ANALYSIS OF OFF-DESIGN OPERATION OF HIGH MACH NUMBER SUPERSONIC TURBOJET ENGINES

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For two hypothetical turbojet engines, one of which was designed for a flight Mach number of 2.5 and the other for 3.0, engine operation during take-off was analyzed for four methods of operating the engines off-design. These off-design operational methods were compared on the basis of engine thrust, engine specific impulse, and the operating requirements which the compressor and turbine must fulfill. The way in which a given engine should be operated off-design depends on the characteristics of its particular compressor and turbine. Because the characteristics of each set of components will differ one from another, no one mode of operation can be selected as best for all engines and operating conditions. This analysis indicates the methods that appear most promising and the way in which they can best be exploited.

If the compressor is capable of operation in a constant-geometry engine at equivalent rotational speeds considerably above the design value, highest engine thrust and highest engine specific impulse are obtained with compressor overspeed operation. Since during such engine off-design operation the turbine operates at its design point, the turbine can be designed very near tolerable operating limits. Turbine stator adjustment in combination with compressor overspeeding is better than turbine stator adjustment alone. Engine equivalent rotational speed should be raised as much as the compressor-surge characteristics will permit, and the additional thrust required should then be obtained by means of a further increase in turbine-inlet temperature in combination with turbine stator adjustment. If the compressor-surge characteristics limit the turbine-inlet temperature to values less than the rated value, compressor-exit bleed can be used to permit an additional rise in turbine-inlet temperature. This bleeding can provide relatively large amounts of cooling air during take-off operation. The turbine operation moves away from limiting blade loading during such off-design operation, and the blade-jet speed ratio varies by a comparatively small amount. For the assumed compressor characteristics, increasing engine thrust by means of exhaust-nozzle adjustment requires an increase in turbine size and thereby results in rather severe penalties in turbine design-point performance.
INTRODUCTION

One of the principal problems in the design of a turbojet engine for use at high supersonic speeds (flight Mach numbers greater than 2) is the incorporation of sufficient operational versatility into the engine to provide good engine performance during both top-speed flight and take-off, or low-speed, flight. High engine thrust under any flight condition is obtained by operating the engine at a limit on turbine-inlet temperature imposed by properties of the engine materials. Production of high engine thrust during both high- and low-speed flight therefore requires that, because of the large variation in ram temperature, the engine be operated over a wide range of engine temperature ratio (ratio of turbine-inlet to compressor-inlet temperature). This variation in engine temperature ratio requires that either the compressor or turbine, or both, be operated under conditions requiring a wide range of internal flow conditions.

In order to achieve satisfactory operation in flight, it was suggested at the NACA Lewis laboratory that high-speed supersonic turbojet engines be designed for the top-speed flight conditions; for other flight conditions (such as take-off) the turbine-inlet temperature, the compressor equivalent rotational speed, and the compressor pressure ratio would be maintained constant by appropriate adjustment of the turbine stator and the exhaust nozzle. Reference 1 showed that this method of engine operation is capable of producing high engine thrust at low flight speeds, but that, for some turbine design conditions, the turbine must operate over a wide range of operating conditions, and the turbine aerodynamic design must be made conservative in order to satisfy the turbine off-design requirements. Because the turbine stress and turbine-tip frontal area are both critical factors for this high-speed application, such compromising of the turbine design is highly undesirable.

For this method of engine operation, the range of conditions over which the engine must operate requires that the process of engine design consider a design range rather than a design point. In practice, that operating condition for which engine performance is most important will be most seriously considered in the engine design; the other operating conditions in the range of design will generally only restrict or qualify the engine design. If the designer desires superlative engine performance during high-speed flight and if he is willing to accept mediocre engine efficiency under other operating conditions in order to realize this superlative high-speed performance, principal emphases in design will be given the high-speed flight condition. Such is the point of view that has been adopted in this report. This high-speed flight condition is herein referred to as the "design-point", and other operating conditions as "off-design".
Analysis of other methods of operating engines off-design is therefore important in order to determine whether or not another method of engine operation would result in production of sufficient engine thrust without compromising the turbine design or without introducing the mechanical complexity of turbine stator adjustment. For any method of operating an engine off its design condition, one of the essential characteristics requisite to obtaining high engine thrust is high turbine-inlet temperature. In addition to turbine stator adjustment, several other operational methods embody this characteristic of high turbine-inlet temperature. The following methods of engine operation off-design were therefore analyzed and compared at the NACA Lewis laboratory: (1) compressor overspeeding, (2) exhaust-nozzle adjustment in combination with compressor overspeeding, (3) compressor-exit bleed with or without compressor overspeeding, and (4) turbine stator adjustment with or without compressor overspeeding. The following factors are used to indicate the relative merits of these methods of off-design operation: engine thrust, engine specific impulse (lb-sec of thrust/lb of fuel), and the operating requirements that the compressor and turbine must fulfill. This analysis considers that high-speed flight is the condition for which the engine is to be designed. Engine take-off operation is the only off-design operating condition analyzed and is considered typical of the off-design operating conditions.

Operation of a compressor at the same actual rotational speed for take-off as for flight at a design Mach number of 2.5 or 3 results in an equivalent rotational speed of 130 or 145 percent, respectively, of the value in flight. Because of the lack of data on compressor characteristics at 130 or 145 percent of design equivalent rotational speed, several simplifying assumptions were made regarding the compressor characteristics during operation at equivalent rotational speeds above the design value (hereinafter called "compressor overspeeding"). In addition to results based on these assumptions, the effect of deviation from these assumptions is discussed.

Full-scale compressors cannot currently be operated at equivalent rotational speeds considerably above their design values without accepting mediocre performance for the design, or flight, conditions. The analysis presented herein shows what is gained in engine performance if both capacity for overspeeding and good design-point performance can be combined in one compressor; it does not consider the revisions necessary in compressor design techniques in order to realize such performance.

SYMBOLS

The following symbols are used in this report. The station numbers are delineated in figure 1.
A  area, sq ft
a  velocity of sound, ft/sec
b  fraction of mass flow bled at compressor exit
E  work, Btu/lb
F  engine thrust, lb
f  fuel-air ratio at exit of primary burner
G  acceleration due to gravity, ft/sec^2
I  engine specific impulse, lb-sec/lb fuel
J  mechanical equivalent of heat, ft-lb/Btu
k  ratio of specific heats for hot gas
M  Mach number
N  rotational speed, rpm
N/\sqrt{\gamma_1}  equivalent rotational speed, rpm
p  pressure, lb/sq ft
R  gas constant, ft-lb/(lb)(^oR)
r  radius, ft
T  temperature, ^oR
U  blade speed, ft/sec
V  absolute velocity, ft/sec
w  weight flow, lb/sec
\Gamma  density of blade metal, lb/cu ft
\gamma  ratio of specific heats for air
\delta  pressure-reduction ratio, p/\text{p}2116
\eta_c  compressor adiabatic efficiency
\[ \begin{align*}
\eta_T & \quad \text{turbine polytropic efficiency} \\
\eta_\infty & \quad \text{local value of compressor polytropic efficiency} \\
\theta & \quad \text{temperature-reduction ratio, } T/518.4 \\
\rho & \quad \text{gas density, lb/cu ft} \\
\sigma & \quad \text{blading stress, lb/sq ft} \\
\psi & \quad \text{stress-correction factor for tapered blades} \\
\end{align*} \]

Subscripts:

\begin{align*}
c & \quad \text{compressor} \\
cr & \quad \text{critical or choking} \\
\text{des} & \quad \text{design, or high-speed flight, condition} \\
eq & \quad \text{equivalent operating condition} \\
h & \quad \text{hub} \\
j & \quad \text{jet} \\
T & \quad \text{turbine} \\
t & \quad \text{tip} \\
x & \quad \text{axial} \\
0 & \quad \text{free stream} \\
1 & \quad \text{compressor inlet} \\
2 & \quad \text{compressor exit} \\
3 & \quad \text{turbine inlet} \\
4 & \quad \text{exit from turbine rotor} \\
5 & \quad \text{exit from exhaust nozzle} \\
\end{align*} 

Superscript:

\[ \begin{align*}
^I & \quad \text{stagnation state} \\
\end{align*} \]
GENERAL DISCUSSION OF OFF-DESIGN OPERATING PROBLEMS

The method of engine operation which is mechanically the simplest is that which does not vary the geometry of the compressor, the turbine, or the exhaust-nozzle throat. Such an engine is herein referred to as a "constant-geometry" engine. Under those conditions for which the exhaust nozzle is choked, such operation of a nonafterburning engine results in constant turbine-exit equivalent weight flow and constant exhaust-nozzle throat area. Under those conditions for which the exhaust nozzle is not choked and its throat area must therefore be adjusted in order to maintain constant turbine-exit equivalent weight flow, the term constant-geometry-engine operation is still employed.

For operation of constant-geometry turbojet engines over a wide range of flight Mach number, two extremes in engine operating mode are usually considered. These are (1) constant rotational speed \( N \) and (2) constant equivalent rotational speed \( N/\sqrt{\theta_1} \). For an engine designed for satisfactory high-thrust operation during take-off, operation with constant rotational speed results in low equivalent rotational speed and therefore low equivalent weight flow during high-speed flight. The turbine-inlet temperature can usually be maintained near the maximum value that the engine materials will permit over the range of flight Mach number. Although this type of operation results in high engine thrust during low Mach number flight (e.g., during take-off), the low equivalent weight flow during high-speed flight results in low engine thrust. If the engine is redesigned to provide high equivalent air flow per unit of engine frontal area during high-speed flight, operation at constant rotational speed during low-speed flight will result in high values of equivalent rotational speed and therefore such high values of compressor-inlet Mach number and compressor equivalent weight flow that the compressor efficiency will decrease. In an extreme case, surging of the compressor will restrict the range of conditions over which constant rotational speed can be maintained. Even though for operation with constant rotational speed the turbine-inlet temperature may be kept constant, there remains the problem of balancing the desirability of high equivalent weight flow during supersonic flight against the undesirability of low compressor efficiency or surging during take-off operation.

The frequently considered alternative of operation with constant equivalent rotational speed avoids some of these operating problems but creates some new ones. If the compressor is operated at a single point on the compressor map (design values of equivalent rotational speed and compressor pressure ratio) for the required range of flight conditions, high equivalent weight flow can be obtained during high-speed flight without sacrificing compressor efficiency during take-off operation (without turbine stator adjustment). On the other hand, such operation with a constant-geometry turbine requires that the engine temperature ratio (ratio of turbine-inlet to compressor-inlet temperature) be
constant. During take-off, this constant value of engine temperature ratio corresponds to a turbine-inlet temperature far below the material limit, with the result that a relatively low engine thrust is obtained during take-off operation.

If a penalty in compressor efficiency be accepted during take-off operation, compressor overspeeding can be employed to produce high engine thrust during take-off without resorting to extreme amounts of adjustment of other engine components such as turbine stator or exhaust nozzle, or to the use of compressor-exit bleed. Within a limited range, operation at increased equivalent rotational speed will result in increased weight flow, a trend that tends to increase engine thrust. The shaded area to the right of the line of design equivalent rotational speed in figure 2 is available as a region in which increased engine thrust might be obtained. In order to exploit this region fully, compressor overspeeding must be employed in combination with some geometry variation of the engine, such as exhaust-nozzle adjustment or turbine stator adjustment.

It is likely that increasing the amount of compressor overspeeding of a constant-geometry engine will eventually cause the compressor to surge. Exhaust-nozzle adjustment offers a possibility of operating an engine at constant equivalent rotational speed and constant turbine-inlet temperature over a range of flight Mach number. The compressor will then operate over a range of compressor pressure ratio at constant equivalent rotational speed. The range over which the compressor must operate is illustrated on the schematic compressor map in figure 3. High-speed flight corresponds to a low engine temperature ratio and thus a low compressor pressure ratio at design equivalent rotational speed. For take-off, the engine temperature ratio and the compressor pressure ratio are both high. The required rise in compressor pressure ratio can be approximated by considering the blade-speed line in figure 3 to be vertical. With this assumption, the compressor pressure ratio will rise, for example, from 4.0 at a flight Mach number of 2.5 to a value of 5.2 during take-off; in an actual case, the rise in pressure ratio will be somewhat less. This type of operation offers high engine thrust during both take-off and high-speed flight but may possibly result in low compressor efficiency and thus low engine specific impulse at high flight speeds.

Another scheme for obtaining thrust increases is the use of compressor-exit bleed. With the compressor operating at its design point for both high-speed flight and take-off, high turbine-inlet temperature may be employed during take-off if gas is removed between the compressor and turbine. Engine thrust can be obtained from both the turbine-exhaust gas and the gas bled between the compressor and turbine. For flight at a Mach number of 2.5, operation with constant turbine-inlet temperature requires bleeding 23 percent of the compressor weight flow during take-off operation if the compressor pressure ratio and equivalent rotational speed are held constant. Exhaust-nozzle adjustment is required for operation with compressor-exit bleed.
Turbine stator adjustment with the compressor operating at its design point has been suggested as a means of realizing some of the advantages of operation at both constant rotational speed and constant equivalent rotational speed. By means of turbine stator adjustment, high compressor equivalent weight flow can be obtained during high-speed flight in combination with high turbine-inlet temperature and high compressor efficiency during take-off. The disadvantages of turbine stator adjustment are (1) mechanical complexity, (2) penalties in turbine efficiency associated with off-design operation, and (3) penalties in turbine design-point performance for some ranges of design conditions (see ref. 1). Exhaust-nozzle adjustment is also required for operation with turbine stator adjustment.

The general problem of engine off-design operation has so far been related only to the compressor, whereas operating problems associated with the turbine also affect the selection of engine operating mode. One turbine operating problem is associated with increasing exhaust-nozzle area. As the exhaust-nozzle area is increased, the turbine-exit equivalent weight flow rises and approaches a limiting value, which is about 10 percent less than the equivalent weight flow that chokes the turbine annulus. Because increasing exhaust-nozzle area corresponds to increasing turbine-exit weight flow, modes of engine operation that require increasing exhaust-nozzle area during off-design operation are limited in their range of application for turbines of a given design configuration. The operating range of an engine requiring exhaust-nozzle adjustment can be extended by redesigning the turbine for increased exit annular area, but such a change in design imposes a penalty on size and centrifugal stress of the turbine and thereby impairs engine design-point performance. In applying turbine stator adjustment in engine operation, several turbine problems will restrict the range of engine operation off-design: (1) The turbine efficiency may fall to an undesirably low value; (2) the turbine may be incapable of passing the required equivalent weight flow at high engine temperature ratios; (3) the turbine might be incapable of producing the required work.

The way in which a given engine should be operated off-design depends on the characteristics of its particular compressor and turbine. Because the characteristics of each set of components will differ from another, no one mode of operation can be selected as best for all engines and operating conditions. There is a need, however, for an indication as to which methods appear to be most promising and the way in which they can best be exploited.
ANALYSIS AND DISCUSSION

Assigned Engine Design Conditions

Since no general way was found to relate changes in engine performance and changes in compressor and turbine operating requirements, the off-design study was restricted to two sets of engine design conditions, which are summarized in the following table:

<table>
<thead>
<tr>
<th>Flight Mach number, $M_0$</th>
<th>Compressor pressure ratio, $p_2'/p_1'$</th>
<th>Turbine-inlet temperature, $T_3'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>2.5</td>
<td>2000</td>
</tr>
<tr>
<td>3.0</td>
<td>3.0</td>
<td>3000</td>
</tr>
</tbody>
</table>

The compressor-inlet axial Mach number $(V_x/a)_1$ and the compressor adiabatic efficiency $\eta_c$ for both conditions are 0.5 and 0.85, respectively.

Assumptions

The compressors were assumed to have the following off-design characteristics: (1) compressor work per pound of air proportional to the blade speed squared, and (2) axial velocity at compressor entrance proportional to blade speed for subsonic values of inlet axial velocity. For a constant-geometry turbine (without turbine stator adjustment), the turbine-inlet equivalent weight flow $w_3\sqrt{\gamma_3/\gamma_2}$ was assumed to be constant. The effect of variation of fuel addition on continuity was neglected. For take-off, the following values of various parameters were assumed:

- Turbine polytropic efficiency, $\eta_T$ ........................................ 0.85
- Burner stagnation-pressure ratio, $p_3'/p_2'$ .............................. 0.90
- Ratio of specific heats for compressor, $\gamma$ ............................... 1.40
- Ratio of specific heats for turbine, $k$ ........................................ 1.30
- Inlet diffuser pressure ratio, $p_1'/p_0'$ .................................. 1.00

The assumed compressor characteristics affect the compressor performance in the following way: The assumption that the compressor work depends on blade speed and not on pressure ratio results in a trend in compressor efficiency such that the higher the compressor pressure ratio at any given equivalent rotational speed, the higher the compressor
efficiency. In an actual compressor, the trend of compressor efficiency for any given equivalent rotational speed is such that the efficiency rises, levels off as the surge line is approached, and then decreases slightly. Thus, the assumption that the compressor work is proportional to the square of the blade speed makes high compressor pressure ratio appear somewhat more favorable than will actually be the case.

The assumption that the compressor-inlet axial velocity is proportional to the blade speed results in a constant rotor-inlet relative angle as the blade speed varies for a compressor with no inlet guide vanes. In an actual case, the rotor-inlet relative angle varies so that the angle of attack on the rotor blades increases at low blade speeds and decreases at high blade speeds. This variation in angle of attack will very likely result in losses in compressor efficiency, especially in the high range of rotor-inlet Mach number. As compressor pressure ratio increases at constant equivalent blade speed, the compressor equivalent weight flow decreases and the surge line is approached. As a result of the assumed relation between inlet axial velocity and blade speed, the equivalent weight flow is independent of the compressor pressure ratio. For these reasons, the assumed compressor characteristics correspond to a smaller rise in equivalent weight flow with rising equivalent blade speed than will actually occur.

Compressor Overspeeding

Compressor characteristics. - Operation of an engine at constant rotational speed over a range of flight-conditions results in variation in compressor equivalent rotational speed. Flight at Mach numbers high enough to result in a ram temperature greater than 518.4° R produces an actual rotational speed greater than the equivalent rotational speed. The manner in which \( \frac{U}{U/N_0^1} \) rises with flight Mach number is shown in figure 4. This ratio is of some consequence for the following reasons: If an engine is operated at constant rotational speed, this ratio is numerically equal to the ratio of the equivalent rotational speed for take-off to the equivalent rotational speed for the design, or flight, condition. The ratio \( \frac{U}{U/N_0^1} \) is therefore a measure of the amount by which, during take-off, the compressor must exceed its design equivalent rotational speed if the engine is to be operated at constant actual rotational speed. For flight Mach numbers of 2.5 and 3.0, this ratio has the values 1.30 and 1.45, respectively; in other words, for constant-rotational-speed operation, these engines will exceed their design equivalent rotational speeds by 30 and 45 percent, respectively. These numbers, along with the results of reference 1, indicate the extreme range of conditions over which high-speed supersonic turbojets may be required to operate.
Within the limits of the simplifying assumptions, the manner in which this overspeeding may be expected to affect the compressor equivalent weight flow \( w_1\sqrt{\theta_1/\delta_1} \) may be determined by considering

\[ w = \rho VA \]

and assumption (2). From this consideration,

\[
\frac{w_1\sqrt{\theta_1/\delta_1}}{(w_1\sqrt{\theta_1/\delta_1})_{des}} = \frac{U/\sqrt{\theta_1}}{(U/\sqrt{\theta_1})_{des}} \left\{ \frac{1 - \frac{y-1}{2} \left(\frac{V}{a_1}\right)^2}{1 - \frac{y-1}{2} \left(\frac{V}{a_1}\right)_{des}^2} \right\}^{\frac{1}{y-1}}
\]

Equation (1) is plotted in figure 5(a) for three values of design compressor-inlet axial Mach number \( (V_x/a)_{1,des} \). For values of design compressor-inlet axial Mach number of 0.3 and 0.5, the ratio of the equivalent weight flows rises continually with rising equivalent blade speed over the range presented in figure 5(a). For a value of design compressor-inlet axial Mach number of 0.7, on the other hand, the equivalent weight flow rises and reaches a maximum at an equivalent blade-speed ratio of 1.37; at this point, the compressor-inlet axial Mach number equals 1.0.

The effect of compressor overspeeding on compressor pressure ratio \( p_2/p_1 \) can be assessed from the equation for continuity of mass flow:

\[
(1-b)(1+f) \frac{w_1\sqrt{\theta_1}}{\delta_1} = \frac{w_3\sqrt{\theta_3}}{\delta_3} \frac{p_2}{p_1} \frac{p_3}{p_2} \sqrt{\frac{T_1}{T_3}}
\]

Under the assumptions that \( (1+f) \), turbine-inlet equivalent weight flow \( w_3\sqrt{\theta_3/\delta_3} \), and burner pressure ratio \( p_3/p_2 \) do not vary, and with the additional condition that the fraction of gas bled \( b \) is constant, equation (2) reduces to

\[
\frac{w_1\sqrt{\theta_1/\delta_1}}{(w_1\sqrt{\theta_1/\delta_1})_{des}} = \frac{p_2/p_1}{(p_2/p_1)_{des}} \sqrt{\frac{T_3/T_1}{T_3/T_1}}
\]

If it is postulated that the turbine pressure ratio \( p_2/p_4 \) is constant, then the turbine work \( W_T \) is proportional to the turbine-inlet temperature \( T_4 \), or
Since
\[ E_T (1-b)(1+f) = E_C \] (5)
and, as assumed,

\[ E_C \sim U^2 \] (6)

then equations (3) to (6) may be combined to yield

\[ \frac{p_2^1/p_1^1}{(p_2^1/p_1^1)_{des}} = \frac{w_1 \sqrt{\theta_1^1/\theta_1}}{w_1 \sqrt{\theta_1^1/\theta_1}_{des}} \frac{U/\sqrt{\theta_1}}{(U/\sqrt{\theta_1})_{des}} \] (7)

Equation (7) is plotted in figure 5(b) for three values of design compressor-inlet axial Mach number \( (V_{x/a})_{des} \): 0.3, 0.5, and 0.7.

For each value, the compressor pressure ratio rises continually, almost linearly, with compressor equivalent blade-speed ratio \( \frac{U/\sqrt{\theta_1}}{(U/\sqrt{\theta_1})_{des}} \).

The ratio of compressor pressure ratios \( \frac{p_2^1/p_1^1}{(p_2^1/p_1^1)_{des}} \) is seen to be independent of the design pressure ratio \( (p_2^1/p_1^1)_{des} \). For a value of design compressor-inlet axial Mach number of 0.5, the compressor pressure ratio rises to 1.55 and 1.82 times the design value for compressor overspeeds of 30 and 45 percent, respectively.

The specified operating conditions will later be shown to duplicate the conditions for constant-geometry-engine operation. By combining the data of figure 5, constant-geometry-engine operating lines may be drawn on compressor-map coordinates as shown in figures 6 and 7. As a result of the assumed relation between blade speed and compressor-inlet axial velocity, the compressor equivalent weight flow \( w_1 \sqrt{\theta_1^1/\theta_1} \) is independent of compressor pressure ratio \( p_2^1/p_1^1 \) (see eq. (1)); the vertical in figure 6. As a typical result from figure 6(b), the compressor pressure ratio varies from 3 to 5.5 as the compressor equivalent blade-speed ratio rises from unity to 1.45 for a compressor design having a design value of compressor-inlet axial Mach number of 0.5 and a design compressor pressure ratio of 3.0. In figure 7, raising the design value of compressor-inlet axial Mach number decreases the rate at which compressor pressure ratio rises with compressor equivalent blade-speed ratio.
Within the limits of the simplifying assumptions, the conditions specified result in certain values of compressor efficiency; that is, the value of compressor efficiency is dependent upon the compressor characteristics already specified. These values of efficiency are indicative of the efficiency that compressors will be required to have during overspeed operation. If a particular range of design conditions requires that the compressor efficiency rise for equivalent rotational speeds above the design value, the compressor operating requirements will be difficult or impossible to fulfill without accepting a penalty in compressor design-point efficiency. If, on the other hand, the required efficiency falls with rising equivalent speed, the compressor requirements are easier to satisfy.

The compressor adiabatic efficiency may be stated as

\[
\eta_c = \frac{\frac{\gamma - 1}{\gamma} \left( \frac{P_2'}{P_1'} \right)^{\frac{\gamma-1}{\gamma}} - 1}{(1+\rho)(1-\beta)\dot{E}_T}
\]  

(8)

By combining equations (5), (6), and (8),

\[
\frac{\eta_c}{\eta_c,\text{des}} = \frac{\frac{\gamma - 1}{\gamma} \left( \frac{P_2'}{P_1'} \right)^{\frac{\gamma-1}{\gamma}} - 1}{\left( \frac{U}{N\theta_1^{\text{des}}} \right)^2}
\]

(9)

Variation in compressor-adiabatic-efficiency ratio \(\eta_c/\eta_c,\text{des}\) along the constant-geometry-engine operating line is plotted in figure 8(a) for four values of design compressor pressure ratio at a compressor-inlet axial Mach number of 0.5; figure 8(b) is a similar plot made for three values of design compressor-inlet axial Mach number at a design compressor pressure ratio of 3.0. In general, required compressor-efficiency ratio decreases with increasing compressor equivalent blade-speed ratio, with the exception of the lowest compressor pressure ratio in figure 8(a). The higher the values of design compressor pressure ratio and design compressor-inlet axial Mach number, the more rapid is the decrease in compressor efficiency with compressor equivalent blade speed. For a design compressor pressure ratio of 3.0 and design compressor-inlet axial Mach number of 0.3 (fig. 8(b)), required compressor efficiency shows initially no change with rising
equivalent blade speed. For a design compressor pressure ratio of 2.0 and design compressor-inlet axial Mach number of 0.5 (fig. 8(a)), the trend of required compressor efficiency with equivalent blade speed reverses, climbing by 5 percent above the design values; after a compressor equivalent blade-speed ratio of 1.2 is obtained, the required compressor efficiency decreases as for the other cases.

In practice, contrary to the assumptions herein, the compressor equivalent blade speed and pressure ratio could not be increased along the constant-geometry-engine operating line almost without limit. Eventually compressor surging would be encountered along the constant-geometry-engine operating line, and engine operation at this or higher blade speeds would not be practical. The value of compressor equivalent blade-speed ratio at which surging is first encountered will depend to some extent on the location of the operating line on the compressor map; or, to express the same thought in another way, if the required compressor efficiency drops along the operating line, surging might be forestalled. If the engine design point is at the point of maximum compressor efficiency, the required compressor efficiency cannot have the rising trend shown in figure 8(a); the compressor will register by surging that a too-high efficiency (or pressure ratio) is required of it. A design-point compressor efficiency less than the maximum could be selected, of course, but that would penalize the design-point performance.

From the foregoing analysis, the selection of values for design compressor pressure ratio and design compressor-inlet axial Mach number appears to affect the trend of required compressor efficiency along the constant-geometry-engine operating line. Compressor designers may find it useful to know something of the trend of compressor polytropic efficiency with compressor equivalent blade speed along the operating line. The required value of compressor polytropic efficiency is a measure of the difficulty of obtaining each small increment of compression. For a small increment of compression,

$$\frac{d(p_2'/p_1')}{p_2'/p_1'} = \eta_{\infty} \frac{\gamma}{\gamma-1} \frac{d(T_2'/T_1')} {T_2'/T_1'}$$

(10)

For the assumed relation between compressor work and blade speed,

$$\frac{T_2}{T_1} - 1 \sim (U/\sqrt{T_1})^2$$

(11)

By combining equations (1), (7), (10), and (11),
\[ \frac{\gamma \eta_{\infty}}{\gamma - 1} \left( 1 - \frac{T_2'}{T_1'} \right) = 1 - \frac{(V_x/a')^2}{2\left[ 1 - \frac{\gamma - 1}{2}\left( \frac{V_x}{a'} \right)^2 \right]} \]  

(12)

The polytropic efficiency \( \eta_{\infty} \) in equation (12) is a local value and not an average value of polytropic efficiency such as is usually considered. Equation (12) is plotted in figure 9 for three values of design compressor-inlet axial Mach number \((V_x/a)_1, \text{des}\). In order to relate compressor temperature ratio \( \frac{T_2'}{T_1'} \) and compressor pressure ratio \( \frac{p_2'}{p_1'} \), an adiabatic efficiency \( \eta_c \) of 0.85 was used in preparation of figure 9; the result of this assignment of adiabatic efficiency is that, even though equation (12) is a general equation, figure 9 must be considered as representing design-point conditions in order to be consistent with the assumptions in the rest of the analysis. Values of local polytropic efficiency \( \eta_{\infty} \) greater than 1.0 were considered to be outside the range of interest and therefore were not plotted in figure 9; examination of equation (12) indicates that as the compressor temperature ratio \( \frac{T_2'}{T_1'} \) approaches 1.0, the local polytropic efficiency \( \eta_{\infty} \) approaches infinity for any finite value of compressor-inlet axial Mach number. Figure 9 shows that for a design compressor-inlet axial Mach number of 0.5, the local polytropic efficiency \( \eta_{\infty} \) is less than 1.0 only for design pressure ratios greater than 2.4, and less than 0.9 for pressure ratios greater than 2.7.

The manner in which the local polytropic efficiency \( \eta_{\infty} \) varies along the constant-geometry-engine operating line can be determined by combining equations (6) and (12):

\[ \frac{\gamma \eta_{\infty}}{\gamma - 1} \left( \frac{T_2'}{T_1'} \right)_{\text{des}} - 1 + \left[ \frac{U/\sqrt{\theta_1'}}{U/\sqrt{\theta_1}} \right]_{\text{des}}^2 = 1 - \frac{(V_x/a')_{\text{des}}^2 \left[ \frac{U/\sqrt{\theta_1'}}{U/\sqrt{\theta_1}} \right]_{\text{des}}^2}{2 \left[ 1 - \frac{\gamma - 1}{2}\left( \frac{V_x}{a'} \right)_{\text{des}}^2 \right]} \]

(13)

For a value of design compressor-inlet axial Mach number of 0.5, equation (13) is plotted in figure 10. For the assumed variation in compressor weight flow and work with compressor blade speed, the local value of polytropic efficiency \( \eta_{\infty} \) varies over a wide range. For a design compressor pressure ratio \( \frac{p_2'}{p_1'} \), the value of local
polytropic efficiency $\eta_p$ is above 1.0 at the design point and diminishes to 1.0 at a compressor equivalent blade-speed ratio $U/\sqrt{\theta_1}$ of 1.10; these high values of local polytropic efficiency are reflected in figure 8(a) as a rise in compressor adiabatic efficiency $\eta_c$ above the design value for compressor equivalent blade-speed ratios slightly greater than 1.0. As the design compressor pressure ratio is increased, the local polytropic efficiency has more practical values; for a compressor design pressure ratio of 2.39, the local polytropic efficiency does not exceed 1.0 for blade-speed ratios greater than 1.0. If the compressor design pressure ratio is 3.0, the local polytropic efficiency is required to be only 0.82 at design blade speed and less than 0.60 for compressor equivalent blade-speed ratios greater than 1.2. Even though the compressor designer may not be able to guarantee a range of overspeed operation, these values of local polytropic efficiency indicate the degree of difficulty of obtaining the required overspeed performance.

Turbine characteristics. - A sufficient number of conditions affecting the compressor and turbine has already been specified to permit determining the turbine design requirements. As was postulated in equation (4), the turbine pressure ratio is maintained constant along the constant-geometry-engine operating line. Under this condition, turbine work $E_T$ is proportional to the turbine-inlet temperature $T_i$. With the compressor, and thus turbine, work proportional to the blade speed squared, the turbine equivalent blade speed $U/\sqrt{\theta_2}$ is constant. Constant turbine pressure ratio and equivalent blade speed correspond to turbine operation at a single point on the turbine performance map; not only is the turbine equivalent operating condition constant as engine speed is changed during take-off operation, but also the turbine operates at this one condition during flight as well.

For this method of operation, the turbine can thus be critical in aerodynamic design with the assurance that during engine off-design operation the turbine operating requirements will not become more severe. As shown in reference 1, some engine design conditions require compromising the turbine design if the engine is equipped with adjustable turbine stators, in order to permit operation at a single point on the compressor map. Compressor overspeeding appears to offer a way to overcome this operational difficulty at least partially.

For operation of a turbine at a single equivalent operating condition, the turbine-exit equivalent weight flow $w'_4\sqrt{\theta_4'/\theta_4}$ is constant. A choked exhaust nozzle that is part of a nonafterburning turbojet engine will have its throat area kept constant in order to pass this constant equivalent weight flow.
Engine performance. - In addition to the manner in which compressor overspeeding affects compressor and turbine operating requirements, the resulting engine performance must also be investigated. In order to simplify the presentation of engine performance, an equivalent operating condition is defined just as in reference 1. If variations in Reynolds number, specific heats, and (l+f) may be neglected, there is, corresponding to the flight or design condition, a homologous condition for take-off such that the following factors are all constant between take-off and flight: engine temperature ratio $T_2/T_1$, compressor pressure ratio $p_2/p_1$, compressor equivalent blade speed $U/N_1$, compressor equivalent weight flow $w_1N_1/\sqrt{\rho_1}$, turbine pressure ratio $p'_2/p'_1$, and turbine equivalent blade speed $U/N_3$. This particular take-off condition is herein referred to as "the equivalent operating condition."

For both engines analyzed, the compressor-inlet axial Mach number was assigned the design value of 0.5, a moderately conservative value. The combination of this compressor-inlet axial Mach number with a compressor-inlet hub-tip radius ratio of 0.5 results in a compressor weight flow of 27.7 pounds of air per second per square foot of compressor-tip frontal area. Reference 2 indicates that, even for extreme turbine design, a one-stage turbine is incapable of handling more than this much air flow at high supersonic speeds. In reference 2, figure 1(c) approximates the conditions for extreme turbine design at a flight Mach number of 3 and a turbine-inlet temperature of 3000°F. If the compressor pressure ratio is 3 and the turbine blade-tip equivalent speed $U_t/\sqrt{\rho_1}$ is 1200 feet per second, the turbine can pass 27.5 pounds of air per second per square foot of turbine-tip frontal area. Because an equivalent tip speed of 1200 feet per second is an actual tip speed of 1740 feet per second under these conditions, it is doubtful that within limits imposed by centrifugal stress a combination of air flow and blade-tip speed even this high could be employed. As reference 2 shows, superior air-handling capacity and lower blade speed could be obtained by using a two-stage turbine. On the other hand, a primary burner designed for an inlet velocity of 150 feet per second cannot handle under the assigned conditions even as much air per unit frontal area as a one-stage turbine. For these reasons, a design value of compressor-inlet Mach number of 0.5 was thought to be high enough for the applications being considered.

For the conditions assumed herein,

$$\frac{F}{F_{eq}} = \frac{w_1N_1/\sqrt{\rho_1}}{(w_1\sqrt{\rho_1/\rho_1})_{des}} \frac{V_5}{V_{5,eq}}$$  \hspace{1cm} (14)
Equations (14) and (15) were evaluated for a range of compressor equivalent blade-speed ratio, and the results are presented in figures 11 and 12. For both the Mach 2.5 and the Mach 3.0 engines, the engine thrust at rated turbine-inlet temperature is more than triple the thrust at the equivalent operating condition. The rises in thrust are a result of changes in three factors: turbine-inlet temperature, weight flow, and compressor pressure ratio. For the Mach 3.0 engine, for example, the rise in thrust ratio \( \frac{F}{F_{eq}} \) from 1.0 to 3.1 results from a 26-percent rise in air flow, a 125-percent rise in thrust per unit air flow due to increased turbine-inlet temperature, and a 9-percent increase in thrust per unit air flow due to increased compressor pressure ratio \( (1.26 \times 2.25 \times 1.09 = 3.1) \). The rises in turbine-inlet temperature, weight flow, and compressor pressure ratio result in a greater increase in engine thrust than could be obtained by increasing turbine-inlet temperature alone and adjusting the turbine stator.

Figure 12(a) shows that, for the Mach 2.5 engine, the effect of a rise in engine thrust above that at the equivalent operating condition is at first to raise the engine specific impulse. Further increase in engine thrust is accompanied by decreasing engine specific-impulse ratio; at rated turbine-inlet temperature for this engine, the value of engine specific impulse returns to its value at the equivalent operating condition. For the Mach 3.0 engine (fig. 12(b)), engine specific impulse immediately drops off in value as the engine thrust is increased; at rated turbine-inlet temperature for this engine, the engine specific impulse has fallen off to 70 percent of its value at the equivalent operating condition.

Since, for constant-geometry-engine operation, the turbine equivalent blade speed \( \frac{U}{N_0} \) is constant, operation with constant turbine-inlet temperature \( T_{3i} \) results in constant blade speed. Constant blade speed corresponds to constant centrifugal stress. An engine operated in this way would thus produce the greatest thrust of which it is capable within limits imposed by material properties. Engine operation would be simplified in that the engine rotational speed would remain constant; and, for those flight Mach numbers high enough to cause choking of the exhaust nozzle, the exhaust-nozzle throat area would be constant.

**Exhaust-Nozzle Adjustment**

It seems highly probable that as the equivalent rotational speed of any constant-geometry engine is continually increased above the design value, an equivalent rotational speed and turbine-inlet temperature will eventually be reached at which compressor surging will occur. Exhaust-nozzle adjustment may offer some relief by permitting operation at even higher thrust levels without surging the compressor. For any given
engine equivalent rotational speed, opening the exhaust-nozzle throat will permit decreasing the turbine-inlet temperature while maintaining constant engine equivalent rotational speed. As equation (3) shows, this will reduce the compressor pressure ratio and thus move the compressor away from the surge limit.

On the other hand, opening the exhaust nozzle will increase the turbine-exit equivalent weight flow and thereby move the turbine toward the blade-loading limit. Limiting blade loading is a condition first investigated in reference 3 and, for blades of conventional design, constitutes a fundamental limit on turbine work. Reference 4 relates the blade-loading limit to the turbine-exit axial Mach number and thereby to the turbine-exit equivalent weight flow. Limiting blade loading manifests itself as an engine operating problem in the following way: If, for an engine to be operated at constant equivalent rotational speed, the engine temperature ratio is gradually decreased, the turbine pressure ratio must be increased by opening the exhaust nozzle in order that the turbine will produce enough torque to keep the engine from slowing down. If this trend is continued, a turbine operating condition will eventually be reached beyond which any further reduction in engine temperature ratio will result in a drop in engine equivalent rotational speed despite an increase in turbine pressure ratio. For this turbine operating condition, the blade-loading limit has been reached. Reference 4 relates the blade-loading limit to the turbine-exit axial Mach number and thereby to the turbine-exit critical area \( A_{cr,4} \). In order to operate within the limiting axial Mach number of 0.7 specified in reference 4, the actual exit annular area must be made at least 10 percent greater than the turbine-exit critical area. Equation (11) of reference 1 states that

\[
\sigma = \frac{\pi U_h^2 \psi}{2G} \left[ \frac{r_t}{r_h} \right]^2 - 1
\]

In order to produce a given amount of work, a certain minimum blade-hub speed is required. For specified values of centrifugal stress \( \sigma \), blade-hub speed \( U_h \), and taper factor \( \psi \), the particular value of hub-tip radius ratio may be determined from this equation. For this particular value of hub-tip radius ratio, the turbine-tip frontal area is directly proportional to the annular area. For this reason, if the requirements of off-design operation dictate that the turbine annular area be increased above the value required for satisfactory design-point operation, the engine thrust per unit turbine frontal area for design-point operation is thereby diminished in inverse proportion to the required value of exit annular area. At the blade-loading limit, the turbine-exit critical area \( A_{cr,4} \) is a direct measure of the exit annular area and thus of the frontal area.
The manner in which exhaust-nozzle adjustment may produce increased engine thrust during take-off may be described as follows: At any given engine equivalent rotational speed, maximum thrust is obtained by operating at the highest turbine-inlet temperature permissible within the surge limit; this means small exhaust-nozzle areas, and compressor pressure ratios as high as compressor surge will permit. On the other hand, at the point on the compressor map where the compressor surge line crosses the constant-geometry-engine operating line, the thrust of a constant-geometry engine reaches a limit. This point is denoted as point A in figure 13, which is a sketch representing the effect of exhaust-nozzle adjustment in combination with compressor overspeeding on engine thrust during take-off. It may be possible to obtain higher engine thrust off the constant-geometry-engine operating line by raising engine equivalent rotational speed. Higher equivalent rotational speed will result in higher equivalent weight flow and thereby possibly higher thrust. If the compressor surge line intersects the constant-geometry-engine operating line at a small acute angle, the turbine-inlet temperature may even possibly be increased within the surge limit. Point A in figure 13(a) illustrates such a situation. Increasing the equivalent rotational speed to operate at point B assures a thrust increase over operation at point A, because both a higher engine temperature ratio and a higher equivalent weight flow are attained. If, on the other hand, the compressor surge line and constant-geometry-engine operating line intersect at a large acute angle, as illustrated by point A in figure 13(b), it cannot readily be seen whether increasing the equivalent rotational speed to operate at point C will yield a thrust increase or not. Any increase in engine thrust in changing operating points from A to C would be contingent upon whether the increase in equivalent weight flow is sufficiently great to offset the decrease in engine temperature ratio. If the slope of the compressor surge line is much less than in figure 13(b), it can easily be imagined that increasing the equivalent rotational speed from point A to C could be to no avail, since the decrease in engine temperature ratio might well offset the rise in equivalent weight flow.

Whether or not exhaust-nozzle adjustment will result in a sufficient increase in engine thrust during take-off to offset the tendency toward limiting blade loading in the turbine thus depends on the compressor-surge characteristics. A quantitative evaluation of the changes in engine thrust and turbine-exit annular area is presented in order that, for a prescribed compressor-surge characteristic, the merits of exhaust-nozzle adjustment may be ascertained.

Turbine design requirements. - For the assumed conditions, a given engine rotational speed corresponds to a given turbine work output $E_T$, the work output being proportional to the blade speed squared. The turbine blade-jet speed ratio is therefore constant for any given engine and independent of exhaust-nozzle adjustment. Operation along the
constant-geometry operating line corresponds to constant turbine equivalent rotational speed. At any compressor equivalent rotational speed, opening the exhaust nozzle requires a decrease in turbine-inlet temperature and thereby increases the turbine equivalent rotational speed. It may be inferred from equation (24) of reference 5 that this rise in equivalent blade speed corresponds to decreasing rotor-entrance Mach number and increasing acceleration across the rotor, both trends being conservative.

The minimum turbine-exit annular area, that value of area at which limiting blade loading is obtained, is directly proportional to the turbine-exit equivalent weight flow. For a stress-limited turbine design, such as will very likely be required for these high-supersonic flight speeds, opening the exhaust nozzle from the setting for take-off operation on the constant-geometry-engine operating line will therefore require an increase in turbine-exit annular area and frontal area. The turbine-exit annular area should be selected to pass satisfactorily the maximum turbine-exit equivalent weight flow that will be encountered over the range of engine operation.

Engine performance. - Lines of constant engine thrust ratio $F/\text{Feq}$ are shown on the compressor map in figure 14 along with the constant-geometry-engine operating line. The selection of an operating condition in figure 14 corresponding to maximum thrust is illustrated in figure 15. One of the lines of constant engine thrust ratio $F/\text{Feq}$ is tangent to the surge line; this point of tangency represents the operating condition at which the maximum engine thrust can be produced by means of exhaust-nozzle adjustment. If one of the lines of constant engine thrust ratio as drawn in figure 14 does not happen to be tangent to a particular surge line, the operating point for maximum thrust can be determined in the following way: If at any given point on the compressor map the surge line has a slope greater than that of the line of constant engine thrust ratio at that point, engine thrust can be increased by adjusting the exhaust-nozzle and raising the engine speed, following the surge line until it becomes parallel with the lines of constant engine thrust ratio.

The variation in turbine-exit critical-area ratio $(A/A_{\text{des}})_{cr,4}$ is shown in figure 16. Since, at the blade-loading limit, both the turbine-exit annular area and turbine-tip frontal area (for a given centrifugal stress in the rotor blades) are directly proportional to the turbine-exit critical-area ratio, the curves in figure 16 also can be interpreted as showing changes in the values of minimum turbine-exit annular area and turbine-tip frontal area. Figure 16 shows, that, for any given compressor equivalent weight flow (a line of constant compressor equivalent blade speed in fig. 16), highest engine thrust is obtained with the smallest usable exhaust-nozzle area.
The thrust-area factor \( \frac{F (A_{des})}{Feq (A_{cr}, 4)} \) can be used to determine under what conditions highest engine thrust per unit turbine-tip frontal area is obtained; the value of the thrust-area factor is proportional to engine thrust per unit turbine frontal area for a stress-limited turbine design. Lines of constant engine thrust-area factor are plotted on the coordinates of the compressor map along with the constant-geometry-engine operating line in figure 17. If at any point on the compressor map the slope of the surge line is greater than the slope of the lines of constant thrust-area factor, higher take-off thrust for a turbine of a given size may be obtained by means of exhaust-nozzle adjustment. A comparison of the slopes of the lines of constant thrust ratio \( \frac{F}{Feq} \) in figure 14 with the lines of constant thrust-area factor \( \frac{F (A_{des})}{Feq (A_{cr}, 4)} \) in figure 17 shows that obtaining an increase in engine thrust per unit turbine size is the more stringent requirement. If an increase in thrust-area factor can be obtained, a higher take-off thrust per unit turbine frontal area can be obtained by exhaust-nozzle adjustment than by adding engines or increasing engine size.

Because increasing the exhaust-nozzle throat area requires an increase in turbine-exit annular area, such operation requires that the turbine design be modified to satisfy the off-design operating requirements. In order to minimize this compromising of the turbine design, the exhaust-nozzle adjustment should be held to the minimum essential to producing the required engine thrust during off-design operation.

The engine specific impulse obtained with exhaust-nozzle adjustment is presented in figure 18. The impulse for any given thrust level is always lower than that obtainable along the constant-geometry-engine operating line.

Compressor-Exit Bleed

The decrease in turbine pressure ratio \( \frac{p_3}{p_4} \) and the increase in turbine-inlet temperature \( T_3 \) associated with bleeding air at the compressor exit might conceivably result in a rise in engine thrust during off-design operation. Previous work in this field supports the point of view that bleeding air at the compressor exit while keeping the compressor at a given operating point results in a reduced turbine pressure ratio. For a case considered in reference 4, the turbine pressure ratio was theoretically decreased from 4.45 to 2.73 by bleeding 28.6 percent of the engine air flow at the compressor exit and raising turbine-inlet temperature from 1100° to 2160° R while operating the compressor at a single operating point. Figure 3 of reference 6 shows that for a constant value of turbine torque, decreasing the turbine equivalent
rotational speed decreases the turbine pressure ratio; equilibrium operation of a compressor map requires a given value of torque independent of the amount of air bled, and rising turbine-inlet temperature at a given engine rotational speed reduces the turbine equivalent rotational speed. The following questions were therefore investigated:

(1) For an engine that has been overspeeded along the constant-geometry-engine operating line as much as the compressor-surge characteristics will permit, how much can the engine thrust be increased by raising the turbine-inlet temperature and bleeding air at the compressor exit?

(2) What is the effect of such operation on the turbine design requirements?

Turbine design requirements. - The decreased turbine pressure ratio with increased amounts of air bled moves the turbine operating condition away from limited blade loading, and therefore the turbine design need not be compromised on this score. Despite the decreasing turbine pressure ratio, the turbine work output per pound of gas passing through the turbine increases as the amount of bleeding is increased; the turbine work increases in this case, even though the turbine pressure ratio drops because of the great rise in turbine-inlet temperature. For the constant power required by the compressor, the turbine must produce a constant amount of power from a mass flow that decreases as the amount of bleeding is increased, whence the increasing turbine work.

This rise in turbine work at constant blade speed lowers the blade-jet speed ratio $U/V_j$. The blade speed is considered to be constant as bleeding is varied, and jet speed is directly proportional to the square root of turbine work. Consequently,

$$\frac{U/V_j}{(U/V_j)_{des}} = \sqrt{1-b} \tag{16}$$

Equation (16) is plotted in figure 19. If 20 percent of the compressor air flow is bled at the compressor exit ($b = 0.20$), the blade-jet speed ratio is decreased to only 89 percent of the design value. Since the curve of turbine efficiency against blade-jet speed ratio is usually still very flat in this region, the turbine efficiency should be relatively unchanged from the design value. Equation (2) can be used to show that bleeding 20 percent of the air results in a considerable temperature rise, that is

$$\frac{T_3}{T_1} \sim \frac{1}{(1-b)^2} \tag{17}$$
Bleeding 20 percent thus corresponds to raising the turbine-inlet temperature by 56 percent, for example, from 1415° to 2210° R.

**Engine performance.** - The bleed air was considered to be used in three ways: (1) It was discarded without producing thrust; (2) air at the total state prevailing at the compressor exit was expanded to atmospheric pressure and used to produce thrust; (3) the bleed air was heated to the turbine-inlet temperature and then expanded to atmospheric pressure in order to produce thrust; in a practical case, this hot gas would be bled at the exit from the primary burner.

The resulting engine thrust is shown in figures 20 and 21 for two kinds of operation, namely, using the bleed air to produce thrust and discarding the bleed air. For the Mach 3.0 engine with 20-percent overspeed, raising the turbine-inlet temperature to the rated value of 3000° R and using the bleed air to produce thrust increases the engine thrust by 28 percent; if the bleed air is discarded, the thrust rise is only 10 percent. The circles in figure 20 represent engine thrust values obtained at 20-percent overspeed if the bleed air is at rated turbine-inlet temperature. For the Mach 3.0 engine, the total thrust rise above the value for no bleed with 20-percent overspeed is 58 percent, the 10-percent increment resulting from the high bleed temperature. These thrust increases are somewhat less than those obtainable by means of turbine stator adjustment and compressor overspeeding.

For the Mach 3.0 engine with a 20-percent overspeed, increasing turbine-inlet temperature to the rated value permits bleeding 17 percent of engine air flow. This air could be used during off-design operation to cool various parts of the engine for which ram air is used during high-speed flight; thus a valuable characteristic of compressor-exit bleed is used despite its poor thrust characteristic.

Figure 22 shows the engine specific impulse that can be obtained with compressor-exit bleed. As may well be expected, compressor-exit bleed has poor impulse characteristics.

**Turbine Stator Adjustment**

Use of turbine stator adjustment for design-point operation of the compressor was investigated in reference 1. This is, of course, not the only way in which turbine stator adjustment could be used. If, for example, the engine equivalent rotational speed were raised above the design value along the constant-geometry-engine operating line, a speed would eventually be reached above which a further speed increase would result in compressor surging. Turbine stator adjustment could be employed in combination with exhaust-nozzle adjustment to keep the compressor operating at this point, and the turbine-inlet temperature could then be increased to obtain even higher engine thrust.
A second way to use turbine stator adjustment is to depart from the constant-geometry-engine operating line after surging has been encountered. Use of higher engine rotational speed would increase the engine weight flow and perhaps thereby increase the engine thrust. Both applications of turbine stator adjustment are considered herein; only the maximum, or design, value of turbine-inlet temperature is employed.

On operating line. - In figure 23, the engine thrust with turbine stator adjustment is compared with that of the constant-geometry engine for a range of compressor equivalent rotational speeds. At all speeds less than rated actual speed, the engine with turbine stator adjustment is capable of producing greater thrust. For the Mach 3.0 engine, this thrust advantage is 225 percent at design equivalent rotational speed and 144 percent at 120 percent of design equivalent speed. The advantage of turbine stator adjustment diminishes at the higher rotational speeds. The rises in weight flow and compressor pressure ratio with rotational speed raise engine thrust even for an engine with turbine stator adjustment. If the compressor is capable of being overspeed by a sufficiently great amount, the greatest engine thrust for take-off is obtained by raising the rotational speed to the design value. For this operating condition, no turbine stator adjustment is required. The scale of turbine-inlet critical-area ratio \( \left( \frac{A}{A_{\text{des}}} \right)_{cr,3} \) on the abscissa shows that for small amounts of compressor overspeed, large changes in turbine-inlet critical-area ratio are required, and at high overspeed only small changes in turbine-inlet critical-area ratio are necessary. For the Mach 3.0 engine at design equivalent rotational speed, the turbine-inlet critical-area ratio \( \left( \frac{A}{A_{\text{des}}} \right)_{cr,3} \) and, concomitantly, the turbine-inlet equivalent weight flow \( \frac{\sqrt{\gamma}}{S_{\text{in}}} \) must be increased by 45 percent in order for the turbine-inlet temperature to be increased to the design value of 3000° R. As reference 1 shows, the actual area must be increased considerably more than 45 percent in order to realize this 45-percent increase in equivalent flow.

The variation in turbine-exit critical area \( A_{cr,4} \) with engine equivalent rotational speed is shown in figure 24. Since both the Mach 2.5 and the Mach 3.0 engines have design conditions that lie a little to the left of the break-even point in figure 2 of reference 1, increasing the engine temperature ratio and adjusting the turbine stator result in initially a small decrease and then an increase in the turbine-exit critical area ratio \( \left( \frac{A}{A_{\text{des}}} \right)_{cr,4} \). Because at low compressor equivalent rotational speeds a large increase in turbine-inlet temperature is required to proceed from constant-geometry-engine operation to operation with turbine stator adjustment at rated turbine-inlet temperature, an increase in turbine-exit critical area results. At high compressor equivalent rotational speeds, only a small rise in turbine-inlet temperature produces the change from constant-geometry-engine operation to
operation with turbine stator adjustment at rated turbine-inlet temperature, with the result that turbine-exit critical area ratios 
\[(A/A_{des})_{cr,4}\] less than unity are obtained and the turbine moves away from limiting blade loading. A rise in the turbine-exit critical area 
\[A_{cr,4}\] above the design value is undesirable, because the turbine design must then be compromised in order to avoid limiting blade loading and thereby to permit satisfactory operation during take-off.

The effect of these changes in turbine-exit critical area on engine thrust per unit turbine size is shown in figure 25; as previously, the thrust-area factor 
\[
\frac{F}{F_{eq}} \frac{A_{des}}{A_{cr,4}}
\] is used to express take-off thrust per unit turbine size. For that part of the operating range for which the turbine-exit critical-area ratio is less than unity, the thrust ratio \(F/F_{eq}\) is also plotted. In this range, the thrust ratio \(F/F_{eq}\) expresses the variation in engine take-off thrust per unit turbine size, because the turbine must be designed for the largest value of \(A_{cr,4}\) encountered over the operating range, and a value less than the design value cannot be exploited during the off-design operation. The size of the thrust-area factor with turbine stator adjustment is always greater than without, the advantage decreasing as engine speed rises.

The variation in engine specific impulse with turbine stator adjustment is presented in figure 26. At low compressor equivalent rotational speeds, the thrust increase yielded by turbine stator adjustment is obtainable at the expense of a decrease in engine specific impulse. At high engine equivalent rotational speeds, not only high engine thrust, but also high engine specific impulse, is obtainable by means of turbine stator adjustment. If the compressor is capable of being overspeed sufficiently, constant engine geometry is superior to turbine stator adjustment without overspeed in the following respects: engine thrust, engine specific impulse, and compromising of the turbine design. Constant engine geometry is inferior to the extent to which the compressor design must be compromised to include the overspeed capacity.

Off operating line. - Lines of constant engine thrust are plotted on compressor-map coordinates in figure 27 in order to show the effect on engine performance of turbine stator adjustment off the constant-geometry-engine operating line. If a compressor is overspeed along the constant-geometry-engine operating line until surging is reached, additional engine thrust can be obtained if the compressor-surge line rises significantly to the right of the constant-geometry-engine operating line. The criterion for greatest thrust is, as before, to choose as an operating condition that point along the surge line at which the surge line becomes tangent to one of the family of lines of constant thrust ratio \(F/F_{eq}\) in figure 27. For any given surge line, considerably
greater rises in engine thrust resulting from departure from the operating line may be obtained with turbine stator adjustment than with exhaust-nozzle adjustment. Figure 28 shows, on the other hand, that these rises in take-off thrust are at the expense of increased turbine-exit critical area and thereby a penalty in turbine design-point performance. Lines of constant thrust-area factor \( \frac{F(A_{\text{des}})}{F_{\text{eq}} \left( \frac{A}{A_{\text{cr}}} \right)} \) are plotted in figure 29 to aid in judging for any given compressor-surge characteristic if any gain in take-off thrust per unit turbine size can be obtained by leaving the operating line. Figure 30 shows that, for any given thrust level, operation off the operating line produces a decrease in engine specific impulse.

**Effect of Changing Compressor Characteristics**

The compressor performance presented in reference 7 was used to determine the effect of using compressor characteristics other than those assumed herein. For a constant-geometry-engine operating line on this compressor map, the turbine-inlet equivalent weight flow \( \dot{m}_{3} \sqrt{\frac{\rho_{3}}{\rho_{0}}} \) and the turbine pressure ratio \( \frac{p_{3}'}{p_{4}'} \) were assumed to be constant. For a constant value of turbine efficiency, engine thrust was computed for operation along an operating line that lies near the surge line; the rotational speed designated as rated in reference 7 was not exceeded for take-off. In figure 31, engine thrust is shown to rise more rapidly with blade speed than for the conditions assumed herein; the principal reason for this is the more rapid rise in weight flow. The trends of engine thrust with rising compressor speed indicated herein are pessimistic in comparison with these results. Even with these characteristics, the predicted trends in engine performance are essentially those based on the assumptions of this analysis.

**Critical Comment**

Of the four types of engine off-design operation considered, compressor overspeeding is the type that appears most promising, because it provided the highest engine thrust during low-speed flight. Another reason is that, as an aid in obtaining superlative turbine design-point performance, the turbine design conditions can be made critical with the assurance that the operating conditions do not tend to become even more critical during engine off-design operation. The principal problem is whether or not compressors can provide much of an overspeed margin without seriously compromising their design-point performances. If, in attempting to provide overspeed margin in a compressor design, the compressor surge line on the compressor map falls below the anticipated level of compressor pressure ratio, small variations in the location of the compressor surge line can be compensated for by small adjustments of the exhaust-nozzle throat area.
Exhaust-nozzle adjustment requires that the turbine be made larger. Ideally, the engine design thrust per unit of turbine-tip frontal area will be inversely proportional to the maximum value of exhaust-nozzle throat area. In practice, exhaust-nozzle adjustment will be somewhat more attractive than the analysis shows, because, in order to provide some margin for error in engine design, the turbine will very likely not be designed at the rotor blade-loading limit. This margin in turbine work capacity can then be exploited to permit some exhaust-nozzle adjustment without sacrificing engine design-point performance. Another reason is that near the surge line the compressor efficiency does not fall as the operating point is moved away from the surge line along a line of constant equivalent rotational speed; instead, there is a plateau of high efficiency followed by a region of decreasing efficiency, the region of decreasing efficiency closely paralleling the trend described herein. In particular, these two factors, the inherent small margin in turbine work capacity and the compressor efficiency plateau, greatly improve the competitive position of exhaust-nozzle adjustment if only small amounts of adjustment are employed.

Use of compressor-exit bleed during take-off and low-speed flight may be required by engine cooling requirements. The high ram-pressure ratio which is available during high-speed flight may permit use of ram air for cooling many parts of the engine, conceivably even the turbine blades. Such use of compressor-exit bleed during low-speed flight results in a thrust penalty in comparison with turbine stator adjustment in combination with compressor overspeeding.

For a certain range of engine design conditions, application of turbine stator adjustment requires that the turbine size be made greater than would be the case if turbine stator adjustment were not employed (see ref. 1). For that range of engine design conditions for which use of turbine stator adjustment does not require increasing the turbine size, one of the principal questions affecting application of turbine stator adjustment is the amount by which the turbine efficiency varies. Reference 1 shows that the rotor-entrance relative flow angle varies from the design value by an amount that increases as the amount of stator adjustment increases. For a given increment of stator adjustment, the effect of this varying angle of incidence on turbine efficiency should be small near the design point and larger for operating conditions considerably removed from the turbine design point. Combining turbine stator adjustment with compressor overspeeding reduces the amount of turbine stator adjustment required to reach the design value of turbine-inlet temperature and thereby decreases any penalty in turbine efficiency associated with turbine stator adjustment.
CONCLUSIONS

For two turbojet engines designed for flight at flight Mach numbers of 2.5 and 3.0, engine operation during take-off was analyzed for four methods of operating the engines off-design. The off-design operational methods were compared on the basis of engine thrust, engine specific impulse, and the operating requirements that the compressor and turbine must fulfill. The way in which a given engine should be operated off-design depends on the characteristics of its particular compressor and turbine. Because the characteristics of each set of components will differ one from another, no one mode of operation can be selected as best for all engines and operating conditions. This analysis indicates the methods that appear to be most promising and the way in which they can best be exploited. The following conclusions about engine off-design operation were drawn from this analysis:

1. If the compressor is capable of operation in a constant-geometry engine at equivalent rotational speeds considerably above the design value, highest engine thrust and highest engine specific impulse are obtained with compressor overspeed operation. Since during such engine off-design operation the turbine operates at its design point, the turbine can be designed very near tolerable operating limits.

2. Turbine stator adjustment in combination with compressor overspeeding is better than turbine stator adjustment alone. Engine equivalent rotational speed should be raised as much as the compressor-surge characteristics will permit, and the additional thrust required should then be obtained by means of a further increase in turbine-inlet temperature in combination with turbine stator adjustment.

3. If the compressor-surge characteristics limit the turbine-inlet temperature to values less than the rated value, compressor-exit bleed can be used to permit an additional rise in turbine-inlet temperature. This bleeding can provide relatively large amounts of cooling air during take-off operation. The turbine operation moves away from limiting blade loading during such off-design operation, and the blade-jet speed ratio varies by a comparatively small amount.

4. For the assumed compressor characteristics, increasing engine thrust by means of exhaust-nozzle adjustment requires an increase in turbine size and thereby results in rather severe penalties in turbine design-point performance.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, September 29, 1953
REFERENCES


Figure 1. - Cross section of turbojet engine showing location of numerical stations.
Figure 2. - Area available on compressor map for overspeed operation.
Engine temperature ratio, \( \frac{T_{3}}{T_{1}} \)

Take-off

Surge line

Flight

Design equivalent rotational speed, \( \frac{N}{\sqrt{\rho_{1}}} \)\text{des}

Compressor equivalent weight flow, \( \frac{w_1\sqrt{\rho_1}}{\theta_1} \)

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Figure 5. - Effects of compressor overspeeding.
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Figure 6. - Continued. Constant-geometry-engine operating line on compressor map, showing effect of design compressor pressure ratio. Design compressor-inlet axial Mach number, 0.50.
Compressor equivalent weight-flow ratio, \( \frac{w/\theta_1/\theta_{1e}}{(w/\theta_1/\theta_{1e})_{des}} \)

Compressor equivalent blade-speed ratio, \( \frac{U/\theta_1}{(U/\theta_{1e})_{des}} \)

(c) Design compressor pressure ratio, 4.0.

Figure 6. - Continued. Constant-geometry-engine operating line on compressor map, showing effect of design compressor pressure ratio. Design compressor-inlet axial Mach number, 0.50.
Design compressor pressure ratio, \( \frac{p_2}{p_1} \)

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Compressor equivalent blade-speed ratio, \( \frac{U/L}{(U/L)_{des}} \)

(b) Mach 3.0 engine.

Figure 11. - Concluded. Effect of compressor overspeeding on engine thrust.
Figure 12. - Variation in engine specific impulse with engine thrust for compressor overspeeding.
(a) Surge line and constant-geometry-engine operating line intersect at small acute angle.

(b) Surge line and constant-geometry-engine operating line intersect at large acute angle.

Figure 13. Sketch representing effect of exhaust-nozzle adjustment in combination with compressor overspeeding on engine thrust during take-off.
Figure 14. - Compressor map for exhaust-nozzle adjustment, showing lines of constant engine thrust.

(a) Mach 2.5 engine.

Compressor equivalent weight-flow ratio, 

\[
\frac{\omega_1 \sqrt{T_1^3 / \theta_1}}{(\omega_1 \sqrt{T_1^3 / \theta_1})_{des}}
\]
Compressor equivalent weight-flow ratio, \( \frac{w_{\text{eq}}}{\rho_1 s_1} \)

\( \frac{(w_{\text{eq}}/\rho_1 s_1)}{(w_{\text{eq}}/\rho_1 s_1)_{\text{des}}} \)

(b) Mach 3.0 engine.

Figure 14. - Concluded. Compressor map for exhaust-nozzle adjustment, showing lines of constant engine thrust.
Engine thrust ratio, $\frac{F}{F_{eq}}$

Constant-geometry-engine operating line

High value

Point of tangency

Surge line

Low value

Compressor equivalent weight flow, $\frac{v_1 \sqrt{\rho_1}}{5_1}$

Figure 15. - Operating condition for maximum thrust.
Figure 16. - Compressor map for exhaust-nozzle adjustment, showing lines of constant engine thrust and turbine-exit critical area.
Figure 16. - Concluded. Compressor map for exhaust-nozzle adjustment, showing lines of constant engine thrust and turbine-exit critical area.
Figure 17. - Compressor map for exhaust-nozzle adjustment, showing lines of constant thrust-area factor.

(a) Mach 2.5 engine.

Compressor equivalent weight-flow ratio, 
\[
\frac{w_i/\rho_i/s_i}{(w_i/\rho_i/s_i)_{des}}
\]

Thrust-area factor, 
\[
\frac{F}{F_{eq}} \left( \frac{A_{des}}{A} \right)_{cr,4}
\]
Figure 17. - Concluded. Compressor map for exhaust-nozzle adjustment, showing lines of constant-thrust-area factor.

(b) Mach 3.0 engine.
Figure 18. - Variation in engine specific impulse with engine thrust and compressor equivalent blade speed for exhaust-nozzle adjustment.
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(a) Mach 2.5 engine.

Figure 20. - Variation in engine thrust with take-off turbine-inlet temperature and compressor equivalent blade speed for compressor-exit bleed. Bleed air used.
(b) Mach 5.0 engine.

Figure 20. - Concluded. Variation in engine thrust with take-off turbine-inlet temperature and compressor equivalent blade speed for compressor-exit bleed. Bleed air used.
Take-off turbine-inlet temperature, $T_{3i}^{CR}$

(a) Mach 2.5 engine.

Figure 21. - Variation of engine thrust with take-off turbine-inlet temperature and compressor equivalent blade speed for compressor-exit bleed. Bleed air discarded.
Figure 21. Concluded. Variation of engine thrust with take-off turbine-inlet temperature and compressor equivalent blade speed for compressor-exit bleed. Bleed air discarded.
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(a) Mach 2.5 engine.
Figure 23. - Concluded. Variation in engine thrust with compressor equivalent blade speed and turbine-inlet critical area for turbine stator adjustment along constant-geometry-engine operating line.
Figure 24. - Variation in turbine-exit critical area with compressor equivalent blade speed for turbine stator adjustment along constant-geometry-engine operating line.
Figure 25. - Variation in thrust-area factor with compressor equivalent blade speed for turbine stator adjustment along constant-geometry-engine operating line.
Figure 26. - Variation in engine specific impulse with engine thrust for turbine stator adjustment along constant-geometry-engine operating line.
Figure 27. - Compressor map for turbine stator adjustment off constant-geometry-engine operating line, showing lines of constant engine thrust.

(a) Mach 2.5 engine.
Compressor equivalent weight-flow ratio, \( \frac{w_f \sqrt{T_f / S_f}}{(w_f \sqrt{T_f / S_f})_{des}} \)

Figure 27. Concluded. Compressor map for turbine stator adjustment off constant-geometry-engine operating line, showing lines of constant engine thrust.
Figure 28. - Compressor map for turbine stator adjustment off constant-geometry-engine operating line, showing lines of constant engine thrust and turbine-exit critical area.

(a) Mach 2.5 engine.
Figure 28. - Concluded. Compressor map for turbine stator adjustment off constant-geometry-engine operating line, showing lines of constant engine thrust and turbine-exit critical area.
Figure 29. - Compressor map for turbine stator adjustment off constant-geometry-engine operating line, showing lines of constant thrust-area factor.

(a) Mach 2.5 engine.
Figure 29. Concluded. Compressor map for turbine stator adjustment off constant-geometry-engine operating line, showing lines of constant thrust-area factor.
Figure 30. - Variation in engine specific impulse with engine thrust and compressor equivalent blade speed for turbine stator adjustment off constant-geometry-engine operating line.
Figure 31. - Effect on engine thrust of overspeeding 10-stage axial-flow compressor of reference 7.
Take-off turbine-inlet temperature, \( T_i \), OR

Compressor equivalent blade-speed ratio, \( \frac{U/\sqrt{\theta_1}}{(U/\sqrt{\theta_1})_{des}} \)

(b) Mach 3.0 engine.

Figure 31. - Concluded. Effect on engine thrust of overspeeding 10-stage axial-flow compressor of reference 7.