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HIGH-ALTITUDE COOLING
II - AIR-COOLED ENGINES

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ADVANCE RESTRICTED REPORT

HIGH-ALTITUDE COOLING

II - AIR-COOLED ENGINES

By David T. Williams

SUMMARY

The heat-transfer theory for air-cooled engines is summarized and an analysis of the cooling pressure drop is made for the case in which the pressure drop is an appreciable fraction of the absolute pressure. A chart is given for the simple determination of the cooling pressure drop predicted on the basis of the usual type of sea-level cooling-correlation tests. The method is applied to predict the variation with altitude of the cooling pressure drop required by a typical engine.

INTRODUCTION

In the studies of references 1 to 3 satisfactory semiempirical methods were developed for correlating the cooling-air mass flow with the operating conditions of an air-cooled engine. Because pressure drop is a more practical variable than mass flow and because the product of relative density and pressure drop (or $\alpha \Delta p$) was shown to be determined by the mass flow, this product was used in place of mass flow as the correlation variable. For application in high-altitude flight, however, the mass flow no longer uniquely determines $\alpha \Delta p$ because of compressibility effects, and an extension of the correlation methods is needed if the required cooling pressure is not to be seriously underestimated. Some discussions of the problem have already been given, together with analyses of the effects (reference 4).

The purpose of the present paper is to summarize the theoretical basis for the compressibility correction and to present a chart by means of which the correction can be made simply and with a minimum of supplemental data. The theory of reference 1 is first outlined and
the differential equation of the cooling-air flow is then set up in a simplified form and solved. The chart is a plot of the ratio of cooling pressure drop to absolute pressure against a single parameter that can be simply computed from the data.

This paper is the second of the series on high-altitude cooling introduced in reference 5.

SYMBOLS

\[ V \] \quad \text{velocity} \\
\[ \rho \] \quad \text{density} \\
\[ \sigma \] \quad \text{relative density} \quad (\rho/0.002378) \\
\[ q \] \quad \text{dynamic pressure} \\
\[ p \] \quad \text{static pressure, absolute} \\
\[ \Delta p \] \quad \text{static-pressure drop} \\
\[ T \] \quad \text{temperature, } ^\circ F \text{ absolute} \\
\[ t \] \quad \text{temperature, } ^\circ F \\
\[ \Delta T \] \quad \text{temperature rise, } ^\circ F \\
\[ G = \rho V \] \\
\[ R \] \quad \text{gas constant} \\
\[ F \] \quad \text{pressure-drop coefficient of fin passage} \\
\[ W \] \quad \text{weight flow of cooling air} \\
\[ M_0 \] \quad \text{weight flow of engine charge air} \\
\[ a_1 \] \quad \text{cylinder internal area} \\
\[ a_0 \] \quad \text{cylinder external area} \\
\[ H \] \quad \text{heat-transfer rate} \\
\[ L \] \quad \text{length of fin passage}
The heat-transfer equations. - In the theory of heat transfer as developed in reference 1, a cylinder wall is considered as a hot body at temperature $T_h$, heated from within by a flow of hot gases at temperature $T_g$, and cooled from without by air at temperature $T_1$. The rate of heat transfer within the cylinder is assumed to vary as the product of the temperature difference $T_g - T_h$ and a heat-transfer coefficient. This internal heat-transfer coefficient varies as some power $m$ of the weight flow of charge air $M_0$. Thus the heat transferred per second is

\[ Q = h \cdot A \cdot \left( T_g - T_h \right) \]

where $Q$ is the rate of heat transfer, $A$ is the surface area, $h$ is the heat-transfer coefficient, $T_g$ is the gas temperature, and $T_h$ is the wall temperature.
\[ H = \bar{B} M_c^{m^1} a_1 (T_g - T_h) \]  \hspace{1cm} (1)

where

\[ a_1 \quad \text{internal area} \]

\[ \bar{B} M_c^{m^1} \quad \text{average internal heat-transfer coefficient} \]

\[ M_c \quad \text{weight flow of charge air} \]

\[ \bar{B}_e, m \quad \text{experimental constants} \]

On the outside, the heat rejected to the cooling air is assumed to vary as the product of the temperature difference \( T_h - T_l \) and some power \( m^1 \) of the weight flow of cooling air \( W \):

\[ H = K' W^{m^1} a_o (T_h - T_l) \]  \hspace{1cm} (2)

where

\[ a_o \quad \text{external cylinder area} \]

\[ K' W^{m^1} \quad \text{average external heat-transfer coefficient} \]

\[ K', m^1 \quad \text{experimental constants} \]

From equations (1) and (2)

\[
\frac{M_c^{m/m^1}}{W \left( \frac{T_h - T_l}{T_g - T_h} \right)^{1/m^1}} = A^1
\]

\[
= \left( \frac{K' a_o}{\bar{B} a_1} \right)^{1/m^1} \]  \hspace{1cm} (3a)
This equation is the fundamental relation for the determination of the weight flow of cooling air under any engine operating condition and at any altitude. If the engine operating conditions are constant, the altitude affects the weight flow \( W \) required for cooling only insofar as it affects \( T_1 \).

**Discussion of the variables of equations (3).** - The effective gas temperature \( T_g \) for a given engine is a function of the operating conditions. This temperature increases, for example, with increase in carburetor-air temperature or with increase in the temperature rise through the blower and thereby affects \( W \) and \( \Delta p \).

Some characteristic curves (fig. 1) show that the required cooling pressure drop (roughly proportional to \( W^2 \)) was nearly doubled when the carburetor inlet-air temperature was increased from 100\(^\circ\) F to 300\(^\circ\) F. As shown in reference 2, \( T_g \) is also sensitive to variations in fuel-air ratio. A plot of the relative sea-level pressure drop required to cool a certain engine at constant indicated horsepower against fuel-air ratio (fig. 2) shows that increasing the fuel-air ratio from 0.08 to 0.10 reduced the required cooling pressure drop by half.

Figure 2 shows that the use of very lean mixtures likewise reduces the required cooling pressure drop. This cooling aid, which might be particularly applicable in the cruising range, is not easily used, however, because, without exact methods of fuel metering and without perfect uniformity of the mixture distribution to the different cylinders, the use of fuel-air ratios so close to the cut-out point becomes dangerous.

The symbol \( T_h \) of equations (3) was originally defined as a mean cylinder-head temperature; however, satisfactory correlations of multicylinder engines have been made with \( T_h \) taken as the mean rear spark-plug-gasket temperature. It is probable that other points
could be used equally well; the values of exponents m
and m' would depend on the point chosen.

The temperature $T_1$ is the external air temperature
corrected for the adiabatic temperature rise resulting
from the airplane speed:

$$T_1 = T_0 + 0.83 \left( \frac{V_o}{100} \right)^2$$

where the temperatures are in °F absolute and $V_o$ is in
fect per second.

Values for $T_g$, $A'$, $m$, and $m'$ are found by analysis
of systematic sea-level test data according to the methods
of reference 1. The exponents $m$ and $m'$ are both
generally about 2/3. The value of $T_g$ for cylinder heads
is normally of the order of 1610°F absolute (1150°F)
when the intake manifold temperature is 80°F and the
fuel-air ratio is 0.08; however, as already indicated,
this value is subject to considerable variation. It is
affected, for example, by changes in back pressure; sea-
level tests showed a decrease of 20 percent in the
required cooling pressure drop when the exhaust manifold
pressure was halved. This effect is of little importance
when a turbosupercharger is used because the back pressure
will be of the order of sea-level pressure in the region
of the critical altitude.

As already remarked, the main effect of altitude on
the required weight flow of cooling air results simply
from the variation of $T_1$ with altitude; that is, for
otherwise constant conditions,

$$\frac{W}{W_{SL}} = \frac{(T_h - T_1)_{SL}^{1/m'}}{(T_h - T_1)^{1/m'}}$$

where the subscript SL refers to sea-level conditions.
Figure 3, based on this equation, shows the variation of
W with altitude for several values of $T_h$. The tempera-
ture of Army air, uncorrected for flight speed, was used
for $T_1$, and $m'$ was taken as 2/3. The reduction in
required weight flow with increasing altitude is seen to be most pronounced when $T_h$ is low. Because cylinder barrels are about 150°F cooler than cylinder heads, the barrels will be expected to overcool at altitude if sufficient cooling pressure drop is maintained for constant head temperature. Actually, the thermal problems of heads and barrels are not quite independent because heat is conducted between them.

In sea-level tests the pressure drop of the cooling air has been shown experimentally to vary as a constant power of its weight flow $W$:

$$\sigma \Delta p \propto \frac{W^{m'}}{n}$$

where $m'/n$ is very nearly 2. Because the pressure drops $\Delta p$ is more easily measured than $W$ and is also of more direct interest in practice, $\sigma \Delta p$ has until now replaced $W$ as a variable in cooling-correlation tests at sea level. A possible form of equation (3) is thus

$$\frac{T_h - T_1}{T_e - T_h} = K \frac{Re^m}{(\sigma \Delta p)^n}$$  \hspace{1cm} (3c)

The modification of this expression necessary for cooling correlation at high altitudes will be next considered.

**PRESSURE DROP**

Air-flow path. - The assumed path of the cooling air through the engine is shown diagrammatically in figure 4, together with the static-pressure drops in the different parts of the path. The air accelerates from the stagnation region 1 in front of the cylinder into the baffle entrance 2; the static pressure falls by an amount $\Delta p_{1-2}$. The air then flows along the fin passages, which are assumed to be of constant cross section. The static-pressure drop $\Delta p_{2-3}$ that occurs along the passages is the sum of the friction pressure drop and the pressure drop necessary to accelerate the air as its density.
decreases along the passages. The air finally flows from the baffle exit into the space behind the cylinder. The abrupt enlargement of the flow passages causes a dissipation of most of the kinetic energy at the baffle exit; that is, the flow into the space behind the engine occurs at essentially constant static pressure. The pressure drop across the engine is thus simply the sum of $\Delta p_{1-2}$ and $\Delta p_{2-3}$.

**Pressure drop at the baffle inlet.** - At low altitudes, for which the entrance Mach numbers are low, the pressure drop $\Delta p_{1-2}$ is $\frac{1}{2} \rho v^2$. For higher entrance Mach numbers, the pressure drop into the baffles is given by the formula for compressible flow. It is assumed that the fraction $b$ of the total heat input occurs without pressure loss in the stagnation region in front of the cylinder; that is, just before the air accelerates into the baffles, its temperature is $T_1 + b\Delta T$, where $T_1$ is given by equation (4) and $\Delta T$ is determined from the heat rejection and the weight flow. The density here is $\rho_1 \left( \frac{T_1}{T_1 + b\Delta T} \right)$.

If the flow into the baffles is isentropic, then, by the Bernoulli equation,

$$v_2^2 = \frac{2\gamma}{\gamma - 1} \frac{p_1}{\rho_1} \frac{T_1 + b\Delta T}{T_1} \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\gamma-1} \right]$$

or

$$\frac{G^2 \left( 1 + b \frac{\Delta T}{T_1} \right)}{p_1 \rho_1} = \frac{2\gamma}{\gamma - 1} \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\gamma-1} \right] \left( \frac{p_2}{p_1} \right)^{2/\gamma}$$

where $G = \rho_2 V_2 = \rho V$ along the passage and is given by the mass flow and the flow area of the baffles. The ratio $p_2/p_1$ is thus given as a function of the term on the left side of this equation; a plot of $p_2/p_1$ against $\frac{G^2 \left( 1 + b \frac{\Delta T}{T_1} \right)}{p_1 \rho_1}$ is given in figure 5.
For convenience in the use of equation (6), figures 6 to 8 show \( T_1, P_1, \) and \( \rho_1 \) for Army air plotted against altitude for several airplane speeds. It was assumed that only 90 percent of the flight dynamic pressure is recovered as static pressure; that is,

\[
P_1 - P_0 = 0.9 (P_s - P_0)
\]

where

- \( P_s \) free-stream stagnation pressure
- \( P_0 \) free-stream static pressure

Pressure drop within the baffles. - The pressure decreases along the fin passages as a result of friction and acceleration:

\[
dP = \frac{1}{2} \rho \dot{V}^2 \frac{F}{L} \, dx + \rho V \, dV
\]

where

- \( x \) distance along passage
- \( L \) length of passage
- \( F \) friction coefficient for passage as a whole (that is, \( \frac{1}{2} \rho \dot{V}^2 F \) would be the pressure drop within the fin passages in absence of temperature and acceleration effects)

This equation is, for convenience, rewritten as follows:

\[
-\rho \, dP = \dot{G}^2 \left( \frac{F}{\dot{G}} \, \frac{dx}{dT} \, dT + \frac{dV}{V} \right)
\]  \hspace{1cm} (7)

In order to simplify the solution, heat is assumed to be rejected uniformly to the cooling air along the passage:

\[
\frac{dx}{dT} = \frac{L}{T_3 - T_2} = \frac{L}{(1 - b)\Delta T}
\]
From the gas law,

\[ p = \frac{RT}{\rho} \]

and from the condition that \( pV \) is a constant (equal to \( G \)),

\[ \frac{dV}{V} = \frac{dp}{\rho} = \frac{dT}{T} - \frac{dp}{p} \]

These two substitutions reduce equation (7) to

\[ dp \left( \frac{T}{p} - \frac{P}{R G^2} \right) = dT \left[ \frac{F}{2(1 - b) AT} \right] + 1 \] (8)

The friction factor \( F \), which is a function of the Reynolds number, may be evaluated for the mean viscosity along the passage and treated as a constant. As a further simplification, the first term \( \frac{T}{p} dp \) is integrated by considering \( T \) constant at its mean value \( \frac{T_1 + T_2}{2} = T_m \). The temperature \( T_2 \) is given by

\[ \frac{T_2}{T_1 + b \Delta T} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} \] (9)

The ratio \( \frac{T_2}{T_1 + b \Delta T} \) is shown in figure 5 as a function of

\[ G^2 \left( \frac{1 + b \Delta T}{T_1} \right) \left( \frac{p_2}{p_1} \rho_1 \right) \]

Also

\[ T_3 = T_2 + (1 - b) \Delta T \]

Integration of equation (8) between stations 2 and 3 gives
The ratio \( \frac{p_3}{p_2} \) is thus determined by the two dimensionless groups \( \frac{p_2}{\text{RG}^2 T_m} \) and \( F + \frac{2(T_3 - T_2)}{T_m} \); this ratio is plotted as a function of these two quantities in figure 9.

The pressure drop follows from the values of \( \frac{p_2}{p_1} \) and \( \frac{p_3}{p_2} \) determined by equations (6) and (10):

\[
\frac{p_1 - p_3}{p_1} = p_1 \left( 1 - \frac{p_2}{p_1} \frac{p_3}{p_2} \right) \tag{11}
\]

In applying the theory when \( b, p_1, \) and \( T_1 \) are given, the three quantities \( G, \Delta T, \) and \( F \) must be known. As an illustration of the process of computing \( p_3 - p_1 \), suppose \( p_1 = 582 \) pounds per square foot, \( T_1 = 456^\circ F \) absolute, and \( p_1 = 0.000744 \) slugs per cubic foot, with \( G = 0.2172 \) slug per foot \(^2\) second, \( \Delta T = 123^\circ F, \)

\( F = 1, \) and \( b = 0.5. \)

First compute

\[
\frac{G^2}{p_1} \left( 1 + \frac{\Delta T}{2T_1} \right) = \frac{(0.2172)^2}{582 \times 0.000744} \left( 1 + \frac{123}{2 \times 456} \right)
\]

\[
= 0.1089 \left( 1 + \frac{0.27}{2} \right)
\]

\[
= 0.1236
\]

From figure 5(a), \( \frac{p_2}{p_1} = 0.9338 \) and \( \frac{T_2}{T_1 + \Delta T} = 0.980 \)
Now,
\[ T_m = T_2 + \frac{1}{4} \Delta T \]

\[ = 0.980 \left( T_1 + \frac{\Delta T}{2} \right) + \frac{1}{4} \Delta T \]

\[ = 0.980 (456 + 61.5) + 30.8 \]

\[ = 537.4^\circ F \text{ absolute} \]

Then

\[ F + 2 \left( \frac{T_3 - T_2}{T_m} \right) = 1 + \frac{123}{537.4} \]

\[ = 1.23 \]

Also

\[ \frac{p_2^2}{RG^2T_m} = \frac{p_2}{p_1} \cdot \frac{p_1 c_1}{G^2} \cdot \frac{T_1}{T_m} \]

\[ = (0.9338)^2 \times \frac{1}{0.1089} \times \frac{456}{537.4} \]

\[ = 6.795 \]

From figure 9, therefore,

\[ \frac{p_3}{p_2} = 0.885 \]
and

\[ P_1 - P_3 = 582(1 - 0.9338 \times 0.985) = 101.2 \text{ pounds per square foot} \]
\[ = 19.5 \text{ inches of water} \]

Simplified chart for pressure-drop determination. The number of independent variables required to fix \( \Delta p \) was found in the computation to be five excluding \( b \), but this number can be reduced. The ratio

\[ \frac{\Delta p}{P_1} = \frac{P_1 - P_3}{P_1} \]
\[ = 1 - \frac{P_2}{P_1} \frac{P_3}{P_2} \quad (12) \]

is completely determined by three dimensionless quantities \( \frac{G^2}{P_1 P_2} \), \( \frac{\Delta T}{T_1} \), and \( F \), if \( b \) is known. It will be assumed for the remaining development that \( b = \frac{1}{2} \). The value of \( \frac{P_2}{P_1} \) is determined by use of equation (6) from the variable

\[ \frac{G^2}{P_1 P_2} \left( 1 + \frac{1}{2} \frac{\Delta T}{T_1} \right) \]

Likewise, \( \frac{P_3}{P_2} \) is found by use of equation (10) from the two variables

\[ \frac{P_2^2}{R \frac{G^2}{T_m}} = \frac{P_2^2}{P_1^2} \frac{P_1 P_1}{P_2} \frac{T_1}{T_m} \quad (13) \]

and

\[ F + \frac{\Delta T}{T_m} \]
The factor $\frac{p_2}{p_1}$ of equation (13) is determined by $G^2/p_1p_1$ and $\Delta T/T_1$. The factor

$$\frac{T_1}{T_m} = \frac{T_1}{\left(\frac{T_1}{2}\right)^{\frac{1}{\gamma}} + \frac{1}{4} \Delta T}$$

is obviously determined by the same variables. Also

$$\frac{\Delta T}{T_m} = \frac{\Delta T T_1}{T_1 T_m}$$

so that $\frac{p_3}{p_1}$, $\frac{p_2}{p_1}$, and hence $\frac{\Delta p}{p_1}$ are known when $G^2/p_1p_1$, $\Delta T/T_1$, and $F$ are known, as was to be proved.

If the number of variables required to find $\frac{\Delta p}{p_1}$ could not be reduced to less than three, an obvious way to simplify the computation of $\frac{\Delta p}{p_1}$ would be to plot it against one of the three dimensionless variables, say $G^2/p_1p_1$, for various values of the other two, $\Delta T/T_1$ and $F$. The use of such a system of curves would be awkward in practice because it would generally require a double interpolation in finding $\frac{\Delta p}{p_1}$. It is therefore proposed to find a variable to replace $G^2/p_1p_1$ such that curves of $\frac{\Delta p}{p_1}$ against the variable will be as nearly as possible coincident for various values of $\Delta T/T_1$ and $F$; that is, an attempt will be made to
express $\Delta p/p_1$ in terms of one variable instead of three.

In its passage through the baffle passages, the cooling air experiences expansion due to heating and pressure drop. At very high total pressures the expansion due to pressure drop is negligible; the cooling pressure drop under such circumstances can be shown to be very roughly

$$\frac{G^2}{2p_1} \left( 1 + \frac{\Delta T}{2T_1} \right) \left( 1 + F + \frac{\Delta T}{T_1} \right)$$

When $\Delta p/p_1$ is plotted against the variable

$$\frac{G^2}{2p_1 p_0} \left( 1 + \frac{\Delta T}{2T_1} \right) \left( 1 + F + \frac{\Delta T}{T_1} \right)$$

for various values of $\Delta T/T_1$ and $F$, the resultant curves as shown in figure 10 are nearly coincident for small values of $\Delta p/p_1$. The curves represent the effect of expansion due to pressure changes alone. In practice, these curves can be used in place of the previously described calculation to compute $\Delta p$. The curves as computed are rigorously correct and are not affected by the approximation used in choosing the form of the variable

$$\frac{G^2}{2p_1 p_0} \left( 1 + \frac{\Delta T}{2T_1} \right) \left( 1 + F + \frac{\Delta T}{T_1} \right)$$

The only error in the use of these curves comes from the approximation required in interpolating for different values of $F$ and $\Delta T/T_1$. Because of the small separation of the various curves, the errors due to interpolation are negligible compared with experimental errors in measuring pressure drops.

Application of theory to experiment. - It is apparent that certain changes from present practice in testing
procedure are required if cooling pressure drops at altitude are to be predicted. In place of the quantity \( \sigma \Delta p \), a true mass-flow index such as \( G \) must be observed and used in the correlation; also \( \Delta T \) and \( F \), which have not ordinarily been used, must be measured.

A more explicit description of a possible procedure is as follows:

A mass-flow index \( (\sigma \Delta p)_1 \) is defined by

\[
(\sigma \Delta p)_1 = \frac{g^2}{2 \rho_0} (1 + F)
\]

(15)

where \( \rho_0 \) is the sea-level density, \( 0.002378 \) slug per cubic foot. The pressure drop \( \Delta p \) in \( (\sigma \Delta p)_1 \) might be observed across a cold engine at sea level; \( (\sigma \Delta p)_1 \) is a true mass-flow index and varies for a particular engine only with \( G \). The variable

\[
\frac{g^2}{2 p_1 \rho_0} \left( 1 + \frac{\Delta T}{2 T_1} \right)
\]

\[
\left( 1 + F + \frac{\Delta T}{T_1} \right)
\]

is then written

\[
\frac{(\sigma \Delta p)_1}{\sigma_1 p_1} \left( 1 + \frac{\Delta T}{2 T_1} \right) \left[ 1 + \frac{\Delta T}{T_1(1 + F)} \right]
\]

A curve of \( F \) against \( (\sigma \Delta p)_1 \) is first made by use of equation (15) and the values of \( G \) and \( \sigma \Delta p \) measured on a cold engine. The usual type of correlation tests are then made but, in place of \( \sigma \Delta p \), the new variable \( (\sigma \Delta p)_1 \) is used to fix the curve of

\[
\frac{\Delta T}{T_1(1 + F)}
\]

against \( \frac{M_e m/n}{T_g - T_h} \). The variable \( (\sigma \Delta p)_1 \) is found from figure 10 and the observed value of \( \Delta p / p_1 \), observed values of \( \Delta T / T_1 \), and \( F \) estimated by use of the curve of \( F \) against \( (\sigma \Delta p)_1 \).
Finally, an experimental curve of $\Delta T/(T_h - T_1)$ against $(\sigma \Delta p)_1$ is made from these values of $(\sigma \Delta p)_1$ and the corresponding measured temperatures.

In order to predict a cooling pressure drop, the required value of $(\sigma \Delta p)_1$ is found from the curve of $T_h - T_1$ against $\frac{m}{\sqrt{n}}$ in the customary way. The values of $\Delta T$ and $F$ are found from the proper curves against $(\sigma \Delta p)_1$, and the variable

$$\frac{(\sigma \Delta p)_1}{\sigma_1 p_1} \left( 1 + \frac{\Delta T}{2T_1} \right) \left[ 1 + \frac{\Delta T}{T_1(1 + F)} \right]$$

is computed. The ratio $\Delta p/p_1$, and hence $\Delta p$, is found by use of figure 10.

In Figure 11 the relations of Figure 10 are plotted in slightly different form. The ordinate is the same but the abscissa is $(\sigma \Delta p)_1/\sigma_1 p_1$.

It is of interest to observe that a correlation curve of the conventional type based on $\sigma_3 \Delta p$, where $\sigma_3$ is the density ratio of the air at the baffle outlet, will predict the required cooling pressure drops within about 5 percent at all altitudes. Because $\sigma_3$ is not easily predicted in practice, the theory as derived is considered preferable for cooling correlation.

Estimation of required cooling pressure drop from conventional correlation data. - If no complete cooling data are available, a rough estimate of the required cooling pressure drop at altitude can be made by use of a conventional cooling correlation.

The solid line of Figure 12 is such a curve for a modern engine; it is a plot of $\frac{(T_h - T_1)}{(T_g - T_h)}$ as a function of the variable $M_e^{1.76}/\sigma \Delta p$. The assumption is now made that a similar curve with the variable $M_e^{1.76}/(\sigma \Delta p)_1$...
would be parallel to, but displaced from, the curve shown; hence, only a single value of \((\sigma\Delta p)_1\) will be sufficient to fix the new correlation curve. If \(\Delta T/T_1\) and \(F\) are known at one engine condition with the corresponding value of \(\Delta p/p_1\), the value of \((\sigma\Delta p)_1\) can be found by use of figure 10 and the point and curve can be located. In predicting cooling pressure drops at altitude, \((\sigma\Delta p)_1\) is found from the assumed new correlation curve and \(F\) is assumed constant. The term \(\Delta T\) is computed from the data of the one test in which \(\Delta T\) was measured by use of the relation

\[
\Delta T \propto (T_h - T_1)(\sigma\Delta p)_1^{n-1/2}
\]

where \(n\) is the slope of the correlation curve. Either figure 10 or figure 11 may then be used to find the cooling pressure drop at altitude.

In case only single-cylinder correlation data are available and the cooling of a multicylinder engine is required to be known, it should be remembered that some spread in temperature is expected among the engine cylinders. In order that no cylinder shall overheat, all but one of them must be overcooled. The spread between the hottest and the coldest cylinders is expected to be as much as 100° F for a modern double-row engine.

Example. - Let it be required to estimate the pressure drop to cool the engine to which figure 12 pertains under the following conditions:

Output, horsepower ................................ 2000
Altitude, feet ........................................ 35,000
Airplane speed, miles per hour ....................... 350
Maximum head temperature, °F ......................... 500
Fuel-air ratio ........................................ 0.105
Carburetor-air temperature, °F ......................... 100

In one test at sea level \(\sigma_1\Delta p\) was found to be 13.8 inches of water for \(t_h = 355° F\), \(t_1 = 81° F\), and
\[ \frac{T_h - T_1}{T_g - T_h} = 0.470. \] The temperature rise observed was 61° F, \( \sigma_1 \) was 0.940, and \( p_1 \) was 398.7 inches of water; \( F \) was found to be approximately equal to \( \sigma_1 \).

From figure 10 with

\[ \frac{\Delta p}{p_1} = \frac{13.8}{0.94 \times 398.7} = 0.0368 \]

\[ \frac{(\sigma \Delta p)_1}{3_1 p_1} \left(1 + \frac{\Delta T}{2T_1}\right) \left(1 + \frac{\Delta T}{T_1} \frac{1}{1 + F}\right) = 0.0353 \]

Therefore

\[ (\sigma \Delta p)_1 = \frac{0.0353 \times 0.94 \times 398.7}{\left(1 + \frac{61}{541 \times 2}\right) \left(1 + \frac{61}{2 \times 541}\right)} = 11.9 \text{ inches of water} \]

and

\[ \frac{M_e^{1.76/\sigma \Delta p}}{M_e^{1.76/(\sigma \Delta p)_1}} = \frac{11.9}{13.8} = 0.862 \]

The curve of \( \frac{M_e^{1.76/\sigma \Delta p}}{M_e^{1.76/(\sigma \Delta p)_1}} \) assumed is shown dotted in figure 12; each abscissa is increased in the ratio 1:0.862 above its value on the solid curve.
For the assumed conditions, assume further $t_h = 450^\circ F$ or $50^\circ F$ below the limiting head temperature of the hottest cylinder. From figure 6, $T_1 = 456^\circ F$ absolute or $t_1 = 4^\circ F$, for the given fuel-air ratio and carburetor-air temperature, $t_g = 1054^\circ F$, as found from the same correlation tests. Then

\[
\frac{T_h - T_1}{T_g - T_h} = \frac{t_h - t_1}{t_g - t_h} = 0.751
\]

and

\[
\frac{M_1}{(\sigma A p)_1} = 3.34
\]

From the engine calibration

\[M_e = 4.25 \text{ pounds per second}\]

so that

\[(\sigma A p)_1 = \frac{(4.25)^1.76}{3.34} = 3.82 \text{ inches of water}\]

Since, for $(\sigma A p)_1 = 11.9$ inches of water, $\Delta T = 61^\circ F$, and since the slope of the correlation curve is $0.321$,

\[\Delta T = 61 \times \left(\frac{3.82}{11.9}\right)^{0.321-1/2} \times \frac{450 + 4}{355 - 81} = 123.7^\circ F\]

From figures 7 and 8

\[p_1 = 582 \text{ pounds per square foot}\]

\[= 112 \text{ inches of water}\]
\[ \rho_1 = 0.000744 \text{ slug per cubic foot} \]

or

\[ \sigma_1 = 0.3128 \]

Hence

\[
\frac{(\sigma_1 \Delta p)}{\sigma_1 p_1} \left( 1 + \frac{\Delta T}{2T_1} \right) \left( 1 + \frac{\Delta T}{T_1} \frac{1}{1 + F} \right)
\]

\[
= \frac{3.82}{0.3128 \times 112} \left( 1 + \frac{123.7}{2 \times 456} \right) \left( 1 + \frac{123.7}{456} \times \frac{1}{1 + 1} \right)
\]

\[ = 0.1405 \]

By use of figure 10, \( \Delta p = \frac{0.175 \times 582}{5.2} = 19.6 \text{ inches of water} \), which is in satisfactory agreement with the example previously computed.

In order to illustrate the altitude effect, the calculations were carried out for a range of altitude up to 50,000 feet with \( t_g = 1054^\circ F \) and \( t_h = 450^\circ F \); \( F \) was assumed constant in the absence of complete data. In order to show the effects of fuel-air ratio and of the permissible head temperature, calculations were also made for \( t_h = 450^\circ F \) and \( t_g = 956^\circ F \), corresponding to a fuel-air ratio of 0.115 for this engine, and for \( t_h = 400^\circ F \) (with \( t_g = 1054^\circ F \)). The results are plotted in figure 13. It is seen that lowering the permissible value of \( t_h \) only 50\(^\circ\) F doubled the required pressure drop, whereas increasing the fuel-air ratio 9.5 percent nearly halved the required pressure drop.

The dotted curve in figure 13 represents the dynamic pressure for an airplane speed of 350 miles per hour. If this curve indicates roughly the pressure available for cooling the engine, it is clear that 2000 horsepower could normally be obtained on this engine at altitudes above 40,000 feet only by using 500\(^\circ\) F as the limiting head temperature of the average cylinder and enriching the fuel-air mixture more than is usual with such engines.
The curves of figure 13 show the advantage of reducing the spread in cylinder-head temperatures and in fuel-air ratio in cases for which engine cooling at high altitudes must be improved.

CONCLUSIONS

1. For accurate presentation of cooling data over the entire altitude range, it is recommended that cooling-air mass flow or some equivalent variable be used instead of pressure drop in the correlation equation.

2. The simple approximate relation between mass flow and pressure drop that applies at sea level is inaccurate at high altitude. Accurate computation of the pressure drop at high altitude requires a knowledge of the temperature rise of the cooling air and the friction coefficient of the fins in addition to the usual information presented for low-altitude cooling.

3. A curve is shown which greatly simplifies the prediction of altitude cooling pressure drop from the correlation data.

4. A method is presented for adapting present incomplete correlation data to predict altitude cooling requirements. Calculations for a modern engine show that the acceleration of the cooling air because of decreased pressure at 35,000 feet causes nearly a 25 percent increase in pressure drop required for cooling.

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REFERENCES


3. Pinkel, Benjamin, and Ellerbrock, Herman H., Jr.: Correlation of Cooling Data from an Air-Cooled Cylinder and Several Multicylinder Engines. NACA Rep. No. 683, 1940.


Figure 1: Effect of carburetor-air temperature on the resultant cooling pressure drop. Values given, relative to pressure drop required for carburetor-air temperature of 100°F.

Figure 2: Effect of fuel-air ratio on relative cooling pressure drop. Values are given relative to pressure drop required for fuel-air ratio of 0.05.
Figure 3: Effect of altitude on cooling air temperature. Low density effect on temperature neglects constant power.
Figure 4.- Diagram of cooling-air flow path and pressure drops.
Figure 5. Correlation for determination of $n_2/N$, and $T_2/(N+64)$. 

At small values of $p_2/p_1$. 

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Figure 6: Effect of altitude on stagnation temperature before engine, Army air.
Figure 7: Effect of altitude on stagnation pressure $p_2$ in front of engine, Army Airs 90 percent ram.
Figure 8 - Effect of altitude on stagnation density \( \rho \) in front of engine. Army air, 80 percent ram.
Figure 9.—Curves for determination of $p_3/p_2$. 

(a) Small values of $p_3/p_2$. 

(b) Large values of $p_3/p_2$. 

Figure 9a, b
Figure 10.- Curves for finding $\Delta p/p$, from $(\sigma_1 \Delta p)/\sigma, p_1, \Delta T/T_1, \text{and } F$. 

$$\frac{G^2}{2a_0} \left(1 + \frac{\Delta T}{T_1}\right) \left(1 + F + \frac{\Delta T}{T_1}\right) = \frac{10}{\xi} \frac{(\sigma_1 \Delta p)}{\sigma, p_1} \left(1 + \frac{\Delta T}{T_1}\right) \left(1 + \frac{\Delta T}{T_1(T_1/F_1)}\right)$$
Figure 11. Alternative curves of $c_p/p_1$ as a function of $(x - p_1)/x$, $n$, $n^*$, and $F$. 
Figure 12.-Cooling correlation curve for the example engine.
Figure 13: Effect of altitude on the required rating for the example of 10,000-lb. Army Y-55 aircraft. Dotted line shows conditions at 4,500-mach flight speed and 3,500-alt. Dotted line shows expected conditions at 3,500-mach flight speed.