NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

REPORT No. 777

THE THEORY OF PROPELLERS
III—THE SLIPSTREAM CONTRACTION WITH NUMERICAL
VALUES FOR TWO-BLADE AND FOUR-BLADE PROPELLERS

By THEODORE THEODORSEN

1944
AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Metric</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unit</td>
<td>Abbreviation</td>
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</tr>
<tr>
<td>Time</td>
<td>t</td>
<td>second</td>
</tr>
<tr>
<td>Force</td>
<td>F</td>
<td>weight of 1 kilogram</td>
</tr>
<tr>
<td>Power</td>
<td>P</td>
<td>horsepower (metric)</td>
</tr>
<tr>
<td>Speed</td>
<td>V</td>
<td>meters per second</td>
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</tbody>
</table>

2. GENERAL SYMBOLS

- $W$: Weight = $mg$
- $g$: Standard acceleration of gravity = 9.80665 m/s$^2$ or 32.1740 ft/sec$^2$
- $m$: Mass = $g$
- $I$: Moment of inertia = mk$^2$. (Indicate axis of radius of gyration $k$ by proper subscript.)
- $\mu$: Coefficient of viscosity

3. AERODYNAMIC SYMBOLS

- $S$: Area
- $S_w$: Area of wing
- $G$: Gap
- $b$: Span
- $c$: Chord
- $A$: Aspect ratio, $b^2/S$
- $V$: True air speed
- $q$: Dynamic pressure, $1/2\rho V^2$
- $L$: Lift, absolute coefficient $C_L = L/qS$
- $D$: Drag, absolute coefficient $C_D = D/qS$
- $D_0$: Profile drag, absolute coefficient $C_D = D_0/qS$
- $D_i$: Induced drag, absolute coefficient $C_D = D_i/qS$
- $D_p$: Parasite drag, absolute coefficient $C_D = D_p/qS$
- $C$: Cross-wind force, absolute coefficient $C = C/qS$
- $i_w$: Angle of setting of wings (relative to thrust line)
- $i_t$: Angle of stabilizer setting (relative to thrust line)
- $Q$: Resultant moment
- $R$: Resultant angular velocity
- $\alpha$: Angle of attack
- $\epsilon$: Angle of downwash
- $\alpha_0$: Angle of attack, infinite aspect ratio
- $\sigma_i$: Angle of attack, induced
- $\gamma$: Flight-path angle
Page 18, figure 7(a): The bottom part of the lowest curve in the lower left-hand corner of the figure should be — — instead of — — — —.
REPORT No. 777

THE THEORY OF PROPELLERS
III—THE SLIPSTREAM CONTRACTION WITH NUMERICAL VALUES FOR TWO-BLADE AND FOUR-BLADE PROPELLERS

By THEODORE THEODORSEN
Langley Memorial Aeronautical Laboratory
Langley Field, Va.
National Advisory Committee for Aeronautics

Headquarters, 1500 New Hampshire Avenue NW., Washington 25, D. C.

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By Theodore Theodorsen

SUMMARY

As the conditions of the ultimate wake are of concern both theoretically and practically, the magnitude of the slipstream contraction has been calculated. It will be noted that the contraction in a representative case is of the order of only 1 percent of the propeller diameter. In consequence, all calculations need involve only first-order effects. Curves and tables are given for the contraction coefficient of two-blade and four-blade propellers for various values of the advance ratio; the contraction coefficient is defined as the contraction in the diameter of the wake helix in terms of the wake diameter at infinity. The contour lines of the wake helix are also shown at four values of the advance ratio in comparison with the contour lines for an infinite number of blades.

INTRODUCTION

Since reference is often made to the wake infinitely far behind the propeller, it is desirable to establish certain relationships between the dimensions of the propeller and those of the wake helix at infinity. The present paper considers the relationship of the propeller diameter and the wake diameter, or the problem of the slipstream contraction.

The discussion is restricted to a consideration of first-order effects, that is, to the determination of the contraction per unit of loading for infinitely small loadings only. It will be seen that the contractions are indeed very small, of the order of a few percent of the propeller diameter, and that the high-order terms are therefore not of concern. The interference velocity accordingly is neglected as small compared with the stream velocity. The wake helix lies on a perfect cylinder and the pitch angle is everywhere the same. It is noted that the assumption of zero loading corresponds to that used by Goldstein for a different purpose.

SYMBOLS

- $R$: tip radius of propeller
- $r$: radius of element of vortex sheet
- $\Delta r$: contraction
- $\Delta r_0$: total contraction or contraction at $\frac{h}{R}=0$
- $\tau$: angle between starting point of spiral line and point $P$
- $H$: pitch of spiral
- $\theta$: angular coordinate on vortex sheet
- $h=H\frac{\theta}{2\pi}$
- $\lambda$: advance ratio $\left(\frac{H}{2\pi R}\right)$
- $x$: ratio of radius of element to tip radius of vortex sheet $\left(\frac{r}{R}\right)$
- $\nu_r$: radial velocity
- $V$: advance velocity of propeller
- $w$: rearward displacement velocity of helical vortex surface
- $\bar{w} = \frac{w}{V}$
- $P$: number of blades
- $\kappa$: mass coefficient
- $c_r = 2\pi \bar{w}$
- $\Gamma$: circulation at radius $x \left(\frac{\pi c_r x V P}{\kappa} K(x)\right)$
- $K(x)$: circulation function for single rotation $\left(\frac{p \Gamma \omega}{2 \pi V \omega}\right)$
- $\omega$: angular velocity of propeller, radians per second
- $y_1$: radial velocity at point $P$ due to a doublet element at $\theta, x$ except for a constant factor
- $y_2$: angle of contraction, except for a constant factor
- $Y_1 = \int_0^1 y_2 \, dx$
- $Y_2$: contour line of contraction, except for a constant factor
- $Y_3$: contour line of contraction, except for a constant factor
- $\frac{c_r \lambda^3}{\kappa} \left(\frac{\lambda^3}{4} Y_3\right)$
- $\frac{\Delta r_0}{R}$: total contraction in terms of radius $\left(\frac{c_r \lambda^3}{\kappa} Y_3\right)$
- $\frac{\Delta r_0 \kappa}{c_r}$: contraction coefficient $\left(\frac{\lambda^3}{4} Y_3\right)$
- $z_1 = \sin \phi \left[\tan^2 \phi \left(\frac{x}{x^2 - z^2}\right) E(k) + 2z^2 [F(k) - E(k)]\right]$
THEORY

The radial velocity is obtained by using the Biot-Savart law and integrating over the entire surface of discontinuity. If \( \Delta r_0 \) is the total contraction, the problem is to determine the ratio \( \frac{\Delta r_0}{R} \) for various numbers of blades at several advance ratios. Simple expressions referring to zero loading are used throughout.

The radial inward velocity \( dv_R' \) at the point \( P \) is calculated. (See figs. 1 and 2.) This velocity results from an element of circulation \( f \, ds \), which is located on a spiral of radius \( r \) that starts in a plane perpendicular to the axis and containing the reference point \( P \). The angle between the starting point of the spiral line and the point \( P \) is designated \( \tau \). The spiral extends below the plane to infinity. If the pitch of this spiral is designated \( H \), the element at a projected angle \( \theta \) from the starting point of the spiral is then at a distance \( h \) below the reference plane where

\[
h = H \frac{\theta}{2\pi}
\]  

(1)

By introducing the nondimensional quantities

\[
\lambda = \frac{H}{2\pi R}, \quad x = \frac{\tau}{R}
\]

(2)

in the Biot-Savart law, the following expression is obtained for the radial inward velocity \( dv_R' \) due to an element on the wake helix of strength \( f \):

\[
dv_R' = \frac{1}{4\pi} \int \frac{f \, d\theta \, \lambda x}{R} \left[ \frac{\theta \cos (\theta + \tau) - \sin (\theta + \tau)}{[1 + x^2 + \lambda^2 \theta^2 - 2x \cos (\theta + \tau)]^{1/2}} \right]
\]

(3)

By differentiating equation (3) with respect to \( x \), the field of a doublet element on the helical vortex sheet is obtained, the doublet element consisting of two neighboring singlet elements each of strength \( f \). Setting \( f \, dx \) equal to \( \Gamma \) and dividing through by the stream velocity \( V \) gives

\[
dv_R' = -\frac{1}{4\pi} \int \frac{\Gamma \, d\theta \, \lambda x}{RV^2} \left[ \frac{\theta \cos (\theta + \tau) - \sin (\theta + \tau)}{[1 + x^2 + \lambda^2 \theta^2 - 2x \cos (\theta + \tau)]^{1/2}} \right]
\]

where \( v_R \) is the radial velocity at the point \( P \).

Equation (4) may be written in the form

\[
dv_R' = -\frac{1}{4\pi} \int \frac{\Gamma \, d\theta \, \lambda y_1}{RV^2}
\]

(5)

where

\[
y_1 = \frac{\theta \cos (\theta + \tau) - \sin (\theta + \tau)}{[1 + x^2 + \lambda^2 \theta^2 - 2x \cos (\theta + \tau)]^{1/2}}
\]

(6)

The function \( y_1 \) is plotted against \( \theta + \tau \) for four values of \( \lambda \) and various values of \( x \) in figures 3 to 6. With

\[
\Gamma = \frac{2\pi V w}{\rho \omega} K(x)
\]

(7)

where

\[
c_s = \frac{2\pi V w}{\rho \omega}
\]

(8)

substitution in equation (5) gives

\[
dv_R' = -\frac{1}{4\pi} \int \frac{\lambda^2 c_s K(x)}{\rho k} y_1 \, d\theta
\]

(9)
If the point $P$ is at a distance $h=H \frac{\theta}{2\pi}$ below the propeller, integrating equation (9) over the wake yields

$$\frac{v_R}{V} = -\frac{\lambda^2 c_2}{4} \int_0^\infty \int_0^1 K(x) \sum_n y_i \, dx \, d\theta$$  \hfill (10)

It is noted that, with equally spaced blades, the function

$$\sum_n y_i$$

is an odd function of $\theta$ and

$$\sum_n \int_0^\infty y_i \, d\theta = 0$$  \hfill (12)

Equation (10) can therefore be rewritten as

$$\frac{v_R}{V} = -\frac{\lambda^2 c_2}{4} \int_0^\infty K(x) \sum_n y_i \, dx \, d\theta$$  \hfill (13)

Let

$$y_i = \frac{K(x)}{p} \sum_n y_i$$

$$Y_1 = \int_0^1 y_2 \, dx$$

$$Y_2 = \int_0^\infty Y_1 \, d\theta$$  \hfill (14)

Values of $Y_1$ and $Y_2$, multiplied by a constant factor for convenience in plotting, are given in tables I to IV for two-blade and four-blade propellers for which $\lambda$ and $\theta$ take on various values. These functions are plotted against $\theta$ in figures 7 and 8.

Equation (13) becomes

$$\frac{v_R}{V} = -\frac{\lambda^2 c_2}{4} Y_2$$  \hfill (15)

Now

$$\frac{v_R}{V} = \frac{dR}{dh} = \frac{dR}{d\theta} = \frac{1}{R^2}$$  \hfill (16)

Therefore

$$\frac{dx}{d\theta} = \frac{\lambda^2 c_2}{4} \frac{v_R}{V} = -\frac{\lambda^2 c_2}{4} Y_2$$  \hfill (17)

whence

$$\Delta r = R_2 - R_1 = \frac{1}{R} \int_{R_1}^{R_2} dx = -\frac{\lambda^2 c_2}{4} \int_{R_1}^{R_2} Y_2 \, d\theta$$  \hfill (18)

If $R_e$ is the radius at the propeller and $R_t$ is the ultimate radius of the wake ($\theta_0=0, \theta_t=\infty$),

$$\Delta r = \frac{c_2 \lambda^2}{4} \int_0^\infty Y_2 \, d\theta = \frac{\lambda^2}{4k} Y_2$$  \hfill (19)

where

$$Y_2 = \int_0^\infty Y_1 \, d\theta$$

Values of $Y_2$ are given in tables V and VI and are plotted in figure 9 for two-blade and four-blade propellers for which $\lambda$ and $\theta$ take on various values.

After all substitutions are made, the complete multiple integral for the total contraction is obtained as

$$\frac{\Delta r}{R} = \frac{c_2 \lambda^2}{4} \int_0^\infty \int_0^\infty K(x) \frac{1}{p} \sum_n y_i(\theta, x) \, dx \, d\theta$$  \hfill (20)

(See figs. 10 and 11.)

**INFINITE NUMBER OF BLADES**

For purposes of comparison, it is useful to obtain the contraction for the case of an infinite number of blades. By resolving the circulation into components parallel to and perpendicular to the axis of the wake, the helical vortices can be replaced by a system of vortices parallel to the axis and another of ring vortices having centers on the axis. Only the ring vortices contribute to the radial velocity.

The field due to a vortex ring of strength $f$ and radius $r=Rx$, located at a distance $h$ below the reference point $P$, is given by Lamb (reference 1, p. 237). In the notation of the present paper it is

$$\psi' = -\frac{fR}{2\pi} \lambda \left( \frac{2-k}{2} \right) F(k) - \frac{2}{k} E(k)$$  \hfill (21)

where $E(k)$ and $F(k)$ are the complete elliptic integrals and

$$k^2 = \frac{x}{(h/R)^2 + (1+k)^2}$$

As before, a doublet ring is obtained by differentiating equation (20) with respect to $x$. By setting

$$f dx = \frac{\Gamma}{h}$$

the following expression is obtained for the field of a doublet ring:

$$\psi'' = \frac{\Gamma R}{8\pi \hbar} k \left( \frac{1}{2} k^2 E(k) + \frac{1}{2} \left( \frac{2F(k) - 2k^2}{1-k^2 E(k)} \right) \right)$$  \hfill (22)

In order to obtain the effect of the entire vortex system, equation (21) is integrated with respect to $h/R$ and $x$ as

$$\psi(h/R) = \int_{-h/R}^{h/R} \frac{\Gamma R^2}{8\pi h} k \left( \frac{1}{2} k^2 E(k) + \frac{1}{2} \left( \frac{2F(k) - 2k^2}{1-k^2 E(k)} \right) \right) dx$$  \hfill (23)

The radial velocity

$$v_R = \frac{1}{R^2} \frac{\partial \psi}{\partial h}$$
Equation (22) may be written in the form
\[
\psi \left( \frac{h}{R} \right) = \frac{R^2}{8\pi H} \int_0^1 \Gamma \int_{-\infty}^\infty \Phi \left( \frac{h}{R}, x \right) d\frac{h}{R} dx
\]
so that
\[
v_e = -\frac{1}{8\pi H} \int_0^1 \Gamma \int_{-\infty}^\infty \Phi \left( \frac{h}{R}, x \right) dx
\]
\[
= -\frac{1}{8\pi H} \int_0^1 \Gamma \int_{-\infty}^\infty \Phi \left( \frac{h}{R}, x \right) dx
\]
(24)

Now
\[
\Delta r = \int \frac{c_1}{R} \frac{dR}{dh}
\]
the following equation is finally obtained:
\[
\Delta r = \frac{c_1}{16\pi^3} \int_0^1 \left( \frac{x^2}{\lambda^2 + x^2} \right)^{\frac{3}{2}} dx \int_{\infty}^\infty \left( \frac{x}{\lambda^2 + x^2} \right) E(k) + \frac{1}{1-k^2} \left[ 2F(k) - \frac{2-k^2}{1-k^2} E(k) \right] d\frac{h}{R}
\]
(28)

For convenience in using the Legendre tables, the second integral is written in the form
\[
\int_{\infty}^\infty z_1 \frac{d\frac{h}{R}}{R}
\]
(29)

where
\[
z_1 = \sin \phi \left[ \tan \phi \left( x - \frac{\lambda^2}{k} \right) E(k) + 2\frac{\lambda^2}{k} [F(k) - E(k)] \right]
\]
and
\[
\left( \frac{h}{R} \right)^2 = \frac{4x}{\sin^2 \phi} - (1+x)^2
\]
or
\[
\phi = \sin^{-1} k
\]
(See fig. 12 for plots of \( z_1 \) against \( h/R \).)

The final expression is
\[
\frac{\Delta r}{R} = \frac{c_1}{16\pi^3} \int_0^1 \left( \frac{x^2}{\lambda^2 + x^2} \right)^{\frac{3}{2}} dx \int_{\infty}^\infty \sin \phi \left[ \tan \phi \left( x - \frac{\lambda^2}{k} \right) E(k) + 2\frac{\lambda^2}{k} [F(k) - E(k)] \right] d\frac{h}{R}
\]
(See table VII and fig. 13.)

INFFINITE NUMBER OF BLADES FOR DUAL ROTATION

The contraction for a dual-rotating propeller with an infinite number of blades is next obtained. In this case \( K(x)=1 \), and the radial velocity is
\[
v_e = -\frac{\Gamma}{2\pi H} \left[ \frac{2}{k} - k \right] F(k) - \frac{2}{k} E(k) \left( \frac{h}{R} \right)^{\frac{3}{2}}
\]
Since the value at the lower limit is zero and \( K(x)=1 \) for an infinite number of blades, it follows by substituting the value of \( \Gamma \) that
\[
\Delta r = \frac{c_1}{4\pi} \int_{\infty}^\infty \left( \frac{h}{R} \right) d\frac{h}{R}
\]
where
\[
w_1 = \left[ \frac{2}{k} - k \right] F(k) - \frac{2}{k} E(k) \left( \frac{h}{R} \right)^{\frac{3}{2}}
\]
(See table VIII and fig. 14.)

CONCLUDING REMARKS

The contraction coefficients are given for two-blade and four-blade single-rotating propellers at four specific values of the advance ratio. The calculations involve triple integrations and are therefore somewhat laborious and susceptible to numerical errors. Until more convenient methods are devised to perform this integration, it is hoped that the values given in this paper will serve the purpose. It is well to notice the small magnitude of the contraction. A four-blade propeller with normal loading and advance ratio is shown to have a total contraction in terms of the radius of less than one percent. The first-order treatment embodied in the paper is therefore adequate for all technical purposes.

LANGLEY MEMORIAL AERONAUTICAL LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., OCTOBER 10, 1944.

REFERENCE

### TABLE I.—FUNCTION $\frac{\lambda^3 Y_i}{4}$ FOR TWO-BLADE PROPELLER

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<th>$\theta$ (deg)</th>
<th>$\frac{\lambda^3 Y_i}{4}$</th>
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### TABLE II.—FUNCTION $\frac{\lambda^3 Y_i}{4}$ FOR FOUR-BLADE PROPELLER

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### TABLE III.—FUNCTION $\frac{\lambda^3 Y_i}{4}$ FOR TWO-BLADE PROPELLER

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<th>$\theta$ (deg)</th>
<th>$\frac{\lambda^3 Y_i}{4}$</th>
<th>$\theta$ (deg)</th>
<th>$\frac{\lambda^3 Y_i}{4}$</th>
<th>$\theta$ (deg)</th>
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### Table V.—Function $\frac{\lambda^3}{4y^3}$ for Two-Blade Propeller

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<th>$\theta$ (deg)</th>
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<th>$\lambda = 5\theta$</th>
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### Table VII.—Contour Lines—Single-Rotating Propeller

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<th>$\alpha_1$</th>
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<tbody>
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### Table VIII.—Contour Lines—Dual-Rotating Propeller

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</table>
FIGURE 3.—The function $v_1$ for $\lambda = \frac{3}{4}$ and four values of $\tau$.
(a) \(x=\frac{3}{4}\).  
(b) \(x=\frac{3}{4}\).  
(c) \(x=\frac{3}{4}\).  
Figure 3.—Continued.
Figure 3.—Concluded.

Figure 4.—The function \( y_1 \) for \( \lambda = \frac{1}{4} \) and four values of \( \tau \).
FIGURE 4.—Continued.
THEORY OF PROPELLERS. III—SLIPSTREAM CONTRACTION

FIGURE 4.—Continued.

(d) \( x = \frac{1}{3}, \frac{1}{2}, \text{and} \frac{3}{4}; r = 270^\circ. \)

(e) \( x = \frac{3}{4}. \)
FIGURE 4.—Concluded.
THEORY OF PROPELLERS. III—SLIPSTREAM CONTRACTION

FIGURE 5.—The function $y_1$ for $\lambda=1$ and four values of $r$.

(a) $z=34$.

(b) $z=36$. 
Figure 5.—Continued.
Figure 5.—Concluded.

Figure 6.—The function $y_1$ for $\lambda = 1/2$ and four values of $r$. 
(b) $z=\frac{1}{4}$.

(c) $z=\frac{3}{4}$.

(d) $z=\frac{3}{4}$, $\frac{3}{2}$, and $\frac{3}{4}$; $r=20^\circ$.

Figure 6.—Continued.
FIGURE 6.—Concluded.
(a) Two-blade propeller. (See table I.)

(b) Four-blade propeller. (See table II.)

FIGURE 7.—The function $\frac{1}{2} \gamma_1$ against $\theta$ for four values of $\lambda$. 
(a) Two-blade propeller. (See table III.)

(b) Four-blade propeller. (See table IV.)

Figure 8.—The function $\frac{\lambda}{2} \psi$ against $\theta$ for four values of $\lambda$. 
(a) Two-blade propeller. (See table V.)

(b) Four-blade propeller. (See table VI.)

Figure 9.—The contour function \( \frac{\Delta R}{R} = \frac{\Delta V}{V} \) against \( \theta \) for four values of \( \lambda \).
THEORY OF PROPELLERS. III—SLIPSTREAM CONTRACTION

Figure 10—Contraction coefficient $\frac{\Delta p}{\rho c^2}$ against $\frac{x}{C}$ for two- and four-blade propellers.

Figure 11—Contraction coefficient $\frac{\Delta p}{\rho c^2}$ against $\frac{x}{C}$ for two- and four-blade propellers.

Figure 12—The function $z$ against $\frac{x}{C}$ for several values of $z$. 
Figure 12.—Contour lines of wake helices. (See table VII.)
Figure 14.—Contour lines of wake for $p = \infty$. Dual rotation. (See table VIII.)
Positive directions of axes and angles (forces and moments) are shown by arrows.

<table>
<thead>
<tr>
<th>Axis</th>
<th>Force (parallel to axis) symbol</th>
<th>Moment about axis</th>
<th>Angle</th>
<th>Velocities</th>
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<td>Symbol</td>
<td>Designation</td>
<td>Symbol</td>
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<td>Rolling</td>
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<td>$Y \rightarrow Z$</td>
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<td>Pitching</td>
<td>$M$</td>
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<td>Normal</td>
<td>$Z$</td>
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<td>$N$</td>
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Absolute coefficients of moment

$$C_l = \frac{L}{\frac{q}{\delta} S}, \quad C_n = \frac{M}{\frac{q}{\delta} S}, \quad C_y = \frac{N}{\frac{q}{\delta} S}$$

4. PROPELLER SYMBOLS

- $D$: Diameter
- $p$: Geometric pitch
- $p/D$: Pitch ratio
- $V'$: Inflow velocity
- $V$: Slipstream velocity
- $T$: Thrust, absolute coefficient $C_s = \frac{T}{\rho n^2 D^4}$
- $Q$: Torque, absolute coefficient $C_q = \frac{Q}{\rho n^2 D^4}$

$$P$$: Power, absolute coefficient $C_p = \frac{P}{\rho n^2 D^4}$

$$C_t$$: Speed-power coefficient $= \frac{s}{\rho V^3}$

$$\eta$$: Efficiency

$$n$$: Revolutions per second, rps

$$\phi$$: Effective helix angle $= \tan^{-1}\left(\frac{V}{2\pi n}\right)$

5. NUMERICAL RELATIONS

1 hp = 76.04 kg·m/s = 550 ft-lb/sec
1 metric horsepower = 0.9863 hp
1 mph = 0.4470 mps
1 mps = 2.2369 mph

1 lb = 0.4536 kg
1 kg = 2.2046 lb
1 mi = 1,609.35 m = 5,280 ft
1 m = 3.2808 ft