RELATION OF "LILIENTHAL EFFECT" TO DYNAMIC SOARING FLIGHT.

By Roderich Fick.

From "Zeitschrift für Flugtechnik und Motorluftschifffahrt," 1924, November 28, and December 12.

July, 1925.
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Otto Lilienthal observed that a flat surface, capable of rotation around a horizontal axis and supported by a counterpoise, oscillates vertically in a natural wind, whereby the momentary mean value of all the positions of said surface always forms an angle of 3-4° above the horizontal. The attempted explanations of this "upward component" long remained fruitless and finally led to doubting the results themselves and regarding them as errors of observation. On the assumption, however, that the observations of the upward component were correct, the turbulence of the wind was the only possible cause of the phenomenon. Neither the Knoller-Betz theory nor the variations of the wind in horizontal flow explained the effect, so long as flat or symmetrical profiles were employed, but, even in these cases, the effect was said to be always produced.

In this article, the phenomenon of the upward component in the case of flat surfaces will be referred to as the "Lilienthal effect." Otto Lilienthal's further observation that a cambered surface, fixed at a suitable angle, experiences a forward thrust toward the wind, I will here distinguish from the "Lilienthal effect" by calling it the "Knoller-Betz effect," because Knoller

* From "Zeitschrift für Flugtechnik und Motorluftschifffahrt," 1924, November 28, pp. 244-246, and December 12, pp. 258-260.
and Betz first satisfactorily explained this effect by means of vertical wind oscillations, at the basis of which, however, there lie air motions which the Lilienthal effect could not produce. The Lilienthal effect is also produced in the case of cambered airfoils. It will be shown that the same cause, which can produce the Lilienthal effect on flat airfoils, considerably strengthens the thrust due to the Knoller-Betz effect.

R. Knoller and Schmauck have already discovered the conditions of motion of the air, which can explain the Lilienthal effect (Flug- und Motortechnik, Vol. III, No. 22, "Die Gesetze des Luftwiderstandes," by R. Knoller; "Beitrag zur Erklärung des Segelfluges" by Schmauck, "Zeitschrift für Flugtechnik und Motorluftschifffahrt," 1913). Neither one, however, discovered absolute continuity, but only approximate or limited. Knoller assumes unsymmetrical waves, in which the downward phases are steeper than the upward phases, while the latter are correspondingly longer, the flow cross-section and velocity being constant. Schmauck, in an extension of the work of Betz, takes into consideration, in addition to the oscillations in direction, the simultaneous oscillations in strength and thereby emphasizes the special case, in which the oscillations in direction and strength coincide in such manner that the upward flow is faster and the downward flow slower. The question now arises as to whether there are meteorological causes for such motion conditions of the air.

Schnell, of Munich, has proposed a method for calculating
the energy of turbulence, necessarily existing near the ground, from the increase in the wind velocity with increase in height above the ground, by friction on ground obstacles (Lecture before the Aviation Convention at Munich in the fall of 1923). According to Schnell, the direction of transmission of the energy of turbulence in the higher layers of air is necessarily vertical and consequently exerts a lift on every object in the air. The transmission itself does not show how the lift is produced on an airfoil. In order to understand this, the flow around the airfoil must be considered. From the Lilienthal effect, we must deduce the conditions of motion of the air, in a way similar to that described by Knoller and Schmauck, and the meteorological cause of such a condition of motion must then be sought in Schnell's vertical direction of transmission.

It is now important to get a clear idea of the magnitude of the Lilienthal effect on the basis of a possible flow pattern and to compare the result with the Knoller-Betz effect alone.

Corresponding to the method of Prof. W. Hoff (Z.F.M., Vol. 13, pp. 276-278), a sinuous wind is taken as the basis, in which, however, the ascending flow velocities are greater than the descending. A streamline must fulfill the condition

$$\beta' = \beta_0 \sin(\lambda k)$$

and simultaneously let

$$v_0 = n v_{oo} \ n > 1,$$
in which $\beta'_0$ is the greatest angle and $\beta''$ the smallest angle formed by the streamline with the horizontal at the distance from an ordinate $l = 0$. $\lambda = 2\pi/L$ and $L$ is the wave length, $v_0$ is the maximum and $v_{oo}$ the minimum flow velocity.

Fig. 1 is the flow diagram which strictly fulfills the continuity conditions. It is produced by shifting all the streamline points the same optional small amount $K$ in a direction which forms the angle $\psi$ with the horizontal. The flow pattern can be continued as far as desired perpendicular to the plane of the diagram.

Since

$$\frac{v_0}{v_{oo}} = \frac{s_{oo}}{s_0}$$

we have

$$\sin(\psi - \beta'_0) = \frac{s_0}{K}$$

or

$$\frac{\sin(\psi + \beta'_0)}{\sin(\psi - \beta'_0)} = \frac{s_{oo}}{s_0} = \frac{v_0}{v_{oo}}.$$

$$\sin(\psi + \beta'_0) = \frac{s_{oo}}{K}.$$

If we put

$$v_0 = n v_{oo}, \quad n > 1,$$

we then have

$$\frac{\sin(\psi + \beta'_0)}{\sin(\psi - \beta'_0)} = n \quad \text{or} \quad \tan \psi = \frac{n + 1}{n - 1} \tan \beta'_0 \quad (1)$$

On the horizontal line $AB$, at the distance $l$ from the origin $l = 0$, the flow velocity is

$$v = v_0 \frac{\sin(\psi - \beta'_0)}{\sin(\psi - \beta'')} \quad (2)$$
and the direction is

$$\beta' = \beta'_0 \sin(\lambda \, l'), \quad l = l' + \Delta l$$

For \( n = 1 \) we have the special case of the Knoller-Betz theory. Formulas (1), (2) and (3) then become

$$\tan \psi = \frac{2}{\omega} \tan \beta'_0 = \infty$$

$$\psi = \frac{\pi}{2} = 90^\circ$$

$$v = v_0 \frac{\cos \beta'_0}{\cos \beta'}$$

$$\beta' = \beta'_0 \sin(\lambda \, l)$$

since \( \Delta l = 0 \), consequently \( l = l' \).

For small values of \( \beta'_0 \), formula (5) approximates

$$v \approx v_0,$$

and hence the velocity of flow is nearly constant.

The flow diagram for \( n = 2 \) is plotted in Fig. 1. We will now endeavor to find what power the same glider can receive from the wind in the flow diagram for \( n = 2 \) and in the flow diagram for \( n = 1 \), \( \beta'_0 \) being 20° and the mean wind velocity being the same in both cases. For \( n = 2 \), we take \( v = 14 \) m/s.

From formula (1) we obtain

$$\psi = 47.5^\circ, \quad \psi - \beta'_0 = 27.5^\circ$$

In Fig. 2 the flow velocities were calculated from formulas (2) and (3) multiplied by the cosine of the corresponding angles.
of flow in terms of \( l \) and we obtain the mean horizontal wind velocity \( v_m \):

\[
v_m = \frac{1}{L} \int_0^L v \cos \beta' \, d l = 9.26 \, \text{m/s}
\]

We will first consider the case of flight against the wind and infinite inertia of the airplane, which is permissible for a sufficiently high frequency. Moreover the airfoil constantly maintains the angle of attack of least drag. The horizontal velocity \( v_F \) of the airplane will then be such that

\[
H_{10} = \int_0^T H_1 \, dt = 0
\]

in which \( T \) is the duration period, \( H_1 \) the drag at the time \( t \), also

\[
H_1 = c_{h_1} v_R^2 \frac{F}{16}
\]

and \( c_{h_1} \) is the drag coefficient.

The present investigation is based on the polar diagram employed by W. Hoff in his article (Z.F.M. 1922, p. 276) and a 16 m² airfoil. For different values of \( v_F \), at the time \( t \), hence at the distance \( l \) from the point \( l = 0 \) (Fig. 1), the angle of attack \( \beta \) of the wind and its velocity \( v_R \), before being affected by the wing, were determined according to

\[
\tan \beta = \frac{v \sin \beta'}{v_F + v \cos \beta'}
\]

and

\[
v_R = \frac{v \sin \beta'}{\sin \beta}
\]
Furthermore, the $c_{h1}$ values for $\beta$ at the time $t$ were taken from Fig. 3, using the curve $c_h$ plotted against $\beta$. These values were multiplied by $v_R^2$ and plotted against $t$ and thus

$$H_{10} = \frac{1}{T_0} \int_{T}^{T} c_{h1} v_R^2 d\ t$$

was obtained.

This rendered it possible to plot $H_{10}$ against $v_F$ and to determine, by the intersection of the connecting curve with the abscissa, the value of $v_F$ for which $H_{10} = 0$. In the case under consideration, it was found to be

$$v_F = 16 \text{ m/s}$$

From Fig. 4 we obtain, for this $v_F$ as the mean value $V_{10}$ of all the vertical forces,

$$V_{10} = \frac{1}{T_0} \int_{T}^{T} c_{v_F} v_R^2 d\ t = 506 \text{ kg}$$

i.e., our 16 m$^2$ airplane would just be able to maintain horizontal flight with a full load of 506 kg. The necessary power for horizontal flight in still air is approximately

$$L = \frac{G}{75} \sqrt{\frac{2 m_g}{\rho \xi'}} = \frac{506 \times 4}{75} \sqrt{\frac{506}{16 \times 250}} = 9.62 \text{ HP.}$$

in which $\xi'$ is the mean flight coefficient, hence the mean of all $c_L^2/c_D^2$ for the same range of angles of attack, which occur in dynamic soaring flight. For our airplane, this flight coefficient is about 250.

If we now take for our basis a flow pattern in which $n = 1$ and the mean wind velocity is the same, i.e., 9.26 m/s, $\beta'_0 = 20^\circ$, 

giving the case of the Betz theory under otherwise the same conditions.

We obtain the flow velocities, at the distance \( l \) from \( l = 0 \), from

\[
v = v_0 \frac{\cos \beta'}{\cos \beta'}
\]

and the mean horizontal magnitude of wind velocity from

\[
v_m = v_0 \frac{\cos \beta'}{\cos \beta'} \cos \beta' = v_0 \cos \beta'
\]

The mean horizontal magnitude of wind velocity is therefore constant and should be 9.26 m/s. It corresponds therefore to the magnitude of wind velocity for \( \beta' = 0^\circ \).

The value of \( v_F \), for which \( H_{10} = 0 \), is found in the same way as when \( n = 2 \) (Fig. 5).

We get \( v_F = 13 \text{ m/s} \).

\[
H_{10} = 0, \quad V_{10} = \frac{1}{T_0} \int_0^T c_{V_1} v_R^2 \, dt = 368 \text{ kg.}
\]

\[
L = \frac{368 \times 4}{75} \sqrt{\frac{368}{16 \times 250}} = 5.95 \text{ HP.}
\]

For \( n = 2 \), the deducible power is therefore about 62% greater than for \( n = 1 \). This was the most favorable case under the Knoller-Betz theory. In the simplest case the deducible power is considerably smaller, but there is a greater percentage increase for \( n = 2 \) as compared with \( n = 1 \), due to the camber of the airfoil.

We will now investigate, for the same two flow patterns, the
simplest case, in which the airfoil remains in a constant negative angle of attack \( \gamma \). We here assume \( \gamma = -4^\circ \).

Since I have at my disposal no better profile, which has been tested at a sufficiently large negative angle of attack, I will employ the profile Rumpler CI from TB II, p. 33, though this profile is very unfavorable for the case in hand. In Fig. 6, \( c_{h1} \) and \( c_{v1} \) are plotted against \( \beta \) for this profile. 8 m/s is found as the airplane velocity corresponding to the flow pattern \( n = 2 \), for which the drag vanishes. In Fig. 7, \( c_{v1} v_R^2 \) and \( c_{h1} v_R^2 \) are again plotted against \( t \) for \( v_F = 8 \) m/s, and we have

\[
H_{10} = \int_0^T c_{v1} v_R^2 \, dt = 0
\]

\[
V_{10} = \frac{1}{T} \int_0^T c_{v1} v_R^2 \, dt = 36 \text{ kg}.
\]

The polar diagram of the profile Rum CI gives the flight coefficient \( \xi = c_L^3/c_{D_{\text{max}}}^2 \) as 22.5. The mean flight coefficient \( \xi' \), therefore, can surely not be over 20, so that, in the most unfavorable case, the deducible power will be

\[
L = \frac{G}{75} \sqrt{\frac{G/F}{\xi'}} = \frac{36 \times 4}{75} \sqrt{\frac{36}{16 \times 20}} = 0.645 \text{ HP}.
\]

In the same manner the result for the flow pattern for \( n = 1 \) is found from Fig. 8 to be

\[
H_{10} = 0, \quad V_{10} = \frac{1}{T} \int_0^T c_{v1} v_R^2 \, dt = 21.8 \text{ kg}
\]

and

\[
L = \frac{21 \times 8 \times 4}{75} \sqrt{\frac{21 \times 8}{16 \times 20}} = 0.303 \text{ HP}.
\]
The increase in the Knoller-Betz effect is therefore, in the simplest case, about 115% for \( n = 2 \), as compared with \( n = 1 \). A considerably greater increase can be expected with better profiles.

The relative data obtained indicate the following:

1. The best modern gliders (such as the Vampyr and Geheimrath) should, according to their dynamic power requirement of about 2.5 HP., be capable of turbulence soaring flight in "the most favorable case" of the Knoller-Betz theory.

Nothing of that kind, however, happened, even in the case of airplanes which were supposed to steer automatically according to the Betz diagram. From this we might conclude either that steering according to "the most favorable case" is impossible, or that the flow diagram, taken as the basis, corresponds in no way to reality.

2. The "simplest case" affords possible power deductions, which, in the evaluation of the flights of the Strolch, Konsul and Hawa 6, must have been already very noticeable. If dynamic effects were not noticed, however, (i.e., if sinking speed and coefficient of glide corresponded, both with wind and in gliding flight with no wind, to those theoretically determined for static soaring flight, we might conclude that turbulence is in no way capable of improving the flight. In the cases here considered, either \( n \) or \( \beta_0 \) or both values must have been assumed too high, or the flow pattern must have been disturbed by other mete-
In opposition to this, however, stands the Lilienthal effect, which still declares that the resultant of all the dynamic forces, at least in flight against the wind with a velocity equal to that of the wind, must be directed upward 3-4°.

We will investigate the wind itself, with reference to its Lilienthal effect, for our case \( n = 2 \). We will assume that the flow pattern moves with such a velocity against the Lilienthal vane that a sufficiently high frequency is produced and that the inertia of the experimental arrangement can be regarded as infinite. The velocity of the flow pattern, in the present investigation may be regarded as independent of the wind velocity. It simply defines the period or frequency (similar to the propagation of water waves without horizontal displacement of the water particles).

We will resolve the wind of the flow pattern, at the time \( t \), into a vertical and a horizontal component. The vertical component \( v_y \) is

\[ v_y = v \sin \beta', \]

and the horizontal component

\[ v_x = v \cos \beta'. \]

The continuity is expressed by

\[ \int_0^T v_y \sin \beta' = 0. \]

If we construct the squares of the flow velocities (hence
the 16-fold pressures, in consideration of

\[ q = \frac{\rho v^2}{3}, \rho = 0.125 \]

and multiply these values by the sine or cosine of the corresponding flow inclinations, thus obtaining

\[ v_y^2 \sin \beta' \text{ and } v_x^2 \cos \beta' \]

and plot these values against values of \( \tau \), we obtain from Fig. 9,

\[ \frac{1}{T} \int_0^T v_y^2 \sin \beta' \, d\tau = 6.0 \text{ and } \frac{1}{T} \int_0^T v_x^2 \cos \beta' \, d\tau = 92.0 \]

and therefrom the Lilienthal effect through

\[ \tan \alpha = \frac{6}{92} \text{ as } \alpha = 3^\circ 44' \]

Hence, for the flow pattern employed, \( n \) and \( \beta_0' \) were so chosen that the magnitude of the Lilienthal effect was limited.

We must therefore conclude:

1. That the flow pattern for \( n = 3 \), taken as the basis, may approximate reality.

2. That steering according to the Betz diagram for "the most favorable case" was not successful.

3. That the simplest case of the Knoller-Betz theory occurs in every soaring flight. The calculated minimum sinking velocity for the static soaring flight is represented by the well known formula

\[ v_s = \sqrt{\frac{2\phi}{\rho \xi}} \xi = \frac{c_L^3}{c_D_{max}} \]
Turbulence of both imperceptible (higher) and perceptible frequency is, however, present in every wind. The latter condition is demonstrated by the need of steering movements to maintain equilibrium. The angle of attack for \( \frac{c_L^2}{c_D_{\text{max}}} \) can, for gliding flight, be continuously maintained only in still air. In a wind, the value of \( \frac{c_L^2}{c_D_{\text{max}}} \) only occurs incidentally, since then even with a stationary airplane, the angle of attack of the wind is subject to continual changes. The sinking speed must therefore be greater in a wind, being approximately \( v_s = \sqrt{\frac{2 \cdot 0}{\rho \cdot \xi'}} \), in which \( \xi' \) is a mean value which denotes, during a period, a maximum value of \( \frac{c_L^2}{c_D^2} \) attained momentarily at time \( t \).

According to the quantitative evaluation of the Betz diagram in "the most favorable case" by Prof. W. Hoff, we would have, in a wind, a 25% greater minimum sinking speed relative to the wind. This did not prove to be the case. Hence:

4. There is no pure static soaring flight. A portion of the flight power is always automatically derived from the turbulence; how much is still to be determined. For the present, the dynamic power seems to increase the diminution of \( \xi = \frac{c_L^2}{c_D_{\text{max}}} \) to \( \xi' \) and to prevent, by steering motions, the diminution of the coefficient of glide. A foothold can be obtained from the data given in this article. One can compare the power requirement of the best gliders e.g., the Hawa 6, which, with 15 m\(^2\) area, is

\[
L_{\text{stat}} = v_s \frac{G}{75} \text{ HP.} = \frac{0.447 \times 145}{75} = 0.866 \text{ HP.}
\]
with the results of Figs. 7-8, which give the possible power on the basis of winds of 0.6 and 0.3 HP. for 16 m² area.

5. There is no use in developing, for the glider profile alone, the highest $c_L^3/c_D^{max}$ and $c_L/c_D^{max}$, but only the highest mean values, since soaring flight is attempted only in a wind. The vibration amplitudes are not yet known. Lilienthal effect does not produce them since, for the wind here tested, $n$ could be greater and $\beta'_0$ smaller, without changing the direction of the resultant of all the dynamic pressures. The most favorable profiles for soaring flight would therefore have to be found by direct measurements of lift and drag on experimental airfoils in a natural wind.

Translation by Dwight H. Miner, National Advisory Committee for Aeronautics.
Fig. 1 Flow diagram for $n=2, \beta' = 20^\circ$

Fig. 2
Fig. 5

Fig. 6 Rumpler CI $\gamma = -4^\circ$