_{\max}|$. Continuing the numerical examples from above, take $d = 30\text{ cm}$, $\theta_c = 60^\circ$, and $\phi = 0^\circ$ to describe the assumed position for the second-detector element and $(0.75, 0.75, 0)\text{ cm}$ for the first-detector element. In this case, $u_f = 25.23\hat{i} - 0.75\hat{j} - 15.00\hat{k}\text{ cm}$ and $|e_{\max}| = 0.60\text{ cm}$. To compute an upper bound, make the uncertainty vector approximately orthogonal to u_f by redefining $e_{1:2} = 0.6\hat{j}\text{ cm}$, yielding $\Delta\alpha_{1:2} = 1.2^\circ$.

To align the NaI elements relative to the center of the NaI ring array, the radial position of each sled can be varied over a range of 15 cm to 35 cm from the disk center. The base of each sled has a friction brake to hold the element at any desired radial position between the extremes. Angle guides on the sleds allow the angle of each NaI element to be varied continuously between 0° and 75° , to match the angle of the element axis with the central scattering angle θ_c . The flexibility of the RCC alignment system results in a rather long, tedious alignment process. For example, changing the system to a different θ_c can take as long as 1 hour for two workers.

A 0.35 m radius alignment disk made of Plexiglas can be mounted on the axle to facilitate positioning of each element. This disk has radial lines inscribed every 22.5° , and inscribed circles with radii from 10 to 35 cm at 5 cm intervals. The radial position of each element is set with an accuracy of $\pm 0.2\text{ cm}$ using the alignment disk, and the azimuthal position and distance from the aluminum disk along the z-axis can be checked and adjusted to within $\pm 0.2\text{ cm}$. (Here, the second-detector ring axis and the z-axis are assumed to be aligned.) These spatial uncertainties are Δx_{er2} , Δy_{er2} and Δz_{e2} , respectively. Uncertainties Δx_{er2} and Δy_{er2} for an element at azimuthal angle ϕ are related, by a

rotation through $-\phi$, to position uncertainties along the basis directions, defined as Δx_{e2} and Δy_{e2} .

$$\begin{aligned}\Delta x_{e2} &= \Delta x_{er2} \cos(\phi) - \Delta y_{er2} \sin(\phi) \\ \Delta y_{e2} &= \Delta x_{er2} \sin(\phi) + \Delta y_{er2} \cos(\phi)\end{aligned}\tag{4-12}$$

The orthogonal position uncertainties Δx_{e2} , Δy_{e2} and Δz_{e2} define the components of the uncertainty vector $\mathbf{e}_{e2}$, with magnitude 0.47 cm. As with the uncertainties in aligning the two detectors, an angular uncertainty, $\Delta\alpha_{e2}$, in the direction of $\mathbf{u}_f$ due to $\mathbf{e}_{e2}$ can be computed from Equation (4-11), with $\mathbf{e}_{1:2}$ replaced by $\mathbf{e}_{e2}$. The position uncertainty of each NaI element is random relative to the position of the NaI ring center, and the other NaI elements. The contribution of $\Delta\alpha_{e2}$ to the total angular uncertainty in $\mathbf{u}_f$ (and thus $\hat{\mathbf{u}}$) is a quadrature sum with $\Delta\alpha_{1:2}$,

$$(\Delta\alpha)^2 = (\Delta\alpha_{1:2})^2 + (\Delta\alpha_{e2})^2\tag{4-13}$$

where $\Delta\alpha$ is the total angular uncertainty. To complete the numerical example of the alignment uncertainty, again take $d = 30$ cm, $\theta_c = 60^\circ$, and $\phi = 0^\circ$ for the second-detector element position and (0.75, 0.75, 0) cm for the first. An upper bound on $\Delta\alpha_{e2}$ can be approximated by defining $\mathbf{e}_{e2} = 0.47\hat{\mathbf{j}}$ cm. Using Equation (4-11) and the same approximations used to bound $\Delta\alpha_{1:2}$ yields $\Delta\alpha_{e2} \leq 0.90^\circ$. Finally, using $\Delta\alpha_{1:2} \leq 1.1^\circ$ from the earlier example and Equation (4-13), $\Delta\alpha$ is found to be less than or equal to 1.4° .

4.4 RCC Data Acquisition Electronics

In this section the data acquisition electronics used in the RCC are described. All of these electronic units were designed by V. Jordanov [Jor94].

Each HPGe array element and each NaI array element has its own pulse processing board. The same board design is used for both types of detectors; a block diagram of this circuit board is shown in Figure 4-8. Inputs to the circuit come from either an HPGe element preamplifier or the anode of a NaI element PMT. The input signal is

passed through a two-stage amplifier. For the HPGe elements only, the signal is first processed by a pole-zero cancellation circuit to shorten the preamplifier tail pulse and correct for distortion in following shaping circuits [Kno89]. In the case of the NaI elements, the first stage of the amplifier (A1) converts the current pulse from the PMT anode into a voltage pulse. A1 has an integration time constant of 24 ns and serves as a coarse gain control on the pulse amplitude. Fine gain amplification is accomplished with a linear amplifier (A2). A baseline restorer circuit (BLR) associated with A2 corrects for baseline shift introduced by decoupling capacitors, or drifts in A2. A detailed schematic diagram of A1, A2 and BLR are shown in Figure B-2 of Appendix B.

The linear output from A2 is split between a "spectroscopy branch" and a "timing branch". The first block in the spectroscopy branch represents a shaping amplifier (A3) with shaping times variable from 1 to 1.5 μ s. The output of A3 is passed to an 8-bit, flash peak digitizer (PD). Differential nonlinearity of the digitizer in PD is typically less than 25% (maximum 50%). The digital output data from PD is sent over a parallel bus to an output data buffer, where it is stored until read by an external circuit.

Figure 4-8. Block diagram of RCC pulse processing electronics [Jor94].

In the timing branch, the signal first passes to a lower level discriminator (LLD). When the fast linear signal exceeds the threshold it enables the constant fraction discriminator (CFD) and all of the subsequent electronics. In CFD, time pick-off from the linear signal is performed, and the leading edge of its logic signal output represents the time of occurrence of the event. Next, the output of CFD is delayed by an 8-bit digitally controlled delay line. The delay is thus adjustable to 1 part in 255 at approximately 1 ns intervals, with an 8 position dip switch. The delayed timing signal is sent to a gate generator (GATE), and its output holds PD open long enough for the maximum of the shaped pulse from A3 to be properly determined. The delayed timing signal is also passed to a coincidence circuit for the NaI elements. Detailed schematic diagrams of the spectroscopy and timing branches are included in Appendix B as Figures B-3 and B-4, respectively. A schematic of power distribution circuits on the pulse processing board is also included as Figure B-5.

For the HPGe elements, the timing signals from all elements are sent to an encoder that generates a timing signal associated with the first element that fires. This first-detector encoder also generates a 4-bit digital position code indicating which element fired first. The timing signal from the first-detector encoder is then passed to an external digital gate and delay generator. Figure 4-9 is a simple block diagram of the entire RCC acquisition system, where the first-detector encoder and the digital gate and delay (DGD) generator are shown. DGD has an 8-bit digital delay line, which can delay the signal in approximately 1 ns intervals. Another 8-bit delay line also allows the gate width to be digitally adjusted in 1 ns increments. Output from DGD is a timing window (tw) that is applied to the coincidence circuit of each NaI pulse processing board. The coincidence is determined using an edge triggering mode. When the leading edge of the NaI element time signal falls within tw, a coincidence is recorded.

As illustrated in Figure 4-9, pulse height data from the buffer of each second-detector element in coincidence with tw is passed to a "mailbox" circuit. The mailboxes

Data from all of the NaI elements in coincidence with a given HPGe event are recorded. Storing all of the second-detector data in this way limits losses of valid scattering events due to simultaneous random coincidences in other elements. As a result, between 1 and 16 event words may be formed for a given coincidence. If more than one event word is formed for a coincidence, they are passed in queue fashion to the output pipeline. To differentiate between different coincidence events from the first detector, the signal ce is also passed to a toggle-trigger, which toggles the highest bit in fdp. Multiple second-detector events are treated as independent events. Some of these events are erroneous but most are eliminated during data analysis with summed energy windows. A new first-detector coincidence is indicated in the data stream when this bit changes. The output pipeline is a 4-byte wide by 512 event deep buffer. Data in the pipeline are read into a personal computer via a 16-bit digital input/output (DIO) board. The acquisition software, which controls data reading operation, is described in Section 4.5.

An 8-bit DIO, shown in Figure 4-9, is used to control a few important parameters of the acquisition electronics. Specifically, outputs from each element of both detectors may be suppressed if their data are not being used for some reason. By turning the detectors off when not in use, dead time from useless events is limited. Also calibrations and diagnostic measurements on individual detector elements may be performed. Delay time in the DGD is also remotely controlled. Control of this delay time allows a coincidence-delay curve [Kno89] to be generated automatically with a software routine described in Section 4.5. Automating the coincidence-delay curve measurement makes it possible to regularly check and adjust the DGD delay, allowing use of narrower time windows and thus reduced random background count rates.

The interface control has three manually selectable modes. The first mode records data only when the two detectors are in coincidence. This mode, used to record image data, has been described above. In a second mode, the coincidence requirement is overridden and all recorded pulse heights from the second-detector array are recorded. In

this "NaI singles" mode, pulse height calibration of the NaI elements can be performed. The single event count rate in the second-detector array may also be monitored with the NaI singles mode by passing the signal ce to a fast external counter. For the third mode, the coincidence requirement is again overridden, but pulse heights from the first-detector array are recorded. Pulse height calibration of the HPGe elements can be performed in this "HPGe singles" mode. The single event count rate of the first-detector array may be monitored by sending the signal tw to a fast external counter, and this measurement is unaffected by the mode of the interface controller.

4.5 RCC Data Acquisition Software

A description of the data acquisition software (referred to hereafter as the RCC application) is presented in this section. The RCC application is implemented on a personal computer (PC) with an Intel 486DX microprocessor running at 33 MHz clock speed. The PC has 8 Mbytes of memory and runs Microsoft Windows v3.1. The application has a graphical user interface (GUI) that is compatible with Windows v3.1 and has been written using Borland C++ v3.1, from which many of the basic GUI routines are drawn. In addition to acquiring data, the RCC application has routines (or "tools") for displaying histograms, counts in individual elements and images among others. Tools for calibrating and performing diagnostics are also incorporated. Additionally, most of the tools needed for data analysis and image reconstructing are included in the RCC code. Acquisition, display, calibration, and diagnostic routines were written by N. Petrick and A. J. Caird [Pet94].

The primary function of the RCC application is to acquire and store data from the output pipeline of the acquisition electronics via a 16-bit DIO. (See Figure 4-9.) A timer interrupt is used to command the computer to read all of the data in the output pipeline every 0.25 s. The data is stored to memory in list mode, as it is read from the pipeline. Data from each event consists of four 8-bit bytes. The lowest four bits of the first byte

contain the position code of the first-detector element. The highest bit in this byte indicates whether these data represent a new first-detector event. The second byte contains the 8-bit pulse height code from the first detector. The lowest four bits of the third byte hold the second-detector position code and the last byte holds the second-detector pulse height.

The computer clock time can be written into the data stream if desired. These "time stamps" occupy 8 bytes in the data stream, and the first two bits of the first byte are set to one to differentiate them from event data. The time stamps allow data to be sorted based on the acquisition time. Thus corrupted event data can be separated from useful data if a malfunction occurs during a data run.

The data stream is written to the computer hard drive at regular intervals (normally every 30 minutes) to minimize data loss in the event of a system failure. After a data acquisition run that ends normally, the data stream can be written to a disk in a binary format with a header describing the total number of counts acquired, the start time and the run duration in real time. This binary file can be read back by the RCC application for analysis at a later time. Backup files are written with the same binary format.

The RCC application has tools that make pulse height calibration of all the elements in both detectors a fast and reliable operation. Frequent pulse height calibrations of the NaI elements were necessary because of significant gain drift in most of these elements. To calibrate, data is recorded from a gamma-ray source of known energy with the RCC interface control set to either of the singles modes. A histogram of the pulse height spectra from all elements in one detector can then be displayed. (This histogram can also be stored on a drive to make a compact record of the calibration.) A peak search algorithm is then used to identify a single peak in the pulse height spectrum of each element. The search algorithm is based on the first derivative of the histogram after applying a Gaussian smoothing. A more commonly used second-derivative algorithm

[Mar67] failed because of the differential nonlinearity in the peak digitizer. Relative intensity thresholds and peak positions can be specified to locate the peak of interest in multi-line spectra. After calibrating both detectors at two energies, the calibration is complete and a "pulse height calibration summary" may be written to disk. The summary file contains energies and channel numbers for each calibration point in all the detector elements.

To determine the proper delay of the time coincidence window (t_w in Figure 4-9) applied to the NaI coincidence circuits, a coincidence-delay curve is measured [Kno89]. A tool in the RCC application automatically acquires the data needed for this curve and displays it as a histogram. The software steps the digital delay code by a fixed interval over a range of delay codes. A unit delay is approximately 1 ns. At each delay time the software dwells for a fixed period and the coincident counts are recorded. The delay code interval, delay code range and delay time are all user-specified before the algorithm is started. An example coincidence delay curve is shown in Figure 4-10.

During coincident data acquisition at modest rates (less than 1000 counts per second), some diagnostic tools are available to monitor the RCC acquisition electronics. The counts in each element of both detectors are displayed as a color map on a schematic representation of the two detectors, and the counts in any element can also be displayed numerically. In addition, the coincident pulse height histograms of either detector may be displayed. Both of these displays are updated at intervals of 5 s or more.

The RCC application includes a tool which computes the summed energy of each event using the pulse height calibration described above. The summed energy spectrum can then be displayed as a histogram. This spectrum is important for data analysis, because the selection of events for image reconstruction is based on the summed energy. The summed energy histogram tool allows display of events from any combination of elements in the two detectors. With this tool, variations in the efficiency and resolution of individual elements can be investigated. For example, the relative counts shown in Figure

Figure 4-10. Example coincidence delay curve measured with a ^{137}Cs point source. Each unit of the delay code is approximately equal to 1 ns. The dashed line shows the delay time code, τ_d extracted from the data.

4-2a were computed from the counts in the summed energy spectrum of each HPGe element.

Data analysis tools are also incorporated into the RCC application. One tool selects events falling in a prescribed summed energy window and sorts them into equal scattering angle bins by detector element pairs. The algorithm for this tool is described in Section 5.2. The resulting angular binned data are the ray-sums (or projections) $\lambda_{\ell\phi k}$ used in Equation (3-39) to generate a laminogram from the data. Additionally, the application has the capability to add data from different runs, subtract background data, and interlace data from different second detector views once it is put in the ray-sum format. The ray-sums can be written to a binary file and read back in the application. In this form the data is much more compact than the list mode data stream.

With the RCC application, images can be reconstructed using the RFL algorithm. Laminogram formation, as computed using Equation (3-39), is implemented as a tool in

this application, and routines for ramp filtering laminograms using any of the filters of Section 3.4.2 are included. The apodizing functions can also be used to low-pass filter noisy images from the iterative reconstruction algorithms. Images can be displayed as false color or gray scale density plots. The false color map roughly follows the colors of a blackbody radiator. Finally, a tool is available to plot horizontal and vertical contours through a displayed image and show the intensity of any pixel in the image.

4.6 RCC System Performance

Four system parameters are described in this section: timing resolution, "summed energy resolution", dead time, and cross-talk between the HPGe array elements. These general parameters have not been included in previous sections on the various RCC subsystems because they are measures of performance for more than one of the subsystems.

In Figure 4-11, a time-to-amplitude converter (TAC) spectrum is shown for the time signal from the NaI detector versus the HPGe detector. This spectrum is generated by Compton scattering ^{137}Cs gamma rays over a wide range of angles from the HPGe detector into the NaI detector. The FWHM of the coincidence peak in Figure 4-11 is 12.7 ns, and the full width at one-tenth maximum (FWTM) is 31.4 ns. The large tail on the coincidence peak toward lower times is due to variable shape in the HPGe pulses, caused by variation in the electric field through the depth of the HPGe elements [Kno89].

To show that the tail on the low side of the coincidence peak is indeed due to the HPGe, timing spectra of a fast plastic scintillator versus a single NaI element and the HPGe detector array versus the same plastic scintillator are shown in Figures 4-12a and 4-12b, respectively. As above, the timing spectrum was generated by Compton scattering ^{137}Cs gamma rays from the plastic scintillator into the other detector over a wide range of angles. A commercial constant fraction discriminator (CFD) unit was used to pick off a time signal from the plastic scintillator with a time resolution of less than 1 ns FWHM. As

Figure 4-11. Time spectrum of ^{137}Cs gamma rays Compton scattered from first RCC detector array to second RCC detector array.

can be seen in Figure 4-12b, the low time tail is present in the coincidence peak between the HPGe detector and the plastic. The width of the HPGe-plastic peak is 10.3 ns FWHM and 28.9 ns FWTM. The coincidence peak between the plastic and the NaI element shows no tailing on the low side. The plastic-NaI peak width is 6.2 ns FWHM and 12.0 ns FWTM. The timing resolution of the NaI elements is related to their pulse height resolutions. The element used to generate Figure 4-12b has one of the worst pulse height resolutions (11.3% at 0.662 MeV). A spectrum measured using an NaI element with better pulse height resolution (8.9% at 0.662 MeV) yields a resolution of 3.9 ns FWHM when timed against the plastic scintillator. The NaI elements are timed against the plastic individually and not as a group because adjusting their delays to all match the plastic would have destroyed their synchronization with the HPGe timing signal.

As discussed in the development of the RCC efficiency model in Section 2.2, only events where the full incident gamma-ray energy is deposited in the camera are useful for

generating images. To select these full energy events, a window is set in the spectrum of summed energies about the incident gamma-ray energy. The width of the summed energy window is determined by the summed energy resolution of the camera. Thus, the summed energy resolution is of critical importance to the performance of the RCC.

The summed energy resolution ΔE can be expressed in terms of the resolution with which recoil electron energy is measured in the first detector ΔE_{re} and scattered gamma-ray energy is measured in the second detector $\Delta E_{\gamma}'$.

$$(\Delta E_s)^2 = (\Delta E_{re})^2 + (\Delta E_{\gamma}')^2 \quad (4-14)$$

The FWHM energy resolution of the first detector is given by ΔE_1 in Equation (4-1), and the FWHM energy resolution of the second detector is given by $R_2 E$ in Equation (4-3). Substituting ΔE_1 for ΔE_{re} and $R_2 E$ for $\Delta E_{\gamma}'$, in Equation (4-14) yields a formula for ΔE_s .

$$\begin{aligned} (\Delta E_s)^2 &= (\Delta E_1)^2 + (R_2 E_{\gamma}')^2 \\ &= (a_1 + a_2 E_{re}) + (b_1 + b_2 E_{\gamma}') \\ &= 10^{-6}(1.067 + 1.068 E_{re}) + 10^{-4}(8.861 + 54.96 E_{\gamma}') \end{aligned} \quad (4-15)$$

where E_{re} , E_{γ}' , and ΔE_s are in units of MeV. The contribution of the first detector is negligible even when the majority of the incident energy is deposited in the first detector. (The extreme case occurs when 3 MeV gamma rays undergo back scattering.) Further, ΔE_s can be expressed in terms of the incident gamma ray energy and the scattering angle. Inserting Equation (2-2) in (4-15) gives

$$\Delta E_s(E_{\gamma}, \theta) = 10^{-2} \sqrt{8.861 + 54.96 \left[\frac{E_{\gamma}}{1 + (E_{\gamma}/0.511)(1 - \cos \theta)} \right]} \quad (4-16)$$

a)

b)

Figure 4-12. Time spectra of ^{137}Cs gamma rays Compton scattered from a) a plastic scintillator into one element of the second RCC detector array, and b) a plastic scintillator into the first RCC detector array.

In Figure 4-13, a histogram of the summed energy spectra including all elements of both detectors is shown as an example. The summed energy histogram is from irradiation of the RCC with an ^{124}Sb point source located on the RCC ring axis. The central scattering angle of the RCC θ_c is 60° . The measured summed energy resolution of the 0.602 MeV peak ($I_\gamma = 98\%$) is 0.059 MeV FWHM, compared to a value of 0.056 MeV predicted from Equation (4-16). The resolution of the 1.691 MeV peak ($I_\gamma = 49\%$) is approximately 0.069 MeV FWHM, while Equation (4-16) yields a resolution of 0.066 MeV. The measured resolutions are larger than the calculated values, due in part, to calibration uncertainties in the NaI elements of approximately ± 1 channel, which in this case is 0.008 MeV. The calibration uncertainty is random from element to element thus broadening peaks in the summed energy histograms. The NaI elements also suffer significant gain drift during irradiation with sources that cause singles count rates of 10^4 counts per second and above. This effect is attributed to *fatigue* of the PMTs [Kno89]. At the start of an irradiation, gain drifts as high as 5% have been noted after a one hour run, with the gain reaching equilibrium after several hours of irradiation. Gain drift acts in concert with the calibration uncertainty to broaden summed energy peaks.

Dead time in the RCC is dominated by the single event count rate in the first detector. All elements in the first detector remain dead until the timing signal is passed from the first-detector encoder to DGD. (See Figure 4-9.) On the other hand, the coincidence circuits of the second-detector elements act independently of one another, and the dead time of an element depends only on the count rate in that element. In measurements performed to date, the average singles count rate in a second-detector element has been less than 15% of the count rate in the HPGe detector as a whole.

Dead time of the first detector, and thus the RCC, has been measured using the decaying source method. The single event count rate from the first-detector array has been recorded periodically over a 30 h span while the RCC was irradiated with a 2 mCi ^{24}Na source with a half life of 15 h. If the dead time is assumed to be nonparalyzable, the

Figure 4-13. Summed energy histogram measured from a ^{124}Sb point source with RCC central scattering angle θ_c of 60° .

following equation relates the measured count rate r ; the initial, true interaction rate g_0 ; the decay constant of the isotope λ_c , and the dead time τ [Kno89].

$$re^{\lambda_c t} = -g_0 \tau r + g_0 \quad (4-17)$$

By creating a plot with $re^{\lambda_c t}$ as the ordinate and r as the abscissa, a straight line should result. The y-intercept of the line is g_0 , and the dead time can be extracted from its slope, $-g_0 \tau$. Figure 4-14a shows a plot of $re^{\lambda_c t}$ versus r derived from measurements of the decaying ^{24}Na source and a linear least squares fit to the data. From the fit, g_0 is $8.71 \times 10^4 \text{ s}^{-1}$ and τ is $2.1 \text{ } \mu\text{s}$.

If a paralyzable model is used for the dead time, r , g_0 , λ_c , and τ are related by [Kno89]:

$$\lambda_c t + \ln(r) = -g_0 \tau e^{-\lambda_c t} + \ln(g_0) \quad (4-18)$$

As with the nonparalyzable model, τ is found from a plot, but now with $\lambda_c t + \ln(r)$ as the ordinate and $e^{-\lambda_c t}$ as the abscissa. The resulting straight line has a y-intercept of $\ln(g_0)$ and a slope of $-g_0 \tau$. In Figure 4-14b, the measured data from the decaying ^{24}Na source is graphed in the form of $\lambda_c t + \ln(r)$ versus $e^{-\lambda_c t}$ and fit. From this fit, g_0 is $8.69 \times 10^4 \text{ s}^{-1}$ and τ is $1.9 \mu\text{s}$.

Both the nonparalyzable and paralyzable models represent the measured dead time data adequately. The nonparalyzable model is used hereafter because the true interaction rate can be computed directly from the measured count rate and the dead time,

$$g = \frac{r}{(1 - r\tau)} \quad (4-19)$$

whereas the true interaction rate can only be computed iteratively from the paralyzable model.

$$r = ge^{-g\tau} \quad (4-20)$$

Measurements of single event rates in the first detector show that there is some cross-talk between the elements of that detector. Cross-talk occurs when a single interaction generates a pulse in two or more elements. In the RCC, a "cross-talk event" causes the first-detector encoder to fire on the first arriving pulse, and generate the position code for the element from which that pulse originated. The exact cause of cross-talk events has not been identified, but there are three possible mechanisms. First, an incident gamma ray could scatter in an element, escape, and interact in another element. Scattering could cause cross-talk between any two elements in the array; however from solid angle considerations, cross-talk between adjacent elements is most likely. A second cross-talk mechanism occurs when a recoil (or photo-) electron escapes from one element into an adjacent element. Because electrons continuously deposit energy as they travel, the cross-talk from electron escape will always involve adjacent elements. The fraction of

a)

b)

Figure 4-14. Graphs of measured data used for dead time characterization of first RCC detector array and linear least squares fits assuming a) non-paralyzable dead time model ($re^{\lambda_{ct}}$ vs. r) and b) paralyzable dead time model ($\lambda_{ct}t + \ln(r)$ vs. $e^{-\lambda_{ct}t}$).

electrons escaping an element depends strongly on the electron range and hence the energy of the incident gamma ray. A third possible cross-talk mechanism is capacitive coupling between the neighboring elements. An interaction near the edge of an element may cause fluctuations in the electric field of the adjacent element, and induce a pulse.

To quantify the fraction of all events which are cross-talk events, measurements were made of single event count rates in four adjacent elements occupying one quadrant of the HPGe array, while they were irradiated with a ^{137}Cs point source. First, the count rate was measured from each element with all other elements turned off. For element i , the total interaction rate g_i was computed from the measured count rate r_i and the dead time τ ($= 2.1 \mu\text{s}$), using Equation (4-19). An "expected" true interaction rate in the four elements combined $\bar{g}_c$ was found by summing the individual interaction rates.

$$\bar{g}_c = \sum_{i=1}^4 g_i \quad (4-21)$$

All four of the elements were then turned on and the singles count rate was measured without moving the source. A "measured" true interaction rate g_c was then computed. The expected interaction rate was found to be 15.6% higher than the measured rate. The difference in $\bar{g}_c$ and g_c is due to cross-talk between adjacent elements which share either an edge or a corner. To expand, when all of the adjacent elements were turned on, only one of the elements involved in a cross-talk event registered a count in the acquisition electronics; the pulse from the other element(s) was lost. When only one element was turned on, all of the events in an element, even those shared with an adjacent element via cross-talk, registered a count. (Of course, only pulses exceeding the LLD can register counts.) Table B-3 provides a summary of the data and calculations described above.

To ensure that the differences found between the expected and measured interaction rates was not due to an erroneous dead time, the measurement was repeated (at similar count rates) using the four elements at the outside corners of the HPGe array,

which are not adjacent to one another. With the four corner elements, $\tilde{g}_c$ was found to be only 1.5% higher than g_c . (See Table B-4 for a summary of this data.) Because the interaction rates closely agree when the four corner elements were used, the larger discrepancy between the interaction rates when the four adjacent elements were used can be attributed to cross-talk.

For the first-detector array as a whole, the fraction of cross-talk events is likely to be higher than for the four elements used above, because there are more common borders per element in the array (three edges per element) than in those four elements (two edges per element). Extrapolating based on the number of edges per element, the percentage of cross-talk events may be as high as 23% in the array. The cross-talk events are significant because they affect the efficiency (ϵ_{acc}) of the RCC. This is particularly true for the latter two mechanisms. An otherwise useful event may be lost because the recoil electron energy is shared between two first-detector elements, but the energy from only one is recorded and the summed energy will not match the incident gamma-ray energy. On the other hand, the capacitive coupling may cause a fast pulse in the adjacent element with little energy loss from the element where the interaction occurred. If the count is registered to the interaction element, the result may be a useful event whose summed energy falls in the desired window. Since the cross-talk mechanism has not been completely explained, the effect of cross-talk can be summarized by saying that some fraction of the cross-talk events, which make up 15.6% to 23% of the total events, are lost from the full energy peak in the summed energy spectrum of the camera.

CHAPTER V

RCC POINT RESPONSE

In this chapter, the point response of the RCC is studied. Techniques used to measure RCC point spread functions (PSFs), analyze data and reconstruct images of the PSFs are described in detail. These techniques are also used to measure and reconstruct images of the extended sources described in Chapter VI. After the techniques and analysis, a section is devoted to contaminating events, and where possible, measurements are compared to the analytic models of contaminating events from Section 2.4. Efficiencies are extracted from the measured PSFs and are compared to the analytic efficiency model from Section 2.2. Measured variations in efficiency with incident gamma-ray energy E_γ , central scattering angle θ_c , first-to-second detector separation distance d and polar angle of the point sources from the RCC ring axis α are compared to model predictions.

The RCC frequency response measured from a point source is compared to a simulation, and resolutions of measured and simulated PSFs are also compared. The FWHM angular resolution recovered with the different image reconstruction techniques is determined from the PSFs. Measures of spatial resolution other than FWHM are also presented because the PSFs may not be well characterized by a Gaussian or other common peaked function. The effect of image noise on the various resolution measures is studied, and the resolution from an off-axis point source is compared to an on-axis source. Measured variations in angular resolution with E_γ , θ_c , d and α are compared to model predictions. Finally, the RCC point response is compared to a hypothetical pinhole camera.

5.1 Measurement Methods

Successful PSF measurement or extended source imaging requires that several critical parameters either be set or calibrated before data are recorded. Among the first parameters set before imaging a particular source are the gains of the pulse processing board amplifiers (A1 and A2 in Figure 4-8). The gain of each element of the first-detector array is adjusted so that the Compton edge of the highest energy line falls between 90 and 95% of the full range of the peak digitizer (PD in Figure 4-8). It is important that the HPGe spectra be spread over as many of the available 256 channels in this way because the channel width is greater than the resolution of the HPGe elements given in Equation (4-2), even at the highest gain settings. The channel width thus dominates the energy resolution in the first-detector elements and, in turn, the energy component of the angular resolution ($\Delta\theta_e$) given in Equation (2-22).

Restrictions on the gain of the NaI elements in the second detector are less severe, because the resolution of the NaI elements, as specified in Equation (4-3), is always a factor of ten larger than the channel width, even at the lowest gains. NaI gain is adjusted so that the full energy peak of the highest energy line falls between 80 and 90% of the full range of PD.

The pulse height of the first-detector elements is calibrated with two gamma-ray lines spanning as much of the energy range as possible. Calibration data are measured with the data acquisition electronics in HPGe singles mode. Linear calibration parameters are computed automatically using the peak search and calibration tools of the RCC application described in Section 4.5. The gain of the HPGe elements is very stable, and recalibration is required only once per day during extended measurements. On the other hand, NaI elements suffer significant gain drifts due to fatigue of their PMTs. Data acquisition runs are typically limited to a duration of 4 hours or less so that the NaI elements can be recalibrated. Linear pulse height calibrations of the NaI elements are

accomplished using the same method as the HPGe elements except that the electronics are set in NaI singles mode.

After an initial calibration of the pulse heights, the LLD (refer to Figure 4-8) can be adjusted to give a desired lower energy threshold $E_{1\min}$. In the first-detector elements, $E_{1\min}$ imposes a lower bound on the scattering angle that can be measured for a particular E_γ . The scattering angle lower bound θ_ℓ can be expressed in terms of E_γ and $E_{1\min}$ by substituting $E_{1\min}$ for E_{re} in Equation (2-6).

$$\cos(\theta_\ell) = 1 - m_0 c^2 \left[\frac{E_{1\min}}{E_\gamma (E_\gamma - E_{1\min})} \right] \quad (5-1)$$

Equation (5-1) implies that each pair of first and second-detector elements has an associated cone with a half-angle θ_ℓ centered about the backprojection direction $\hat{u}$ where that pair of elements has zero sensitivity.

Figure 5-1 shows "blind spots" that result from projecting zero-sensitivity cones with half-angle $\theta_\ell = 15^\circ$ onto an orthogonal source plane at a distance z_s of 10 cm from the first detector. The shaded area in Figure 5-1 is the blind spot from the second-detector element at azimuthal angle $\phi = 0^\circ$, with $\theta_c = 60^\circ$, projected through the middle of the first-detector array. Blind spots from the other 15 second-detector elements, at azimuthal increments of 22.5° , are indicated with contour lines. Images of sources falling in one of the blind spots will be distorted and the resolution will be degraded by the absence of information in one (or more) of the second-detector elements. In Figure 5-1, there is a circular region with a radius (r_{df}) of 10 cm where each second-detector element has a nonzero sensitivity, and sources are imaged free of distortion. r_{df} can be expressed in terms of θ_c , θ_ℓ and z_s .

$$r_{df} = z_s \tan(\alpha_{\max}) \quad (5-2)$$

where,

Figure 5-1. Contours showing "blind spots" from the 16 second-detector elements, with the blind spot of the element at $\phi=0^\circ$ shaded. The RCC central scattering angle θ_c is 60° , the threshold scattering angle θ_t is 15° , and the source plane to detector spacing z_s is 10 cm.

$$\alpha_{\max} = \theta_c - \theta_t \quad (5-3)$$

The circle described by r_{df} is a simplification of the actual field of view (FOV) of the RCC. Unlike the center of the first-detector array used for backprojection in Figure 5-1, the quantization location of each element is not on the RCC ring axis. The FOV from each first-detector element will form an eccentric circle, and the FOV radius of the camera as a whole, r_{FOV} , is reduced by approximately the distance from the center to the outer corner of the first-detector array, h_1 .

$$r_{FOV} = r_{df} - h_1 \quad (5-4)$$

The lower energy threshold in the second-detector elements E_{2min} may place an upper bound θ_u on the measured scattering angles if E_{2min} is greater than the backscattered gamma-ray energy E_{bs} . Substituting, $E_\gamma - E_{2min}$ for E_{re} in Equation (2-6) gives

$$\cos(\theta_u) = \begin{cases} 1 - m_0 c^2 \left[\frac{E_\gamma - E_{2min}}{E_\gamma (E_{2min})} \right] & \text{if } E_{2min} > E_{bs} \\ -1 & \text{if } E_{2min} \leq E_{bs} \end{cases} \quad (5-5)$$

where,

$$E_{bs} = \frac{E_\gamma}{1 + 2(E_\gamma / m_0 c^2)} \quad (5-6)$$

To keep the FOV imposed by E_{2min} consistent with that imposed by E_{1min} , the value of E_{2min} must be set such that

$$\begin{aligned} \theta_u &= \theta_c + \alpha_{max} \\ &= 2\theta_c - \theta_l \end{aligned} \quad (5-7)$$

While Equation (5-7) ideally gives the same FOV from the second-detector threshold as that imposed by the first-detector threshold, the significant downward gain drift in the NaI PMTs acts to effectively increase E_{2min} . To prevent FOV reduction as a result of this gain drift, the minimum gain is estimated for a projected run, and E_{2min} is set to E_{bs} for the lowest energy E_γ of interest, based on this minimum gain,

$$E_{2min} = \left(\frac{G_{min}}{G_i} \right) E_{bs} \quad (5-8)$$

where G_{min} is the calibration slope at the minimum gain and G_i is the calibration slope when E_{2min} is set. The calibration zero offset is assumed to be 0 in Equation (5-8).

By FOV considerations alone, the values of $E_{1\min}$ and $E_{2\min}$ should be as low as possible. However, the lowest pulse heights in both detectors are filled with electronic noise and background radiation (such as X-rays). Setting the thresholds too low results in high random background rates with increased dead time and high rates for other corrupting event types.

Setting the coincidence timing between the two detectors requires that the timing signal delay in each element be synchronized. To achieve this synchronization, the time spectrum of each HPGe element is first measured versus a plastic scintillator.

(Equipment and the technique used for these measurements is described in Section 4.6.)

Delays of the HPGe timing signals are adjusted via the digital delay on the pulse processing board so that the centroids of the coincidence peaks match one another to within ± 2 ns. After adjusting all of the HPGe element delay times, the time spectrum of each NaI element is measured versus the entire HPGe array. Delay times of the NaI elements are then adjusted so that their coincidence peaks all match to within ± 2 ns.

Once the delay time of each element of both detectors is adjusted, the time spectrum of one detector versus the other is measured with all elements active; an example is shown in Figure 4-11. The width of the time coincidence window (signal $\underline{tw}$ in Figure 4-9) is adjusted to be approximately as wide as the FWTM of the coincidence peak in the two detector time spectrum. Nominally, $\underline{tw}$ was set to 30 ns during most data acquisition runs. Since the coincidence circuitry operates in an edge triggering mode, the width of $\underline{tw}$ is equal to the resolving time $2\tau_r$.

The final timing parameter to consider is τ_d , the delay time of $\underline{tw}$. To determine the proper τ_d , a coincidence delay curve is measured using the technique described in Section 4.5; an example curve is shown as Figure 4-10. From the coincidence delay curve, the delay times where the coincident count rate falls to one-fifth its peak value are computed, and τ_d is set to the average of these two times.

Mechanical alignment of the RCC and the source are the last critical adjustments that must be made prior to acquiring image data. The HPGe detector is first aligned relative to its cart and then the NaI detector elements are aligned relative to the second-detector disk. The two detector carts are then aligned with one another. Finally, the source is positioned relative to the RCC. Alignment techniques and uncertainties in the RCC alignment are detailed in Section 4.3.

Once the above critical parameters are set, the remaining steps in acquiring image data are straightforward. Before a data run is started, the total number of counts and the run duration, T , must be entered into the RCC application. The total number of counts that can be recorded during a single run is limited to approximately 1.1×10^6 by the memory of the computer. Fatigue-induced gain drift of the NaI element PMTs limits T to about 2 hours when the singles count rate in each NaI element is above 10^4 counts per second. At these high count rates, the pulse heights of the NaI elements must be recalibrated between each run. A short test run is done before an image data run to determine the coincidence count rate. From the test count rate, the limitations on T and the maximum total counts can be determined, and specified in the RCC application accordingly. The interval for writing time stamps into the data stream can also be specified, as well as whether the counts in each detector are to be displayed.

During a data run, all critical parameters are recorded in a laboratory notebook, and the counts are periodically monitored (if this feature has been specified) to ensure that each element is recording data. After the acquisition run ends normally, the data stream is stored to disk.

Multiple runs are often necessary to record enough data to form an image. As mentioned above, the NaI detectors must be recalibrated between each run by removing the imaging source and replacing it with standard sources. Upon completing the recalibration procedure, the standard sources are removed and the imaging source is restored to its original position.

When the imaging source is sufficiently intense, a significant fraction of the coincident events result from random (or chance) coincidences. The contribution of the random coincident events to the total event count rate can be measured by changing τ_d to "spoil" the coincidence. For the coincidence-delay curve in Figure 4-10, only random coincidence events are accepted when $\tau_d < 80$ ns or $\tau_d > 176$ ns. After spoiling the coincidence by adjusting τ_d , the count rate is tested and the steps followed to record image data are repeated to record random background data. The duration of the random background run (T_b) is set to minimize statistical uncertainty in the image data after the random background is subtracted away. For a fixed total measurement duration, the optimum value for T_b in terms of the image data duration, T_d is [Kno89]

$$T_b = T_d \sqrt{r_b/r_d} \quad (5-9)$$

where r_d and r_b are the coincidence count rates for recording image data and random background data, respectively.

The number of second-detector elements in the ring array M_ϕ needed to properly sample a source space for reconstruction with the RFL algorithm can be estimated using Equation (3-61) from Section 3.4.4. For spatially extended sources, M_ϕ is often greater than the 16 elements of the RCC. M_ϕ can be increased by recording image data with the second-detector ring rotated to several different azimuthal positions. The azimuthal angle of the array is specified by the angle ϕ_0 of NaI element number zero relative to the positive x-axis. For example, consider the case where M_ϕ is 64 and the angular interval between azimuthal samples is $360^\circ/64$ or 5.63° . Since the azimuthal angle between second-detector elements Φ_2 is 22.5° , image data are recorded at four different values of ϕ_0 : 0° , 5.6° , 11.3° and 16.9° , to uniformly distribute the 64 azimuthal samples. Durations of the image data runs T_d are chosen so that the product of the number of gamma rays emitted during the run and the live fraction ($1-m\tau$) is approximately the same at each ϕ_0 .

5.2 Data Analysis

This section is devoted to taking measured image and random background data stored in list mode and generating a laminogram on a square source plane. Steps in this data analysis process include: selecting events based on their summed energies, binning the data according to scattering angle in the form of ray-sums, combining data from different measurements, and forming a laminogram from the ray-sums.

5.2.1 Selecting Full Energy Events and Forming Ray-Sums

The first step in forming an image from RCC data is selecting events whose summed energy E_s falls in a specified window. The upper and lower bounds on the energy window E_h and E_l are related to the incident gamma-ray energy E_γ and a width factor E_w .

$$\begin{aligned} E_l &= E_\gamma - E_w \\ E_h &= E_\gamma + E_w \end{aligned} \tag{5-10}$$

E_w is a function of the summed energy resolution ΔE_s from Equation (4-16) evaluated at E_γ and θ_c

$$E_w = (\Delta E_s + \bar{m})1.1 \tag{5-11}$$

where $\bar{m}$ is the average energy width of one channel in the NaI detectors. Assuming the summed energy peak has a Gaussian shape and $\Delta E_s = E_w$, the energy window encompasses 98% of the peak area. The extra channel $\bar{m}$ in Equation (5-11) is added because peak searches on the NaI pulse height spectra are only accurate to ± 1 channel, and the factor of 1.1 is used to account for broadening from poorly resolved, low scattering angle events and gain drift in the NaI elements. Figure 5-2 shows the energy window used to select events from the 0.603 MeV line of an ^{124}Sb point source. For this measurement θ_c is 60° , and ΔE_s is 0.055 MeV. The value of $\bar{m}$ is 0.0074 MeV and the resulting E_w is 0.068 MeV.

In order to test whether the summed energy of a particular event falls in the specified summed energy window, the energy deposited in each of the two elements is computed from their respective pulse heights and calibrations. Computing the energies is complicated by the differential nonlinearity of the peak digitizers in the pulse processing boards. For the HPGe elements the differential nonlinearity was measured to an accuracy of $\pm 3\%$ per channel. The measured differential nonlinearities and the pulse height calibrations are combined to assign an energy width to every channel of each HPGe element, along with a zero intercept energy for each element. Correcting for the differential nonlinearity in the HPGe elements is important because ΔE_{re} also determines the energy contribution to the angular resolution $\Delta\theta_e$, and ΔE_{re} is dominated by the width of the HPGe pulse height channels. On the other hand, the channel width in the NaI spectra is much smaller than the energy resolution. Consequently, differential nonlinearity has little effect on knowledge of the measured value of E_γ' and no correction is made.

For an event falling in a summed energy window, its HPGe pulse height is sorted into one of 256 histograms, depending in which element of each detector the interaction occurred. These histograms are precursor to the ray-sums $\lambda_{\ell\phi k}$ introduced in Sections 3.3 and 3.5. Unlike the $\lambda_{\ell\phi k}$, which are functions of θ , the histograms $\lambda_{\ell\phi i}$ are functions of the recoil electron energy E_{re} , where i represents the pulse height channel. In Figure 5-3, histograms of the pulse height spectra are shown for a HPGe element in coincidence with two different NaI elements. In that figure, the gray columns show spectra of the HPGe pulse height from a ^{124}Sb point source with no summed energy requirements, while black columns show $\lambda_{\ell\phi i}$ from events whose summed energies fall between 0.535 MeV and 0.671 MeV. Note the significant reduction of the continua in $\lambda_{\ell\phi i}$ versus the original spectra.

After selecting events based on their summed energies, the resulting $\lambda_{\ell\phi i}$ are rebinned according to scattering angle to form $\lambda_{\ell\phi k}$. The scattering angles are divided

Figure 5-2. Summed energy histogram from ^{124}Sb point source in gray. Energy window centered about $E_\gamma = 0.602$ MeV with width $E_w = 0.068$ MeV is highlighted in black.

into M_θ bins of equal width, where the minimum and maximum scattering angles are chosen to cover the source region of interest and M_θ is computed from Equation (3-57) to ensure proper sampling for the RFL algorithm. In Equation (3-29), the ray-sum for a particular scattering angle bin defined by $\theta_{k-1} < \theta < \theta_k$ is computed from $\lambda_{\ell\phi}(\theta)$, which is a differential distribution in θ . (Note that the subscript ℓ has been added to $\lambda_\phi(\theta)$ to identify the first-detector element.) The windowed HPGe pulse height histograms can be treated as a differential distribution in E_{re} , $\lambda_{\ell\phi}(E_{re})$, if the finite width of the pulse height channels is temporarily disregarded. Thus,

$$\lambda_{\ell\phi}(\theta)d\theta = \lambda_{\ell\phi}(E_{re})dE_{re} \quad (5-12)$$

Substituting Equation (5-12) into Equation (3-29) yields

$$\begin{aligned}
\lambda_{\ell\phi k} &= \frac{E_{re}(\theta_k)}{E_{re}(\theta_{k-1})} \int \lambda_{\ell\phi}(E') dE' \\
&= \int_0^{E_{re}(\theta_k)} \lambda_{\ell\phi}(E') dE' - \int_0^{E_{re}(\theta_{k-1})} \lambda_{\ell\phi}(E') dE' \\
&= I_{\ell\phi}(E_{re}(\theta_k)) - I_{\ell\phi}(E_{re}(\theta_{k-1}))
\end{aligned} \tag{5-13}$$

E_{re} is given as a function of θ in Equation (2-4). Since the pulse height histograms are actually discrete, not continuous as in Equations (5-12) and (5-13), summed distributions $I_{\ell\phi i}$ are generated from the pulse height histograms.

$$I_{\ell\phi i} = \sum_{i'=0}^i \lambda_{\ell\phi i'} \tag{5-14}$$

The continuous values of $I_{\ell\phi}$ used in Equation (5-13) are estimated by linear interpolation from $I_{\ell\phi i}$.

5.2.2 Subtracting Random Background

The measured random background data are subtracted from image data in the ray-sum format. The random background ray-sums $\lambda_{\ell\phi kb}$ must be weighted before subtracting them from the image data ray-sums $\lambda_{\ell\phi kd}$ so that the subtracted background counts correspond to the number of random coincidence events measured during the image data run

$$\lambda_{\ell\phi k} = \lambda_{\ell\phi kd} - W_b \lambda_{\ell\phi kb} \tag{5-15}$$

where, W_b is the ratio of measured random coincidence events in the image data C_{rcd} to events in the background C_{rcb} .

C_{rcd} cannot be found directly from the measurement. If the source does not suffer significant decay during the duration of the measurement, W_b is simply equal to the ratio

Figure 5-3. HPGc pulse height spectra of one HPGc element in coincidence with two different NaI elements. Gray columns show histograms before summed energy windowing, and black columns show histograms after windowing corresponding to $\lambda_{\ell\phi_1}$.

of the durations, T_d/T_b . However, if the source does decay during the measurement, computation of W_b is more difficult. An expression for W_b is derived below for a measurement in the presence of a single-isotope decaying source.

Whether random events are accumulated during a data run or a background run, their rate is the same, assuming an identical source. The general symbol C_{rc} is adopted to express the random events accumulated during a run of duration T , beginning at time t , and is given by

$$C_{rc} = \int_t^{t+T} r_{rc}(t') dt' = 2\tau_r \int_t^{t+T} r_1(t') r_2(t') dt' \quad (5-16)$$

where r_{rc} is the random coincidence count rate, and r_1 and r_2 are the singles count rates in the first and second detector, respectively. ($2\tau_r$ is the resolving time.) The count rate in

the first detector can be expressed in terms of interaction rate and dead time τ by rearranging the nonparalyzable dead time model of Equation (4-19).

$$r_1(t) = \frac{g_1(t)}{1 + \tau g_1(t)} \quad (5-17)$$

The interaction rate, $g_1(t)$, is proportional to the activity of the source and Equation (5-17) can be rewritten as

$$r_1(t) = \frac{g_{10}e^{-\lambda_c t}}{1 + \tau g_{10}e^{-\lambda_c t}} \quad (5-18)$$

where g_{10} is the interaction rate corrected to time $t = 0$, and λ_c is the decay constant of the source. Because the dead time of the RCC is dominated by the dead time of the first detector (as noted in Section 4.6) $r_2(t)$ can be expressed as

$$r_2(t) = g_2(t) = g_{20}e^{-\lambda_c t} \quad (5-19)$$

Substituting Equation (5-18) and (5-19) into Equation (5-16) yields

$$C_{rc} = K_2 \int_t^{t+T} \frac{e^{-2\lambda_c t'}}{1 + K_1 e^{-\lambda_c t'}} dt' \quad (5-20)$$

where,

$$\begin{aligned} K_1 &\equiv \tau g_{10} \\ K_2 &\equiv 2\tau_r g_{10} g_{20} \end{aligned} \quad (5-21)$$

Integrating and rearranging Equation (5-21) gives:

$$C_{rc} = \frac{K_2}{\lambda_c K_1^2} \left\{ K_1 e^{-\lambda_c t} (1 - e^{-\lambda_c T}) - \ln \left[\frac{1 + K_1 e^{-\lambda_c t}}{1 + K_1 e^{-\lambda_c (t+T)}} \right] \right\} \quad (5-22)$$

From Equation (5-22), the factor W_b can be derived for an image data run of duration T_d beginning at time t_d and a random background run of duration T_b beginning at time t_b .

$$W_b = \frac{K_1 e^{-\lambda_c t_d} (1 - e^{-\lambda_c T_d}) - \ln \left[\frac{1 + K_1 e^{-\lambda_c t_d}}{1 + K_1 e^{-\lambda_c (t_d + T_d)}} \right]}{K_1 e^{-\lambda_c t_b} (1 - e^{-\lambda_c T_b}) - \ln \left[\frac{1 + K_1 e^{-\lambda_c t_b}}{1 + K_1 e^{-\lambda_c (t_b + T_b)}} \right]} \quad (5-23)$$

Because the constant K_2 drops out of Equation (5-23), only the single event rate in the first detector must be measured to determine W_b .

By neglecting the dead fraction, the denominator of the integrand in Equation (5-20) is equal to unity and the complex expression of Equation (5-23) can be replaced by

$$W_b \cong e^{-2\lambda_c (t_d - t_b)} \frac{1 - e^{-2\lambda_c T_d}}{1 - e^{-2\lambda_c T_b}} \quad (5-24)$$

When the dead fraction is small ($K_1 < 0.1$) and difference in start times is less than one half of $t_{1/2}$ for the isotope (i.e. $|t_d - t_b| < t_{1/2}/2$), neglecting the dead fraction results in an error of less than 2.5%. An additional advantage of using Equation (5-24) versus (5-23) is that no measurement of r_1 is required.

Due to limitations on the duration of image data runs, it is often necessary to record multiple runs using the same camera geometry to get adequate counts for forming an image. Ray-sums from these multiple runs are summed after subtraction of the random background. A tool exists in the RCC for subtracting weighted random background ray-sums from image data ray-sums and for summing multiple image data ray-sums. Any ray-sums combined with this tool must have identical scattering angle bins.

5.2.3 Combining Runs from Different Angular Views

In Section 5.1, the technique for imaging with M_ϕ greater than the existing 16 NaI elements is discussed. In order to create a smooth laminogram from ray-sums measured at different ϕ_0 , the ray-sums must be corrected to a common number of measured Compton

scattering events C_{cc} . Defining the number of measurements taken with different values of ϕ_0 to be M_q and selecting one of the measurements to serve as a basis to all of the others, the normalized ray-sums $\tilde{\lambda}_{\ell\phi kq}$ for measurement q are

$$\tilde{\lambda}_{\ell\phi kq} = W_q \lambda_{\ell\phi kq} \quad (q = 1, 2, \dots, M_q - 1) \quad (5-25)$$

where the basis measurement for the normalization is arbitrarily chosen to be $q = 0$.

The number of Compton scattering events C_{cc} measured during a particular run is an integral of the measurable count rate r_{cc} over the duration of the run.

$$C_{cc} = \int_t^{t+T} r_{cc}(t') dt' = \int_t^{t+T} \frac{g_{cc}(t')}{1 + \tau g_1(t')} dt' \quad (5-26)$$

In Equation (5-26), C_{cc} has been expressed as an integral whose integrand is a function of the true interaction rate of Compton scattering events $g_{cc}(t)$ and the dead time due to the count rate in the first detector. Since both $g_{cc}(t)$ and $g_1(t)$ are proportional to the source activity, Equation (5-26) must be rewritten when the source activity decays during the run.

$$C_{cc} = g_{cco} \int_t^{t+T} \frac{e^{-\lambda_c t'}}{1 + K_1 e^{-\lambda_c t'}} dt' \quad (5-27)$$

where g_{cco} is $g_{cc}(t)$ corrected to time $t = 0$, and K_1 is defined in Equation (5-20).

Integrating Equation (5-27) and rearranging yields

$$C_{cc} = \frac{g_{cco}}{\lambda_c K_1} \left\{ \ln \left[\frac{1 + K_1 e^{-\lambda_c t}}{1 + K_1 e^{-\lambda_c (t+T)}} \right] \right\} \quad (5-28)$$

From Equation (5-28), the normalization factor for run q with duration T_q and starting time t_q relative to the basis run $q = 0$ with duration T_0 and starting time t_0 can be expressed as

$$W_q = \frac{C_{cco}}{C_{ccq}} = \frac{\ln \left[\frac{1 + K_1 e^{-\lambda_c t_0}}{1 + K_1 e^{-\lambda_c (t_0 + T_0)}} \right]}{\ln \left[\frac{1 + K_1 e^{-\lambda_c t_q}}{1 + K_1 e^{-\lambda_c (t_q + T_q)}} \right]} \quad (q = 1, 2, \dots, M_q - 1) \quad (5-29)$$

The dead time can be neglected in Equation (5-27) to give a simpler expression for W_q as done for the random background weighting factor W_b .

$$W_q \cong e^{-\lambda_c (t_0 - t_q)} \frac{(1 - e^{-\lambda_c T_0})}{(1 - e^{-\lambda_c T_q})} \quad (q = 1, 2, \dots, M_q - 1) \quad (5-30)$$

Limitations on the applicability of Equation (5-30) are the same as those for the simplified expression of W_b in Equation (5-24) with approximately the same errors.

Normalized ray-sums from measurements made at M_q different values of ϕ_0 are combined in a procedure referred to as "interlacing". The sets of ray-sums are first ordered by increasing ϕ_0 and then they are reordered according to

$$\tilde{\lambda}_{\ell\phi'k} = \tilde{\lambda}_{q\ell\phi k} \quad (\phi' = 0, 1, \dots, 16M_q - 1) \quad (5-31)$$

where,

$$\phi' = \phi(M_q - 1) + q \quad (\phi = 0, 1, \dots, 15, q = 0, 1, \dots, M_q - 1) \quad (5-32)$$

The interlacing procedure described by Equations (5-31) and (5-32) is implemented as a tool in the RCC application. After interlacing, all M_q sets of ray-sums are ordered by increasing azimuthal angle of the second-detector element and can be used in backprojection-summation as if they originated from a ring Compton camera with a second detector consisting of $M_\phi = 16M_q$ elements.

From the ray-sums, laminograms can be generated using the backprojection-summation described in Section 3.3, which is also implemented as a tool in the RCC

application. The source plane geometry must be specified before the backprojection-summation algorithm can be run. The source plane geometry includes source to first detector distance z_s (measured from the middle of the HPGe crystal, in cm), the minimum and maximum extent of the plane in both the x and y directions (again in cm), and the number of pixels in both the x and y directions. The RCC application can presently handle up to 128×128 pixels in the source space. By exploiting the 8-fold symmetry of the RCC, the backprojection-summation algorithm is limited to square source spaces centered on the ring axis with square pixels.

5.3 Measurement of Contaminating Events

In this section measurements of contaminating events are shown and their impact on the useful imaging events is discussed. In the following section, measured efficiencies for the RCC are reported, but it is necessary to first remove contaminating events with significant count rates to achieve accurate measures of the efficiency.

Generally, random coincidence events have been the largest fraction of the total contaminating event rate. In Figure 5-4a, summed energy spectra of the raw image data and weighted random background data of a ^{64}Cu point source are shown on a semi-log plot. The ^{64}Cu isotope decays by positron emission with subsequent release of 0.511 MeV positron annihilation photons. These 0.511 MeV photons result in a strong full energy peak in the summed energy spectrum of the raw data. There is also a coincident peak in the summed energy spectrum that ranges from 0.68 to 0.76 MeV due to absorption of the full energy of one annihilation photon and absorption of the other emitted photon after it undergoes a large-angle Compton scatter in the shielding surrounding the source. (A 5 cm thick lead wall is placed around the source to shield those in the laboratory and in adjacent rooms from gamma-ray exposure.) Above this "backscatter coincidence" peak, the weighted random background closely matches the raw data. The weighting factor is determined from Equation (5-24). The upper edge in the random background spectrum at

a)

b)

Figure 5-4. Histograms showing a) summed energy spectra of measured raw data and weighted random background and a model of the random background spectral shape from a ^{64}Cu point source and b) a net data spectrum after subtracting weighted random background from the raw data. RCC parameters include $z_s = 10$ cm, $d = 30$ cm and $\theta_c = 60^\circ$.

0.85 MeV corresponds to a 0.511 MeV photon being absorbed in one detector while an independent 0.511 MeV photon backscatters in the other detector and escapes. The shape of the ^{64}Cu random background spectrum is modeled using Equation (2-36), and the model is illustrated with a solid line. The discrepancy between the modeled and measured background spectra is due to three features of the pulse height response functions of the detectors which were difficult to model analytically. The first feature is the strength and shape of the backscatter peak. The backscatter event rate is assumed to be 12.5% of the incident rate and the shape is described in Appendix C. Secondly, the lower level threshold is modeled as a one channel cut off in both detectors, with the same cutoff energy for each detector; in the real system the spectrum falls to zero over a significant energy range and the threshold is slightly different for each element. Finally, multiple scattering events which fill the valley between the Compton edge and the full energy peak were not incorporated into the model. Comparisons of modeled and measured response functions are shown as Figures C-1 and C-2 in Appendix C.

A summed energy spectrum of the ^{64}Cu net data is shown as a histogram in Figure 5-4b. The net data is the difference between the raw and weighted random background data shown in 5-4a. In the net data spectrum, partial energy deposition (PED) events from the 0.511 MeV photons cause the "bump" between the 0.33 and 0.37 MeV. The upper energy is approximately equal to the sum of the 0.170 MeV recoil electron energy from a 60° Compton scatter of a 0.511 MeV photon in the first detector followed by the scattered gamma ray (with $E_\gamma' = 0.341$ MeV) suffering a Compton backscatter in the second detector in which a recoil electron of 0.195 MeV results. The lower edge of this "bump" corresponds to a 60° scatter of a 0.511 MeV photon in the first detector followed by another scatter in the second detector that deposits enough energy to fall just above the lower level threshold of 0.16 MeV in that detector. If one attempted to image gamma rays in the 0.33 to 0.37 MeV energy range in the presence of annihilation photons, PED events from those photons would act as contaminating events.

Photons which undergo multiple scattering in the first detector and are absorbed in the second detector result in contaminating multiple scattering events which are estimated by Singh [Sin83a] to make up 8% of the full energy peak in Figure 5-4b. Events with energies above 0.37 MeV and below the full energy peak in Figure 5-4b result from photons which undergo multiple scattering interactions in either detector and at least a single scatter and escape in the opposite detector. Events where a recoil electron penetrates out of one of the detector elements also show up in this part of the energy spectrum. Below the 0.33 MeV lower edge of the PED "bump" in that figure, events result from photons which undergo large angle scattering from shielding material surrounding the source.

The summed energy spectrum of ^{64}Cu in Figure 5-4b is relatively simple with the 0.511 MeV full energy peak suffering only random coincidence and multiple scattering contaminating events. A much more complex spectrum results from a ^{24}Na source, which emits 1.37 and 2.75 MeV gamma rays in cascade. Each of the contaminating events identified in Section 2.4 is possible with a ^{24}Na source. Figure 5-5 shows summed energy spectra from a ^{24}Na line source. (Spectra from a point source are not shown here because all measured point source spectra had too few counts to clearly exhibit the features of either the raw data or background spectrum clearly.) A full energy peak from the 1.37 MeV gamma rays is clearly discernible as is the 2.75 MeV full energy peak. Since the ^{24}Na isotope emits the two cascade gamma rays within picoseconds of one another [Led78], each decaying nucleus emits a total of 4.12 MeV in gamma-ray energy within the 30 ns resolving time of the camera. Thus, the weighted random background spectrum in Figure 5-5a matches the raw data above the total emitted energy of 4.12 MeV because only random coincidence events are possible above this energy. Also, the difference between the raw data and the random background spectra in the energy range from 2.75 and 4.12 MeV is due only to cascade coincidence events, where one of the ^{24}Na gamma ray pair interacts in one of the detectors and the second gamma ray interacts in the other detector.

A solid line indicates the modeled shape of the random coincidence spectrum. The agreement is much better than for the ^{64}Cu random coincidence spectrum, in part, because the low energy backscatter peak has less impact at the high energies of the ^{24}Na gamma rays. (The ratio of backscattered to unscattered event rates is again assumed to be 12.5%.) Also, the relative separation between the Compton edge and full energy peak is smaller and the multiple scattering events not included in the model are less important than for the ^{64}Cu .

In Figure 5-5b, the summed energy histogram of the net data is shown on a linear scale. (Note that the linear scale has been multiplied by two for energies above 1.7 MeV.) The region between 3 and 4 MeV, where only cascade coincidence events are possible, is identified; however, these events are present at energies down to the threshold of about 0.25 MeV. A model for the cascade coincidence events developed from Equation (2-37) predicts that the number of events falling between the energies of 3.5 and 4 MeV will be 28% of the events between the energies of 3 and 3.5 MeV. In the net spectrum of Figure 5-5, the total number of events between 3.5 and 4 MeV is 3344 or 27% of the 12208 events between 3 and 3.5 MeV, which is closely predicted by the model. As with the random coincidence model, this cascade coincidence model requires that the response functions of the two detectors be accurately represented. The cascade coincidence model predicts that the event rate with summed energies between 3 and 4 MeV is 6.4% of the total rate when a 12.5% intensity backscatter peak is included; whereas, it predicts a relative rate of 12.7% if the backscatter is neglected. Thus, the model indicates that between 123,000 and 244,000 of the 799,000 events in the net spectrum are due to cascade coincidence. Because the backscattered gamma-ray energies are low (below 0.3 MeV for both 1.37 and 2.75 MeV incident gamma rays), predicted cascade event rates at low energies are sensitive to their inclusion in the model, while event rates at the highest energies are largely unaffected.

Apart from the random and cascade coincidence events, other types of contaminating events are illustrated in Figure 5-5. Backscatter events dominate the net

a)

b)

Figure 5-5. Histograms showing a) summed energy spectra of measured raw data and weighted random background and a model of the random background spectral shape from a 5 cm long ^{24}Na line source and b) a net data spectrum after subtracting weighted random background from the raw data. RCC parameters include $z_s = 10$ cm, $d = 30$ cm and $\theta_c = 60^\circ$.

data spectrum of Figure 5-5b between the summed energies of 0.3 and 0.7 MeV, and PED events from the 1.37 MeV gamma rays are prominent between the minimum and maximum possible energies of 0.73 and 1.20 MeV, respectively. PED events from 2.75 MeV gamma rays can deposit energies between 1.82 and 2.58 MeV, but are not as readily identified as the PED events from 1.37 MeV gamma rays, because they are obscured by other types of events. It is important to note that PED events from the 2.75 MeV line do not fall in the summed energy window for selecting the 1.37 MeV gamma rays, which is typically less than 0.19 MeV wide.

There are two other features of Figure 5-5b worth noting. One is a triangular shaped peak at about 1.9 MeV which is the sum of three different interaction sequences. The most probable sequence is due to a pair production event from a 2.75 MeV gamma ray in one detector, in which both annihilation photons escape the camera (referred to as a *double escape*), coupled with a coincident 1.37 MeV gamma ray, which backscatters externally and is subsequently absorbed in the other detector. The minimum summed energy from this sequence is about 1.95 MeV. Another sequence requires that the full energy of a 1.37 MeV gamma ray is absorbed in one detector while an annihilation photon created externally by the coincident 2.75 MeV gamma ray is fully absorbed in the other detector for a summed energy of 1.88 MeV. The third contribution is the lower edge of the PED events from the 2.75 MeV gamma rays described above.

The other feature of interest is a peak at 2.24 MeV due to a double escape from a 2.75 MeV gamma ray interaction in one detector where one of the annihilation photons is absorbed in the other detector, and the remaining photon escapes the camera. With the presence of this peak in the summed energy spectrum, one might anticipate that the 2.75 MeV peak would contain contaminating single escape coincidence events as described in Section 2.4, where one of the annihilation photons is absorbed by the detector element in which the pair production occurred and the remaining photon is absorbed in the other detector. While the pulse height spectra of the individual elements all have very

prominent double escape peaks, there are no indications of the single escape peaks necessary for the single escape coincidence.

The last type of contaminating event from Section 2.4 to consider is reverse scattering, where incident gamma rays are scattered from the second detector and are subsequently absorbed in the first detector. Analytic modeling of these events predicts that 1.37 MeV gamma rays of ^{24}Na will undergo reverse scattering at about 2% of the rate of the forward scattering events from a point source in the geometry described for Figures 5-4 and 5-5. A 1.37 MeV gamma ray undergoing a reverse scattering in this geometry deposits a recoil electron energy of 1.150 MeV in the second detector with the remaining 0.219 MeV deposited in the first detector. No peaks are found at the appropriate energy in the pulse height spectra of either detector, corroborating the predictions of the model. Similarly small reverse scattering event rates are predicted from the other gamma rays discussed in this section and no peaks are found in these spectra either.

To review the analysis of summed energy spectra in Figures 5-4 and 5-5, the prevalent type of contaminating event is due to random coincidence. These events can be accurately measured and subtracted from the raw image data, but with a commensurate increase in noise over raw image data with no random coincidence events. Of the remaining types of contaminating events, only PED and cascade coincidence events are measured in significant quantities. Further, it is shown by energy considerations that PED events from 2.75 MeV gamma rays from ^{24}Na cannot fall in the summed energy window for selecting 1.37 MeV gamma rays.

5.4 RCC Efficiency: Measurement from Point Sources and Comparison to Analytic Model Results

Measurements of ϵ_{icc} , the intrinsic efficiency of the RCC, have been made at a range of incident photon energies E_γ using annihilation photons from ^{64}Cu , and gamma rays from ^{137}Cs , ^{124}Sb and ^{24}Na . The energies and intensities of photons from these

Table 5-1. Principle emitted photon energies and intensities (in photons emitted per decaying nucleus) for radioisotopes used to measure the RCC point response and in 2D phantoms described in Chapter VI, as compiled by Lederer and Shirley [Led78].

Isotope	E_γ (MeV)	I_γ
^{64}Cu	0.5110	0.386
	1.3461	0.006
^{65}Zn	0.5110	0.029
	1.1155	0.508
^{137}Cs	0.6617	0.850
^{124}Sb	0.6027	0.983
	0.6458	0.072
	0.7228	0.113
	1.6910	0.490
	2.0910	0.056
^{24}Na	1.3686	1.000
	2.7541	1.000

radioisotopes are listed in Table 5-1, along with ^{65}Zn used only for extended sources. In addition to the measurement of variation with energy, variation of ϵ_{icc} with the geometric factors of central scattering angle θ_c , first-to-second detector spacing d , and the polar angle of the source relative to the RCC ring axis α have been measured, and each variation is compared to analytic model predictions. The continuous model curves show how the efficiency varies between the discrete measurements and indicates how the RCC behaves beyond the measured range. Also because values of ϵ_{icc} computed from the model are used in backprojecting ray-sums and in iterative reconstruction algorithms, it is necessary to demonstrate the validity of the model.

5.4.1 Variation of Efficiency with Incident Energy

Variation of ε_{icc} with E_γ is shown in Figure 5-6 at two different values of θ_c . For both sets of measurements the geometric parameters z_s and d are 10 and 30 cm, respectively. In part a, θ_c is 45° and the measurements are made at energies of 0.603 and 1.69 MeV using ^{124}Sb gamma rays, at 0.662 MeV using ^{137}Cs gamma rays and at energies of 1.37 and 2.75 MeV using ^{24}Na gamma rays. In part b, θ_c is 60° and the measurements are made at 0.511 MeV using annihilation photons from ^{64}Cu in addition to the gamma-ray energies of part a. The value of ε_{icc} is determined from the measurements using the following relation

$$\varepsilon_{\text{icc}} = \frac{\bar{C}_d - W_b \bar{C}_{\text{rcb}}}{(\Omega_1/4\pi)N_\gamma} \quad (5-33)$$

where, Ω_1 is the solid angle subtended by the first-detector array relative to the point source, and W_b is the random coincidence background weighting factor from Equation (5-23). $\bar{C}_d$ is the number of coincident events falling in a summed energy window with bounds given in Equation (5-10) that are recorded during an image data run using all elements of both detectors. Additionally, a restriction on the value of E_{re} excludes from those events that scatter at angles less than $\theta_c - 18^\circ$ and greater than $\theta_c + 18^\circ$. The energies corresponding to these scattering angle limits are designated E_{rel} and E_{reh} , respectively. $\bar{C}_{\text{rcb}}$ is the number of events measured during the accompanying random coincidence background run that survive the same selection process. N_γ is the total number of gamma rays emitted during the image data run of duration T_d . For all of the point source measurements the dead time is negligible and N_γ can be computed from

$$N_\gamma = A_0 e^{-\lambda_s(t-t_0)} (1 - e^{-\lambda_s T_d}) \quad (5-34)$$

a)

b)

Figure 5-6. Comparison of measured variation of ϵ_{icc} , intrinsic efficiency, with incident gamma-ray energy, E_γ to analytic model predictions at two values of θ_c : a) 44° and b) 60° . Corrections of measured efficiencies are made using modeled contaminating event efficiencies. RCC parameters z_s and d were fixed at values of 10 cm and 30 cm. No error bars are shown due to small counting statistics uncertainties of less than 3% for every measurement.

where A_0 is the source activity at time t_0 and t is the run start time. Raw data for Figure 5-6 is tabulated in Appendix C as Table C-1.

Comparison of the measured and predicted efficiencies in Figure 5-6 shows that the model under-predicts the measured values at energies of 0.603 and 1.37 MeV for $\theta_c = 45^\circ$ and closely matches at these energies for $\theta_c = 60^\circ$; however, the model consistently over predicts at all other energies for both values of θ_c . This pattern suggests a significant contribution from one or more types of contaminating events at 0.603 and 1.37 MeV. Because both the 0.603 and 1.37 MeV gamma rays are measured in the presence of higher energy gamma rays, as seen in Table 5-1, the possibility of contaminating PED events exist. Table 5-2 shows that the 2.75 MeV (energy E_γ^b) gamma rays of ^{24}Na cannot be confused with the 1.37 MeV gamma rays (energy E_γ^a), because the recoil electron energy (E_{re}^b) deposited in the first detector by gamma rays of energy E_γ^b is higher than the window set on E_{re} for selecting the E_γ^a events. The efficiency for PED contaminating events ϵ_{aped} is thus zero. For the same reason PED events from the intense 1.69 MeV gamma rays of ^{124}Sb cannot contribute counts to the 0.603 MeV window. In fact, E_{re}^b at θ_c is higher than E_γ^a in both cases. However, Table 5-2 also shows that secondary lines with energies of 0.646 and 0.723 MeV may cause PED events that are confused with those from the 0.603 MeV line. A 2.0910 MeV line in ^{124}Sb deposits too much recoil electron energy to contribute counts to either the 0.603 or 1.69 MeV windows.

Cascade coincidence contaminating events are possible for both ^{124}Sb and ^{24}Na , because 50% of the 0.603 MeV gamma rays of ^{124}Sb are emitted in cascade with a 1.69 MeV gamma ray and all of the 1.37 MeV gamma rays of ^{24}Na are emitted in cascade with a 2.75 MeV gamma ray. In Table 5-3, a summary of the modeled efficiencies for PED, cascade coincidence and also reverse scattering contaminating events relative to the modeled ϵ_{acc} are listed for each combination of E_γ and θ_c in Figure 5-6. The intrinsic total contaminating event efficiency ϵ_{itce} is estimated by multiplying the sum of the

Table 5-2. Recoil electron energies resulting from Compton scattering of ^{124}Sb and ^{24}Na gamma rays showing the possibility that the summed energy window for gamma rays of energy E_γ^a include PED contaminating events from gamma rays emitted at the higher energy of E_γ^b . If E_{re}^b is less than E_{re}^a , PED contaminating events are possible.

Isotope	θ_c (deg)	E_γ^a (MeV)	θ_{min} (deg)	θ_{max} (deg)	E_{re}^a at $\theta=\theta_{max}$ (MeV)	E_γ^b (MeV)	E_{re}^b at $\theta=\theta_c$ (MeV)	$E_{re}^b < E_{re}^a$?
^{124}Sb	44	0.6027	26	62	0.2361	0.6458	0.1745	Yes
						0.7228	0.2117	Yes
						1.6910	0.8320	No
						2.0910	1.1399	No
^{124}Sb	44	1.6910	26	62	1.0885	2.0910	1.1399	No
^{124}Sb	60	0.6027	48	72	0.2911	0.6458	0.2501	Yes
						0.7228	0.2994	No
						1.6910	1.0540	No
						2.0910	1.4045	No
^{124}Sb	60	1.6910	48	72	1.2240	2.0910	1.4045	No
^{24}Na	44	1.3686	26	62	0.8120	2.7541	1.6860	No
^{24}Na	60	1.3686	48	72	0.9300	2.7541	2.009	No

contaminating event efficiencies by the modeled value of ϵ_{icc} and is used to compute the corrected measurements of Figure 5-6.

$$\epsilon_{itce} = \epsilon_{icc} \left[(\epsilon_{aped}/\epsilon_{acc}) + (\epsilon_{acce}/\epsilon_{acc}) + (\epsilon_{arev}/\epsilon_{acc}) \right] \quad (5-35)$$

The relative PED efficiency is marked with an "n/a" in Table 5-3 where no higher energy gamma rays exist to cause contaminating events, while those cases where the PED events are eliminated by thresholds on the data, are set to zero. (These events are also identified by a "No" in the last column of Table 5-2.) Computation of ϵ_{aped} is based on Equation (2-35), and the response of the NaI elements of the second detector which includes a

backscatter peak with an intensity equal to 12.5% of the unscattered intensity, as described in Appendix C.

The summed energy window limits used to select the 0.603 MeV gamma rays of ^{124}Sb for the $\theta_c = 44^\circ$ measurement are 0.531 and 0.674 MeV. This window includes contributions from full energy depositions from the 0.646 and 0.723 MeV lines, which have summed energy resolutions of 0.059 and 0.061 MeV, respectively. Assuming a Gaussian shape to the summed energy peaks, about 87% of the 0.646 MeV events and 3.0% of the 0.723 MeV events will fall in this summed energy window, and the predicted value of ϵ_{acc} at these two energies are 90% and 77% of the 0.603 MeV efficiency, respectively. The contribution of these events can be cast as another term in Equation (5-35) of the form $\epsilon/\epsilon_{\text{acc}}$ with a magnitude of 0.0602, which is included in the totals of Table 5-3. Similarly, the relative contribution of these events to the 0.603 MeV summed energy window for $\theta_c = 60^\circ$ is computed to be 0.0585.

The open triangles of Figure 5-6 represent the measured values of ϵ_{icc} after subtracting predicted total contaminating event rate ϵ_{itce} . At energies of 1.37 and below, the efficiency model slightly over predicts the correction measurements. This over prediction is expected due to event loss from cross talk between elements in the HPGe array, which is not included in the model. At energies above 1.37 MeV, the over prediction is more severe, degrading with increasing E_{re} . For these higher energy photons, values of E_{re} range from 0.83 MeV for $E_\gamma = 1.37$ MeV at $\theta_c = 45^\circ$ up to 2.1 MeV for $E_\gamma = 2.75$ MeV and $\theta_c = 60^\circ$ with the corresponding ratios of predicted to corrected measured efficiencies ranging from 1.01 to 1.88. This indicates that the simple technique to account for recoil electron penetration in the efficiency model is inadequate. However, this technique is a significant improvement compared to neglecting the recoil electron penetration all together, where the ratio balloons to 3.28 for $E_\gamma = 2.75$ MeV and $\theta_c = 60^\circ$.

Table 5-3. Modeled contaminating event absolute efficiencies from point sources relative to modeled values of ϵ_{icc} for the RCC with $z_s = 10$ cm and $d = 30$ cm. *Includes full energy contributions from 0.646 and 0.723 MeV gamma rays of ^{124}Sb , described in the text.

E_γ (MeV)	Isotope	θ_c (deg)	$\frac{\epsilon_{aped}}{\epsilon_{acc}}$	$\frac{\epsilon_{acce}}{\epsilon_{acc}}$	$\frac{\epsilon_{arev}}{\epsilon_{acc}}$	Sum $= \frac{\epsilon_{itce}}{\epsilon_{icc}}$
0.6027	^{124}Sb	44	0.0748	0.0809	0.0080	0.2239*
0.6617	^{137}Cs	44	n/a	n/a	0.0089	0.0089
1.3686	^{24}Na	44	0	0.1693	0.0183	0.1876
1.6910	^{124}Sb	44	0	0.0870	0.0220	0.1090
2.7541	^{24}Na	44	n/a	0.2468	0.0314	0.2782
0.5110	^{64}Cu	59	n/a	n/a	0.0080	0.0080
0.6027	^{124}Sb	60	0.0664	0.1042	0.0097	0.2388*
0.6617	^{137}Cs	59	n/a	n/a	0.0108	0.0108
1.3686	^{24}Na	60	0	0.1119	0.0207	0.1326
1.6910	^{124}Sb	60	0	0.1638	0.0246	0.1884
2.7541	^{24}Na	60	n/a	0.2323	0.0345	0.2668

There are some approximations in the analytic model, which should be mentioned here. The model computes the intrinsic efficiency for a single first-detector element centered on the ring axis of the camera with all events scattering at θ_c . Since the first detector of the RCC consists of 16 elements and none of them reside on the ring axis, the measured efficiency incorporates a range of scattering angles. For the source and camera configuration in Figure 5-6, the scattering angle through the quantization positions of the HPGe and NaI elements varies from 52.9° to 67.0° when $\theta_c = 60^\circ$ and from 37.5° to 52.5° when $\theta_c = 45^\circ$. The model also only computes the efficiency for a single NaI element at $\phi = 0^\circ$, and the efficiency of the entire RCC is found simply by multiplying the efficiency for a single element by the total number of elements as indicated by Equation (2-15). Because of variations in the geometry, intrinsic differences between the elements, and

uncertainties in pulse height calibration, timing calibration and alignment, the efficiencies of the individual elements will not be identical. (A significant difference in the relative efficiency of the HPGe detectors and a correction factor is discussed in Section 4.1.) Raw counts of the individual NaI elements measured during a image data run with an on-axis point source are listed in Table C-2 of Appendix C. The counts are from the window about the 0.603 MeV gamma rays measured from a ^{124}Sb source with $z_s = 10$ cm, $d = 30$ cm, and $\theta_c = 60^\circ$. The reduced χ^2 [Bev92] of this set of raw counts is 1.63. The probability that a reduced χ^2 larger than this value would be found with 14 degrees of freedom (one NaI element was turned off) is between 0.05 and 0.1, indicating significant contributions to the variance beyond the Poisson distribution. However, the relative standard deviation in the raw counts is less than 3% and variations due to the effects described above are on the same order. In addition, the measurements at $\theta_c = 60^\circ$ with the ^{124}Sb and ^{24}Na isotopes employed only 15 of the 16 NaI elements and the measurements at $\theta_c = 45^\circ$ with the ^{137}Cs source only used 8 NaI elements. These measured efficiencies are normalized upward to give values that are consistent with and comparable to the measurements with the full array of 16.

5.4.2 Variation of Efficiency with Geometric Factors

A geometric factor which has a large impact on ϵ_{icc} is the RCC crystal separation distance d . The variation of ϵ_{icc} with d is shown in Figure 5-7 for the 0.662 MeV gamma rays from ^{137}Cs . No corrections are made to these measured values for contaminating events because only reverse scattering events will affect the measurement, and their effect is predicted to be quite small. The model closely predicts the measured efficiency at $d = 17.5, 30$ and 45 cm, both in magnitude and trend. The factors P_c , f_2 and $\epsilon_{\text{ip}2}$ are each affected by changing d . P_c is affected because the limits on the integral over scattering angle in Equation (2-14) vary with d . In Equation (2-15) f_2 is shown to vary approximately as d^{-2} , and this is the dominant factor in the efficiency variation with d . The model for $\epsilon_{\text{ip}2}$

Figure 5-7. Measured and predicted variation of ϵ_{icc} with crystal separation distance d for 0.662 MeV gamma rays from an on-axis ^{137}Cs point source. Other geometric factors were fixed at $\theta_c = 45^\circ$ and $z_s = 10$ cm. Error bars represent the standard deviation due to counting statistics.

varies with d via the mean chord length $\bar{C}_2$ through the detector (see Equation (2-16)). Because $\bar{C}_2$ increases with d , the value of ϵ_{ip2} also increase which is clearly illustrated in Table A-2.

There is significant variation in ϵ_{icc} with the central scattering angle θ_c of the RCC, as illustrated in Figure 5-8 where measurements at $\theta_c = 30^\circ$, 45° and 60° with 0.662 MeV gamma rays from ^{137}Cs are compared to predictions from the analytic model. As in Figure 5-7, no corrections for contaminating events are applied to these measurements. At $\theta_c = 60^\circ$, the predicted magnitude of ϵ_{icc} decreases to 68.7% of its value at $\theta_c = 30^\circ$. This decrease is due to the components P_c and f_2 . The ratio of P_c at $\theta_c = 60^\circ$ to its value at $\theta_c = 30^\circ$ is 0.714, and is dominated by the differential Compton scattering cross section $d\sigma/d\Omega$ embedded in P_c . The ratio of f_2 between these two angles is 0.587, because the area subtended by the angular scattering range in the denominator of Equation (2-15) increases with θ_c for θ_c between 0° and 90° . The component ϵ_{ip2} counteracts the other two

Figure 5-8. Measured and predicted variation of ϵ_{icc} with central scattering angle θ_c for 0.662 MeV gamma rays from an on-axis ^{137}Cs point source. Geometric factors d and z_s were held fixed at 30 cm and 10 cm. Vertical error bars represent the standard deviation due to counting statistics, and horizontal error bars show range of scattering angles present in the measurement.

and increases by a factor of 1.64 between $\theta_c = 30^\circ$ and $\theta_c = 60^\circ$. For a fixed E_γ , the scattered gamma-ray energy E_γ' decreases with θ_c causing both the linear attenuation coefficient μ_2 and peak-to-total ratio r_{pt} to increase. Equation (2-16) shows that ϵ_{ip2} increases with both μ_2 and r_{pt} resulting in its increase with θ_c .

The prediction of ϵ_{icc} diverges slightly from the measurements as θ_c increases.

This discrepancy is likely due to interaction of scattered gamma rays in the cryostat and other material around the HPGe detector which are not included in the model. The path of the scattered gamma rays through these materials is proportional to $1/\cos(\theta_c)$.

The most important applications of the analytic efficiency model is in computation of backprojection weights and transition probability matrices for iterative image reconstruction. For the model to be useful in these applications it must accurately represent the efficiency of the RCC for off-axis gamma-ray sources. In Figure 5-9

Figure 5-9. Measured and predicted variation of ϵ_{icc} with scattering angle for the 0.603 MeV gamma rays from an off-axis ^{124}Sb point source. Off-axis measured values are determined from individual NaI elements and multiplied by 16 for direct comparison to on-axis measured values from Figure 5-6. Geometric factors for the off-axis measurement include $z_s = 8.7$ cm, $\alpha = 29^\circ$, $d = 30$ cm, and $\theta_c = 59^\circ$. Vertical error bars represent the standard deviation due to counting statistics, and horizontal error bars show range of scattering angles in the measurement.

efficiencies measured from the 0.603 MeV gamma rays of an off-axis ^{124}Sb point source are compared to the on-axis model. The point source is at a distance 9.9 cm from the center of the HPGe array with polar angle $\alpha = 29^\circ$ and azimuthal angle $\beta = 0^\circ$, such that $z_s = 8.7$ cm.. With the off-axis source, the scattering angle θ through the center of the HPGe array varies among the NaI elements from $\theta = 30^\circ$ for the element at azimuthal angle $\phi = 180^\circ$ up to $\theta = 90^\circ$ for $\phi = 0^\circ$. (Azimuthal angles β and ϕ are both measured counter-clockwise from the positive x-axis.)

The "off-axis efficiencies" in Figure 5-9 are computed using Equation (5-33) and counts from individual NaI elements. Predicted absolute efficiencies ϵ_{acc} are computed from the on-axis model with $z_s = 9.9$ cm to match the distance from the source to the center of the first detector, and the predicted intrinsic efficiencies shown in the figure are

computed by dividing ε_{acc} by Ω_1 , which is proportional to $\cos(\alpha)$, to be consistent with the calculation of ω_{ij} in Equation (3-41). As in Figure 5-8, the vertical error bars represent the standard deviation due to counting statistics and the horizontal error bars show the range of scattering angles for each NaI element. Also note that none of the measured values are corrected for contaminating events.

The measured off-axis efficiencies compare favorably with predictions from the on-axis model, which supports use of the on-axis model to compute off-axis backprojection weights ω_{ij} in Section 3.3.

5.5 RCC Resolution: Measurement from Point Sources and Comparison to Simulation and Analytic Model Results

In order to study the resolution of an imaging system, appropriate measures of the resolution must be selected among the many that are used. The most common measure of resolution in an image is the full width at half maximum (FWHM) of a contour through the center of the imaging system's PSF. Many of the referenced authors have used this resolution measure [Bar81, Nor79, Sim78, Sin83b]. The spatial FWHM is convenient for this work because it is readily compared to the FWHM angular resolution $\Delta\theta_t$ predicted with the analytic model of Chapter II. The relation between the spatial FWHM measured from a radial contour and $\Delta\theta_t$ is given by

$$\Delta\theta_t = \text{atan}[r_h/z_s] - \text{atan}[r_\ell/z_s] \quad (5-37)$$

where r_ℓ and r_h are radial positions (also given by (x_ℓ, y_ℓ) and (x_h, y_h) in Cartesian coordinates) where the PSF falls to half its maximum intensity. For a contour tangential to a circle of radius r_o , where r_o is the radial position of the peak.

$$\Delta\theta_t = \text{atan}\left[\frac{r_h}{\sqrt{z_s^2 + r_o^2}}\right] - \text{atan}\left[\frac{r_\ell}{\sqrt{z_s^2 + r_o^2}}\right] \quad (5-38)$$

If the source is on-axis, the contour is symmetric about r_0 and $\Delta r (= r_h - r_l)$ is less than $z_s/5$, Equation (5-37) and (5-38) both simplify to

$$\Delta\theta_t \cong \frac{\Delta r}{z_s} \quad (5-39)$$

where $\Delta\theta_t$ is in units of radians. In determining the FWHM, the half maximum is always taken to be one-half of the maximum pixel intensity in the PSF on the discrete source plane. The positions of the half maxima are estimated by linear interpolation from the PSF contours, which are chosen to pass through the maximum pixel.

While the FWHM is widely used as a measure of resolution in a PSF and will be used extensively here, it does not reflect the blurring caused in the enhanced tails in laminograms or by other PSFs that are broader than a Gaussian function. An alternative measure of resolution is the spectral resolution (SR) introduced by Lecomte and coworkers [Lec84], who determine the spectral resolution from a contour through a continuous PSF.

$$SR = \frac{\left[\int f(r) dr \right]^2}{\int [f(r)]^2 dr} \quad (5-40)$$

Since the PSFs of interest are discrete, Equation (5-40) can be recast in discrete form.

$$SR = \frac{\left[\sum_{n=0}^{N-1} f(n) \right]^2 \Delta r}{\sum_{n=0}^{N-1} [f(n)]^2} \quad (5-41)$$

where N is the number of samples and Δr is the sample interval. If the contour is taken along the x - or y -axis of the image, Δr is equal to the pixel width Δx and N is the number of pixels in that dimension. The absolute value of the pixel intensity is used in the numerator so that SR reflects the noise due to small negative fluctuations possible in the RFL algorithm. In the limit that the PSF is contained completely in one pixel, which is

equivalent to a Kronecker delta, SR reduces to Δr , the width of one pixel. In the opposite extreme, where the PSF is uniformly flat, SR becomes $\Delta r N$, the width of the entire source plane. For comparison with the FWHM, the SR of a continuous Gaussian distribution is equal to its full width at 20.8% of the maximum or 3.545 times the standard deviation. Since the FWHM of a Gaussian is 2.356 times its standard deviation, the ratio SR:FWHM is 1.51 for a Gaussian function.

Norton and Linzer introduce an additional measure of resolution, which they call the PSF figure-of-merit G [Nor79]. They compute G from a continuous PSF on a circular reconstruction space of radius $r_{\max}$.

$$G = \frac{|f(r_0, \theta_0)|}{\left\{ \int_0^{2\pi} \left[\int_0^{r_{\max}} |f(r, \theta)|^2 r dr \right] d\theta \right\}^{1/2}} \quad (5-42)$$

where $f(r_0, \theta_0)$ is the intensity of the PSF at its center. Note that the denominator is proportional to the root-mean-square (RMS) average of the PSF intensity for a fixed source size, and thus G is related to the peak to RMS average ratio. Modifying Equation (5-42) for a discrete image on a rectangular source space of size $N_1 \times N_2$, G becomes

$$G = \frac{\max[f(n'_1, n'_2)]}{\left\{ \sum_{n_1=0}^{N_1-1} \left[\sum_{n_2=0}^{N_2-1} |f(n_1, n_2)|^2 \right] \right\}^{1/2}} \quad \begin{cases} 0 \leq n'_1 < N_1 \\ 0 \leq n'_2 < N_2 \end{cases} \quad (5-43)$$

In Equation (5-43), the fact that the maximum of the PSF occurs at or near the source position has been used to simplify computation of the numerator. Note that G incorporates the entire PSF in the resolution measure unlike SR which only gives the resolution for a contour. There is a possibility with SR, just as there is with FWHM, that the chosen contour is not representative of the PSF as a whole, giving a misleading

resolution. Using G as a resolution measure eliminates this possibility. In the limit that the PSF is a Kronecker delta function, $G = 1$, and in the opposite extreme, where the PSF is spatially uniform, G is equal to $(N_1 N_2)^{-1/2}$. Note that G increases as the resolution improves, while SR and FWHM decrease, a potential source of confusion.

Using these measures of resolution, the quality of PSFs from the different image reconstruction techniques defined in Chapter III can be compared. Forming the laminogram is considered first because it is an intermediate step in the RFL algorithm; it provides a convenient first estimate for the iterative algorithms; and it is a crude reconstruction algorithm in its own right. Resolution measures from the laminogram PSFs will be presented as a benchmark to show the improvement in image quality achieved with other algorithms

5.5.1 Laminogram Frequency Response

Most of the PSFs considered here are drawn from on-axis point sources. Laminograms of these sources are generated on a discrete source plane of 64 by 64 pixels, with each pixel being 0.2 cm by 0.2 cm in size. The ray-sums consist of 64 angular bins for each pair of detector elements (again one from the first detector and one from the second) covering a 36° range of scattering angles centered about θ_c . This angular range completely covers a circular region 6.4 cm (32 pixels) in diameter at the center of the source space, located a distance $z_s = 10$ cm away from center of the first detector. The spatial width of the angular bins is approximately 0.1 cm at the ring axis. Other pertinent parameters will be described with the data as presented. The algorithm for forming a laminogram ($\hat{f}_{bj}$) from a set of RCC ray-sums (λ_i) and backprojection weights (ω_{ij}) is described by Equation (3-39). The weights for each ray-sum are normalized by the total solid angle of the angular bin as given in Equation (3-41).

In Figure 5-10 contours through the modulation transfer function (MTF) of a measured laminogram are compared to those from a simulation. The measured data are

from 0.603 MeV gamma rays of a ^{124}Sb point source near the ring axis. Each detector pair has 64 ray-sums spanning the scattering angle range from 26° to 62° . The simulation uses the same angular bin structure as the measured data. These simulations are performed by forward projecting a simulated source on a discretized source plane. The result is a set of simulated ray-sums spanning all detector pairs and up to 64 scattering angle bins. The forward projection is performed according to Equation (3-67) using forward projection weights (w_{ij}) from Equation (3-70). The laminogram is formed according to Equation (3-39), as above.

In Figure 5-10 the horizontal contour through the MTF closely follows the $1/\rho$ envelope. Note that this simulation has oscillations which diverge further from the envelope than the simulations with $\theta_c = 60^\circ$ in Figure 3-7 of Section 3.2. These oscillations are due to the more highly curved paths of integration for the RCC when $\theta_c = 44^\circ$, making the response of the RCC less like a straight-ray tomograph than when $\theta_c = 60^\circ$. The horizontal and vertical contours through the measured MTF fall off more severely with increasing ρ than the simulated MTF. This difference in frequency response results from several factors not incorporated in the simulation, including finite angular resolution of the RCC, presence of contaminating events, and finite source size.

Figure 5-11a compares simulated and measured ray-sums for one detector pair from the data used in Figure 5-10: differences in the two sets of ray-sums are apparent. In order to make the simulated ray-sums a better representation of the measured data, the effects of finite total angular resolution and contaminating events must be added. Figure 5-11a also includes a curve showing the simulated ray-sums after blurring with a Gaussian function to model resolution and adding continua to model contaminating events. The Gaussian function used in the blurring has an area equal to the total area of the simulated ray-sums, and its peak is at the centroid of the scattering angles. The width of the Gaussian is 4.5° FWHM based on predicted and measured resolutions which will be shown below. The simulated contaminating event continuum below the Gaussian peak

Figure 5-10. Comparison of measured and simulated MTFs for the RCC. The measured data are from the 0.603 MeV gamma rays of an on-axis ^{124}Sb point source. RCC geometry parameters include $\theta_c = 44^\circ$, $d = 30$ cm and $N_2 = 32$ elements. The source spacing is $z_s = 10$ cm. This same geometry is used in the simulation. Remaining source space parameters are described in the text. Only the horizontal contour through the simulated MTF is shown because the vertical contour is identical.

has a height equal to 2% of the peak area, and continuum height above the peak is 1% of the peak area. Linear interpolation is used to smoothly transition between these two continua over a region extending ± 3 standard deviations from the centroid. With the Gaussian blurring and added continua, the simulated ray-sums now compare favorably with the measured ones.

After blurring and adding continua to the entire set of simulated ray-sums used in Figure 5-10, a simulated laminogram is computed. The horizontal contour through the MTF of this laminogram is now compared to that of the measured MTF in Figure 5-11b. While the blurred simulation is not in perfect agreement with the measurement, it does demonstrate that the finite angular resolution simulated with the Gaussian blurring and the contaminating events simulated by the continua are responsible for deviation of the measured MTF from the ideal $1/\rho$ response.

a)

b)

Figure 5-11. Illustration of finite resolution and contaminating events on frequency response of the RCC. a) Comparison of simulated ray-sums, before and after blurring and adding continua, to measured ray-sums from one detector pair. b) Comparison of horizontal contour through simulated MTF after blurring to a contour through measured MTF. The measurement is described in Figure 5-10.

5.5.2 Reconstruction with the Ramp-Filtered Laminogram Algorithm

Significant deviation of the measured frequency response from $1/\rho$, like that shown in Figures 5-10 and 5-11, persists in the measured MTFs of all the different camera geometries and incident source energies considered in this work. Because the frequency response falls off rapidly at higher frequencies, a filter should be chosen for the RFL algorithm that minimizes loss of useful high frequency components, while eliminating the highest frequencies where noise dominates. The RECT filter would be an ideal choice, except for the resulting Gibb's phenomenon described in Section 3.4.2. The next best choice is a high-order BUTR filter, which closely follows the RECT filter to intermediate frequencies then rapidly, but smoothly, drops to zero. Figure 3-9b includes both the RECT filter with $v_{\max} = 0.5$ and a 12th order BUTR filter ($n = 12$) with $v_c = 0.33$, where this filter is indeed shown to possess the desired properties. This "standard" BUTR filter will be used as the primary filter for RFL reconstruction of point sources throughout the chapter. The PARZ filter, which attenuates the high frequency components more severely than any other filter, will be used widely to show the range of results possible among the different filter types.

The laminograms, whose MTFs are shown in Figure 5-11b, have been reconstructed using the standard BUTR filter to serve as examples of the PSFs resulting from the ramp-filtering algorithm. Figure 5-12 shows surface plots of these reconstructed PSFs on the entire 64 by 64 pixel source plane on which the laminograms are formed. While the 64 by 64 pixel source space is shown for completeness, the valid reconstruction area falls inside the center 32 by 32 pixel region, because the reconstruction is done according to Rowland's technique for eliminating filtering artifacts, as described in Section 3.4. The FWHM of the simulated PSF in part a is 0.83 cm in both the horizontal and vertical directions and is close to the FWHM of the measured PSF at 0.86 cm in both

Figure 5-12. Surface plots showing a) a simulated PSF and b) a measured PSF reconstructed using the RFL algorithm with a BUTR filter where $n = 12$ and $v_c = 0.33$. The source space consist of 64 by 64 square pixels of size 0.2 by 0.2 cm. The measured data is from the 0.603 MeV gamma rays of an on-axis ^{124}Sb point source. Remaining RCC geometry and source parameters are described in Figure 5-10.

directions. The spatial resolution of the simulation corresponds to $\Delta\theta_c = 4.75^\circ$. This value is only 6% larger than the FWHM of the Gaussian function used in blurring the ray-sums; thus, the angular resolution is well preserved under this image reconstruction algorithm.

The figure-of-merit G is 0.321 for the simulation in Figure 5-12a compared to 0.298 for the measurement in part b. The spectral resolution SR is 1.38 cm for the simulation and 1.50 cm for the measurement. (Note that these measures were drawn from the central 32 by 32 pixel regions of the source planes.) The ratio of SR to FWHM for the simulation is 1.66. This value is about 10% larger than that of a Gaussian, indicating that contours through this PSF are weighted more heavily in their tails than a Gaussian distribution. The measured SR to FWHM ratio is somewhat larger at 1.74. All three resolution measures show the measurement to have worse resolution than the simulation. The measured FWHM is about 4% "worse" than the simulation, while its G and SR are about 8% worse. The larger differences in G and SR are due to noise in the data, which has little impact on the FWHM.

In Figure 5-12, the background of the simulated PSF is noticeably smoother than in the measurement. None of the radial or azimuthal ringing found in the simulated PSF of Figure 3-10a, where the ray-sums are not blurred, is present here. The smoother background in the blurred simulation illustrates the natural low-pass filtering provided by the finite camera resolution that is discussed in Section 3.4. From Equations (3-56) and (3-60), 32 angular bins and 73 second-detector positions are predicted to be necessary to avoid aliasing in the simulated PSF, but there are no signs of aliasing in this PSF with 64 angular bins and only 32 second-detector elements, illustrating the improved azimuthal sampling provided by the multi-element first detector. The RCC geometry for the measurement is identical to the simulation; thus, variations in the measured background are not a result of aliasing but random variations in the ray-sum counts and unaccounted for variations in the camera sensitivity.

The RFL algorithm has been applied to six measured PSFs. The measurements are chosen to cover a range of incident gamma-ray energies, RCC geometry parameters and contributions from contaminating events. In Table 5-4 the FWHM resolutions are shown for PSFs reconstructed using the standard BUTR filter and a PARZ filter. The FWHM resolution of the laminograms is also provided for comparison. The BUTR reconstruction reduces the FWHM by factors ranging between 1.70 and 1.92 relative to the laminogram, while the PARZ reconstruction reduces the FWHM by factors from 1.41 to 1.61. As anticipated, reconstruction with both filters results in much better FWHM resolution than the laminogram, and the BUTR filter gives significantly better resolution than the PARZ filter. Simulations indicate that if the point source is not centered in one of the pixels, the reported FWHM may be 0.033 cm (17% of the pixel width) larger than the FWHM from a centered source. There is no correlation between the horizontal and vertical positions, so the uncertainty in the average FWHM is $0.033 / \sqrt{2}$ or ± 0.023 cm: differences smaller than this value are not significant.

Table 5-4. FWHM resolutions (in cm) measured from six PSFs. Resolutions are listed for five different reconstruction algorithms: the laminogram from backprojection and summing, ramp-filtering of the laminogram using both a BUTR filter with $v_c = 0.33$ and $n=12$ and a PARZ filter with $v_{\max} = 0.5$, and ART reconstruction shown with and with out applying a Butterworth window ($v_c = 0.33$ and $n=12$) as a low pass filter. Parameters of the reconstructions are described in the text. Each tabulated FWHM value is an average of horizontal and vertical contours through the peak of the PSF.

Isotope	E_γ (MeV)	θ_c (deg)	N_2	Reconstruction Method				
				Laminogram	Ramp-filtering		ART	
					BUTR filter	PARZ filter	unfiltered	low-pass filtered
Sb-124	0.6027	44	16	1.63	0.85	1.01	0.82	0.84
Sb-124	1.691	44	16	1.24	0.71	0.88	0.72	0.73
Cs-137	0.6617	44	16	1.58	0.92	1.03	0.87	0.91
Cs-137	0.6617	59	16	1.57	0.86	1.02	0.84	0.87
Sb-124	0.6027	44	32	1.64	0.86	1.01	0.85	0.86
Sb-124	1.691	44	32	1.31	0.77	0.91	0.81	0.79

Values of SR recorded from reconstruction of the same six measured PSFs are reported in Table 5-5. Note that the number of net counts in each set of measured ray-sums is reported in this table, and the isotope identifier has been removed. The rate of the subtracted random background counts never exceeds 3.5% of the gross count rate for any of these measurements. The BUTR filter reduces SR by factors ranging from 2.64 to 3.09 relative to the laminogram, and the PARZ filter reduces SR by factors between 2.50 and 2.71. The smallest values in these ranges of improvement ratios correspond to the 1.69 MeV data set recorded with $N_2 = 16$, which only has 8005 net counts. For the BUTR filtered PSFs, the SR improvement ratio correlates with the number of counts in the data set: sets with the fewest counts have the largest relative statistical variation, indicating that SR is sensitive to image noise. The relationship between statistical variation in the

Table 5-5. Spectral resolutions (SR in cm) determined from the PSFs presented in Table 5-4. Resolutions are listed for the five different reconstruction algorithms described in Table 5-4. Each tabulated SR value is an average of horizontal and vertical contours through the peak of the PSF.

E_γ (MeV)	θ_c (deg)	N_2	Net Counts	Reconstruction Methods				
				Laminogram	Ramp-filtering		ART	
					BUTR filter	PARZ filter	unfiltered	low-pass filtered
0.6027	44	16	33829	4.63	1.51	1.71	1.48	1.48
1.6910	44	16	8005	3.83	1.45	1.53	1.20	1.23
0.6617	44	16	20394	4.30	1.54	1.66	1.48	1.47
0.6617	59	16	14358	4.37	1.53	1.69	1.49	1.46
0.6027	44	32	75944	4.63	1.50	1.72	1.55	1.52
1.6910	44	32	14551	3.93	1.41	1.51	1.31	1.32

net counts, image noise and each of the resolution measures is studied in Section 5.5.4, where the number of counts are varied in reconstruction of data from one measurement. The SR improvement ratio for the PARZ reconstructions also correlates with the number of net counts but not as strongly as the BUTR reconstructions.

The ratio SR:FWHM can be used to compare the shape of the PSF contours to a Gaussian. As pointed out earlier, SR:FWHM is 1.51 for a continuous Gaussian function, while a distribution with larger tails has a larger ratio. Statistical variation in the PSF contour also acts to increase this ratio. The average SR:FWHM ratio of the laminograms is 2.88 or about 91% larger than for the Gaussian, quantifying the blurring of the laminograms. The average SR:FWHM ratio of the BUTR and PARZ filter reconstructions are much smaller at 1.81 and 1.68. Although these ratios are larger than the Gaussian ratio by 20% and 11% respectively, they represent a significant improvement over the laminogram.

Table 5-6. PSF figure of merit G determined from the measured PSFs presented in Table 5-4. Resolutions are listed for the five different reconstruction algorithms described in Table 5-4.

Isotope	E_γ (MeV)	θ_c (deg)	N_2	Reconstruction Method				
				Laminogram	Ramp-filtering		ART	
					BUTR filter	PARZ filter	unfiltered	low pass filtered
Sb-124	0.6027	44	16	0.113	0.297	0.255	0.309	0.301
Sb-124	1.691	44	16	0.154	0.362	0.299	0.389	0.367
Cs-137	0.6617	44	16	0.123	0.280	0.250	0.293	0.283
Cs-137	0.6617	59	16	0.121	0.283	0.250	0.288	0.284
Sb-124	0.6027	44	32	0.116	0.298	0.256	0.297	0.297
Sb-124	1.691	44	32	0.146	0.344	0.289	0.342	0.338

In Table 5-6, the PSF figure of merit G is recorded for each of the data sets and reconstruction algorithms of Table 5-4. This resolution measure indicates that the BUTR reconstruction improves the resolution by factors ranging from 2.28 to 2.63 and the PARZ reconstruction by factors from 1.94 to 2.26. The G resolution improvement ratios fall neatly between those listed for the FWHM and the SR, and appears to offer a balanced resolution gauge incorporating both the peak width and weighting of the tails; whereas, the FWHM provides no information about tails or image noise, and SR strongly reflects the impact of PSF tails and noise.

In all three of the PSF resolution tables presented above, it is evident that ramp-filtering with a PARZ filter always significantly improves the resolution relative to the laminogram; however, the resulting resolution is worse than that from the BUTR filter, in all cases. It is apparent that the PARZ filter discards too much high frequency information in these PSFs, and that the resolutions present in the BUTR reconstructions better represents the resolution capability of the RCC.

Since the data sets presented in Tables 5-4 through 5-6 are included to show the system response to a range of incident gamma-ray energies and for different RCC geometries, it is important to discuss the differences between the resulting resolution measures. The first two sets are drawn from different windows on the same summed energy histogram from a ^{124}Sb point source with $\theta_c = 44^\circ$. The 0.603 MeV PSF has significantly worse resolution than the 1.691 MeV resolution by all measures. There are two likely causes for the poorer resolution at the lower energy: significant contaminating events in the 0.603 MeV summed energy window and large variation of the energy component of the angular resolution $\Delta\theta_e$ with incident gamma-ray energy. The relative contribution of contaminating events is predicted in Table 5-3 to be somewhat larger for the 0.603 MeV gamma rays than the 1.691 MeV gamma rays. The increased magnitude and perhaps the type of contaminating events may contribute to the difference in resolution. The value of $\Delta\theta_e$ is indeed expected to decrease with increasing energy as predicted with Equations (2-22) and (2-23) and shown in Figure 2-6. The measured resolutions are compared to the analytical model later in this section where the difference is found to be greater than predicted.

There are several contaminating event types which are possible for ^{124}Sb , as documented in Table 5-3. To help identify the cause of the poorer resolution in the 0.603 MeV PSF, it can be compared to the PSF of ^{137}Cs point source measured with $\theta_c = 44^\circ$. There is little effect from contaminating events in ^{137}Cs , which emits a single gamma ray of energy 0.662 MeV. Note that the resolution of the low energy ^{124}Sb laminogram is consistently worse than that of the ^{137}Cs laminogram. This is unexpected because the ^{137}Cs point source is actually a 0.48 cm diameter disk, while the ^{124}Sb source is irregularly shaped, but less than 0.3 cm in extent. The larger size of the ^{137}Cs source does manifest itself after application of the ramp filter, with the G and FWHM values now indicating worse resolution than the ^{124}Sb source. However, the SR values are comparable between the two sources, pointing to the presence of enhanced tails in the

^{124}Sb PSF, which may be an effect of contaminating events. There is also evidence that the poorer resolution at 0.603 MeV relative to 1.691 MeV is due to $\Delta\theta_e$. If the source sizes are assumed to add in quadrature with the FWHM of the PSF, subtracting the quoted sizes from the FWHM of the BUTR filtered PSFs yields "corrected" resolutions of about 0.79 cm for both. Thus, it appears that the change in PSF resolution between the two energies is due to both variation in $\Delta\theta_e$ and contaminating events.

Comparing the two ^{137}Cs PSFs, the FWHM resolution is better at $\theta_c = 59^\circ$ than at $\theta_c = 44^\circ$, but the other two measures give indistinguishable results. Thus, based on above discussions of these resolutions measures, the $\theta_c = 59^\circ$ PSF has a narrower peak, but appears to have larger (or noisier) tails.

Comparing the resolutions for the $N_2 = 16$ and $N_2 = 32$ cases, the FWHM resolution of the 1.691 MeV PSF is significantly better for $N_2 = 16$. There is no theoretical reason for the resolution to get worse as the number of second-detector elements is increased, particularly in this case, where the $N_2 = 32$ set includes some the data from the $N_2 = 16$ set interlaced with another set taken at $\phi_0 = 11.3^\circ$. Broadening of the $N_2 = 32$ PSF is likely due to misalignment of the source between the measurement of the two data sets, which were taken on different days. This misalignment is not as noticeable in the 0.603 MeV sets because the resolution is worse at that energy. While the FWHM is larger for $N_2 = 32$, the SR is smaller indicating that the PSF tails are either smaller or less noisy. The $N_2 = 32$ sets have more net counts with lower relative standard deviations, and based on the assumed relation between relative standard deviation and net counts, these PSFs are expected to have less noise. The reduced noise in the $N_2 = 32$ sets is also related to sampling. Figure 5-12b is a surface plot of the BUTR filter reconstructed PSF from the 0.603 MeV, $N_2 = 32$ data set, which is shown to provide adequate sampling for the entire 64 by 64 pixel source space. For a simulation done with parameters identical to the simulation of Figure 5-12a, except that $N_2 = 16$, only the center 30 pixel diameter region is adequately sampled. As a result, there is some azimuthal ringing at the edges of the 32 by

32 pixel source space used to calculate the resolution, increasing image noise in the $N_2 = 16$ PSFs and thus increasing SR.

5.5.3 Reconstruction with Iterative Algorithms

Tables 5-4 through 5-6 also include resolutions of PSFs reconstructed with the ART algorithm described in Section 3.5. The formulation of this algorithm is given in Equation (3-68), and it includes a non-negativity constraint on the image and the damping factor κ_i given in Equation (3-69). Recalling that the ART algorithm is simply an iterative matrix inversion routine, it is useful to recognize that the problem is over-specified by a factor of at least 4: there are 4096 unknown pixel intensities which are determined from 16384 ($=16 \times 16 \times 64$) ray-sums for the $N_2 = 16$ configurations. As a consequence of the over specified matrix, the value of κ_i will be smaller than that used in the exactly specified or square matrix case. For the $N_2 = 16$ configurations, a maximum κ_i of 0.03 is found to give good results. Since few of the ray-sums in the PSF data sets actually contain valid events from the point source, their weights are enhanced by choosing $\kappa_1/\kappa_2 = 2$; thus $\kappa_1 = 0.02$ and $\kappa_2 = 0.01$. For the $N_2 = 32$ configurations, the maximum value of κ_i is reduced by 2, while maintaining the same ratio between κ_1 and κ_2 to give $\kappa_1 = 0.01$ and $\kappa_2 = 0.005$. The algorithm is stopped after iteration k if

$$\left(e_{\text{rms}}^{(k-1)} - e_{\text{rms}}^{(k)} \right) / e_{\text{rms}}^{(k-1)} < 0.001 \quad (5-44)$$

To reach this convergence criteria with the six PSFs of Table 5-4, between 6 and 10 iterations are required.

Even with the small damping factors chosen here, the PSFs reconstructed with the ART algorithm still suffer from some salt-and-pepper noise. A low pass filter is applied to the image from the final iteration to reduce this noise, and to facilitate comparison with the RFL algorithm results, a high order Butterworth window with the same parameters as the standard BUTR filter ($v_c = 0.33$ and $n = 12$) is used.

Both the direct and iterative reconstruction techniques are shown in Tables 5-4 through 5-6 to successfully transform the blurry laminogram PSFs, with their $1/r$ tails, into narrower, Gaussian-like distributions. Generally, there is good agreement between resolutions from the RFL algorithm with the standard BUTR filter, and from the ART algorithm after applying an equivalent low-pass filter. The agreement between the RFL and iterative algorithms provides some validation of the RFL algorithm, because it has not been used before in reconstructing Compton camera images, while the ART algorithm has been used successfully [Sin90]. The exception to this general agreement comes from the SR measurement. The non-negativity constraint used to improve convergence of the ART algorithm is responsible for this difference, because pixels that would have negative magnitudes and contribute by increasing SR are set to zero. Further comparisons between the techniques will be made in Chapter VI using extended sources.

Iterative SIRT and MLE algorithms have also been investigated. The SIRT algorithm is very similar to the ART. The difference between these algorithms lies in how computed corrections are applied to the image. For ART, the estimated image is updated as the correction term for each ray-sum is computed, whereas, in SIRT the correction terms are summed in a separate array and their average is applied to the image only at the end of each iteration. The SIRT algorithm is supposed to yield a smoother image than ART, but it converges more slowly. The SIRT algorithm is stopped using the same criteria as the ART algorithm, but due to the slow convergence, a relaxation parameter is not used.

The RMS error and resolution of a reconstructed PSF are compared for ART and SIRT as a function of iteration in Figure 5-13. The data set used is from the ^{137}Cs point source measurement with $\theta_c = 59^\circ$ included in Tables 5-4 through 5-6. In part a, the improvement in the RMS error of the SIRT reconstruction after 23 iterations is seen to be only half that achieved with the ART algorithm in only 9 iterations. In part b, the resolution of the SIRT PSF only approaches the resolution of the PARZ filtered PSF, while

the resolution of the ART PSF quickly reaches and slightly exceeds the resolution of the BUTR filtered PSF. In addition to having a poor G, the tails of the SIRT PSF are more prominent than the PARZ filtered PSF. The SIRT algorithm appears to have little to offer in comparison to the other algorithms. If a very smooth iterative reconstruction is desired, a better result can be achieved by low-pass filtering the last ART PSF with a Parzen function, and with less than half the computations.

The MLE algorithm was developed by Dogan [Dgn93a] to run on either a CRAY-YMP or an Apollo workstation; its computation requirements are too great for it to be implemented on a personal computer like the others. As a consequence, only a limited number of reconstructions could be done using this algorithm. One MLE reconstruction has been done using ray-sum data from two ^{137}Cs point sources (one on-axis and one 5.8 cm off-axis) simultaneously collected with the RCC. The camera parameters include $\theta_c = 60^\circ$, $d = 30$ cm, $N_2 = 16$ and $z_s = 10$ cm. After 20 iterations, the on-axis point source reaches a FWHM resolution of 0.90 cm on a grid of 0.4 cm by 0.4 cm pixels, slightly better than the 0.92 cm value reported for the BUTR reconstruction in Table 5-4. Had the MLE reconstruction been done with a grid of smaller pixels and in the absence of the off-axis source, perhaps a better resolution would have been achieved, but simply obtaining the quoted results required 5 hours of CPU time on the CRAY. At this computational cost, further point source studies were not warranted.

5.5.4 Effect of Image Noise on Resolution Measures

The impact of image noise on the resolution measures, SR in particular, is mentioned above, and the topic of image noise is introduced in Section 3.1. Image noise is often reported quantitatively as the RMS uncertainty s_r in the image of a flat object, which is large relative to the resolution,

$$s_r = \frac{\left[\sum_{i=1}^{N_o} (f_i - \bar{f})^2 / N_o \right]^{1/2}}{\bar{f}} \quad (5-45)$$

where N_o is the number of pixels in the object, f_i is the intensity of the i^{th} pixel and $\bar{f}$ is the mean pixel intensity. For the point sources of concern in this chapter, s_r cannot be computed from Equation (5-45). However, Budinger and coworkers [Bud79] report that for either direct or iterative reconstruction of emission computed tomography images that

$$s_r \propto \frac{(N_o)^{3/4}}{(C_d)^{1/2}} \quad (5-46)$$

where C_d is the total number of counts in the data used to form the image. Since this group assumed that the counts are Poisson distributed, the $1/(C_d)^{1/2}$ term in Equation (5-46) is equal to σ_{C_d}/C_d where σ_{C_d} is the standard deviation of C_d . Thus, the relative RMS uncertainty between two images of identical, large, flat objects is specified by the fractional uncertainty σ_{C_d}/C_d of the total number of counts. The requirement that the image object be large and flat is only necessary to provide an adequate number of samples for computing s_r directly from the data; therefore, the relationship between the fractional uncertainty in the counts and the image noise can be extended to identical objects which have arbitrary size and intensity distributions. This implies that the magnitude of σ_{C_d}/C_d can serve as a useful gauge of the image noise in reconstructed PSFs.

For the RCC, net counts after background subtraction are used to form the ray-sums. The net counts C_n are computed from the gross counts C_d and random background counts C_{rcb} using the weighting factor from Equation (5-24).

$$C_n = C_d - W_b C_{rcb} \quad (5-47)$$

Both C_d and C_{rcb} are Poisson distributed and the fractional uncertainty can be computed using standard error propagation techniques.

a)

b)

Figure 5-13. Comparisons of a) RMS error e_{rms} and b) resolution figure of merit G as functions of iteration k for reconstruction of an example PSF with the ART and SIRT algorithms. Resolutions are also compared to RFL algorithm results from Table 5-6. The data are from a ^{137}Cs point source with RCC parameter $\theta_c = 59^\circ$. Other measurement parameters are described in Table 5-6.

Figure 5-14. Measures of resolution in reconstructed PSFs versus fractional uncertainty σ_n/C_n for four subsets of data drawn from the same measurement. Data is from a the 0.603 MeV gamma rays of a ^{124}Sb point source on the RCC ring axis. The RCC parameters include $\theta_c = 44^\circ$, $d = 30$ cm, $N_2 = 16$ and $z_s = 10$ cm.

$$\frac{\sigma_n}{C_n} = \frac{(C_d + W_b^2 C_{rcb})^{1/2}}{C_d - W_b C_{rcb}} \quad (5-48)$$

Based on Equations (5-46) and (5-48), the effect of noise on the resolution measures can be studied by varying the number of net counts from a single data set. The 0.603 MeV ^{124}Sb data set from Table 5-4 with $N_2 = 16$ is used along with three subsets of this 5402 s long run, which contains 33829 net counts with a fractional uncertainty of 0.56%. The three subset of 305 s, 605 s and 1825 s in duration result in net counts of 1913, 3791, and 11403 with fractional uncertainties of 2.32%, 1.65% and 1.00%, respectively. The random background counts make up about 3% of the gross counts in this measurement. These four sets of ray-sums have been reconstructed using the RFL algorithm with the standard BUTR filter, and resolutions extracted from these PSFs are plotted versus fractional uncertainty in Figure 5-14. The magnitude of SR is shown to increase monotonically with σ_n/C_n , and among the resolution measures, it shows the largest variation over the range of σ_n/C_n values. This substantiates the observation from

Figure 5-15. Radial contours through MTFs of laminograms from the 0.603 MeV gamma rays of an on-axis ^{124}Sb point source showing the frequency distribution of noise. The laminograms are reconstructed from data taken during one measurement and are identical except for the number of counts in the ray-sums and the commensurate difference in the fractional uncertainty σ_n/C_n in these counts.

Table 5-5 that SR is very sensitive to image noise. The value of G shows that the resolution is degraded by 9.8% at the highest fractional uncertainty but is nearly constant at lower fractional uncertainties. The FWHM simply fluctuates as the fractional uncertainty increases, by as much as 8% from its value at $\sigma_n/C_n = 0.56\%$. These fluctuations contribute to uncertainty in the measured FWHM for data sets with higher fractional uncertainties. The results of this study of noise in the PSFs indicate that a fractional uncertainty of less than 1.65% is needed to get consistent resolution measures.

In Section 3.1, the fact that Poisson noise will dominate the signal at high frequencies in the MTF of the laminogram from a large, uniform object is discussed. It was asserted there that in the laminogram of any object, not just a large uniform one, the noise increases relative to the signal with increasing frequency. To prove this assertion, MTFs of laminograms are computed from the ^{124}Sb data sets with 0.56% and 2.32% fractional uncertainties from Figure 5-14. Radial contours through these MTFs are compared in Figure 5-15. The MTFs are nearly identical at low frequency, but the

magnitude of the high fractional uncertainty MTF increases relative to the other. Because these images are drawn from the same data set, the only plausible explanation for the divergence at high frequency is the increased statistical noise in the MTF with high fractional uncertainty. Since noise dominates the signal at higher frequencies in point source and in large, flat objects, it can reasonably be assumed that any other object will exhibit the same behavior.

5.5.5 Off-Axis Resolution

All data presented thus far are measured from on-axis point sources. It is important to investigate the deviation of the off-axis PSF from the on-axis characteristics, because Fourier filtering, inherent in the RFL algorithm, assumes that the system response is invariant to a shift in the source position. With its curved lines of integration and spatially varying bin width, the PSF of the RCC is spatially variant. This spatial variation has already been demonstrated via simulation in Figure 3-10. However, as pointed out in Section 3.1, many straight-ray tomography systems also suffer from spatially variant PSFs.

A comparison between the on-axis and off-axis point response can be made by viewing the surface plots presented in Figure 5-16. In both measurements, the second-detector element at $\phi = 270^\circ$ (positioned vertically below the first detector) could not be used. A design flaw prevented this element from being properly placed with $\theta_c > 55^\circ$, but a modification was made before any extended source images were taken. The missing ray-sums from this element pass horizontally through the on-axis source position. As a result, the on-axis PSF in Figure 5-16 is asymmetric with a horizontal FWHM of 0.85 cm and a vertical FWHM of 0.91 cm. The average FWHM at 0.88 cm is in good agreement with the measured values for similar energy gamma rays in Table 5-4. SR is 2.36 and 2.27 cm in the horizontal and vertical directions, respectively, and G is 0.260. These resolutions are worse than the results presented earlier because the laminogram is formed on a larger 128

by 128 pixel plane (pixel width remains 0.2 cm) and is truncated to 64 by 64 pixels to accommodate the off-axis source. If the source space is truncated to 32 by 32 pixels, agreement with the resolutions of Table 5-5 and Table 5-6 is good. A simulated PSF employing the same parameters as those of Figure 5-16a (including $N_2 = 15$) and a Gaussian blurring with $\Delta\theta = 5^\circ$ yields 0.88 and 0.96 cm for the horizontal and vertical FWHM. This simulation gives similar resolutions as measured by SR and G and also reproduces the oscillations of the measured PSF.

The off-axis point source of Figure 5-16b is positioned 4.8 cm away from the ring axis, and the source plane is at $z_s = 8.7$ cm. The polar angle α of the point source relative to the ring axis is then 29° , and the total distance from the center of the first detector to this source is then 9.9 cm, close to the distance of the on-axis source at 10 cm. The horizontal (or radial) FWHM increases by 27% from the on-axis value to 1.08 cm, while the vertical (or tangential) FWHM decreases by 8% to 0.84 cm for an average FWHM of 0.96 cm. SR values in this off-axis PSF are 2.40 cm in the horizontal direction and 1.78 cm in the vertical direction, and G is 0.245. A simulation of this off-axis PSF yields horizontal and vertical FWHM resolutions of 1.03 cm and 0.90 cm. The other resolution measures are also in reasonable agreement between measured and simulated PSFs. The shape of the peak and the low intensity oscillations in Figure 5-16 are reproduced in that simulation.

The angular resolution corresponding to the horizontal FWHM is computed using Equation (5-37) to be 4.87° in the on-axis PSF and 5.45° in the off-axis PSF. Thus, the angular resolution increases by only 12% compared to 27% for the spatial resolution, illustrating that most of the increase in spatial resolution is due to the RCC localization cones intercepting the source plane obliquely at the off-axis location while they intercept it normally at the on-axis position. The angular resolutions corresponding to the vertical FWHM is computed using Equation (5-38) to be 5.21° for the on-axis PSF and 4.84° for the

Figure 5-16. Surface plots comparing reconstructed PSFs measured from a) an on-axis, ^{124}Sb point source and b) an off-axis, ^{124}Sb point source. The RCC parameters include $E_\gamma = 0.603$ MeV, $\theta_c = 59^\circ$, $d = 30$ cm and $N_2 = 15$. The source space distance is $z_s = 10$ cm in a) and $z_s = 8.7$ cm in b). The RFL algorithm with a standard BUTR filter is used to reconstruct these PSFs.

off-axis PSF. The average angular resolutions thus agree within 3%, less than the uncertainty in the measured FWHM resolutions.

While the angular resolution variation with the polar angle α is shown to be small, the spatial resolution variation is significant, and may cause distortions in extended images reconstructed with the RFL algorithm. However, the PSFs of several PET devices with ring-shaped detector arrays show a similar radial elongation at off-axis positions [Der88, Lec90, Evn91]. In one device the PSF is 71% wider in the radial direction than in the tangential near the edge of the FOV. For each of these devices, ramp-filtering has been used successfully to reconstruct images. Ultimately the importance of the spatially variant PSF of the RCC will be born out in the extended images of Chapter VI.

5.5.6 Resolution Variation with Energy and Geometric Parameters

In the introduction, some demanding aspects of gamma-ray imaging at a nuclear industrial site are identified, among them is imaging of polychromatic fields over the 0.5

to 3.0 MeV energy range. Variation in the efficiency of the RCC over this range in E_γ is shown in Figure 5-6 of Section 5.4, and here variation of the total angular resolution $\Delta\theta_t$ with E_γ is shown in Figure 5-17 for central scattering angles $\theta_c = 44^\circ$ (part a) and $\theta_c = 60^\circ$ (part b). This figure also compares measured values of $\Delta\theta_t$ to predictions from the analytic model of Section 2.3. RCC parameters common to all measurements include $d = 30$ cm and $z_s = 10$ cm. For all measurements at $\theta_c = 44^\circ$, N_2 is 16, while for measurements at $\theta_c = 60^\circ$ and E_γ of 0.603, 1.37, 1.69 and 2.75 MeV, N_2 is 15. In discussions regarding Figure 5-16a the average FWHM for $N_2 = 15$ and $N_2 = 16$ are found to be comparable. Two measurements shown in part b of Figure 5-17 at E_γ of 0.511 and 0.662 MeV have $N_2 = 16$ but θ_c is 59° . The difference in resolution between scattering angles of 59° and 60° is negligible as will be shown later in this section. Error bars in this figure reflect the 0.047 cm uncertainty in determining the average spatial FWHM described in the discussion of Table 5-4. The PSFs are reconstructed using the RFL algorithm with the standard BUTR filter, and measured $\Delta\theta_t$ values are found using Equation (5-39) with the average FWHM taken as Δr .

Note that predictions are made for each measured E_γ in Figure 5-17, but that these predicted resolutions do not smoothly vary with E_γ . In the measurements the gain has been varied such that the energy width of the channels in the first detector ranged between 0.0037 and 0.0102 MeV. Because these widths are significantly larger than the FWHM energy resolution of the first detector (see Equation (4-2)), they are taken to be the energy resolution in predicting $\Delta\theta_e$ and are responsible for the discontinuous variation of these predicted resolutions. The measured and predicted values of $\Delta\theta_t$ shown in this figure along with the average channel widths are listed in Table C-3 of Appendix C. That table also lists the spatial resolution measures, net data counts and fractional uncertainty for each measured PSF. Continuous curves are included in this figure to show how $\Delta\theta_t$ is expected to vary with E_γ had the channel widths been held fixed at 0.0102 MeV and that

a)

b)

Figure 5-17. Comparison of measured and predicted total angular resolution $\Delta\theta_t$ as a function of incident energy E_γ at central scattering angles of a) $\theta_c = 44^\circ$ and b) $\theta_c = 60^\circ$. Measured $\Delta\theta_t$ values had varying first-detector energy resolutions and predicted $\Delta\theta_t$ values shown as triangles employ the energy resolution of the measurement. The continuous curve shows how the total resolution is predicted to vary if the first-detector resolution is held fixed at the maximum measured value. Error bars reflect the uncertainty in determining the spatial FWHM.

the total resolution is dominated by the geometric component $\Delta\theta_g$ over the entire energy range.

The measured and predicted values of $\Delta\theta_t$ in Figure 5-17 generally agree within the uncertainty for energies of 1.37 MeV and above. In part b, the prediction does not overlap the measurement at energies of 1.37 and 2.75 MeV. The measurement at $E_\gamma = 2.75$ MeV is drawn from a data set where the fractional uncertainty of the net counts σ_n/C_n is greater than 2.5%, and is quite noisy as a result. A PSF with a similar value of σ_n/C_n is shown in Figure 5-14 to have a FWHM which varies by 8% from the least noisy PSF. As a result of the large noise in this PSF, the uncertainty in the measured resolution at this energy is larger than reflected by the error bars. There is no obvious reason why the measured resolution is high at $E_\gamma = 1.37$ MeV in part b. Discrepancies in the resolutions at 0.511 MeV and 0.662 MeV at both scattering angles are due to the sizes of the ^{64}Cu and ^{137}Cs "point" source at 0.4 cm and 0.48 cm, respectively. The resolution measured from the 0.603 MeV gamma rays of ^{124}Sb is larger than predicted because it suffers from a large number of contaminating events. PED and full energy events from the low intensity 0.646 MeV and 0.723 MeV gamma rays of ^{124}Sb may be particularly responsible for these broad PSFs, because they cause peaks in the pulse height spectra of the first detector near the peak from the gamma ray of interest. Other types of contaminating events such as cascade coincidence and random coincidence create flat continua in the first-detector pulse height spectra that are removed with the ramp filter.

To this point all of the PSFs have been reconstructed based on a priori knowledge of E_γ . Likewise, predictions of $\Delta\theta_e$ have been computed using Equation (2-22), which assumes no (significant) uncertainty in E_γ . However, the value of E_γ can be determined with enough accuracy from measured summed energy spectra such that there is little impact on $\Delta\theta_t$. As an example, consider the PSF from the 1.3686 MeV gamma rays of ^{24}Na at $\theta_c = 60^\circ$, where the centroid of the measured energy peak is at 1.3606 MeV. Using this a posteriori measured energy to form a laminogram instead of the known energy and

Figure 5-18. Comparison of measured and predicted total angular resolution ($\Delta\theta_t$) as a function of scattering angle θ at a fixed E_γ of 0.662 MeV. Other RCC parameters are fixed at $d = 30$ cm and $z_s = 10$ cm. Measured resolutions are derived from RFL reconstructions with $N_2 = 16$ and also a single second element backprojection method described in the text. Vertical error bars reflect uncertainty in the spatial FWHM, and horizontal error bars show range of θ due to off-axis positions of first-detector elements. Predicted values of energy and geometry components $\Delta\theta_e$ and $\Delta\theta_g$ are also shown.

applying the standard BUTR filter, gives a PSF with an average FWHM of 0.779 cm and thus $\Delta\theta_t = 4.46^\circ$. Using the known E_γ , gives a PSF with an average FWHM of 0.777 cm and $\Delta\theta_t = 4.45^\circ$. The close agreement between these measured resolutions is predicted. An expression is derived in Equation (2-24) to predict $\Delta\theta_e$ when the incident gamma-ray energy is uncertain. Taking the difference of 0.008 MeV between the measured and known values of E_γ as the uncertainty in E_γ , $\Delta\theta_t$ is predicted to be 4.20° , and using the known E_γ and Equation (2-22) for $\Delta\theta_e$, the predicted $\Delta\theta_t$ is 4.15° . This example demonstrates that the RCC is capable of imaging incident gamma rays of unknown energy without the assistance of another spectrometer.

Resolution is shown as a function of scattering angle in Figure 5-18, where the scattering angle has been changed by varying θ_c . The measured resolution has been

determined using the RFL algorithm on ray-sums from the 0.662 MeV gamma rays of ^{137}Cs , also by backprojecting ray-sums from a single element of the second detector.

These single-element backprojections were measured when the Compton camera was in a demonstration configuration with only 8 elements in the second-detector ring: the RFL algorithm

gives poor results with so few detectors. To determine $\Delta\theta_t$ from the single-element backprojection, the spatial FWHM is found from a contour passing perpendicularly through the backprojected ray-sums from a single element of the second detector and all of the first-detector elements. From the FWHM and Equation (5-39), $\Delta\theta_t$ is computed.

Figure 5-19 shows an example of the spatial FWHM determined from single-element backprojection.

The "single b.p." measured resolutions in Figure 5-18 are the average of at least three independent single-element backprojections with second-detector elements at $\phi = 0^\circ$, 90° and 180° . At $\theta = 30^\circ$ and $\theta = 45^\circ$, a fourth single-element backprojection from $\phi = 270^\circ$ is also incorporated. These measured resolutions agree with the RFL results to within the quoted uncertainties. All measured values of $\Delta\theta_t$ are consistently higher than predictions by about 20% due to the 0.48 cm diameter of the ^{137}Cs disk source. If, as done earlier, the source size is subtracted by assuming that it adds in quadrature with the FWHM, the average measured $\Delta\theta_t$ falls to 4.4° . This correction brings the measured resolutions into agreement with the predictions which do not incorporate the source size.

The predicted geometric and energy components of the angular resolution are included in Figure 5-18 with $\Delta\theta_g$ being dominant for the scattering angles between 10° and 90° . There is little variation in $\Delta\theta_g$ over the displayed range of θ , and as a result, $\Delta\theta_t$ varies by less than 0.7° . This consistency in the angular resolution is advantageous for off-axis sources which involve a range of scattering angles, and is responsible for the good agreement in angular resolution noted in Figure 5-16 between on-axis and off-axis sources.

Figure 5-19. a) Gray-scale image of backprojected ^{137}Cs events in one NaI detector with $\theta_c = 45^\circ$, $\phi = 0^\circ$ and $z_s = 10$ cm. Pixels are 0.2 cm wide. b) Horizontal and vertical contours through the image showing the spatial FWHM.

In Section 5.4 the intrinsic efficiency is shown to depend strongly on the RCC crystal separation distance d . Figure 5-20 shows that $\Delta\theta_t$ also has a strong dependence on d . Measured resolutions in that figure are derived from the average of four single-element backprojections of ray-sums from the 0.662 MeV gamma rays of a ^{137}Cs point source. RCC parameters held fixed include $\theta_c = 45^\circ$ and $z_s = 10$ cm. Discrepancies between measured and predicted values of $\Delta\theta_t$ are again due to the 0.48 cm diameter of the "point" source, and again by assuming that the source size adds in quadrature with the spatial FWHM, the source size can be removed from the measure $\Delta\theta_t$. After subtracting the source size, the measured resolutions at $d = 17.5$ cm and 45 cm are 6.7° and 3.6° in good agreement with the predicted resolutions of 6.6° and 3.2° . Figure 5-20 includes predicted values of $\Delta\theta_g$, computed from Equation (2-18), which drives the variation of $\Delta\theta_t$ with d . Because $\Delta\theta_g$ and its components are generally below 10° , Equation (2-18) can be rewritten using the approximation $\tan(\Delta\theta) \cong \Delta\theta$.

$$(\Delta\theta_g)^2 \cong (2\Delta\theta_{W_1})^2 + (2\Delta\theta_{L_1})^2 + (2\Delta\theta_{R_2})^2 \quad (5-49)$$

Figure 5-20. Comparison of measured and predicted $\Delta\theta_t$ as a function of d . Remaining parameters are held fixed at $E_\gamma = 0.662$ MeV, $\theta_c = 45^\circ$, and $z_s = 10$ cm. Measured resolutions are derived using single-element backprojection and error bars reflect the spatial uncertainty in determining the FWHM. Predicted values of $\Delta\theta_t$ and $\Delta\theta_g$, and the three geometry components, $\Delta\theta_{W1}$, $\Delta\theta_{L1}$, and $\Delta\theta_{R2}$, are shown.

Thus, $2\Delta\theta_{W1}$ represents the contribution of the first-detector element width to the geometric component of the angular resolution; likewise, $2\Delta\theta_{L1}$ and $2\Delta\theta_{R2}$ represent contributions of the first-detector element length and second-detector element radius to $\Delta\theta_g$. The predicted contributions to $\Delta\theta_g$ from these components are also shown in Figure 5-20 where the second-detector element radius is seen to be the dominant factor in the total angular resolution.

5.6 Comparison to a Pinhole Camera

To provide a simple comparison, the measured point response of the RCC is compared to a hypothetical imaging device with a pinhole aperture collimating the detector of an Anger camera, called a "pinhole camera". Most of the sources in this study have been placed 10 cm from the first detector of the RCC, so the same distance is taken for the source to pinhole spacing. For simplicity the pinhole to detector distance is taken

as 10 cm also. Thus, full size images of the source will fall on the detector. A common Anger camera detector of 0.95 cm thick NaI(Tl) is assumed with an intrinsic spatial resolution of 0.3 cm. For the RCC with $\theta_c = 60^\circ$ and $d = 30$ cm, the resolution of a 0.511 MeV PSF has been measured to be 0.79 cm after correcting for the size of the ^{64}Cu source. With the specified spacing between the source, pinhole and detector, a point at the center of the source plane will cast a shadow on the detector that is twice the diameter of the pinhole. Assuming that the total image resolution in the source plane is the quadrature sum of the pinhole shadow diameter and the intrinsic resolution, a pinhole diameter of 0.37 cm gives a total resolution of 0.79 cm.

Now that a camera with a resolution equivalent to the RCC has been specified, the efficiencies can be compared. For this Anger camera to image multi-energy gamma-ray fields like the RCC, only full energy events can be accepted. The intrinsic peak efficiency ϵ_{ipa} of the 0.95 cm thick NaI crystal is approximately 0.12 for 0.511 MeV photons. This efficiency is interpolated from values for 0.63 cm thick and 1.27 cm thick crystals provided by Anger and Davis [Ang64]. The solid angle of the pinhole relative to the center of the source plane Ω_{ph} is 1.08×10^{-3} Sr, so the absolute peak efficiency ϵ_{apa} is 1.03×10^{-5} about a factor of 11 higher than the efficiency ϵ_{acc} of the RCC, which has been measured as 9.54×10^{-7} at 0.511 MeV.

A more likely detector for a pinhole camera measuring 0.511 MeV photons is a 2.5 cm thick NaI crystal like those used in the PET machine PENN-PET. These detectors have an intrinsic resolution of 0.6 cm [Mue86]. To match the total resolution of the RCC, the pinhole diameter must be reduced to 0.26 cm giving a solid angle of 5.31×10^{-4} Sr. The value of ϵ_{ipa} for this detector is 0.365 at 0.511 MeV [Ang64], yielding an ϵ_{apa} of 1.54×10^{-5} , 16 times that of the RCC. Thus, in imaging a point source, the RCC would have to dwell 16 times longer than the hypothetical Anger camera with the 2.5 cm thick detector. The comparison worsens when extended sources are considered. The RCC results in noisier images than the simple pinhole camera because of the image reconstruction process.

Singh and coworkers found that about a factor of two more photons must be collected with a Compton camera than a mechanically collimated camera to generate an image of a flood-field phantom with equal resolution and noise [Sin90]. They used an ART algorithm similar to the one used in this work, which is somewhat noisier than the RFL algorithm. As a result, the RCC would have to dwell approximately 32 times longer than the hypothetical pinhole camera to image a flood-field phantom at 0.511 MeV.

The efficiency comparisons are not favorable for the RCC at these low energies; however, the relative efficiency of the RCC improves at the highest energies. For the RCC with $\theta_c = 60^\circ$, the efficiency falls by a factor of 14 from 0.511 MeV to 2.75 MeV. No comparable studies have been found for an Anger camera crystal at these energies; however, the intrinsic peak efficiency of a 2.5 cm diameter by 2.5 cm long cylindrical NaI crystal decreases by about a factor of 18 at 2.75 MeV relative to 0.511 MeV [Kno89]. What remains undetermined is the degradation of the intrinsic resolution of the Anger camera detector due to scattering of the 2.75 MeV gamma rays in the detector. Anger and Davis have computed that 86% of full energy events from 0.511 MeV gamma rays deposit all their energy within 0.5 cm of their initial interaction location in a 2.5 cm thick NaI scintillator [Ang64]. This fraction drops to 83% at 0.662 MeV [Ang64] and will continue to fall as the energy increases, because the photoelectric absorption cross section decreases relative to the Compton cross section with increasing energy beyond 0.662 MeV. The intrinsic resolution of the camera will deteriorate as events from high energy gamma rays deposit larger fractions of their energy further from the initial interaction point. Without benefit of computations or measurements it is difficult to determine exactly how much the resolution will change from 0.511 MeV. To place an upper limit on the pinhole camera efficiency at 2.75 MeV the resolution is assumed to be unchanged from 0.511 MeV, and the pinhole diameter is held fixed, yielding an efficiency about 25 times that of the RCC.

The RCC efficiency can be improved by increasing the number of first detector elements, while the camera is limited to one pinhole. Specific recommendations for

increasing the RCC efficiency are given in Section 7.2.1. In addition to the efficiency there are other severe problems with this pinhole camera designed to image 2.75 MeV gamma rays. Consider a lead pinhole collimator which allows a 20 cm diameter FOV at the source plane and is 9.5 cm thick to attenuate 99% of the 2.75 MeV gamma-rays. There will large contrast loss near the center of this collimator because the necessary 9.5 cm thickness can only achieved at distances beyond 5 cm from the collimator center, and the camera will have little utility. In addition, a cone-shaped collimator meeting these specifications has a mass of 135 kg. Thus, a pinhole camera with an efficiency about 25 times higher than that of the RCC at 2.75 MeV has been hypothesized, but due to these difficulties such a device is impractical.

CHAPTER VI

RCC IMAGING CHARACTERISTICS

Although the point response of the RCC has been presented in detail in Chapter V, it is necessary to image spatially extended sources to fully demonstrate the capabilities and characteristics of any imaging device. To quantify the capabilities of the RCC, resolution and smoothness have been measured from images of five spatially-extended, two-dimensional sources (or phantoms): a ^{24}Na line source, a ^{24}Na bar source, a ^{65}Zn bar source, a multi-energy phantom and a ^{24}Na flood-field phantom. Where possible, the spatial FWHM is reported for these images as a measure of the RCC resolution. The smoothness of the images is reported using the RMS uncertainty s_r , as defined in Equation (5-45), for portions of each image. The RFL algorithm is used to reconstruct all of the images, and is compared to the iterative techniques in some cases for validation. One of the measured images is compared to a simulation to help identify the cause of any artifacts, as well as showing how statistical variations in the ray-sums affect image quality.

The ^{24}Na line source is a 5.1 cm long by 0.2 cm wide wire and is sketched in Figure 6-1a. This line source was positioned 10 cm from the RCC first detector during the imaging measurement (as were all of the measurements presented in this chapter), and it is essentially a one-dimensional line, because the 0.2 cm width is much smaller than the FWHM resolution of the RCC at 0.6 cm or greater. The line source is an important transition between the zero-dimensional point sources of Chapter V and the remaining four sources of this chapter which are truly two-dimensional.

The next source considered is a ^{24}Na bar source made of three parallel wires and shown in Figure 6-1b. This source demonstrates resolution behavior off the RCC ring axis, and the ability of the RCC to image a two-dimensional source of 2.75 MeV gamma rays

Figure 6-1. Sketches of spatially extended sources: a) ^{24}Na line source, b) ^{24}Na bar source, c) ^{65}Zn bar source, d) multi-energy phantom and e) ^{24}Na flood-field phantom.

which have an energy near the top of the energy range of interest. Another bar source consisting of four ^{65}Zn wires was also imaged; it is sketched in Figure 6-1c. This source demonstrates that gamma-ray emitting sources separated by distances approaching the system resolution (about 0.9 cm at this energy) can indeed be resolved along their entire length.

Another source consisting of ^{65}Zn wires arranged in the shape of a "U" and ^{64}Cu wires arranged in the shape of an "M" is sketched in Figure 6-1d. This multi-energy phantom demonstrates the ability of the RCC to spectrally separate gamma rays emitted by (and generate images of) spatially extended sources. Comparisons with the ^{65}Zn bar source allow study of effects due to the presence of the ^{64}Cu gamma-rays.

Finally, a large disk-shaped source was imaged. The outside diameter of this water-tight phantom is 14 cm with two smaller disks of 3.5 cm diameter inside the larger disk, as shown in Figure 6-1e. The large disk was filled with an aqueous solution of ^{24}Na ; one smaller disk was filled with a ^{24}Na solution having twice the specific activity of that in the large disk, and the other small disk was filled with water. The two small disks form "hot" and "cold" spots inside the background of the larger disk, and provide a means for measuring contrast recovery.

Gamma-ray energies and intensities for radioisotopes used in these sources are included in Table 5-1.

6.1 ^{24}Na Line Source

The line source provides a one-dimensional transition between the point spread functions (PSFs) measured in the previous chapter and the remaining sources of this chapter which are all two-dimensional. Much as reconstructed images of point sources are known as point spread functions, images of a line source are known as *line spread functions* [Bar81]. Possible variations in the resolution along the length of the line can be determined from profiles taken across the line spread function. Also, the MTF can be determined from a contour through the DFT of the line spread function. This contour must be perpendicular to the direction of the line and through the position of the DC component [Bar81].

The ^{24}Na line source is 5.1 cm long by 0.2 cm wide in the imaging plane and is 0.6 cm deep in the direction perpendicular to the imaging plane. This source geometry results

from six 2.5 cm long by 0.2 cm diameter wires with pairs of the wires placed end-to-end and the pairs then stacked three deep perpendicular to the image plane. At the beginning of the data run the total source activity was 8.10 MBq. The activity decayed to 1.12 MBq at the end of the run, about 42 hours later. Variation in the activity was measured between 0.5 cm sections of one of the 2.5 cm long wires using an HPGe detector with calibrated efficiency, and the standard deviation was found to be 9.0%.

During the imaging measurements the source was positioned horizontally 10 cm in front of the center of the first detector and 0.2 cm above the ring axis. The RCC system configuration and image plane parameters are given in Table 6-1. This table shows that the angular range of the ray-sums used to reconstruct images of the 1.37 and 2.75 MeV gamma rays from the line source falls well within the angular range ($\theta_l < \theta < \theta_u$) imposed by the lower energy thresholds on the two detectors (E_{1min} and E_{2min}). The angular width of the ray-sums $\delta\theta$ listed in this table is 1° , which has been chosen so that the spatial width of the ray-sums remains smaller than the pixel width of 0.2 cm out to a radius of r_{FOV} . By choosing $\delta\theta$ this way, it is possible to make direct comparison of resolutions between the line spread functions and the PSFs. In this table, the number of ray-sums M_θ is shown to be 56. Using Equation (3-57) with $a = r_{FOV}$, the requisite number of M_θ is found to be only 17, less than one-third of the number used. Similarly, the number of second detector positions M_ϕ required is computed to be 57 from Equation (3-61). The value of M_ϕ actually used is 32, significantly less than the requirement value. While it appears that this would result in severe undersampling in the azimuthal direction, 32 second detector positions are shown to provide adequate sampling out to radius $a = 6.4$ cm in the simulation of Figure 5-12a.

In Figure 6-2 a vertical contour through the MTF from the 1.37 MeV gamma rays of the line source is compared to radial contours through the MTF from the 1.37 MeV gamma rays from a ^{24}Na point source. The configuration of the RCC for the point source measurement is approximately the same as those for the line source listed in Table 6-1. It

is apparent from the figure that the frequency response of the RCC to the line source is in general agreement with the frequency response to the point source. Differences between the MTF of the line and point sources are primarily due to the fact that M_ϕ is 32 for the line source measurement and only 15 for the point source. In fact, the difference between the two radial contours in the point source MTF are at least as great as the differences between the line and point source MTFs. The differences in the point source contours are due to asymmetries associated with having only 15 elements in the second detector. (See Figure 5-16 and associated text for further explanation.)

Reconstructed images of both the 1.37 and 2.75 MeV gamma rays emitted by the line source are shown as surface plots in Figure 6-3. These images are reconstructed using the RFL algorithm with the standard BUTR filter ($v_c = 0.33$ and $n = 12$). The resolution can be measured along the length of the line source by taking vertical contours at each horizontal position. The line source is centered horizontally on the ring axis and approximately covers the range $-2.5 \text{ cm} \leq x \leq 2.5 \text{ cm}$. The mean FWHM resolution of the 1.37 MeV image is computed to be 0.830 cm with a standard deviation of 0.042 cm over the slightly reduced range $-2.3 \text{ cm} \leq x \leq 2.3 \text{ cm}$. The FWHM resolution of the point source at this energy is $0.779(\pm 0.023)$ cm, which agrees with the resolution measured from the line source to within the uncertainty in the measurements. The mean FWHM resolution in the 2.75 MeV image (over the same range of x) is $0.757(\pm 0.075)$ cm, and is larger than the FWHM resolution of $0.645(\pm 0.023)$ found from the point source. It is likely that the resolution determined from the point source is erroneously small due to the small number of counts recorded in that measurement.

The FWHM resolution of both the low and high energy images of the line source are shown as a function of x in Figure 6-4. The vertical position of the peak center y_c is also shown as a function of x , and is found by taking the average of the positions where the peak falls to half its intensity.

Table 6-1. RCC system parameters for ^{24}Na line source measurement and source space parameters used in image reconstruction.

Description	Symbol	Magnitude	Unit
Source spacing	z_s	10	cm
Detector spacing	d	30	cm
Central scattering angle	θ_c	58	deg
FOV radius needed to image source without distortion	r_{FOV}	3.7	cm
Polar angle corresponding to r_{FOV}	α	27	deg
Lower energy threshold in first detector	$E_{1\text{min}}$	0.10	MeV
Lower bound scattering angle for E_γ^a (=1.37 MeV)	θ_ℓ^a	13.9	deg
Lower bound scattering angle for E_γ^b (=2.75 MeV)	θ_ℓ^b	7.3	deg
Min. scattering angle used	θ_{min}	30	deg
Lower energy threshold in second detector	$E_{2\text{min}}$	0.15	MeV
Upper bound scattering angle for E_γ^a (=1.37 MeV)	θ_u^a	180	deg
Upper bound scattering angle for E_γ^b (=2.75 MeV)	θ_u^b	180	deg
Max. scattering angle used	θ_{max}	86	deg
Summed energy window for E_γ^a	E_s^a	1.282 to 1.454	MeV
Summed energy window for E_γ^b	E_s^b	2.661 to 2.847	MeV
Ray-sum angular width	$\delta\theta$	1	deg
Number of ray-sums	M_θ	56	
Number of second detector positions	M_ϕ	32	
Pixel width	Δr	0.2	cm
Number of pixels	N_x, N_y	64

Figure 6-2. Vertical contour through MTF of the ^{24}Na line source laminogram compared to radial contours through MTF of a ^{24}Na point source laminogram.

$$y_c = (r_h + r_l)/2 \quad (6-1)$$

r_h and r_l are defined in Equation (5-37). Figure 6-4 shows that there is no measurable variation in resolution at positions away from the ring axis, where $x = 0$. There is more variance in the high energy image resolution than in the low energy image resolution because the high energy ray-sums contain fewer counts, and the relative uncertainty in the net counts (σ_n/C_n) is correspondingly larger. Based on the values of y_c in Figure 6-4, the source remained fixed during the duration of the measurement.

With the line source positioned horizontally, an average resolution can be computed more efficiently by averaging the pixels at each vertical position, and extracting the FWHM resolution only once. When an average contour of the low energy image is computed over the range $-2.3 \text{ cm} \leq x \leq 2.3 \text{ cm}$, the FWHM resolution and peak center are 0.831 cm and 0.177 cm. These values are indistinguishable from the mean values found from the individual contours reported above. For the high energy gamma rays, the FWHM resolution and y_c are 0.760 cm and 0.198 cm, again indistinguishable from the mean of the individual contours. This simple method of finding the resolution and peak

Figure 6-3. Surface plots showing measured images of a) 1.37 MeV and b) 2.75 MeV gamma-rays emitted by a ^{24}Na line source reconstructed using the RFL algorithm with the standard BUTR filter. RCC system, source and source space are described in the text and Table 6-1.

position will prove to be useful in analysis of all the bar source images in the following two sections.

Another "point-like" attribute of the line source is that the spectral resolution can be measured from each contour along its length. The mean SR over the horizontal range $-2.3 \text{ cm} \leq x \leq 2.3 \text{ cm}$ in the low energy image is $1.60(\pm 0.08) \text{ cm}$. To make this measurement compatible with those taken from the point source, the vertical range extends over $-3.1 \text{ cm} \leq y \leq 3.1 \text{ cm}$. Thus the ratio SR:FWHM is $1.93(\pm 0.14)$, which is slightly smaller than the value of 2.18 measured for the point source. This ratio is indicative of the shape of the peak as discussed in Section 5.5 where it is introduced and where the ratio of 1.51 is computed for a Gaussian function. SR of the average contour is 1.45 cm with a SR:FWHM of 1.74. This ratio is significantly smaller than the mean value and within 15% of the ideal Gaussian peak. The large improvement in the SR of the average contour over the mean SR of the individual contours is due to reduced noise in the average contour, which is achieved by averaging fluctuations in the individual

Figure 6-4. Graph of FWHM resolutions and peak center positions as a function of horizontal position in the reconstructed images of both low energy (1.37 MeV) and high energy (2.75 MeV) gamma rays emitted by the ^{24}Na line source shown in Figure 6-3.

contours. For the high energy image, the mean SR is $1.90(\pm 0.16)$ cm, and the ratio is $2.53(\pm 0.33)$ in agreement with the ratio of 2.27 from the point source. The value of SR in the average contour of the high energy image is 1.37 cm for a ratio of 1.79. From these resolution measures, the ^{24}Na line spread function is seen to closely approximate a Gaussian function in shape, and the noise sensitivity of the SR, noted in discussion of the PSF measurements, is confirmed.

The narrow vertical width of the line source has been exploited to compute image resolution and other point-like characteristics of the RCC response. Here the fact that the length of the line source is large compared to the resolution is used to compute the RMS uncertainty s_r in the peak intensity of the image. The value of s_r is computed using Equation (5-45), and the first step is determination of the average intensity $\bar{f}$. Because this line source is positioned horizontally, $\bar{f}$ can be determined by averaging along the rows where the vertical contours are at their peak value. Since $y_c \approx 0.20$ cm, s_r is

computed for the rows of pixels centered at $y = 0.1$ cm and $y = 0.3$ cm. The horizontal range of the pixels used in $\bar{f}$ must exclude those pixels with intensities affected by resolution effects from the ends of the line source; this is accomplished by using pixels with x-coordinates at least 3 standard deviations in resolution (σ_r) away from either end of the source. The reconstructed peak is approximately Gaussian, so the value of σ_r can be computed from

$$\sigma_r = \text{FWHM}/2.356 \quad (6-2)$$

For the low and high energy images, $3\sigma_r$ is 1.05 cm and 0.96 cm, respectively. To be conservative and to allow for comparisons with images having poorer resolution, only pixels which are more than 1.5 cm from the edge of the source are used. Therefore, $\bar{f}$ and s_r are computed using only those 10 pixels with their horizontal center in the range $-0.9 \text{ cm} \leq x \leq 0.9 \text{ cm}$. The values of s_r at $y = 0.1$ cm and 0.3 cm are 0.022 and 0.030 for an average of 0.026. In the high energy image, the average s_r for the same two rows of pixels is 0.055. A comparison of the average s_r measurements indicates that the high energy image in Figure 6-3 is noisier than the low energy image, which is apparent from visual inspection. This result is also expected based on the relative magnitude of σ_n/C_n , which is significantly larger for the high energy ray-sums than for the low energy.

The length of the line source can be estimated from these images by finding the horizontal positions where the image falls to half its peak intensity. The value of $\bar{f}$, used in computing s_r above, is taken as the peak intensity. The length of the source in the low energy image is 5.13 cm at $y = 0.1$ cm and 5.15 cm at $y = 0.3$ cm for an average of 5.14 cm. In the high energy image, the average length is 5.24 cm. These values are in excellent agreement with the actual length of 5.1 cm.

Both the resolution and noise in the line source images have been investigated. Being able to measure both quantities in one image is unique because it is not possible to measure noise in the PSFs such as those in Chapter V, where the noise level is simply

Table 6-2. Variation in parameters characterizing the resolution, noise and peak shape with filter type and maximum frequency for the high energy ^{24}Na line spread function reconstructed using the RFL algorithm.

Filter	Mean Values				Values from Ave. Contour				$\bar{s}_r$	L_t (cm)
	$\langle \text{FWHM} \rangle$ (cm)	s_{fwhm} (cm)	$\langle y_c \rangle$ (cm)	s_{yc} (cm)	FWHM (cm)	y_c (cm)	SR (cm)	SR: FWHM		
BUTR										
Standard	0.757	0.076	0.194	0.031	0.760	0.198	1.36	1.79	0.055	5.24
SHEP										
$v_{\text{max}} = 0.5$	0.767	0.092	0.197	0.036	0.782	0.201	1.44	1.84	0.093	5.23
HAMM										
$v_{\text{max}} = 0.5$	0.846	0.057	0.203	0.022	0.844	0.203	1.45	1.72	0.044	5.25
HANN										
$v_{\text{max}} = 0.5$	0.856	0.054	0.203	0.021	0.853	0.203	1.46	1.71	0.041	5.26
PARZ										
$v_{\text{max}} = 0.5$	0.932	0.045	0.204	0.018	0.930	0.204	1.57	1.69	0.032	5.28
PARZ										
$v_{\text{max}} = 0.4$	1.012	0.038	0.203	0.017	1.011	0.204	1.68	1.67	0.028	5.30
PARZ										
$v_{\text{max}} = 0.3$	1.167	0.027	0.201	0.013	1.168	0.201	1.92	1.64	0.023	5.31
PARZ										
$v_{\text{max}} = 0.2$	1.521	0.023	0.196	0.015	1.521	0.197	2.36	1.55	0.013	5.33

inferred from σ_n/C_n . On the other hand, it is not always possible to directly measure resolution in two-dimensional images; difficulties arise in deconvolving effects of the resolution. This unique aspect of the line source makes it an ideal source for studying the effect of different filters on the resolution, peak shape and noise in images produced with the RFL algorithm.

In Table 6-2, variation of the mean resolution $\langle \text{FWHM} \rangle$ and mean vertical peak position $\langle y_c \rangle$ are listed to show how resolution varies when the SHEP, HAMM, HANN or

PARZ filter (each with $v_{\max} = 0.5$) is used in place of the standard BUTR filter. The magnitude of $v_{\max}$ is also varied down to 0.2 with the PARZ filter to study its effect on resolution. As above these mean values are computed over those pixels with horizontal centers restricted by $-2.3 \text{ cm} \leq x \leq 2.3 \text{ cm}$. Values of FWHM and y_c extracted from the average contour are also shown for comparison, and it is apparent that the average contour values and the mean values are indistinguishable. Peak shape is specified in this table by the ratio SR:FWHM using SR and FWHM values determine from the average contour. Image noise reported in the table is characterized by $\bar{s}_r$ which is an average of the RMS uncertainty at $y = 0.1 \text{ cm}$ and $y = 0.3 \text{ cm}$, and each value of s_r is computed over the range of pixels with horizontal centers restricted by $-0.9 \text{ cm} \leq x \leq 0.9 \text{ cm}$. While s_r provides a useful estimate of variance due to noise in these images, it does not account for covariance among the pixels. See Section 7.1.3 for further discussion. Finally, the horizontal length of the line is included to demonstrate that there is little variation in the length as the reconstruction filter is changed.

The parameters reported in Table 6-2 are for reconstruction of the high-energy gamma-ray data, which is used because it gives a sharper but noisier image of the line source compared to the low energy gamma rays. As a result, it is easier to distinguish changes in resolution and RMS uncertainty. First comparing the resolution of the image reconstructed with the SHEP filter to the BUTR filtered image, the resolution is essentially the same but the image is almost twice as noisy ($\bar{s}_r$ is 0.093 vs. 0.055). The HANN filter gives a larger FWHM resolution than the BUTR; however, these resolutions still agree within the uncertainty of the measurements. The HANN filter does reduce the image noise by 25% when compared with the BUTR filter. The HAMM filter gives results that are virtually identical to those of the HANN. The PARZ filter yields an image with a resolution that is 23% larger than that of the BUTR, and is the only one that gives a resolution that is definitively different from that produced with the BUTR filter. The PARZ filtered image is also 42% less noisy than the BUTR image. Concerning the shape

of the peak, the ratio SR:FWHM becomes smaller as the image noise decreases and as the resolution becomes larger. Among the filter types the SHEP filter has the largest ratio and it is by far the noisiest image, while the PARZ has the smallest ratio (only 12% larger than for a Gaussian) and is significantly less noisy than the others.

The changes in resolution and image noise with decreasing maximum frequency of the PARZ filter shown in Table 6-2 are expected: the resolution becomes worse and the image becomes smoother. The peak shape also becomes more nearly Gaussian as the $v_{\max}$ is reduced. The only unanticipated result is that the changes in resolution and noise are greatest when $v_{\max}$ is reduced from 0.3 to 0.2, and changes between the other values of $v_{\max}$ are significantly smaller.

Of the five types of filters studied, only three of them: the standard BUTR, the HANN and the PARZ, are necessary to cover the measured ranges of resolution and noise. The SHEP and HAMM filter will not be used further. For reducing noise below the level of the PARZ with $v_{\max} = 0.5$, a PARZ filter with a lower maximum frequency can be used.

6.2 ^{24}Na Bar Source

The first of the two-dimensional images to be investigated consists of three ^{24}Na line sources placed parallel to one another to form a bar source. A dimensioned sketch of this source is included as Figure 6-1b. Figure 6-5 is a full-size autoradiograph of this source taken by placing a piece of Polaroid film directly on the source for 16 min. The film is then developed as it would be for a photograph and yields a full size image of the source.

This bar source serves to demonstrate several capabilities of the RCC. Comparison of on and off-axis resolutions will reveal any degradation in resolution away from the ring axis. In Figure 5-16, resolution degradation is shown to occur in an off-axis point source, but most of the degradation is explained by spatial broadening of the angular bins as they intersect the source plane away from the ring axis. This source also provides and opportunity to test the RCC capabilities at 2.75 MeV, an energy near the upper end of

Figure 6-5. Full size autoradiograph of ^{24}Na bar source.

the energy range of interest. The effect of varying the ray-sum counts by a factor of 50 on the image noise $\bar{s}_r$ in RFL reconstructed images is studied. A complimentary study is made to show how the resolution varies with the number of counts when the reconstruction filter is modified to keep $\bar{s}_r$ fixed. A comparison is also made between bar source images reconstructed with the RFL algorithm and the ART algorithm.

A table describing the RCC system parameters, the source and the source space are included in Appendix D as Table D-1. The activity of the source was 30.3 MBq at the start of the measurement and decayed to 1.52 MBq at the end of the run 64 hr later.

Figure 6-6 shows surface plots of the laminogram from the low energy gamma rays emitted by the bar source and a RFL reconstruction using a HANN filter with $v_{\max} = 0.5$. This figure clearly demonstrates the improvement in resolution achieved by applying a ramp-filter to this two-dimensional laminogram. The HANN filter is used for this reconstruction, instead of the standard BUTR, to make the average RMS uncertainty along the three bars $\bar{s}_r$ near that of the line source image. The HANN-filtered image

Figure 6-6. Surface plots showing a) the measured laminogram of 1.37 MeV gamma rays emitted by a ^{24}Na bar source and b) the image of the 1.37 MeV gamma rays reconstructed by filtering the laminogram shown in part a with a HANN filter having $v_{\max} = 0.5$. RCC system, source and source space are described in the text and Table D-1.

shown in the figure has an $\bar{s}_r$ of 0.046 versus 0.066 for a BUTR-filtered version. The value of $\bar{s}_r$ reported here is the average of s_r computed along the two most intense horizontal rows for each of the three bars, and over the horizontal range - $0.9 \text{ cm} \leq x \leq 0.9 \text{ cm}$. Thus computation of $\bar{s}_r$ for the bar source is consistent with its computation for the line source except that the value is averaged over six rows instead of two. It is again possible to compute s_r along each line because the bars are nearly horizontal. The bar source image is noisier than the line source image, not because σ_n/C_n is larger, but because the image is spread over three times as many pixels. In fact σ_n/C_n for the low energy bar source ray-sums is 0.193% compared to 0.280% for the low energy line source. Using Equation (5-46) the ratio of noise in the bar source image to the line source image is predicted to be

$$\frac{s_{r\text{bar}}}{s_{r\text{line}}} = \left(\frac{N_{0\text{bar}}}{N_{0\text{line}}} \right)^{3/4} \left(\frac{\sigma_n/C_{n\text{bar}}}{\sigma_n/C_{n\text{line}}} \right) = 1.57 \quad (6-3)$$

Figure 6-7. Gray scale density plots showing measured images of a) 1.37 MeV and b) 2.75 MeV gamma rays emitted by a ^{24}Na bar source reconstructed using the RFL algorithm with a HANN filter having, $v_{\text{max}} = 0.5$. RCC system, source and source space are described in the text and Table D-1 of Appendix D.

where $N_{0\text{bar}}/N_{0\text{line}}$ is set to 3. Thus this equation predicts that the bar source noise will be larger than the line source noise. The bar source is found to be noisier than the line source, as predicted, but the measured ratio is 2.54, considerably larger than the prediction. This disagreement indicates that the model given in Equation (5-46) for large, flat sources may not be accurate for comparing discontinuous sources with differing sizes, such as the line and bar source.

Gray scale density plots of both low and high energy bar source images are shown in Figure 6-7. Both images are reconstructed using a HANN filter with $v_{\text{max}} = 0.5$; therefore, the data used to generate the density plot in Figure 6-7a are identical to that represented by the surface plot in Figure 6-6b. The gray scale density plot provides a more complete view of the image than the surface plot, and will be used to present the remaining images.

An average vertical contour is computed over the horizontal range $-2.3 \text{ cm} \leq x \leq 2.3 \text{ cm}$ for both of the images of Figure 6-7. The FWHM and peak positions extracted from each line of both images are listed in Table 6-3, and are compared to the

mean values $\langle \text{FWHM} \rangle$ and $\langle y_c \rangle$. These comparisons confirm that the mean of the values extracted from the individual contours and those extracted from the average contour are in agreement, as was noted for the line source. There is a consistent degradation in the FWHM resolution of the off-axis bars compared to the on-axis bar. While the differences in the $\langle \text{FWHM} \rangle$ are smaller than the standard deviations in the high energy image, the fact that the increase is consistent indicates that the degradation is physical. The angular resolution of the bars $\Delta\theta$ can be estimated from the measured values of the FWHM and y_c using Equation (5-37) evaluated at $x = 0$, the horizontal center of the source. The values of $\Delta\theta$ listed for the high energy image in Table 6-3 are in good agreement. The low energy image shows even better agreement with one of the off-axis bars having slightly lower angular resolution than the on-axis bar, and the relative difference between off-axis and on-axis values of $\Delta\theta$ is about half the difference in the spatial FWHM. The relative uncertainty in the reported values of $\Delta\theta$ is approximately equal to that of the $\langle \text{FWHM} \rangle$, and as a result the values of $\Delta\theta$ in the high energy image agree well within the uncertainty. There is thus no discernible difference in $\Delta\theta$ between the on and off-axis bars.

The FWHM resolution of the center bar of the high energy image reported in Table 6-3 agrees with the resolution of the HANN filtered line spread function reported in Table 6-2. Also, the value of $\bar{s}_r$ for the high energy bar source is 2.13 times the value of $\bar{s}_r$ reported for the HANN filtered line spread function. This ratio is in somewhat better agreement with the predicted ratio from Equation (6-3) than is the ratio for the low energy images. The validity of using $\bar{s}_r$ as a measure of image noise is supported by visual comparison of the surface plot in Figure 6-6b and those in Figure 6-3. Values of $\bar{s}_r$ indicate that the low energy bar source image in Figure 6-6b is noisier than the low energy line source of Figure 6-3a, and this can be confirmed by comparing the variation along the peaks in both figures, as well as in the low intensity regions. Furthermore, values of $\bar{s}_r$

Table 6-3. Parameters characterizing the spatial resolution, angular resolution and noise of low energy (1.37 MeV) and high energy (2.75 MeV) images of the ^{24}Na bar source reconstructed using the RFL algorithm with a HANN filter having $v_{\text{max}} = 0.5$.

	Mean Values				Values from Ave. Contr.			$\bar{s}_r$	L_ℓ
	$\langle \text{FWHM} \rangle$	s_{fwhm}	$\langle y_c \rangle$	s_{yc}	FWHM	y_c	$\Delta\theta$		
	(cm)	(cm)	(cm)	(cm)	(cm)	(cm)	(deg)		(cm)
Low Energy									
Upper bar	--	--	--	--	1.090	2.391	5.90	0.061	5.18
Center bar	--	--	--	--	1.015	0.242	5.82	0.037	5.14
Lower bar	--	--	--	--	1.024	-1.799	5.68	0.039	5.15
Average	--	--	--	--	1.043	--	5.80	0.046	--
High Energy									
Upper bar	0.945	0.101	2.450	0.063	0.973	2.448	5.26	0.078	5.38
Center bar	0.864	0.075	0.236	0.036	0.856	0.233	4.91	0.063	5.35
Lower bar	0.971	0.119	-1.852	0.060	0.948	-1.862	5.25	0.121	5.31
Average	0.927	0.058	--	--	0.926	--	5.14	0.087	--

indicate that Figure 6-6b is smoother than the high energy line spread function of Figure 6-3b, and visual comparison supports this conclusion.

Spatial resolution measured from the high energy gamma rays reported in Table 6-3 is better than the resolution measured from the low energy gamma rays. This same variation in spatial resolution is found in the line spread and point spread functions measured from ^{24}Na sources. While none of the measured differences in resolution is

significantly larger than the uncertainty in the measurements, the fact that it appears in all three makes it likely that the difference is physical. These resolution differences are not predicted by the analytic resolution model, and they may result from the effect of contaminating events in the low energy image. This topic will be addressed further in Section 6.3.

In Figure 6-7, differences in the spatial resolution between the two images is not readily apparent; however, the high energy image is obviously noisier than the low energy image. This noted variation in the noise is supported by the ratio of $\bar{s}_r$ between the high and low energy images, which is 1.91. The fractional uncertainty in the high energy data σ_n/C_n is 0.384%. Since the two images cover the same number of pixels and σ_n/C_n is 0.193% in the low energy data, Equation (5-46) predicts this ratio to be 1.99, in excellent agreement with the measurement. Thus, Equation (5-46) accurately predicts the noise ratio between images from identical sources with different fractional uncertainties but does not predict this ratio well between different source geometries.

Table 6-3 also includes the lengths of the bars, which are in good agreement with the actual value of 5.1 cm. The method of determining the length has been described in Section 6.1.

The image shown in Figure 6-6a is generated from 522,000 gross counts collected during a 41.5 hr measurement time. An additional 13.0 hr was dedicated to measuring the random background. The long duration of this measurement is due, in part, to the short 15 hr half-life of the ^{24}Na isotope. If the source had not significantly decayed from its initial activity, the same number of counts could have been collected in about 12 hr, and the background measurement could have been reduced to 9 hr. Even without source decay these are very long measurement times, much too long for the industrial applications envisioned for the RCC. It is worthwhile then to investigate how severely image quality is degraded as counting time, and thus the number of counts, are decreased. To perform this investigation, images are reconstructed from three subsets of the 1.37

Table 6-4. Summary of data and reconstructed image parameters of four subsets of the ^{24}Na bar source data. The images are reconstructed using the RFL algorithm with a HANN filter having $v_{\text{max}} = 0.5$. The FWHM is an average over all three bars.

Data Measurement Duration (min)	Net Counts	σ_n/C_n (%)	FWHM	$\bar{s}_r$
2490	492,000	0.193	1.043	0.046
360	90,500	0.378	1.035	0.103
90	23,500	0.718	1.020	0.110
30	11,400	1.000	1.021	0.225

MeV ^{24}Na bar data. The duration of the data measurement time for these three subsets is listed in Table 6-4 along with the duration for the entire data set. To simplify the analysis, all of the background data is used to correct each of the subsets with W_b adjusted accordingly. In addition to the measurement duration, net counts C_n and fractional uncertainty of the net counts σ_n/C_n are also given in Table 6-4.

Each data subset is reconstructed using the RFL algorithm and a HANN filter with $v_{\text{max}} = 0.5$. The FWHM resolution is reported for each subset, representing the average of the values for all three bars. The FWHM for each bar is computed from an average contour over the range $-2.3 \text{ cm} \leq x \leq 2.3 \text{ cm}$. Note that there is little change in FWHM resolution as the number of counts decreases in agreement with PSF results shown in Figure 5-14. However, there is a significant increase in $\bar{s}_r$ as the counts decrease and consequently, as σ_n/C_n increases. Equation (5-46) indicates that $\bar{s}_r$ should be proportional to σ_n/C_n , and Figure 6-8 shows that these points are well represented by a line with zero intercept computed using linear regression.

Figure 6-8. Variation of noise $\bar{s}_r$ in bar source images as a function of the fractional uncertainty in the data σ_n/C_n . A best fit straight line with zero intercept shows that $\bar{s}_r$ is approximately proportional to σ_n/C_n as predicted in Equation (5-46). The images are reconstructed using a RFL algorithm and a HANN filter with $v_{\max} = 0.5$.

Images reconstructed from the data in Table 6-4 with $\sigma_n/C_n = 0.378\%$ and 1.000% are shown in Figure 6-9 as gray scale density plots. The data subset with $\sigma_n/C_n = 0.378\%$ closely matches the value for the entire high energy data set, and the resulting value of $\bar{s}_r$ is within 25% of the value measured from the high energy image. Note the similarities in variations in the peak and background intensities between these two images shown in Figure 6-9a and Figure 6-7b. The quality of the image with $\sigma_n/C_n = 0.378\%$ shown in Figure 6-9b is quite poor as indicated by a measured $\bar{s}_r$ value of 0.225; the bars in this image appear to be discontinuous and fluctuations in the background are more intense than some points along the bars.

To improve the quality of the images in Figure 6-9, the noise must be reduced. In a laminogram the magnitude of the signal decreases the increasing frequency, and thus

Figure 6-9. Gray scale density plots of images reconstructed from subsets of the 1.37 MeV ^{24}Na bar data with the RFL algorithm using various reconstruction filters. a) Subset with $\sigma_n/C_n = 0.378\%$ reconstructed using a HANN filter with $v_{\max} = 0.5$. b) Subset with $\sigma_n/C_n = 1.000\%$ also reconstructed using a HANN filter with $v_{\max} = 0.5$. c) Subset with $\sigma_n/C_n = 0.378\%$ reconstructed using a PARZ filter with $v_{\max} = 0.26$. d) Subset with $\sigma_n/C_n = 1.000\%$ reconstructed using a PARZ filter with $v_{\max} = 0.245$.

the signal to noise ratio declines with frequency. Further, the frequency where the noise begins to dominate the signal decreases as σ_n/C_n increases, which has been shown for the laminograms of point sources in Figure 5-15. To reduce the image noise, the highest frequencies, where the noise dominates the signal, must be eliminated. Using a PARZ reconstruction filter in place of the HANN filter de-emphasizes the high frequencies and reduces noise. This is demonstrated explicitly for the ^{24}Na line source in Table 6-3.

Further reduction in the noise of the line source images is achieved by reducing $v_{\max}$ of the PARZ filter.

PARZ filters with reduced values of $v_{\max}$ are used to reconstruct the three subsets of the low energy ^{24}Na bar data. The filter for each image is chosen to give an $\bar{s}_r$ of 0.046, which is equal to the value of the full data set reconstructed with the HANN filter. Density plots of the images reconstructed in this way with $\sigma_n/C_n = 0.378\%$ and 1.000% are shown as Figures 6-9c and 6-9d for comparison with the images reconstructed with the HANN filter. A consequence of reducing noise by eliminating the highest frequencies is a degradation of the resolution, and Figure 6-10 shows the average FWHM solution of the three bars as a function of σ_n/C_n . The value of $v_{\max}$ for the PARZ reconstruction filters is 0.260, 0.245 and 0.245 for the images with σ_n/C_n of 0.378%, 0.718% and 1.000%, respectively.

Figure 6-10 shows that a five-fold increase in σ_n/C_n , resulting from a 83-fold decrease in imaging time, results in a bar source image with the same noise and only a 36% increase the spatial resolution. This is an important result for industrial applications where large areas or many different items may need to be surveyed in a short time, and some loss of resolution may be acceptable.

As the FWHM resolution approaches the spacing between the bars, it becomes impossible to accurately measure the FWHM directly from the data. Instead the average contours are fit to a function $f_a(y)$ of the form

$$f_a(y) = \frac{I}{\sigma_g \sqrt{2\pi}} \left\{ \exp \left[-\frac{1}{2} \left(\frac{y-y_\ell}{\sigma_g} \right)^2 \right] + \exp \left[-\frac{1}{2} \left(\frac{y-y_c}{\sigma_g} \right)^2 \right] + \exp \left[-\frac{1}{2} \left(\frac{y-y_h}{\sigma_g} \right)^2 \right] \right\} \quad (6-4)$$

which is the sum of three Gaussian functions centered at y_ℓ , y_c and y_h with a common intensity I and standard deviation, σ_g . In Figure 6-11a, an average vertical contour through the low energy, HANN filtered image of the full data set is compared to a non-

Figure 6-10. Variation of average FWHM resolution in images of the 1.37 MeV gamma rays emitted from the ^{24}Na bar source as a function of the fractional uncertainty in the data σ_n/C_n . The reconstruction filter was varied to keep $\bar{s}_r$ constant at 0.046.

linear regression fit of the data to Equation (6-4). For this higher resolution image, the centers of the three bars are at -1.798 cm, 0.242 cm, and 2.388 cm, and σ_g is 0.441 cm with a corresponding FWHM resolution of 1.039 cm. The measured values listed in Table 6-3 are nearly identical to these fitted parameters. In Figure 6-11b, it is apparent that the measured parameters are in error, while the fitted parameters are in good agreement with the data.

To provide a comparison of the RFL algorithm with the ART algorithm, the full set of low energy data measured from the ^{24}Na bar source is reconstructed using ART. While several comparisons are made between these algorithms using point sources (see Table 5-4 through 5-6), this comparison is more demanding because both resolution and image noise can be determined directly. Variation in the RMS error of the ray-sums $e_{\text{rms}}^{(k)}$ with iteration is shown in Figure 6-12. Average values of the FWHM resolution and noise, as

a)

b)

Figure 6-11. Normalized average vertical contours through reconstructed images of 1.37 MeV gamma rays emitted from the ^{24}Na bar source compared to parametric fits of the data. The data have fractional uncertainties σ_n/C_n of a) 0.193% and b) 1.000%. The fitted parameters are determined either directly from the data or by non-linear regression.

Figure 6-12. Variation of RMS error e_{rms} , FWHM spatial resolution and noise $\bar{s}_r$ as a function of iteration number k for ART reconstruction of the 1.37 MeV gamma rays emitted from the ^{24}Na bar source,

measured by $\bar{s}_r$, are also shown as functions of iteration. At the last iteration the FWHM is 0.937 cm and $\bar{s}_r$ is 0.104. In addition to the fact that the convergence criteria of $(e_{rms}^{(k-1)} - e_{rms}^{(k)})/e_{rms}^{(k)} < 0.001$ is reached, trends in the values of FWHM and $\bar{s}_r$ indicate little reason for further iteration: the resolution improvement is only 0.2% between the 8th and 9th iteration, while the noise increases by 6%.

The resolution in the final ART images is about 10% better than the image formed using the RFL algorithm and a HANN filter with $v_{max} = 0.5$, but the noise is 2.4 times larger. A gray scale density plot of the ART image from the final (9th) iteration is shown in Figure 6-13a; it is sharp but noisy as indicated by the measured parameters. If this final image is low pass filtered using a Hann window with $v_{max} = 0.5$ to match the RFL image, the FWHM increases to 1.022 cm and $\bar{s}_r$ decreases to 0.049. These values are very close to those measured from the RFL image, and a gray scale density plot of the image is shown as Figure 6-13b for comparison.

Figure 6-13. Gray scale density plot of final image of 1.37 MeV gamma rays emitted from the ^{24}Na bar source reconstruct using the ART algorithm a) before and b) after low-pass filtering using a Hann window with $v_{\text{max}} = 0.5$.

6.3 ^{65}Zn Bar Source

The ^{65}Zn bar source serves to demonstrate the capability of the RCC to resolve spatially extended sources separated by distances approaching the system spatial resolution. The spatial resolution is approximately 0.9 cm at this gamma-ray energy ($E_\gamma = 1.12$ MeV) and source spacing ($z_s = 10$ cm). The source also provides an opportunity to measure the spatial resolution at an energy near the lower energy of ^{24}Na without the presence of contaminating events from a higher energy line.

This bar source consists of four line sources placed parallel to one another. Each bar is 5 cm long by 0.1 cm wide by 0.1 cm deep. The bars are oriented vertically with the left-most bar 2.5 cm to the left of the ring axis (at $x = -2.5$ cm). The pairs are separated by distances of 1.1 cm, 1.5 cm and 2.1 cm from left to right. The total source activity of the ^{65}Zn bar source was 65.0 MBq at the beginning of the run and it suffered less than 0.5% decay by the end of the run, approximately 38 hours later. The RCC parameters and

Figure 6-14. Gray scale density plot showing measured image of 1.12 MeV gamma rays emitted by a ^{65}Zn bar source reconstructed using the RFL algorithm with a HANN filter having $v_{\text{max}} = 0.5$. RCC system, source and source space are described in the text and Table D-2.

source space parameters are similar to those used for the ^{24}Na bar source and are listed in Table D-2 of Appendix D.

An image of the ^{65}Zn bar source is shown as a gray scale density plot in Figure 6-14. The image is reconstructed using the RFL algorithm with a HANN filter having $v_{\text{max}} = 0.5$, and it demonstrates that the RCC can indeed resolve two bars separated by slightly more than the resolution of the system over their entire length. The data used to create the image includes 284,100 net counts C_n with $\sigma_n/C_n = 0.234\%$, and the noise in the image with $\bar{s}_r = 0.056$, similar to that of the low energy ^{24}Na bar source image of Figure 6-7a. Since the bars of the ^{65}Zn source are oriented vertically, s_r is computed for the two peak columns of each bar over the range from $-0.9 \text{ cm} \leq y \leq 0.9 \text{ cm}$. It is possible to compute s_r in this way because all of the bars in the source are nearly vertical.

An average horizontal contour of the image in Figure 6-14 is computed over the range $-2.3 \text{ cm} < x < 2.3 \text{ cm}$, and from it the average FWHM is determined by making a nonlinear fit of the average contour data to an equation of the form

$$f_a(y) = \frac{I}{\sigma_g \sqrt{2\pi}} \left\{ \exp \left[-\frac{1}{2} \left(\frac{x-x_\ell}{\sigma_g} \right)^2 \right] + \exp \left[-\frac{1}{2} \left(\frac{x-x_{\ell c}}{\sigma_g} \right)^2 \right] \right. \\ \left. + \exp \left[-\frac{1}{2} \left(\frac{x-x_{rc}}{\sigma_g} \right)^2 \right] + \exp \left[-\frac{1}{2} \left(\frac{x-x_r}{\sigma_g} \right)^2 \right] \right\} \quad (6-5)$$

which is similar to Equation (6-4) with a term added to account for the fourth bar. The data and fit are compared in Figure D-1 of Appendix D. The average FWHM resolution determined from the fit ($= 2.356\sigma_g$) is 0.926 cm. This fitted, average value is in excellent agreement with the mean FWHM of $0.913(\pm 0.095)$ cm determined for the right-most bar from horizontal contours in the range $-2.3 \text{ cm} < x < 2.3 \text{ cm}$. The fitted horizontal centers of the bars are, from left to right, $x_\ell = -2.51 \text{ cm}$, $x_{\ell c} = -1.38 \text{ cm}$, $x_{rc} = 0.15 \text{ cm}$ and $x_r = 2.21 \text{ cm}$. Note that for both the right-most and right-center bars the fit and the data are in good agreement. There is no apparent change in the resolution at positions away from the ring axis.

The average FWHM resolution determined for the ^{65}Zn bar source from the fit is 11% smaller than the 1.043 cm FWHM of the low energy ^{24}Na bar source image reported in Table 6-3; however, the energy of the ^{65}Zn gamma rays is similar to that of the low energy ^{24}Na gamma rays (1.12 MeV compared to 1.37 MeV), and the resolution of the two images is predicted to be within 1.4% by the analytic resolution model. On the other hand, the resolution of the ^{65}Zn bar source image is in excellent agreement with the FWHM of the high energy ^{24}Na bar source image and a slightly larger difference of 1.9% is predicted between these images. These comparisons support evidence presented earlier that the resolution for imaging the low energy gamma rays of ^{24}Na is degraded by contaminating events that are not present in measurements of the high energy ^{24}Na gamma rays or the ^{65}Zn gamma rays. Unfortunately, the models developed for estimating contaminating event rates do not point to any of the identified contaminating event types as a cause of the degraded resolution. The relative rates for contaminating events in the extended

source images is expected to be similar to those computed for on-axis point sources in Table 5-3, where the relative rate of contaminating events is predicted to be smaller for the low energy ^{24}Na gamma rays than for the high energy gamma rays. As with the point sources, the possibility of confusing partial energy deposition (PED) events from the high energy gamma rays with full energy deposition of the low energy gamma rays is zero, even with the expanded range of scattering angles used for the ^{24}Na bar source.

The ^{65}Zn isotope decays to ^{65}Cu by two modes: predominantly electron capture resulting in the 1.12 MeV gamma rays imaged in Figure 6-14 and positron emission the remaining 1.46% of the time [Led78]. The intensity of the 1.12 MeV gamma rays is 0.508, and each positron emission results in the creation of two annihilation photons for an effective intensity of 0.029 (see Table 5-1). The net counts measured due to annihilation photons emitted by the ^{65}Zn bar source (and falling in the summed energy window $0.446 \text{ MeV} < E_s < 0.576 \text{ MeV}$) is 12% of the number falling in the summed energy window set about the gamma-ray peak ($1.040 \text{ MeV} < E_s < 1.192 \text{ MeV}$). The annihilation photons contribute no contaminating events to the summed energy window used to collect the gamma rays for imaging this bar source, but they do interfere with an image measured from the multi-energy phantom of the next section.

6.4 Multi-Energy Phantom

A multi-energy phantom consisting of two sources emitting different energy photons is used to demonstrate the ability of the RCC to spectrally separate photons of different energies and generate independent images of them. One of the sources is made of 0.1 cm diameter wires of ^{65}Zn in a 5.0 cm long by 2.4 cm wide U-shaped configuration. The other source is made of 0.02 cm diameter ^{64}Cu wires arranged in the shape of an "M", which is 5.0 cm high by 4.9 cm wide. A sketch of this phantom is included as Figure 6-1d, and a full-size autoradiograph is shown in Figure 6-15.

Figure 6-15. Full size autoradiograph of multi-energy phantom.

A table describing the RCC system, the source and the source space parameters used in generating images of this source is included in Appendix D as Table D-3. The activity of the ^{65}Zn source was 41.1 MBq at the beginning of the run, and its activity only decayed by 0.36% over the course of the 31 hr measurement. The ^{64}Cu source activity was 56.1 MBq at the start of the run and decayed to 10.5 MBq by the end.

The half-lives of the two isotopes used in this phantom are very different: 12.7 hr for ^{64}Cu versus 244 days for ^{65}Zn . As a result, the method devised to correct for the random background from a single decaying isotope in Sections 5.1 and 5.2 does not apply. To correct for the random background in this measurement, a short background count was taken at the beginning and end of each data run. The random coincidence rate for the data run r_{rcd} is estimated as the average of the two short counts. The random background correction factor W_b is then approximated as the ratio of r_{rcd} to the random background rate of the corresponding background measurement r_{rcb} . To limit uncertainty in the value of W_b determined with this method, the length of the data runs was kept short (less than 2

Figure 6-16. Gray scale density plots showing measured images of a) 1.12 MeV gamma rays emitted by the U-shaped source and b) 0.511 MeV gamma rays emitted by the M-shaped source in the multi-energy phantom. This image is reconstructed using the RFL algorithm with a HANN filter having $v_{\max} = 0.5$. RCC system, source and source space are described in the text and Table D-3.

hr.) with four data runs at both azimuthal positions, and a random background was collected between each pair of data runs.

An image of the ^{65}Zn gamma rays emitted by the U-shaped source is shown in Figure 6-16a as a gray scale density plot. This image is reconstructed using the RFL algorithm with a HANN filter having $v_{\max} = 0.5$. Spatial resolution in this image is computed for each of the vertical uprights of the "U" from an average contour over the range $-0.9 \text{ cm} \leq y \leq 2.3 \text{ cm}$. This range is reduced from that used with the ^{65}Zn bar source to minimize effects from the horizontal section along the bottom. The FWHM resolution of the left upright is 0.926 cm and the center is at $x = -1.32 \text{ cm}$. The right upright has a FWHM of 0.908 cm and is centered at 1.15 cm. The average FWHM is then 0.917 cm, in agreement with the value of 0.926 cm found for the ^{65}Zn bar source. The corresponding angular resolution of these uprights is 5.17° . The separation distance between the uprights as measured from the image is 2.47 cm, in agreement with the actual width of 2.4 cm.

The average contour through the uprights of the "U" has been fit with two Gaussian functions having the same standard deviation and intensity, after Equations (6-4) and (6-5). The fitted FWHM resolution is 0.926 cm and a comparison of the data and the fit is shown in Appendix D as Figure D-2. The ratio of the peak intensity in the measured contour to the valley (or minimum intensity) between the peaks is 12.8, while the peak-to-valley ratio in the Gaussian fit is 71.4. The difference between the two is due in part to the noise in the measurement. This noise is enhanced by the fact that the absolute value of each pixel is used to form the image from the RFL algorithm. However, the minimum between the peaks is 2.7 times more intense than the background lying outside the outline of the "U". It is difficult to attribute the increased background inside the "U" to a particular cause, but two possibilities are errors in W_b as determined from the modified method described above, and a heavier weighting in the tails of the point response than the Gaussian function. This second possible cause is contradicted by the good agreement between Gaussian fits and the average contours of the line and bar sources.

To characterize the noise in the image, the RMS uncertainty s_r is computed for the two upright sections of the "U" using exactly the same technique applied to the line and bar sources in the preceding sections of this chapter. The average noise in the two uprights $\bar{s}_r$ is 0.0497. This noise level is lower than that of the ^{65}Zn bar source, where $\bar{s}_r = 0.0563$. The value of σ_n/C_n in the "U" data is 0.24% compared to 0.23% for the bar source, but the "U" is less noisy because it only occupies about two-thirds the number of pixels.

An image of the annihilation photons emitted by the M-shaped ^{64}Cu source is shown as a gray scale density plot in Figure 6-16b. This image has been reconstructed using the same RFL algorithm used for the ^{65}Zn "U", and its resolution is determined from an average horizontal contour over the range $-2.3 \text{ cm} \leq y \leq -1.1 \text{ cm}$. The FWHM resolution of the left leg of the "M" is 1.24 cm and its peak center is at $x = -2.51 \text{ cm}$. For the right leg,

the FWHM is 1.21 cm, and the peak center is at $x = 2.35$ cm. The distance between the two legs is 4.86 cm, in agreement with the actual value of 4.9 cm. However, the average FWHM of 1.221 cm corresponds to an angular resolution of 6.63° , which is 28% larger than that of the "U". While a slightly worse angular resolution is predicted for the RCC at 0.511 MeV than at 1.12 MeV, the difference between these measured values is not accounted for by the analytical resolution model. The degraded resolution in the image of the ^{64}Cu "M" may result from contaminating events, as discussed more fully below.

The value of σ_n/C_n in the 0.511 MeV ray-sums is 0.21%, but an accurate measure of $\bar{s}_r$ could not be made because too many of the pixels along the length of its legs are affected by the tails from the diagonal sections. The peak-to-valley ratio of the average contour is 6.4, considerably worse than that of the "U", although the separation between the legs of the "M" is about twice that between the uprights of the "U". Further, the ratio of the background intensity inside the outline of the "M" to the background outside is about 3.5. The increased background inside the "M" is caused by contaminating events.

Event count rates recorded from measurement of the multi-energy phantom are summarized in Table 6-5 for summed energy windows about both photon energies. The true coincidence rates reflect the difference between the measured total count rate for all event types and the measured random background rates. The fraction of random events in the 0.511 MeV window is substantially larger than the fraction in the 1.12 MeV window. As a result, errors in the value of W_b determined by the modified method described earlier in this section are magnified in the ^{64}Cu image compared to the other. Such errors would degrade the resolution and peak-to-valley ratio of the two images, and affect the ^{64}Cu image to a greater degree.

Event rates of the other types of contaminating events are estimated using the point source models presented in Section 2.4. The double scattering rates are estimated from computations by Singh, who found the ratio of double to single scattering events to be 0.077 at 0.5 MeV [Sin83a]. Thus, the fraction of all events which are double scattered

Table 6-5. Average event count rates in the RCC during measurement of the multi-energy phantom. *Indicates computed count rate.

Event Type	Count Rate (cps)	
	0.511 MeV Window	1.12 MeV Window
All	21.50	7.97
Random coincidence	6.63	1.79
True coincidence	14.87	6.18
Double scattering*	1.15	< 0.48
Reverse scattering*	0.12	0.11
Other sources*	0.74	n/a
Partial energy deposition*	0	n/a

is 0.071. The rate of double scattering events in the 0.511 MeV window reported in Table 6-5 is the product of this fraction and the true coincidence count rate. The same fraction multiplied by the true coincidence rate in the 1.12 MeV window provides an upper bound on the double scattering rate at this energy.

The rate of PED events in the 0.511 MeV window due to partial energy deposition from the 1.12 MeV gamma rays is zero, because the minimum recoil electron energy E_{re} deposited by these gamma rays is larger than the upper edge of the window set on E_{re} (0.260 MeV) to collect the annihilation photons over the specified range of scattering angles. (Refer to Table 5-2 and associated text for further discussion on eliminating PED events by limiting scattering angle.) The minimum possible scattering angle from the ^{65}Zn "U" source through the quantization positions of any pair of detector elements is 37.6° , and with the measured angular resolution of 5.17° the minimum measurable angle is 3σ below this at 31.0° . The value of E_{re} for a 31.0° scatter of a 1.12 MeV photon is 0.265 MeV, above the upper threshold of 0.260 MeV. The PED event rate is listed as "n/a" in the 1.12 MeV window because annihilation photons cannot deposit enough energy to fall in

that window, and the intensity of the 1.35 MeV gamma-ray line from ^{64}Cu is so small that it can be neglected.

The reverse scattering event rate is computed to be 0.8% of the Compton coincidence rate from an on-axis point source of annihilation photons when the RCC is configured with $\theta_c = 59^\circ$. This point source result is used to estimate the reverse scattering event rate in the 0.511 MeV window reported in Table 6-5. The reverse scattering rate in the 1.12 MeV window is computed the same manner, based on a computed ratio of 1.7% from the point source model. Neither the reverse scattering nor the other two computed contaminating event rates vary enough between the two windows to account for the degradation in the image of the ^{64}Cu "M".

A final type of contaminating event in the ^{64}Cu image must be addressed: the annihilation photons emitted by the ^{65}Zn source. The relative rate of net counts (or true coincidence counts as they are labeled in Table 6-5) from the annihilation photons to the 1.12 MeV gamma rays in the ^{65}Zn bar source is 0.12. The relative rates from the "U" source can be assumed to be the same because the geometry of these two sources are similar and the RCC configuration is identical. Multiplying this ratio by the true coincidence rate in the 1.12 MeV window gives the estimated rate due to "other sources" for the 0.511 MeV window in Table 6-5. These events will definitely contribute to the intensity of the background inside the outline of the "M" and thus reduce the peak-to-valley ratio. These events may also contribute to the degraded resolution measured for this image.

Images of the ^{64}Cu "M" reconstructed with the iterative ART and MLE algorithms are included in Figure 6-17. Both algorithms began with a laminogram as the initial estimate of the source intensity. The ART image is reconstructed on the same source plane as the RFL image shown in Figure 6-16b with 128 by 128 pixels of size 0.2 cm by 0.2 cm. The relaxation parameters and convergence criterion are the same as the ART reconstruction of the ^{24}Na bar source shown in Figure 6-12. This algorithm required 9

Figure 6-17. Gray scale density plots showing measured image of 0.511 MeV gamma rays emitted by the M-shaped source in the multi-energy phantom. In a) the image is reconstructed using the ART algorithm and is low-pass filtered with a Hann window having $v_{\max} = 0.5$ and is displayed on a grid of 48 by 48 pixels of size 0.2 cm by 0.2 cm. In b) the image is reconstructed using the MLE algorithm and is displayed on a grid of 33 by 33 pixels of size 0.3 cm by 0.3 cm.

iterations to converge using the same laminogram used to form Figure 6-16b as the initial estimate of the source intensity. The MLE algorithm also used a laminogram as the initial estimate, which saves one iteration compared to starting with a uniform field for both of these iterative algorithms. The ART reconstructed image of the "M" is low pass filtered with a Hann window having $v_{\max} = 0.5$ to reduce high-frequency noise. The MLE image in Figure 6-17b is reconstructed on a source plane with 33 by 33 pixels of size 0.3 cm by 0.3 cm. The convergence criterion for the MLE is the same as the ART algorithm, and it converged after 20 iterations. The image shown in the figure is from the 15th iteration; it is shown instead of the final image because it is much smoother with little loss in resolution.

The FWHM of the average contour through the legs of the "M" in the ART image (over the range $-2.3 \text{ cm} \leq y \leq -1.1 \text{ cm}$) is 1.19 cm for the left leg and 1.18 cm for the right leg. The peak positions of the two legs are $x = -2.51 \text{ cm}$ and $x = 2.34 \text{ cm}$ and thus are separated by 4.85 cm. The resolutions and position of the legs in the ART image are

nearly identical to those of the filtered image, which provides another confirmation of the validity of the RFL algorithm. However, the peak-to-valley ratio of 4.7 is worse than the ratio in the filtered image, indicating that the intensity of the background is higher in the ART image. Background in the ART images is expected to be lower than in the ramp-filtered images because the negative pixel intensities are set to zero to improve convergence in the ART algorithm; whereas, negative intensities in the ramp filtered images are set to their absolute values. Perhaps more iterations would bring this ratio in agreement with or below that of the ramp-filtered image.

The MLE image has somewhat poorer resolution than the other images. For an average horizontal contour over the range $-2.4 \text{ cm} \leq y \leq -1.2 \text{ cm}$, the FWHM is 1.43 cm for the left leg and 1.61 cm for the right leg, giving an average resolution of 1.52 cm. The peak positions of the legs are $x = -2.49 \text{ cm}$ and $x = 2.22 \text{ cm}$, and thus are separated by 4.71 cm. The peak-to-valley ratio in the average contour is 4.26 which is close to that of the ART algorithm. It is difficult to quantify the noise in images of this M-shaped source, but visually the intense regions of the MLE image appear to be noisier than the others, while the background regions appear smoother.

The poorer resolution measured from the MLE image relative to the ramp-filtered and ART images is due in part to the larger pixels used in its reconstruction. The larger pixels are used to reduce computation time, but even with fewer pixels in the MLE image (a total of 1089 versus 16384 for the other algorithms), 20 iterations required approximately 10 hours of CPU time on a CRAY YMP. This computational expense is high compared to the iterative ART algorithm, which required 4.5 hours on a PC (with a 33 MHz Intel 486DX microprocessor and 8 Mbytes of memory), and is far too high for the portable instrument envisioned for nuclear industrial applications. The RFL algorithm was nearly an order of magnitude faster than the ART algorithm on the same PC, requiring 26 min. to form the laminogram and 28 s to perform the filtering. (The ART reconstruction time also includes the time for laminogram formation.) For the ultimate

application, even the reconstruction time of the RFL algorithm may be an order of magnitude too long; however, about 19 min. of the laminogram formation time is spent computing a matrix of backprojection weights (ω_{ij}), which could be determined in advance if the gamma-ray energy to be imaged can be anticipated. Also the speed of the algorithms implement on the PC can be approximately doubled if the computer memory is increased to 24 Mbytes, because they rely heavily on a block of hard disk configured as virtual memory.

6.5 ^{24}Na Flood-Field Phantom

This flood-field phantom consists of three water-tight disks filled with aqueous solutions of ^{24}Na . A large outer disk is 14 cm in diameter and two smaller disks with inside diameters of 3.5 cm are positioned inside the larger disk, and each disk has a depth of 1.5 cm. A sketch of the source is shown in Figure 6-1e. At the beginning of the imaging run the areal activity density in the large disk was approximately 0.36 MBq/cm², while one of the small disks had about twice the areal density at 0.76 MBq/cm² and the other small disk was filled with water and had zero activity. The total activity of ^{24}Na was 44 MBq at the beginning of the run, and it decreased by 52% over the 16 hr duration of the measurement. When filled with these solutions the phantom effectively consisted of a hot and cold spot within a larger, circular background region with relative intensities of 2.1, 0 and 1.0, respectively. One difficulty with the phantom not shown in the sketch is the 0.6 cm thick wall that makes up the curved surfaces of the small disks. These walls act to enlarge the cold spot to 4.7 cm in diameter and add a concentric, cold ring around the hot spot.

In addition to demonstrating the ability of the RCC to identify regions of high and zero intensity in the presence of a background, this phantom provides an opportunity to study other parameters important to image quality. One such parameter is the contrast retained in the image of the phantom relative to the phantom itself, which reflects the

ability of the RCC to quantify the intensity of a source in the presence of a significant background. Another parameter of importance is the image noise which is easily determined from the phantom due to its size and shape. Finally, the ability of the RCC to reproduce the flat intensity profile of the large disk can be studied.

A table describing the RCC system, the source and the source space is included in Appendix D as Table D-4. The pixels used in reconstructing this image are 0.4 cm by 0.4 cm, twice as large as those used for any other source. The major reason for this difference in pixel size is that the number of counts collected in the imaging run is not sufficient to reconstruct the image at the higher frequencies found in the finer meshed source space. The net counts in the 1.37 MeV data image is only 187,000 with a fractional uncertainty of 0.35%. The total area of this phantom is equal to 960 of the 0.4 cm pixels. For comparison, the low energy ^{24}Na bar source data has a fractional uncertainty of 0.193% and subtends only 200 of the 0.2 cm pixels used in its reconstruction (assuming each bar is 2 pixels wide). Thus, the image of this phantom will be noisier than the low energy ^{24}Na bar source, even when reconstructed on the coarser source grid. The maximum radial extent of this phantom is approximately twice that of any of the other sources. To keep the azimuthal sampling consistent with those images, the number of second detector positions is increased to 64, in accordance with Equation (3-61).

An image of the 1.37 MeV gamma rays emitted by the flood-field phantom is shown as a gray scale density plot in Figure 6-18a. This image is reconstructed using the RFL algorithm with a PARZ filter having $v_{\text{max}} = 0.5$. It is apparent that the quality of this image is not as good as those shown in earlier sections. Noise is largely responsible for the poorer image quality. With its coarse pixels, the resolution of this image is poorer than the others, but it cannot be measured directly. An estimate of the FWHM resolution is found to be 1.33 cm by reconstructing an image of the low energy gamma rays from the ^{24}Na line source on a grid of 0.4 cm by 0.4 cm pixels and using the same reconstruction filter.

Figure 6-18. Gray scale density plots showing images of the 1.37 MeV gamma rays emitted by the ^{24}Na flood-field phantom reconstructed using a) the RFL algorithm with a PARZ filter having $v_{\text{max}} = 0.5$ and b) the ART algorithm after low-pass filtering with a Parzen window also having $v_{\text{max}} = 0.5$.

In Figure 6-18b an ART reconstruction of the 1.37 MeV gamma rays from the flood-field phantom is shown after low pass filtering with a Parzen window having $v_{\text{max}} = 0.5$. Inside the boundaries of the large disk, the two images are very similar. The hot and cold spot are reproduced in both images, and both images are noisy with discontinuous representations of the large disk boundary. There is a significant difference between the cold region outside the large disk boundary in the two images. In the filtered image, this cold region contains much more noise than in the ART image.

The difference between the two images in Figure 6-18 may be due to a large number of contaminating events that deposit small energies in the first detector and are placed in ray-sums for small scattering angles in accordance with Equation (2-6). The large number of low angle contaminating events is illustrated in a graph of ray-sum counts measured from the flood-field phantom versus scattering angle in Figure D-3. For the eight element pairs used to form this graph, the phantom subtends angles ranging from 25.3° to 92.8° . Thus, counts in the range below the minimum scattering angle from 12° through 24° are due only to contaminating event noise, and counts above this

minimum from 28° to 40° include noise and phantom signal. The ratio of counts below the minimum angle to counts in the range from 28° to 40° is 0.69. At high scattering angles the ratio of noisy counts above the maximum angle (94° through 106°) to counts just below the maximum (78° through 90°) is significantly lower at 0.44.

When the ray-sums measured from this phantom are used to form a laminogram on the extended source space specified for the RFL algorithm in Section 3.4.1, the laminogram contains two concentric disks: one corresponding to the large disk of the phantom with a radius of 7.0 cm and another with a radius of about 11 cm. From Equations (5-2) and (5-3), the radius of larger disk corresponds to the θ_c of 12° in the ray-sums and is thus related to the noise in the low angle ray-sums. The laminogram of the phantom is shown on the extended source plane in Figure D-4a in Appendix D, where the 11 cm radius disk due to the low angle noise can readily be seen. An RFL reconstruction of the laminogram of Figure D-4a which retains the 11 cm radius "noise disk", and is shown in Figure D-4b. Figure 6-18a is simply a truncated version of Figure D-4b, and the noise in its cold region is simply part of the "noise disk".

While the RFL algorithm retains the noise disk, the ART algorithm moves image intensity from the noise disk to pixels at the outer edges of the extended source space. In fact, Figure D-5a of Appendix D shows that the noise disk is nearly eliminated after a single iteration of the ART algorithm, while the intensity is significantly enhanced at the edges. To illustrate how the ART algorithm might move intensity to the edges of a source space, consider two sets of ray-sums: one from a single second-detector element at $\phi = 0^\circ$ and the other from a second-detector element at $\phi = 90^\circ$. Assume that there is only one of the noisy low-angle ray-sums, and it is in the $\phi = 90^\circ$ set. This ray-sum is designate n with λ_n counts. In a laminogram formed by backprojecting both sets of ray-sums, pixels intercepted by ray-sum n will include intensity that does not correlate with counts in the $\phi = 0^\circ$ ray-sums ($\lambda_i, i = 1 \dots M_0$). Also assume that the angular range of the ray-sums only

covers about half of the source space width. See as an example Figure 3-1, where the range of scattering angles from $\theta = 30^\circ$ to $\theta = 90^\circ$ covers about half the source space width.

At the start of the first ART iteration in the example, the laminogram intensity is forward projected into the $\phi = 0^\circ$ ray-sums, according to Equation (3-67), to form q_i . For those $\phi = 0^\circ$ ray-sums that share pixels with ray-sum n , the uncorrelated intensity will cause q_i to be greater than λ_i , and the ART algorithm will subtract intensity from all pixels intercepted by these ray-sums. At the end of the first iteration, the modified image intensity will be forward projected into the $\phi = 90^\circ$ ray-sums to find q_j ($j = 1 \dots M_\theta$). Due to the earlier adjustment in intensity, q_j will be less than λ_j , and the ART algorithm will add counts to those pixels intercepted by the $\phi = 90^\circ$ ray-sums, including ray-sum n . Those pixels in the $\phi = 90^\circ$ ray-sums not intercepted by the $\phi = 0^\circ$ ray-sums will have increased intensity after the first iteration. As the ART algorithm proceeds, the uncorrelated intensity of the noisy ray-sum is iteratively added and subtracted from the image, resulting in a gradual transfer from the pixels intercepted by both sets of ray-sums to pixels at the edges of the source space that are intercepted by only the $\phi = 90^\circ$ ray-sums. The transfer of intensity in later iterations is illustrated in Figure D-5b, where the intensity of the noise disk is suppressed below all other regions of the image after the algorithm converges. This supports the contention that image intensity from uncorrelated events is moved to pixels that are intercepted by the fewest number of ray-sums.

Like the example presented above, pixels lying in the RCC FOV, which include the phantom and the noise disk in Figure D-4a, are intercepted by ray-sums from all pairs of RCC detector elements. On the other hand, pixels lying at the edges of the source space are only intercepted by a fraction of the ray-sums due to "blind spots". It is hypothesized that the intensity shift from the noise disk to the source space edges, noted in the ART reconstruction of the phantom, results from the same iterative addition and subtraction of uncorrelated intensity that is illustrated in the example.

Figure 6-19. Gray scale density plots showing a) a discrete model of the flood-field phantom and b) a simulated image of the phantom reconstructed using the RFL algorithm with a PARZ filter having $v_{\max} = 0.5$.

To assist in analysis of the noisy measured images, a simulated image of this phantom is also reconstructed. In Figure 6-19a, a discrete model of the phantom is shown as a gray scale density plot. This model is computed by overlaying the outline of the phantom on a grid of 0.4 cm by 0.4 cm pixels and assigning an intensity to each pixel that is proportional to the product of the intensity and area of the phantom that intersects it. Special care has been taken to include the cold walls of the small disks in the model. The intensity ratio between the hot spot and the background is the intended 2.0 versus the value of 2.1 actually achieved for the phantom. A simulation is done by first forward projecting the phantom model into the ray-sums used for reconstructing the measured data, blurring the ray-sums with a Gaussian function having a FWHM of 5° , and then forming a laminogram from these noiseless simulated ray-sums. Figure 6-19b shows an image reconstructed from these ray-sums using the RFL algorithm. The reconstruction filter is the same PARZ filter with $v_{\max} = 0.5$ used for the measured image of Figure 6-18a. As in the measured image, the cold ring around the hot spot is reproduced in the simulated image, and the background intensity is not fully recovered in the narrow region where the cold spot passes closest to the edge of the background disk.

Horizontal and vertical contours through the discrete model of the phantom, the simulated image and the two measured images are include in Figure 6-20. The horizontal contours represent an average of the two central rows of pixels in the image, and the vertical contour is an average of the two central columns. The RFL algorithm yields negative intensities in the center of the cold spot of both the measured and simulated images. Thus a constant factor is added to these contours such that the minimum intensity in the cold spot is zero, and in all contours the intensity is normalized such that the average background is unity. The average background intensity is computed from a total of 96 pixels inside the large disk which are 2.2 cm, or further, from the edge of the large disk or the outside walls of the small disks.

For the measured images, the relative uncertainty in the average background s_r is computed over the same 96 pixels used in finding the average background and has values of 0.0851 for the filtered image and 0.101 for the ART image. The larger noise in the ART image is consistent with comparisons done with the ^{24}Na bar source and the M-shaped ^{64}Cu source. The inverse of the s_r computed here is commonly referred to as the signal-to-noise ratio (SNR). The SNR has the value of 11.8 for the filtered image and 9.90 for the ART image.

Contrast C is commonly defined as the difference in intensity ΔI from a background level $\bar{I}$ [Mac83, Sin90].

$$C = \frac{|\Delta I|}{\bar{I}} \quad (6-6)$$

The ability to visualize objects in a noise-free environment is related to contrast, but is also affected by noise. It has been shown by Rose [Ros48], among others, that an object in a noisy image is detectable if the ratio of C to SNR is greater than 3. Some authors have also defined this as a signal-to-noise ratio [Mac83, Sin90]. Here it is defined as the contrast-to-noise ratio CNR. Recognizing that $\text{SNR} = \bar{I}/\sigma_I$, where σ_I is the standard deviation of the background level, CNR can be computed from

a)

b)

Figure 6-20. Contours through the digital model of the flood-field phantom and through reconstructed images of the phantom. The contours are taken through a) the horizontal center and b) the vertical center of the images.

$$\text{CNR} = \frac{C}{\text{SNR}} = \frac{\Delta I}{\sigma_I} \quad (6-7)$$

The contrast of both the hot and cold spot of the phantom is designed to be 1.0, as is illustrated in the horizontal contour of the digital model included in Figure 6-20. The actual contrast in the cold spot is 1.0 as intended but the hot spot contrast is 1.1. From the measurements, C is 0.953 in the ART image and 0.931 in the filtered image with the peak of the horizontal contour taken as the hot spot intensity. Thus 86% of the contrast is recovered in the ART image and 85% of the contrast is recovered in the filtered image. These values of contrast recovery agree within the uncertainty in the data. In the image of the simulated reconstruction, C of the hot spot is 0.932. This contrast recovery of 93% is low for a noiseless image, indicating that the peak intensity is affected by the poor resolution in the image. In fact, when the simulated laminogram is reconstruction with a sharp RECT filter to improve the resolution, C increases to 0.991. If enough data had been collected to warrant a sharper reconstruction filter, the contrast recovery in the measured images could be increased similarly. The contrast of the cold spot in the filtered images is meaningless, because the minimum of the cold spot has purposely been set to zero. However in the ART image, the cold spot contrast is 0.841. The cold spot intensity is computed by taking an average of horizontal contour values between $y = 3$ cm and $y = 5.4$ cm, representing the two minima in the cold spot. The CNR of the measured hot spots is 10.9 in the filtered image and 9.44 in the ART image, while the CNR of the cold spot in the ART image is 8.33. Since these ratios are all approximately three times Rose's minimum detectability criterion, the objects should be easily detected, and visual inspection of Figure 6-18 confirms this prediction.

The degree to which images retain the flat intensity profile over the whole of the large disk is shown in the vertical contours of Figure 6-20b. The simulated image has a

very flat contour except at the edges ($|y| \geq 4.6$ cm), where the intensity falls off due to the finite resolution, and at the center, where the presence of the small disks affects the intensity. Contours through the two measured images appear to follow the simulated contour, but with a good deal of superimposed noise. The variation of the measured contours from the simulated one is consistent with the measured noise levels.

The vertical contours of Figure 6-20b show that the size of the phantom in the simulated and measured images is in good agreement with the actual size, represented by the model contour. Defining the edge of the image as the point where the contour falls to half maximum and computing the half maximum relative to the average of the minimum intensities on either end of the contour, the width of each image agrees with the model to within a pixel width. The agreement between the measured horizontal contours and the model contour is not as good. The left half of the image appears to be shifted outward by approximately one pixel, while the right edge also appears to be shifted outward by about one pixel width. The simulation indicates no such shift, so it is not possible to infer the cause of the shifts from the noisy measured images alone. However, a worst-case interpretation would ascribe these 7% variations to the shift-variant point response of the RCC.

In summary, the images measured from the 1.37 MeV gamma rays emitted by the flood-field phantom give a contrast recovery of 85% or better for the hot spot and 84% for the cold spot. The SNR of the image produced by the RFL algorithm is 11.8, while the SNR of the ART image is 9.90. Vertical contours through the measured images and a simulated image show that the flat intensity profile of the phantom is reproduced over its entire length. The contrast and noise of the two measured images are similar, but the ART image has considerably less noise outside the boundary of the phantom. This image is the first case where the ART algorithm has generated a significantly better image than the RFL algorithm.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

In this work, a Compton scatter camera based on a novel ring-shaped second detector has been devised for nuclear industrial applications, and the advantages of a Compton scatter camera for imaging in an industrial setting have been described. The ring-shaped shaped second detector makes the RCC different from previous Compton cameras and offers the advantages of a fast Fourier-based reconstruction algorithm and reduction of unwanted events which are either not scattered in the first detector or suffer small angle scatters.

Analytic models for the resolution, efficiency and contaminating event rates in the RCC from on-axis point sources have been developed and used in designing the RCC. One direct Fourier-based image reconstruction algorithm, the RFL algorithm, has been implemented in a computer code along with the iterative ART and SIRT algorithms. Based on these reconstruction algorithms, a code for simulating the response of the RCC to a planar source has been developed. Following predictions from the analytical models, the RCC was built around the first detector, which had previously been used in a Compton camera designed for medical applications [Sin83a, Sin90]. Parameters critical to the performance of the RCC have been measured; the response of the camera to point sources has also been measured, and imaging characteristics of the RCC have been determined by imaging spatially extended sources.

This chapter is divided into two sections. Section 7.1 includes a summary of critical system parameters and related conclusions; a description of unique measurement methods and data analysis techniques used; comparisons between the three image reconstruction algorithms described above and an iterative MLE algorithm; a summary of

RCC point response measurements with comparisons to the analytic models and simulations, and imaging characteristics of the RCC drawn primarily from measurements of spatially extended sources. Section 7.2 includes recommendations for future work to improve the RCC system performance, measurement methods, data analysis techniques, analytic models, laminogram simulations and image reconstruction algorithms.

7.1 Summary and Conclusions

7.1.1 System Parameters

The RCC consists of five primary components: a first detector, a second detector, a mechanical support and alignment system, data acquisition electronics and software. Performance parameters of each component are summarized here along with system performance parameters that depend on more than one component.

The first-detector array of the RCC consists of 16 rectangular HPGe solid state radiation detectors in a planar array. The average energy resolution of the elements has been measured to be 1.1 keV at 0.122 MeV and 1.3 keV at 0.662 MeV. Relative efficiencies of these elements vary by up to 12%, and corrections are made for these variations during image reconstruction.

The second-detector array consists of a ring of 16 cylindrical NaI(Tl) scintillators. The average resolution of these elements has been measured to be 10.3% at 0.662 MeV. This resolution is poor compared to a typical small NaI(Tl) crystal, which can have a resolution as low as 6% at 0.662 MeV [Kno89]. The average resolution has been measured to vary from 11.8% at 0.511 MeV to 6.3% at 1.69 MeV. The PMTs used in the RCC suffer large gain drifts when exposed to intense sources: drifts as high as 5% per hour have been noted. These drifts act to degrade summed energy resolution and vary critical energy thresholds, making frequent energy calibrations necessary.

A two piece support system holds the elements of both detectors in alignment with one another and the source. The total angular uncertainty resulting from this system is estimated to be less than or equal to 1.4° . The second-detector support allows for a wide range of θ_c and d ; it also includes a wheel so that second detector can be rotated to simulate larger arrays. One consequence of this flexibility is that alignment of the second-detector elements is a slow, tedious process. The sleds that hold the second-detector elements were modified to allow all 16 elements to be used when θ_c is large, but a small systematic error in determining the placement of these new sleds resulted in all of the extended source and some of the point source measurements having θ_c of 58° or 59° when 60° was intended. The impact of this systematic error is negligible.

Each element of both detectors has its own pulse processing electronics consisting of a timing branch and spectroscopy branch. The electronics allow for coincident image data collection, pulse height calibration and timing calibration. Software for acquiring data from the electronics and controlling the electronics is a Microsoft Windows application implemented on a personal computer. This application has all of the routines needed to measure and generate images from the RCC including automated calibration, data acquisition, run time diagnostics, data storage, data analysis, and image reconstruction.

Timing resolution between the two detectors for selecting coincident events is typically 12.7 ns FWHM and 31.4 ns FWTM and is dominated by time pick-off in the first-detector elements. To generate images of different energy gamma rays (and thus different radioisotopes), events are selected based on the summed energy deposited in the two detectors. The resolution of the summed energy histogram is a function of both E_γ and θ , and has been measured to be 9.8% at 0.602 MeV and 4.1% at 1.69 MeV when θ is 60° . Dead time in the RCC is dominated by the single event count rate in the first detector. Using a nonparalyzable model, the system dead time has been measured as 2.1 μ s.

Measurements of count rates in the first detector show that as many as 23% of full energy events may be lost due to cross-talk in the first detector.

7.1.2 Measurement Methods and Data Analysis Techniques

There are several methods in performing imaging measurements with the RCC that are not commonly used in other gamma-ray spectroscopy or imaging systems. The automated methods for calibrating the pulse heights and timing in the 32 total elements of the RCC are described. In addition, methods for aligning the source and detectors, and optimizing data and background collection times have been described. The lower level thresholds in both detectors are critical because they impose a finite FOV on the RCC. Outside this FOV events are lost from one or more of the second-detector elements, resulting in a distorted image. Methods for selecting and setting the energy thresholds have been described, along with ways of compensating for gain drifts in the second-detector elements which reduce the FOV.

A technique for selecting measured events based on their summed energy has been described, as have the calculations necessary to form ray-sums from the measured data. These calculations include a correction for the differential nonlinearity of the spectroscopy electronics in the first-detector elements. Methods for subtracting the random background and combining different second-detector views are developed for imaging a decaying source.

7.1.3 Image Reconstruction Algorithms

The direct RFL algorithm for reconstructing RCC algorithms is based on emission computed tomography. In emission computed tomography, the intensity of a radiation source is integrated over straight rays through the source, while the RCC integrates over curved paths through the source. The ring shape of the second detector results in 16 or more integration paths which intersect at each point in the source plane to be imaged, which are similar to those of a straight-ray tomography device. However, the shape of the

RCC integration paths changes depending on the position in the image plane. In straight-ray tomography, Fourier transforms of the integrated data in the ray-sums is related to the Fourier transform of the source intensity by the Central Slice Theorem. This theorem does not directly apply to the ray-sums of the RCC, but others have successfully used Fourier techniques to reconstruct images from devices that integrate over curved paths. Two such systems have been reviewed. Simulations of laminograms formed from on and off-axis point sources show that the frequency response of the RCC closely follows a $1/\rho$ fall off, which is characteristic of straight-ray tomography. The ramp filter of the RFL algorithm serves to compensate for this frequency fall off.

A method for forming laminograms necessary for the RFL algorithm is developed mathematically in Chapter III. Likewise, a method for applying a ramp filter to the laminogram is also developed there. Several apodizing functions for smoothly truncating high frequencies in the laminogram are described, and equations are provided for estimating the radial and azimuthal sampling necessary to avoid aliasing.

An iterative ART algorithm is developed along with a closely related SIRT algorithm. An iterative MLE algorithm developed by another [Dgn93a] is also described. These iterative algorithms have been used by others [Sin90, Bre90, Heb87] to reconstruct Compton camera images, and provide comparisons by which the performance of the RFL algorithm has been gauged. It is important to note that the comparisons made between image reconstruction algorithms in this work are of a practical nature. Mathematically rigorous comparisons require these iterative algorithms be run to complete convergence for a well characterized solution. Comparisons made here are based on the need to efficiently produce images with good visual characteristics, such as low noise and high resolution, and thus the number of iterations was limited to a small number.

Reconstruction of a series of PSFs using the RFL algorithm with the standard BUTR filter results in a marked improvement in resolution compared to the laminogram itself. The improved resolution is reflected not only in the FWHM but also the spectral

resolution and PSF figure of merit. Less improvement in resolution is found with the PARZ filter than with the standard BUTR, although the resolution is much better than the laminogram. ART reconstruction of these PSFs gives somewhat better resolution than the RFL reconstruction with the standard BUTR filter. When the ART PSFs are low pass filtered with the same Butterworth function used to apodize the ramp filter, the ART and ramp-filtered PSFs are in excellent agreement. The ratio SR:FWHM in the PSFs shows that they are weighted somewhat heavier in the tails than a continuous Gaussian function, but they are much closer to a Gaussian shape than the laminogram itself. This ratio, like the SR resolution measure itself, is very sensitive to noise in the PSF.

The SIRT algorithm has been found to give a much worse resolution than the ART for a PSF while requiring 2.5 times more iterations to converge. Due to this poor performance the SIRT algorithm has not been investigated further. An image of two point sources has been reconstructed with the MLE algorithm with a slightly better resolution than the RFL and ART algorithms. However, the high computation cost of this algorithm prohibited its further use in reconstructing point source data.

Images of the 2.75 MeV gamma rays emitted from the ^{24}Na line source have been reconstructed using the RFL algorithm with five different filters introduced in Section 3.4. The standard BUTR filter yields the best resolution, and the PARZ filter with $v_{\max} = 0.5$ yields the worst resolution but gives the least noise. The HANN filter produces an image with resolution and noise characteristics that fall almost halfway between the extremes of the BUTR and the PARZ filters. The SHEP filter gives an image with slightly worse resolution and much worse noise than the standard BUTR, and the HAMM filter yields an image that is similar to the HANN filter in both resolution and noise. The effect of $v_{\max}$ on image quality has also been studied by varying its value from 0.5 down to 0.2 in increments of 0.1 in reconstructing the 2.75 MeV gamma rays from the ^{24}Na line source with the RFL algorithm. The resolution deteriorates as $v_{\max}$ is reduced from 0.5, but the noise decreases.

Spatially extended sources reconstructed with the RFL, ART and MLE algorithms have also been compared. The 1.37 MeV gamma-ray data from the ^{24}Na bar source have been reconstructed with both the RFL algorithm and the ART algorithm. After low pass filtering the ART image with the same function used to apodize the ramp filter, the ART image is about 2% sharper and 7% noisier than the RFL image. Images of the 1.37 MeV gamma rays emitted by flood-field phantom have also been reconstructed with these two algorithms. Contrast in the low pass filtered ART image is comparable to the RFL image, while the noise in the ART image is somewhat greater. The RFL image includes significant noise outside the boundary of the phantom, but the ART image does not have this artifact. This noise is thought to be caused by low energy contaminating events in the measured ray-sums. The ART algorithm moves the noise intensity to the outer edges of the image plane, while the RFL algorithm does not. Images of the 0.511 MeV annihilation photons emitted by the ^{64}Cu "M" in the multi-energy phantom have been reconstructed with the RFL, ART and MLE algorithms. Resolution in the ART and RFL images is similar, but the MLE image is about 28% worse. The peak to valley ratio in contours through the legs of the "M" (which is indicative of the contrast) is similar for the two iterative algorithms, but is about 36% better for the direct algorithm.

In reconstructing the ^{64}Cu "M", the MLE algorithm required about 7.5 hours on a CRAY YMP; the ART algorithm required 4.5 hours on a PC, and the RFL algorithm ran for 26 min. on the same PC. Due to its speed, the RFL algorithm is much more useful than the iterative algorithms for reconstructing images in an industrial setting. The most significant disadvantage the direct RFL algorithm compared to the iterative algorithms is the noise outside of the boundary of the flood-field phantom. Another disadvantage of the RFL algorithm is the requirements for radial and azimuthal sampling necessary to avoid aliasing, because both the number of angular bins and second-detector positions increase linearly with FOV radius a . This same requirement does not apply to the iterative algorithms. However, this is not a severe problem because the number of second detector

elements needs to be increased to improve efficiency, as discussed in Section 7.2.1. The recommended number of elements is 128, which will properly sample to roughly twice the maximum radius of 7.0 cm (at $z_s = 10$ cm) imaged in this work.

7.1.4 Point Response

Analytic models of the resolution, efficiency and contaminating event rates have been developed for on-axis point sources. The resolution and efficiency model closely follow models developed by Singh [Sin83a]. The resolution model includes components due to the finite geometrical size of the detector elements $\Delta\theta_g$ and the finite energy resolution of the device $\Delta\theta_e$. Expressions are derived for $\Delta\theta_e$ in three cases: when E_γ is known a priori, when E_γ is determined with some uncertainty, a posteriori, and when E_γ is determined on an event-by-event basis again with uncertainty. This model predicts a FWHM angular resolution. Models of several types of contaminating events have been developed to predict the absolute efficiency or count rate of these events for a particular summed energy window. Identified contaminating event types include multiple scattering in the first detector, reverse scattering, single escape coincidence, partial energy deposition, random coincidence and cascade coincidence.

Intrinsic efficiencies of the RCC have been measured for a range of E_γ , and the geometry parameters θ_c and d . When θ_c is 60° , the measured efficiency ranges from 2.86×10^{-4} at 0.511 MeV down to 2.34×10^{-5} at 2.75 MeV. These efficiencies have been corrected for measured random background, and when corrected for predictions of all other types of contaminating events, the efficiencies range is from 2.83×10^{-4} at 0.511 MeV to 1.56×10^{-5} at 2.75 MeV. The model consistently over-predicts the corrected measurement by 0.5% to 25% when E_γ is 1.37 MeV or below. This discrepancy is likely due to absorption in the cryostat of the first detector and cross-talk in the first detector, neither of which is included in the model. At higher energies the discrepancy is more severe; the model over-predicts the efficiency by up to 90%. Inaccuracy in the model at these energies is most

likely due to an inadequate model of recoil electron penetration of the first-detector elements. Variations of the measured efficiency with θ_c and d are closely predicted by the model. Measured off-axis efficiencies are reasonably predicted using the on-axis efficiency model after correcting for the solid angle.

The FWHM angular resolution $\Delta\theta_t$ has been determined from a series of PSFs using a single element backprojection technique and reconstructions with the RFL algorithm using the standard BUTR filter. When θ_c is 60° , $\Delta\theta_t$ ranges from 4.98° FWHM at 0.602 MeV to 3.69° at 2.75 MeV. Comparison of these measured resolutions to those predicted by the model shows good agreement when E_γ is at 1.37 MeV or above. The model under-predicts the resolution at lower energies. This disagreement is due to problems with the measurements: the ^{64}Cu and ^{137}Cs sources are too large, and there are too many contaminating events interfering with the ^{124}Sb measurement. The predicted variation in $\Delta\theta_t$ is less than 20% over the entire energy range. This stability in resolution with energy is due to the fact that the resolution is dominated by $\Delta\theta_g$. Variation in $\Delta\theta_t$ with θ_c is measured to be small, and the model reflects this stability. Both measurements and predictions indicate that $\Delta\theta_t$ varies strongly with d .

Measurements have been made of the random coincidence events, and all images presented in this work have been corrected using the measured random background. Comparisons between predictions of the random coincidence events and measured spectra from a ^{64}Cu point source and a ^{24}Na line source shown in Section 5.3 are favorable. The predicted shape of the energy spectrum for cascade coincidence events from a ^{24}Na line source closely matches a measured spectrum. Based on this prediction 30% of the counts in this spectrum are estimated to be cascade coincidence events. No measurements have been made to validate the models of other contaminating event types.

The point response models provide useful tools for designing a Compton camera, and they have been used to optimize the RCC geometry. For example, based on the resolution model the most effective way to change the resolution is by varying d ; however,

the efficiency model shows that improving the resolution in this way comes at a considerable cost in efficiency. In another example, the area of the RCC FOV can be increased by about a factor of 3 by increasing θ_c from 45° to 60° . The resolution model shows that at an E_γ of 0.662 MeV there is no penalty in resolution and only a 20% loss in efficiency when θ_c is changed in this way.

The analytic models are also useful when used in concert with an algorithm for simulating laminograms formed with the RCC. The simulated laminograms of on and off-axis point sources have been used in Section 3.2.3 to show that the frequency response of the RCC falls approximately as $1/p$. Efficiencies used in this simulation had been calculated with the analytic efficiency model. In Section 5.5.2, a simulation is shown to closely reproduce the measured MTF and reconstructed PSF from an on-axis point source. The agreement between the measurement and the simulations is only achieved after the simulated ray-sums are blurred with a Gaussian function and a contaminating event continua are added. The angular width of the Gaussian blurring function had been predicted with the resolution model. These simulations also proved useful in studying the effectiveness of the RFL algorithm in reconstructing both point sources and spatially extended sources.

A point source positioned 29° off the ring axis has also been imaged to study the difference between the off-axis PSF and those measured on the ring axis. While the on-axis PSFs are symmetric, the off-axis PSF reconstructed with the RFL algorithm becomes elongated in the radial direction with the spatial FWHM 29% broader in the radial direction than in the tangential. Ideally the PSF should be space invariant, but the PSFs of several successful PET devices show the same radial elongation at off-axis positions [Der88, Lec90, Evn91].

The useful FOV of the RCC will be limited by the radial elongation of the PSF, which becomes more severe as the source polar angle is increased beyond 29° . While this distortion has only been measured with the RFL algorithm here, it is primarily due to the

localization cones intersection the source plane obliquely at off-axis positions and will also affect the iterative algorithms. The sampling requirements will also act to limit the useful FOV as its radius is increased beyond twice the maximum measured in this work.

The point response of the RCC has been compared to hypothetical pinhole camera with a 2.5 cm thick NaI(Tl) detector. At 0.511 MeV the pinhole camera is about 16 times more efficient than the RCC for imaging a point source and 32 times for a flood-field phantom. At 2.75 MeV the pinhole camera is estimated to be up to 25 times more efficient than the RCC for a flood-field phantom. In Section 7.2.1, recommendations are made for scaling up the RCC efficiency.

7.1.5 Image Characteristics

There are several important characteristics of an imaging system that can not be determined from the point response alone. The resolution variation across the image can be determined more readily than from a series of point source measurements, the noise can actually be measured, not simply inferred from the uncertainty in the counts, and recovery of source contrast and shape can also be measured.

The minimum resolution reported for a spatially extended source imaged with the RCC is 0.76 cm from the image of the 2.75 MeV gamma rays emitted by a ^{24}Na line source. This image has been reconstructed using the RFL algorithm with the standard BUTR filter. The corresponding angular resolution at the ring axis is 4.4° . There is no measurable change in the resolution along the length of this source, which extends 2.5 cm from the ring axis to a polar angle of 14° . This unchanging tangential resolution is consistent with the off-axis PSF measurement. In the image of the 2.75 MeV gamma rays emitted by a ^{24}Na bar source, the resolution of the center bar, which passes through the ring axis, is 0.86 cm or 4.9° . The image has been reconstructed with the RFL algorithm using a HANN filter having $v_{\text{max}} = 0.5$, and the resolution agrees with an image of the line source reconstructed with the same filter. As with the line source there is no measurable

change in the resolution along the length of each bar; however, there is a 12% increase in the FWHM spatial resolution of the off-axis bars. The angular resolution at the center of the off-axis bars is 5.3° corresponding to a 7% increase from the on-axis value. Thus, degradation in the radial resolution is primarily due to the localization cones intersecting the source plane obliquely at off-axis positions and is consistent with the radial resolution measured from an off-axis point source.

There is some loss in resolution in the 1.37 MeV gamma-ray image of all ^{24}Na sources relative to the 2.75 MeV image. A similar result is found for any photons imaged in the presence of higher energy gamma rays. This resolution degradation is likely due to contaminating events, but a particular event type has not been identified.

The resolution of spatially extended images reconstructed with the ART algorithm is better than those reconstructed with the RFL algorithm. However, after the ART images are low-pass filtered with the same frequency function used to apodize the ramp filter in the RFL algorithm, the resolution is indistinguishable from that of the corresponding RFL images.

The RMS uncertainty s_r has been used as a measure of noise in reconstructed images of spatially extended sources. A model given by Budinger indicates that in tomographic images s_r is proportional to σ_n/C_n , the fractional uncertainty in the data used to form the image [Bud79]. For the ^{24}Na line source s_r has been computed along the two most intense rows in the image, and the average of the two rows $\bar{s}_r$ is reported to characterize the noise of this source. For the bar sources and the ^{65}Zn "U" of the multi-energy phantom, s_r is computed for the most two intense rows (or columns) of each bar, and an average of all these values is reported as $\bar{s}_r$. In each of these images, only pixels unaffected by resolution effects from the source edge are used in computing s_r .

There is a difficulty with using s_r as a measure of the variance in each image: it does not indicate covariance among the image pixels. There is covariance between the pixels in any image recorded with the RCC, because the ray-sums are independent

random variables, but each represents an integral over many pixels. The proper way to estimate image variance and covariance is by comparison of images reconstructed from several independent data sets measured under identical conditions; however, such measurements were not practical here due to the long data acquisition times required.

To study the impact of noise on image quality, the number of counts in the 1.37 MeV data from the ^{24}Na bar source has been varied and the image reconstructed with the RFL algorithm. Four different data sets have been generated with the number of net counts C_n ranging from 492,00 down to 11,400 and corresponding values of σ_n/C_n varying from 0.193% up to 1.00%. When images are reconstruction using a HANN filter with $v_{\max} = 0.5$, the resolution varies by less than 3% but $\bar{s}_r$ varies nearly in proportion with σ_n/C_n . Thus, proportionality between s_r and σ_n/C_n predicted by Budinger's relation in Equation (5-46) is found to apply to sources having the same shape. Equation (5-46) also predicts the effect of the number of source pixels on the noise in flood-field sources, but this relation does not hold for predicting the relative noise between bar sources with different areas. When the same four sets of data from the ^{24}Na bar source are reconstructed with different filters selected to keep the value of $\bar{s}_r$ fixed, the resolution increases from 1.04 cm FWHM at $\sigma_n/C_n = 0.193\%$ to 1.42 cm at $\sigma_n/C_n = 1.00\%$. This result has important implications for rapid imaging in industrial applications: a five-fold increase in σ_n/C_n , resulting from an 83-fold decrease in imaging time, yields an image with equivalent noise and only a 36% increase in the FWHM resolution.

The ART algorithm has been found to yield much noisier images than the RFL algorithm. For example, the ART reconstructed image of the 1.37 MeV gamma rays from the ^{24}Na bar source has an $\bar{s}_r$ of 0.104. After low-pass filtering using a Hann window with $v_{\max} = 0.5$, $\bar{s}_r$ is reduced to 0.049, which is somewhat larger than the value of 0.046 found in the RFL reconstructed image of this data.

The effect of noise on resolution in an on-axis PSF has also been studied by varying σ_n/C_n by a factor of four. No significant change in the FWHM resolution is noted

as σ_n/C_n increase; however, the resolution measure G is slightly degraded over this range, and SR degrades significantly. Both FWHM and G are less dependent on image noise than SR, and thus provide more reliable indications of image resolution.

The shapes and sizes of the line, bar and multi-energy sources, which are all made up of series of line segments, are well reproduced in the RCC images. The center of the lines in the images fall within 0.2 cm of their actual positions, and the lengths of the line segments measured in the images are all within 0.3 cm of the actual lengths; however, the lengths in the images are always larger than the actual values, which could be due, in part, to variations in alignment during different data runs. For the ^{24}Na flood-field phantom, the 14 cm diameter of the large disk is closely reproduced in vertical contours through both RFL and ART reconstructed images. However, in horizontal contours passing through the hot and cold spots, the diameter of the images is about 0.8 cm too large. The apparent growth in the measured images is not reproduced in a noiseless simulation, so the variation in the horizontal width of the measured images may simply be a result of noise.

Contrast recovery in the hot spot of the flood-field phantom is 85% in the RFL image and 86% in the ART image. Contrast recovery in the cold spot cannot be determined for the RFL image, but is measured to be 84% in the ART image. The flat profile of the vertical contour through the phantom is reproduced in the measured images from both algorithms to within the variation due to image noise.

Measurements have been made of a multi-energy phantom with two sources that emit different energy photons: the two photon energies have been separated spectroscopically, and independent images of the two sources have been generated. These images demonstrate the ability of the RCC to image a multi-energy gamma-ray field from spatially extended sources, something that had not previously been demonstrated with a Compton scatter imaging device. The resolution and noise in the image of the 1.12 MeV gamma rays emitted by the ^{65}Zn "U" are at least as good as the image of the ^{65}Zn bar

source which had been imaged alone. The image of the 0.511 MeV annihilation photons emitted by the ^{64}Cu "M" has worse resolution and contrast than expected, due in part to a small fraction of ^{65}Zn decays which also result in 0.511 MeV annihilation photons

7.2 Recommendations for Further Work

Based on the conclusions drawn in the previous section, several recommendations are made to improve the performance of the RCC and for related research which may lead to other improvements. The recommendations are divided into four areas: RCC system improvements, measurement methods, analytic models and simulations, and image reconstruction techniques.

7.2.1 RCC System Performance

For most of the spatially extended sources, the RFL reconstructed images have been blurred by the smooth reconstruction filters used to reduce noise due to the small number of counts collected. The impact of reducing the crystal separation distance d on the spatially extended images should be investigated, because it may be possible to generate images with similar resolution and noise in a shorter period of time. To illustrate, a reduction in d from 30 cm to 22.5 cm increases the predicted intrinsic efficiency by about 73% at all energies if $\theta_c = 60^\circ$, while the predicted resolution at 2.75 MeV is increased from 4.1° to 5.2° and at 0.511 MeV the resolution increases from 5.1° to 6.0° .

By increasing θ_c to 75° , the FOV of the RCC can be expanded. For example, the radius of distortion free FOV was 10.7 cm in imaging the 1.37 MeV gamma rays emitted by the flood-field phantom with $\theta_c = 59^\circ$. With the threshold in the first detector $E_{1\text{min}}$ held fixed, the FOV radius expands to 19.5 cm if $\theta_c = 75^\circ$. In addition to increasing the FOV area by a factor of 3.5, increasing the scattering angle also results in straighter paths of integration on average. The fraction of E_γ transferred to the recoil electron will increase

on average, improving the summed energy resolution. The disadvantages of increasing θ_c to 75° in this case appear insignificant, including a predicted 2% decrease in ϵ_{icc} , a 2% increase in $\Delta\theta_t$, and a reduction in E_{2min} from 0.28 MeV to 0.26 MeV, which will increase the random background slightly. But, due to the increased recoil electron energy, these electrons will be more likely to escape out of the first detector. Based on this example, it is recommended that the RCC be evaluated with θ_c at 75° to test the validity of these predicted changes in performance.

The efficiency of the RCC is quite low, as emphasized by the comparison to a pinhole camera in Section 5.6. Because the present system is considered a prototype, the low efficiency is not too troubling. To increase the efficiency for a ring Compton scatter camera that might be fielded, the number of elements in the second-detector ring should be increased from 16 to 128, and the number of first-detector elements should be increased from 16 to 36. These changes will increase the absolute efficiency by a factor of 18, but with a commensurate increase in the cost and complexity of the system.

Another recommended improvement for increasing the efficiency is to increase the size of the HPGe elements in the first detector. The present elements are 0.5 cm x 0.5 cm x 0.6 cm, and the recommended size is 1.0 cm x 1.0 cm x 1.2 cm, which is predicted to increase the absolute efficiency by a factor ranging from 6 at 0.5 MeV to 10 at 3.0 MeV. The increased efficiency at high energies is due in part to a reduction in the fraction of recoil electrons escaping from the first-detector elements. There are also some negative impacts of increasing the size of the first detector: angular resolution will deteriorate somewhat with predicted increases in $\Delta\theta_t$ ranging up to 24%, and Singh computes that the fraction of events undergoing double scatters will increase by 30% due to the increased thickness of the elements [Sin83a]. In addition, the increased size will worsen the time resolution with a resulting increase in the fraction of random background events; however, time resolution improvements described below will more than offset this increase.

For further increase in the RCC efficiency, the existing NaI(Tl) elements in the second-detector ring can be replaced with bismuth germanate (BGO) elements having the same active dimensions. The projected improvement in ϵ_{icc} with BGO elements is a result of increased peak efficiency ϵ_{ip2} , which is difficult to determine without performing measurements or Monte Carlo simulations. The 0.95 cm diameter by 5.08 cm long crystals used here are not a standard size, but their volume is only 12% larger than a standard 2.54 cm by 2.54 cm crystal. In addition the ratio of volume to surface area which effects the peak to total ratio is similar between these two sizes. To compare the efficiencies of the NaI(Tl) and BGO crystals, consider gamma rays of energies of 0.5 and 3.0 MeV undergoing scatters of 60° in the RCC with $d = 30$ cm. The scattered gamma ray energies are 0.34 and 0.76 MeV, and from the intrinsic peak efficiency model in Appendix A, ϵ_{ip2} is 0.39 at 0.34 MeV and 0.12 at 0.76 MeV. Interpolation of the measured efficiencies for a 2.54 cm by 2.54 cm BGO crystal by Wilson and Hewitt [Wln85] gives intrinsic peak efficiencies of 0.95 and 0.49 at these energies. Thus, the RCC efficiency will increase by a factor ranging from 2.4 to 4.3 depending on the incident E_γ . The energy resolution is generally worse for BGO than for NaI(Tl), but the present RCC elements have a poor resolution averaging 10.3% at 0.662 MeV. Small BGO crystals commonly have resolutions better than 12% at 0.662 MeV [Wln85, Bic94]. The time resolution with the BGO detectors will be somewhat worse than with the NaI(Tl) crystals because the decay time of the scintillation light is 300 ns in BGO versus 230 ns in NaI(Tl) [Kno89].

If the above suggestions are all implemented (increasing the number of elements, increasing the HPGe element size and replacing the NaI elements with BGO), the total efficiency is predicted to increase by factors ranging from 260 to 760 above that of the present RCC. With these changes the efficiency for 2.75 MeV gamma rays would be 30 times higher for imaging a flood-field phantom using the Compton camera than using the pinhole camera with a 2.5 cm thick detector described in Section 5.6.

The number of contaminating events could be reduced in the RCC if the present NaI(Tl) detectors were replaced with new, high quality NaI(Tl) detectors of the same size with significantly better resolution. If the average resolution can be reduced to 7% at 0.662 MeV, the summed energy resolution will also be improved by about 32%, and the summed energy window width can be reduced by about 1/3. By reducing the window width, the number of random background events, cascade coincidence events and PED events will all be reduced by about 1/3. Improved energy resolution will also result in better timing resolution for the second-detector elements. Finally new second-detector elements would presumably suffer less gain drift than those in the RCC resulting in sharper lower energy thresholds, which will reduce contaminating events and decrease dead time in these elements. Of course, new NaI(Tl) elements will not increase the efficiency as the recommended BGO crystals would.

Whether NaI(Tl) or BGO is selected, any new scintillation detectors incorporated in the RCC must include high count-rate PMTs to limit the deleterious effects of gain drift.

Random background from low energy gamma rays can be reduced by adding 0.5 cm thick lead shielding around the second-detector elements. For gamma rays with energies of 0.6 MeV and below, more than half will interact in the shielding. If the shielding covers only that half of the curved detector surface facing the source plane, 110 g of lead is need for each second-detector element and a total of 14 kg is needed for a 128 element ring.

Random background can be further reduced in the RCC if the time pick-off circuits for the HPGe elements were to be replaced with *amplitude and rise-time compensated* (ARC) circuits. For small germanium crystals like those in the present RCC, the FWHM time resolution has been computed to be less than 100 ps when an energy of 0.511 MeV is deposited [Boh73, Lla73]. For crystals with a volume of 1 cm³ like those recommended above, the timing increases to 120 ps, and when lower energy events are included the timing increases to 150 ps. Taking a conservative estimate of 1 ns timing in the HPGe

elements and 4 ns timing using the current electronics with new NaI(Tl) elements having 7% energy resolution, the total FWHM time resolution will be 4.1 ns. This time resolution is better than a factor of three smaller than the measured resolution of the current system. The time window width could then be reduced by the same factor, and the random background would be decreased by approximately a factor of three.

Dead time in the RCC is dominated by the count rate in the HPGe detector, because the entire detector is dead for 2.1 μ s each time one of the elements fires, and this situation will be exacerbated if recommended increases in the number and size of first-detector elements is implemented. To reduce the events lost to dead time, the number of time coincidence circuits needs to be increased. For the current system, the timing signals from the 16 HPGe could be routed to four different digital gate and delay circuits in groups of four, and the four time windows could be passed through a fast "or" before being applied to the coincidence circuits of the second-detector elements. Some modifications to the position codes and the interface control would also be necessary. For an upgraded system with 36 elements in the first detector and 128 elements in the second, significantly more circuits would be needed. Also for an upgraded system, computer-controllable lower level threshold circuits are recommended.

Time spent aligning the second-detector element sleds could be reduced if the present manual system of guide bars and locking screws were replaced with a motorized system. The reliability of positioning these elements would also be improved. If the second-detector ring were expanded to 128 elements, the present system would have to be redesigned to accommodate them.

7.2.2 Measurement Methods and Data Analysis Techniques

The method of normalizing the random background presented in Section 5.2.2 is only valid for a decaying source with a single isotope. For imaging multi-energy gamma-ray fields, such as that from the multi-energy phantom of Section 6.4, this methodology

has to be abandoned. To image multi-energy fields from unknown sources, it is recommended that the duration of the image data and random background runs be reduced to a fraction of the shortest anticipated isotope half-life and the acquisition mode be alternated between data and background. Using this method, decay of the gamma-ray producing isotopes can be neglected between a data run and the ensuing background run; however, this will only be practical if the acquisition modes are alternated automatically. Fortunately, the RCC is presently set up to control τ_d , the time delay between the two detectors, and alternating these modes is simply a matter of writing software to automatically change τ_d . It may also be desirable to modify the software to automatically generate ray-sums from the data as it is acquired and likewise perform a weighted subtraction of the background as it is acquired. This modification will increase image reconstruction speed, but only when E_γ is known beforehand.

Images of gamma rays measured in the presence of higher energy gamma rays have been shown to have degraded resolution, and it appears that this is caused by one of the contaminating event types, but the specific type could not be determined. It is recommended that measurements be designed and carried out to determine the magnitude of contaminating events other than random coincidence events. As an example, cascade coincidence events could be measured by shielding the second-detector elements from the first detector to eliminate direct Compton scattered events. Also, reverse scattering events could be measured by shielding the first detector from incident gamma rays from a source, while the second detector is fully exposed to the source. These measurements may lead to methods for removing these contaminating event types from measured ray-sums.

It may be possible to measure and correct for contaminating event types without making extra measurement runs, as is presently done to correct for the random coincidence events. For a well resolved peak in a summed energy spectrum, it may be possible to estimate the number of contaminating events in the summed energy window of

width E_w set about that peak from background windows set on either side of this data window. The maximum width needed for the background windows is $E_w/2$, which can be reduced based on the relative counts between the data and background windows in a manner analogous to the duration of random background runs set by Equation (5-9). Ray-sums could then be formed from the sum of the two background windows to estimate the contribution of the contaminating events to the data window, and these ray-sums could be subtracted from the data ray-sums. This method will only be effective if the contaminating events are dominated by those types with slowly varying summed energy spectra such as random coincidence, cascade coincidence and PED. It is recommended that this method be tested by forming ray-sums from the data and background windows described above set on the summed energy spectrum of a measured random background run. If these ray-sums prove to be similar, further investigation of this background window method may be warranted.

7.2.3 Analytic Models and Simulations

The analytic efficiency model for on-axis point sources fails to closely predict the measured results for incident gamma rays with high energies that undergo high angle scatters in the first detector. This failure appears to be due to recoil electron penetration of the elements, which is not adequately modeled. To improve this model, it is recommended that Monte Carlo simulations be carried out to determine how the fraction of escaping electrons vary with incident energy and scattering angle. The escaping fraction computed from the Monte Carlo simulations can be fit as a function of the recoil electron energy and incorporated into the analytic model.

The contaminating event models should be compared to the results of the measurements recommended above and modified as necessary to match the measured results. These models may be more useful than the present models in identifying the type of contaminating event causing the resolution loss for images measured in the presence of

higher energy gamma rays. Further, accurate models may be useful in developing methods for removing these contaminating events from measured image data.

The algorithm for simulating laminograms would be more useful if noise effects could also be included in the simulations. To incorporate noise effects in these simulations, a Monte Carlo algorithm can be added to the code to randomize counts in the simulated ray-sums according to the Poisson distribution. After validation with measured images, noisy simulated laminograms could be used to predict the imaging time required to achieve a desired noise level in an image.

7.2.4 Image Reconstruction

Laminogram formation is the longest part of the RFL algorithm, and the speed of laminogram formation can be increased by reducing the number of subpixels used. It is recommended that the effect of reducing the number of subpixels on image resolution be studied to find an optimum.

Unlike the sources imaged in this study, the depth z_s of a given source from the camera will not be known in general, and if the image is reconstructed at the wrong depth some blurring will occur. For sources at large z_s , where the spatial resolution becomes much larger than the size of the entire first-detector array, significant errors in determining z_s have little effect on the resolution. For example, consider an on-axis point source 1 m from the first detector. At this distance the present first-detector array only subtends an angular range of 1.6° along its diagonal, and a 50% error in z_s results in an angular error of 0.8° in the image. Adding 0.8° in quadrature with a 4° RCC resolution increases the resolution to only 4.1° . However, if z_s is small or the size of the first-detector array is made larger, errors in z_s will become significant. In such cases, z_s can be estimated with an algorithm developed by Dogan and coworkers [Dgn93b], which uses a cone intersection technique to determine the depth of a source. The depth of an off-axis point source 10 cm from the source has been determined from measured data using with

techniques with a FWHM uncertainty of 1.8 cm. One difficulty with this algorithm is that it is only effective in determining the distance to point or line sources. Alternatively, an independent method of determining source distance could be incorporated into a mobile camera. Laser-based range measurement devices, commonly used today in surveying, could suit this purpose.

In the limit that z_s is much larger than the size of the first-detector array ($z_s \gg 2W_1\sqrt{N_1}$), it may be possible to reconstruct images on a spherical surface centered at the middle of the first detector. Reconstructing on a sphere neglects the finite size of the first detector, but imposing the requirement for large z_s makes the impact on the angular resolution negligible. The advantage of reconstructing on a sphere is that the width of the ray-sums remains fixed and the reconstructed image resolution should be space invariant. It may be possible to devise a direct image reconstruction algorithm where the ray-sums are filtered before backprojecting them onto the sphere.

APPENDICES

APPENDIX A

ANALYTIC MODELS: DERIVATIONS AND DETAILS

A.1 Average Thickness $\tilde{S}$ of RCC First Detector Element Viewed at Polar Angle θ_c from Ring Axis

The average thickness S of a parallelepiped shaped first detector element changes with the polar angle θ_c from which it is viewed. $\tilde{S}(\theta_c)$ can be computed by dividing the element volume by the surface area $A_{\perp}$, perpendicular to the "viewing direction" $\hat{v}$, from the center of the first detector element to the center of the second. (See Figure 2-4.) $A_{\perp}$ is equal to the dot product of $\hat{v}$ with the normal to each visible surface $\hat{n}_i$ multiplied by the surface area A_i .

$$A_{\perp} = \sum_{i=1}^N A_i (\hat{n}_i \cdot \hat{v}) \quad (A-1)$$

For simplicity, $\tilde{S}(\theta_c)$ is only computed for cases when two surfaces are visible. As an example, in Figure 2-4 restricting the views to the plane of the paper leaves two surfaces of the first element visible,

$$\tilde{S}(\theta_c) = \frac{V}{A_1(\hat{n}_1 \cdot \hat{v}) + A_2(\hat{n}_2 \cdot \hat{v})} \quad (A-2)$$

where, V is the element volume. When the element shown in Figure 2-4 is viewed in back from the ring axis, $\tilde{S}(\theta_c = 0^\circ) = 2L_1$; when viewed from above, $\tilde{S}(\theta_c = 90^\circ) = 2W_1$, and when viewed from $\theta_c = 45^\circ$, $\tilde{S}(\theta_c) = 2\sqrt{2}(W_1L_1)/(W_1 + L_1)$.

A.2 Parametric Fits to Interaction Coefficients

An important use of the analytic models in Chapter II is to predict the variations in efficiency with incident and scattered gamma-ray energies. Accurate equations of the linear attenuation coefficient (μ) and the Compton interaction coefficient (σ) in the detector materials are necessary. These coefficients have been computed using a code developed by Berger [Ber83], and the coefficient logarithms have been fit with a third-order polynomial in the gamma-ray energy E_γ .

$$\mu = \exp \left[a_0 + a_1 (\ln E_\gamma) + a_2 (\ln E_\gamma)^2 + a_3 (\ln E_\gamma)^3 \right] \quad (\text{A-3})$$

The following table shows the parameters for the coefficients of interest, with the coefficients in cm^{-1} and the energies in MeV. Over the energy range from 0.2 to 2.8 MeV, the fitted values matched Berger's to better than 0.1% for the Compton interaction coefficient and 3.5% for the linear attenuation coefficient.

Table A-1. Polynomial fits to interaction coefficients as a function of energy for materials used in the RCC to an equation of the form $\exp(a_0 + a_1(\ln E_\gamma) + a_2(\ln E_\gamma)^2 + a_3(\ln E_\gamma)^3)$.

Material	Coeff. Type	a_0	a_1	a_2	a_3
NaI	μ	-1.5677	-0.51371	0.17057	-0.091051
NaI	σ	-1.6162	-0.48419	-0.060456	0.0020181
Ge	μ	-1.2075	-0.46284	0.042082	-0.029923
Ge	σ	-1.2116	-0.48580	-0.058124	0.00043289

A.3 Intrinsic Peak Efficiency Model for RCC Second Detector Elements

In Section 2.2, the intrinsic peak efficiency of the second detector, ϵ_{ip2} , was given in Equation (2-16). In that equation, $\bar{C}_2$ is the mean chord length of the second detector element. Shapiro and Netea [Sha85] give an analytic expression for the mean chord length through a cylindrical detector from a coaxial point source

$$\bar{C}_2 = \left\{ L_2 \ln \left[\frac{\sqrt{1 + (\rho + 2\omega)^2}}{(\rho + 2\omega)} \right] + R_2 \left[\arctan\left(\frac{1}{\rho}\right) - \arctan\left(\frac{1}{(\rho + 2\omega)}\right) \right] - d \ln \left[\frac{(\rho + 2\omega)}{\rho} \sqrt{\frac{(1 + \rho^2)}{1 + (\rho + 2\omega)^2}} \right] \right\} / \left[1 - \frac{\rho}{\sqrt{1 + \rho^2}} \right] \quad (A-4)$$

where,

$$\rho = \frac{d - L_2/2}{R_2} \quad (A-5)$$

$$\omega = \frac{L_2}{2R_2} \quad (A-6)$$

Values of the peak-to-total ratio r_{pt} were determined at several incident gamma-ray energies by measuring the intrinsic peak efficiency ϵ_{ip2} of the NaI elements used in the second detector and dividing by an intrinsic total efficiency ϵ_{it2} computed from

$$\epsilon_{it2} = 1 - e^{-\mu_2^0 \bar{C}_2} \quad (A-7)$$

Measurements were made at a range of source to detector distances, d , to determine if the change in $\bar{C}_2$ was properly modeled. d is the distance from the source position to the middle of the NaI crystal. r_{pt} was averaged over the different values of d . The average for each energy was then fit as a function of energy with an equation of the form

$$r_{pt} = b_0 E^{b_1} \quad (A-8)$$

Table A-2. Variation of peak-to-total ratio, r_{pt} , for RCC second detector elements as a function of energy, and comparison between measured and fit values.

E_γ (MeV)	d (cm)	ϵ_{ip2}	$\bar{C}_2$ (cm)	$1 - e^{-\mu_2 \bar{C}_2}$	r_{pt}		
					Measured	Average	Fitted
0.279	10	0.534	3.04	0.874	0.611	--	--
0.279	30	0.616	4.29	0.946	0.651	--	--
0.279	--	--	--	--	--	0.631	0.621
0.662	10	0.137	3.04	0.563	0.244	--	--
0.662	25	0.178	4.15	0.677	0.263	--	--
0.662	50	0.195	4.59	0.713	0.273	--	--
0.662	--	--	--	--	--	0.260	0.250
0.835	10	0.0922	3.04	0.509	0.181	0.181	0.195
1.116	10	0.0610	3.04	0.458	0.133	--	--
1.116	25	0.0818	4.15	0.565	0.145	--	--
1.116	--	--	--	--	--	0.139	0.144
1.691	15	0.0446	3.62	0.457	0.0976	--	--
1.691	25	0.0510	4.15	0.504	0.0988	--	--
1.691	50	0.0522	4.59	0.540	0.0967	--	--
1.691	--	--	--	--	--	0.0977	0.0927

The results of the least-squares fit were $b_0 = 0.1614$ and $b_1 = -1.056$ when E is in MeV.

The data and fit are shown in Table A-2.

A.4 Selecting Quantization Positions for RCC Elements

In selecting the best position to represent all of the interaction in a given detector element, the volume centroid of that element has been used. This position is referred to as the quantization location. For elements on both detectors, the relatively small faces will be uniformly irradiated, hence their quantization locations should be centered on the faces. However, the interactions will not be uniform through the depth of the elements, because the incident gamma rays are attenuated as they penetrate into the element. The mean interaction depth $\langle x \rangle$ is a better choice for the depth of the quantization location than the average, $\bar{x}$. In an element of depth, L , with a total interaction cross section, μ , $\langle x \rangle$ is:

$$\begin{aligned} \langle x \rangle &= \frac{\int_0^L x e^{-\mu x} dx}{\int_0^L e^{-\mu x} dx} \\ &= \frac{1}{\mu} - L \left(\frac{e^{-\mu L}}{1 - e^{-\mu L}} \right) \end{aligned} \tag{A-9}$$

For cases relevant to the RCC, $\bar{x}$ and $\langle x \rangle$ are nearly identical. If the 0.6 cm deep Ge crystals making up the RCC first detector are irradiated with 0.5 MeV gamma rays $\mu \approx 0.6 \text{ cm}^{-1}$ and from Equation (A-9) $\langle x \rangle = 0.28 \text{ cm}$. When considering other geometric uncertainties in the RCC, this is indistinguishable from $\bar{x} = 0.3 \text{ cm}$. At higher energies, where μ is smaller, $\langle x \rangle$ will be even closer to $\bar{x}$, because $\langle x \rangle$ goes to $L/2$ as μ goes to zero. In the second detector, the position of the quantization location along its length does not affect the angular resolution in the analytic model and has little impact even in realistic situations.

APPENDIX B

RCC ELECTRONICS DIAGRAMS, MECHANICAL DRAWINGS AND MEASURED PARAMETERS

B.1 Detailed Electronics Diagrams

The high voltage divider circuit used to power the PMTs in the second detector is discussed in Section 4.2. A detailed schematic drawing of this circuit is shown in Figure B-1. Figures B-2, B-3, B-4 and B-5 show detailed schematic diagrams of the pulse processing electronics described in Section 4.4. Figure B-2 is a schematic of the two-stage amplifier; Figure B-3 is a schematic of the spectroscopy branch; Figure B-4 is a schematic of the timing branch, and Figure B-5 is a schematic of the power distribution circuits on pulse processing board.

Figure B-1. Detailed schematic diagram of high voltage divider circuit for RCC second detector [Jor93].

Figure B-2. Detailed schematic diagram of two-stage fast amplifier in RCC pulse processing electronics [Jor93].

Figure B-3. Detailed schematic diagram of spectroscopy branch in RCC pulse processing electronics [Jor93].

Figure B-5. Detailed schematic diagram of power distribution in RCC pulse processing electronics [Jor93].

B.2 Drawings of Second Detector Sled

Mechanical drawings of the sled used to position each element of the second detector are included in this section. These sleds are described in Section 4.3. Figure B-6 is a drawing of the sled assembly. Figures B-7 and B-8 show details of the sled base and the sled top, respectively. A drawing of the rail on which the sleds ride and details of the sled angle adjustment are shown in Figure B-9.

Figure B-6. Drawing of second detector sled assembly.

Sled Bottom - All pieces from $1/4" \times 1/2"$ Al bar stock
Scale 1" - 2" (unless otherwise noted)

Assembly - Top View

Assembly - End View

Sled Rail Piece - Top View

Sled Rail Piece - End View
Scale 2" - 1"

Figure B-7. Drawing of second detector sled base.

Figure B-8. Drawing of second detector sled top

Figure B-9. Drawing of rail for second detector sled.

B.3 Measured System Parameters

As described in Section 4.1, the pulse height resolution of each element of the first detector array was measured at energies of 0.122 and 0.662 MeV. The measurements were made by passing the preamplifier output signal to a commercial shaping amplifier, an inverting amplifier, and a multi-channel analyzer. Table B-1 shows the results of these measurements.

Table B-1. First detector pulse height resolution data.

Element Electronics Channel Number	⁵⁷ Co (122 keV) FWHM (keV)	¹³⁷ Cs (662 keV) FWHM (keV)
0	0.99	0.79
1	0.98	1.36
2	0.99	1.02
3	0.99	1.20
4	1.24	1.36
5	1.05	1.43
6	1.05	1.39
7	1.22	1.21
8	0.98	1.22
9	1.22	1.57
10	1.16	1.48
11	1.01	0.98
12	1.32	1.53
13	1.04	1.60
14	1.13	1.10
15	1.04	1.53
Average*	1.095	1.332

*Average excludes channel 0, which is judged to be erroneous due to the small pulse height resolution at 662 keV.

2	3	12	14
0	1	13	15
7	5	9	8
6	4	11	10

Figure B-10. Sketch of HPGe detector array mapping pulse processing electronics channel number to array position.

Figure B -10 is a sketch of the HPGe detector array viewed from the front of the RCC which maps the channel numbers of first detector pulse processing electronics to their position in the array. These channel numbers are used in Table B-1.

Table B-2 lists measured FWHM widths of full energy peaks in the pulse height spectra of the second-detector elements. The spectra were gathered by irradiating the second detector with point gamma-ray sources of ^{124}Sb , ^{137}Cs , ^{22}Na and ^{65}Zn . A linear fit to the average resolution of the elements as a function of incident gamma-ray energy is shown in Figure 4-3.

Tables B-3 and B-4 include the data measured to quantify the cross-talk in the HPGe detector array. Cross-talk is discussed in detail in Section 4.5. The data in Table B-3 was measured from adjacent elements: 0, 1, 2 and 3. (See Figure B-10.) Data in Table B-4 was measured from the four corner elements: 2, 6, 10 and 14.

Table B-2. Second detector FWHM peak widths in keV for several incident gamma-ray energies.

Channel Number	Incident Gamma-Ray Energy (keV)					
	511	603	662	1116	1275	1691
0	54.6	60.5	59.2	78.6	78.6	95.1
1	70.9	85.1	80.2	93.1	71.9	134.7
2	67.2	68.8	73.3	77.9	72.0	98.1
3	57.9	63.6	63.5	69.5	86.7	116.3
4	55.4	69.3	64.7	71.7	85.6	101.2
5	58.2	68.2	60.8	72.1	73.0	101.7
6	60.2	77.4	66.7	91.3	72.8	108.3
7	61.1	75.0	72.7	95.9	89.1	126.4
8	59.6	69.9	62.9	84.3	56.8	90.0
9	58.8	64.0	64.8	73.9	85.2	104.5
10	58.1	70.9	63.4	76.8	81.5	113.1
11	65.2	72.0	68.7	75.3	52.6	96.6
12	65.8	71.0	72.1	83.1	71.9	95.5
13	61.4	76.7	70.5	81.3	86.8	105.7
14	49.1	60.2	68.6	75.2	80.7	111.3
15	64.9	76.0	74.7	89.0	92.9	107.1
Average	60.5	70.5	67.9	80.5	77.4	106.6

Table B-3. Measured cross-talk in adjacent first-detector elements.

Channel Number	Measured Counts (per 100 s)	Measured Count Rate, r (cps)	True Inter. Rate, g (cps)	Uncert. in g (cps)	Ratio $\bar{g}_c/g_c$	Uncert. in Ratio
Measured Single Element Count Rates						
			$\bar{g}_i$			
0	128116	1281.2	1284.6	3.6	--	--
1	124308	1243.1	1246.3	3.5	--	--
2	121437	1214.4	1217.5	3.5	--	--
3	118154	1181.5	1184.5	3.4	--	--
Estimated Combined Element Count Rates						
			$\bar{g}_c$			
0,1	--	--	2530.9	5.0	--	--
0,3	--	--	2469.1	4.9	--	--
0,1,2	--	--	3748.4	6.1	--	--
0,1,2,3	--	--	4932.9	7.0	--	--
Measured Combined Element Count Rates						
			$\bar{g}_c$			
0,1	235087	2350.9	2362.5	4.9	1.071	0.003
0,3	239680	2396.8	2408.9	4.9	1.025	0.003
0,1,2	336005	3360.1	3383.9	5.8	1.108	0.003
0,1,2,3	423035	4230.4	4268.3	6.6	1.156	0.002

Table B-4. Measured cross-talk in spatially separated first-detector elements.

Channel Number	Measured Counts (per 100 s)	Measured Count Rate r (cps)	True Inter. Rate g (cps)	Uncert. in g (cps)	Ratio $\bar{g}_c/g_c$	Uncert. in Ratio
Measured Single Element Count Rates						
			$\bar{g}_i$			
2	121437	1214.4	1217.5	3.5	--	--
6	119874	1198.7	1201.8	3.5	--	--
10	92243	922.4	924.2	3.0	--	--
14	90990	909.9	911.6	3.0	--	--
Estimated Combined Element Count Rates						
			$\bar{g}_c$			
2,6	--	2407.0	2419.2	4.9	--	--
10,14	--	1828.8	1835.9	4.3	--	--
2,6,10	--	3320.1	3343.5	5.8	--	--
2,6,10,14	--	4217.4	4255.1	6.5	--	--
Measured Combined Element Count Rates						
			$\bar{g}_c$			
2,6	239018	2390.2	2402.2	4.9	1.007	0.003
10,14	182016	1820.2	1827.1	4.3	1.005	0.003
2,6,10	328670	3286.7	3309.5	5.8	1.010	0.002
2,6,10,14	415676	4156.8	4193.4	6.5	1.015	0.002

APPENDIX C

POINT RESPONSE DATA

C.1 Modeled Pulse Height Response of NaI(Tl) Elements

The pulse height response of the NaI(Tl) elements of the second detector array is included in the contaminating event models for random coincidence events, cascade coincidence events and partial energy deposition events. The model uses a Gaussian function to represent the full energy peak with its standard deviation σ equal to FWHM/2.356, and the Compton continuum is represented by a rectangle.

Figure C-1 is a comparison of the modeled pulse height response to the 0.662 MeV

Figure C-1. Comparison of modeled and measured pulse heights one NaI(Tl) element irradiated with a ^{137}Cs source.

Figure C-2. Comparison of modeled and measured pulse heights of one NaI(Tl) element irradiated with 1.12 MeV gamma rays from a ^{65}Zn source. The 0.511 MeV annihilation photons emitted by the source are not included in the model.

gamma rays of ^{137}Cs to a measurement made with one of the NaI(Tl) elements. The ratio of the full energy peak area to the continuum area is given by $r_{\text{pt}}/(1-r_{\text{pt}})$ where r_{pt} is the peak-to-total ratio. The simple rectangular shape recommended by Gardner [Gar76] has been modified to account for resolution broadening of the Compton edge and a backscatter peak. The upper edge of the Compton continuum rectangle is replaced by a 2.356σ wide triangle that falls to half its height at the Compton edge energy. The backscatter events are represented by a triangle added on top of the Compton continuum. The upper edge of this triangle corresponds to the energy of a full energy gamma ray after suffering a 70° Compton scatter. The lower edge of the backscatter peak triangle is set by the lower level threshold of the detector, and the area of this triangle is free parameter used to fit the data. This "backscatter peak" also includes X-rays and noise in the detector.

The model accurately reproduces most of the features of the pulse height response to 0.662 MeV gamma rays in Figure C-1 and 1.12 MeV gamma rays in Figure C-2. One obvious difference is the valley between the Compton edge and the full energy peak, which is filled by multiple scattering events in the measured spectrum. Another difference is the finite width of the lower level threshold. Note that the 1.12 MeV gamma rays of ^{65}Zn are accompanied by 0.511 MeV annihilation photons which are not incorporated in the model.

C.2 Measured Efficiency and Resolution Data

Table C-1. Data for efficiency versus energy at two values of θ_c . Intrinsic efficiencies are plotted in Figure 5-6.

θ_c (deg)	E_γ (MeV)	N_2	C_n	Number of gamma-rays emitted	ϵ_{acc}	ϵ_{icc}
44	0.6027	16	33829	2.67e+10	1.27e-06	3.99e-04
44	0.6617	16	20394	2.23e+10	9.16e-07	2.88e-04
44	1.3686	16	5742	1.45e+10	3.96e-07	1.24e-04
44	1.6910	16	2578	1.33e+10	1.94e-07	6.10e-05
44	2.7541	16	1362	1.45e+10	9.43e-08	2.96e-05
59	0.5110	16	66697	7.32e+10	9.11e-07	2.86e-04
60	0.6027	15	55270	6.18e+10	9.54e-07	3.00e-04
59	0.6617	16	14358	2.22e+10	6.47e-07	2.03e-04
60	1.3686	15	7132	2.60e+10	2.93e-07	9.19e-05
60	1.6910	15	4217	3.08e+10	1.46e-07	4.59e-05
60	2.7541	15	1695	2.60e+10	6.96e-08	2.19e-05

Table C-2. Variation in NaI(Tl) element counts measured from an on-axis ^{124}Sb point source with $\theta_c = 60^\circ$

Element Number	Gross Data Counts	Variance
0	2018	3999.3
1	1879	5739.6
2	1990	1227.8
3	1875	6313.9
4	1995	1587.2
5	2007	2708.2
6	1941	183.9
7	2001	2147.4
8	1870	7269.3
9	2017	3799.5
10	1940	220.8
11	1883	5178.2
12	--	--
13	1962	56.9
14	2015	3653.0
15	1930	638.1
	Total Counts	Variance
	29323	3194.5
	Average Counts	Std. Deviation
	1954.9	56.5

Table C-3. Spatial and angular resolutions for measured point spread functions reconstructed using RFL algorithm with the standard BUTR filter showing variations with central scattering angle θ_c and incident gamma-ray energy E_γ . Listed measured and predicted total angular resolutions $\Delta\theta_t$ are shown in Figure 5-17. The listed average bin width of the first detector elements is used to compute the predicted $\Delta\theta_t$. Other RCC parameters include $z_g = 10$ cm and $d = 30$ cm.

θ_c (deg)	E_γ (MeV)	N_2	C_n	σ_n/C_n (%)	G	SR horiz. (cm)	SR vert. (cm)	SR ave. (cm)	FWHM horiz. (cm)	FWHM vert. (cm)	FWHM ave. (cm)	Meas. $\Delta\theta_t$ (deg)	Bin Width (keV)	Predict. $\Delta\theta_t$ (deg)
44	0.6027	16	33829	0.56	0.298	1.51	1.44	1.47	0.85	0.85	0.85	4.85	5.80	4.23
44	0.6617	16	20394	0.71	0.280	1.55	1.53	1.54	0.92	0.91	0.92	5.23	5.79	4.19
44	1.3686	16	5742	1.32	0.326	1.34	1.33	1.34	0.78	0.68	0.73	4.16	10.20	4.12
44	1.6910	16	8005	1.19	0.362	1.54	1.36	1.45	0.71	0.71	0.71	4.06	5.80	4.07
44	2.7541	16	1362	2.71	0.317	1.91	1.84	1.87	0.67	0.73	0.70	3.98	10.20	4.07
59	0.5110	16	66697	0.39	0.298	1.54	1.62	1.58	0.87	0.84	0.86	4.90	3.69	4.16
60	0.6027	15	55270	0.46	0.289	1.63	1.68	1.65	0.84	0.90	0.87	4.98	5.78	4.25
59	0.6617	16	14358	0.84	0.283	1.49	1.56	1.53	0.88	0.85	0.86	4.92	3.68	4.10
60	1.3686	15	7132	1.26	0.321	1.53	1.87	1.70	0.72	0.84	0.78	4.45	10.19	4.15
60	1.6910	15	4217	1.66	0.331	1.53	2.01	1.77	0.62	0.81	0.72	4.09	5.78	4.05
60	2.7541	15	1695	2.64	0.329	1.48	1.44	1.46	0.60	0.69	0.65	3.69	10.19	4.07

APPENDIX D

SPATIALLY EXTENDED SOURCE DATA

D.1 Tables of Imaging Parameters

Table D-1. RCC system parameters for ^{24}Na bar source measurement and source space parameters used in image reconstruction.

Description	Symbol	Magnitude	Unit
Source spacing	z_s	10	cm
Detector spacing	d	30	cm
Central scattering angle	θ_c	59	deg
FOV radius needed to image source with out distortion	r_{FOV}	4.7	cm
Polar angle corresponding to r_{FOV}	α	31	deg
Lower energy threshold in first detector	$E_{1\text{min}}$	0.10	MeV
Lower bound scattering angle for E_γ^a (≈ 1.3686 MeV)	θ_ℓ^a	13.9	deg
Lower bound scattering angle for E_γ^b (≈ 2.7541 MeV)	θ_ℓ^b	6.8	deg
Min. scattering angle used	θ_{min}	26	deg
Lower energy threshold in second detector	$E_{2\text{min}}$	0.32	MeV
Upper bound scattering angle for E_γ^a (≈ 1.3686 MeV)	θ_u^a	103	deg
Upper bound scattering angle for E_γ^b (≈ 2.7541 MeV)	θ_u^b	114	deg
Max. scattering angle used	θ_{max}	90	deg
Summed energy window for E_γ^a	E_s^a	1.282 to 1.454	MeV
Summed energy window for E_γ^b	E_s^b	2.661 to 2.847	MeV
Ray-sum angular width	$\delta\theta$	1	deg
Number of ray-sums	M_θ	64	
Number of second detector positions	M_ϕ	32	
Pixel width	Δr	0.2	cm
Number of pixels	N_x, N_y	128	

Table D-2. RCC system parameters for ^{65}Zn bar source measurement and source space parameters used in image reconstruction.

Description	Symbol	Magnitude	Unit
Source spacing	z_s	10	cm
Detector spacing	d	30	cm
Central scattering angle	θ_c	59	deg
FOV radius needed to image source with out distortion	r_{FOV}	4.7	cm
Polar angle corresponding to r_{FOV}	α	31	deg
Lower energy threshold in first detector	$E_{1\text{min}}$	0.05	MeV
Lower bound scattering angle for E_γ (≈ 1.1155 MeV)	θ_ℓ	11.9	deg
Min. scattering angle used	θ_{min}	28	deg
Lower energy threshold in second detector	$E_{2\text{min}}$	0.30	MeV
Upper bound scattering angle	θ_u	104	deg
Max. scattering angle used	θ_{max}	92	deg
Summed energy window for E_γ	E_s^a	1.040 to 1.192	MeV
Ray-sum angular width	$\delta\theta$	1	deg
Number of ray-sums	M_θ	64	
Number of second detector positions	M_ϕ	32	
Pixel width	Δr	0.2	cm
Number of pixels	N_x, N_y	128	

Table D-3. RCC system parameters for multi-energy phantom measurement and source space parameters used in image reconstruction.

Description	Symbol	Magnitude	Unit
Source spacing	z_s	10	cm
Detector spacing	d	30	cm
Central scattering angle	θ_c	59	deg
FOV radius needed to image source with out distortion	r_{FOV}	4.8	cm
Polar angle corresponding to r_{FOV}	α	31	deg
Lower energy threshold in first detector	E_{1min}	0.035	MeV
Lower bound scattering angle for E_γ^a (≈ 0.5110 MeV)	θ_ℓ^a	22	deg
Lower bound scattering angle for E_γ^b (≈ 1.1155 MeV)	θ_ℓ^b	9.9	deg
Min. scattering angle used	θ_{min}	28	deg
Lower energy threshold in second detector	E_{2min}	0.23	MeV
Upper bound scattering angle for E_γ^a (≈ 0.5110 MeV)	θ_u^a	103	deg
Upper bound scattering angle for E_γ^b (≈ 1.1155 MeV)	θ_u^b	140	deg
Max. scattering angle used	θ_{max}	92	deg
Summed energy window for E_γ^a	E_s^a	0.446 to 0.576	MeV
Summed energy window for E_γ^b	E_s^b	1.040 to 2.192	MeV
Ray-sum angular width	$\delta\theta$	1	deg
Number of ray-sums	M_θ	64	
Number of second detector positions	M_ϕ	32	
Pixel width	Δr	0.2	cm
Number of pixels	N_x, N_y	128	

Table D-4. RCC system parameters for ^{24}Na flood-field phantom measurement and source space parameters used in image reconstruction.

Description	Symbol	Magnitude	Unit
Source spacing	z_s	10	cm
Detector spacing	d	30	cm
Central scattering angle	θ_c	59	deg
FOV radius needed to image source with out distortion	r_{FOV}	10.7	cm
Polar angle corresponding to r_{FOV}	α	46.9	deg
Lower energy threshold in first detector	$E_{1\text{min}}$	0.06	MeV
Lower bound scattering angle for E_γ (≈ 1.1155 MeV)	θ_ℓ	10.6	deg
Min. scattering angle used	θ_{min}	11	deg
Lower energy threshold in second detector	$E_{2\text{min}}$	0.28	MeV
Upper bound scattering angle	θ_u	117	deg
Max. scattering angle used	θ_{max}	109	deg
Summed energy window for E_γ	E_s^a	1.289 to 1.448	MeV
Ray-sum angular width	$\delta\theta$	2	deg
Number of ray-sums	M_θ	49	
Number of second detector positions	M_ϕ	64	
Pixel width	Δr	0.4	cm
Number of pixels	N_x, N_y	128	

D.2 Nonlinear Parametric Fits to Contours through Images

Figure D-1. Normalized average horizontal contour through reconstructed image of gamma rays emitted from the ^{65}Zn bar source compared to a parametric fit of the data. The fitted parameters are determined by non-linear regression.

Figure D-2. Normalized average horizontal contour through reconstructed image of gamma rays emitted from the ^{65}Zn "U" source compared to a parametric fit of the data. The fitted parameters are determined by non-linear regression.

D.3 Figures Illustrating Cold-Region Noise in Flood-Field Phantom

Figure D-3. Example ray-sum data from ^{24}Na flood-field phantom showing enhanced noise below minimum angle subtended by phantom due to contaminating events. The counts are from 8 selected pairs of detector elements including the second-detector elements at $\phi = 90^\circ$ and $\phi = 180^\circ$ and the central 4 elements of the first detector.

Figure D-4. Gray scale density plots of ^{24}Na flood-field phantom showing a) the laminogram on the extended 128x128 pixel source space and b) the RFL reconstruction with a PARZ filter having $v_{\max} = 0.5$ on the extended 128x128 source space.

Figure D-5. Gray scale density plots of ^{24}Na flood-field phantom images reconstructed with the ART algorithm after a) after 1 iteration and b) after 9 iterations. Both images are shown on the extended 128x128 pixel source space and are low-pass filtered with a with a Parzen window having $v_{\max} = 0.5$.

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