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METHODS AND CHARTS FOR COMPUTING STABILITY DERIVATIVES OF A V-BOTTOM PLANING SURFACE

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METHODS AND CHARTS FOR COMPUTING STABILITY DERIVATIVES OF A V-BOTTOM PLANING SURFACE

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SUMMARY

Methods and charts are presented for computing stability derivatives of a longitudinally straight V-bottom planing surface representing the forebody of a seaplane float or a flying-boat hull without chine flare. The charts for computing hydrodynamic derivatives were used in calculating the trim limit of stability for angles of dead rise of 10°, 20°, and 30°. The lower trim limit of stability of a seaplane planing on the forebody was calculated from measurements of lift, resistance, trimming moment, and wetted length of planing surfaces and was found to be in good agreement with experimental values of the trim limit of stability.

The velocity derivatives Z_w and M_w were computed as a function of the draft, while the component due to the effect of vertical velocity on the trim was neglected. A comparison of the measured results with the calculated results indicated that the trim component was of minor importance and that better accuracy was obtained by neglecting it in the present calculations.

INTRODUCTION

The methods that are conventionally used in the analysis of aerodynamic stability have been successfully employed in investigations of the stability of a seaplane in the planing condition. (See references 1 and 2.) The determination of both aerodynamic and hydrodynamic stability derivatives has been rather difficult. In particular, the evaluation of the hydrodynamic components of the velocity derivatives (variations of lift and trimming moment with both linear and angular velocity) has involved assumptions of doubtful validity. These methods have given results that agree qualitatively with the results of investigations of the low-angle type of porpoising in which the afterbody is not involved. Applications of the methods to investigations of the high-angle type of porpoising appear more difficult and are not considered in the present report.

The purpose of the present investigation was to compare values of the hydrodynamic derivatives as calculated from general test data with the values measured from records of the disturbed motions of a planing body and to develop a procedure for evaluating the derivatives more accurately than heretofore from general test data on planing surfaces. The four derivatives Z_z, Z_w, M_θ, and M_q were measured from the damped oscillations of planing bodies. The derivative Z_w was calculated and compared with the values obtained experimentally. Of the four remaining hydrodynamic derivatives, Z_θ and M_z were obtained directly from general test data; Z_q and M_w were measured but not with sufficient accuracy for comparison with the calculations. As a check on the overall accuracy of the methods and data, the lower trim limit of stability was computed for three different angles of dead rise and was compared with the results of tests of dynamic models.

SYMBOLS

The coefficients used in the present report are defined as follows:

C_{L_p} planing lift coefficient $\left(\dfrac{L_p}{\frac{1}{2}\rho u^2 b^2}\right)$

C_{R_p} planing resistance coefficient $\left(\dfrac{R_p}{\frac{1}{2}\rho u^2 b^2}\right)$

C_p center-of-pressure coefficient $(s/W.L.)$

C_d draft coefficient (d/b)

C_V speed coefficient $(V/\sqrt{gb})$

C_Δ load coefficient (Δ/wb^3)

C_{Δ_o} initial load coefficient (Δ_o/wb^3)

where

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L_p lift, pounds

ρ density of water, slugs per cubic foot (1.97 slugs/cu ft for water in NACA tank No. 1)

u horizontal velocity, feet per second

b beam of hull, feet

R_p resistance, pounds

s distance from trailing edge to center of pressure, feet

W.L. wetted length from trailing edge measured along keel, feet

d draft, feet

V speed, feet per second

g acceleration of gravity, feet per second per second

w specific weight of water, pounds per cubic foot (63.5 lb/cu ft for water in NACA tank No. 1)

Δ load, pounds

Δ_o initial load, pounds

Other symbols used in this report are defined as follows:

k_1 horizontal distance of center of pressure forward of center of gravity

k_2 vertical distance of center of pressure below center of gravity

I moment of inertia of seaplane about transverse axis through center of gravity, slug-feet^2

m mass of seaplane, slugs

Z force along OZ-axis, equal to lift but opposite in sign

M trimming moment about transverse axis through center of gravity, positive when tending to raise bow, pound-feet

P period of oscillation of planing surface after disturbance, seconds

q angular velocity about center of gravity, radians per second

r distance of perpendicular from center of gravity to keel forward of trailing edge, feet

t time, seconds

V_R resultant velocity of forebody due to small vertical velocity w impressed upon horizontal velocity u

w vertical velocity, feet per second

z vertical displacement from a condition of steady planing, positive downward, feet

z_o distance of pivot above water level

$z_1 \ldots z_n$ maximums on curve of disturbed motion of planing surface

τ trim, radians

θ angle of rotation about center of gravity, radians

δ logarithmic decrement $\left(\log \frac{z_1}{z_2}\right)$

$$Z_z = \frac{1}{m}\frac{\partial Z}{\partial z}$$

$$Z_w = \frac{1}{m}\frac{\partial Z}{\partial w}$$

$$Z_\theta = \frac{1}{m}\frac{\partial Z}{\partial \theta}$$

$$Z_q = \frac{1}{m}\frac{\partial Z}{\partial q}$$

$$M_z = \frac{1}{I}\frac{\partial M}{\partial z}$$

$$M_w = \frac{1}{I}\frac{\partial M}{\partial w}$$

$$M_\theta = \frac{1}{I}\frac{\partial M}{\partial \theta}$$

$$M_q = \frac{1}{I}\frac{\partial M}{\partial q}$$

CALCULATION OF DERIVATIVES

Equations of Motion

Perring and Glauert (reference 1) demonstrated that the fore-and-aft degree of freedom may be neglected in an analysis of porpoising and that the motion may be treated as one having freedom in only trim and rise. In the conventional methods of stability analysis, the oscillations are assumed to be small. Maclaurin's series is used to obtain expressions for the force and the moment and all derivatives of order higher than the first are neglected. The seaplane is assumed to be traveling at a constant forward speed under conditions of static equilibrium. An arbitrary disturbance of small amplitude is assumed and the equations of motion that describe the resulting small oscillations may be investigated either to determine the actual motions or to determine whether the motions diverge in amplitude or converge to zero. If the axes move with the seaplane and are taken with the origin at the initial position of the center of gravity, with OX forward and always parallel to the water surface and with OZ vertical and positive downward (fig. 1), the equations of motion following a disturbance are

$$m\frac{d^2z}{dt^2} = z\frac{\partial Z}{\partial z} + w\frac{\partial Z}{\partial w} + \quad \frac{\partial Z}{\partial \theta} + q\frac{\partial Z}{\partial q} \tag{1}$$

$$I \frac{d^2\theta}{dt^2} = z \frac{\partial M}{\partial z} + w \frac{\partial M}{\partial w} + \theta \frac{\partial M}{\partial \theta} + q \frac{\partial M}{\partial q} \qquad (2)$$

The division of equation (1) by m and of equation (2) by I and the use of the notations

$$Z_z = \frac{1}{m}\frac{\partial Z}{\partial z}; \quad M_z = \frac{1}{I}\frac{\partial M}{\partial z}; \text{ and so forth}$$

give the equations for the acceleration

$$\frac{d^2 z}{dt^2} = zZ_z + wZ_w + \theta Z_\theta + qZ_q \qquad (3)$$

$$\frac{d^2\theta}{dt^2} = zM_z + wM_w + \theta M_\theta + qM_q \qquad (4)$$

Basic Data for Computing Derivatives

Data from reference 3 have been plotted in a convenient form that neglects the effect of Froude's number on lift, resistance, and the position of the center of pressure. Lift coefficient, resistance coefficient, and center-of-pressure coefficient are plotted against draft coefficient; $\frac{\text{lift coefficient}}{\text{draft coefficient}^2}$, $\frac{\text{resistance coefficient}}{\text{draft coefficient}^2}$, and center-of-pressure coefficient are plotted against trim. These plots are given in figures 2 to 19 as faired curves for angles of dead rise of 10°, 20°, and 30°. Values for the draft were computed from measurements of wetted lengths.

Displacement Derivatives

Derivative Z_z.- The derivative Z_z may be obtained from a plot of C_{L_p} as a function of C_d with τ as the parameter. The force Z is equal to the lift L_p but opposite in sign and, at constant τ and u, $dz = dd$. Hence, for unit mass,

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$$Z_z = \frac{1}{m}\frac{\partial Z}{\partial z} = -\frac{1}{m}\frac{\partial L_p}{\partial d} = -\frac{1}{m}\frac{\partial C_{L_p}}{\partial C_d}\frac{\frac{1}{2}\rho u^2 b^2}{b}$$

and Z_z may be evaluated if the mass moving vertically is substituted in the equation and the slope $\dfrac{\partial C_{L_p}}{\partial C_d}$ is obtained from figure 2, 3, or 4.

<u>Derivative Z_θ</u>.- The derivative Z_θ is proportional to the rate of change of the force Z with change in θ. A change in θ implies a change in draft as well as in trim τ; hence,

$$Z_\theta = \frac{1}{m}\frac{\partial Z}{\partial \theta} = \frac{1}{m}\left(\frac{\partial Z}{\partial d}\frac{dd}{d\theta} + \frac{\partial Z}{\partial \tau}\frac{d\tau}{d\theta}\right)$$

and, because $\dfrac{d\tau}{d\theta} = 1$,

$$Z_\theta = Z_z\frac{dd}{d\theta} - \frac{1}{m}\frac{\partial C_{L_p}}{\partial \tau}\frac{1}{2}\rho u^2 b^2$$

In order to obtain $\dfrac{dd}{d\theta}$, which equals $\dfrac{dz}{d\theta}$, the equation relating d and θ is differentiated. From figure 1, if z_o is the distance of the pivot above the water level,

$$d = r \sin \theta + p \cos \theta - z_o$$

where r is the distance of the pivot forward of the trailing edge and p is the distance of the pivot above the keel. Then,

$$\frac{dd}{d\theta} = r \cos \theta - p \sin \theta$$

and, for small angles,

$$\frac{\partial d}{\partial \theta} = r - p\theta$$

The slope $\frac{\partial C_{L_p}}{\partial \tau} \frac{1}{{C_d}^2}$ may be obtained from figure 5, 6, or 7. Figures 5, 6, and 7 were obtained by cross-plotting figures 2, 3, and 4, respectively.

Trimming moments may be plotted directly as a function of draft or trim to obtain M_z or M_θ, but separate plots are required for each velocity to be investigated. If the definitions of planing coefficient, resistance coefficient, and center-of-pressure coefficient are used and if the effect of Froude's law is neglected, these moment derivatives may be obtained at any speed from a plot of C_p as a function of C_d and τ.

Derivative M_z.- The expression for trimming moment is

$$M = L_p k_1 - R_p k_2$$

where k_1 is the horizontal distance of the center of pressure forward of the center of gravity and k_2 is the vertical distance of the center of pressure below the center of gravity. In order to obtain M_z, this expression is differentiated with respect to d, with τ held constant:

$$M_z = \frac{1}{I} \frac{\partial M}{\partial d} = \frac{1}{I} \left(L_p \frac{\partial k_1}{\partial d} + k_1 \frac{\partial L_p}{\partial d} - R_p \frac{\partial k_2}{\partial d} - k_2 \frac{\partial R_p}{\partial d} \right)$$

From figure 1,

$$k_1 = (s - r) \cos \tau + p \sin \tau$$

and

$$k_2 = p \cos \tau - (s - r) \sin \tau$$

By differentiation,

$$\frac{\partial k_1}{\partial d} = \frac{C_p}{\tau} + \frac{d}{\tau}\frac{\partial C_p}{\partial d}$$

and

$$\frac{\partial k_2}{\partial d} = -C_p - d\frac{\partial C_p}{\partial d}$$

The partial derivative $\frac{\partial R_p}{\partial d}$ may be evaluated by measuring the slopes of the curves in figures 8 to 10; likewise, $\frac{\partial C_p}{\partial d}$ is evaluated as a slope in figures 14 to 16.

<u>Derivative M_θ</u>.- The derivative M_θ is analagous to Z_θ and may be evaluated as the sum of two variables:

$$M_\theta = \frac{1}{I}\frac{\partial M}{\partial \theta} = \frac{1}{I}\left(\frac{\partial M}{\partial d}\frac{\partial d}{\partial \theta} + \frac{\partial M}{\partial \tau}\frac{d\tau}{d\theta}\right)$$

where

$$\frac{\partial M}{\partial \tau} = L_p\frac{\partial k_1}{\partial \tau} + k_1\frac{\partial L_p}{\partial \tau} - R_p\frac{\partial k_2}{\partial \tau} - k_2\frac{\partial R_p}{\partial \tau}$$

Differentiating k_1 and k_2 with respect to τ gives

$$\frac{\partial k_1}{\partial \tau} = -\frac{dC_p}{\tau^2} + \frac{d}{\tau}\frac{\partial C_p}{\partial \tau} + r\tau + p$$

$$\frac{\partial k_2}{\partial \tau} = -p\tau - d\frac{\partial C_p}{\partial \tau} + r$$

The partial derivative $\frac{\partial R_p}{\partial \tau}$ may be evaluated from figures 11 to 13, and $\frac{\partial C_p}{\partial \tau}$ is obtained from figures 17 to 19.

$$\frac{d\tau}{dw} = \frac{1}{u}$$

For unit mass and unit moment of inertia, the derivatives may be expressed in the form

$$Z_w = \frac{W.L.}{u} Z_z + \frac{1}{u} Z_\theta$$

$$M_w = \frac{W.L.}{u} M_z + \frac{1}{u} M_\theta$$

As will be shown in the following paragraphs, experimental data indicate that the components of Z_w and M_w which contain Z_θ and M_θ are of minor importance and for the present purpose may be neglected.

<u>Derivatives Z_q and M_q</u>.- In order to evaluate Z_q and M_q, it is assumed that an angular velocity q may be considered to have the same effect upon the planing surface as the sum of the effects due to a vertical and a horizontal velocity - that is,

$$Z_q = \frac{1}{m}\frac{\partial Z}{\partial q} = \frac{1}{m}\left(\frac{\partial Z}{\partial u}\frac{du}{dq} + \frac{\partial Z}{\partial w}\frac{dw}{dq}\right)$$

$$M_q = \frac{1}{I}\frac{\partial M}{\partial q} = \frac{1}{I}\left(\frac{\partial M}{\partial u}\frac{du}{dq} + \frac{\partial M}{\partial w}\frac{dw}{dq}\right)$$

Because

$$\frac{du}{dq} = k_2$$

and

$$\frac{dw}{dq} = -k_1$$

the two derivatives can be evaluated if $\frac{\partial Z}{\partial u}$ and $\frac{\partial M}{\partial u}$ are determined. By the use of

$$L_p = C_{L_p} \frac{1}{2} \rho u^2 b^2$$

$$\frac{\partial L_p}{\partial u} = C_{L_p} \rho u b^2 = \frac{2L_p}{u}$$

In order to obtain $\frac{\partial M}{\partial u}$, slopes are taken from a plot of moment as a function of speed at constant load. In the expression

$$M = k_1 L_p - k_2 R_p$$

L_p is known, k_1 and k_2 have already been calculated, and R_p may be found by the use of figures 8 to 10 because C_d is known.

EVALUATION OF ROUTH'S DISCRIMINANT AND DETERMINATION OF CRITICAL TRIM

If the hydrodynamic and aerodynamic derivatives have been found for a given load, speed, and trim, the derivatives are substituted in the equations of motion. Routh's well-known criterions for the motion to be stable are that B, C, D, E, and R all be greater than zero, where

$$B = -(Z_w + M_q)$$

$$C = -(M_\theta + Z_z - M_q Z_w + M_w Z_q)$$

$$D = Z_z M_q - Z_q M_z + Z_w M_\theta - Z_\theta M_w$$

$$E = Z_z M_\theta - Z_\theta M_z$$

and

$$R = BCD - D^2 - B^2 E$$

The lower limit of stability may be found when B, C, D, E, and R have been evaluated for a number of angles.

EXPERIMENTAL EVALUATION OF DERIVATIVES

The hydrodynamic derivatives Z_z, Z_w, M_θ, and M_q were measured by the "free-oscillation method" that is sometimes used in measuring aerodynamic derivatives. A model composed of a planing surface and a tail plane (fig. 21), which represent the forebody and the horizontal tail of a seaplane (described in reference 4), was first towed free to move vertically and locked in trim and was then towed free to trim but locked in draft. In each case, the model was disturbed momentarily and a time history of the subsequent oscillations was obtained. If the oscillations are assumed to be those of a damped linear oscillator, the equation of motion with the model locked in trim is

$$m \frac{d^2 z}{dt^2} = Z_w \frac{dz}{dt} + Z_z z$$

where

Z_w damping factor $\left(2m \frac{\delta}{P}\right)$

Z_z displacement factor $\left[m \frac{(4\pi^2 + \delta^2)}{P^2}\right]$

and

P period of one oscillation

δ logarithmic decrement $\left(\log \frac{z_1}{z_2}\right)$

z_1, z_2 two consecutive maximums on curve

A typical trace of the damped motion in draft with locked trim is given in figure 22. Analagous equations apply when the model is free to trim but locked in draft.

$$I \frac{d^2\theta}{dt^2} = M_q \frac{d\theta}{dt} + M_\theta \theta$$

where

$$M_q = \frac{2I\delta}{P}$$

and

$$M_\theta = I\,\frac{(4\pi^2 + \delta^2)}{P^2}$$

A limited number of records were taken to obtain these four derivatives and the results are compared with the corresponding calculated values of the derivatives in tables I and II. The results given in table II show good agreement between the measured values of Z_w and the value calculated by the formula

$$Z_w = \frac{1}{m}\,\frac{W.L.}{u}\,Z_z$$

The agreement is better and the calculations are simpler than if the term $\frac{1}{m}\,\frac{1}{u}\,Z_\epsilon$ is added.

LOWER TRIM LIMIT OF STABILITY

The methods and charts for computing the derivatives were applied to the specific problem of calculating the lower trim limit of stability for angles of dead rise of 10°, 20°, and 30°. The calculations were generally similar to those of reference 4 but were more extensive and employed the method of calculating the velocity derivatives previously described herein. The calculations were made for a model with a horizontal tail plane and without a wing. The dimensions, the mass, the moment of inertia, and other characteristics assumed for the model were the same as those of reference 4 and were as follows:

Mass, slugs	5.05
Moment of inertia, slug-feet^2	5.2
Beam, feet	1.33
Center of gravity, fraction beam above keel	1.26
Center of gravity, fraction beam forward of T.E.. .	.38
Tail area, square feet.	3.47
Aspect ratio of tail plane	3.4
Tail arm (measured from c.g. to quarter-chord point), feet	3.93
Elevator area, square inches	492
Elevator chord, percent total chord	48

Table III presents the results of the calculations for one set of conditions: namely, angle of dead rise of 30°, speed of 40 feet per second, and load of 60 pounds. The critical trim - that is, the trim at which the calculations indicate a transition from stability to instability, was determined at two speeds for four loads and for two angles of dead rise, 20° and 30°. The calculated values are plotted in figure 23. Graphs showing all the hydrodynamic derivatives used in these computations are available, on request, from the National Advisory Committee for Aeronautics. The faired curves of figure 23 are from reference 4 and were determined experimentally. The calculated values agree with the experimental values within the probable limits of experimental error, except at a speed of 30 feet per second and a load of 100 pounds. The discrepancy for this one point may be due to the fact that the positions of the center of pressure were obtained by extrapolating the data in reference 3 from 80 pounds to 100 pounds. Also, the assumption that Froude's law of comparison may be neglected probably introduces significant errors at the low speeds and heavy loads.

CONCLUDING REMARKS

Methods and charts are presented herein for computing the stability derivatives of a longitudinally straight V-bottom planing surface representing the forebody of a seaplane float or a flying-boat hull without chine flare. The methods are believed to be generally applicable in computing stability derivatives of a single planing surface and the charts for angles of dead rise of 10°, 20°, and 30° are believed to be satisfactory approximations for calculations of the lower trim limit of stability when the straight portion of the forebody forward of the step is the only planing area involved. In particular, the results indicate that the velocity derivatives may be calculated satisfactorily as functions of the draft with trim constant while the component due to the effect of vertical velocity on the trim is neglected.

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TABLE I

COMPARISON OF MEASURED VALUES OF DERIVATIVES Z_z AND Z_w

WITH CALCULATED VALUES

[Model without tail; angle of dead rise, 30°]

Speed (fps)	Load (lb)	Trim (deg)	Z_z Measured	Z_z Calculated	Z_w Measured	Z_w Calculated $C_1(Z_\tau)_{W.L.}$ (1)	Z_w Calculated $C_2(Z_\tau)_d$ (2)	Z_w Calculated $C_3(Z_d)_\tau$ (3)
30	60	6	-101	-98	-8.0	-12.2	-3.4	-7.6
		8	-116	-108	-5.5	-7.1	-1.1	-5.6
		10	-117	-114	-4.3	-5.7	-1.7	-4.6
		12	-142	-130	-3.9	-4.8	-2.7	-4.0
30	100	6	----------	----------	----------	-18.6	-6.1	-13.3
		8	-137	-143	-10.6	-12.5	-1.9	-9.9
		10	-148	-146	-7.8	-9.5	-2.8	-7.6
		12	-154	-153	-5.7	-5.6	----------	----------
45	60	6	-150	-143	-4.5	-6.9	-2.3	-4.9
		8	-179	-175	-3.9	-5.0	-1.6	-4.0
		10	-178	-172	-2.5	-3.1	-.6	-3.2
45	100	6	-188	-183	-8.4	-13.4	-4.0	-8.0
		8	-215	-211	-7.1	-7.6	-2.7	-6.7
		10	-233	-230	-4.6	-6.2	-2.0	-5.3
		12	-249	-252	-5.1	-4.0	-1.7	-4.5

[1] $C_1(Z_\tau)_{W.L.}$ = Constant × Z_τ with wetted length constant (reference 1).

[2] $C_2(Z_\tau)_d = \frac{\partial Z}{\partial \theta}\frac{d\tau}{dw}$ with draft constant (reference 2).

[3] $C_3(Z_d)_\tau = \frac{\partial Z}{\partial z}\frac{dz}{dw}$ with trim constant.

TABLE II

COMPARISON OF MEASURED VALUES OF DERIVATIVES M_θ AND M_q

WITH CALCULATED VALUES

Speed (fps)	Load (lb)	Trim (deg)	M_θ Measured	M_θ Calculated	M_q Measured	M_q Calculated
Model without tail; angle of dead rise, 30°						
30	60	8.0	-16.0	-25.2	-2.0	-2.0
		11.8	-19.5	-19.8	-1.0	-1.3
		9.4	-16.2	-20.0	-1.5	-1.3
30	100	10.4	-25.7	-13.0	------	------
		------	------	------	-4.0	-1.6
45	60	10.8	-19.5	-19.8	------	------
Model with tail; angle of dead rise for measured M_θ and M_q, $22\frac{1}{2}°$; angle of dead rise for calculated M_θ and M_q, 20°						
30	60	8.2	------	------	-2.35	-2.2
		8.6	------	------	-2.0	-2.1
		7.4	------	------	-3.8	-2.3
30.3	60	8.2	-22.8	-30.0	------	------
		8.6	-25.0	-32.0	------	------
		7.4	-26.7	-30.0	------	------
30	100	10.0	------	------	-3.5	-2.3
		9.0	------	------	-2.6	-3.6
30.3	100	10.0	-23.0	-76.0	------	------
		9.0	-27.4	-60.0	------	------
40	60	7.0	-49.2	-42.0	-3.3	-2.4
		7.8	-38.7	-40.0	-3.7	-2.4
40	100	8.4	-42.9	-50.0	-3.0	-2.6

TABLE III

VALUES OF STABILITY DERIVATIVES AND COEFFICIENTS IN STABILITY EQUATION

FOR A PLANING SURFACE HAVING AN ANGLE OF DEAD RISE OF 30°

[Speed, 40 fps (C_V = 6.11); load, 60 lb (C_Δ = 0.40); beam, 1.33 ft; mass, 5.05 slugs (C_{Δ_o} = 1.08); moment of inertia, 5.2 slug-ft^2; c.g. = 1.26 beam above keel and 0.38 beam forward of T.E.; tail area, 3.47 sq ft; aspect ratio of tail plane, 3.4; tail arm, 3.93 ft]

Trim (deg)	Components of derivatives	Stability derivatives								Coefficients in stability equation				Routh's discriminant,
		Z_z	Z_w	Z_θ	Z_q	M_z	M_w	M_θ	M_q	B	C	D	E	p R
4	Hydrodynamic	-92.7	-7.38	-298	6.40	113	8.99	150.2	-9.45	19.1	-7.5	1930	21,500	-118.0 × 10^5
	Aerodynamic[a]	0	0	0	0	0	-.456	-18.2	-2.25					
	Total	-92.7	-7.38	-298	6.40	113	8.53	132	-11.7					
6	Hydrodynamic	-131	-5.71	-148	2.06	73.0	3.18	20.4	-2.01	9.97	147	798	10,500	-5.07
	Total	-131	-5.71	-148	2.06	73.0	2.72	2.2	-4.26					
8	Hydrodynamic	-142	-4.13	-70.6	.410	32.7	.952	-19.6	-.48	6.86	191	566	7,680	.60
	Total	-142	-4.13	-70.6	.410	32.7	.496	-37.8	-2.77					
10	Hydrodynamic	-156	-3.65	-86.5	.080	16.6	.388	-14.4	-.33	6.23	198	499	6,526	1.14
	Total	-156	-3.65	-86.5	.080	16.6	-.068	-32.6	-2.58					

[a]Aerodynamic components of derivatives are the same for all angles of trim and dead rise.

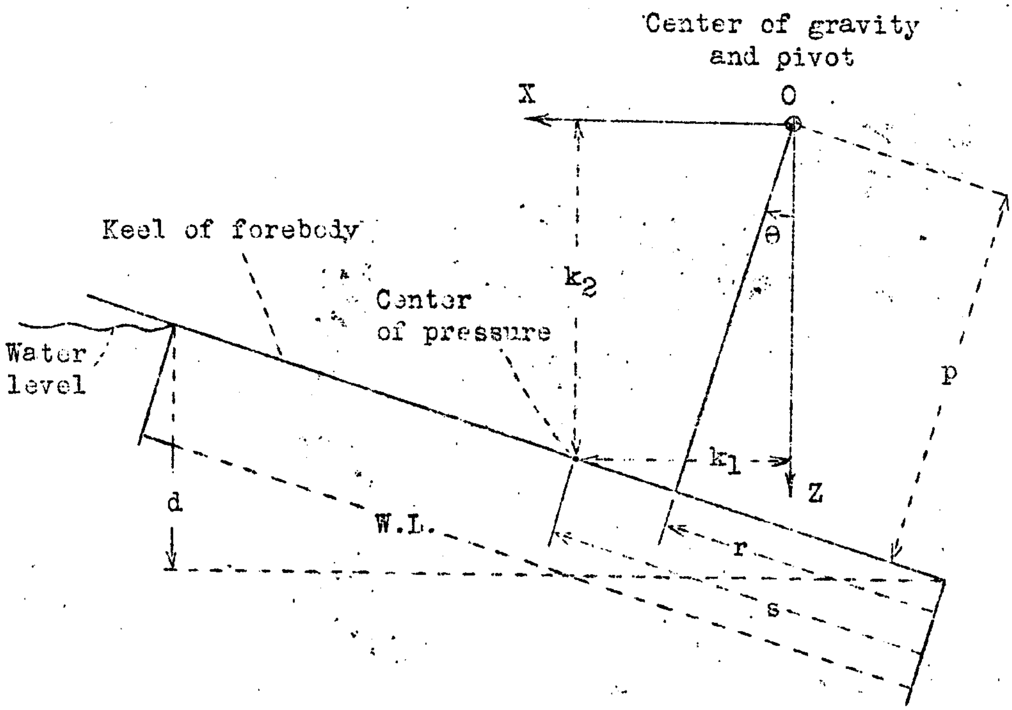

Figure 1.- Geometric relationships on a forebody.

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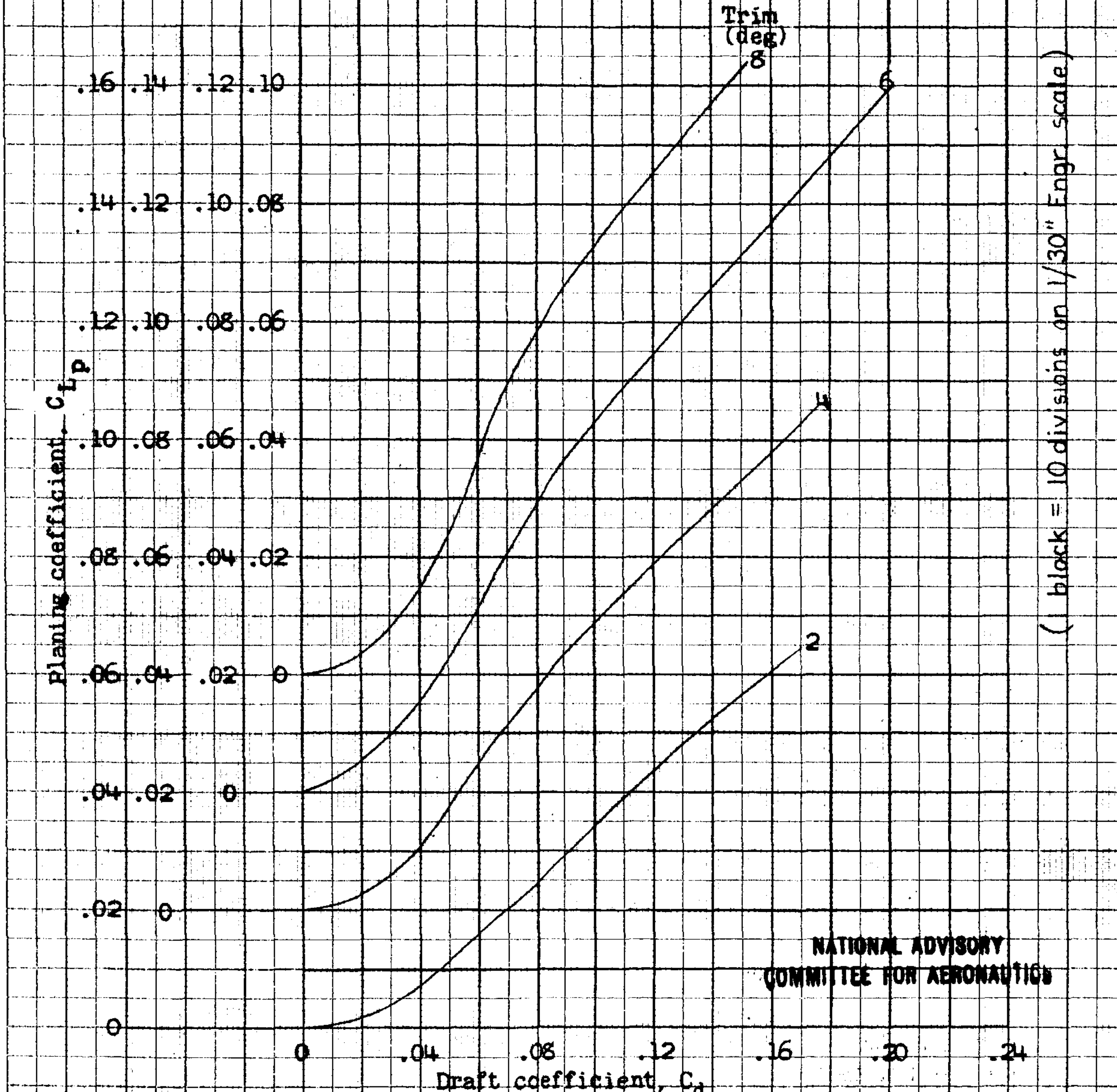

Figure 2.- Variation of lift coefficient with draft coefficient. Angle of dead rise, 10°. $C_{L_p} = \frac{L_p}{\frac{1}{2}\rho u^2 b^2}$; $C_d = \frac{d}{b}$.

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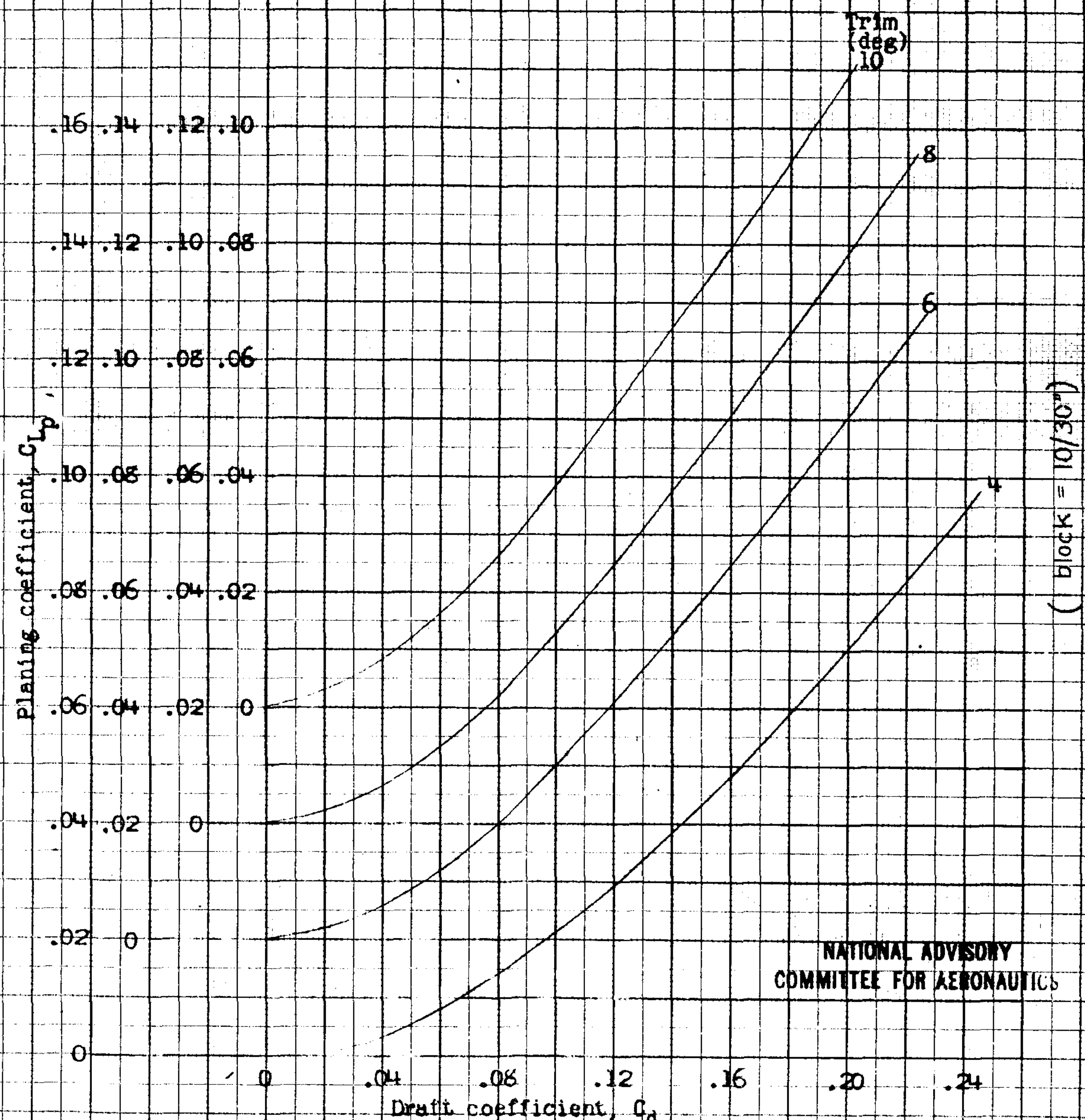

Figure 3.- Variation of lift coefficient with draft coefficient. Angle of dead rise, 20°. $C_{L_p} = \frac{L_p}{\frac{1}{2}\rho u^2 b^2}$; $C_d = \frac{d}{b}$.

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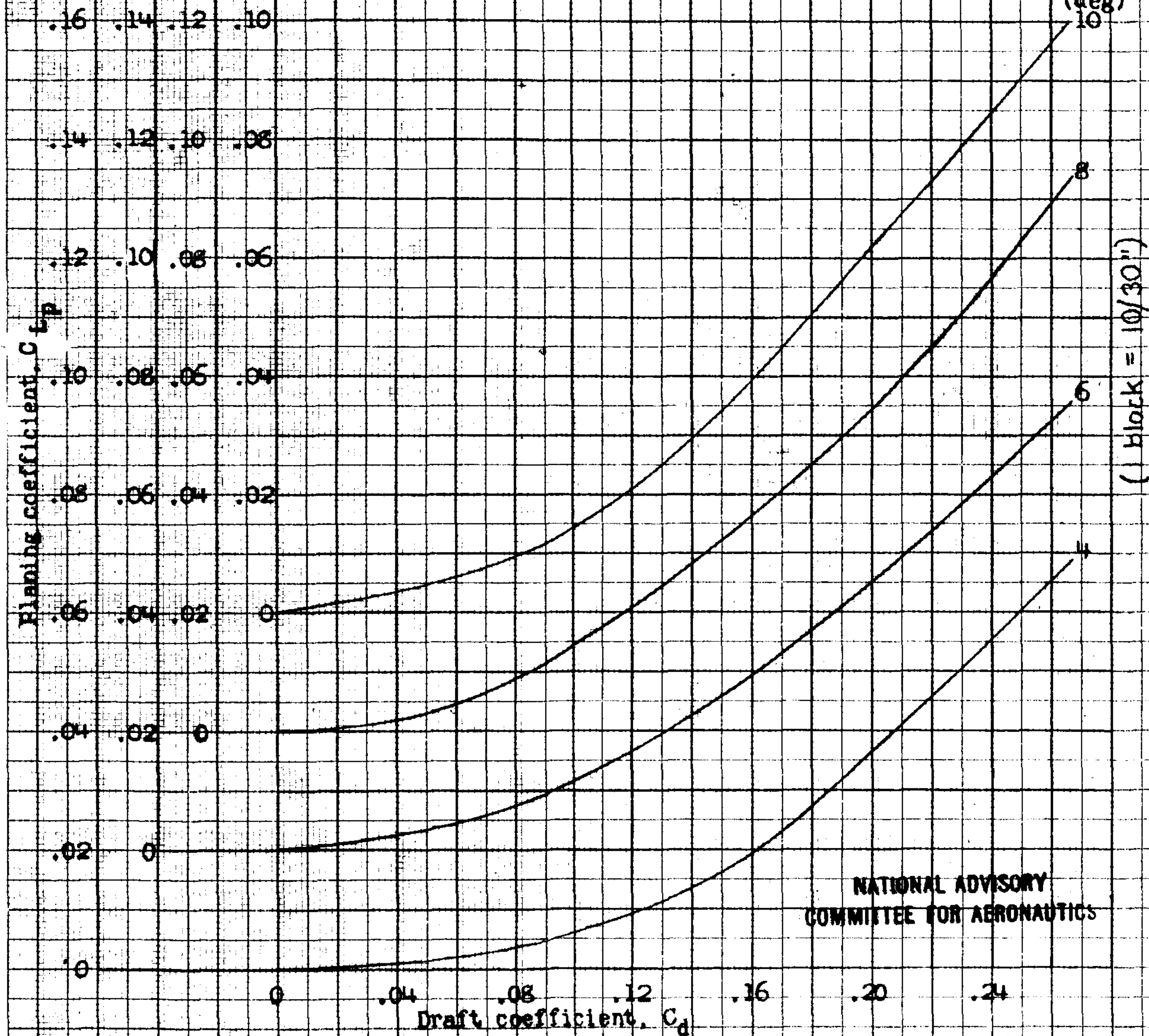

Figure 4.- Variation of lift coefficient with draft coefficient. Angle of dead rise, 30°. $C_{L_p} = \frac{L_p}{\frac{1}{2}\rho u^2 b^2}$; $C_d = \frac{d}{b}$.

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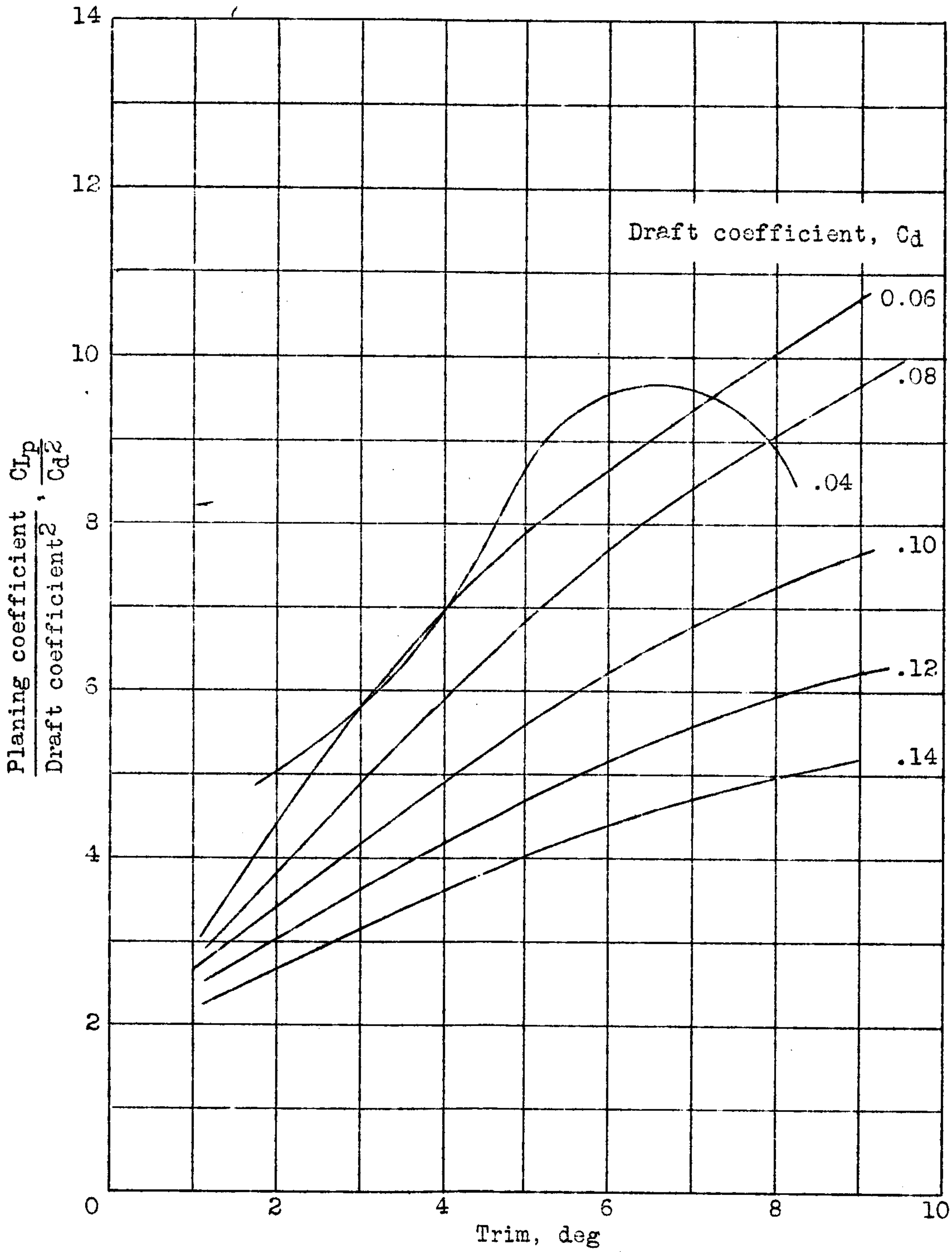

Figure 5.- Variation of $\frac{\text{planing coefficient}}{\text{draft coefficient}^2}$ with trim. Angle of dead rise, 10°.

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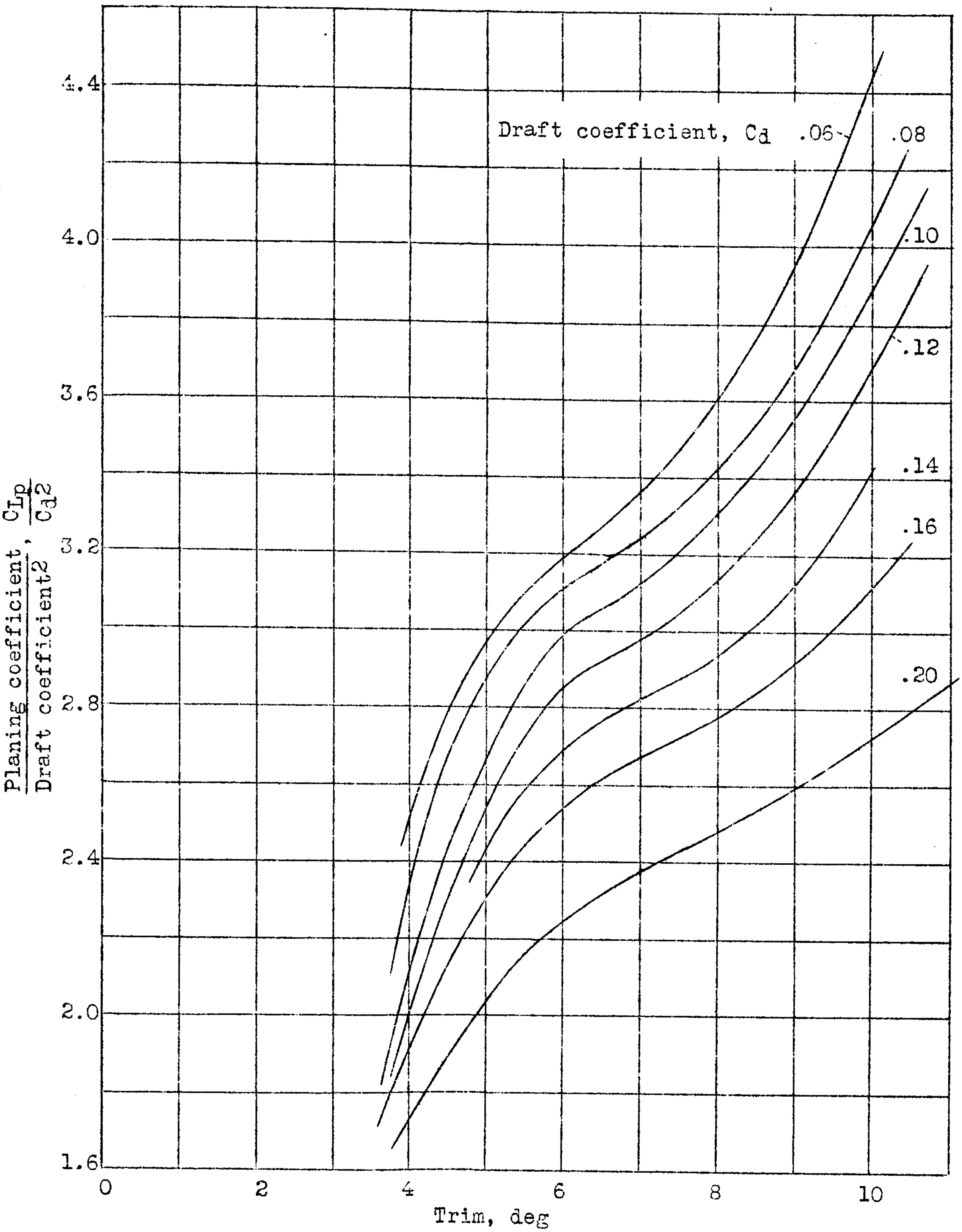

Figure 6.- Variation of $\frac{\text{planing coefficient}}{\text{draft coefficient}^2}$ with trim. Angle of dead rise, 20°.

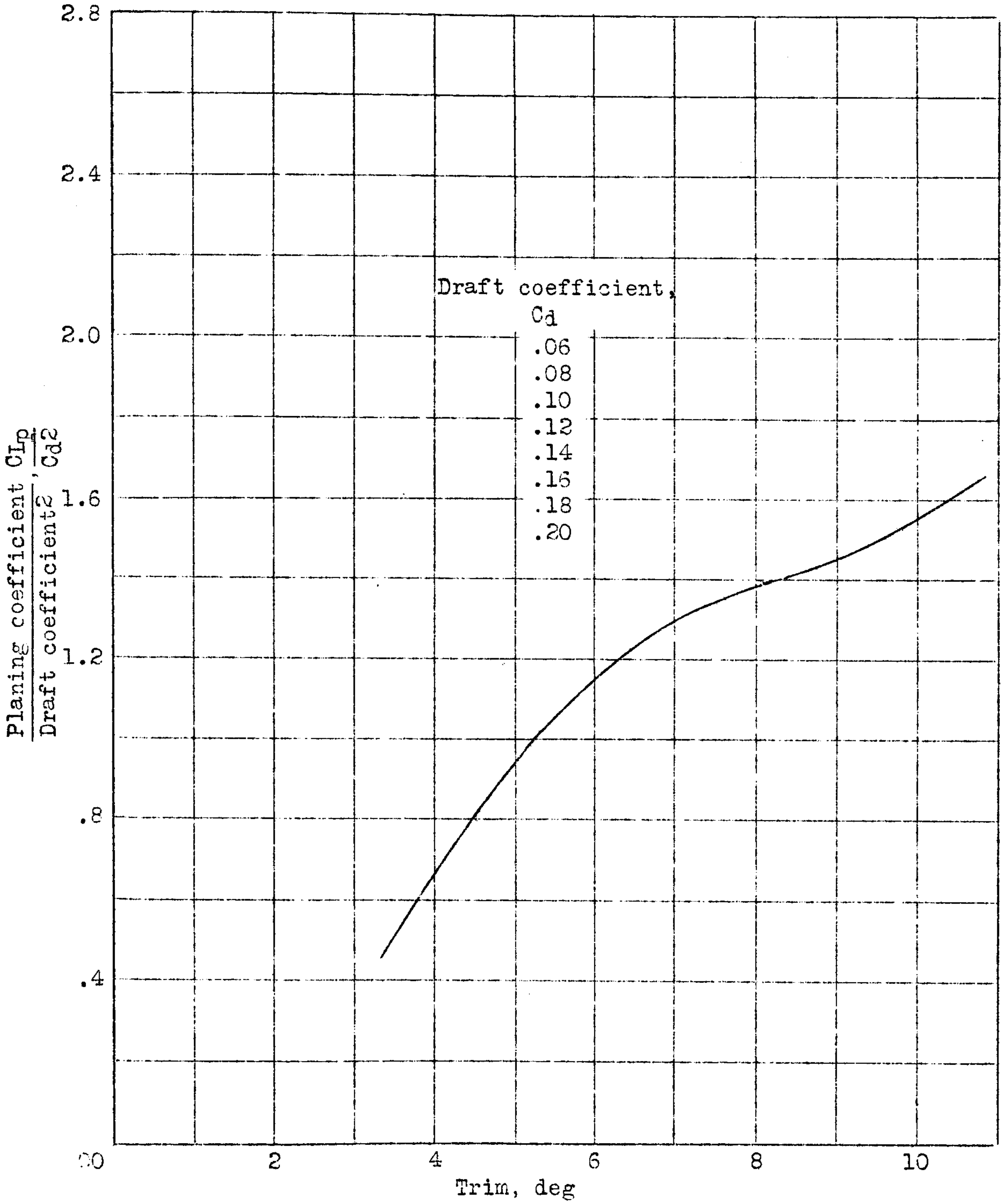

Figure 7.- Variation of $\frac{\text{planing coefficient}}{\text{draft coefficient}^2}$ with trim. One curve for all values of C_d. Angle of dead rise, 30°.

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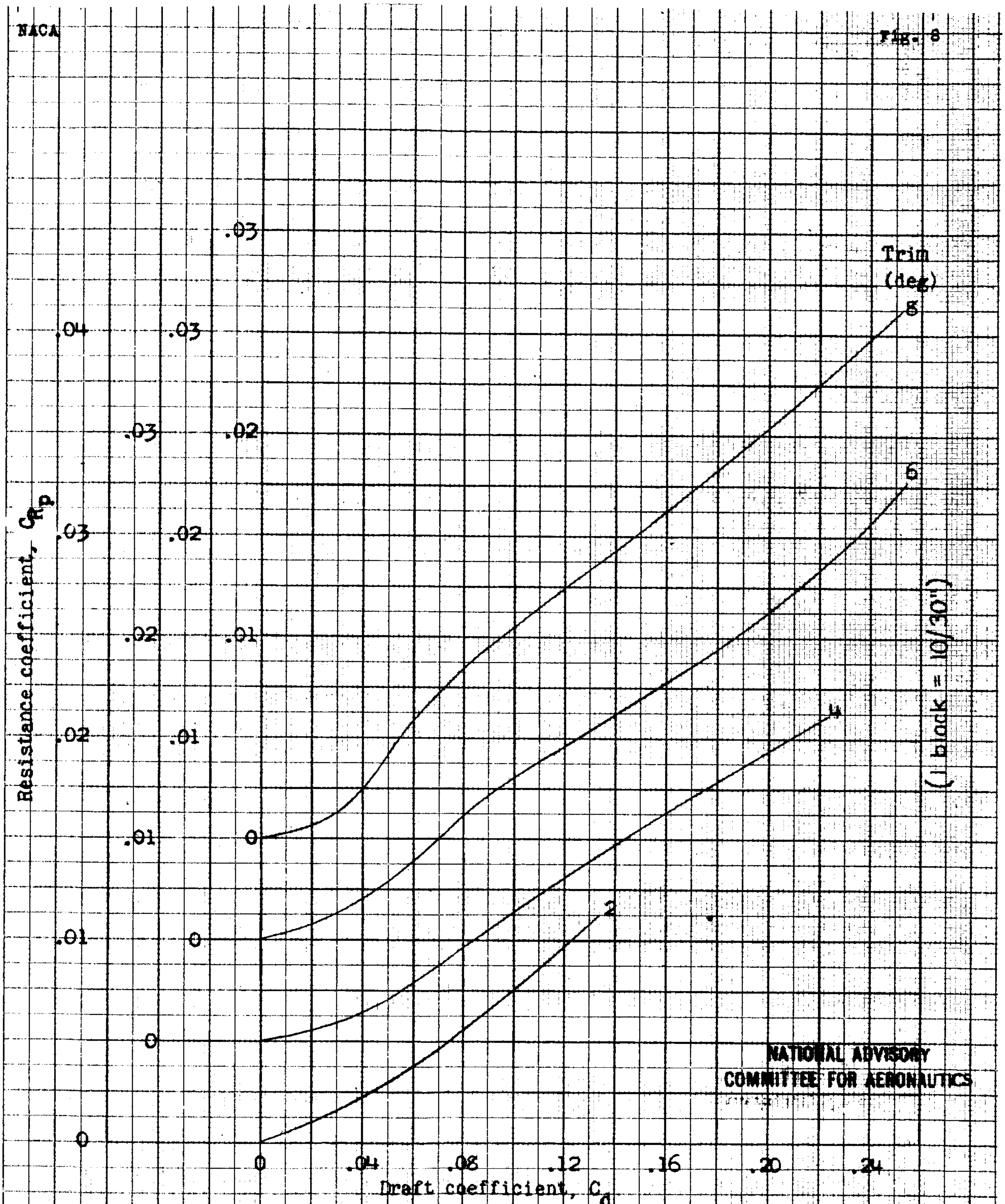

Figure 8.- Variation of resistance coefficient with draft coefficient. Angle of dead rise, 10°. $C_{R_p} = \frac{R_p}{\frac{1}{2}\rho u^2 b^2}$; $C_d = \frac{d}{b}$.

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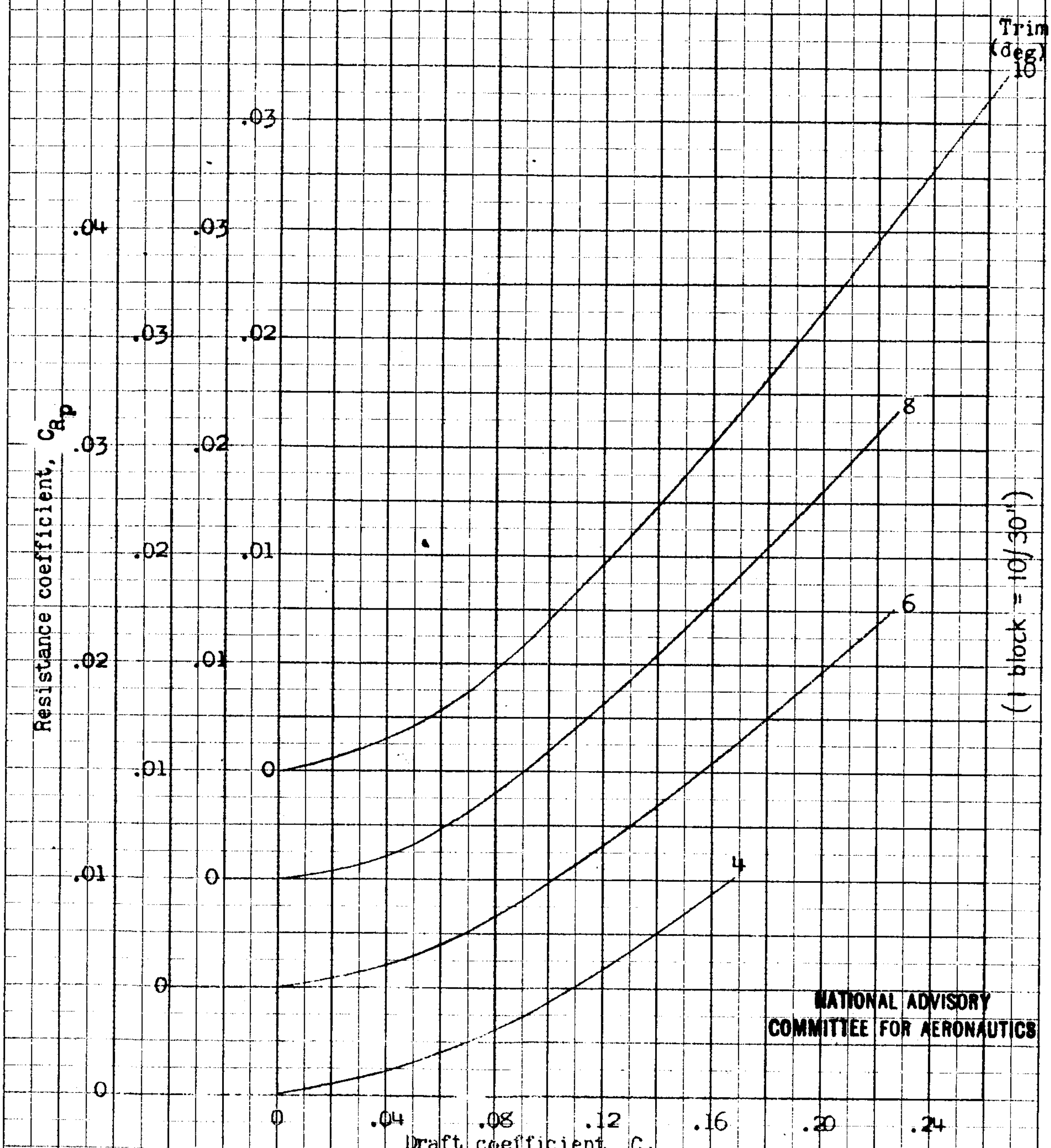

Figure 9.- Variation of resistance coefficient with draft coefficient. Angle of dead rise, 20°. $C_{R_p} = \frac{R_p}{\frac{1}{2}\rho u^2 b^2}$; $C_d = \frac{d}{b}$.

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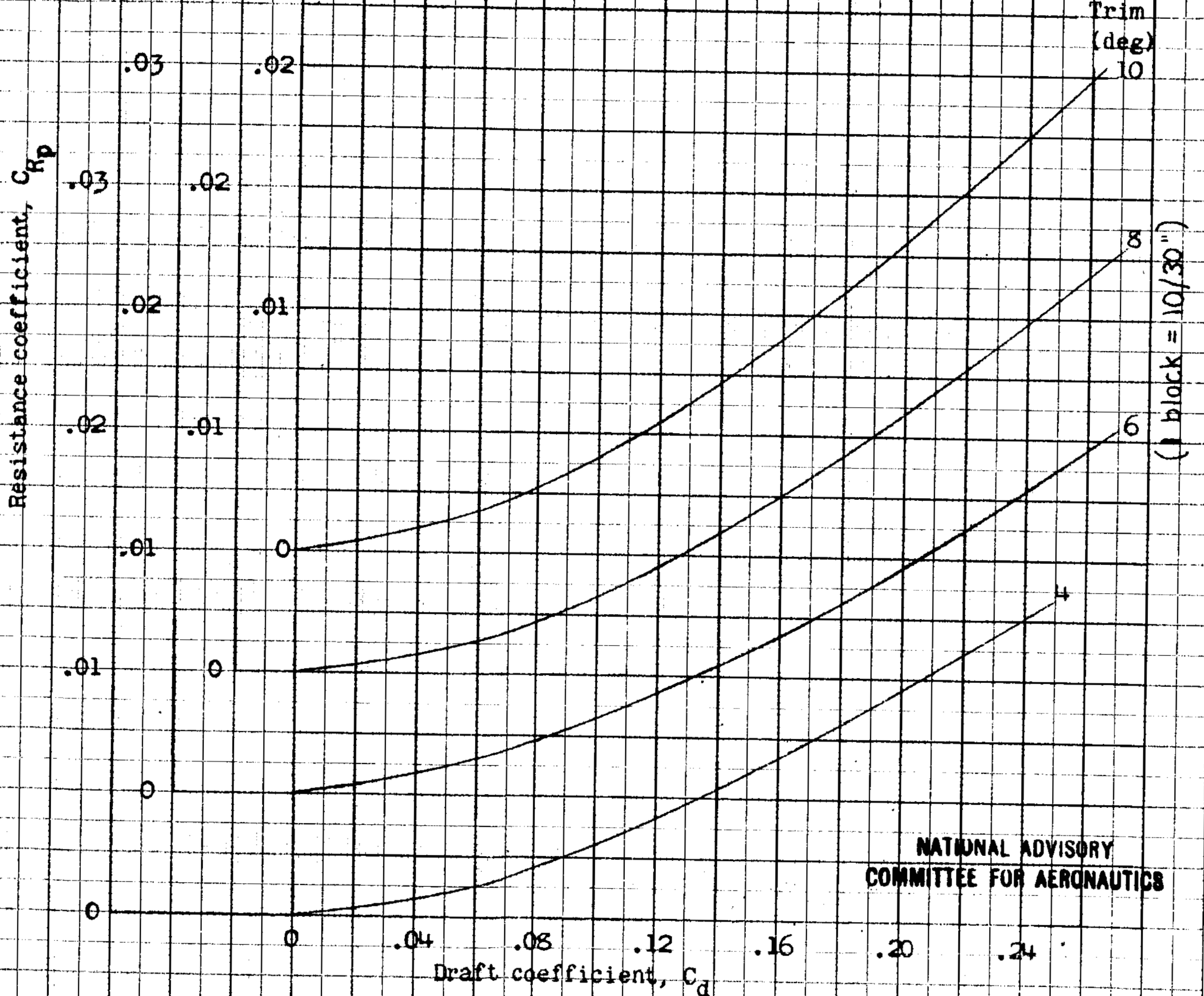

Figure 10.- Variation of resistance coefficient with draft coefficient. Angle of dead rise, 30°. $C_{R_p} = \frac{R_p}{\frac{1}{2}\rho u^2 b^2}$; $C_d = \frac{d}{b}$.

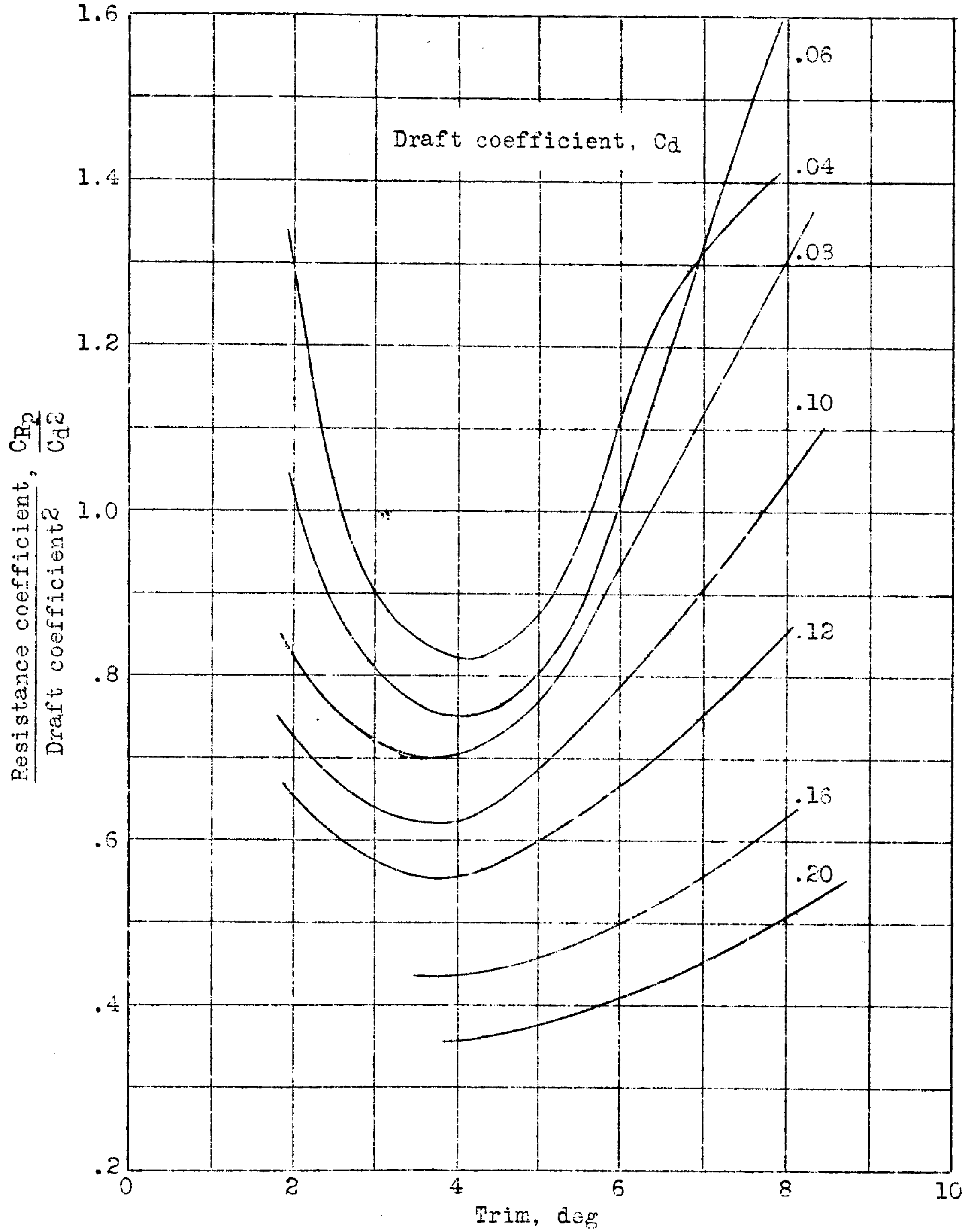

Figure 11.- Variation of $\frac{\text{resistance coefficient}}{\text{draft coefficient}^2}$ with trim. Angle of dead rise, 10°.

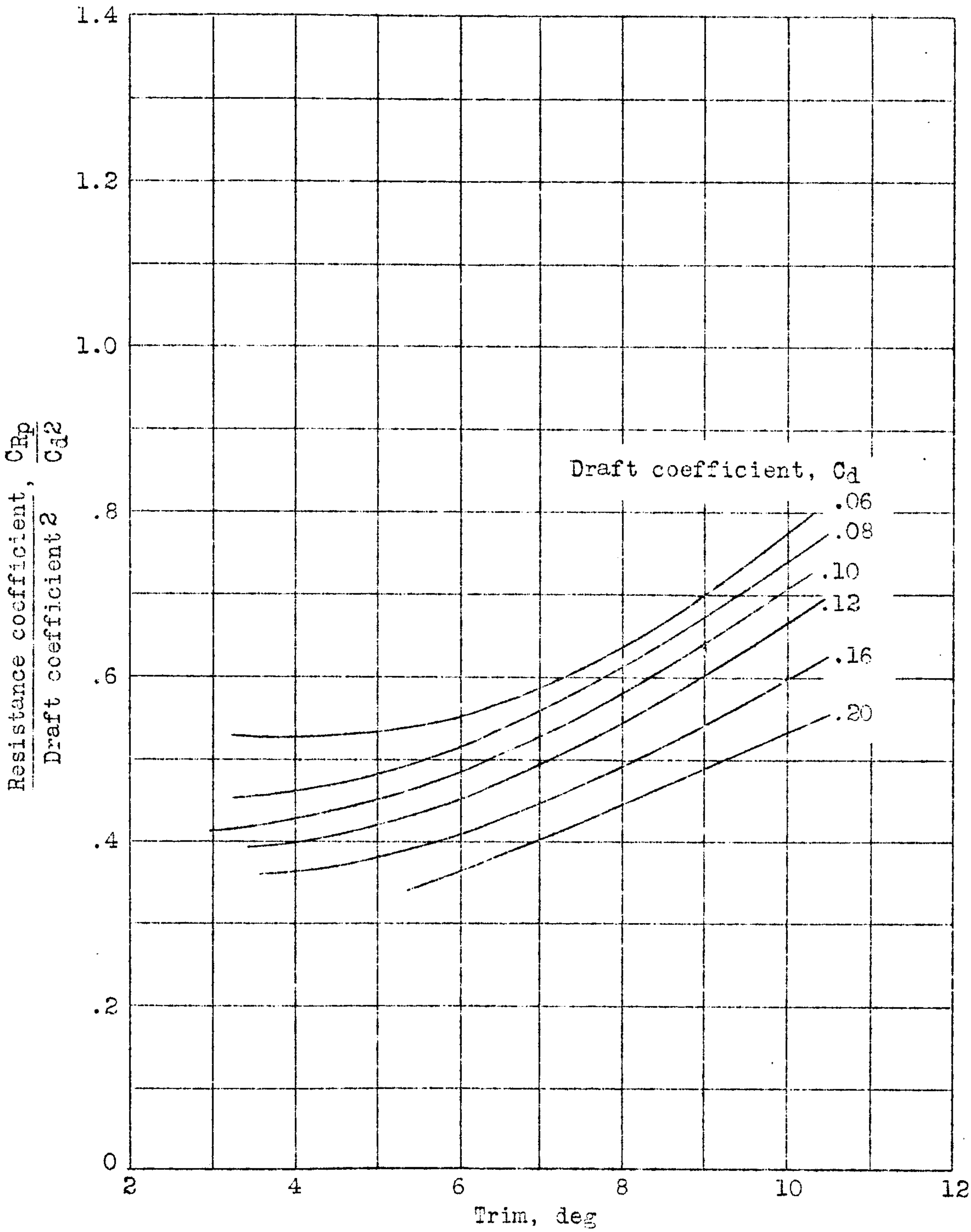

Figure 12.- Variation of $\frac{\text{resistance coefficient}}{\text{draft coefficient}^2}$ with trim. Angle of dead rise, 20°.

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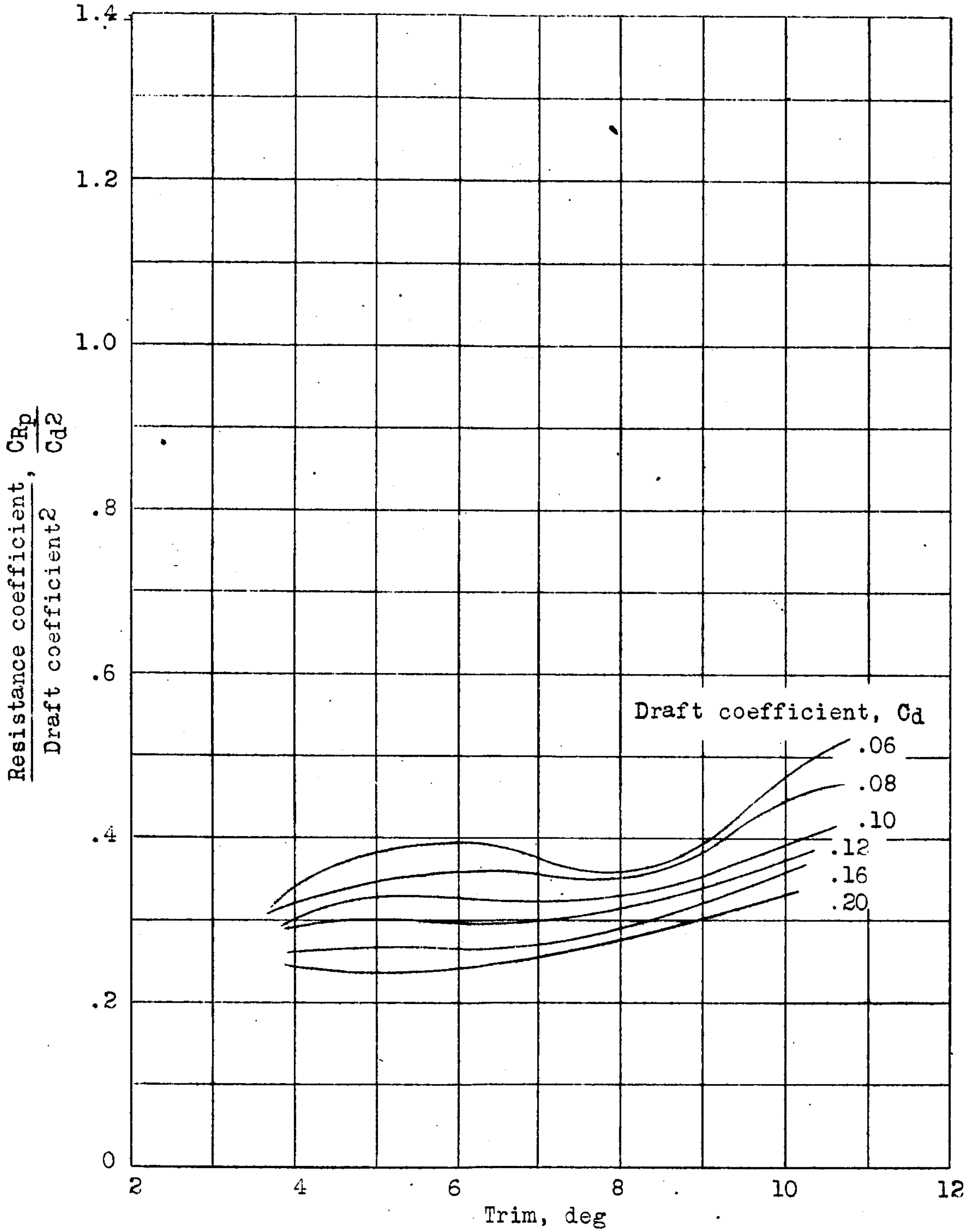

Figure 13.- Variation of $\frac{\text{resistance coefficient}}{\text{draft coefficient}^2}$ with trim. Angle of dead rise, 30°.

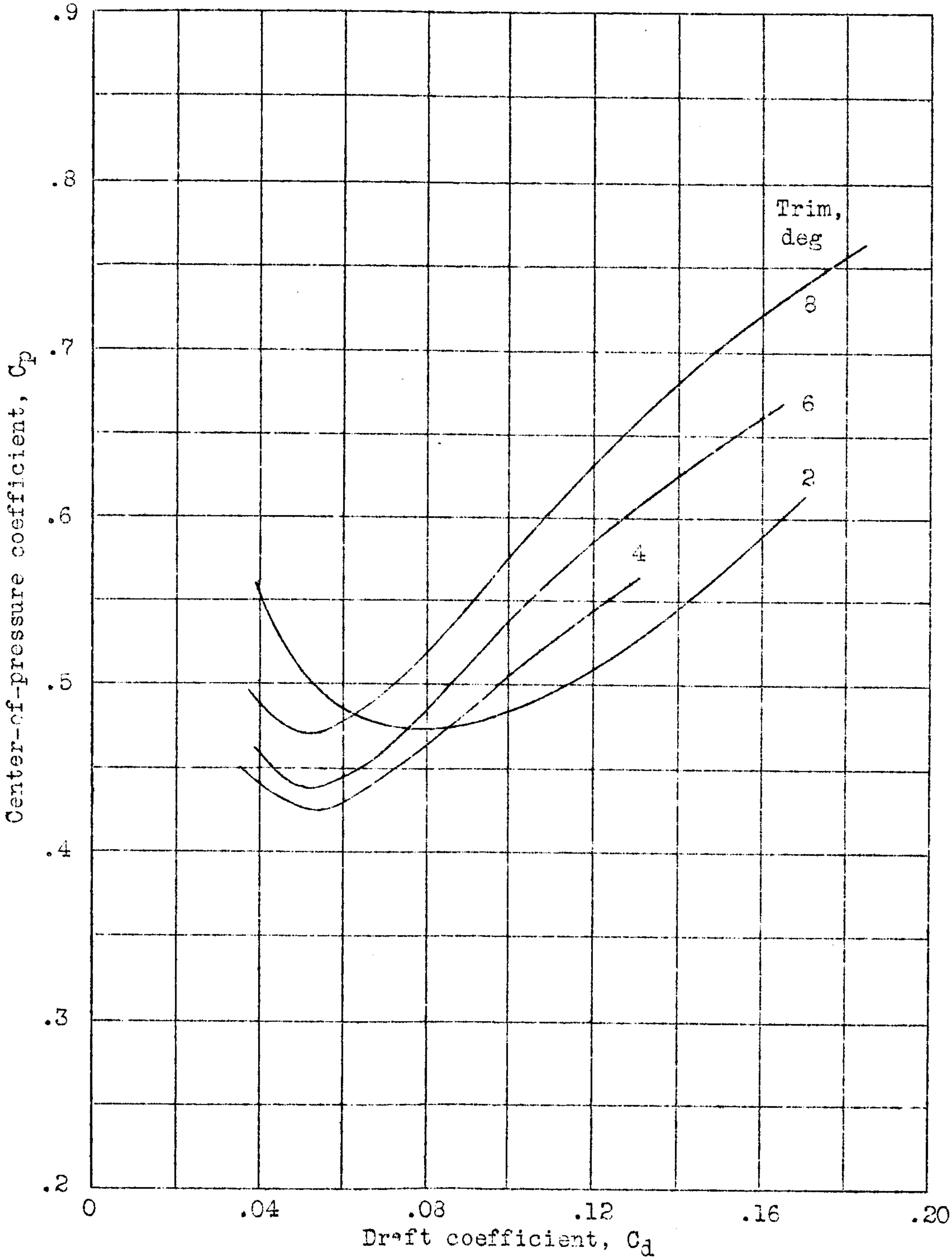

Figure 14.- Variation of center-of-pressure coefficient with draft coefficient. Angle of dead rise, 10°.
$C_p = s/W.L.$; $C_d = d/b$.

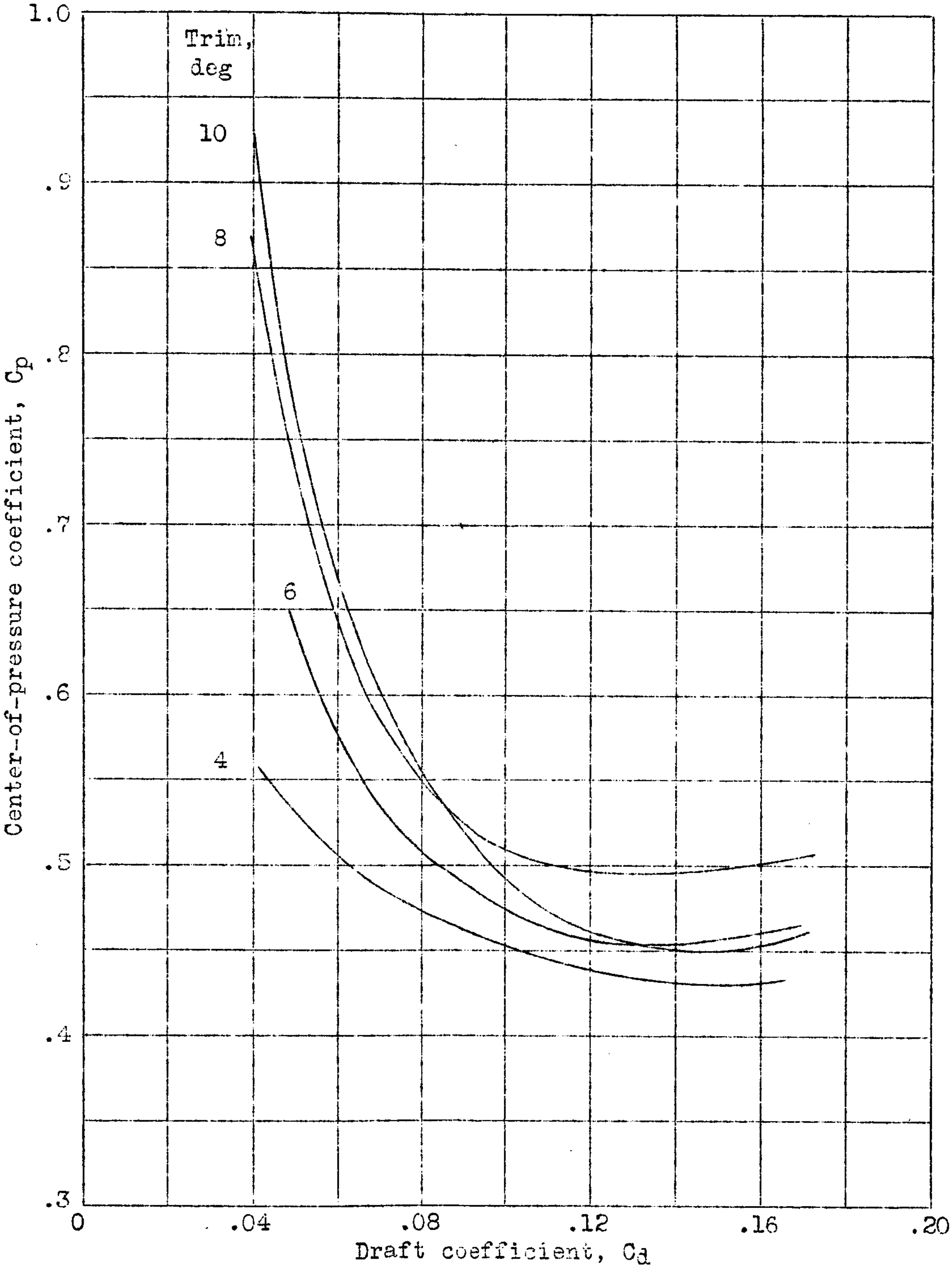

Figure 15.- Variation of center-of-pressure coefficient with draft coefficient. Angle of dead rise, 20°.
$C_p = s/W.L.$; $C_d = d/b$.

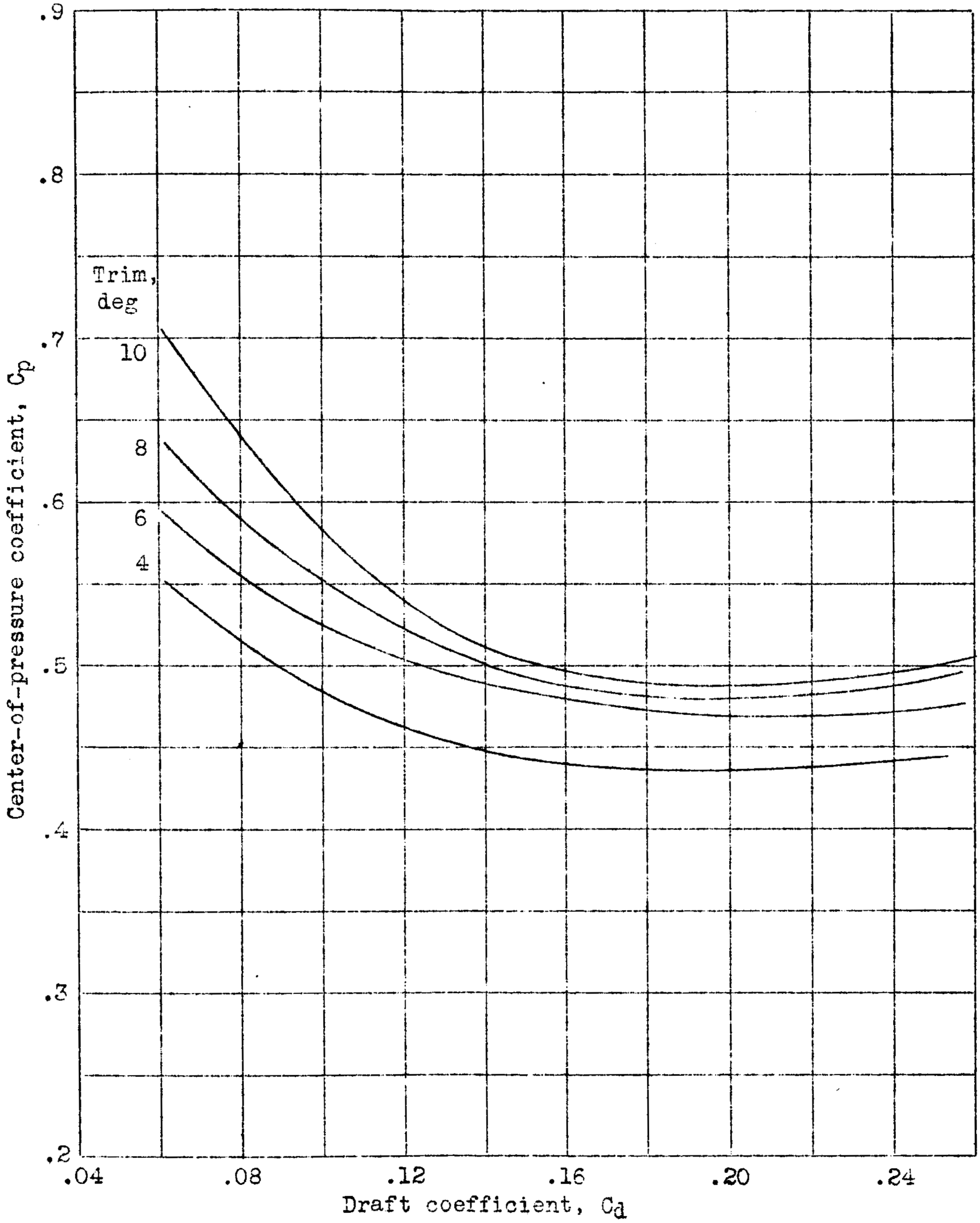

Figure 16.- Variation of center-of-pressure coefficient with draft coefficient. Angle of dead rise, 30°. $C_p = s/W.L.$; $C_d = d/b$.

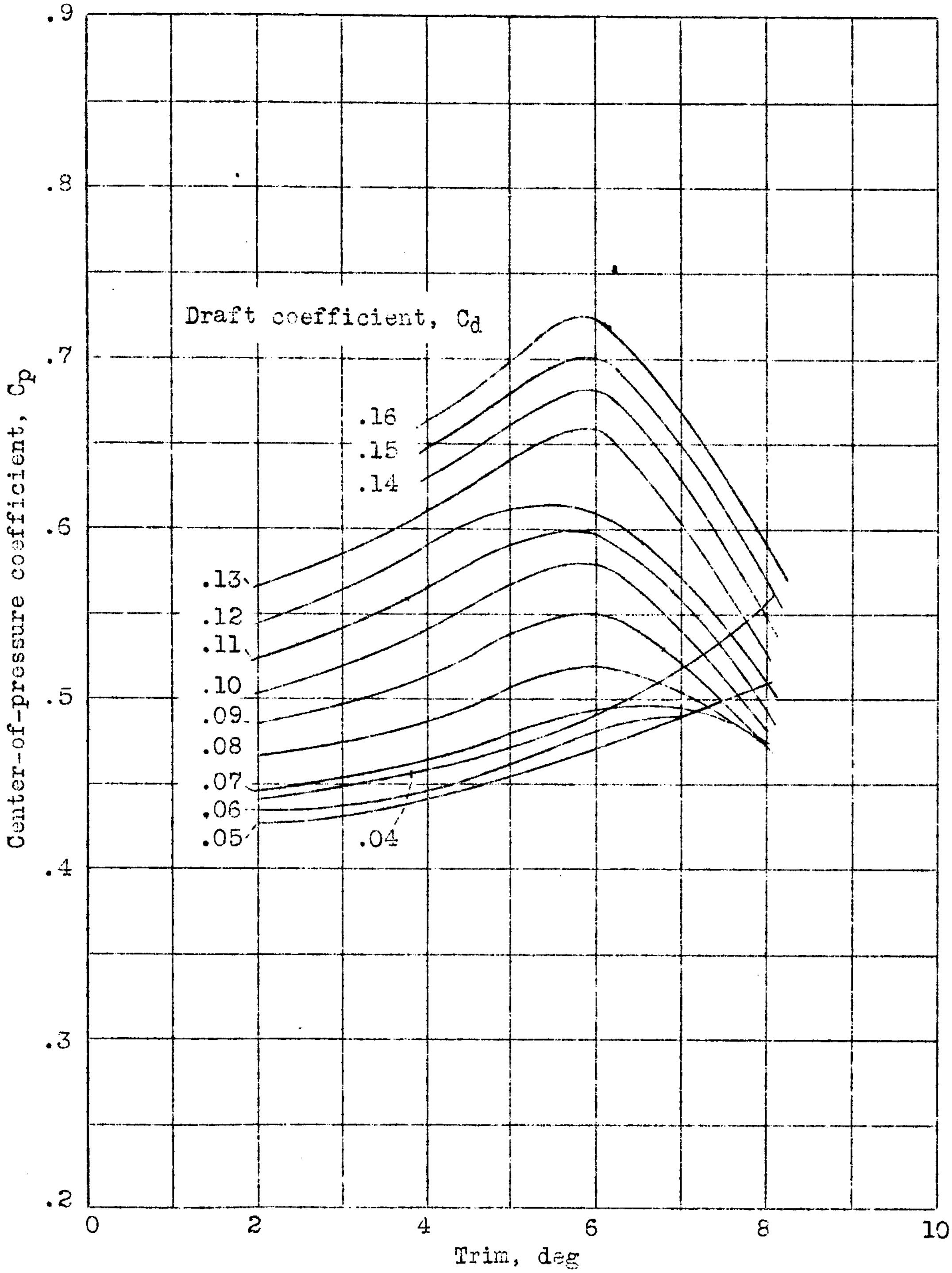

Figure 17.- Variation of center-of-pressure coefficient with trim. Angle of dead rise, 10°. $C_p = s/W.L.$; $C_d = d/b$.

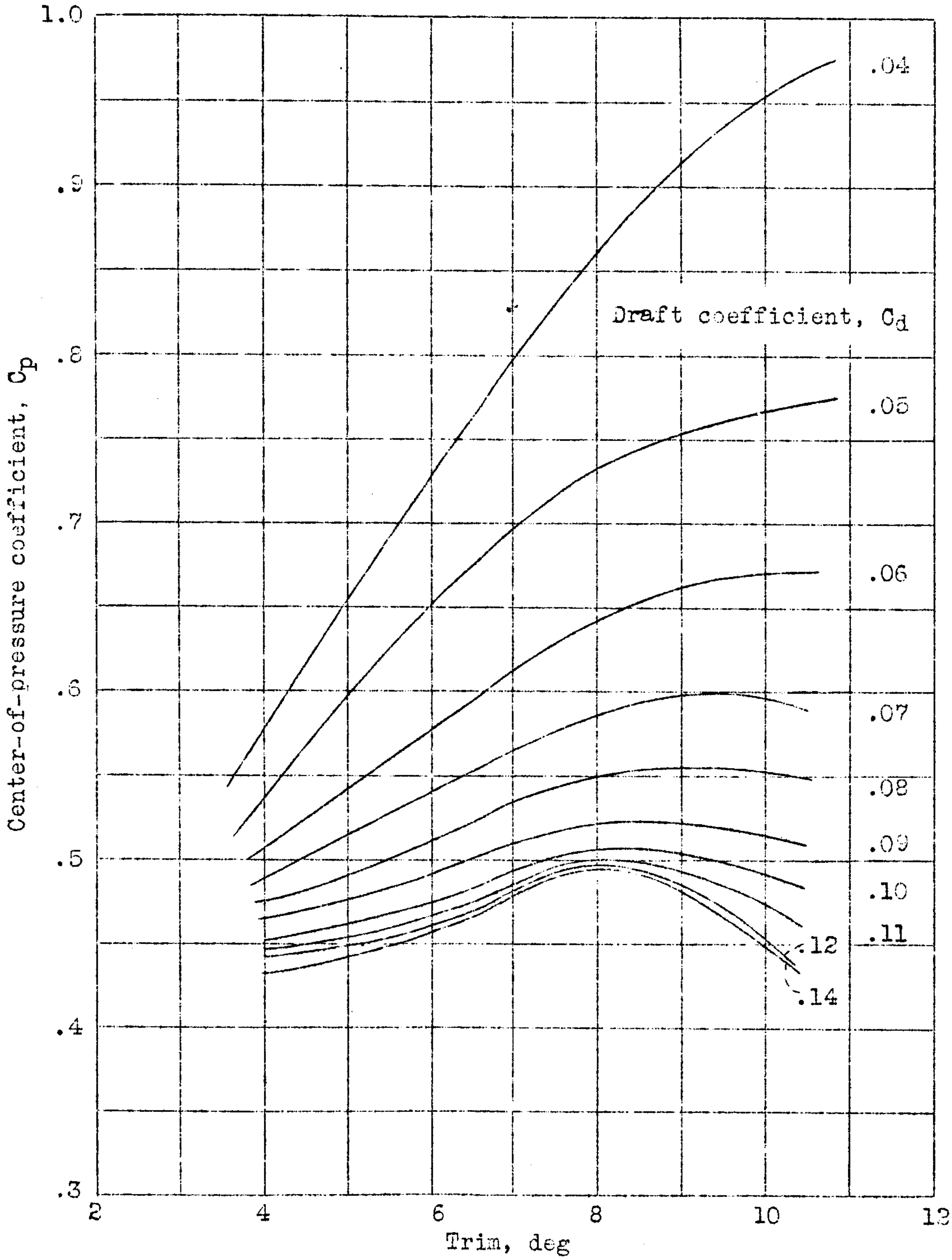

Figure 18.- Variation of center-of-pressure coefficient with trim. Angle of dead rise, 20°. $C_p = s/W.L.$; $C_d = d/b$.

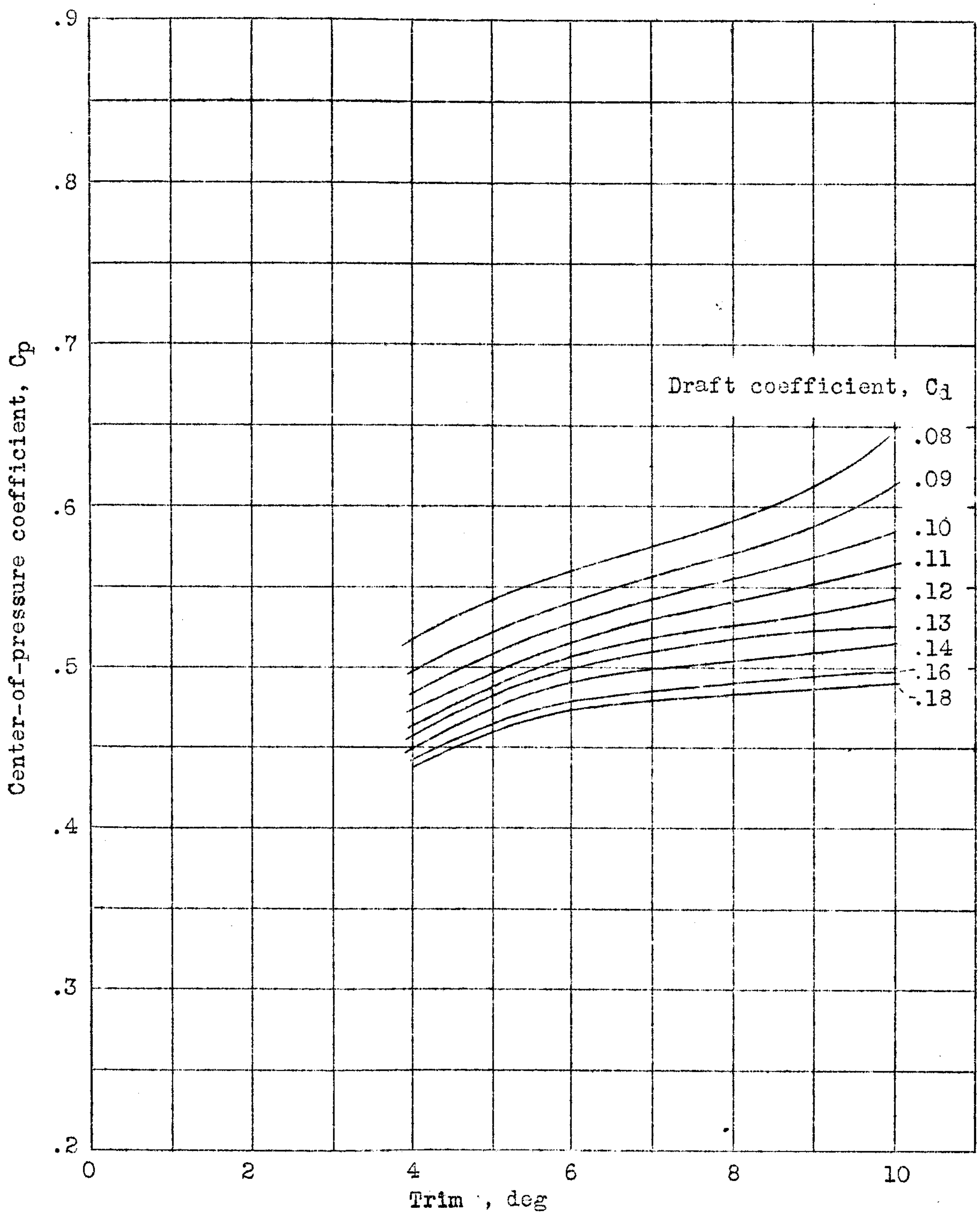

Figure 19.- Variation of center-of-pressure coefficient with trim. Angle of dead rise, 30°. $C_p = s/W.L.$; $C_d = d/b$.

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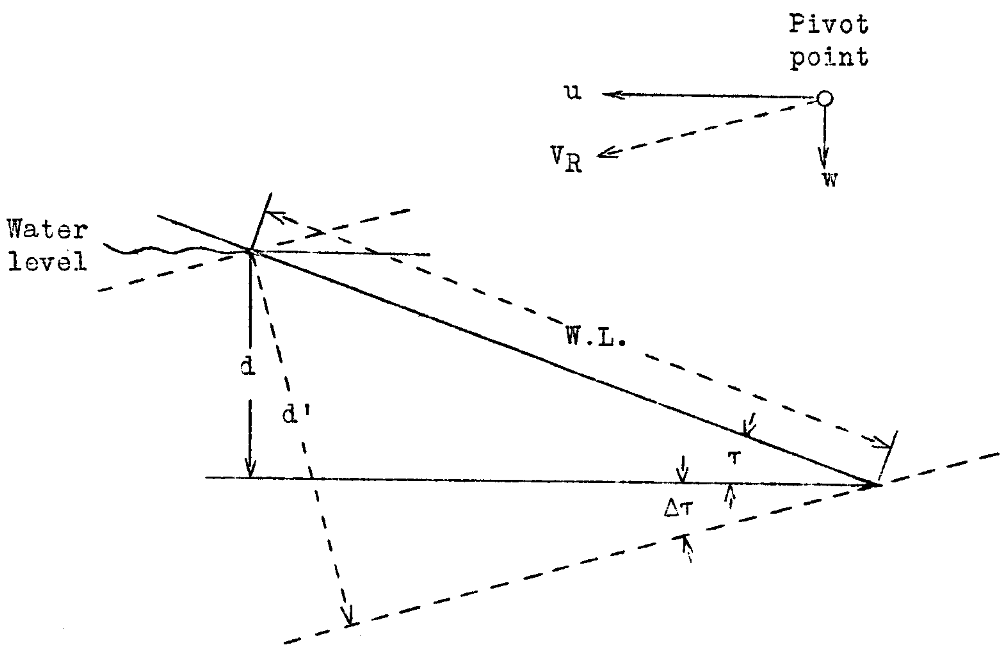

Figure 20.- Resultant draft and trim of a planing forebody due to a small vertical velocity w impressed upon the horizontal velocity u.

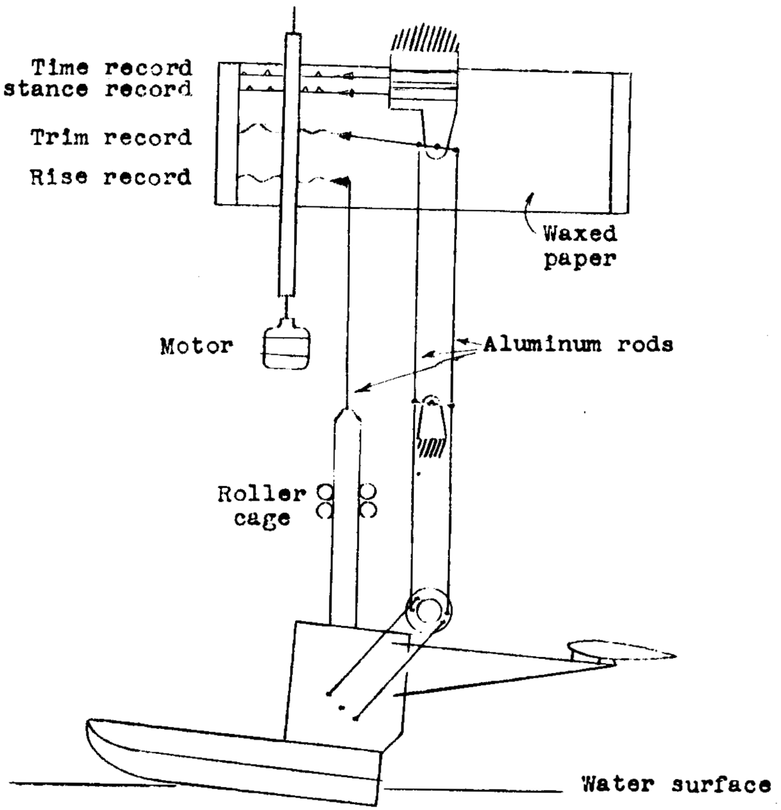

Figure 21.- Trim-and rise-recording apparatus.

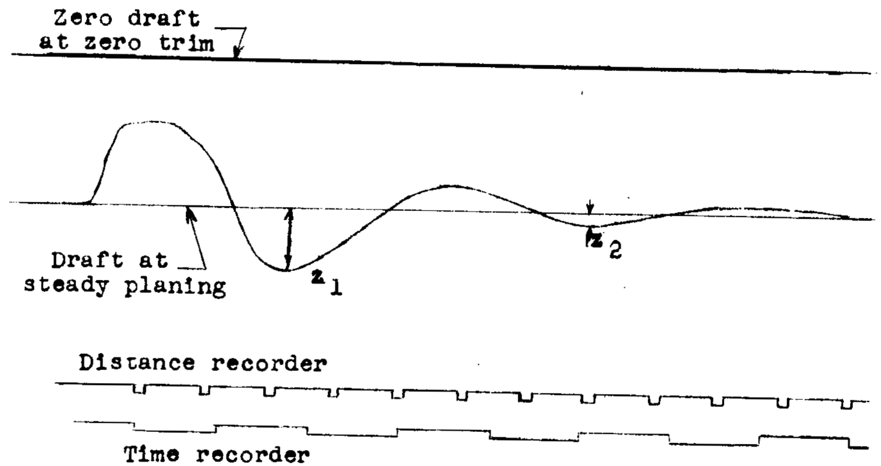

Figure 22.- Typical oscillation of a planing surface about a position of planing equilibrium following a displacement in draft.

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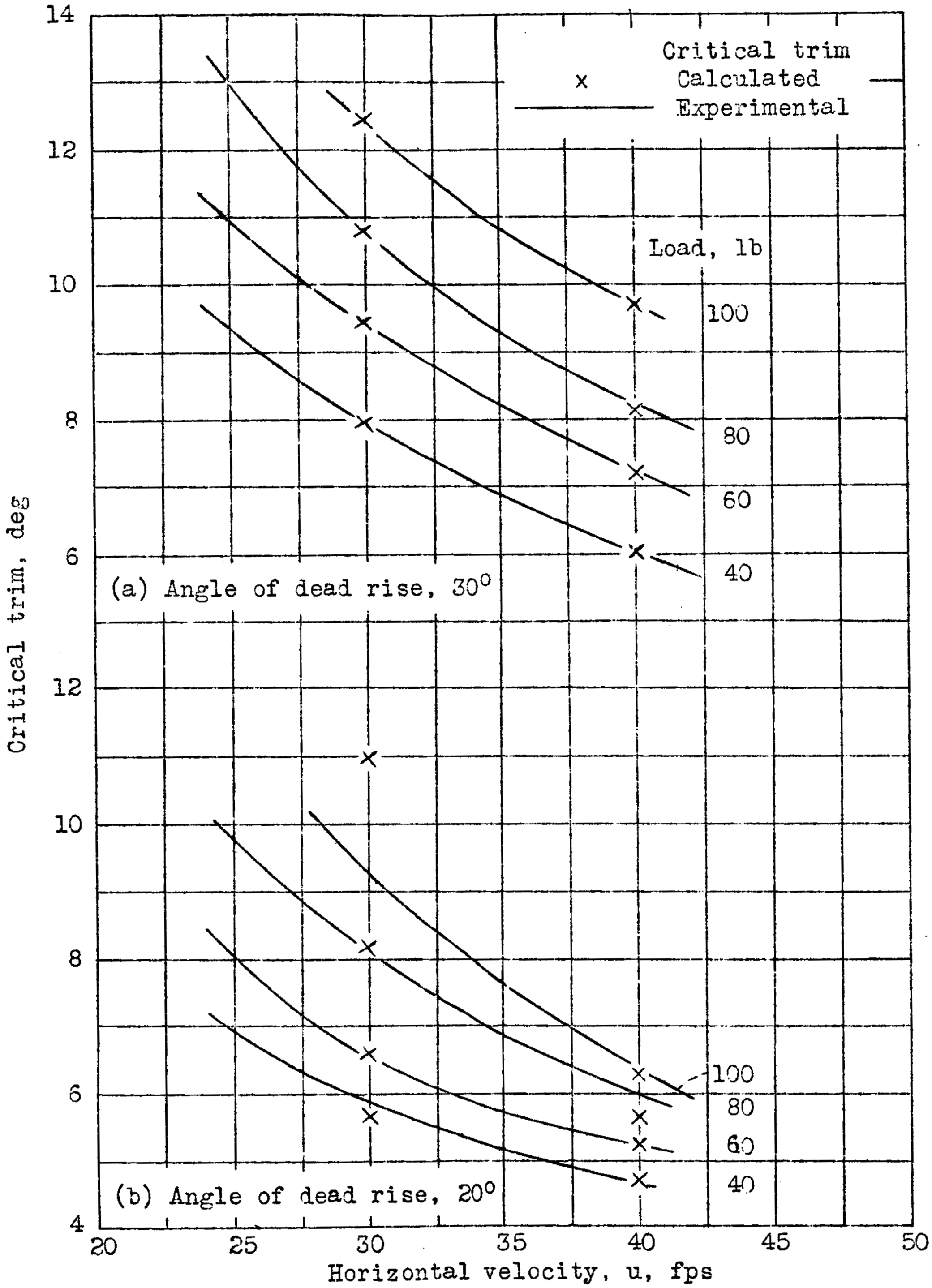

Figure 23.- Comparison of calculated critical trim with experimental critical trim. Model with tail.