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INFLUENCE OF RADIAL ELECTRIC FIELD
ON ALFVÉN-TYPE INSTABILITIES

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quasi-translational invariance,¹¹ and particle orbit modification^{12,13} emerge. Hence, it is obviously valuable to clarify the frequency identification issue in at least the linear analysis of this important problem.

This Brief Communication deals with shear Alfvén fluctuations which include various ballooning/kink instabilities and toroidal Alfvén eigenmodes¹⁴ (TAE). The expected Doppler shift formula for the eigenfrequency can be heuristically estimated by simply using the previous theories [without $E_r^{(0)}$] in a moving frame where $E_r^{(0)}$ is Lorentz transformed away. In a frame moving with $\mathbf{V}_E = \mathbf{E}_r^{(0)} \times \mathbf{B}_0 / B_0^2$,

$$\mathbf{E}_{\text{eff}}^{(0)} = \mathbf{E}_r^{(0)} + \mathbf{V}_E \times \mathbf{B}_0 = 0, \quad (1)$$

in the usual classical limit. Accordingly, the eigenfrequency, ω_{Th} , from the theories without $E_r^{(0)}$ is related to the observed frequency in a lab frame, ω_{Lab} by

$$\omega_{\text{Th}} = \omega_{\text{Lab}} - \mathbf{k} \cdot \mathbf{V}_E, \quad (2)$$

where $\mathbf{k}$ is the wave vector of the fluctuation considered. With the heuristic result as a guide, we now proceed with a proper derivation of Eq. (2) from the governing equations of shear Alfvén fluctuations. At the simplest level, shear Alfvén waves can, of course, be derived from ideal MHD equations. However, for application to present day tokamak plasmas, a kinetic approach is clearly needed. The ion distribution function, for example, deviates significantly from a Maxwellian. In this work, we use a frozen-in-flux constraint for simplification of electron dynamics and a moderate- β ordering is adopted in dealing with the gyrokinetic equation for ions. The radial electric field $E_r^{(0)}$ is included in this analysis, and the associated modification of the frequency of fluctuation is determined.

In a moderate- β plasma with $\delta\mathbf{B}_\perp \gg \delta\mathbf{B}_\parallel$, the electric field can be expressed as $\mathbf{E} = -\partial_t \mathbf{A} - \nabla\Phi$, where $\mathbf{A} = \delta A_\parallel \mathbf{b}$, $\mathbf{b} = \mathbf{B}_0/B_0$, $\Phi = \Phi_0 + \delta\Phi$, and $\mathbf{E}_r^{(0)} = -\nabla\Phi_0$ is in the radial direction. Then, the frozen-in-flux constraint, $\mathbf{E} \cdot \mathbf{B} = 0$ ($\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}_\perp$) becomes

$$-\partial_t \delta A_\parallel - \mathbf{b} \cdot \nabla \delta\Phi = (1/B_0) \delta\mathbf{B}_\perp \cdot \nabla (\Phi_0 + \delta\Phi). \quad (3)$$

Using the vector identity, $\delta\mathbf{B}_\perp \cdot \nabla\Phi_0 = \nabla \delta A_\parallel \cdot \mathbf{b} \times \nabla\Phi_0$, Eq. (3) in the linear limit can be expressed as,

$$(\partial_t + \mathbf{V}_E \cdot \nabla) \delta A_\parallel = -\mathbf{b} \cdot \nabla \delta\Phi. \quad (4)$$

This result can be viewed as a time evolution equation for $\delta\mathbf{B}_\perp$. Note that $\mathbf{E}_r^{(0)}$ enters through $\mathbf{V}_E \cdot \nabla$ term thereby producing a Doppler shift, $\omega_{\text{Th}} \Rightarrow \omega_{\text{Lab}} - \mathbf{k} \cdot \mathbf{V}_E$.

Now, we consider the ion gyrokinetic equation written in gyrocenter variables (μ , $V_\parallel$, $\mathbf{R}$). Assuming a moderate value of β , $\delta\mathbf{B}_\parallel$ is ignored in comparison to $\delta\mathbf{B}_\perp$. After a linearization,¹⁵

$$\begin{aligned} \{\partial_t + (\mathbf{V}_E + \mathbf{V}_d + V_\parallel \mathbf{b}) \cdot \nabla\} \delta f = & -(1/B_0) \mathbf{b} \times \nabla (\delta\Phi - V_\parallel \delta A_\parallel) \cdot \nabla F_0 \\ & -(1/M_i B_0) (\mathbf{B}_0 + \delta\mathbf{B}_\perp) \cdot [\nabla \{\mu B_0 + e(\Phi_0 + \delta\Phi)\} + \partial_t \delta A_\parallel \mathbf{b}] \partial F_0 / \partial V_\parallel. \end{aligned} \quad (5)$$

The notations here are of standard form, and some nonlinear terms have been retained in the last term in order to more clearly depict the over-all structure of the equation. In examining Eq. (5), first note that the linear propagator on the left hand side has a term $(\partial_t + \mathbf{V}_E \cdot \nabla)$.

In addition, note that the following two terms on the right hand side, $(1/B_0) \delta \mathbf{B}_\perp \cdot \nabla e\Phi_0 + e\partial_t \delta A \parallel \mathbf{b} \cdot \mathbf{b}$ can be combined into the single term, $e(\partial_t + \mathbf{V}_E \cdot \nabla) \delta A \parallel$. From these observations, the correspondence rule for frequency, $\omega_{Th} \Rightarrow \omega_{Lab} - \mathbf{k} \cdot \mathbf{V}_E$, again emerges. This more formal derivation of the Doppler shift result is nevertheless still subject to some validity constraints. Specifically, in the presence of $E_r^{(0)}$, the equilibrium distribution function F_0 should produce a mean ion flow, $\mathbf{U}$ which is compatible with the radial force balance, $E_r^{(0)} = (1/Z_i n_i e) \nabla P_i - \mathbf{U} \times \mathbf{B}_0$. By taking the zeroth and the first velocity moment of Eq. (5), it is indeed the case that our conclusion regarding the Doppler shift remains valid provided the $E_r^{(0)}$ dependence of $\mathbf{U} \cdot \mathbf{b}$ is weak.

Finally, we demonstrate an application of the results of the present analysis to a possible interpretation of recent magnetic fluctuation observations in the Princeton Beta Experiment-Modified¹⁶ (PBX-M) tokamak. A new feature of ELM-related activity⁵ observed by the Mirnov coil array is an intense outward ballooning fluctuation that appears simultaneously with the D_α increase and lasts for about 5 ms. The observed frequency feature consisting of a series of toroidal harmonics from $n=3$ to 8 can be fitted by an empirical relation, $f_n = f_0 + n f_1$, where $f_1 = 20\text{-}30$ KHz, and $f_0 = 30\text{-}50$ KHz. One candidate for explaining the ballooning structure and this frequency dependence is the beam-driven kinetic ballooning mode which can be excited below the ideal ballooning limit via resonances with energetic ions. To be more specific, the energetic particle continuum mode in Ref. 17 has an eigenfrequency $\omega \approx \omega_{bm}$ or ω_{tm} , the bounce or transit frequency of beam ions at the maximum beam injection energy. We note that the more typical MHD gap mode branch of the kinetic ballooning mode¹⁸ has a frequency $\omega \approx \omega_{*pi}$ which depends linearly on the mode number. However, Ref. 17 does not include $E_r^{(0)}$ in the analysis, and the fast reciprocating probe measurements at the edge of the PBX-M device show the existence of an appreciable $E_r^{(0)}$.¹⁹ When this effect is taken into account, our Doppler-shift

formula, Eq. (2), predicts $\omega_{\text{Lab}} = \omega_{\text{Th}} + \mathbf{k} \cdot \mathbf{V}_E$. Here, it is interesting to point out that ω_{Th} is independent of the toroidal mode number n and can be readily calculated from the parameters given in Ref.17. The $\mathbf{k} \cdot \mathbf{V}_E$ term just corresponds to nf_1 . When the PBX-M parameters are used for these quantities, Eq. (2) is found to be in reasonably good agreement with the empirical frequency formula.⁵

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