SAND99-2840J

Uncertainty Propagation in Calibration of Parallel Kinematic Machines

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1. Introduction

Over the last decade, multi-axis machine tools and robots based on parallel kinematic mechanisms (PKMs) have been developed and marketed worldwide. Positional accuracy in these machines is controlled by accurate knowledge of the kinematic parameters which consists of the joint center locations and distances between joint pairs (Figure 1.1). Since these machines tend to be rather large in size, the kinematic parameters (joint center locations, and initial strut lengths) are difficult to determine when these machines are in their fully assembled state. Work recently completed by the University of Florida and Sandia National Laboratories has yielded a method for determining all of the kinematic parameters of an assembled parallel kinematic device. This paper contains a brief synopsis of the calibration method created, an error budget, an uncertainty analysis for the recovered kinematic parameters and the propagation of these uncertainties to the tool tip.

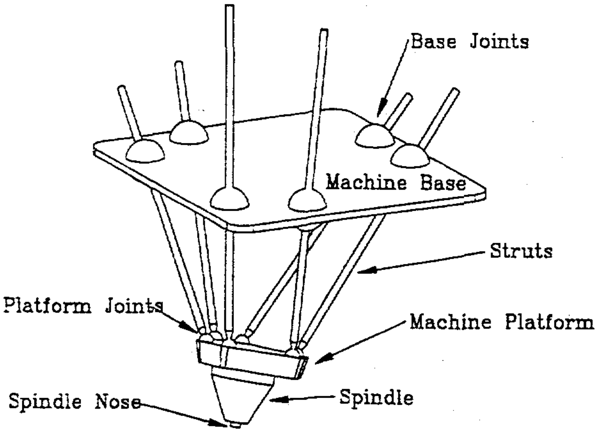

Figure 1.1: A 12-joint (6-6) Stewart platform six DOF device.

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DISCLAIMER

2. Sequential Determination Method

The proposed method has four parts: (1) locating the central reference frame (R) and machine frame (M), (2) identifying the joint center locations, (3) determining the spindle nose location and centerline orientation, and (4) determining the initial strut lengths.

2.1 Locating the Central Reference Frame (R)

Three gage points (r_1, r_2, and r_3) are secured at three corners of the worktable, creating a single fixed coordinate reference frame (R) to which all future coordinate measurements collected in other floating reference frames will be transformed (Figure 2.1). Utilizing a spatial coordinate measuring device such as a laser tracker, the locations of the R gage points, the plane of the worktable, the X-axis and the desired location of the machine origin are measured. The homogeneous transformation (HTM) between the R system and the machine system (M) is calculated.

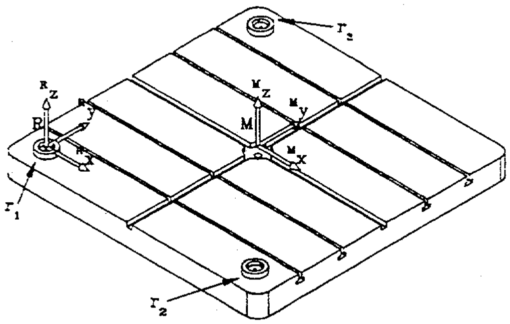

Figure 2.1: R and M reference frames.

2.2 Joint Center Location Identification

Two gage points (SR and SL) for the spatial measuring device are affixed to the strut in question, one to each side of the strut and equal distance from the strut centerline such that the two gage points and the strut centerline lie in the same plane. Three gage points are affixed to the platform which define a coordinate system (PR). The platform is moved along an arbitrary predetermined path holding the strut in question at an arbitrary fixed length and varying the lengths of the other five struts. As the platform moves, the fixed-length strut rotates in its joints (Figures 2.2 and 2.3) tracing out a sphere. At specific locations along the path, the machine motion is paused, and the spatial coordinates of the strut gage and platform gage points are measured. Once all of the coordinates are collected, the locations of the base and platform joint centers may be determined by fitting the transformed gage point coordinates to the equation of a sphere. Assuming the joints produce spherical motion, the center of the calculated best-fit sphere is the center of rotation of the joint in question. This method is repeated for each of the five remaining struts to recover all 12 of the base and platform joint center locations.

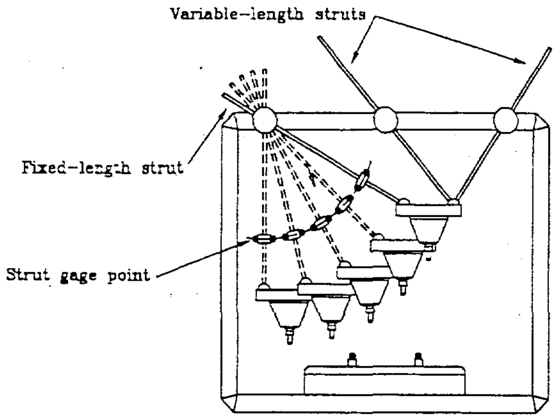

Figure 2.2: To an observer stationary relative to the M system, a point on the fixed length strut rotates about the base joint center.

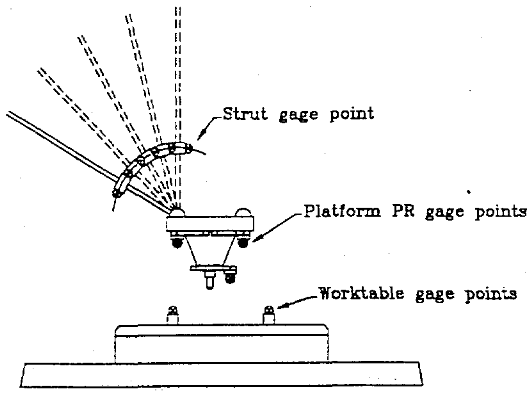

Figure 2.3: To an observer stationary relative to the PR system, a point on the fixed length strut rotates about the platform joint center.

2.3 Locating the Spindle Nose

A fixture holding a gage point is attached to a tool holder. The toolholder is mounted in the spindle and rotated slowly by hand. At approximately 30-40 different locations, the motion is stopped and the coordinates of the gage point are measured relative to the PR system. The coordinates are fit to a plane, and then to a circle in the best fit plane (Figure 2.4). The unit normal vector of the best-fit plane is the orientation vector of the spindle relative to the PR system. The coordinates of the center of the circle projected along the positive direction of the normal vector by the offset distance of the fixture and tool holder are the coordinates of the center of the spindle nose. Since the platform joint locations and the spindle location and orientation are known relative to the same system, the locations of the platform joints may be expressed relative to the spindle nose.

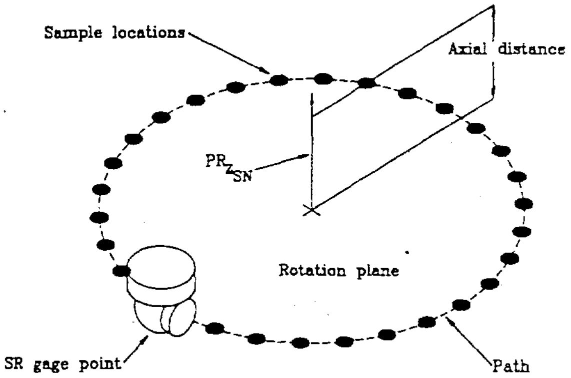

Figure 2.4: Spindle axis identification.

2.4 Determining the Initial Strut Lengths

The platform is retracted to its home position. The coordinates of the PR gage points are measured. Knowing the locations of the base joints and the platform joints relative to the PR system, the straight line distances between the each of the six strut joint pairs may calculated. This process of homing the platform, measurement of the PR system, and calculation of the straight line distances is repeated several times. The initial strut reference length for each strut is taken to be the mean of the calculated strut lengths for each strut over all of the repetitions.

3. Error Budget

The sequential method presented in the previous section relies heavily upon constancy of the fixed strut length, and upon repeated coordinate measurements of the strut, platform gage points and central reference frame gage points. In order to estimate the amount of uncertainty present in the recovered kinematic parameters and the propagation of the kinematic parameter uncertainties to the tool tip, an error budget for the constant strut length and measurement device must be constructed.

Factors identified to influence the length of the strut between the fixture and the base joint ("Base" in Table 3.1) are: base joint motion sphericity, strut axial flexibility, thermal effects, strut length command mismatch, and the least count servo motion. Between the fixture and the platform joint ("Platform" in Table 3.1) are: joint motion sphericity, strut axial flexibility, and thermal effects. Four distinct scenarios were considered (Table 3.1): (1) best estimate of the uncertainty present in the real system; (2) assumes uncertainty in the machine with perfect measurements; (3) assumes uncertainty in the measruements with a perfect machine; and (4) more optimistic version of #1.

Table 3.1: Scenario input uncertainties.

Scenario	Base (mm)	Platform (mm)	Measurement (mm)
1	0.016	0.015	0.025
2	0.016	0.015	0.000
3	0.000	0.000	0.025
4	0.010	0.010	0.013

4. Uncertainty in Kinematic Parameters

Propagation of strut length and measurement uncertainties to the recovered kinematic parameters was done by a Monte Carlo simulation (Figure 4.1). Each scenario was subjected to 10000 iterations. First, the reverse kinematic solver calculated the nominal strut lengths for the pose in each particular fixed-length strut toolpath given the set of nominal kinematic parameters. For each iteration, a small amount of uniformly distributed, random strut length uncertainty was added to the strut lengths. The new pose of the platform and gage point locations were calculated. Uniformly distributed, random measurement uncertainty was added to the gage point locations. Finally, the joint center locations and initial strut lengths were determined from the perturbed gage point locations. The mean and standard deviation of the sphericity and the center error distance (CED) were also calculated over all of the iterations for each scenario. Table 3.2 shows the results for strut 1, which are representative of the results for the other 5 struts. Table 3.3 contains the results of the propagation of the machine and measurement uncertainties to the initial strut lengths were calculated.

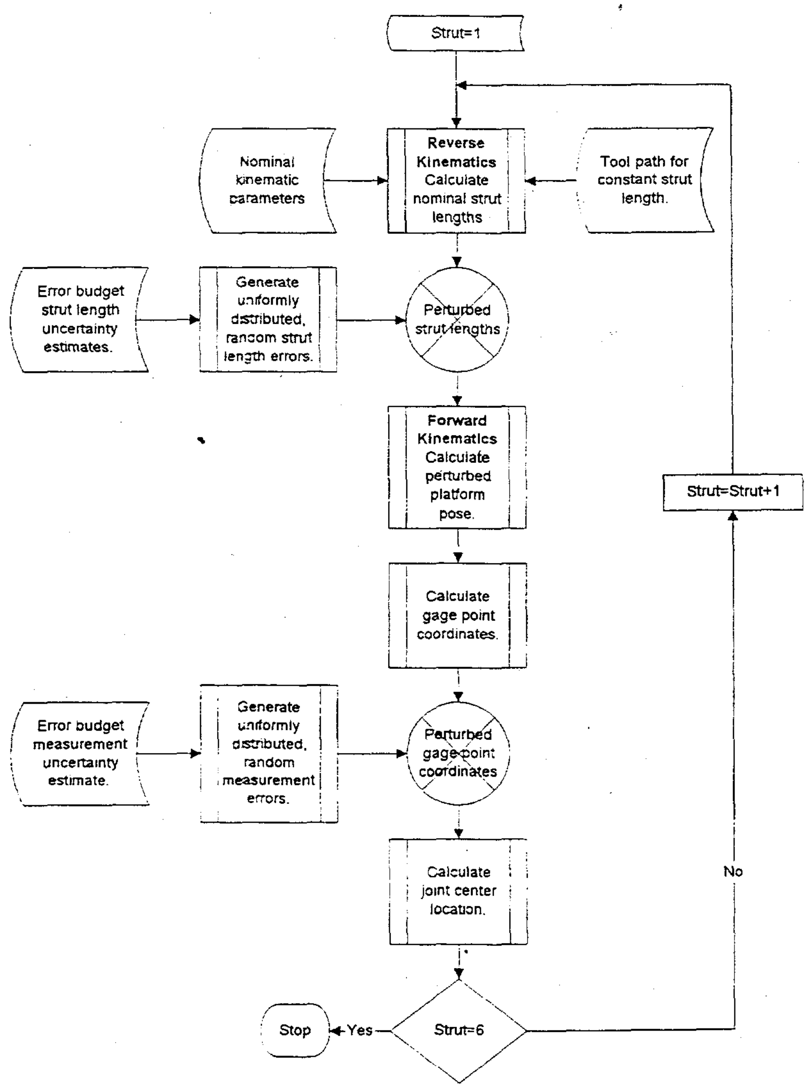

Table 3.2: Joint center uncertainty.

	Scenario Number	Base Joint Mean (mm)	Base Joint σ (mm)	Platform Joint Mean (mm)	Platform Joint σ (mm)
Sphericity	1	0.045	0.008	0.064	0.014
	2	0.029	0.004	0.027	0.003
	3	0.033	0.006	0.057	0.013
	4	0.026	0.005	0.035	0.007
CED	1	0.067	0.044	0.098	0.064
	2	0.048	0.031	0.045	0.028
	3	0.050	0.032	0.086	0.057
	4	0.039	0.025	0.054	0.035

Table 3.3: Uncertainty in the initial strut lengths.

Scenario	$\sigma_{Strut\ 1}$ (mm)	$\sigma_{Strut\ 2}$ (mm)	$\sigma_{Strut\ 3}$ (mm)	$\sigma_{Strut\ 4}$ (mm)	$\sigma_{Strut\ 5}$ (mm)	$\sigma_{Strut\ 6}$ (mm)
1	0.139	0.162	0.122	0.134	0.287	0.160
2	0.066	0.071	0.060	0.067	0.123	0.076
3	0.123	0.147	0.106	0.118	0.257	0.141
4	0.077	0.089	0.067	0.075	0.157	0.088

5. Uncertainty in Machine Motion

A Monte Carlo simulation was used to propagate the uncertainties in the calibration to the tool tip. First, path plans for six circular traces were developed (Table 3.4). Second, the inverse kinematics calculated the nominal poses of the mechanism for each path from the nominal parameters. Third, 1000 sets of parameters were selected from the 10000 previously simulated. For each set, scenario and pose in each circular path, the error between the nominal and the recovered simulated parameters were calculated and projected on to the strut lines of action. Additional uniformly, randomly distributed strut length uncertainties were added. The forward kinematics determined the perturbed tool tip locations for each path. Using the perturbed tool tip locations the best-fit circle was calculated for each path.

Table 3.4: Locations of circular traces.

Circle Number	Nominal Circle Locations (mm) X	Y	Z	Radius
1	0	0	180	150
2	200	200	180	150
3	-200	200	180	150
4	-200	-200	180	150
5	200	-200	180	150
6	0	0	180	300

For each circular trace and uncertainty scenario, the mean and standard deviation of the X and Y circle center coordinates, radii and circularities were calculated over all of the iterations. Results for circle #1 for all four scenarios is shown in Table 3.5 and are representative of the other five traces.

Table 3.5: Circle #1 location uncertainty.

Scenario	$\overline{X}$ (mm)	σ_X (mm)	$\overline{Y}$ (mm)	σ_Y (mm)	$\overline{R}$ (mm)	σ_R (mm)	$\overline{\text{Circ}}$. (mm)	σ_{Circ} (mm)
1	0.000	0.038	0.001	0.038	150.000	0.002	0.115	0.008
2	-0.001	0.018	0.000	0.018	150.000	0.001	0.115	0.008
3	0.001	0.033	-0.001	0.032	150.000	0.001	0.003	0.002
4	-0.001	0.021	0.000	0.021	150.000	0.001	0.074	0.005

6. Conclusions

The theoretical of the proposed sequential determination method for identifying the kinematic parameters in fully assembled Stewart platform type PKMs appears to show that it is a viable technique.

Monte Carlo simulations show both the machine and measurement uncertainties appear to have a large impact on the correct identification of the joint centers and the initial strut lengths. Uncertainties in the machine strut lengths does not appear to have as great of an effect on the recovered parameters.

The Renishaw ball bar circles used to evaluate the success of the calibration appear to show that the correct kinematic parameters were identified, although a great improvement in machine performance did not appear to have occurred. However, a rigorous application of the ASME B5.54 standard is warranted to decisively show improvements in positional accuracy.