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RESEARCH MEMORANDUM

MATCHED PERFORMANCE CHARACTERISTICS OF A 16-STAGE

AXIAL-FLOW COMPRESSOR AND A 3-STAGE TURBINE

By John J. Rebeske, Jr., and James F. Dugan, Jr.

Lewis Flight Propulsion Laboratory
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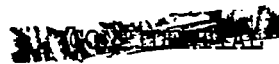
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FLOW COMPRESSOR AND A 3-STAGE TURBINE

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SUMMARY

Component performance data for one modification of a 16-stage compressor and a 3-stage turbine were used to determine the matched performance characteristics of an engine having these components. The turbine power-limit line and equilibrium operating lines for exhaust-nozzle areas of 360, 395, 420, 600, and ∞ square inches were determined. Matched performance characteristics were determined for leakages of 0, 2, and 5.5 percent. Operation in the surge region was investigated for one available experimental point.

The compressor surge line and the infinite exhaust-nozzle-area equilibrium operating line restricted the region of matched compressor-turbine operation. Some means, such as compressor-exit bleed, would be required for this engine to accelerate through the intermediate speed range. Operation with a choked exhaust-nozzle area greater than 467 square inches resulted in turbine limiting loading and low turbine efficiency with a corresponding reduction in engine thrust. Turbine efficiency varied between 78 and 82 percent for low-speed operation along the 600- and 420-square-inch exhaust-nozzle-area equilibrium operating lines. The compressor efficiency varied between 58 and 66 percent. For operation above 83 percent design speed along the 395 and 360 square inch operating lines, the turbine efficiency varied between 81.5 and 83 percent, whereas the compressor efficiency varied between 80 and 83 percent. For a specified compressor speed and turbine-inlet temperature, exhaust-nozzle area increased, compressor pressure ratio decreased, and engine thrust decreased with increasing leakage. Consideration of one experimental point in the surge region showed that a match point was impossible because the turbine pressure ratio corresponding to the required power exceeded the available compressor pressure ratio.

INTRODUCTION

As part of a study of high-pressure-ratio multistage axial-flow compressors and turbines an investigation was conducted at the NACA Lewis laboratory to determine matched performance characteristics of one version
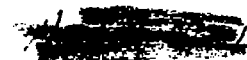
of a 16-stage compressor with a 3-stage turbine for a high pressure ratio turbojet engine. The performance characteristics of several versions of the compressor are given in references 1 and 2. The compressor performance presented in reference 2 as configuration C was chosen for this matching study, because for the modifications investigated, this compressor gave a good compromise in compressor efficiency, pressure ratio, surge characteristics, and weight flow over a range of equivalent speeds from 30 to 100 percent of design. The performance characteristics of this compressor are matched to the performance characteristics of the 3-stage turbine described in reference 3.

The purpose of this matching study is:

(1) To determine how well the component performance characteristics of a particular compressor and turbine are matched in an engine, and to point out problems encountered in matching a single-spool compressor with a turbine in a high-pressure-ratio engine

(2) To determine the effect of leakage between compressor and turbine on engine performance

(3) To investigate engine operation at one point in the surge region

Equilibrium operating lines are determined from the component performance characteristics for assumed engine operating conditions of zero flight speed, various effective exhaust-nozzle areas, and various values of leakage between the compressor and the turbine. Engine operation along these equilibrium operating lines from 50 to 100 percent of design speed, and the effect of leakage on predicted engine operation are discussed.

SYMBOLS

The following symbols are used in this report:

A   area, sq in.

C_v velocity coefficient

f   ratio of fuel flow to air flow

g   acceleration due to gravity, 32.174 ft/sec^2

ΔH stagnation enthalpy change, Btu/lb

J   mechanical equivalent of heat, 778.16 ft-lb/Btu

K   constant, $60J/2\pi$, (sec)(ft-lb)/(min)(Btu)

M   Mach number

N   rotative speed, rpm

P_a power for accessories and friction losses, ft-lb/min

p   static pressure, lb/sq ft

p'   stagnation pressure, lb/sq ft

R   gas constant, ft-lb/(lb)($^{\circ}$R)

T'   stagnation temperature, $^{\circ}$R

V   velocity, ft/sec

V_{cr} critical gas velocity, $\sqrt{(2\gamma/\gamma + 1)\ gRT'}$, ft/sec

w   weight flow, lb/sec

w_l leakage weight flow, lb/sec

β function of gamma, $\dfrac{\gamma_0}{\gamma_e}\left[\dfrac{\left(\dfrac{\gamma_e+1}{\gamma_e}\right)^{\dfrac{\gamma_e}{\gamma_e-1}}}{\left(\dfrac{\gamma_0+1}{\gamma_0}\right)^{\dfrac{\gamma_0}{\gamma_0-1}}}\right]$

Γ torque, lb-ft

γ ratio of specific heats

δ ratio of stagnation pressure to pressure at NACA standard sea-level conditions, p'/2116 (lb)/(sq ft)

η efficiency

θ stagnation temperature ratio, T'/518.4, $^{\circ}$R

θ_{cr} squared ratio of critical velocity to critical velocity at NACA standard sea-level conditions, $\left(\dfrac{V_{cr}}{V_{cr,0}}\right)^2$

Subscripts:

0   NACA standard sea-level conditions

1   compressor inlet

2   compressor outlet

3   turbine inlet

4   turbine outlet

5   exhaust-nozzle outlet

c   compressor

cr   critical

e   equivalent operating conditions

t   turbine

PROCEDURE

Equilibrium operation of a compressor and a turbine as a directly coupled unit in a turbojet engine requires that four conditions be satisfied:

(1) The compressor rotative speed must equal the turbine speed.

(2) The air weight flow into the compressor plus the fuel weight flow added in the burners minus any leakage between the compressor and the turbine must equal the weight flow into the turbine, when no air is bled from the engine.

(3) The turbine power, or torque, must equal the power, or torque, required to drive the compressor and the auxiliary equipment and to overcome mechanical losses.

(4) The over-all pressure ratio resulting from ram, compressor compression, and pressure loss in the burners must equal the over-all pressure ratio across the turbine, the tail pipe, and the jet nozzle.

These conditions for equilibrium operation of a compressor and a turbine in a jet engine are restated in the following equations (see appendix A for equivalent turbine parameters):

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$$\frac{T'_3}{T'_1} = \left(\frac{N_c/\sqrt{\theta_1}}{N_t/\sqrt{\theta_3}}\right)^2 \qquad (1)$$

$$\frac{w_t N}{60\delta_3} = \left(1 + f - \frac{w_l}{w_c}\right)\frac{\delta_2}{\delta_3}\frac{w_c N}{60\delta_2} \qquad (2)$$

$$\frac{\Gamma_t}{\delta_3} = \frac{Kw_t\Delta H_t}{N\,\delta_3} = \frac{Kw_c\Delta H_c}{N\,\delta_2}\left(\frac{\delta_2}{\delta_3}\right) + \frac{P_a}{N\,\delta_3} = \frac{\Gamma_c}{\delta_3} + \frac{P_a}{N\,\delta_3} \qquad (3)$$

$$\frac{p'_1}{p_0}\frac{p'_2}{p'_1}\frac{p'_3}{p'_2} = \frac{p'_3}{p'_4}\frac{p'_4}{p'_5}\frac{p'_5}{p_0} \qquad (4)$$

In calculating the engine performance from the component character-istics and the foregoing equations, NACA standard sea-level conditions at the compressor inlet were assumed in addition to the following conditions:

Ram-pressure ratio, p'_1/p_0 . . . . 1.0
Burner pressure ratio, p'_3/p'_2 97
Stagnation-pressure loss in tail pipe at $M_5=1.0$, p'_5/p'_4 97
Fuel-air ratio constant over range of T'_3, f 02
Ratio of specific heats for gas flow in turbine and tail
    pipe, γ 1.32
Torque to drive auxiliaries, $P_a/N\,\delta_3$, lb-ft 3.0

No attempt was made to correct the cold-air turbine performance for change in rotor-blade clearance and stator-blade area with turbine-inlet temperature.

Equations (1) to (3) define possible equilibrium operating points for a given compressor and turbine. However, before a simultaneous solution of these equations was made, the cold-air turbine character-istics reported in reference 3 were corrected for the γ change from cold air to hot gas. The method used is described in appendix A. The general method used to solve equations (1) to (3) is presented in ref-erence 4. The compressor and the turbine performances are shown in plots of the torque parameter (equation (3)) against a weight-flow parameter (equation (2)) for various percentages of design equivalent

compressor and turbine speed in figures 1 and 2, respectively. Inter-
sections of compressor and turbine speed lines define possible operating
points. In order to satisfy the speed condition $N_c = N_t$, the ratio of
the turbine-inlet to the compressor-inlet temperature was calculated for
each possible operating point from equation (1). The temperature
ratios T_3'/T_1' were then plotted against the turbine weight-flow param-
eter $w_t N/60\delta_3$ for lines of constant equivalent compressor speed. Val-
ues of $w_t N/60\delta_3$ were thus determined for lines of constant turbine-
inlet temperature and compressor speed. Corresponding values of the
compressor weight-flow parameter $w_c N/60\delta_2$ could then be calculated
from equation (2) for the assumed values of fuel-air ratio, burner pres-
sure drop, and leakage. Lines of constant turbine-inlet temperature
thus determined are plotted in figure 3 in which the compressor pressure
ratio is shown as a function of $w_c N/60\delta_2$ for lines of constant com-
pressor speed and contours of compressor efficiency. The lines of con-
stant T_3' thus represent the simultaneous solutions of equations (1)
to (3). The fourth condition of exhaust-nozzle area must now be satis-
fied to determine an operating line for the engine.

Determination of engine operating lines for given exhaust-nozzle
areas. - For a given turbine-inlet temperature, compressor pressure
ratio, compressor speed, and weight-flow parameter from figure 3, the
corresponding turbine pressure ratio, outlet temperature T_4', and tur-
bine weight flow may be calculated by equating the torque and the weight-
flow parameters of figures 1 and 2, respectively.

When the compressor and the turbine pressure ratios are known and
the conditions of ram-pressure ratio p_1'/p_0, burner pressure drop p_3'/p_2',
and pressure drop in the tail pipe p_4'/p_5' are assumed, the pressure
ratio across the exhaust-nozzle outlet p_5'/p_0 may be calculated from
equation (4). If the pressure ratio p_5'/p_0 is supercritical, an effec-
tive exhaust-nozzle area may be calculated from the following equation:

$$A_5 = 144 \ \frac{w_t \sqrt{T_5'}}{p_5'} \sqrt{\frac{R}{\gamma g}} \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{5}$$

For the subcritical case, a tail-pipe efficiency was calculated that
agreed with the assumed value p_4'/p_5' for choke flow in the exhaust
nozzle, and a corresponding finite value of A_5 was calculated. This
method is presented in appendix B. The values of A_5 thus calculated
were plotted against $w_c N/60\delta_2$ for lines of constant T_3'. The values
of $w_c N/60\delta_2$ thus determined for constant values of A_5 and T_3' are
plotted in figure 3.

In determining the operating line for an infinite exhaust-nozzle area, perfect diffusion was assumed; therefore $p_4'/p_5' = 1.0$ and $p_5'/p_0 = 1.0$.

Determination of turbine power limit. - In order to further define the limits of engine operation, a turbine power-limit line was determined. From a plot of Γ_t/δ_3 against turbine pressure ratio p_3'/p_4' for lines of constant $N/\sqrt{\theta_3}$, the values of Γ_t/δ_3 and p_3'/p_4' were determined for the condition of limiting blade loading (zero slope). The line representing these maximum values of Γ_t/δ_3 is shown in figure 2. The intersection of compressor-speed lines (fig. 1) with the maximum-torque or turbine power-limit line of figure 2 determines the value of $w_t N/60\delta_3$ corresponding to the maximum turbine power. The turbine power-limit line can thus be plotted on figure 3 by use of equation (2).

Determination of engine thrust for zero flight velocity. - For zero flight velocity the thrust equation is

$$\text{thrust} = \frac{w_t (V_5)}{g} \qquad (6)$$

where V_5 is obtained from

$$V_5 = C_v \sqrt{2g \frac{R\gamma}{\gamma-1} T_4' \left[1 - \left(\frac{p_0}{p_5'}\right)^{\frac{\gamma-1}{\gamma}} \right]} \qquad (7)$$

RESULTS AND DISCUSSION

Engine operation with modified compressor and three-stage turbine. - The matched performance of the compressor of reference 2 designated as configuration C with the three-stage turbine of reference 3 is presented in figure 3(a) for an assumed value of $w_l/w_c = 0.00$. The compressor performance is presented in terms of the compressor pressure ratio against a compressor weight-flow parameter for lines of constant percentages of compressor equivalent design speed. Also shown is the compressor surge line. Superimposed on this compressor map are lines of constant turbine-inlet temperature, the turbine power-limit line, and lines which represent equilibrium engine operation for exhaust-nozzle areas of 360, 395, 420, 600, and ∞ square inches. The region of possible engine operation without bleed lies to the right of the compressor surge line and to the left of the infinite-area and turbine power-limit lines. Operation in the region to the left of the surge line is usually undesirable, or even impossible when the turbine pressure ratio corresponding to the required turbine power exceeds the

available compressor pressure ratio. Above approximately 83 percent speed, the infinite-area operating line coincides with the turbine power-limit line. Operation to the right of the turbine power-limit line is impossible, because in this region the power required exceeds the power available from the turbine. Below approximately 83 percent speed, operation in the region between the infinite-area line and the turbine power-limit line is impossible for $p_1'/p_0 = 1.0$, because the turbine pressure ratio corresponding to the required turbine power exceeds the available compressor pressure ratio.

At 50 percent design speed, only the 600-square-inch and the infinite-area operating lines lie below the compressor surge line. As the engine speed is increased, these operating lines enter the surge region between 70 and 76 percent equivalent design speed. All operating lines emerge from the surge region between 79 and 84 percent of design speed. The infinite-area operating line enters the surge region at approximately 76 percent design speed and emerges from the surge region at approximately 79 percent of the design speed. Therefore, acceleration to design speed cannot be achieved by merely operating with a variable exhaust-nozzle area. In order to accelerate through the intermediate speed range where the dip in the surge line occurs, some means such as bleed must be employed. At speeds above 85 percent design, the operating lines lie well to the right of the surge region. It is interesting to note that the operating lines in this region are nearly vertical (essentially constant $w_c N/60\delta_2$). Because the constant turbine equivalent speed lines in the torque parameter against weight-flow parameter plot are vertical in this region (fig. 2), the turbine is operating at a constant value of $N/\sqrt{\theta_3}$. The operating line for an exhaust-nozzle area of 600 square inches approaches the turbine power-limit line at compressor speeds greater than 85 percent of compressor equivalent design speed, because the turbine-outlet flow area in the plane of the trailing edges is approximately 467 square inches (annulus area minus trailing-edge blockage corrected for the assumed stagnation-pressure loss in the tail pipe). When the exhaust-nozzle area is equal to or greater than 467 square inches and the available pressure ratio p_5'/p_0 is supercritical, the turbine-outlet area will choke; this is the condition of limiting loading for the turbine (reference 5). Consequently, the turbine power-limit line represents the equilibrium operating line for all exhaust-nozzle areas equal to or greater than 467 square inches for choke flow in the exhaust nozzle. At 100 percent design speed and design turbine-inlet temperature (2160° R), the compressor pressure ratio is 8.99 and the exhaust-nozzle area is 362 square inches.

Effect of leakage on engine operation. - The effects of various specified values of leakage w_l/w_c on engine operation are shown in figures 3(a) to 3(c). A compressor map with superimposed equilibrium

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operating lines, turbine-inlet-temperature lines, and the turbine power-limit line is shown in figure 3(b) for 2 percent leakage. These lines have shifted to the right, or away from the surge region. Figure 3(c) shows the compressor map for 5.5 percent leakage. The equilibrium operating lines, the turbine-inlet-temperature lines, and the turbine power-limit line are further away from the surge region than they were for 2 percent leakage operation. Both figures 3(b) and 3(c) show that the shift in turbine-inlet-temperature lines is greater than the shift in equilibrium operating lines and turbine power-limit lines. For a specified compressor speed and turbine-inlet temperature, the exhaust-nozzle area increased and compressor-pressure ratio decreased with increasing leakage. The shift of the equilibrium operating lines away from the surge region is in the direction required for acceleration through the intermediate speed range around the dip in the compressor surge line. Figure 3(c), however, shows that even with 5.5 percent leakage, the 600-square-inch exhaust-nozzle-area line still intersects the compressor surge line. The leakage required to shift the 600-square-inch exhaust-nozzle-area equilibrium operating line so that it would not intersect the surge line was not computed, because of the adverse effect of leakage on engine thrust at design speed.

The previous discussion shows, in a qualitative manner, what happens to engine operation with various amounts of leakage between the compressor and the turbine. The quantitative effects of leakage for 100 percent design speed on thrust, exhaust-nozzle area, and compressor pressure ratio are shown in figure 4 as a function of turbine-inlet temperature T_3'. For case I (no leakage) and design turbine-inlet temperature T_3' (2160° R), the compressor pressure ratio is 8.98 and falls progressively to 8.80 and 8.48 for cases II and III (2 and 5.5 percent leakages), respectively. The decrease in compressor pressure ratio for a constant speed and turbine-inlet temperature may be predicted from the condition of continuity between the compressor and the turbine for choke flow in the turbine. The exhaust-nozzle areas necessary to maintain constant speed at design turbine-inlet temperature are 362, 368, and 400 square inches for cases I, II, and III, respectively; these values represent a 1.6 percent increase for case II and a 10.5 percent increase for case III. The decreases in thrust for cases II and III at design turbine-inlet temperature are 3.6 and 4.7 percent.

On the basis of these results, it was concluded that, for optimum engine performance, leakage should be minimized to obtain maximum thrust at design speed. If necessary, as is the case of this engine, some means such as compressor-outlet bleed should be used to accelerate through the intermediate speed range.

The lowest turbine-inlet temperature for each case represents engine operation at the power-limit line for the turbine. It was

assumed that a turbine pressure ratio of 4.8 was sufficient to attain
limiting loading of the turbine (fig. 2). Therefore, any further reduc-
tion in turbine-inlet temperature below the minimum values shown
(fig. 4) would cause the engine speed to drop. At this condition of
limiting loading a further increase in exhaust-nozzle area would cause
a decrease in engine thrust. Although p_3'/p_4' is large enough for lim-
iting loading, p_5'/p_0 is less than the critical pressure ratio required
to choke the exhaust-nozzle areas. Consequently, the exhaust-nozzle
areas shown are greater than the 467 square inches calculated for limit-
ing loading and choke flow in the exhaust nozzle.

Component efficiencies. - For case II $(w_l/w_c = 0.02)$, the compo-
nent efficiencies were calculated along engine operating lines for
exhaust-nozzle areas of 600, 420, 395, and 360 square inches and over
a range of equivalent engine speeds from 50 to 100 percent design. The
variation of the component efficiencies with engine speed is shown in
figure 5.

Low-speed efficiencies are shown for exhaust-nozzle areas of 600
and 420 square inches, because only the larger values of A_5 allow
engine operation in the low-speed range. The compressor efficiency
ranges from 58 to 66 percent. These low efficiencies for low-speed
operation are characteristic of high-pressure-ratio multistage axial-
flow compressors. During low-speed operation of such compressors, the
outlet stages operate in a low-efficiency turbining region and the inlet
stages operate in a low-efficiency high-angle-of-attack region. The
turbine efficiency ranges from 78 to 82 percent.

For engine operation near design speed, the exhaust-nozzle area
can be reduced to some value near 467 square inches. For choke flow
in the exhaust nozzle, an area of approximately 467 square inches
would allow operation along the turbine power-limit line at a turbine
pressure ratio just sufficient to attain limiting loading of the tur-
bine rotor blades. The use of larger exhaust-nozzle areas would allow
a higher turbine pressure ratio for the same limiting turbine work and
therefore cause the turbine efficiency to decrease with a corresponding
reduction in engine thrust.

For operation above 83 percent design speed along the 395- and
360-square-inch operating lines (fig. 5), the compressor efficiency
varies between 80 and 83 percent whereas the turbine efficiency varies
between 81.5 and 83 percent.

Operation in surge region. - In this report, the compressor surge
line has been used as a limit line restricting the region of engine
operation. In some engines, operation in the surge region is impossible.
Such is believed to be the case for the engine discussed in this report.

One operating point in the surge region was obtained during the testing of a compressor differing from the compressor of this report only in that the 14th through 16th rotors had a lower solidity. This point is shown in figure 6 in which compressor equivalent torque is plotted against compressor weight-flow parameter for constant values of compressor equivalent speed. The incipient surge point at 58 percent equivalent design speed and the point in the surge region were matched with the three-stage turbine for $w_l/w_c = 0.02$ and the engine parameters are listed in the following table:

	$\dfrac{\Gamma_c}{\delta_2}$ (lb-ft)	T_3' (°R)	$\dfrac{p_3'}{p_4'}$	$\dfrac{p_2'}{p_1'}$	A_5 (sq in.)
58 Percent speed, incipient surge	2195	1433	1.95	2.13	422
58 Percent speed, point in surge	2380	1878	1.89	1.66	---

Comparison of these two points shows, for the point in surge, an increase in equivalent compressor torque, an increase in turbine-inlet temperature, a decrease in turbine pressure ratio, and a decrease in compressor pressure ratio. For the point in surge, the required turbine pressure ratio exceeds the compressor pressure ratio so that equilibrium operation at this point is impossible. It is believed that engine operation is impossible for all points in the surge region.

SUMMARY OF RESULTS

In an investigation of the matched performance characteristics of a 16-stage compressor and a 3-stage turbine, the following results were obtained:

1. The compressor surge line and the infinite exhaust-nozzle-area equilibrium operating line restricted the region of matched compressor-turbine operation.

2. For a specified compressor speed and turbine-inlet temperature, exhaust-nozzle area increased, compressor pressure ratio decreased, and engine thrust decreased with increasing leakage.

3. Some means, such as compressor-outlet bleed, would be required for this engine to accelerate through the intermediate speed range.

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4. Operation with a choked exhaust-nozzle area greater than 467 square inches resulted in turbine limiting loading and low turbine efficiency with a corresponding reduction in engine thrust.

5. Turbine efficiency varied between 78 and 82 percent for low-speed operation along the 600- and 420-square-inch exhaust-nozzle-area equilibrium operating lines. The compressor efficiency varied between 58 and 66 percent. For operation above 83 percent design speed along the 395- and 360-square-inch exhaust-nozzle-area operating lines, the turbine efficiency varied between 81.5 and 83 percent whereas the compressor efficiency varied between 80 and 83 percent.

6. Consideration of one experimental point in the surge region showed that a match point was impossible because the turbine pressure ratio corresponding to the required power exceeded the available compressor pressure ratio.

CONCLUDING REMARKS

Although engine leakage between the compressor and the turbine moves equilibrium engine operating lines away from the compressor surge line and thus gives a larger margin for engine acceleration, it causes a decrease in engine thrust at design speed and turbine-inlet tempera-ture. Therefore, for optimum engine thrust, leakage should be minimized and some method, such as engine bleed, should be employed to allow rapid engine acceleration from idle to design speed.

Lewis Flight Propulsion Laboratory
      National Advisory Committee for Aeronautics
            Cleveland, Ohio

APPENDIX A

CORRECTION OF COLD-AIR TURBINE DATA TO HOT-GAS CONDITIONS

The method used to determine turbine characteristics for hot-gas conditions is presented in the appendix of reference 6. A mean value of γ of 1.32 was chosen and assumed to remain constant over the entire operating region of the turbine.

Determination of equivalent torque and pressure ratio. - If the first two terms of equation (3) are rewritten in terms of equivalent turbine variables, the following expression for equivalent torque is obtained:

$$K \frac{w_t \sqrt{\theta_{cr,3}}}{\delta_3} \beta \frac{\sqrt{\theta_{cr,3}}}{N} \frac{\Delta H_t}{\theta_{cr,3}} = \frac{\Gamma_t}{\delta_3} \beta \tag{A1}$$

When the equivalent torque for cold-air and hot-gas conditions are equated and solved for $(\Gamma_t/\delta_3)_e$

$$\left(\frac{\Gamma_t}{\delta_3}\right)_e = \left(\frac{\Gamma_t}{\delta_3}\right)_0 \frac{\beta_0}{\beta_e} \tag{A2}$$

The torque values $(\Gamma_t/\delta)_0$ for given equivalent turbine speed and overall turbine pressure ratio, presented in figure 7 of reference 3, were multiplied by the ratio β_0/β_e and plotted against the corresponding pressure ratio corrected for the assumed change in γ. These curves were extrapolated to the condition of limiting loading (zero slope).

Determination of equivalent weight-flow parameter $w_t N/60\delta_3$. - If the left side of equation (2) is rewritten in terms of equivalent turbine variables

$$\frac{w_t \sqrt{\theta_{cr,3}}}{60\delta_3} \beta \frac{N}{\sqrt{\theta_{cr,3}}} = \frac{w_t N}{60\delta_3} \beta \tag{A3}$$

When this parameter is equated for hot and cold conditions and solved

$$\left(\frac{w_t N}{60\delta_3}\right)_e = \left(\frac{w_t N}{60\delta_3}\right)_0 \frac{\beta_0}{\beta_e} \tag{A4}$$

The values for $(w_t N/60\delta_3)_0$ were calculated from the data presented in figure 6 of reference 3 and multiplied by the ratio β_0/β_e corresponding to the assumed change in γ. A plot of $(\Gamma_t/\delta_3)_e$ against $(w_t N/60\delta_3)_e$ for lines of constant equivalent speed and pressure ratio was then made (fig. 2). Because of the assumption that γ remains constant over the entire operating region of the turbine, all critical subscripts can be removed and the equivalent performance parameters may be expressed in terms of temperature ratios only.

APPENDIX B

CALCULATION OF EXHAUST-NOZZLE AREA FOR

SUBCRITICAL PRESSURE RATIO

For an assumed value of $p_5'/p_4' = 0.97$, the following equation may be written:

$$\frac{p_5'}{p_0} = \frac{p_4'}{p_0}\frac{p_5'}{p_4'} \tag{B1}$$

For choke flow at the exhaust-nozzle throat, p_5'/p_0 and M_4 are thus determined. The ratio of $(M_5/M_4)^2$ is proportional to the efficiency of the tail pipe and is assumed constant for all subcritical pressure ratios. After the value of M_4 is obtained from p_4'/p_0, M_5 is determined and the corresponding value of $(A/A_{cr})_5$ may be obtained.

The critical exhaust-nozzle area $A_{cr,5}$ may be calculated from equation (5) by using $T_4' = T_5'$; A_5 for the subcritical case is

$$A_5 = \left(\frac{A}{A_{cr}}\right)_5 A_{cr,5} \tag{B2}$$

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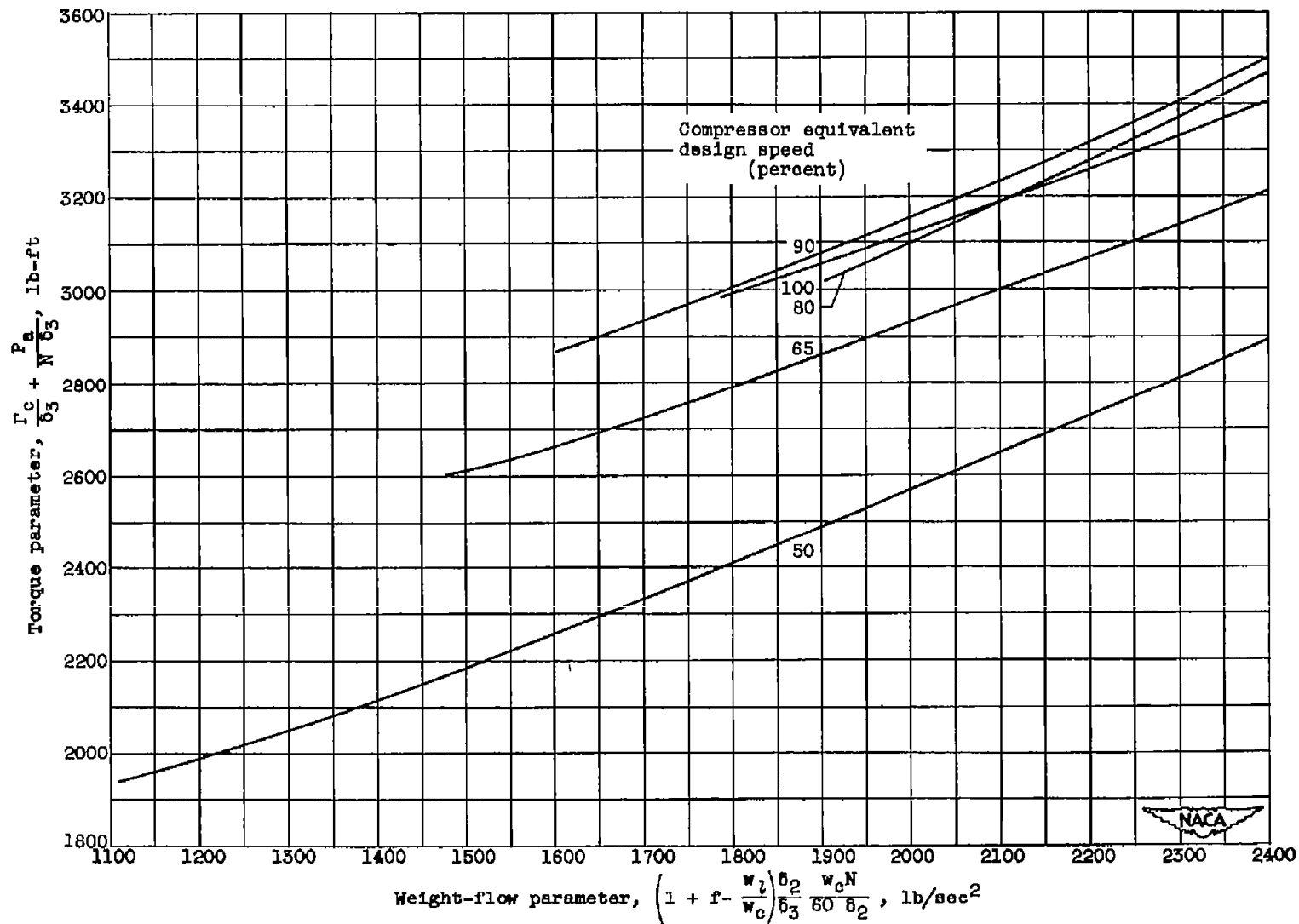
Figure 1. - Performance of 16-stage compressor.
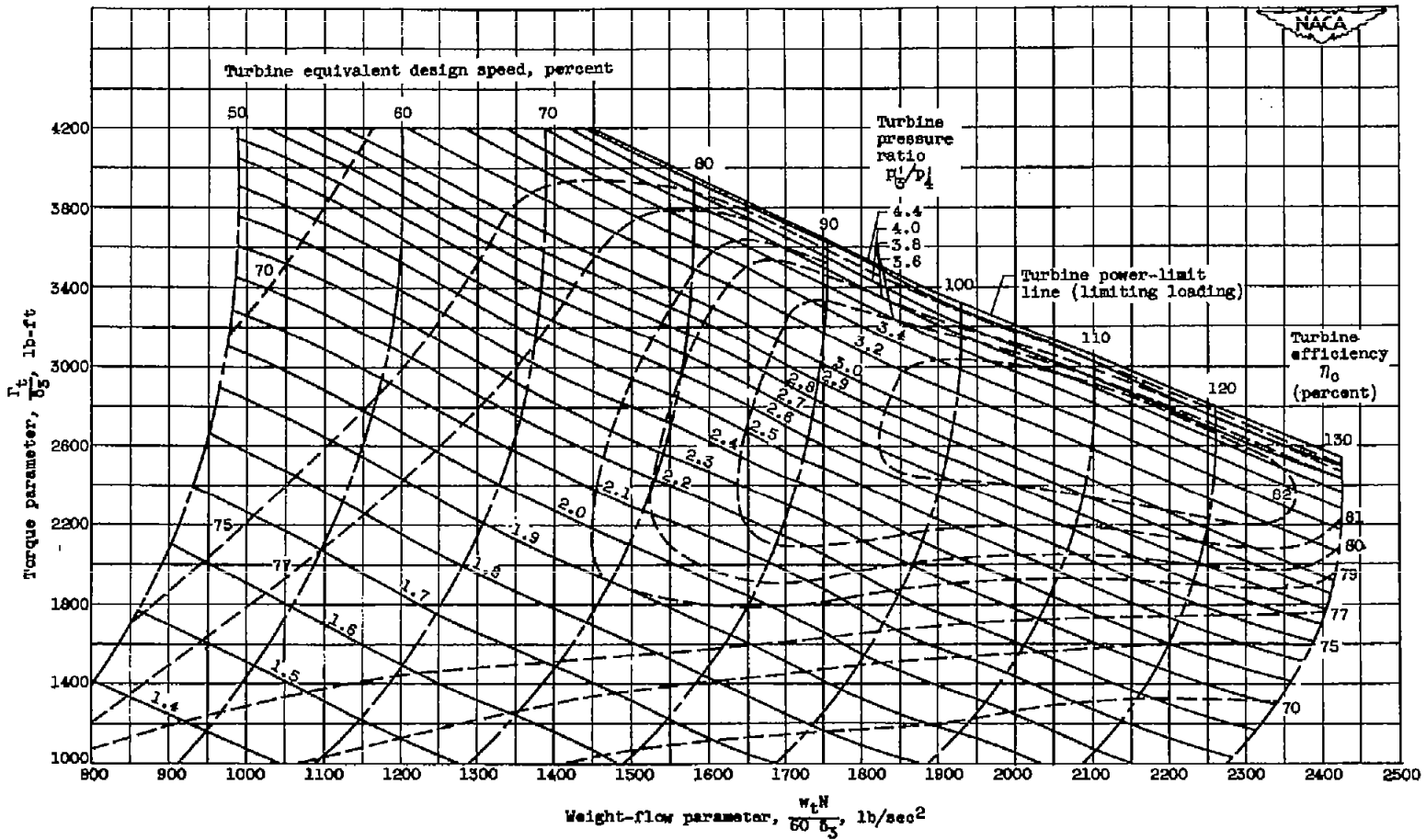
Figure 2. - Performance of 3-stage turbine. Ratio of specific heats, 1.32.
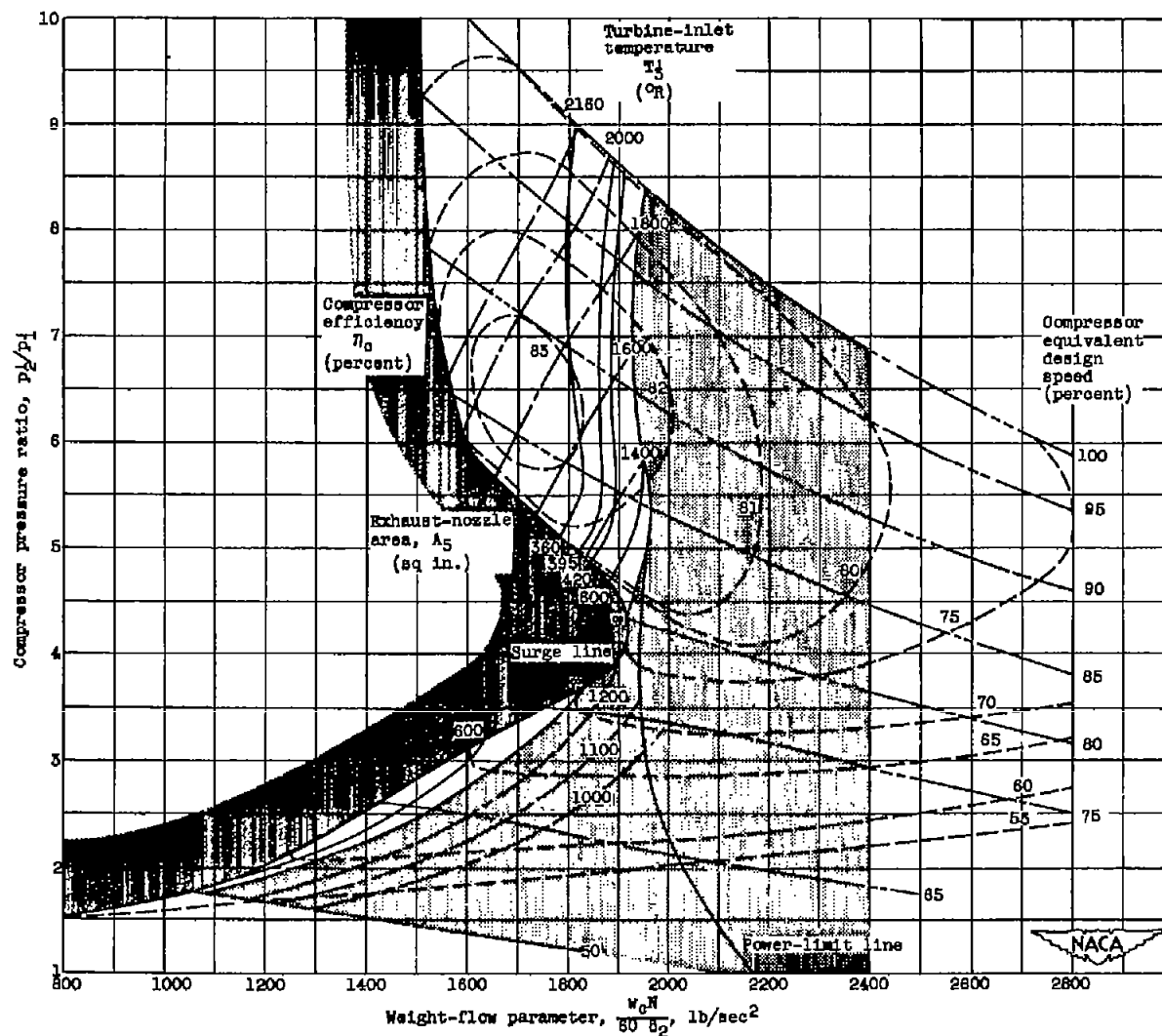
(a) No leakage.

Figure 3. - Matched performance characteristics of 16-stage compressor and 3-stage turbine.
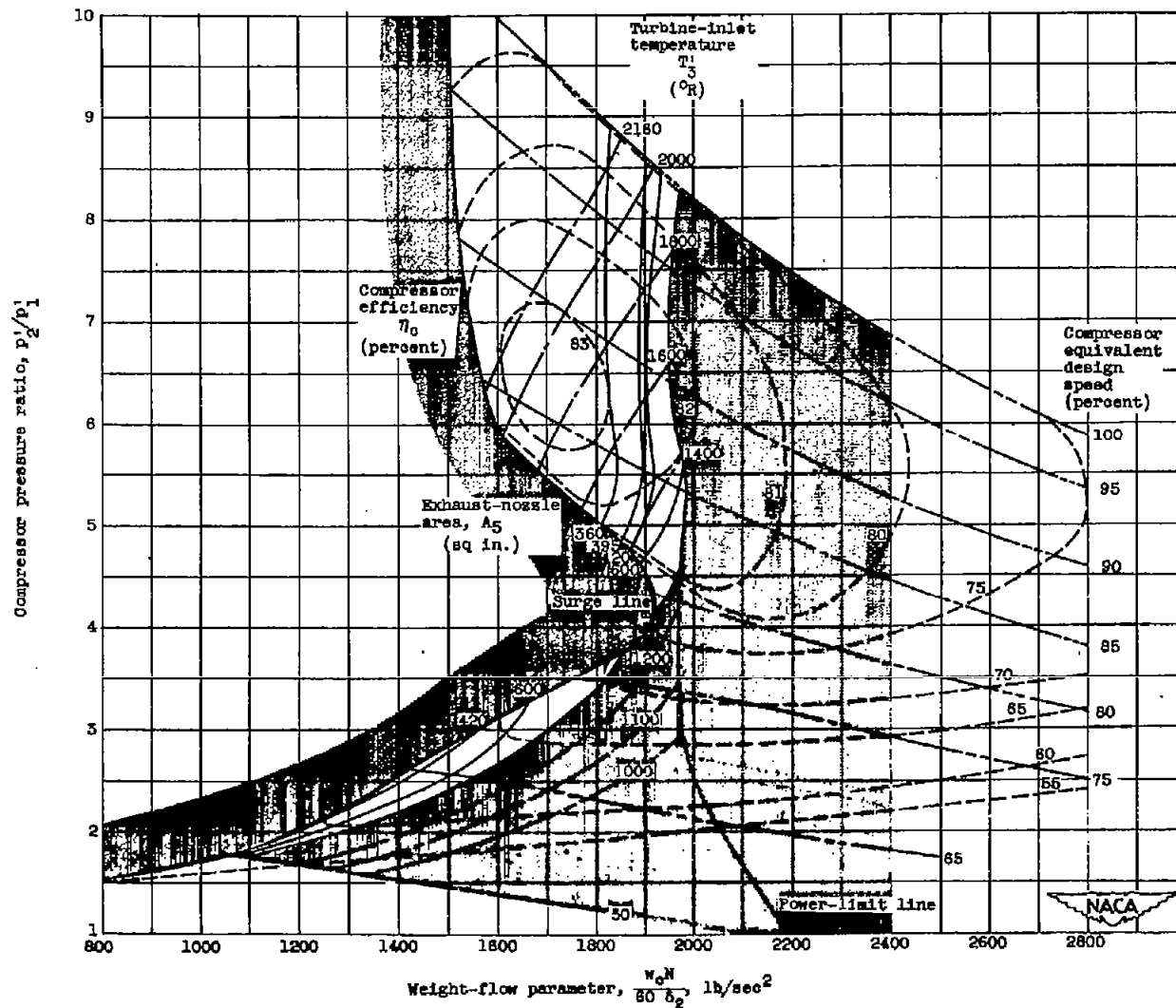
(b) Leakage, 2 percent.

Figure 3. - Continued. Matched performance characteristics of 15-stage compressor and 3-stage turbine.
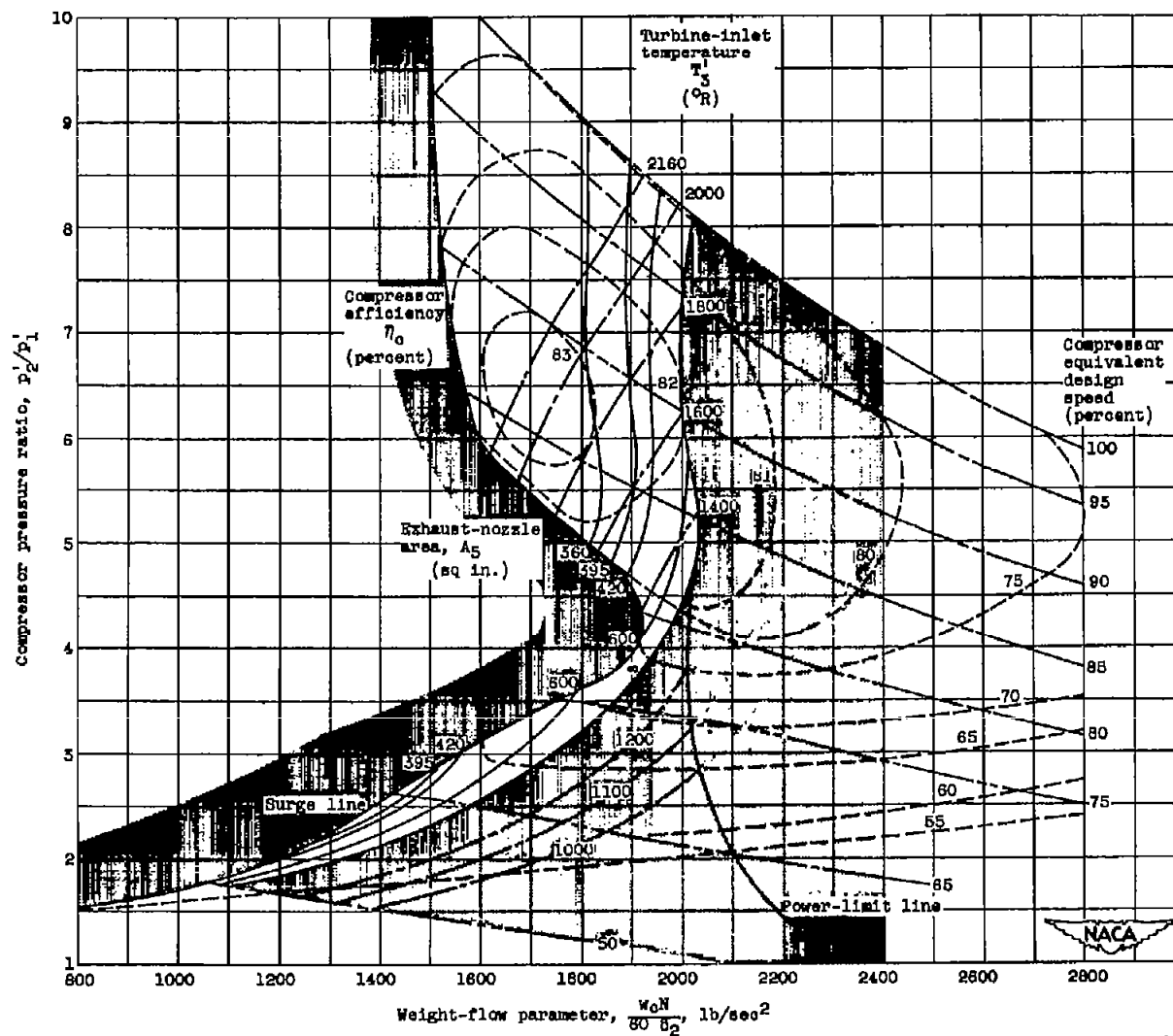
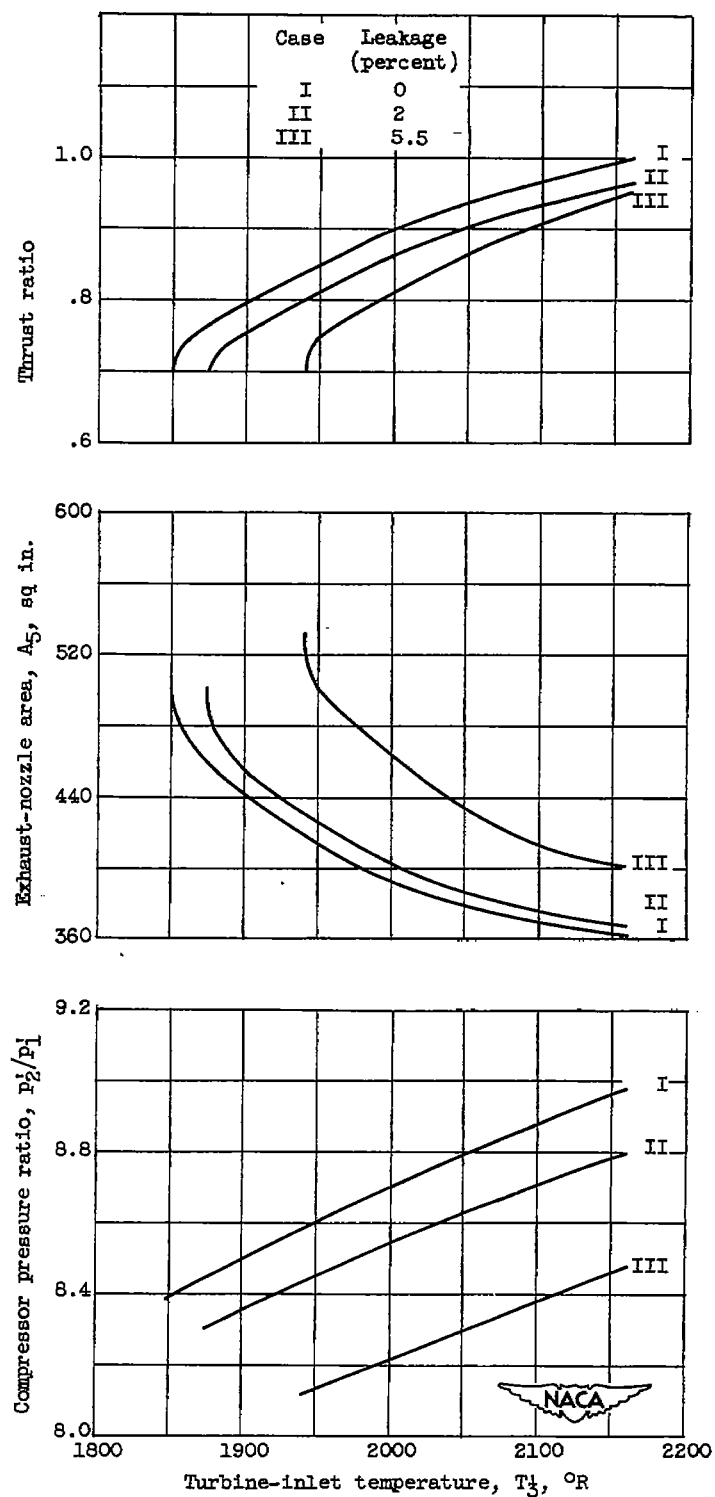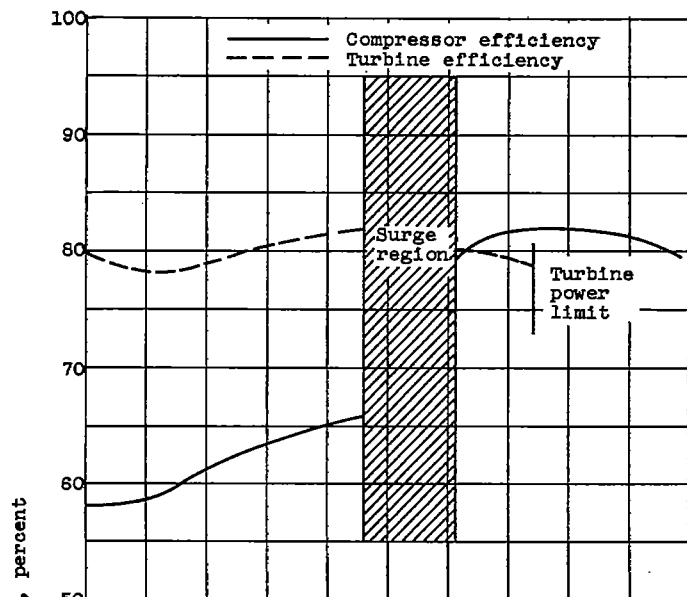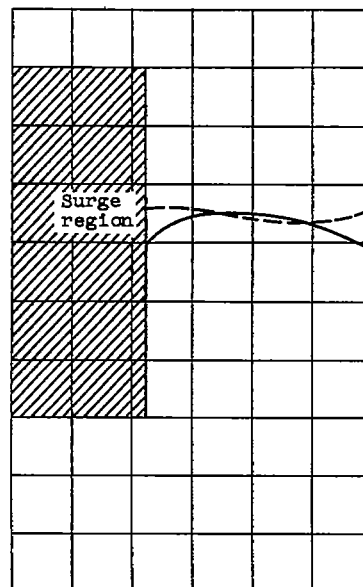(c) Leakage, 5.5 percent.

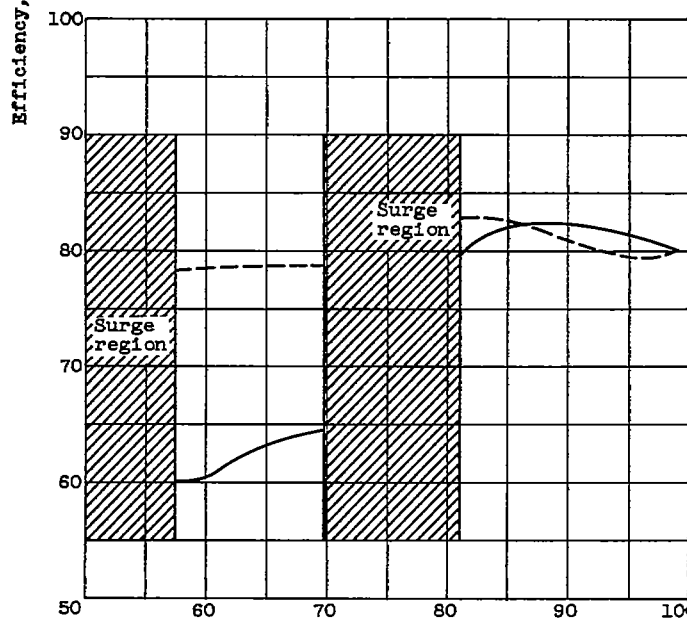Figure 3. - Concluded. Matched performance characteristics of 16-stage compressor and 3-stage turbine.

Figure 4. - Effect of leakage on engine performance at
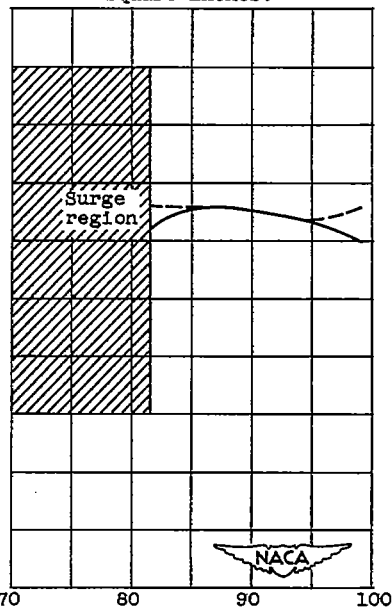design speed.

(a) Exhaust-nozzle area, 600 square inches.

(c) Exhaust-nozzle area, 395 square inches.

(b) Exhaust-nozzle area, 420 square inches.

(d) Exhaust-nozzle area, 360 square inches.

Figure 5. – Efficiency variation along equilibrium operating line. Leakage, 2 percent.
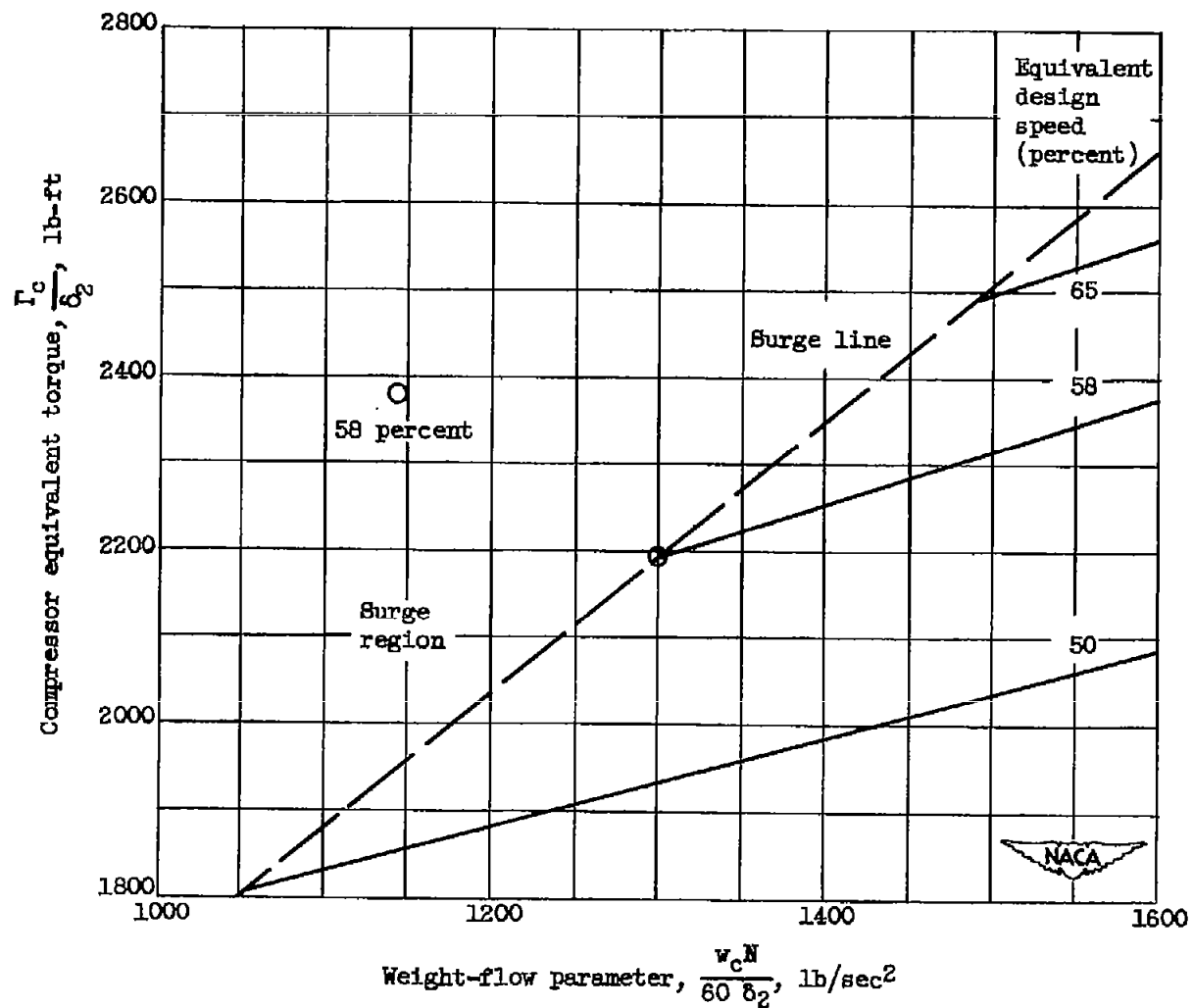
Figure 6. - Compressor performance for point in surge at 58 percent equivalent design speed. Stator-blade angle decreased 3° in 12th through 15th stages and reduced solidity in 14th through 16th rotors.