REPORT No. 749

PROPELLER CHARTS FOR THE DETERMINATION OF THE ROTATIONAL SPEED FOR THE MAXIMUM RATIO OF THE PROPULSIVE EFFICIENCY TO THE SPECIFIC FUEL CONSUMPTION

By David Biermann and Robert N. Conway

SUMMARY

A set of propeller operating efficiency charts, based on a coefficient from which the propeller rotational speed has been eliminated, is presented. These charts were prepared with data obtained from tests of full-size metal propellers in the NACA propeller-research tunnel. Working charts for nine propeller-body combinations are presented, including results from tests of dual-rotating propellers.

These charts are to be used in the calculation of the range and the endurance of airplanes equipped with constant-speed propellers in which, for given flight conditions, it is desired to determine the propeller revolution speed that gives the maximum ratio of the propulsive efficiency to the specific fuel consumption. The coefficient on which the charts are based may be written in the form of a thrust coefficient or a thrust-power coefficient.

A method of using the charts is outlined and sample computations for a typical airplane are included.

INTRODUCTION

In the calculation of the range and the endurance of airplanes equipped with constant-speed propellers, the thrust horsepower required for flight at a given gross weight, airspeed, and altitude is known. It is desired to determine, from the propeller-engine operating characteristics, the propeller rotational speed that gives the maximum ratio of the propulsive efficiency to the specific fuel consumption. The usual propeller coefficients are insufficient for use in such calculations because knowledge of both the engine power and the rotational speed is required and these quantities are unknown.

In order to facilitate the calculations, the Bureau of Aeronautics, Navy Department, requested the NACA to prepare working plots of a thrust coefficient that is independent of propeller revolution speed. The present report contains such working charts for nine propeller-body combinations and includes results obtained from tests of dual-rotating propellers. A method that can be used for the determination of the maximum value of the ratio of the propulsive efficiency to the specific fuel consumption is also included.

SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>power of engine, foot-pounds per second</td>
</tr>
<tr>
<td>$T_e$</td>
<td>effective thrust $(T - \Delta D)$, pounds</td>
</tr>
<tr>
<td>$T$</td>
<td>total propeller thrust</td>
</tr>
<tr>
<td>$\Delta D$</td>
<td>increased drag of body due to propeller slipstream</td>
</tr>
<tr>
<td>$W$</td>
<td>weight, pounds</td>
</tr>
<tr>
<td>$n$</td>
<td>propeller rotational speed, rps</td>
</tr>
<tr>
<td>$L/D$</td>
<td>lift-drag ratio</td>
</tr>
<tr>
<td>$V$</td>
<td>airspeed, feet per second</td>
</tr>
<tr>
<td>$D$</td>
<td>propeller diameter, feet</td>
</tr>
<tr>
<td>$\rho$</td>
<td>mass density of air, slugs per cubic foot</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>mass density of air at sea level (0.002378)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>relative density of air $(\rho/\rho_0)$</td>
</tr>
<tr>
<td>$q$</td>
<td>dynamic pressure $(\frac{1}{2}\rho V^2)$</td>
</tr>
<tr>
<td>$C$</td>
<td>engine specific fuel consumption, pounds per brake horsepower-hour</td>
</tr>
<tr>
<td>$Q$</td>
<td>engine torque, foot-pounds</td>
</tr>
<tr>
<td>$C_p$</td>
<td>speed-power coefficient $(\frac{4PV}{\pi D^2})$</td>
</tr>
<tr>
<td>$V/nD$</td>
<td>advance-diameter ratio</td>
</tr>
<tr>
<td>$T_e'$</td>
<td>thrust coefficient $(\frac{T_e}{\sigma D^2 \sqrt{\rho}})$</td>
</tr>
<tr>
<td>$C_p'$</td>
<td>power coefficient $(\frac{\rho}{\sigma D^2 \sqrt{\rho}})$</td>
</tr>
<tr>
<td>$P_e$</td>
<td>power disk-loading coefficient $(\frac{8P}{\pi D^2 \sqrt{\rho}})$</td>
</tr>
</tbody>
</table>

DISCUSSION

In the calculation of the range and the endurance of airplanes equipped with constant-speed propellers it is desired to determine, from the engine-propeller operating characteristics, the propeller speed that gives the maximum ratio of the propulsive efficiency to the specific fuel consumption. The thrust horsepower required for a given flight at a given gross weight, airspeed, and altitude is already known.

The usual propeller coefficients of power, thrust, and propulsive efficiency that are defined as
are insufficient for this purpose because neither the
engine power nor the engine speed is known.
In order to solve these special problems with the least
work it is necessary to use plots of propeller coefficients
that do not involve the rotational speed. Suitable
forms of the coefficients may be obtained from the usual
thrust or power coefficients as follows:

\[ T_c = \frac{C_p}{(V/nD)^3} \frac{T}{\rho D^3 V^2} \]

\[ C_p' = \frac{C_p}{(V/nD)^3} \frac{P}{\rho D^3 V^2} \]

Because the brake horsepower is unknown, the equa-
tion for \( C_p' \) should be multiplied by \( \eta \) to obtain a thrust-
power coefficient:

\[ \eta C_p' = \eta \frac{C_p}{(V/nD)^3} \frac{P}{\rho D^3 V^2} \]

The two general forms of the coefficient are \( \eta P/\rho D^3 V^2 \)
and \( T_c/\rho D^3 V^2 \), which may be converted into engineering
units as follows:

\[ \eta \frac{P}{\rho D^3 V^2} = \frac{73300 \text{ thp}}{\sigma D^4 (mph)^3} \]

and

\[ \frac{T_c}{\rho D^3 V^2} = \frac{195.4 \ T_c}{\sigma D^4 (mph)^3} \]

**PROPELLER DATA**

The propeller charts were prepared from data ob-
tained in the NACA 20-foot propeller-research tunnel. The tests that provided the data were made with tractor
propellers of approximately 10-foot diameter operating
in conjunction with various representative body types. A list of the charts and the basic propeller data are
given in the following table; principal model dimensions
are given directly on the propeller charts.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Propeller drawing number</th>
<th>Total number of blades</th>
<th>Body condition</th>
<th>Approximate propeller diameter (ft)</th>
<th>Rotation</th>
<th>Blade angle range at 0.75 R (deg)</th>
<th>Source of propeller data, reference number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2</td>
<td>3106-4</td>
<td>3</td>
<td>Streamline body, with spinner, on symmetrical wing</td>
<td>10</td>
<td>Single</td>
<td>30 to 65</td>
<td>1</td>
</tr>
<tr>
<td>3, 4</td>
<td>3106-A-3</td>
<td>3</td>
<td>Streamline body, with spinner</td>
<td>11.26</td>
<td>...do</td>
<td>30 to 65</td>
<td>1</td>
</tr>
<tr>
<td>5, 8</td>
<td>3106-4</td>
<td>4</td>
<td>Streamline body, with Spinner, on symmetrical wing</td>
<td>12</td>
<td>...do</td>
<td>30 to 65</td>
<td>2</td>
</tr>
<tr>
<td>9, 10</td>
<td>3106-4</td>
<td>6</td>
<td>...do</td>
<td>10</td>
<td>...do</td>
<td>30 to 65</td>
<td>2</td>
</tr>
<tr>
<td>11, 12</td>
<td>3106-5, 6, 7, 8</td>
<td>6</td>
<td>...do</td>
<td>10</td>
<td>...do</td>
<td>30 to 65</td>
<td>2</td>
</tr>
<tr>
<td>13, 14</td>
<td>3106-5</td>
<td>3</td>
<td>Liquid-cooled engine nacelle, with spinner</td>
<td>10</td>
<td>Single</td>
<td>15 to 65</td>
<td>3</td>
</tr>
<tr>
<td>15, 17</td>
<td>5866-6</td>
<td>3</td>
<td>Radial engine nacelle</td>
<td>10</td>
<td>...do</td>
<td>15 to 65</td>
<td>4</td>
</tr>
<tr>
<td>16, 18</td>
<td>5866-9</td>
<td>4</td>
<td>Liquid-cooled engine nacelle, with spinner</td>
<td>10</td>
<td>...do</td>
<td>15 to 65</td>
<td>6</td>
</tr>
</tbody>
</table>

**PROPELLER CHARTS**

Two charts are provided for each of the propeller-body
combinations. One chart contains curves of \( V/nD \)
plotted against \( T_c \) between 0 and 0.10 with contour
lines of constant \( \eta \). The other chart contains curves of
\( T_c \) between 0 and 0.40 and curves of \( \eta \) plotted against
\( V/nD \). The same quantities may be read on either type
of plot but greater accuracy is obtained by the use of the
first type for the range of \( T_c \) values that it covers. The
second type of chart is included because it offers greater
convenience in extending the range of \( T_c \) values.

**USE OF CHARTS**

It is possible that propeller charts of the type given
in this report may have a number of uses that are not
anticipated at the present time. Also, it is quite
likely that different methods can be devised for using
the charts in solving any specific problem. The chief
object of these charts is, however, to furnish a means of
determining the optimum propeller speed for given
flight conditions to obtain the maximum ratio of the
propulsive efficiency to the engine fuel consumption.
A method of obtaining the optimum propeller speed is
presented in the following outline.
Figure 1.—Chart of $V/nD$ against $T_x$ for three-blade propeller S154-6. Streamline body, with spinner, on symmetrical wing; single rotation. Propeller data taken from reference 1.
Figure 2.—Chart of $\eta$ against $V/nD$ for three-blade propeller 618A-3. Streamline body, with spinner, on symmetrical wing; single rotation. Propeller data taken from reference 1.

Figure 4.—Chart of $\eta$ against $V/nD$ for three-blade propeller 618A-3. Streamline body, with spinner; single rotation. Propeller data taken from reference 1.
Figure 5.—Chart of $V/nD$ against $T_h$ for three-blade propeller 6106A-8. Streamline body, with spinner; single rotation. Propeller data taken from reference 1.
FIGURE 5—Chart of $V/nD$ against $T_r$ for four-blade propeller 815-6. Streamline body, with spinner, on symmetrical wing; single rotation. Propeller data taken from reference 2.
Figure 6.—Chart of \( \eta \) against \( V/nD \) for four-blade propeller 3156-4. Streamline body, with spinner, on symmetrical wing; single rotation. Propeller data taken from reference 2.

Figure 7.—Chart of \( \eta \) against \( V/nD \) for six-blade propeller 3156-4. Streamline body, with spinner, on symmetrical wing; single rotation. (The 50° efficiency curve was interpolated.) Propeller data taken from reference 2.
Figure 7.—Chart of $V/nD$ against $\tau$ for six-blade propeller $110-6$. Streamline body, with spinner, on symmetrical wing; single rotation. Propeller data taken from reference 3.
Figure 9.—Chart of $V_D$ against $T_e$ for two two-blade propellers 8150-4 and 8156-4. Streamline body, with spinner, on symmetrical wing; dual rotation. Propeller data taken from reference 2.
Figure 12.- Chart of $\eta$ against $V/nD$ for two three-blade propellers 3156-6 and 3160-6. Streamline body, with spinner, on symmetrical wing; dual rotation.
Propeller data taken from reference 2.
Maximum Ratio of Propulsive Efficiency to Specific Fuel Consumption
Figure 13.—Chart of $\frac{V}{nD}$ against $T_{\theta}$ for three-blade propeller 860-6. Liquid-cooled engine model, with spinner; single rotation. Propeller data taken from reference 3.
Figure 1A: Chart of \( \eta \) against \( V/nD \) for three-blade propeller S66-8. Liquid-cooled engine nacelle, with spinner, single rotation. Propeller data taken from reference 2.
Figure 16.—Chart of \( V/nD \) against \( T_s \) for four-blade propeller 5666-9. Liquid-cooled engine nose, with spinner; single rotation. Data taken from reference 5.
Given: Aerodynamic characteristics of airplane and flight conditions:
(1) $L/D$ for speed of flight and altitude (or thrust power for speed of flight and altitude)
(2) Weight, $W$
(3) Airspeed, $V$
(4) Diameter of propeller, $D$
(5) Density at altitude, $\rho$
(6) Engine gear ratio

Characteristics of engine, including:
(1) Brake horsepower for altitude of flight as a function of engine speed and manifold pressure
(2) Specific fuel consumption as a function of engine speed and manifold pressure or torque

To obtain: Propeller speed to effect the maximum ratio of propulsive efficiency to specific fuel consumption
Solution based on thrust coefficient, $T_e$:
(1) Determine effective thrust $T_e$ from $L/D$ and $W$
(2) Solve for thrust coefficient
$$T_e = \frac{T_e}{\rho D^2 V^2} = \frac{195.4 \ T_e}{\sigma D^3 (mph)^3}$$
(3) For several values of propeller speed $n$, solve for $V/nD$

(4) For several values of $V/nD$ computed under step (3), read from propeller chart values of propulsive efficiency $\eta$
(5) For several values of $n$, compute propeller torque $Q = \frac{T_e V}{2\pi n}$

(6) For several propeller values of $n$, compute $n/n_{\text{rated}}$ and $Q/Q_{\text{rated}}$ ($n_{\text{rated}}$ and $Q_{\text{rated}}$ correspond to rated engine speed and torque values with gear ratio taken into account)

(7) From fuel-consumption data, determine values of engine specific fuel consumption $C$ corresponding to engine speeds and torques computed under step (6)
(8) For several values of $n$, compute and plot values of $\eta/C$
(9) From curve obtained in step (8), determine $n$ for maximum value of $\eta/C$

Solution based on thrust coefficient, $\eta C_P$:
(1) Determine thrust power $\eta P$ or thrust horsepower from curves of power against speed of given data
(2) Solve for $\eta C_P' = \frac{\eta P}{\rho D^2 V^2} = \frac{73300 \ lhp}{\sigma D^3 (mph)^3}$
(3) The rest of the solution corresponds to steps in the solution based on thrust coefficient, except that \( Q = \frac{n^2}{2\pi \eta} \)

**SAMPLE CALCULATION**

A calculation for a sample airplane is herein performed to demonstrate the use of the charts. Values of \( n \) corresponding to the maximum value of \( \eta/C \) were determined for three flight speeds; the computations for only the condition of flight at maximum \( L/D \) of the airplane are included. Range computations have also been made for the three flight speeds based on a specific fuel load.

In addition, range computations were made for the same flight speeds but with the engine torque and speed adjusted according to the relation \( Q \propto n^3 \) to demonstrate the value of adjusting the engine speed to obtain the maximum engine-propeller operating efficiency. (The relation \( Q \propto n^3 \) corresponds to the operation with a fixed-pitch propeller.)

The two charts necessary for the calculations are included: One chart is a plot of \( L/D \) against airspeed for the sample airplane (fig. 19); the other chart is a fuel-consumption chart (fig. 20). The fuel-consumption chart was prepared from data taken during NACA tests of a 1340-5 cylinder from a Pratt & Whitney 1340-SIH1-G engine.

Calculations were made for three conditions of flight, namely: maximum \( L/D \), 50-percent power, and 75-percent power.

**Given:**  
- Weight: gross = 17,500 pounds; gas load = 5930 pounds  
- Power plant: two 700-horsepower engines; engine speed = 2380 rpm, geared 16:9  
- Propellers: two three-blade Hamilton Standard 3155-6 propellers; diameter = 11 feet; propeller speed = 1340 rpm  
- Maximum velocity = 210 miles per hour at 8100 feet  
- Flight altitude: 10,000 feet

**Solution for flight at maximum \( L/D \) based on thrust coefficient \( T_e \) (calculations are shown for one point on fig. 21; other values are given in table 1):**

1. Maximum \( L/D = 15.1 \) (from fig. 19)  
2. \( V \) for maximum \( L/D = 116 \) miles per hour = 170 feet per second  
3. \( T_e = \frac{17500}{L/D} = 1160 \) pounds or 580 pounds per engine
4. \( \frac{195.4 \times 580}{0.738 \times 121 \times (1.346 \times 10^5)} \) = 0.0945
5. If it is assumed that \( n = 20 \) rps,  
   \( \frac{170}{20 \times 11} = 0.773 \)
6. For \( V/nD = 0.773 \) and \( T_e = 0.0945, \eta = 0.823 \) (fig. 1)  
7. \( Q = \frac{580 \times 170}{2 \pi \times 20 \times 0.823} = 953 \) foot-pounds  
8. \( n_{rated} = 22.33 \) rps,  
   \( Q_{rated} = \frac{550 \times 700}{2 \pi \times 22.33} = 2750 \) foot-pounds  
   At \( n = 20 \) rps,  
   \( \frac{170}{20 \times 11} = 0.897 \)
   and \( Q/Q_{rated} = 0.346 \)

**Figure 19:** Variation of \( L/D \) with airspeed for sample airplane.

**Figure 20:** Effect of speed on minimum fuel consumption of several torque values. The 1340-5 cylinder.

(7) From figure 20, by extrapolation,  
   at \( n/n_{rated} = 0.897 \) and \( Q/Q_{rated} = 0.346, \)  
   \( C = 0.600 \) pound per brake horsepower per hour

(8) \( \frac{0.823}{0.600} = 1.37 \)
(9) \( \eta/C \) is plotted against \( n \) in figure 21, together with the corresponding values of \( V, C, \) and \( \eta \), which have been added as a matter of interest. From this figure it is seen that the maximum value of \( \eta/C \) occurs at \( n = 13.7 \).

The results of similar computations for flight at 50-percent and 75-percent power are given in table I.

The range for the three flight speeds was calculated from the Breguet range formula:

\[
\text{Range, miles} = 863 \frac{L}{D} \log \frac{W_g}{W_e},
\]

where \( W_g \) is the gross weight and \( W_e \) is the gross weight loss the gas load.

In the Breguet formula, \( L/D \) is assumed constant and the values of \( \eta/C \) are average during flight; whereas, the values of \( \eta/C \) as used in the problem are for the start of flight.

For maximum \( \eta/C \) at maximum \( L/D \),

Range, miles = 863 \times 15.1 \times 1.62 \times 0.181 = 3820

With the data from table I, the range at 50-percent power was computed as 3680 miles and at 75-percent power the range was 2940 miles. The variation of the range with the airspeed is given in figure 22. For comparison, figure 22 contains a plot of the range for the same airplane equipped with both controllable and fixed-pitch propellers, the fixed-pitch propeller being set for the condition of high-speed flight. Because the gain in range by the use of controllable-pitch propellers set for maximum engine-propeller efficiency was relatively small for the conditions assumed in the sample computations, it would appear that the optimum blade-angle setting is fairly close to the high-speed setting. The stating of such a generality is hazardous, however, chiefly because the fuel-consumption data herein presented may not apply to engines in general.

**TABLE I**

**CALCULATION OF \( \eta/C \)**

<table>
<thead>
<tr>
<th>Flight conditions</th>
<th>( \eta/C )</th>
<th>( \eta/C )</th>
<th>( \eta/C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L/D = 15.1 )</td>
<td>78</td>
<td>82.0</td>
<td>89.3</td>
</tr>
<tr>
<td>( V = 115 \text{ mph} )</td>
<td>17</td>
<td>17.1</td>
<td>18.1</td>
</tr>
<tr>
<td>( T = 1100 \text{ lb} )</td>
<td>15</td>
<td>15.3</td>
<td>16.2</td>
</tr>
<tr>
<td>( \eta = 0.0065 )</td>
<td>12</td>
<td>12.3</td>
<td>13.1</td>
</tr>
</tbody>
</table>

**REFERENCES**