Kinetic Theory and Boundary Conditions for Flows of Highly Inelastic Spheres.

Quarterly Progress Report

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Introduction

In the last quarter, we focused on steady, fully developed, gravity-driven flows of identical, smooth spheres down bumpy inclines, with flow depths greater than one particle diameter, solid fraction profiles everywhere less than .65, and dimensionless granular temperature $T(y=\beta)$ at the top between 0 and 10. For prescribed boundary bumpiness ($r=1$ and $\Delta=.414$), restitution coefficients of the boundary ($e_w=.5$) and the flow particles ($e=.5$), and angles of inclination ($\phi=25.5^\circ$, 27.505° , 28° , and 34°), we determined what appeared to be *complete* range of $T(y=\beta)$ (within $0 < T(y=\beta) \leq 10$) for which such flows can be maintained. For the two lower inclinations ($\phi=25.5^\circ$ and 27.505°), we found that there was a nonzero minimum and a finite maximum flow rate between which steady, fully developed flows can be maintained. However, for the two upper inclinations ($\phi=28^\circ$ and 34°), although there was a nonzero minimum flow rate for each, *it appeared* that there was no maximum flow rate that limited the occurrence of steady, fully developed flows.

In this quarter, we consider in greater detail the same parameter values ($r=1$, $\Delta=.414$, and $e=e_w=.5$) and focus on inclinations near the apparent transition from the those for which there exists a maximum flow rate to the inclinations for which there appeared to be no such maximum. Using careful numerical computations, we demonstrate that the transition is real and we determine the angle at which it occurs. We have devoted the time to these issues because the existence of cases in which there are *no maximum flow rates* that limit the occurrence of steady flows may be of great practical value when it is necessary to steadily transport extremely high volumes of granular materials.

In what follows, we use the notation established in our last three quarterly reports.

Results and Discussion

Here we consider the case in which $r=1$, $e=e_w=.5$, $\Delta=.414$, and choose six angles $\phi=27.3^\circ$, 27.5° , 27.52° , 27.55° , 27.75° , and 28° near the apparent transition from the those angles for which there exists a maximum flow rate to those angles for which there appeared to be no such maximum. In addition, we focus on values of $T(y=\beta)$ (between 0 and .8) near the apparent minimum values suggested by the figures in our last quarterly report. We do so because the flow rate increases rapidly with decreasing values of $T(y=\beta)$ near the minimum values of $T(y=\beta)$. In this report we determine whether a distinction can be made between those angles for which the flow rate increases to a finite maximum and those angles for which the flow rate increases without bound.

In Figure 1, we show for each angle the variations of β , m , $\dot{m}$, $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T(y=\beta)$ between 0 and .8. For $\phi=27.3^\circ$ and 27.5° , as $T(y=\beta)$ decreases from .8, the flow rates increase to an identifiable finite maximum values and then decrease until the assemblies become too dense to flow. For these inclinations, there are no gaps in the range of $T(y=\beta)$ for which steady, fully developed flows can be found. At these angles, the occurrences of steady fully-developed flows are limited by finite maximum flow rates.

However, for $\phi=27.52^\circ$, 27.55° , 27.75° , and 28° , we find that between the extreme values of $T(y=\beta)$ for which steady, fully developed flows can be found, there is a sub-range $T' \leq T(y=\beta) \leq T^*$ for which steady, fully developed flows can not be found. Most importantly, the flow rates apparently become infinite as $T(y=\beta)$ approaches T' from below or T^* from above. Consequently, inclinations for which such a gap in $T(y=\beta)$ occurs are those at which steady, fully developed flows are not limited by a maximum flow rate.

As $T(y=\beta)$ approaches T^* from above, the flow rates increase dramatically, while the flows do not become too dense to prevent the assemblies from flowing. For values of $T(y=\beta)$ between T^* and T' , integrations proceed from the top of the flow indefinitely downward without satisfying the basal boundary condition. However, at no point in the integrations does the solid fraction exceed .65. As $T(y=\beta)$ decreases away from T' , the flow rates at first decrease, and the depth-averaged solid fraction eventually increases until the assemblies become too dense to flow. Based on these observations, we believe that, independent of such artificial restrictions as $v \leq .65$, the theory predicts that there is a real gap in the values of $T(y=\beta)$ for which steady, fully-developed flows can be found, and that the flow rates corresponding to the cases $T(y=\beta)=T^*$ and $T(y=\beta)=T'$ are unbounded. Interestingly, it appears that the upper and lower limits, T' and T^* , of the gap depend on inclination angle ϕ , and that the size of the gap increases as the inclination increases from the angle at which the gap first appears.

The results shown in Figure 1 suggest that the transition from those angles for which the flow rate increases to a finite maximum and those angles for which the flow rate increases without bound occurs between $\phi=27.5^\circ$ and 27.52° . In what follows, we determine the transition angle with more precision and further demonstrate the qualitative differences between those flows that occur below the transition angle and those that occur above the transition angle. In Figures 2, 3, 4, 5, and 6 we show the variations of β , m , $\dot{m}$, $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with the difference $T(y=\beta)-T^*$, for $\phi=27.5^\circ$, 27.505° , 27.51° , 27.75° , and 28° , respectively.

Of greatest interest here are the variations of $\dot{m}$ and $\bar{v}$ with the difference $T(y=\beta)-T^*$. Figures 2 and 3 demonstrate that when $\phi=27.5^\circ$ and 27.505° , for example, the flow rates increase at diminishing rates as $T(y=\beta)$ approaches T^* from above (i.e. as the positive difference $T(y=\beta)-T^*$ becomes smaller). This suggests that the theory would predict a finite maximum rate (corresponding to the limit $T(y=\beta)=T^*$) that limits the occurrence of steady, fully developed flows. Because there is no gap in the range of $T(y=\beta)$ for

which steady, fully developed flows can be found, the values of T^* for $\phi=27.5^\circ$ and $\phi=27.505^\circ$ are taken to be the values of $T(y=\beta)$ at which the "spikes" occur in the variations of flow rate with $T(y=\beta)$, shown in the uppermost right-hand panels of Figure 1 of this report and Figure 2 of our last quarterly report.

By contrast, Figures 4, 5, and 6 demonstrate that when $\phi=27.51^\circ$, 27.75° , and 28° , the flow rates increase nearly linearly with $-\log[T(y=\beta)-T^*]$. This suggests that the flow rates would become unbounded in the limiting case $T(y=\beta)=T^*$. To check that the increase in flow rate does not diminish, we have observed the linear relationship between flow rate and the quantity $-\log[T(y=\beta)-T^*]$ when the difference $T(y=\beta)-T^*$ is as small as 10^{-7} when $\phi=27.75^\circ$ (Figure 5), and when the difference is as small as 10^{-9} when $\phi=28^\circ$, (Figure 6).

Based on these results, we conclude that when $r=1$, $e=e_w=.5$, and $\Delta=.414$, the transition between those angles for which the flow rate increases to a finite maximum and those angles for which the flow rate increases without bound occurs at inclinations ϕ between 27.505° and 27.51° .

Figure Captions

- Figure 1: The variations of β , m_t , $\dot{m}$, $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T(y=\beta)$ for $\phi=27.3^\circ, 27.5^\circ, 27.52^\circ, 27.55^\circ, 27.57^\circ$, and 28° when $r=1$, $e=e_w=.5$, and $\Delta=.414$.
- Figure 2: The variations of β , m_t , $\dot{m}$, $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T(y=\beta) - T^*$ for $\phi=27.5^\circ$, when $r=1$, $e=e_w=.5$, and $\Delta=.414$.
- Figure 3: The variations of β , m_t , $\dot{m}$, $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T(y=\beta) - T^*$ for $\phi=27.505^\circ$, when $r=1$, $e=e_w=.5$, and $\Delta=.414$.
- Figure 4: The variations of β , m_t , $\dot{m}$, $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T(y=\beta) - T^*$ for $\phi=27.51^\circ$, when $r=1$, $e=e_w=.5$, and $\Delta=.414$.
- Figure 5: The variations of β , m_t , $\dot{m}$, $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T(y=\beta) - T^*$ for $\phi=27.75^\circ$, when $r=1$, $e=e_w=.5$, and $\Delta=.414$.
- Figure 6: The variations of β , m_t , $\dot{m}$, $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T(y=\beta) - T^*$ for $\phi=28.0^\circ$, when $r=1$, $e=e_w=.5$, and $\Delta=.414$.

Figure 1: The variations of β , m_l , m , $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T(y=\beta)$ for $\phi=27.3^\circ, 27.5^\circ, 27.52^\circ, 27.55^\circ, 27.57^\circ$, and 28° when $r=1$, $e=e_w=.5$, and $\Delta=.414$.

$$\phi = 27.5^\circ$$

Figure 2: The variations of β , m_t , m , $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T(y=\beta) - T^*$ for $\phi=27.5^\circ$, when $r=1$, $e=e_w=.5$, and $\Delta=.414$.

$$\phi = 27.505^\circ$$

Figure 3: The variations of β , m_t , m , $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T(y=\beta) - T^*$ for $\phi=27.505^\circ$, when $r=1$, $e=e_w=.5$, and $\Delta=.414$.

$$\phi = 27.51^\circ$$

Figure 4: The variations of β , m_t , m , $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T(y=\beta) - T^*$ for $\phi=27.51$, when $r=1$, $e=e_w=.5$, and $\Delta=.414$.

$$\phi = 27.75^\circ$$

Figure 5: The variations of β , m_t , m , $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T(y=\beta) - T^*$ for $\phi=27.75^\circ$, when $r=1$, $e=e_w=.5$, and $\Delta=.414$.

$$\phi=28.0^\circ$$

Figure 6: The variations of β , m_t , $\bar{m}$, $\bar{T}$, $\bar{v}$, E , $\bar{u}$, v , and $u(y=\beta)$ with $T(y=\beta) - T^*$ for $\phi=28.0^\circ$, when $r=1$, $e=e_w=.5$, and $\Delta=.414$.