EFFECT OF CERTAIN COMBINATIONS OF WALL CONTOURING AND DESIGN EXIT VELOCITY DISTRIBUTION ON PREDICTION OF TURBINE-NOZZLE MASS FLOW

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RESEARCH MEMORANDUM

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SUMMARY

An investigation was conducted to determine if certain combinations of design nozzle-exit velocity distribution and wall contouring, employed to increase annulus area, can alter the three-dimensional flow characteristics such that nozzle mass flow cannot be predicted by ordinary two-dimensional design techniques. Four nozzle configurations with different wall contours and velocity distributions were analyzed in the axial-radial plane (axial symmetry was assumed) to determine the effect of the three-dimensional flow characteristics on the maximum mass flow. The accuracy of the method of analysis was established by comparing the maximum mass flow obtained by analysis with the maximum obtained experimentally.

The results of the analysis indicated that two-dimensional design techniques were inadequate for use in predicting mass flow for the nozzle configuration designed for a nontwisted rotor and having a convex inswept inner wall. It was therefore concluded that certain combinations of nozzle-exit velocity distribution and wall contouring employed to vary annulus area can alter the three-dimensional flow characteristics to a point where two-dimensional design techniques are inadequate for predicting mass flow. Consideration of the three-dimensional flow characteristics must therefore be included in the design of nozzles to insure satisfactory turbine performance.

INTRODUCTION

The increased capabilities of compressors in developing high pressure ratios and passing high specific mass flows and the resulting increase in the severity of turbine requirements indicate that single or multistage turbines designed to power these compressors may in many instances employ nozzles within which there is a divergence of annulus area.

Present-day turbines are usually designed on a two-dimensional basis. However, if a substantial divergence in annulus area in the nozzles is required, it becomes questionable as to whether or not the nozzle mass
flow and the exit velocity distribution resulting from equilibrium requirements can be predicted by the conventional two-dimensional design procedures in which simple radial equilibrium is assumed at the entrance and exit. If the nozzles were to pass more than design mass flow, the turbine might reach limiting loading before design specific work could be obtained. If the nozzles were to pass less than design mass flow, the point of maximum turbine efficiency might be moved to a relatively unimportant position on the performance map. Also, this condition could cause mismatching between the compressor and the turbine in a turbojet engine and might force the compressor operation into the surge region. In either case, the velocity distribution might be considerably different from design, thus introducing poor angles of incidence and additional losses.

A mass-flow problem of this type occurred in the investigation of a turbine rotor utilizing a diverging annulus area designed on a two-dimensional basis (ref. 1). A three-dimensional analysis of the flow through this turbine rotor (ref. 2) indicated that the rotor would not pass the design mass flow. This was confirmed by the performance of this turbine as reported in reference 1. The results of the three-dimensional analysis applied to another turbine rotor designed for the same application indicated that design mass flow could be passed, and this was experimentally verified (ref. 3). Thus, it was concluded that the method of analysis (ref. 2) although approximate is adequate in determining the mass flow characteristics of turbine rotors.

The purpose of this investigation was to determine if certain combinations of nozzle-exit-velocity distribution and wall contouring can alter the three-dimensional flow characteristics such that the nozzle mass-flow cannot be predicted by two-dimensional design techniques. As mentioned previously, the effect of the three-dimensional flow characteristics on the actual velocity distribution at the nozzle exit can also be important, as variations from design can introduce losses due to poor angles of incidence. However, this investigation is restricted to the mass-flow study. Two nozzles having inswept inner walls, one designed for free-vortex flow and one designed for a nontwisted rotor, were investigated by the analytical method of reference 2, as well as experimentally, to determine their maximum mass flow. The design exit velocity distribution for a nozzle designed for a nontwisted rotor has the characteristic of the product of the tangential component of velocity and wheel speed increasing from hub to tip as compared with a constant value for free-vortex designs. This increase results in an inward shift of mass flow as compared with an outward shift for free-vortex designs. For comparative purposes, two other nozzle configurations with cylindrical walls, one a free-vortex design and the other designed for a nontwisted rotor, were also investigated. The accuracy of the analytical method will be shown by comparing the results of the analytical investigation with experimental results.
SYMBOLS

The following symbols are used in this report:

\[ a_{cr} \]  velocity of sound at a Mach number of unity

\[ K \]  constant

\[ r \]  radius, in.

\[ r_t \]  tip radius, in.

\[ U \]  blade velocity, ft/sec

\[ V_u \]  tangential velocity, ft/sec

\[ V_z \]  axial velocity, ft/sec

\[ \rho \]  density, lb/cu ft

Superscript:

'  total state

DESCRIPTION OF NOZZLES INVESTIGATED

The four nozzle configurations investigated herein will be designated A, B, C, and D. The following table summarizes the important characteristics of each nozzle:

<table>
<thead>
<tr>
<th>Designation</th>
<th>Type of velocity distribution</th>
<th>Wall configuration</th>
<th>Inlet hub-radius ratio</th>
<th>Outlet hub-radius ratio</th>
</tr>
</thead>
</table>
| A           | Free-vortex design  
\[ U V_u \text{, constant}\] | Cylindrical outer wall  
\[ Convex insweep of inner wall\] | 0.70 | 0.68 |
| B           | Nontwisted-rotor design  
\[ U V_u \text{, increasing from hub to tip}\] | Cylindrical outer wall  
\[ Convex insweep of inner wall\] | 0.70 | 0.67 |
| C           | Free-vortex design  
\[ U V_u \text{, constant}\] | Cylindrical inner and outer walls | 0.70 | 0.70 |
| D           | Nontwisted-rotor design  
\[ U V_u \text{, increasing from hub to tip}\] | Cylindrical inner and outer walls | 0.78 | 0.78 |
The design procedure for nozzle A is presented in reference 3 and a sketch of the blade sections is shown in figure 1(a). Nozzle B was designed for the same application as nozzle A but with non-twisted-rotor velocity diagrams (see ref. 4). The method of obtaining the blade shapes was the same as that used for nozzle A. The blade sections for nozzle B are shown in figure 1(b). Nozzle C was a nozzle blade from a commercial jet engine, and nozzle D was designed for the turbine reported in reference 5. The blade shapes of these two nozzles are shown in figures 1(c) and 1(d), respectively.

METHOD OF ANALYSIS

An analysis of each nozzle configuration was made using the method presented in reference 2, which takes into account three-dimensional flow effects, to determine the maximum mass flow that could be passed. The primary assumptions made in the analysis are:

(a) Isentropic flow

(b) Axially symmetric flow

(c) Flow follows blade mean camber surface

The equations presented in reference 2 are valid for either a rotating or a stationary blade row. In the analysis of the four nozzle configurations, the wheel-speed terms were merely set equal to zero.

The streamline estimate was made for a given nozzle; the design mass-flow distribution upstream and downstream of the nozzle was used to divide the mass flow into equal parts and to fair streamlines between corresponding points. In a complete analysis, the streamline configuration would be modified in an iteration process to maintain continuity between streamlines. However, in the analysis of the nozzles, only the original streamline assumption was used, because the maximum mass flow obtained in the first trial solution is sufficiently accurate (ref. 2). The estimated streamlines and the resultant choking orthogonal (orthogonal that limits the mass flow) for the four nozzles are shown in figure 2.

APPARATUS AND INSTRUMENTATION

The apparatus and instrumentation used in the investigation of nozzles A and B are the same as described in references 1 or 3 except that the turbine rotor assembly was replaced by a wood fairing piece which was contoured approximately the same as the rotor hub. The experimental setup used to investigate nozzles C and D is described in
reference 5. The performance of nozzles C and D was obtained from the over-all turbine investigations, the results of which indicated that both nozzles were choked at the turbine-design operating conditions.

The accuracy of the measured mass flow is estimated to be within ±0.01.

RESULTS AND DISCUSSION

The four nozzle configurations were analyzed to determine the effect of the design exit velocity distribution and wall contouring on the nozzle mass-flow characteristics. Before the results of the analysis are discussed, the accuracy of the method of analysis will be demonstrated by a comparison of the maximum mass flow obtained analytically with that obtained experimentally.

Comparison of Analytical and Experimental Values of Mass Flow

The experimentally obtained values of maximum mass flow and the values of maximum mass flow computed from the analysis for the four nozzles are compared in column 1 of table I. The ratios of the maximum mass flows can be considered flow coefficients, as isentropic flow with no boundary-layer effects is assumed in the analysis. When a flow coefficient of 0.97 (which can be considered a representative value for turbine nozzles) is applied to the analytically obtained value of maximum mass flow, the experimental maximum mass flow can be predicted to within 1 percent. Thus, the analytical method gave reliable values of maximum mass flow for the nozzles with contoured inner walls as well as for the nozzles with straight cylindrical walls.

Results of Analysis

Nozzle A. - The streamline assumption and the choking orthogonal used in the analysis of nozzle A is shown in figure 2(a). The effect of the insweep of the inner wall was found to offset the outward shift in mass flow (characteristic of free-vortex design) such that the choking orthogonal was positioned very close to the trailing edge over most of the blade span. The results of the analysis including the 0.97 flow coefficient indicated that a maximum of 103 percent design mass flow could be passed for nozzle A (column 2, table I). At design total-to-static pressure ratio across the nozzles, design mass flow was passed (column 3, table I) as this nozzle was designed to operate at slightly less than choking flow.
There are two reasons why the two-dimensional design technique was sufficient as far as obtaining design mass flow for this nozzle is concerned:

(a) The three-dimensional choking orthogonal was located close to the trailing edge over most of the blade span, being inside the passage only near the inner wall. Thus, the three-dimensional flow area approached the two-dimensional design area closely.

(b) Nozzle A was designed to pass the greatest specific mass flow near the tip as indicated in figure 3 which is based on the design radial variation of specific mass flow at the nozzle exit. The area under the curve is proportional to the nozzle mass flow. Because the ineffective flow area occurred at the inner wall where the nozzle was designed to pass the least specific mass flow, the effect of the decreased flow area on the mass flow was small.

Nozzle B. - The streamline assumption and the resultant choking orthogonal for nozzle B are shown in figure 2(b). The effect of the insweep of the inner wall is seen to combine with the inward shift of mass flow (characteristic of nozzles designed for nontwisted rotor) such that the choking orthogonal shifted considerably inside the nozzle over most of the blade span. The results of the analysis including the 0.97 flow coefficient indicated that the maximum mass flow of only 96 percent design could be passed (column 2, table I). This nozzle was also designed to operate below the choking condition at the design total-to-static pressure ratio, and it was found experimentally that only 94 percent design mass flow was passed (column 3, table I).

The maximum mass flow was also calculated using the two-dimensional design throat area; choking at all radii and the 0.97 flow coefficient were assumed. The results of this calculation indicated a maximum mass flow of 101 percent design. A comparison between this result and the result of the three-dimensional analysis illustrates the effect of the three-dimensional flow characteristics on the nozzle maximum mass flow.

There are two reasons why the two-dimensional design technique was not satisfactory as far as predicting design mass flow is concerned:

(a) The position of the choking orthogonal caused by the inner-wall contour and the inward shift of mass flow caused the three-dimensional flow area to be considerably less than the two-dimensional design flow area.

(b) Nozzle B was designed to pass the greatest specific mass flow near the inner wall (see fig. 3); hence, the loss in flow area at the inner wall represented a considerable loss in mass flow from design.

Nozzles C and D. - The streamline assumptions and the resultant choking orthogonals for nozzles C and D are shown in figures 2(c) and 2(d). It can be seen that the choking orthogonal for nozzle D did not
lie at the trailing edge but inside the blade over the entire span. The location of the choking orthogonal can be attributed to the fact that the blade thickness increased from the trailing edge faster than the cosine of the mean camber angle decreased; as a result, flow area was minimum inside the blade passage (fig. 2(d)). The results of the analysis including the 0.97 flow coefficient indicated 99 and 101 percent design mass flow could be passed for nozzles C and D, respectively, (column 2, table I). Thus, the maximum mass flow of approximately design is predicted for both nozzles.

Discussion

The analytical results presented herein, which were verified by experimental findings, have indicated that, for certain types of nozzles which employ an increasing annulus area from entrance to exit, a conventional two-dimensional design technique may be inadequate with respect to obtaining design mass flow. A comparison of the results obtained with the four nozzles shows that the velocity distribution as well as the inner wall curvature contributes to the three-dimensional choking effect. For a wheel-type nozzle \((V_u = Kr^{-1})\) with the same type of inner wall curvature as nozzles A and B, the mass flow deficiency should be greater than that obtained for the nontwisted-rotor design, because the radial inward shift of mass flow would be greater. In this case, the mass flow of the nozzle could not be predicted by two-dimensional techniques because the convex inswept inner wall and the radial inward shift of the mass flow would cause the choking orthogonal to occur well within the nozzle passage at the inner wall, thereby reducing the effective flow area.

This same effect could occur for a vortex-design nozzle \((V_u = Kr^{-1})\) or a super-vortex design \((V_u = Kr^{-2})\) that employed a cylindrical inner wall and a convex outswept outer wall. The combination of the wall curvature and the radial outward shift of mass flow might cause the choking orthogonal to occur within the nozzle passage at the outer wall and reduce the effective flow area.

The analytical and experimental results obtained with nozzles C and D can be compared to show the effect of radial mass-flow shift for cylindrical-walled nozzles. From these results, it would appear that, regardless of the type of velocity distribution employed and the resulting radial mass-flow shift, conventional two-dimensional design techniques should be adequate with respect to predicting the mass flow for cylindrical-walled nozzles.

It must be remembered that the analytical method described herein is useful only in obtaining the maximum mass flow. If the nozzles are designed to choke, the analytical value of maximum mass flow corrected for the 0.97 coefficient can be compared directly with the design mass flow to determine whether or not the nozzles will perform satisfactorily
as far as mass flow is concerned. If the nozzles are not designed to
choke, the analytical results can be used only as a guide in determining
if large deviations in mass flow from design will occur.

SUMMARY OF RESULTS

Four turbine nozzle configurations were investigated by three-
dimensional analysis and experimentally to determine if certain combina-
tions of design nozzle-exit velocity distribution and wall contouring
can alter the three-dimensional flow characteristics such that the mass
flow cannot be predicted by two-dimensional design procedures. The
results can be summarized as follows:

1. Two-dimensional design techniques were found to be satisfactory
for the free-vortex design with the convex inswept inner wall as far as
mass flow is concerned. The three-dimensional analysis indicated that
the outward shift of the mass flow balanced the effect of the insweep of
the inner wall such that the choking orthogonal occurred very close to
the trailing edge over most of the blade span.

2. Two-dimensional design techniques were found to be unsatisfactory
for predicting the mass flow of a nozzle designed for a nontwisted rotor
with a convex inswept inner wall. This fact could be attributed to the
combination of the inward shift of the mass flow and the effect of the
insweep of the inner wall. The choking orthogonal was located consid-
erably upstream of the trailing edge of the blade; thus, the mass flow was
considerably less than that predicted by two-dimensional techniques.

3. Two-dimensional design techniques were found to be adequate with
respect to predicting the mass flow for both the free-vortex and the
nontwisted-rotor nozzles with cylindrical walls, although the choking
orthogonal for the nontwisted-rotor design was found in the analysis to
lie within the nozzle passage.

4. The maximum mass flows obtained by analysis including a flow
coefficient of 0.97 were within 1 percent of the experimentally obtained
maximum mass flows for all four nozzles.

CONCLUSION

From the experimental and analytical results presented herein, it
can be concluded that certain combinations of design nozzle-exit velo-
city distribution and wall contouring, employed to vary the annulus area,
can alter the three-dimensional flow characteristics to a point where
two-dimensional techniques are inadequate for predicting mass flow. Consideration of the three-dimensional flow characteristics must therefore be included in the design of nozzles to insure satisfactory turbine performance.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, April 16, 1953

REFERENCES


### TABLE I. - COMPARISON OF DESIGN, EXPERIMENTAL, AND ANALYTICAL MASS-FLOW VALUES

<table>
<thead>
<tr>
<th>Nozzle</th>
<th>Ratio of maximum mass flow obtained experimentally to maximum mass flow obtained by analysis</th>
<th>Ratio of maximum mass flow obtained by analysis including 0.97 flow coefficient to design mass flow</th>
<th>Mass flow at design total-to-static pressure ratio, percent design</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.97</td>
<td>1.03</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
<td>.96</td>
<td>.96</td>
<td>94</td>
</tr>
<tr>
<td>C</td>
<td>.98</td>
<td>.99</td>
<td>100</td>
</tr>
<tr>
<td>D</td>
<td>.97</td>
<td>1.01</td>
<td>100</td>
</tr>
</tbody>
</table>
Hub, $r/r_t = 0.68$

Mean, $r/r_t = 0.84$

Tip, $r/r_t = 1.00$

(a) Nozzle A.

Figure 1. - Nozzle-blade passages and shapes.
(b) Nozzle B.

Figure 1. - Continued. Nozzle-blade passages and shapes.
Figura 1. - Continued. Nozzle-blade passages and shapes.
Figure 1. - Concluded. Nozzle-blade passages and shapes.
Figure 2. - Side view of nozzles showing position of estimated streamlines and resultant choking orthogonal.
Figure 2. - Concluded. Side view of nozzles showing position of estimated streamlines and resultant choking orthogonal.
Figure 3. - Radial mass-flow distribution at nozzle exit for nozzle A (free-vortex design) and nozzle B (nontwisted-rotor design).