NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

REPORT 1138

STUDY OF INADVERTENT SPEED INCREASES
IN TRANSPORT OPERATION

By HENRY A. PEARSON

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National Advisory Committee for Aeronautics

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SUMMARY

Some factors relating to inadvertent speed and Mach number increases in transport operation are discussed with the object of indicating the manner in which they might vary with different qualities of the airplane and the minimum margins required to guard against reaching unsafe values. The speed increments and the margins required under several assumed conditions are investigated. The results indicate that, on a percentage basis, smaller margins should be required of high-speed airplaners than of low-speed airplanes to prevent overspeeding in inadvertent maneuvers. The possibility of exceeding placard speed in prolonged descents is illustrated by computations for typical transport airplanes. Equations are suggested that allow estimates to be made of the necessary speed margins.

INTRODUCTION

In order to guard against inadvertent increases in airspeed, flight regulations limit the effective airplane cruising speed to a fixed percentage (80 percent) of the design or demonstrated speed. For lack of better information, the same limit has been applied to the Mach number. As a result of these regulations, low-altitude propeller-driven airplanes tend to be limited by the indicated airspeed placard so that a margin of 20 percent is maintained on the speed for structurally safe flight but with the possibility of a greater margin on the Mach number where adverse compressibility effects occur. On the other hand high-altitude high-speed jet-powered transports might be expected to be limited in cruising by the Mach number placard which results in a 90 percent margin on Mach number and a greater margin on the structurally safe indicated speed.

Plans for the development of turbojet transports have renewed interest in the problem of selecting satisfactory limits for airplane operating speeds that will insure against exceeding values of either Mach number or the dynamic pressure for which the airplane can be expected to remain controllable and structurally sound. In order for such transports to be economically feasible, however, they must necessarily be operated nearer the maximum level-flight speed than the older propeller-driven airplanes and excessive margins cannot be tolerated.

For these reasons reexamination of the problem of selecting the limiting operating speeds appears desirable. This report presents an analysis for determining the margins required under several assumed conditions and is confined mainly to the physical aspects of the problem.


SYMBOLS

The following symbols are used throughout the present report:

- $A$: aspect ratio, $b^2/S$
- $a$: speed of sound, fps
- $b$: wing span, ft
- $C_D$: airplane drag coefficient, $\text{Drag}/\text{qS}$
- $C_D$: profile drag coefficient
- $C_L$: airplane lift coefficient, $\text{Lift}/\text{qS}$
- $C_{L,e}$: airplane lift-curve slope per radian
- $C_{e}$: tailplane lift-curve slope per radian
- $c$: wing chord, ft
- $C_{m0}$: pitching-moment coefficient of wing-fuselage combination at zero lift
- $\alpha$: tail angle of attack, radians
- $d$: horizontal distance from given object, or distance between tail-off aerodynamic center and center of gravity, ft
- $d_e$: downwash factor
- $e$: span efficiency factor
- $f, f_s$: compressibility factors defined in reference 1
- $g$: acceleration of gravity, 32.2 ft/sec$^2$
- $M$: Mach number, $V/a$
- $q$: dynamic pressure, $\frac{1}{2} \rho V^2$, lb/sq ft
- $S$: wing area, sq ft
- $S_t$: tail area, sq ft
- $T$: engine thrust, lb
- $t$: time, sec
- $V$: true air velocity, fps
- $V_{e}$: true velocity, fps except with subscript mph
- $V_c$: calibrated airspeed (the airspeed related to differential pressure by the accepted standard adiabatic formula used in the calibration of differential-pressure airspeed indicators and equal to true airspeed for standard sea-level conditions), fps
- $W$: airplane weight, lb
- $w_p$: weight of shifted payload, lb
- $x_p$: distance through which payload is shifted, ft
- $x_t$: distance from airplane center of gravity to tail hinge line, ft
- $L/D$: airplane lift-drag ratio
- $h$: altitude, ft
...
If the sudden failure is assumed to impose a load-factor increment of $-2$ (a value that would normally be within the strength limits of the wing), equations (4) may be rewritten

$$\frac{\Delta V}{V} \approx 1000 \left(\frac{t}{V}\right)^2$$  \hspace{1cm} (5a)

$$\frac{\Delta M}{M} \approx 1000 \left(\frac{t}{Ma}\right)^2$$  \hspace{1cm} (5b)

so that, if a maximum fractional increase of not more than 0.1 in speed is to be obtained, the ratio $t/V$ should be less than 1/100. For example, for a transport traveling 350 feet per second, recovery should be started within $3\frac{1}{2}$ seconds. Inasmuch as the time for initiating a recovery would be a constant and independent of the initial speed, it appears that a smaller speed margin should be required for high-speed airplanes than for low-speed airplanes to guard against an autopilot failure.

**Symmetrical Normal Gust**

Equation (3) also serves as a starting point for arriving at estimates of the margins required to cover the speed gained in encountering gusts normal or parallel to the flight path. Assume that an airplane with neutral stability encounters a negative gust normal to the flight path for which the intensity increases linearly to a peak in 10 chords (10c), or a distance $H$, and decreases thereafter linearly to 0 in the same distance. This type of gust-intensity distribution is one commonly investigated in gust-load calculations. If unsteady lift and alleviation defects are omitted, the following simple load-factor relations apply

$$\Delta n_{max} = \frac{C_L \rho U V}{2 W/S} = 2 \Delta n_{at}$$ \hspace{1cm} (6)

The altitude loss for this case is

$$\Delta h = \frac{\Delta n_{max} g t^2}{2}$$

where

$$t = \frac{20c}{V}$$

so that

$$\Delta h = \frac{C_L \rho U V^2 g}{2 W/S} \left(\frac{20c}{V}\right)^2$$ \hspace{1cm} (7)

and from equations (2) and (7)

$$\frac{\Delta V}{V} = \frac{\Delta M}{M} = \frac{1}{2} \frac{C_L \rho U V^2}{W/S} \left(\frac{20c}{V}\right)^2$$ \hspace{1cm} (8)

In this case the percentage increase in speed or Mach number varies inversely with the cube of the initial speed or Mach number, directly with the square of the distance occupied by the gusts, and directly with the gust velocity. If typical maximum values are substituted in equation (6), it may be seen that the percentage increase in speed in encountering a single normal gust is likely to be small, providing the airspeed is not initially so low that the gust causes the airplane to stall. For a succession of down gusts the increase in speed would be larger than for a single down gust but even so the percentage gain would be less with the faster airplane than with the slower one.

**Symmetrical Horizontal Gust**

For horizontal gusts that occur in level flight, the increases in airspeed and Mach number would be instantaneous and are likely to be relatively higher than those due to the normal gust. In this case the fractional increase in airspeed or Mach number is given directly by the equation

$$\frac{\Delta V}{V} = \frac{\Delta M}{M} = \frac{U}{V}$$ \hspace{1cm} (9)

A horizontal gust would usually be of relatively short duration and would not be expected to be too critical in its effects; however, in a descent from altitude through an area of wind shear, more prolonged gust effects might be encountered. Although the magnitudes of the velocity and the gradients in such an area of wind shear are not definitely known, it appears that the gust velocity for use in equation (9) is either less than or, at most, equal to the value of 50 feet per second commonly used.

**Shifting Payload**

A gain in speed or in Mach number can occur if a shift in weight were to occur as when several passengers walk from the rear to the front of the cabin and the pilot fails to retrim the airplane. If it is assumed (a) that the aerodynamic coefficients are linear and do not vary with Mach number over the range being considered, (b) that the speed gain occurs in an atmosphere of constant density equal to that of the initial altitude, and (c) that the pilot does not move either the elevator or throttle before a new equilibrium condition is established, then the following derivation may be made. By designating the initial conditions by the subscript 1 and new trim conditions by subscript 2, the equilibrium equations can be written as

$$W = C_L \alpha_0 q_{1} S$$ \hspace{1cm} (10)

$$\left( C_m \frac{c}{x_i} + C_{L_{\alpha}} \frac{d_1}{x_i} - C_{L_{\alpha_{1}}} \frac{S_i}{S} - C_{Lk} \right) q_{1} S = 0$$ \hspace{1cm} (11)

After a change in moment $w_{x_p}$ due to a shift in payload the new equilibrium equations are

$$W = C_L \alpha_{0_{2}} q_{2} S$$ \hspace{1cm} (19)

$$\left( C_m \frac{c}{x_i} + C_{L_{\alpha_{2}}} \frac{d_2}{x_i} - C_{L_{\alpha_{1}}} \frac{S_i}{S} - C_{Lk} \right) q_{2} S = 0$$ \hspace{1cm} (13)

where the distance $d_2$ is related to the distance $d_1$ through the equation

$$d_2 = d_1 - \frac{w_{x_p} S}{W}$$ \hspace{1cm} (14)

If equation (13) is subtracted from equation (11) and if $q_{2} S 
eq 0$, there is obtained

$$C_{L_{\alpha}} \left( \alpha_{1} \frac{d_1}{x_i} - \alpha_{2} \frac{d_2}{x_i} \right) - C_{L_{\alpha_{1}}} \frac{S_i}{S} (\alpha_{1} - \alpha_{2}) = 0$$ \hspace{1cm} (15)
By noting that

\[ \alpha_i = \left(1 - \frac{d\xi}{d\alpha}\right) \alpha_1 \]

and that

\[ \alpha_2 = \frac{\alpha_2}{\alpha_1} \]

equation (15) can be written as

\[ \left[ C_{L_{\alpha}} \frac{d\alpha_1}{dx} - C_{L_{\alpha}} \frac{S_i}{S} \left(1 - \frac{d\xi}{d\alpha}\right) \right] \alpha_1 - \left[ C_{L_{\alpha}} \frac{d\alpha_2}{dx} - C_{L_{\alpha}} \frac{S_i}{S} \left(1 - \frac{d\xi}{d\alpha}\right) \right] \alpha_2 = 0 \tag{16} \]

By using equations (10) and (12), equation (16) can be rearranged as follows:

\[ \frac{\alpha_1}{\alpha_2} = \frac{q_2}{q_2} \frac{V_2^2}{V_1^2} = M_2^2 = \frac{C_{L_{\alpha}} \frac{d\alpha_2}{dx} - C_{L_{\alpha}} \frac{S_i}{S} \left(1 - \frac{d\xi}{d\alpha}\right)}{C_{L_{\alpha}} \frac{d\alpha_1}{dx} - C_{L_{\alpha}} \frac{S_i}{S} \left(1 - \frac{d\xi}{d\alpha}\right)} \tag{17} \]

Substituting \( d_2 \) (from eq. (14)) into equation (17) and rearranging gives

\[ \frac{V_2}{V_1} = M_2 \sqrt{1 + \frac{(\frac{w_p}{W}) \frac{x_p}{x_i}}{C_{L_{\alpha}} \frac{S_i}{S} \left(1 - \frac{d\xi}{d\alpha}\right) - \frac{d_i}{x_i}}} \tag{18} \]

If \( V_2 \) is set equal to \( V_1 + \Delta V \) and if second-order terms are neglected,

\[ \Delta V \approx \frac{1}{2} \left( \frac{w_p}{W} \right) \frac{x_p}{x_i} \Delta M \frac{\alpha_{L_{\alpha}}}{\alpha_1} \left(1 - \frac{d\xi}{d\alpha}\right) - \frac{d_i}{x_i} \]

Equation (19) indicates that the percentage increase in speed is independent of the initial speed and depends on the aerodynamic and geometric characteristics of the airplane. It shows that the ratio \( \Delta V/V \) increases as the ratio of the change in trim moment to the airplane moment about the hinge line increases \((\frac{w_p x_p}{W x_i})\) and that the ratio \( \Delta V/V \) increases as the static stability (given by the denominator) decreases. The speed increase given by equation (19) is the equilibrium value reached under the assumption made and would correspond to a condition in which the airplane is in a slight but steady glide angle. The transient values of \( \Delta V/V \) may be lower or higher than the trim value given by equation (19) depending upon when the pilot takes appropriate action to retrim. The largest value of \( \Delta V/V \) should occur at a time about equal to one-quarter of the period of the phugoid oscillation of the airplane (roughly \( \frac{1}{16} \) sec). Since this motion is lightly damped, the maximum value reached could be very nearly equal to twice that given by equation (19).

As an example of the quantities involved for the case of a typical present-day four-engine transport, a change in either \( \Delta V/V \) or \( \Delta M/M \) equal to 0.025 would be obtained from equation (19) if 500 pounds of passengers moved from the most rearward position to the front seats.

Most current airplanes capable of operating in or near the transonic speed range are limited to operation below what is commonly called a buffeting boundary. A typical buffeting boundary for a fighter airplane, designated airplane 1, is shown in figure 1. The data for this curve are taken from reference 2. The part of the curve to the right of the dashed vertical line is associated with a separation caused by compressibility on some main component of the airplane; whereas that part of the curve to the left of the vertical line is the usual \( C_{L_{\alpha}} \) curve. Although the boundary to the right of the vertical line can be and has been crossed during special tests with small military airplanes, to do so not only subjects the airplane to large oscillating forces but also places it in a region where stability and control difficulties occur. At present a quantitative evaluation of either the forces or handling qualities of airplanes beyond or at this boundary is not possible. For these reasons, transport airplanes should not be operated at the boundary and some margin appears to be necessary to permit mild maneuvers at cruising speed without the possibility of reaching the buffeting boundary.

In the high-speed cruising range, which is mainly of interest in the present study, the slopes of the available buffeting boundaries are relatively steep. Thus, some idea of the margins which should be maintained can be obtained, even though, as stated previously, the boundary cannot be predicted too accurately. Information on the buffeting boundaries was obtained by analyzing the results for several airplanes, including that given in figure 1, to show the margin required in Mach number in order that a steady 45° bank could be executed at cruising speed without exceeding the boundary. Execution of a 45° bank without loss of altitude requires that the airplane lift coefficient be increased by the factor \( \sqrt{2} \). Thus, if, for example, in figure 1 the original
cruising lift coefficient \( C_L \) were 0.5 the lift coefficient in the bank would be approximately 0.71. As shown in figure 1, the margin in Mach number required would then be about 0.035. This point is illustrated in figure 2 by the circular symbol on the curve for airplane 1. Repeating this procedure at other lift coefficients and for other airplanes gave the results shown in figure 2 in which the abscissa represents the over-all airplane \( C_L \) at cruising speed and the ordinate is the Mach number margin from the buffeting boundary that should be maintained if mild maneuvering at cruising speed is to be permitted. In general, the Mach number at which the airplane would buffet in level flight is best determined from flight demonstration tests and the margins of figure 2 would be applied to this Mach number. For the present at least, a conservative estimate of the margin required to permit maneuvering appears to be given by any one of the following:

\[
\Delta M = \frac{C_L}{12} \text{ allows 30° bank}
\]

\[
\Delta M = \frac{C_L}{10} \text{ allows 45° bank}
\]

\[
\Delta M = \frac{C_L}{7} \text{ allows 60° bank}
\]

The relation \( \Delta M = \frac{C_L}{10} \) represents an envelope of most of the data shown in figure 2 for the 45° bank. The relations given for other angles of bank were determined in a similar manner.

**Figure 2.** Mach number margin required to execute 45° bank without crossing the buffet boundary.

**MARGIN REQUIRED FOR TEMPERATURE INVERSION**

Temperature inversions are known to exist in the atmosphere and the altitude range over which inversions may occur varies from hundreds to thousands of feet. Within such inversions fairly localized gradients of 10° F per thousand feet are not uncommon. A change in the temperature of 10° F corresponds to a change of about 1.3 percent in the speed of sound.

During steep descents with small airplanes operating near critical Mach number, adverse compressibility effects such as buffeting or stability changes are sometimes inadvertently encountered. These occurrences have been correlated with measured temperature inversions so that, in some types of research testing, a margin of about 0.015 in Mach number has been necessary in order to avoid inadvertently reaching the buffeting boundary. Since the airplanes on which such experience has been obtained had critical Mach numbers of about 0.75, a suitable margin to guard against the effects of a temperature inversion during a descent may be obtained from the relation

\[
\frac{\Delta M}{M} = 0.02
\]

The constant 0.02 in this relation would be associated with a somewhat larger temperature gradient than the 10° F mentioned earlier.

**SPEED GAINS DURING PROLONGED DESCENTS**

Statistical data have shown that the probability of exceeding the placard speeds is greatest in prolonged descents. Some of the reasons for such overspeeding (exceeding placard speed), such as meeting schedules or encountering an emergency, are obvious, whereas other less obvious reasons could conceivably be linked with the operation of either the engine or cabin pressurization system.

Overspeeding in the case of an emergency cannot be rationalized as a pilot would take whatever risks were required. Even introducing automatically operated devices such as brake flaps would not positively prevent overspeeding unless these flaps provided sufficient braking to keep the terminal velocity in a steep dive below the placard value. Because jet engines must be operated at a higher percentage of power at high altitudes than piston engines in order to avoid a "flame out," this characteristic could offer an excuse for overspeeding in a descent in case the engines could not be restarted easily. In such a case a pilot, if pressed for time, might not tolerate the low rates of descent which would be forced on him by operating the engine at a relatively high percentage of power. Similarly, an airplane having a cabin pressurized by an exhaust-driven turbosupercharger might also offer an excuse for overspeeding since some engine power would be required during a descent in order to maintain cabin pressure. The obvious remedies in both these instances would be to provide positive means of restarting jet engines in flight and the avoidance of pressurization by exhaust-driven superchargers; otherwise, air brakes would be necessary in order to compensate for the undesirable engine thrust and the weight component in a descent.

In addition to these somewhat unusual and possibly outmoded cases, overspeeding might also occur if the pilot were
to follow some fixed plan of descent without making due allowances for airplane characteristics. Conceivably, a
descent from altitude could be made according to a number
of predetermined plans such as: at constant indicated air-
speed, at constant Mach number, at a constant true airspeed,
at constant glide-path angle, at constant rate of change of
absolute altitude, or at constant rate of change of cabin
pressure altitude. In each of these plans there would be a
steady decrease in the potential and total energy involved.
All plans, however, would be characterized by conditions
which could be treated by equations (1) to (3). For instance,
the initial phase of the descent before reaching steady con-
ditions may be considered transient as may be any subsequent
deviations from the main path due to overcontrolling or
inattentiveness on the part of the pilot. These additional
speed changes over and above that called for by the adopted
plan can be treated as before by using equation (2) or its
equivalent
\[
\frac{dV}{dt} = -g \frac{dh}{V} \quad (20)
\]
In equation (2) the maximum possible percentage increase
for a given \( h \) below the intended descent path varied in-
versely as the square of the average speed along the path.
Equation (20) indicates that the rate of change of speed is
directly proportional to the rate of descent and inversely
proportional to the speed; thus the higher the initial speed
the greater the time available for the pilot to prevent a unit
increase in speed by detecting and checking a unit rate of
descent above the intended value.

The equations of motion for some of the plans of descent
which might be used are given in general form, where the
asterisk is used with the symbols to identify the parameters
held constant.

For a descent at a constant small glide-path angle for
which the weight may be considered equal to the lift, the
equation of motion is
\[
\frac{dV}{dt} = -g \left( \sin \gamma + \frac{T}{W} - \frac{1}{L/D} \right) \quad (21)
\]
Since by definition
\[
\sin \gamma = \frac{dh}{dt} \frac{dh}{V} = \frac{M\alpha}{M}
\]
and, from reference 1,
\[
V_\gamma = V_\gamma \frac{\rho}{f \sqrt{\rho_0}} \quad (23)
\]
the equation of motion for a descent at a constant calibrated
airspeed is (calibrated airspeed is the pilots' indicated
airspeed when the airspeed system has no error)
\[
\frac{dV}{dt} = -g \left( \frac{dh}{dt} f_\gamma \frac{\rho}{f \sqrt{\rho_0}} + \frac{T}{W} - \frac{1}{L/D} \right) \quad (24)
\]
Similarly, for a descent at constant Mach number the equation is
\[
\frac{dV}{dt} = -g \left( \frac{dh}{dh} dt f_\gamma \frac{T}{M^2 \alpha} + \frac{1}{L/D} \right) \quad (25)
\]
and for a constant rate of change of altitude based on stand-
ard conditions
\[
\frac{dV}{dt} = g \left[ \frac{(dh/dt)^2}{V^2} + \frac{T}{W} - \frac{1}{L/D} \right] \quad (26)
\]
Alternate forms of equations (21) to (26) may be obtained
by substituting relations involving the coefficients of lift and
drag
\[
D = C_D qS = C_D \frac{\rho}{2} V^2 S = C_D \frac{\rho q^2}{2} \rho^2 S \quad (27)
\]
where \( C_D \) may be expressed as
\[
C_D = C_D 0 + \frac{C_L^2}{\epsilon_{\pi A}} = C_P 0 + \left( \frac{W \cos \gamma}{qS^2} \right) \frac{1}{\epsilon_{\pi A}} \quad (28)
\]
In general \( T/W \) will be some function of \( V \) and \( h \) for a
given engine speed or throttle setting and \( L/D \) will be a
function of Mach number \( M \) and \( C_L \).

**ILLUSTRATIVE EXAMPLES AND APPLICATION OF RESULTS**

**AIRPLANE CHARACTERISTICS AND CONDITIONS**

In order to integrate equations (21), (24), (25), and (26)
to obtain the speed gain with a given descent plan, a step-by-
step solution must be made. Since several of the principal
variables are either nonlinear or are complicated functions
of other variables, an infinite number of solutions would
exist so that no general charts can be given. However, in
order to illustrate the potential speed gains that may occur
in following various plans for descent, examples are given
for three typical transport airplanes designated airplanes
A, B, and C. The airplane characteristics and conditions
assumed to illustrate the application of the formulas are
summarized in table I.

Airplane A is representative of a propeller-driven airplane
of about ten years ago with a nonpressurized cabin. In the
element, this airplane is assumed to start a descent from

<table>
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<tr>
<th>TABLE I.—PERTINENT AIRPLANE CHARACTERISTICS AND CONDITIONS USED IN EXAMPLES</th>
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<tr>
<td><strong>Quantity</strong></td>
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<tr>
<td></td>
</tr>
<tr>
<td>Weight, lb...</td>
</tr>
<tr>
<td>Wing loading, lb/sq ft</td>
</tr>
<tr>
<td>Thrust/Weight...</td>
</tr>
<tr>
<td>Initial conditions:</td>
</tr>
<tr>
<td>Altitude, ft...</td>
</tr>
<tr>
<td>( V_\gamma ) mph...</td>
</tr>
<tr>
<td>( M )...</td>
</tr>
<tr>
<td>( q, ) lb/sq ft...</td>
</tr>
<tr>
<td>( \gamma ), radians...</td>
</tr>
<tr>
<td>( \Delta h, ) ft/min...</td>
</tr>
<tr>
<td><strong>a</strong> Descent shown both with engine power assumed constant and with zero thrust.</td>
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<tr>
<td><strong>b</strong> Descent shown with zero thrust only.</td>
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</table>
10,000 feet with an initial true airspeed of 200 miles per hour for two thrust conditions. In one case the engine power and propeller efficiency are assumed to be constant during the descent so that the thrust weight ratio varies inversely with the speed according to the relation $T/W = \frac{18}{V_{mn}}$. In the other case the thrust is assumed to be zero. The variation of $L/D$ with airplane lift coefficient $C_L$ is given in figure 3 (a) where the initial condition from which the computations were started is represented by the circular point. Since the speed range of airplane $A$ is quite low, no Mach number effects on the $L/D$ curve were assumed. Also since it was assumed that no cabin pressurization was used, the rate of change of altitude was kept low for the various descent plans.

Airplane $B$ is a pressurized-cabin, propeller-driven airplane typical of some present-day transports. For this airplane the descent was assumed to start at 20,000 feet from a true cruising speed of 350 miles per hour. Rates of descent of 1000 and 2000 feet per minute were assumed. As with airplane $A$, two cases were considered: one with zero power and one in which the engine power and propeller efficiency were assumed constant in the descent. For this case, the total thrust-weight ratio was given by $T/W = 23.1$. The assumed variation of $L/D$ with $C_L$ and Mach number is given by the curves in figure 3 (b), which were derived from tests of a current transport configuration. From supplementary curves it was established that critical Mach number occurred around $M=0.65$; the initial conditions are represented by the point on the $M=0.50$ curve.

Airplane $C$ used in the examples is an assumed swept-wing, pressurized-cabin, turbojet airplane capable of cruising at 30,000 feet at 500 miles per hour or a Mach number of about 0.75. The $L/D$ curves for this airplane are given in figure 3 (c). The critical Mach number for airplane $C$ was established as being around 0.81. For the case of flight with power on, the variation of thrust for the speed and altitude range of interest was assumed to be given by the equation $T=19000-0.3h$. Results of wind-tunnel model tests and jet engine tests were used to estimate the $L/D$ and thrust relations.

For airplane $A$, which was not pressurized, the value $\frac{dh}{dt}=500$ feet per minute is slightly above the upper limit permissible in current practice regarding the effect of rate of change of pressure on passenger comfort, whereas the 1000 feet per minute might apply with only crew members aboard. For airplane $B$ in which the cabin was assumed to be pressurized to 10,000 feet, the rate of descent was chosen as 1,000 feet per minute which would give a total descent time of 20 minutes. With this rate, a total time of about 23 minutes would be required to raise the cabin pressure from 10,000 feet to sea level at a cabin rate not exceeding that corresponding to a change of 0.4 inch of mercury per minute. (A rate not exceeding 0.4 inch of mercury per minute represents current practice. This rate corresponds to values of $\frac{dh}{dt}$ equal to 370, 500, 700, and 1000 feet per minute at sea level, 10,000, 20,000, and 30,000 feet, respectively.) For airplane $C$ also with the 10,000-foot pressurized cabin, descent rates of 1,000 and 2,000 feet per minute were chosen.

![Figure 3](image-url) - Variation of lift-drag ratio with lift coefficient for airplanes of example.
At the lower rate, the cabin and outside pressure would just be equalized shortly before landing, whereas at the higher rate some ground time would be required.

**CALCULATIONS FOR SPECIFIC DESCENT PLANS**

Calculations were made for the various specific descent plans for airplanes A, B, and C and the results are presented in figures 4 to 6. In the computations the transition from the cruising condition to the specific condition planned for the descent was assumed to be instantaneous. In the figures, the altitude is plotted against calibrated airspeed for each plan of descent. Ticks are added to each curve to indicate the elapsed time in minutes. The values of $\frac{dh}{dt}$ used in the calculations were based on values intended to provide reasonable passenger comfort and adequate cabin pressurization.

**Descent at constant $V_c$.**—A descent at constant calibrated airspeed would be represented in figures 4 to 6 by vertical lines. For the range of conditions considered, constant calibrated airspeed corresponds closely to a constant dynamic pressure $q$. Therefore, during such a descent the airplane lift coefficient and lift-drag ratios would remain nearly constant except for Mach number effects on these quantities.

The difference between a descent at a constant calibrated airspeed and one at a constant dynamic pressure $q$, if such a descent could be made, may be obtained by noting the deviation of line $D$ in figures 4 to 6 from a vertical line through the initial point. From the deviations shown, it appears that in a descent at constant calibrated airspeed the dynamic pressure would be expected to increase slightly.

**Descent at constant $M$.**—Curve $C$ of figures 4 to 6 shows that the descent at constant $M$ would result in an increase in calibrated speed and hence in the dynamic pressure $q$. On a percentage basis, the increase in calibrated speed is successively greater with airplanes A, B, and C mainly because the altitude range covered is greater. Regardless of the initial cruising speed, the increase in true airspeed during a constant Mach number descent in a standard atmosphere would not exceed 10 percent; however, for airplane C, the value of $q$ would increase about 3.5 times during the descent from 30,000 to 2,000 feet. This increase in $q$ for descent at constant Mach number would probably be excessive from structural considerations, so that this plan is a less practical one than the descent at constant calibrated airspeed.

![Figure 4](image1.png) **Figure 4.**—Velocity-altitude relations for various descent plans with airplane A.

![Figure 5](image2.png) **Figure 5.**—Velocity-altitude relations for various descent plans with airplane B.
The curves given in figures 4 to 6 for the descent at constant $M$ apply to either the zero thrust or power-on condition, since the pilot would adjust the throttle and glide angle as required in order to maintain the values selected. In fact, equations (24) and (25) cannot be integrated for the power-on cases considered in this report unless throttling or some brake devices are used.

Descent at constant rate of change of altitude.—The computations for airplane A (curve $B$, fig. 4) show that, with constant power setting, the calibrated speed in a descent at constant rate of change of altitude would be increased by about 24 percent in 5 minutes, of which about half would occur within the first minute. For the power-off case (curves $G$ and $H$) the airplane would stall in trying to maintain a constant rate of change of altitude of either 500 or 1000 feet per minute. Thus it appears that, in the event of sudden engine failure, rates of descent higher than 1000 feet per minute would necessarily prevail regardless of passenger comfort.

The fact that the increases in speed measured in present-day transport operations have generally been less than 10 percent means that some throttling is used during the descent. For airplane A, computations would show that if the engines were immediately throttled to about two-thirds of the cruising power (that is, about four-ninths of rated power) a descent could be made at about 500 feet per minute without a substantial increase in speed.

The increase in speed for airplanes B and C at the constant rates of descent chosen are, on a percentage basis, about the same as for airplane A with about half the final maximum increase occurring in the first minute. Thus, it appears that, if the present placard speed which limits cruising operation to 80 percent of the design or demonstrated speed is to be raised with future transports, provisions must be made for reducing the "effective" airplane $L/D$ ratio either by engine throttling or by use of aerodynamic braking.

Descent along constant flight-path angle.—The curves labeled A, E, and F in figures 4 to 6 for descents at constant flight-path angle indicate slightly greater increases in speed and consequently greater rates of descent than the curves for constant rate of descent (curves $B$, $G$, and $H$), even though the flight-path angle $\gamma$ was selected on the basis that it be equal to the rate of descent divided by the initial airspeed. Of interest are the small glide-path angles involved which seldom exceed more than 2°. These small angles are in approximate agreement with statistical measurements which have seldom indicated glide-path angles in a descent of over 5°.

**Composite Plan**

Although the specific plans discussed would probably not be followed throughout a descent without modifications, they are useful in indicating the problem and in pointing out safe procedures to be followed.

It is possible that, unless the cabin were capable of being pressurized to sea-level pressure up to the highest cruising altitude, passenger comfort not only could influence the type of descent plan but also could affect the placard speeds. As stated previously, present practice is to limit the rate of change of cabin pressure to about 0.4 inches of mercury per minute which corresponds to a value of $\frac{db}{dt}=1000$ feet per minute at 30,000 feet and 370 feet per minute at sea level.

For structural reasons future transports will continue to be designed to withstand some maximum dynamic pressure $q$ or its equivalent in airspeed. This maximum dynamic pressure could either be one which is arbitrarily selected as a design point or one to which the airplane must be demonstrated. Transport airplanes of the immediate future will also be limited by some Mach number which is not to be exceeded if stability and control troubles as well as buffeting are to be avoided. These considerations will in general require that a composite plan of descent be adopted.

In order to illustrate these limits some of the results for airplane C given in figures 3 (c) and 6 can be used. It is assumed that wind-tunnel tests of a model or flight demonstrations have shown that, at low lift coefficients, adverse compressibility effects begin at $M=0.81$ (represented by $C'$) and that this value of $M$ should not be exceeded. The structure is assumed to have been designed or demonstrated...
to withstand the loads at a dynamic pressure corresponding to a calibrated airspeed of 422 miles per hour. These extreme operational limits which should not be exceeded are shown in figure 7 by the heavy dashed line having two segments and labeled “design limit.” The short upper segment is a part of the curve representing the 0.81 Mach number limit, whereas the lower segment is a part of the curve representing the dynamic-pressure limit. For comparison, lines B and C from figure 6 are also shown from which it is seen that neither the descent at a constant value of $\frac{dh}{dt}=1000$ nor at a constant $M=0.738$ could be followed throughout without exceeding these limits.

**REQUIRED MARGINS**

In order to allow for the possibility of inadvertent increases in speed and Mach number, some margin from the limits shown by the heavy dashed line is required. Although equations have already been given from which the margins required under a single condition may be obtained, the question arises as to the probability of separate events occurring together. The speed gains resulting from avoiding obstacles, from encountering normal down gusts, and from shifting payload have been omitted in determining the required margins because either they have been shown to be small or the probability of these events occurring simultaneously with other more important events is remote. Deliberate overspeeding beyond established planned speeds has also been omitted because of its psychological aspect.

With these possibilities eliminated, the combinations of events that might have reasonable probability of occurrence are

1. A mild maneuver during descent at operational speed (30° bank) in a region of temperature inversion where horizontal gusts of moderate size (15 fps) exist.

2. Autopilot failure resulting in $-2g$ increment in load factor with 5 seconds delay in recovery in a region of moderate horizontal gusts.

Consideration of these possibilities indicates that a guide to the required margins in Mach number from the operational limits might be obtained from the equation

$$\Delta M = C + 15 \left( \frac{M}{V} \right)_{crus} + 0.02 M_{crit} + 0.02 \quad (29)$$

where the first term is the margin on the buffet boundary at the airplane cruising lift coefficient, the second term is a margin allowing for a 15-foot-per-second horizontal gust at cruising speed, the third term allows for a possible temperature inversion, and the last term takes into consideration the spread in critical Mach number for a series of airplanes of the same type.

The reduction from the design indicated airspeed is based on the second possibility and would be given by an equation of the type

$$\Delta V = 45 + \frac{25000}{V_{design}} \quad (30)$$

where the first term is a combined one allowing for horizontal gusts and variations between airplanes and the second term allows for the possibility of an autopilot failure. In equation (30) $V_{design}$ is considered to be the true airspeed in feet per second corresponding to the design indicated airspeed at the lowest altitude at which the autopilot would be used.

If these suggested equations were applied to airplane C, the reduction from the critical Mach number of 0.810 would be 0.072 or 8.9 percent and would yield a maximum operational Mach number of 0.738. If 10,000 feet is assumed to be the lowest altitude at which flight with the autopilot would occur, the reduction from the true design airspeed of 484 miles per hour would be 80 feet per second or about 55 miles per hour. This new airspeed (484 mph - 55 mph) would correspond to an indicated airspeed of 374 miles per hour, which represents a margin of 11.5 percent on the design indicated speed of 422 miles per hour. The operational limits obtained in this manner are given by the solid heavy line in figure 7.

It should be remembered that the constants appearing in equations (29) and (30) for calculating the required margins are estimated and should be checked at the first opportunity with flight experiences.

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**Figure 7.—Operational and design limits for composite descent. (Airplane C.)**
CONCLUDING REMARKS

As a result of a study to estimate the speed margins that should be allowed to provide for inadvertent speed increases in transport operation, the following trends are indicated:

1. As the cruising speeds of transports increase, the percentage margins required to avoid inadvertent speed gains caused by gusts, autopilot failure, and so forth should decrease.

2. The descent plan of future transport airplanes will probably be a composite one in which the Mach number will be used to furnish the limit in the beginning of the descent and the indicated airspeed will furnish the limit during the later stages.

3. The necessity of including aerodynamic-braking devices will become increasingly important with future transports if reasonable rates of descent are to be attained without large increases in either the airspeed or Mach number. The size and the projected area of such brakes should, however, be coordinated with the requirements of passenger comfort and the type of cabin pressurization used.

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REFERENCES
