RESEARCH MEMORANDUM

STUDY OF THE PRESSURE RISE ACROSS SHOCK WAVES REQUIRED TO SEPARATE LAMINAR AND TURBULENT BOUNDARY LAYERS

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NACA = RM - L52C21 - c.235

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS
WASHINGTON
May 5, 1952
STUDY OF THE PRESSURE RISE ACROSS SHOCK WAVES REQUIRED TO SEPARATE LAMINAR AND TURBULENT BOUNDARY LAYERS

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SUMMARY

A dimensional study and an experimental investigation have been made on the pressure rise across shock waves required to cause separation of the boundary layer on a flat plate. The interaction of shock wave and boundary layer was investigated experimentally when the boundary layer was caused to separate from the surface of a tube of large diameter compared with the boundary-layer thickness, by means of a collar mounted on the tube. The investigation was conducted in a Langley blowdown jet at a Mach number of 3.03, for a Reynolds number range from about $2 \times 10^6$ to $19 \times 10^6$.

The dimensional study, based on certain simplifying assumptions, indicates that the critical pressure rise across a shock wave which just causes separation of the boundary layer is proportional to the skin friction: The available experimental data on flat plates indicate that the critical pressure rise varies as the Reynolds number to the $-\frac{1}{2}$ power for laminar boundary layers and as the Reynolds number to the $-\frac{1}{5}$ power for turbulent boundary layers; therefore, these results are in agreement with the prediction of the dimensional study. The Mach number effect on the critical pressure coefficient for turbulent boundary layers appears to follow that which is predicted for the skin-friction coefficient on a flat plate. The significance of the results obtained is discussed relative to certain practical design problems, such as supersonic-diffuser design.

INTRODUCTION

Increasing interest has been shown in recent years concerning the phenomena associated with the interaction of shock waves and boundary layers. A comprehensive review of the present status of the problem
from both experimental and theoretical considerations is given in reference 1. Experimental investigations show that the state of the boundary layer, that is, whether the boundary layer is laminar or turbulent, largely determines the resulting shock-wave configuration and the upstream influence of the shock wave on the boundary layer. (See references 1 to 3.) The studies up to the present time have been concerned primarily with the differences in shock-wave pattern for interaction with laminar and turbulent boundary layers; however, it was desired in this investigation to determine the conditions under which a boundary layer separates when a shock wave impinges upon it. Such information would have widespread application in aerodynamic problems, especially in the design of efficient supersonic diffusers and air inlets and in the alleviation of flow separation on airfoils and bodies. Some experimental data are available from pressure distributions on flat plates in which separation is induced by interaction of shock waves and boundary layers (references 1 to 6); however, these data are limited in scope, and the effects of Mach number and Reynolds number have not been determined. This paper presents the results of a dimensional study of the problem along with systematic wind-tunnel measurements of the effects of Reynolds number on the pressure rise across shock waves which cause separation of the boundary layer on a flat plate.

The experimental investigation was conducted in a Langley blowdown jet at a Mach number of 3.03, for a Reynolds number range from about $2 \times 10^6$ to $19 \times 10^6$. The boundary layer in these tests appeared to be fully turbulent, except perhaps for the lowest Reynolds number data presented. The boundary layer investigated was on the surface of a 2.94-inch-diameter tube which was mounted in the center of the 8.5-inch test section of the jet. The boundary layer was caused to separate from the surface of the tube by means of a collar mounted on the tube which induced interaction of the shock wave and boundary layer ahead of it. (See fig. 1.) The distance from the collar to the leading edge of the tube was varied in order to change the Reynolds number at which the shock-induced separation took place. These experimental results were compared with the predictions of the present study and with the published results of previous investigations.

**SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$R_x$</td>
<td>Reynolds number $(u_1x/\nu_1)$</td>
</tr>
<tr>
<td>$R_6$</td>
<td>Reynolds number $(u_15/\nu_1)$</td>
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local skin-friction coefficient \( \left( \frac{2\tau}{\rho_1u_1^2} \right) \)

pressure coefficient \( \frac{p_2 - p_1}{\frac{1}{2}\rho_1u_1^2} \)

velocity in the \( x \) direction

longitudinal distance from leading edge of tube to intersection of shock wave and boundary layer

axis normal to tube

kinematic viscosity \( (\mu/\rho) \)

coefficient of viscosity

mass density

total stress

static pressure

Mach number

dynamic pressure

boundary-layer thickness

factor

factor

Subscripts:

1  free stream

2  behind the shock wave

w  wall value

crit  critical
DIMENSIONAL STUDY OF SHOCK-INDUCED SEPARATION

When considering the interaction of a boundary layer and a shock wave, it is useful to remember that if the infinite pressure gradient that the shock wave represents could extend all the way to the wall there would certainly be a reverse flow (separation) in the gas layers close to the surface. The nature of the boundary layer, however, is such that the pressure difference across the shock is spread out in the lower levels of the boundary layer both in front of and behind the shock wave. For the purpose of this discussion it seems logical to assume, at least as a first approximation, that the extent of this spread at the wall is proportional to the boundary-layer thickness $\delta$. If this is so, it will be instructive to consider the effect of a shock wave having a pressure rise from $P_1$ to $P_2$ on the lowest levels of a boundary layer of thickness $\delta$. If these lowest levels comprise a thickness $\alpha \delta$, where $\alpha$ is a small quantity, and the pressure rise $P_2 - P_1$ is spread at the surface over a distance $\beta \delta$, the boundary-layer picture will be as shown:

Now, if the boundary layer is not to separate, the rate at which momentum is transferred into the small rectangle with sides $\alpha \delta$ and $\beta \delta$ by the shearing forces in the boundary layer must tend to balance the rate at which the pressure rise seeks to take momentum out of the rectangle. If the velocity that enters the front of the rectangle is small (as it is near the wall) compared with the change in velocity that can be induced by the pressure rise $P_2 - P_1$, and if, in order to
have no separation, the change in $u$ must also be small to prevent reverse flow, then the condition for which separation just occurs is approximately that the change in momentum per unit time induced by the pressure rise $p_2 - p_1$ must just equal the momentum induced per unit time in the element by the action of shear. Since it is logical to assume that the amount of momentum being transferred across both the upper and lower surfaces of the element considered is proportional to the initial wall shearing stress upon entering the element, the net amount of momentum that remains in the element is also proportional to the initial shear stress. Thus,

$$\left(p_2 - p_1\right)\delta \propto \tau_w\delta$$

so that

$$\left(\frac{\Delta p}{\delta}\right)_{\text{crit}} \propto c_f$$

In general, for laminar layers,

$$c_f \propto R_\delta^{-1}$$

and for turbulent layers with a $\frac{1}{7}$-power velocity profile,

$$c_f \propto R_\delta^{-1/4}$$

For boundary layers on flat plates, equations (3) and (4) become, respectively,

$$\left(\frac{\Delta p}{\delta}\right)_{\text{crit}} \propto R_x^{-1/2}$$

and

$$\left(\frac{\Delta p}{\delta}\right)_{\text{crit}} \propto R_x^{-1/5}$$

Since the derivation of equations (5) and (6) and the start of the experimental investigation, a paper by Stewartson (reference 7) has come to the attention of the authors. The considerably more detailed analysis of reference 7 leads to the inference that the dimensionless pressure rise required to produce separation would be of the order of
for the laminar boundary layer. It is interesting to note that by the simple assumptions of the present study a result is obtained which is very close to that indicated by Stewartson's more detailed analysis.

EXPERIMENTAL TECHNIQUE

Apparatus, Methods, and Tests

The experimental part of this investigation was conducted in a Langley M = 3.03 blowdown jet having a rectangular test section approximately 8.5 inches high and 10 inches wide. This two-dimensional nozzle was connected by way of a settling chamber to a supply of dry compressed air and controlled by a valve in such a manner that the chamber pressure could be held constant at any desired value. All the tests were made at a settling-chamber pressure of 134.7 pounds per square inch absolute and at a stagnation dew point which eliminated any effect of condensation. The Reynolds number of the tests was about $1.87 \times 10^6$ per inch.

Inasmuch as it was desired in these tests to eliminate the influence that the side walls of the tunnel normally exert on the interaction of shock waves and boundary layers on flat plates which span the tunnel test section, the tests were made on a tube with a wall thin enough not to choke the entering flow (fig. 1) which was mounted symmetrically about the center line of the test section of the jet. The radius of the tube (1.47 inches) was about 12 times the thickness of the boundary layer predicted by the use of reference 8 at the largest value of $x$ obtained in the present investigation. It is believed, therefore, that the test conditions are essentially the same as would be obtained on a flat plate in two-dimensional supersonic flow.

The boundary layer was caused to separate from the surface of the tube by means of a collar attached to the tube which induced the desired interaction of shock wave and boundary layer upstream of the collar. This method of inducing interaction with boundary-layer separation was used in references 4 to 6 and appeared very convenient for the present tube arrangement. The two collars investigated projected 0.15 inch and 0.30 inch above the surface of the tube. The 0.15-inch collar was investigated because it is of the order of the calculated boundary-layer thickness on the tube at the greater distances from the leading edge of the tube. The 0.30-inch collar was investigated to determine the effects of the greater collar height on the shock-wave patterns at small distances from the leading edge of the tube. The Reynolds number (based upon the longitudinal distance from the leading edge of the tube
to the point of incidence of the shock wave with the boundary layer was varied by changing the longitudinal location of the collar on the tube. The maximum possible distance from the leading edge of the tube to the collar was 11 inches for the present arrangement. Shadowgraphs were made of the interaction of shock wave and boundary layer in order that the shock angle in the immediate vicinity of the interaction could be measured, and the pressure rise across the shock was thus determined from the shock angle and known free-stream Mach number.

Accuracy of Measurements

At least two shadowgraphs were taken for each test condition in order to provide a check on the measurements of shock angle obtained. The shadowgraphs were magnified 10 times in a profile projector, and the shock angles were measured from the magnified pictures in order to obtain maximum accuracy. It is estimated that the values of $\Delta p/q_1$ presented herein are accurate to within ±5 percent.

RESULTS AND CORRELATION

Test Results

The results of the tests at a Mach number of 3.03 are given in table I and in the typical shadowgraphs of figure 2. As shown in table I for the tests with 0.15-inch and 0.30-inch collars, the pressure rise across the shock wave for separated boundary layers generally decreased slightly with increase in Reynolds number for a Reynolds number range from about $2.24 \times 10^6$ to $19.05 \times 10^6$. The data show that the shock-wave patterns were similar for the two collar heights investigated throughout the Reynolds number range of the tests (fig. 2). The test results further show that the distance from the leading edge of the collar to the apparent location of the intersection of the shock wave with the boundary layer was essentially constant throughout the Reynolds number range for each collar. This distance was about 0.8 inch for the 0.15-inch collar and about 1.5 inches for the 0.30-inch collar.

The slight disturbances extending outward from the tube surface, noted in some instances for the high Reynolds number tests, resulted from scars on the tube surface due to the screw-type locking device used for the collars; however, these disturbances are not considered to have affected appreciably the results obtained.
Correlation with Other Results

The variations of \((\Delta p/q_1)_{\text{crit}}\) with Reynolds number \(R_x\) obtained at a Mach number of 3.03 are presented in figure 3 for the two collars. Included on this plot are the available data from other sources for both laminar and turbulent boundary layers. The unpublished data points given in figure 3 were obtained on a circular-arc airfoil \((M = 1.37)\) and on a wedge at negative angles of attack \((M = 1.2)\) by means of an interferometer technique and a test facility similar to that described in reference 9. Most of the data from other sources (references 1, 2, and 5) are given in the form of pressure distributions along flat plates which experience interaction of shock wave and boundary layer, and the method of determining the pressure rise across the shock wave for both laminar and turbulent boundary layers is indicated in the following sketches of the typical pressure distributions obtained:

For interaction of shock waves and turbulent boundary layers the pressure rise across the shock wave which causes separation is easily determined, as shown in the sketch. For laminar boundary layers, however, the complex shock-wave patterns produce a pressure distribution with the pressure rise in two steps. Except for very weak shock waves, the strength of the incident shock wave is much greater than the critical pressure of separation of the laminar boundary layer, and so a small shock wave which will just cause laminar separation moves ahead of the main incident shock wave. The pressure rise for the laminar case is, therefore, taken at the knee of the first step of the pressure distribution. The boundary layer downstream of this point is turbulent and must withstand the large pressure rise of the main incident shock wave.

Except for an apparent transition region of \(0.8 \times 10^6 < R_x < 3 \times 10^6\), the pressure rise across a shock wave required for separation of the
boundary layer may be represented by a curve which varies as \( R_x^{-1/2} \) for laminar boundary layers and as \( R_x^{-1/5} \) for turbulent boundary layers (fig. 3). This result is very similar to the well-known variation of skin-friction coefficient with Reynolds number obtained on a flat plate (see, for example, reference 10) and, therefore, the experimental results verify the prediction made earlier in this paper.

Although the available data are rather limited, the general trend of the data suggests that at any particular Reynolds number the critical pressure coefficient is decreased with increase in Mach number (fig. 3). In an attempt to determine whether or not the Mach number effect on the critical pressure coefficient is of about the same order of magnitude as that noted for the skin-friction coefficient (as is predicted by the dimensional study), the results of reference 8 concerning the extension of the skin-friction law from incompressible to compressible flow have been applied to the data of figure 3 for turbulent boundary layers. The unpublished-data points of figure 3 have not been used in this study inasmuch as these data were not obtained on flat plates and do not give an accurate enough indication of the local skin friction for the present purpose. The study was made by assuming that the order of magnitude of the effect of Mach number on critical pressure ratio was the same as on skin friction (here evaluated at \( R_0 = 100,000 \) from reference 8) and obtaining the curves for critical pressure ratio against Reynolds number for Mach numbers 1 and 2 (dashed lines in fig. 4) from a \( -\frac{1}{5} \)-power curve faired through the experimental data for Mach number 3.03 (solid line in fig. 4). The results of the study are given in figure 4 in the form of lines which vary as \( R_x^{-1/5} \), superimposed upon the available experimental-data points. As shown in figure 4, the Mach number effect on the critical pressure coefficient does appear to follow that which is predicted for the skin-friction coefficient for turbulent boundary layers. It may be noted that the pressure rise for two points obtained from reference 4 are lower than those reported, since an attempt has been made to reevaluate the pressure rise closer to the point of intersection of the shock wave and boundary layer by examination of the published photographs.

At the present time there are not enough data available for the laminar boundary layer to justify any statement as to the effect of Mach number on the critical pressure ratio.
If it is assumed that the criterion proposed, namely, that the critical pressure rise is proportional to the skin friction, is correct, then certain general conclusions can be drawn as to the nature of flows involving boundary layer and shock interactions.

Shock Configurations at Transonic Speeds

When an airfoil section is tested at a Mach number in excess of its critical speed, a shock wave will exist on the surface. In most cases normally encountered, the strength of this shock wave lies in a range extending from something less than will separate a turbulent boundary to something just more than will cause turbulent separation. In general, it is far more than can be sustained by a laminar boundary layer. Thus, if the boundary layer on the airfoil is laminar, a small shock wave which causes laminar separation moves ahead of the main shock wave and establishes itself at some position where its strength is that which is just required to separate the laminar boundary layer at that point. The boundary layer downstream of this point is generally turbulent. Whether it will reattach itself or not depends on many things (nearness to the remaining shock, strength of the remaining shock, Reynolds number, etc.); however, through the remaining shock it must pass, and in general the appearance of this interaction is much like that in the normal turbulent case. These factors contribute to the formation of the lambda shock pattern. At high Reynolds numbers, if the flow is laminar the strength of the first leg of the lambda shock will be small, whereas if the Reynolds number is decreased the strength of this first leg of the shock wave will increase. Thus, it is conceivable that at low enough Reynolds numbers for the laminar flow case, there would be no lambda shock. It is also conceivable that at high enough Reynolds numbers, where the pressure rise that can be sustained by a turbulent layer is small, the shock wave will cause separation ahead of its usual position and the shock pattern may have an appearance similar to that usually associated with laminar boundary layers.

Supersonic Flaps and Controls

In many applications when it is desired that a flap be deflected upon a wing at supersonic speeds, the pressure distribution over the wing is favorable (for instance, if the wing has a circular-arc profile) so that the boundary layer ahead of the flap is laminar, especially in wind-tunnel tests (see reference 11). If the flap is deflected under such conditions the resultant pressure rise may separate the boundary
layer ahead of the deflected surface. It is also evident that the resulting flow at the flap juncture will depend considerably on Reynolds number, with no separation occurring at low enough Reynolds numbers and the separation effect increasing with increase in Reynolds number, except where an increase in Reynolds number might cause transition ahead of the flap juncture. The application of a roughness strip sufficiently far ahead of this point on small-scale tests should be of considerable help in simulating the full-scale flows, providing, of course, that the full-scale flow is not still a laminar flow at the flap juncture.

Bodies with Laminar Flow

In many cases where the pressure distributions on bodies are such as to maintain laminar flow near the base of the model at very large Reynolds numbers (such as was encountered in reference 12), even the very small pressure rise caused by the shock wave existing near the base of the model may cause separation. Such a condition is that represented by the case of the highest laminar boundary layer shown in figure 3, where a dimensionless pressure rise \( \left( \frac{\Delta p}{q_1} \right)_{\text{crit}} \) of 0.012 caused separation of the boundary layer.

Supersonic Diffusers

Possibly the most important use of the results of this investigation will be in the field of supersonic-diffuser design. Four general conclusions may be drawn:

1. It is desirable to keep the Reynolds number of the supersonic portion of the diffuser low. Thus in some cases it might prove advisable to break one large and long diffuser into an array of many very short diffusers of the same shape.

2. It is generally desirable to have turbulent boundary layers at low Reynolds numbers. Thus artificial transition may be useful unless the Reynolds number is so low that the laminar layer will tolerate almost as large a pressure rise as a turbulent layer.

3. It will be desirable to keep the pressure rise resulting from coalesced compression waves less than the critical value at any point and, preferably, to impinge the resulting wave on any surface at as low a Reynolds number as possible.

4. It is evident that, unless the supersonic Mach number is very low at the position of the normal shock wave in the diffuser, the critical pressure rise of a normal turbulent boundary layer will be exceeded.
Use of Vortex Generators or Turbulence Increasers

In view of the limiting conditions pointed out in the preceding section, it is necessary to discuss the possibility of increasing the critical pressure rise for separation by the use of vortex generators or some other turbulence-inducing device. Taking the view of the present study, a vortex generator may be thought of as preventing shock separation at a given pressure rise by increasing the local skin friction; thus, the best vortex generator for a given application will be one which gives the greatest increase in turbulence at some desired point for the increase in boundary-layer thickness it causes. In order to investigate the relative merits of various schemes for adding turbulence to the boundary layer, a technique similar to that used in the experimental portion of the present investigation could be used. If several sets of vortex generators to be investigated are set around the tube at a certain distance from the leading edge and the collar which induces separation is moved back and forth behind the vortex generators, the shock angle at the edge of the region of boundary layer and shock interaction may be obtained. This information can be used to tell how effective each set of generators was relative to each other set at each station downstream from the generators. A systematic series of such tests should enable the selection of the vortex generators to be used to overcome a given shock interaction problem at a given $R_b$, both as to geometrical shape and as to position of the vortex generators relative to the interaction to be overcome. Of course, there is at every $R_b$ a limit to what can be accomplished in this way, but it is believed that the value of the critical pressure rise may be increased appreciably over its normal value.

CONCLUSIONS

1. A dimensional study of the interaction of shock waves and boundary layers, based on certain simplifying assumptions, indicates that the critical pressure rise across a shock wave which just causes separation of the boundary layer is proportional to the skin friction.

2. The available experimental data from flat-plate tests at constant Mach number indicate that for laminar boundary layers

$$\left(\frac{\Delta p}{q_1}\right)_{crit} \propto R_x^{-1/2}$$
and for turbulent boundary layers

\[
\left( \frac{\Delta p}{q_1} \right)_{\text{crit}} \propto R_x^{-1/5}
\]

Therefore, these results are in agreement with the prediction of the dimensional study.

3. The Mach number effect on the critical pressure coefficient for turbulent boundary layers appears to follow that which is predicted for the skin-friction coefficient on a flat plate.

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REFERENCES


### TABLE I

**SUMMARY OF TEST RESULTS**

\[ M_1 = 3.03 \]

<table>
<thead>
<tr>
<th>Collar height (in.)</th>
<th>x (in.)</th>
<th>Shock angle</th>
<th>( \frac{\Delta p}{q_1 \text{crit}} )</th>
<th>( R_x \times 10^6 )</th>
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Figure 1.- Isometric drawing of tube-collar arrangement used for shock-wave - boundary-layer interaction. Outside diameter of tube, 2.94 inches; inside diameter, 2.76 inches.
Figure 2. Shadowgraphs of interaction of shock wave and boundary layer. $M_1 = 3.03; x$ is in inches.
Figure 2.- Concluded.

(b) 0.30-inch collar.

Figure 2.- Concluded.
Figure 3.- Variation with Reynolds number of critical pressure coefficient across shock waves which cause separation of the boundary layer.
Figure 4. - Effect of Mach number on the variation with Reynolds number of critical pressure coefficient across shock waves which cause separation of the turbulent boundary layer.