RESEARCH MEMORANDUM

LOITERING AND RANGE PERFORMANCE OF TURBOJET-POWERED AIRCRAFT DETERMINED BY OFF-DESIGN ENGINE CYCLE ANALYSIS

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SUMMARY

The loitering and range performance of airplanes equipped with several different turbojet engines was analytically investigated by applying the results of off-design cycle analyses to specific airplane characteristics. The method of off-design cycle analysis is presented herein and is verified by a check with experimental data. The engines investigated were selected to show the effect of pertinent design variables. Most of the results presented are for two methods of engine operation, constant tail-pipe nozzle area with variable engine speed and constant engine speed with variable tail-pipe nozzle area. Airplane characteristics representative of both straight-wing and swept-wing airplanes were considered.

For all engines considered, the loitering and the range fuel flows obtained with rated tail-pipe nozzle area, variable engine speed operation were within 2 or 3 percent of optimum fuel flow obtainable with any type of engine operation. Operation at constant engine speed, variable tail-pipe nozzle area generally resulted in a loitering or range fuel flow higher than that for operation at rated area, variable engine speed.

For rated tail-pipe nozzle area, variable engine speed operation, increasing the rated compressor pressure ratio from 5 to 10 decreased the optimum loitering and the optimum range fuel consumptions approximately 15 percent, while increasing the rated turbine-inlet temperature or shifting the location of peak compressor efficiency had little effect on the optimum fuel consumptions.

The optimum loitering altitude for all engines and airplanes investigated was between approximately 25,000 and 35,000 feet. The corresponding optimum flight Mach numbers were approximately 0.4 to 0.65. In general, the optimum range fuel consumption occurs at 3000 to 5000 feet higher altitude and at approximately 0.15 higher flight Mach number than the optimum loitering fuel flow. The rate of burning fuel at the optimum
loitering flight condition is about 10 percent less than that at the optimum range flight condition.

INTRODUCTION

Two common methods of level-flight airplane operation are loitering and cruising, or range, operation. The purpose of the loitering portion of a flight is endurance. An interceptor may loiter while awaiting the approach of an enemy bomber or an airplane of any type may loiter or "stack" while awaiting permission to land. For either of these loitering applications, it is necessary only to remain aloft. Therefore, for the loitering portion of a flight, the fuel flow expressed in pounds per hour is of interest.

The range or cruising portion of a flight differs from the loitering portion in that it is necessary to traverse a certain distance rather than simply to remain aloft. The fuel flow expressed in pounds per mile is of interest for the range or cruising portions of a flight. For either type of flight it is desirable, of course, that the fuel consumption be a minimum.

The purpose of this report is to investigate analytically the effect of engine design and method of engine operation on the loitering and range performance of turbojet-powered aircraft. Because for these types of flight the engines are operated at less than maximum thrust, the determination of engine performance by means of a design-point cycle analysis is not possible. A simplified off-design cycle analysis was therefore developed at the NACA Lewis laboratory in order to compute engine performances over the necessary range of flight conditions and thrust levels. The resulting off-design analysis presented herein utilizes a minimum of experimentally determined component characteristics. Engine thrust and fuel flow values used in computing the airplane loitering and range performance were determined by means of this off-design cycle analysis for several different engine designs. The engine designs were selected to illustrate the effect of the engine speed at which peak compressor efficiency occurs, the shape of the mass-flow-engine-speed curve, the rated compressor pressure ratio, the rated turbine-inlet temperature, and the compressor type. By applying these thrust and fuel flow values to a given set of aerodynamic characteristics, the optimum flight conditions for loitering and range performance were determined. In addition, the effect of engine design and method of engine operation on loitering and range performance was ascertained.
The following symbols are used in this report:

- **A**: area (sq ft)
- **C_D**: drag coefficient
- **C_d**: discharge coefficient, ratio of actual to ideal mass flow
- **C_j**: jet thrust coefficient, ratio of actual to ideal jet thrust
- **C_L**: lift coefficient
- **c_p**: specific heat at constant pressure, Btu/(lb)(°R)
- **c_v**: specific heat at constant volume, Btu/(lb)(°R)
- **D**: drag (lb)
- **F**: thrust (lb)
- **f**: fuel-air ratio
- **g**: acceleration of gravity (ft/sec²)
- **ΔH**: enthalpy change (Btu/lb)
- **J**: mechanical equivalent of heat, 778(ft-lb)/Btu
- **K**: constant
- **L**: lift (lb)
- **M**: Mach number
- **N**: engine speed, rpm
- **P**: total pressure (lb/sq ft absolute)
- **p**: static pressure (lb/sq ft absolute)
- **R**: gas constant (ft-lb)/(lb)(°R)
- **S**: compressor slip factor, $gJ\Delta H_c/U^2$
- **T**: total temperature, °R
- **U**: rotor tip speed (ft/sec)
- **V**: velocity (ft/sec)
\( W_a \) air flow (lb/sec)
\( W_f \) fuel flow (lb/hr)
\( W_f' \) fuel rate (lb/mile)
\( W_g \) gas flow (lb/sec)
\( w \) airplane gross weight (lb)
\( \gamma \) ratio of specific heats, \( c_p/c_v \)
\( \delta \) ratio of total pressure to NACA standard sea-level pressure, \( P/2116 \)
\( \Theta \) ratio of total temperature to NACA standard sea-level temperature, \( T/519 \)
\( \eta \) efficiency

Subscripts:
0 free stream
2 compressor inlet
3 compressor outlet
4 turbine inlet
5 turbine outlet
6 tail-pipe nozzle exit
act actual
b combustion chamber
c compressor
id ideal
j jet
n net
r rated
ANALYSIS

Engine Performance

The determination of loitering and range performance requires evaluation of the thrust and fuel flow of an engine at conditions other than the engine design point inasmuch as these flights require less than maximum thrust output. For the present analysis these performance factors are determined by means of an off-design cycle analysis.

Component characteristics: Of primary importance in such an analysis is the performance of the various components of the engine. Characteristics of the engine components such as mass flow and efficiency cannot readily be determined by analytical means and must therefore be evaluated from experimental data. For the purpose of this report, two sets of compressor characteristics are used - one representative of axial-flow and one of centrifugal-flow compressors. Two variations in compressor corrected air flow with corrected engine speed assumed for the axial-flow engines are shown in figure 1. Both coordinates of the figure are nondimensionalized through division by their respective sea-level static rated values. For all axial-flow engines, the corrected air flow is assumed to be a function of corrected engine speed only, and independent of the compressor pressure ratio, altitude, or flight Mach number. The curves presented in figure 1(a) are based on two presently used axial-flow turbojet engines.

The variation of axial-flow compressor efficiency with corrected engine speed which was assumed in the analysis is presented in figure 1(b). Compressor efficiency divided by the maximum compressor efficiency is plotted against corrected engine speed divided by the corrected engine speed at which maximum efficiency occurs. This plot was found to approximately generalize all available axial-flow-compressor data. Use of such a compressor efficiency variation assumes that the compressor efficiency is independent of compressor pressure ratio. In actual operation, the compressor efficiency of an axial-flow engine varies somewhat with compressor pressure ratio. This variation is small, however, especially at high engine speeds where the largest variations in pressure ratio are encountered. Further justification of this assumption is presented in a later section in which calculated and experimental engine performance are presented.
A constant combustion efficiency \( \eta_c \) of 0.95 was assumed in the analysis. The pressure ratio across the combustion chamber \( P_4/P_3 \) was also assumed constant at 0.97. The turbine nozzle was assumed choked over the entire range of operating conditions and the turbine efficiency was assumed constant for any engine design. These assumptions of turbine performance have been found to give good results for engine speeds as low as 60 percent of rated corrected engine speed.

The tail-pipe nozzle was considered to have a constant discharge coefficient \( C_d \) throughout the analysis. A constant effective velocity coefficient \( C_j \) of unity was used for all cases except those otherwise noted. The flow in the tail pipe was assumed isentropic \((P_6 = P_5 \text{ and } T_6 = T_5)\).

The inlet diffuser was assumed to recover 0.99 of the total pressure at static conditions \((P_2/P_0 = 0.99)\). At other flight Mach numbers the inlet diffuser was assumed to recover 0.9 of the difference between freestream total and ambient static pressures \((P_2 = P_0 + 0.9 \times (P_0 - P_0))\).

For a centrifugal-flow engine the compressor pressure ratio, corrected air flow, and corrected engine speed are not independent variables. The centrifugal-flow-compressor characteristics used in the present analysis are presented in figure 2, in which the compressor pressure ratio is plotted as a function of corrected air flow and corrected engine speed. The last two factors are divided by their respective rated values.

The compression work of a centrifugal-flow machine is essentially independent of the compressor pressure ratio at a given engine speed and is proportional to the square of the rotor tip speed. It is therefore convenient to define the compression work of a centrifugal-flow compressor in terms of a slip factor

\[
S = \frac{g \Delta H_c}{u^2} = \frac{g \Delta H_c/\theta_2}{U_r^2 \left(\frac{N/\sqrt{\theta_2}}{(N/\sqrt{\theta_2})_r}\right)^2}
\]

A constant slip factor of 0.93 was assumed for the centrifugal-flow compressor in this report. Experimental data indicate that the assumption of a constant slip factor is valid for a wide range of compressor pressure ratios and for corrected engine speeds above 70 percent of rated. The remainder of the components of the centrifugal-flow engine were assumed the same as those of the axial-flow engine.

Method of off-design cycle analysis. - By use of the component characteristics presented in the preceding section, the complete off-design performance of a turbojet engine can be determined in the
following manner: The assumptions of choked turbine nozzles and constant pressure drop across the combustion chamber result in a relation among the compressor pressure ratio, compressor air flow, and turbine-inlet temperature ratio. By means of this and other thermodynamic relations, the pressure and temperature ratios across the engine can be determined for any corrected engine speed and turbine-inlet temperature ratio. For any flight Mach number the engine thrust, fuel flow, and tail-pipe nozzle area are obtainable for each engine temperature and pressure ratio. The details of the off-design cycle analysis are presented in the appendix.

Using the method of analysis presented, the performance of five axial-flow and one centrifugal-flow engine was computed over a range of flight Mach numbers and engine operating conditions. The axial-flow-engine designs were chosen to illustrate the effect of changes in the shape of the mass-flow - engine-speed curve, shift in the corrected engine speed at which peak compressor efficiency occurs, changes in rated turbine-inlet temperature, and changes in rated compressor pressure ratio. Design features of each engine are presented in table I. The results for engine A serve a two-fold purpose. First, when compared with the results of engine B, the effect of a shift in the mass-flow curve can be illustrated because the remainder of the component characteristics are similar to those of engine B. The different turbine efficiencies of engines A and B will prevent a check on the magnitudes of the fuel flow curves for the two engines, but should not prevent at least a qualitative check on the effect of the shape of the mass-flow curve. Second, experimental data are available for an engine with component characteristics nearly equal to those of engine A. Thus, the method of analysis can be checked experimentally.

The effect of the location of peak compressor efficiency can be determined by comparing the results of engines B and C. The effect of increasing the rated turbine-inlet temperature can be obtained by comparing the results of engines D and B. A comparison between a high pressure ratio engine and a typical present-day engine can be had from engines E and B. The 2100° R rated turbine-inlet temperature assumed for the high pressure ratio engine is considered to be consistent with present design trends. A comparison of a typical axial-flow and a typical centrifugal-flow engine is available from engines B and F.

Comparison with experimental data. - A comparison between the experimentally determined engine fuel flow of an engine similar to engine A and the calculated fuel flow for engine A is presented in figure 3(a). The corrected fuel flow is presented as a function of corrected engine speed for tail-pipe nozzle area ratios of 1.0, 1.078, and 1.221. The data are for a flight Mach number of 0.2 and an altitude of 5000 feet. The agreement between the calculated and experimental results is excellent.
A comparison between the calculated and experimentally determined variation of corrected net thrust with corrected engine speed is presented in figure 3(b). Analytical results are presented for two values of jet thrust coefficient $C_j$, 1.0 and 0.95. The agreement between the experimentally determined data and the calculated data for $C_j$ of 0.95 is excellent.

In reference 1; experimental data on the jet thrust coefficients actually obtained on engine A are presented (also called engine A in reference 1). For the range of nozzle pressure ratios encountered in figure 3(b), thrust coefficients of approximately 0.94 to 0.96 were obtained experimentally. The use of a coefficient of 0.95 on the analytical results should therefore be expected to improve the thrust check with experimental data.

Reference 1 also indicates that the value of $C_j$ may vary somewhat with nozzle design and with the pressure ratio across the nozzle. For a well-designed nozzle the value of $C_j$ may be between 0.99 and 1.0 for nozzle pressure ratios above 2.5. Since for most loitering and range flights a rather high nozzle pressure ratio is encountered, a jet thrust coefficient of 1.0 was used in the remainder of this report.

**Airplane Performance**

**Aerodynamic characteristics.** - The airplane aerodynamic characteristics used in the present analysis are presented in figure 4, in which lift coefficient is plotted against drag coefficient for constant values of flight Mach number. Characteristics typical of a current straight-wing airplane are presented in figure 4(a) and characteristics typical of a current swept-wing airplane are presented in figure 4(b).

These aerodynamic characteristics are assumed valid for a range of wing loading $w/A_W$ and power loading $w/F_{n,r}$. The assumption that the aerodynamic characteristics are independent of wing loading implies that the gross weight of the airplane is varied while the external configuration is held fixed. This may be accomplished by varying the fuel load or the pay load.

Varying the power loading without changing the aerodynamic characteristics of the airplane requires that various size engines be fitted into a given fuselage size. There is obviously a limit as to the size engine that can be installed in a given airplane, and in practice the aerodynamic characteristics may vary somewhat with power loading. However, qualitative effects of power loading on airplane loitering and range performance can be shown by assuming the airplane characteristics independent of power loading.
Computation of loitering and range performance. - For a given airplane, wing loading, and altitude there is a flight Mach number at which the airplane drag and, consequently, the required engine thrust are minimum. At this point the airplane lift-drag ratio is a maximum. Also, for a given engine design and size there is a value of thrust for which the specific fuel consumption $W_F/F_n$ is a minimum. The minimum fuel flow at a given altitude, or the maximum loiter time at that altitude for a given amount of fuel, occurs at the flight Mach number at which the specific fuel consumption divided by the lift-drag ratio is a minimum, that is, where $W_f/W = W_F/F_n \frac{1}{L/D}$ is a minimum. At this flight Mach number and altitude the specific fuel consumption is not necessarily minimum nor is the lift-drag ratio necessarily maximum, but rather the ratio of the two quantities is a minimum. There is one altitude, however, where minimum specific fuel consumption and maximum lift-drag ratio occur at the same flight Mach number. The engine fuel flow is a minimum here, and this flight condition is said to be the optimum loitering condition.

The loitering calculations were made as follows: The lift coefficient required for level flight is determined from

$$C_L = \frac{w/A_w}{\frac{1}{2} \gamma_0 P_0 M_0}$$

By use of this lift coefficient and the flight Mach number, the drag coefficient and, consequently, the lift-drag ratio are determined from the aerodynamic characteristics, since $\frac{L}{D} = \frac{C_L}{C_D}$.

For any engine design the static sea-level thrust per pound of air $(F_n/W_a)_r$ is known. Then, for a given power loading,

$$\frac{F_n/S_2}{(W_a\sqrt{\theta_2/S_2})_r} = \frac{S_2}{L/D} \frac{S_2}{F_n} \left[ \frac{w}{F_n} \right] \left(\frac{F_n}{W_a}\right)_r$$

The off-design cycle calculations are used to determine the corrected fuel flow $W_F/(W_a\sqrt{\theta_2/S_2})_r$, corresponding to the above thrust value for any method of engine operation (constant engine speed, constant tail-pipe
nozzle area, and so forth). The loitering fuel consumption in units of pounds of fuel per hour per ton of airplane is determined from

\[
2000 \frac{W_f}{W} = \frac{2000 \sqrt{\beta_2}}{(s_2)_r \left(\frac{F_n}{W_a}\right)_r \left(\frac{w}{F_n}\right)_r \left(\frac{W_a}{\sqrt{\beta_2}}\right)_r}
\]

Flights in which range or distance traveled is of primary importance present a slightly different problem than those whose sole purpose is to remain aloft. For range considerations, the fuel consumption may be expressed in terms of pounds of fuel consumed per mile per ton of airplane. This range fuel consumption is given by the following expression:

\[
\frac{W'_f}{W} = \frac{W_f / F_n}{V_0 L/D} = \frac{W_f}{V_0 w}
\]

Thus, the range fuel consumption in pounds per ton-mile may be obtained by dividing the loitering fuel consumption in pounds per ton-hour by the flight velocity in miles per hour.

RESULTS AND DISCUSSION

Loitering Performance

A typical variation of fuel flow with flight Mach number is shown in figure 5 for an altitude of 35,000 feet. The results are for a straight-wing airplane with a wing loading of 60 pounds per square foot and a power loading of 3 pounds per pound of rated thrust. Engine A is installed in the airplane. Curves are presented for three modes of engine operation - variable tail-pipe nozzle area operation at both rated engine speed and 0.9 rated engine speed, and variable engine speed operation at rated tail-pipe nozzle area. The fuel flow is a minimum at a flight Mach number of approximately 0.58 for all modes of engine operation.

By repeating the calculation of figure 5 for a range of altitudes, it is possible to determine the minimum fuel flow at each altitude, the flight Mach number which yields this minimum fuel flow at each altitude, and the altitude which results in minimum fuel flow.

In order to illustrate the effect of engine design and method of engine operation, the loitering performance was computed for the six turbojet engines presented in table I. Results are also presented to show the effect of wing loading, power loading, and aerodynamic characteristics on loitering performance.
The variation of flight Mach number and loitering fuel flow with altitude for engine A is presented in figure 6(a). Three methods of engine operation are considered - rated engine speed, 0.9 rated engine speed, and rated tail-pipe nozzle area. These curves were obtained by cross-plotting the minimum points on curves similar to those in figure 5 for several altitudes. Therefore each point on the curves of figure 6(a) represents the minimum fuel flow obtainable with a given method of engine operation at that particular altitude. The points on the flight Mach number curve represent the flight Mach number at which minimum fuel flow is obtained at each altitude. A single curve was obtained for all methods of engine operation.

Optimum loitering altitude varies from 32,000 feet for rated area, variable speed operation to 37,000 feet for rated engine speed, variable area operation, while the corresponding flight Mach numbers vary from 0.55 to 0.60.

The difference in optimum loitering altitude results from the fact that the minimum specific fuel consumption occurs at a higher corrected thrust for rated engine speed operation than for rated tail-pipe area operation. When the corrected thrust is high, it is necessary to fly at a high altitude in order that the uncorrected thrust will just equal the airplane drag. As previously mentioned, the flight Mach number for minimum fuel flow is also higher for the rated engine speed case than for the rated area case because there is a lift coefficient for which the lift-drag ratio is maximum; in order to fly at this lift coefficient at the higher altitude rated engine speed optimum point, it is necessary to fly faster than at the lower altitude constant area optimum point. The intersection of the rated engine speed and rated area curves represents approximately the absolute ceiling of the airplane, since at this point the engine speed and area are identical for the two modes of operation.

The minimum loitering fuel flow for rated engine speed, variable-area operation is approximately 22 percent higher than the minimum fuel flow for rated area operation. Reducing the engine speed to 0.9 of rated results in a minimum fuel flow approximately 10 percent higher than the minimum rated-area fuel flow. The higher fuel flows accompanying the constant-speed cases result from the fact that lower compressor efficiencies are encountered at rated and 0.9 rated engine speed than at the reduced engine speed associated with the rated area operation (fig. 1(b)).

Figure 6(b) presents the tail-pipe nozzle areas and engine speeds required for the three modes of engine operation previously mentioned. At an altitude of 20,000 feet the tail-pipe nozzle area for rated engine speed operation is approximately 50 percent larger than rated area. Some difficulty may be encountered in the construction of a variable-area exhaust nozzle which could operate properly over a much wider range of
areas. Therefore loitering flights below 20,000 feet might not be possible at rated engine speed.

The engine speed corresponding to minimum fuel-flow operation with a rated area nozzle is approximately 0.77 of rated. If it were suddenly necessary to obtain maximum thrust as quickly as possible, some time lag would be introduced while the engine was being accelerated from 0.77 to rated engine speed with a rated area nozzle. However, with the engine operating at rated engine speed and a larger tail-pipe nozzle area, maximum thrust could be obtained almost instantaneously by reducing the nozzle area, suggesting the possible necessity of a compromise between loitering fuel flow and time required to obtain maximum thrust. A two-position nozzle that could be opened during the time the engine was accelerating might also be used to reduce the time required to obtain maximum thrust.

From the preceding figures it can be seen that the loitering fuel flow obtained with rated tail-pipe nozzle area operation is lower than that obtained with either of the constant engine speed cases. To specify in any simple or general manner the method of operation that would result in the lowest possible fuel flow is impossible. The variation of loitering fuel flow with engine speed for three altitudes is presented in figure 7. Rated tail-pipe nozzle area points are indicated by circles on the curves. From this figure, it can be seen that the fuel flow obtained with rated tail-pipe nozzle area operation is approximately equal to the minimum fuel flow obtainable at each altitude. The trends exhibited in this figure are general for all engines investigated. For all engines of table I, the rated tail-pipe nozzle area fuel flow was within 2 or 3 percent of the minimum obtainable fuel flow.

It can also be seen from figure 7 that the fuel flow obtained with rated engine speed operation is higher than that obtained at any other engine speed (within the range of interest). It can therefore be concluded that the fuel flow obtained with any constant engine speed method of operation is bracketed between the fuel flow obtained with rated tail-pipe nozzle area operation and that obtained with rated engine speed operation. For this reason the remainder of the results are presented for only the last two methods of engine operation.

The preceding figures are based on the assumption that the component characteristics are not affected by changes in altitude. In actual operation, both the compressor efficiency and the corrected air flow at a given corrected engine speed decrease somewhat as altitude is increased. This depreciation in compressor performance is attributed to changes in Reynolds number (reference 2). A spot check was made to determine the effect of compressor depreciation on loitering performance. Representative variations with altitude of peak compressor efficiency and corrected air flow at rated corrected engine speed were used. The results indicated that the optimum loitering altitude was shifted downward about 2000 feet and the fuel flow at this optimum point was increased.
approximately 4 percent. The intersection of the rated engine speed and rated tail-pipe nozzle area curves was shifted downward approximately 4000 feet. Since these results did not appreciably alter the trends previously presented, the compressor performance was considered independent of altitude in the remainder of the report.

In actual engine operation, combustion efficiency may vary somewhat with altitude, engine speed, and tail-pipe nozzle area. However, within the present range of conditions the assumption of a constant combustion efficiency does not constitute a large sacrifice in accuracy. Furthermore, the comparison of methods of engine operation is even more valid since at a given thrust value the conditions in the combustion chamber which influence the combustion efficiency are about the same for all methods of engine operation considered. The results presented herein could easily be modified to include any prescribed variation of combustion efficiency by multiplying the fuel flow values by the ratio of the assumed combustion efficiency of 0.95 to any prescribed combustion efficiency.

Comparison of various engine designs. - The loitering performance of all six engines of table I operating in straight-wing airplanes with a wing loading of 60 pounds per square foot and a power loading of 3 pounds per pound is presented in figure 8. Figure 8(a) presents the loitering performance for rated tail-pipe nozzle area, variable engine speed operation and figure 8(b), the performance for rated engine speed, variable tail-pipe nozzle area operation.

For rated tail-pipe nozzle area, variable engine speed operation, which represents approximately the optimum method of engine operation for all engines considered, increasing the rated compressor pressure ratio from 5 to 10 decreased the optimum loitering fuel flow approximately 15 percent. Because of its lower compressor efficiency, the optimum loitering fuel flow for the centrifugal-flow engine F was approximately 18 percent higher than that of a similar axial-flow engine B. The other curves on figure 8(a) indicate that increasing the rated turbine-inlet temperature, shifting the location of peak compressor efficiency, or changing the shape of the mass-flow - engine-speed curve did not appreciably affect the optimum loitering fuel flow. For any of these engines (A, B, C, and D) the engine speed, altitude, and flight Mach number can be varied until an optimum fuel flow is reached. As a result, the value of optimum fuel flow for these engines does not vary appreciably from one engine to another.

Variation in engine design has a greater effect on loitering performance for rated engine speed operation than for rated area operation. Figure 8(b) indicates that shifting the location of peak compressor efficiency to a higher corrected engine speed as well as increasing the rated compressor pressure ratio lowers the loitering fuel flow, because this shifting of the location of peak compressor efficiency results in a
higher compressor efficiency at rated engine speed. The combined low compressor efficiency at rated engine speed and the low turbine efficiency of engine A result in an optimum loitering fuel flow for engine A about the same as that for the centrifugal-flow engine (F).

**Effect of method of engine operation.** - As previously indicated, results for any other method of engine operation generally fall between the rated area and rated engine speed results. The spread between the rated area and the rated engine speed curves is therefore a qualitative indication of the spread that may be expected between any method of operation and the optimum.

In order to facilitate the comparison of methods of engine operation, the results of both methods of engine operation from figures 8(a) and 8(b) are reproduced on the same figure for each engine in figure 9.

The difference in fuel flow between rated engine speed and rated area operation for engine B can be seen from figure 9(a). This difference results mainly from the different compressor efficiencies accompanying the two methods of operation.

For an engine such as engine C (fig. 9(b)), which has the peak compressor efficiency located at a higher engine speed than engine B, the operating points for the two methods of operation straddle the peak compressor efficiency. For engine C there is little difference between the fuel flow for the two methods of operation.

Increasing the rated turbine-inlet temperature from 2000° R to 2500° R decreases spread in fuel flows from one method of operation to another, as can be seen by comparing figures 9(c) and 9(a). The spread between the optimum of the rated tail-pipe nozzle area curves and the optimum of the rated engine speed curves is almost half as great for the high temperature engine D (fig. 9(c)) as for the low temperature engine B (fig. 9(a)). It may therefore be concluded that the method of engine operation has less effect on fuel flow for an engine with a high turbine-inlet temperature than for an engine with a low turbine-inlet temperature.

The spread between the rated engine speed and the rated area curves is almost 50 percent greater for the high pressure ratio engine (fig. 9(d)) than for the low pressure ratio engine (fig. 9(a)). Thus, method of engine operation is more important for a high pressure ratio engine. The greater spread of the fuel flow curves for the high pressure ratio engine results from the large effect of compressor efficiency at the higher pressure ratio.

A comparison of figures 9(e) and 9(a) indicates that the effect of method of engine operation at the optimum conditions is less for a centrifugal-flow engine than for a similar axial-flow engine.
Effect of wing loading and power loading. - The effect of wing loading and power loading on the optimum loitering performance is presented in figure 10. The results are for engine B installed in a straight-wing airplane. Figure 10(a) presents results for rated tailpipe nozzle area operation and 10(b), for rated engine speed operation. Curves are presented for three power loadings, 2, 3, and 4 pounds per pound, and for wing loadings from 30 to 90 pounds per square foot. The fuel flows represent the minimum points of curves similar to those of figure 9(a). The altitude and flight Mach numbers at which these minimum fuel flows occur are also plotted on figure 10.

The lowest loitering fuel flows occur at low wing loadings and high power loadings (that is, small engines). For a given-size airplane, increasing the gross weight (and wing loading) increases the optimum loitering fuel flow because of the increase in fuel flow in pounds per hour per ton of airplane and also because of the increase in airplane weight.

Increasing the wing loading increases the optimum loitering Mach number and decreases the optimum loitering altitude. Increasing the power loading results in a decrease in both the optimum altitude and flight Mach number.

The effect of method of engine operation on the optimum loitering fuel flow is more important at low power loadings than at high power loadings as can be seen by comparing figures 10(a) and 10(b). Also, in general, the method of engine operation has a greater effect at high wing loadings than at low wing loadings.

Effect of aerodynamic characteristics. - The loitering performance of a swept-wing airplane with a wing loading of 60 and a power loading of 3 is presented in figure 11. The airplane is equipped with engine B. There is little difference between the optimum altitude for a swept- and a straight-wing airplane, as is seen by comparing figures 11 and 9(a). However, the flight Mach numbers corresponding to the optimum loitering altitudes are higher for a swept-wing airplane than for a straight-wing airplane, being 0.57 for rated area operation and 0.64 for rated engine speed operation, as compared with 0.51 and 0.58 for the straight-wing airplane.

For all altitudes, the fuel flow of the swept-wing airplane is higher than that of the straight-wing airplane because the lift-drag ratio of the swept-wing airplane is lower than that of the straight-wing airplane.

The minimum fuel flow for rated engine speed operation of the swept-wing airplane is only about 10 percent higher than the rated area minimum fuel flow, while for the straight-wing airplane the rated engine speed fuel flow was 18 percent higher than the rated area fuel flow.
This difference results from the lower lift-drag ratio of the swept-wing airplane. Accompanying this lower lift-drag ratio is a higher required thrust. A relatively high engine speed is required for constant area operation to produce this thrust. At this higher engine speed the compressor efficiency for constant area operation is more nearly equal to the compressor efficiency at rated engine speed. Therefore, the fuel flows of the two methods of engine operation are more nearly equal for the swept-wing than for the straight-wing airplane.

Range Fuel Consumption

The variation in range fuel consumption with flight Mach number at an altitude of 35,000 feet is presented in figure 12 for a straight-wing airplane with a wing loading of 60 pounds per square foot and a power loading of 3 pounds per pound. The airplane is equipped with engine A. Figure 12 was obtained by dividing each point on the loitering fuel flow curves (fig. 5) by the product of its respective abscissa and the speed of sound (that is, by the flight velocity). Figure 5 is reproduced together with figure 12 for convenience.

Obviously, the minimum range fuel consumption at any altitude occurs at a higher flight Mach number than does the minimum loitering fuel consumption. The method of engine operation also affects the flight Mach number for minimum range fuel consumption. The flight Mach number for minimum range fuel consumption is lower for methods of engine operation which result in lower values of fuel consumption.

Cross-plotting curves similar to figure 12 makes possible the determination of the minimum range fuel consumption at any altitude and the altitude at which minimum range fuel consumption is obtained. Figures 13 to 17 present the results of such calculations for engines and airplanes and so forth corresponding to those for which the loitering fuel flow was presented in figures 6 to 11. The range fuel consumption results are, in general, similar to the previously discussed loitering fuel flow results; therefore only a brief discussion of the range results will be made to point out how they differ from the loitering results.

The altitude and the flight Mach number which result in minimum range fuel consumption (or maximum range for a given fuel load) are both higher than the corresponding optimum loitering conditions. For engine A (fig. 13) the optimum range altitudes are approximately 4000 feet higher than the optimum loitering altitudes, and the optimum range flight Mach numbers are approximately 0.15 higher than the corresponding loitering values.

The method of engine operation has less effect on range fuel consumption than on loitering fuel consumption. The spread between the optimum rated engine speed results and the optimum rated area results is
about 30 percent less for range consideration than for loitering considerations. At a given altitude, the optimum range flight Mach number is slightly higher for rated engine speed operation than for rated area operation. The difference between the flight Mach numbers for the two methods of engine operation becomes less as altitude is increased.

The effect of engine design for the range results (fig. 15) is similar to that for the loitering results (fig. 8). Varying the wing loading and power loading has a different effect on the range results (fig. 16) than on the loitering results (fig. 10). For the range results, at a given power loading there is a wing loading for minimum range fuel consumption. Increasing the power loading increases the wing loading for minimum range fuel consumption. Increasing the wing loading also increases the optimum range Mach number and decreases the optimum range altitude while increasing the power loading decreases both the optimum range altitude and the Mach number.

A comparison of the rate at which fuel is consumed in a range or cruising flight with the rate at which fuel is consumed in a loitering flight is of interest. The fuel flow in pounds per ton-hour corresponding to the optimum range point can be obtained by multiplying the optimum range fuel flow in pounds per ton-mile by the flight velocity at that point. This fuel flow is generally about 10 percent higher than the optimum loitering fuel flow. Thus changing from the flight condition for best range to the flight condition for best endurance decreases the rate of burning fuel by about 10 percent.

SUMMARY OF RESULTS

A method of analytically determining the off-design performance of a turbojet engine is presented and is applied to the problem of determining the loitering and range performance of turbojet-powered aircraft. A comparison is made between the calculated and the experimentally determined thrust and fuel flow of a typical axial-flow turbojet engine. The agreement between the experimental results and those calculated by means of the off-design cycle analysis is excellent.

For all engines considered, the loitering and the range fuel flows obtained with rated tail-pipe nozzle area, variable engine speed operation were within 2 or 3 percent of the optimum fuel flow obtainable with any type of engine operation. Operation at constant engine speed and variable tail-pipe nozzle area generally resulted in loitering or range fuel flows that were higher than those for operation with rated tail-pipe nozzle area and variable speed.

For rated tail-pipe nozzle area, variable engine speed operation, increasing the rated compressor pressure ratio from 5 to 10 decreased the optimum loitering and the optimum range fuel consumptions by about
15 percent. Increasing the rated turbine-inlet temperature or shifting the location of peak compressor efficiency had little effect on the optimum fuel consumptions.

The optimum loitering altitude was generally between approximately 25,000 and 35,000 feet. The corresponding optimum flight Mach numbers were approximately 0.4 to 0.65. In general, the optimum range fuel consumption occurred at 3000 to 5000 feet higher altitude and at approximately 0.15 higher flight Mach number than the optimum loitering fuel flow. The rate of burning fuel at the optimum loitering flight condition is about 10 percent less than at the optimum range point.

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National Advisory Committee for Aeronautics
Cleveland, Ohio
APPENDIX - OFF-DESIGN CYCLE ANALYSIS

The component characteristics presented in a previous section can be used to determine the complete off-design performance of a turbojet engine in the following manner:

The assumptions of choked turbine nozzles and constant pressure ratio across the combustion chamber result in the following relation among the compressor pressure ratio, corrected air flow, and turbine-inlet temperature ratio (neglecting the change in fuel-air ratio with temperature):

\[
\frac{P_3}{P_2} = K \frac{W_a \sqrt{\theta_2/\theta_2}}{(W_a \sqrt{\theta_2/\theta_2})_r} \sqrt{\frac{T_4}{T_2}}
\]  

(1)

The value of \( K \) may be determined for any engine at static sea-level rated conditions where the compressor pressure ratio and turbine-inlet temperature are known. The corrected air flow factor is equal to unity at this point.

If the constant of proportionality \( K \) is known, any one of the remaining factors in the above equation can be calculated if the other two are known. For example, the corrected air flow factor of an axial-flow engine is known for any corrected engine speed. With this value of corrected air flow the compressor pressure ratio can be determined for a range of values of turbine-inlet temperature ratios.

If the enthalpy drop of the fuel mass passing through the turbine is assumed equal to the bearing and accessory power, the enthalpy drop of the air passing through the turbine must be equal to the enthalpy rise of the air passing through the compressor:

\[
\Delta H_c = \Delta H_t
\]  

(2)

\[
\frac{c_p, c}{\eta_c} \left[ \frac{\gamma_c - 1}{\gamma_c} \right] \frac{T_2}{(P_3/P_2)} = c_p, t (T_4 - T_5)
\]  

(3)

Rearranging gives the engine temperature ratio as

\[
\frac{T_5}{T_2} = \frac{T_4}{T_2} - \frac{c_p, c}{c_p, t \eta_c} \left[ \frac{\gamma_c - 1}{\gamma_c} \right] \frac{(P_3/P_2)}{(P_3/P_2)} - 1
\]  

(4)
From the definition of turbine efficiency,

$$\eta_t = 1 - \frac{T_5}{T_4}$$

$$1 - \left(\frac{P_5}{P_4}\right)^{\frac{1}{\gamma_t - 1}}$$

and the energy balance of equation (3), the pressure ratio across the turbine $P_5/P_4$ is obtained.

$$\frac{P_5}{P_4} = \left\{1 - \frac{c_{p,t}}{c_{p,t} + 1} \frac{T_{2}}{T_{4}} \frac{\gamma_t - 1}{\gamma_t} \left(\frac{P_3}{P_2}\right)^{\frac{1}{\gamma_t - 1}}\right\} \frac{\gamma_t}{\gamma_t - 1}$$

The pressure ratio across the engine $P_5/P_2$ is obtained from

$$\frac{P_5}{P_2} = \left(\frac{P_3}{P_2}\right) \left(\frac{P_4}{P_3}\right) \left(\frac{P_5}{P_4}\right)$$

The engine temperature and pressure ratios, equations (4) and (7), are independent of altitude and flight Mach number. These ratios, together with the ram pressure ratio $P_2/P_0$, determine the jet thrust per pound of gas flow.

For a choked convergent nozzle,

$$\frac{F_j}{W_s \sqrt{\theta_2}} = \sqrt{\frac{519 R}{\gamma_j g}} \left(\frac{\gamma_j + 1}{2}\right)^{\frac{1}{\gamma_j - 1}} \left(\frac{P_0}{P_6}\right)^{\frac{1}{2}} \left(\frac{T_6}{T_2}\right)^{\frac{1}{2}}$$

or for an unchoked nozzle,

$$\frac{F_j}{W_s \sqrt{\theta_2}} = \sqrt{\frac{2 \gamma_j R}{g(\gamma_j - 1)}} \left[1 - \left(\frac{P_0}{P_6}\right)^{\frac{1}{\gamma_j - 1}}\right] \left(\frac{T_6}{T_2}\right)^{\frac{1}{2}}$$
The corrected net thrust per pound of air is equal to

\[
\frac{F_n}{W_a \sqrt{\theta_2}} = (1+\text{fact}) \frac{F_d}{W_a \sqrt{\theta_2}} - \frac{V}{g \sqrt{\theta_2}}
\]

and the corrected net thrust per pound of sea-level static rated air flow is

\[
\frac{F_n/\theta_2}{(W_a\sqrt{\theta_2/\theta_2})_r} = \frac{F_n}{W_a \sqrt{\theta_2/\theta_2}} \frac{W_a \sqrt{\theta_2/\theta_2}}{(W_a \sqrt{\theta_2/\theta_2})_r}
\]

Thus, for any corrected engine speed, turbine-inlet temperature ratio, and flight Mach number, the engine net thrust is obtained in a form which is independent of altitude and engine air handling capacity.

The ideal fuel-air ratio may be obtained from reference 3 using the engine inlet temperature \(T_2\) and the temperature rise across the engine

\[\Delta T = T_2 \left(\frac{T_5}{T_2} - 1\right)\]

The actual fuel-air ratio is obtained from the ideal fuel-air ratio and the combustion efficiency

\[(f)_{\text{act}} = \frac{(f)_{\text{id}}}{\eta_b}\]

The fuel-air ratio is a function of the engine-inlet temperature \(T_2\). However, the corrected fuel-air ratio \(\frac{f}{\theta_2}\) which is used in computing the corrected fuel flow is nearly independent of \(T_2\). Therefore, for the purpose of this analysis, \(T_2\) was assumed constant at a value of 440° R. The corrected fuel flow can then be expressed as

\[
\frac{W_f/\theta_2 \sqrt{\theta_2}}{(W_a \sqrt{\theta_2/\theta_2})_r} = \left(\frac{519}{440}\right) (3600)(f)_{\text{act}} \frac{W_a \sqrt{\theta_2/\theta_2}}{(W_a \sqrt{\theta_2/\theta_2})_r}
\]
Thus the engine fuel flow is expressed in a manner that is independent of altitude and engine air handling capacity.

The tail-pipe nozzle area required for any operating point can be determined in the following manner: From continuity of flow for a choked convergent nozzle, the effective exit area can be expressed as

$$A_6 = \frac{(W_a \sqrt{b_2/b_2})(1+f_{act})}{2116 \sqrt{\frac{b_2}{519 R}}} \frac{\gamma + 1}{2} \frac{T_6/T_2}{P_6/P_2}$$

or for an unchoked nozzle,

$$A_6 = \frac{(W_a \sqrt{b_2/b_2})(1+f_{act})}{2116} \frac{\gamma + 1}{2} \frac{T_6/T_2}{P_6/P_0} \frac{519(\gamma - 1)R}{2\gamma J}$$

The effective tail-pipe nozzle area, expressed as a fraction of sea-level static rated effective area, is found by taking the ratio of one of the above expressions evaluated at any engine operating point to one of the expressions evaluated at sea-level static rated conditions. The expression used in either part of this ratio depends on whether the tail-pipe nozzle is choked at that particular condition. Since the nozzle discharge coefficient $C_d$ was assumed constant, the effective nozzle-area ratio is identical to the physical nozzle-area ratio.

A slightly different analysis must be used on the centrifugal-flow engine because the compressor work of this type of engine is expressed in terms of a compressor slip factor instead of compressor efficiency and compressor pressure ratio. For the centrifugal-flow compressor used in this analysis, the compressor work is

$$\frac{\Delta h_c}{b_2} = \frac{N/\sqrt{b_2}}{(N/\sqrt{b_2})_e}^2 \frac{U_r^2}{gJ}$$

A rated tip speed $U_r$ of 1536 feet per second was used in this analysis. The energy balance between the compressor and turbine is

$$\frac{T_2}{519} \left( \frac{\Delta h_c}{b_2} \right) = c_p t (T_4 - T_5)$$
and the engine temperature ratio is

\[
\frac{T_5}{T_2} = \frac{T_4}{T_2} - \frac{1}{519 \frac{c_p}{\gamma_2}} \left( \frac{\Delta h_c}{\gamma_2} \right)
\]

In a similar manner, the pressure ratio across the turbine is

\[
\frac{P_5}{P_4} = \left(1 - \frac{T_2}{519 T_4 \frac{c_p}{\gamma_2} \eta_t} \frac{\Delta h_c}{\gamma_2} \right) \frac{\gamma_t}{\gamma_t - 1}
\]

The remainder of the centrifugal-flow analysis is identical with the axial-flow analysis.

REFERENCES


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Figure 1. - Axial-flow-compressor characteristics.
Fraction of rated corrected engine speed
\[ \frac{\text{Fraction of rated corrected air flow, } \frac{W_a\sqrt{\theta_2/\theta_2}}{(W_a\sqrt{\theta_2/\theta_2})_r}}{\text{Compressor pressure ratio, } P_3/P_2} \]

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