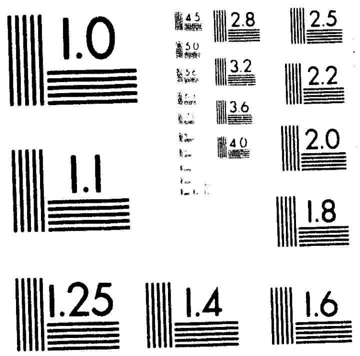
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REPORT

Stability of Bootstrap Current Driven Magnetic Islands in Stellarators

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Abstract

The stability of magnetic island producing perturbations due to fluctuations in the bootstrap current in stellarator configuration is examined. The stability criterion depends on the sign of the derivative of the rotational transform, the pressure gradient and the direction of the equilibrium bootstrap current which is determined by the structure of $|\mathbf{B}|$. It is found that quasi-helically symmetric stellarator configurations with $p'/\iota' < 0$ are unstable to the formation of bootstrap current driven magnetic islands. The stability of conventional stellarator configurations depends upon the field structure.

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Fluctuating neoclassical bootstrap currents provide a mechanism for the resistive reconnection of magnetic field lines and the concomitant magnetic island formation in tokamak plasmas.[1,2] This mechanism is potentially important in developing transport models based on interacting magnetic islands,[3] and in understanding the origin of macroscopic island formation in high performance tokamak plasmas.[4] Since bootstrap currents also are predicted to exist in stellarator configurations, a natural question to ask is: do we expect this mechanism for island formation also to be present in stellarators. In this brief communication, we answer this question and predict how the magnitudes of these islands are modified by the $|\mathbf{B}|$-spectrum and how they compare to the islands predicted to be unstable in tokamak configurations.

Much effort has gone into developing methods to understand and improve the quality of magnetic surfaces in stellarator configurations. The effect of plasma pressure on the magnetic structure and the appearance of magnetic islands has received considerable attention in recent years.[5-14] Most analytic predictions of pressure induced islands are based on magnetohydrodynamic models where resonant Pfirsch-Schlüter currents are predicted to provide a mechanism for island formation.[5,6,8] In particular, Refs. 6 and 8 attempt to make a connection between the predicted saturated island widths and the resistive interchange properties of the plasma equilibrium. However, in long mean-free-path plasmas neoclassical effects become important and tend to dominate the pressure gradient/curvature driven terms.

In this communication, we derive the amplitude of a self-sustained bootstrap current induced magnetic island in a stellarator plasma. The calculation is made using a boundary layer theory similar to the method used in Refs. 1, 5, 6, and 8. In this method a small island width is assumed at a particular rational surface. The self-consistently produced plasma currents are derived in the island region. The "interior" solution is then asymptotically matched to the "exterior" solution through Ampere's Law. The procedure is similar to deriving the nonlinear evolution of tearing instabilities in tokamak plasmas.[15] For simplicity and clarity of presentation, we will not include resonant Pfirsch-Schlüter current, resitive interchange and vacuum field error effects, however all of these terms should be present in a more complete theoretical calculation.

For nested magnetic flux surfaces, the equilibrium magnetic field can be written as $\mathbf{B}_o = \nabla\psi \times \nabla\theta + \iota(\psi)\nabla\zeta \times \nabla\psi$, where ψ is the magnetic flux function, θ and ζ are the poloidal and toroidal angles, respectively, and ι is the rotational transform. We consider a magnetic island perturbation of the form $\mathbf{B}_1 = \nabla\zeta \times \nabla\psi_1$, where $\psi_1 = \psi_{sx}\cos(m\theta - n\zeta)$ is a symmetry breaking magnetic perturbation resonant with the magnetic surface $\iota = n/m$. We presume that the perturbation does not vary rapidly in the radial direction, analogous to the constant-ψ assumption of tearing mode theory.[16] It is convenient to transform the angle coordinates to an angle resonant with magnetic perturbation given by $\alpha = \theta - (n/m)\zeta$. The entire magnetic field is now given by

$$\mathbf{B} = \nabla\psi \times \nabla\alpha + \nabla\zeta \times \nabla\psi^* \ , \tag{1}$$

where the helical flux function is given by

$$\psi^* = \int d\psi\,(\iota - \frac{n}{m}) - \psi_{sx}\cos(m\alpha) \cong \frac{1}{2}\iota' x^2 - \psi_{sx}\cos(m\alpha) \ , \tag{2}$$

$\iota' = d\iota/d\psi$ is evaluated at the rational surface, and $x = \psi - \psi_s$ is the distance in flux space away from the rational surface $\iota(\psi_s) = n/m$. The helical flux function, Eq. (2), describes a magnetic island configuration with island half-width w given by $w = 2\sqrt{\psi_{sx}/\iota'}$. The magnetic perturbation satisfies an Ampere's Law given by $\mathbf{B}\cdot\nabla \times (\nabla\zeta \times \nabla\psi_1) = \mathbf{j}_1\cdot\mathbf{B}$, where $\mathbf{j}_1$ is the perturbed plasma current.

An asymptotic analysis of the perturbed Ampere's Law yields an equation for the island width. As in the tearing mode problem, the exterior solution ignores the effects of inertia and dissipative terms such as resistivity and electron viscosity. As such, the exterior solution is determined through the exterior kink equation, which produces a jump in the logarithmic derivative of the exterior vector potential. We presume that the value of the equilibrium bootstrap current is not large enough to cause a resistive tearing instability. Therefore, we will take the matching parameter to be given by $\Delta' = -2m/r_s$, indicating tearing stability.

The island amplitude is now deduced from determining the interior solution, near the rational surface. The interior solution is matched to the exterior solution by integrating the parallel plasma current through the island region:

$$\int_{-\infty}^{+\infty} dx \oint \frac{d\alpha}{2\pi} \cos(m\alpha) \, \frac{j_1 \cdot \mathbf{B}}{\mathbf{B}\cdot\nabla\zeta} \frac{1}{|\nabla\psi|} = \frac{1}{2}\Delta'\psi_{sx} = -\frac{m}{r_s}\psi_{sx} \,, \tag{3}$$

which can be derived from Ampere's Law.

We will compute the plasma current by solving the drift kinetic equation for the plasma species and computing the needed moment from $j_{||} = \sum_s q_s \int dv \, v_{||} f_s$. The electron drift kinetic equation is written

$$\frac{v_{||}}{B} \mathbf{B}\cdot\nabla f + \mathbf{v}_d \cdot\nabla f = C(f) \,, \tag{4}$$

where the magnetic drift is written in the low-β approximation as

$$\mathbf{v}_d = \frac{v_{||}}{B} \nabla \times \frac{v_{||}\mathbf{B}}{\Omega_e} \,, \tag{5}$$

Ω_e is the electron gyroradius, and the curl operator is taken with fixed particle magnetic moment $\mu = m_e v_\perp^2/2B$ and energy. The collision operator is taken to be a Lorenz pitch angle scattering operator,

$$C(f) = \nu \frac{\xi}{B} \frac{\partial}{\partial \lambda} (\lambda \xi \frac{\partial f}{\partial \lambda}) , \tag{6}$$

where $\lambda = v_\perp^2 / B v^2$, $\xi = \sigma(1 - \lambda B)^{1/2}$, $\sigma = \mathrm{sgn}\, v_{||}$, $\nu = \nu_e(v_t/v)^3$, v_t is the electron thermal velocity, and ν_e is the electron collision frequency. An important distinction between tokamak and stellarator configurations is the magnetic drift velocity since it depends on the $|\mathbf{B}|$ spectrum. In this work we write this spectrum

$$|\mathbf{B}| = \sum_{MN} B_{MN}\, e^{(iM\theta - iN\zeta)} = B_{00}(\psi)[1 + \sum_{MN} \varepsilon_{MN} \cos(M\theta - N\zeta + \varphi_{MN})], \tag{7}$$

where φ_{MN} is a phase factor.

Equation (4) is solved for with two small parameters.[1,3] We define, $\gamma = (\nu_e R / v_t)$ and $\delta = (\rho_{\theta e}/w)$, where $\rho_{\theta e}$ is the poloidal electron gyroradius. We also take $w/a \sim \gamma^2$ where a is the minor radius. To order δ^0, Eq. (4) can be solved by letting the leading order distribution function be a Maxwellian with the density and temperature that are functions of the helical flux function ψ^*, given by Eq. (2). To leading order, the electron density and temperature equilibrate along the modified magnetic field lines. This results in the plasma profiles taking on the topology of the magnetic island.

To first order in δ, the drift kinetic equation is given by

$$\mathbf{B}\cdot\nabla f_1 - \mathbf{B} \times \nabla(\frac{v_{||}}{\Omega_e})\cdot\nabla f_0 = 0 , \tag{8}$$

where the collisional term is higher order in γ. If the magnetic field spectrum is given by a single helicity, i. e. $|\mathbf{B}| = B_{00}[1 + \varepsilon_{MN}\cos(M\theta - N\zeta)]$, the perturbed distribution function is given by

$$f_1 = - \frac{M}{M\iota - N} \frac{G v_{||}}{\Omega_e} \frac{\partial f_o(\psi^*)}{\partial x} + g_1 , \tag{9}$$

where $G = \mathbf{B}\cdot(\partial x/\partial\zeta)$ is evaluated on the rational flux surface and g_1 can be determined from drift kinetic equation to $O(\delta\gamma)$. The solution for g_1 is given by

$$g_1 = -\sigma v \frac{G}{\Omega_e} \langle \partial_x f_o(\psi^*) \rangle \, I(\lambda) \,, \tag{10}$$

where the brackets denote an average over the modified magnetic surfaces,

$$I(\lambda) = \Theta(\lambda_c - \lambda) \int_{\lambda_c}^{\lambda} \frac{B_{00} d\lambda}{2|\xi|} \,, \tag{11}$$

and $\lambda_c = 1/B_{00}(1 + \varepsilon_{MN})$. In the single magnetic helicity case, the bootstrap current in the vicinity of the island can then be computed and is given in the limit that ε_{MN} is small (the large aspect ratio assumption) by

$$j_{||} = -1.46\sqrt{\varepsilon_{MN}} \frac{M}{M\iota - N} \frac{G}{B_{00}} [T_e \langle \partial_x n \rangle + kn \langle \partial_x T_e \rangle] \,, \tag{12}$$

where k is a positive constant and we have ignored ion temperature gradient for simplicity.

Equation (12) is valid as long as $N/M \neq n/m$. In the special case that the spectral harmonic of $|\mathbf{B}|$ is resonant with a rational surface in the plasma, Eq. (8) can be solved for f_1 and is given by

$$f_1 = -\frac{G}{\Omega_e} \frac{df_0}{d\psi^*} [v_{||} - \sigma v I(\lambda)] \,. \tag{13}$$

The difference between this singular case and that described by Eqs. (9) and (10) is that the derivative of the leading order Maxwellian is taken with respect to ψ^*, not x. This leads to an island current given by

$$j_{||} = -1.46\sqrt{\varepsilon_{mn}} \frac{G}{B_{00}} [T_e \frac{dn}{d\psi^*} + kn \frac{dT_e}{d\psi^*}] \,, \tag{14}$$

Since ψ^* is double valued, the direction of this current changes sign on either side of the island region.

For a more general three-dimensional configuration, Eq. (8) cannot be simply analytically integrated. However, solving for f_1 is qualitatively similar to the computation of the equilibrium value of the bootstrap current in stellarator configurations.[17-21] The effect of the island is important in establishing the flux surfaces in real space near the island, but has a small effect in changing the velocity space properties of the plasma. For this reason, the magnitude of the bootstrap current can be given as a function only as a function of $|\mathbf{B}|$ and ι. Following the suggestion of Boozer and Gardner,[21] the value of the bootstrap current near the island is given by

$$j_{||} = -\Delta_o \frac{G}{B_{00}}[T_e\langle\partial_x n\rangle + kn\langle\partial_x T_e\rangle] - 1.46\frac{\varepsilon_{mn}}{\sqrt{b}}\frac{G}{B_{00}}[T_e\frac{dn}{d\psi^*} + kn\frac{dT_e}{d\psi^*}], \quad (15)$$

where the factor Δ_o is the factor that contains information about the magnetic spectrum and is given by

$$\Delta_o \cong \frac{1.46}{\sqrt{b}} \sum_{MN}{}' \frac{M\varepsilon_{MN}}{M\iota - N}, \quad (16)$$

where b is the total fractional variation of $|\mathbf{B}|$ on the flux surface, the sum is over all helicities except that part of $|\mathbf{B}|$ resonant with the rational surface which is described by the last term in Eq. (15).

The kinetic theory calculation restricts the density and temperature profiles to be functions of ψ^*. In order to obtain the form of the plasma gradients in the interior region, a simplified transport equation can be solved.[22] By neglecting plasma particle and heat sources in the island region, the continuity of thermal and particle flux reveals a simple description of the plasma gradients. Within the island separatrix the profiles are flattened and outside the separatrix,

$dn/d\psi^* = dn/d\psi\ [\oint d\alpha\ (2\pi)^{-1}\ \partial_x\psi^*]^{-1}$ where $dn/d\psi$ is the "radial" derivative evaluated in the exterior region.[22]

Now that the parallel plasma current near the rational surface is derived, Eq. (15) can be inserted into Eq. (3) so that the width of the magnetic island can found. After integrating the parallel current, Eq. (3) reduces to

$$\psi_{sx}[\frac{m}{r_s} - k_o\frac{\Delta_o}{w}\frac{2\mu_o p'G}{\iota'\mathbf{B}\cdot\nabla\zeta\,|\nabla\psi|}] = 0\ , \tag{17}$$

where Δ_o is given by Eq. (15) and k_o is a positive constant of order unity. Since the last term in Eq. (14) changes signs across the rational surface it does not contribute to the matching integral. Equation (16) gives a condition for having the bootstrap current cause the formation of a magnetic island. The instability condition is given by

$$\Delta_o\frac{p'}{\iota'} > 0\ . \tag{18}$$

If the instability criterion, Eq. (18), is violated the predicted value for the radial extent of the magnetic island induced by the fluctuating bootstrap current is given by

$$\frac{w_{sat}}{|\nabla\psi|} \cong \frac{k_o r_s}{m}\frac{\iota^2\Delta_o}{\iota'}\frac{2\mu_o p'}{B_\theta^2}\ , \tag{19}$$

where the approximation $|\nabla\psi|^2\mathbf{B}\cdot\nabla\zeta/G \cong B_\theta^2/\iota^2$ is made.

The stability criterion for the formation of bootstrap current driven magnetic islands in toroidal plasmas is given by Eq. (18). Since $\Delta_o = 1.46q\sqrt{\varepsilon} > 0$ for tokamak plasmas, this criterion is violated for normal operation with $p'q' < 0$. For quasi-helically symmetric stellarators, the bootstrap parameter is given by $\Delta_o = 1.46\sqrt{\varepsilon_h}\ M/(M\iota - N)$, where M and N are the poloidal and toroidal mode numbers of the helical symmetry. For the proposed HSX device,[23] $N > M\iota$ so that $\Delta_o < 0$ and quasi-helically symmetric stellarators with $p'/\iota' < 0$ are unstable to these modes. In conventional stellarators with $\Delta_0 > 0$ (the toroidal field-strength variation dominates the helical field strength variation), these modes are stable when $p'/\iota' < 0$.

For unstable cases, the predicted magnitude of the island width scales as $w \cong \Delta_0 \iota^2 \beta_\theta / m\iota'$, so that for quasi-helical configurations the saturated island width is reduced by the factor $M/(N - M\iota)$ compared to a tokamak with similar poloidal beta and shear length. If neoclassical bootstrap current driven magnetic islands are important in reactor relevant tokamak plasmas,[24] they should also have an impact on the operation of reactor grade quasi-helically symmetric plasmas, but at a level reduced by the factor $M/(N - M\iota)$.

Acknowledgments

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