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CALCULATION OF WING SPARS OF VARIABLE CROSS-SECTION AND LINEAR LOAD.

By Léon Kirste.

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CALCULATION OF WING SPARS OF VARIABLE CROSS-SECTION AND LINEAR LOAD.*

By Léon Kirste.

The calculation of wing spars of constant cross-section and load has been thoroughly treated by a large number of authors. Such is not the case, however, regarding the calculation of wing spars whose section and linear load diminish toward the ends, as in wings of trapezoidal contour and decreasing section.

Since the employment of such wings is becoming more general it seems opportune to indicate a method which enables the attainment of the requisite degree of accuracy and is easy to apply.

For the sake of illustration, let us consider an element AB of a girder (Fig. 1) subjected to a bending load p (kg per linear meter) and a longitudinal load C (kg).

Let us assume, as is customary for the establishment of bending formulas, that the inclination of the bend is small with respect to the horizontal.

On passing from section A to section B and projecting the forces on the plane of the latter, we obtain, for the change in the shearing stress,

$$dT = p\ dx + C \sin d\varphi.$$

* From "L'Aeronautique," January, 1925, pp. 25-26.

On the other hand, we have

$$\sin d\varphi = \frac{dx}{\rho} \quad \text{and} \quad \frac{I}{\rho} = \frac{M}{EI}$$

Hence

$$dT = p\ dx + C\ \frac{M\ dx}{EI}$$

which may be written

$$\frac{dT}{dx} = p + \frac{CM}{EI}$$

Now the change in the shearing stress, in a girder stressed only in flexure, is nothing but the linear load, applied at the given point. We may therefore conclude that, in a girder simultaneously subjected to both transverse and longitudinal loads, the resultant bending moments are identical with those which take place in the same girder when subjected to the same transverse loads increased, at every point, by the quantity CM/EI.

It is understood that the bending moment M, which enters into this formula, is the resultant moment due to the stress composed of the two loads. Since this is just the resultant moment we are seeking, the method consists in proceeding by an integration or successive approximations, beginning with the bending moment corresponding to the sole transverse loads. Since the latter is the more general, we will demonstrate it by an example.

Let ABC represent the given wing spar (Fig. 2), jointed at A and held at B by a stay or oblique strut, the portion BC forming an overhang.

The reaction of the support B occasions a compression and, unless it passes through a neutral fiber, also a bending moment. The latter, combined with the moment due to the overhang load, gives the moment M_B at the point B.

The compression in the portion AB is increased as a result of the reactions of the diagonal bracing inside the wing. In general, one exterior element or section corresponds to two or three interior. In this case, the diagram of the longitudinal stresses will have steps, as in Fig. 2. If, on the contrary, a covering of metal or plywood confines the stresses in the plane of the wing, the diagram of the longitudinal stresses will be a continuous curve.

The distribution of the transverse loads depends on many factors, namely, the contour of the wing, variations in the wing section, variations in the angle of attack, marginal losses, etc. It will therefore constitute a curve whose ordinates decrease toward the free end.

We have already assumed that the cross-section of the spar decreases. Hence the same will be true of the moment of inertia, as shown in Fig. 2.

With the aid of these data and a Cremona diagram, we can trace curve I of the bending moments due to the transverse loads alone (Fig. 3).

With these moments of simple flexure, we determine curve I of the theoretical loads MC/EI, which gives the first correction of the transverse loads.

With the amplified loads, obtained by the superposition of the transverse loads and first theoretical loads, we trace curve II of the theoretical loads.

On repeating these operations two or three times, we generally find that the corrections keep on diminishing, so that we ultimately obtain a moment curve which is practically the definitive curve.

If, on the contrary, the corrections should continue to increase, this would be an indication that equilibrium is impossible. In this event there would be produced the phenomenon of instability well known in aviation, namely buckling, which is due to insufficient moments of inertia.

It may be asked whether, on knowing the analytical expressions for the different curves, an integration would not give the desired result more quickly than tracing the successive moment curves.

Since, however, the longitudinal stress and the moment of inertia are not constant, integration is hardly possible, because any linear variation leads to a differential equation of the second order, with linear coefficients which produces a development in series.

Moreover, the graphic solution renders it possible to take account of the local variations in the moment of inertia, load, etc., due to fillings of the wing spar, notches in the wing, local stresses, etc., and which cannot be expressed by an analytical curve.

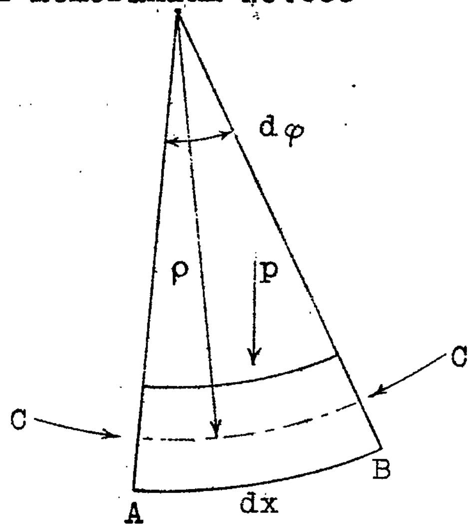

Fig.1

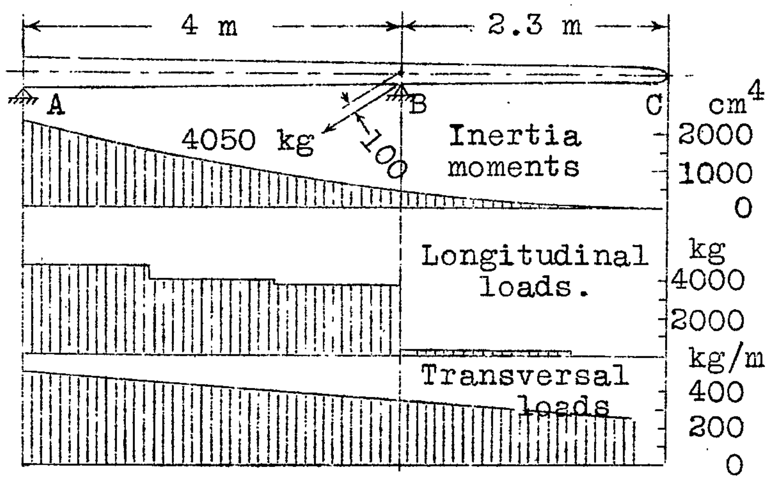

Fig.2

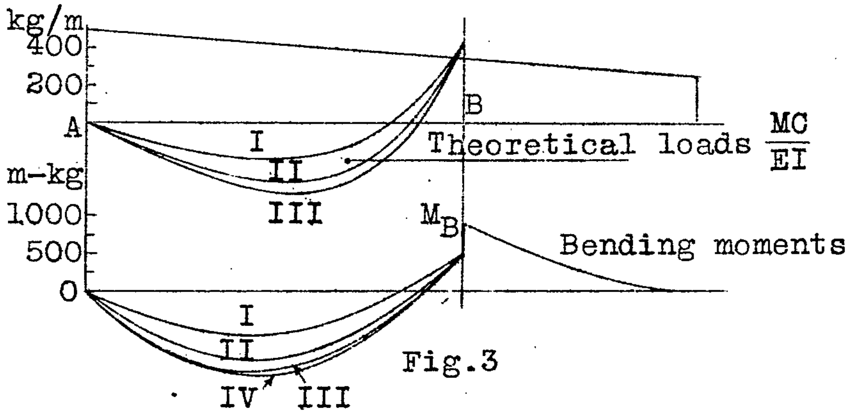

Fig.3