RESEARCH MEMORANDUM

COMPARISON OF HOVERING PERFORMANCE OF HELICOPTERS
POWERED BY JET-PROPULSION AND
RECIPROCATING ENGINES

By Virginia L. Brightwell, Max D. Peters, and J. C. Sanders

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SUMMARY

Maximum hovering time, or time that a helicopter can sustain itself without motion, was calculated for helicopters powered by a reciprocating engine, by a Nernst turbine rotor, by ram-jet engines at the tips of the rotor blades, and by pulse-jet engines at the tips of the rotor blades. The calculations showed that the conventional reciprocating engine permitted much longer hovering time than the jet-propulsion engines investigated, but because the jet-propulsion engines were lighter than the reciprocating engine, the jet-propelled helicopters could lift greater disposable loads. Of the jet engines considered, the pulse-jet engine allowed the longest hovering time, which was about 15 percent of the hovering time possible with the reciprocating engine.

INTRODUCTION

The helicopter, unique in its ability to sustain itself without motion, is useful for inspection, observation, and photographic purposes. Although helicopters powered by jet-propulsion engines have recently been considered, at the present time a limited amount of performance data is available. Previous analytical studies of application of jet-propulsion drives to helicopter rotors are reported in reference 1 and in unpublished data from the University of Southern California. In reference 1, consideration was given to ram jets on the rotor tips and to a rotor with tip jets supplied with air fed from a compressor mounted in the airframe. The range of the helicopter powered by jets supplied by an engine-driven compressor was estimated to equal that of a conventional helicopter, but the range of the helicopter powered with ram jets was considerably less. Marquardt and DeVault of the University of Southern
California studied tip-located ram jets and pulse jets as well as tip jets aspirated by a hub-located, engine-driven compressor or by a hub-located, rotor-driven compressor. It was found that the system using tip-located ram jets would be economically feasible for cruising endurance less than 1 hour, the tip-located intermittent system and the rotor-driven compressor system for endurance less than 2 hours, and the engine-driven compressor system for endurance from 2 to 4 hours. For these periods, such systems appeared to be able to carry approximately twice the payload of a conventional helicopter of similar size at comparable cost.

In this analysis, conducted at the NASA Cleveland laboratory, calculations were made to compare the fuel consumptions of three jet-propulsion engines and one reciprocating engine that were used to power helicopters for hovering flight at sea level. The jet-propulsion engines are a ram jet and a pulse jet (reference 1) and, in addition, a hollow-blade rotor, or Nernst turbine, with burners near the blade tips. The effects of changes in several limiting variables were studied and values were selected to give low fuel consumption. Reasonable values were assumed for the rotor diameter, rotor tip speed, and fuel consumption for the reciprocating engine. The comparisons for low forward speeds would probably be similar to those found for hovering.

**SYMBOLS**

The following symbols and abbreviations are used in this analysis:

- $A_2$: cross-sectional area of burner, (sq ft)
- $A_r$: required cross-sectional area of engine, (sq ft)
- $B$: number of rotor blades, (3)
- $C$: coefficient
- $c$: chord of rotor blade, (ft)
- $c_p$: specific heat at constant pressure, (Btu/(lb·°F))
- $d$: rotor diameter, (ft)
- $F$: thrust, (lb)
- $f/a$: fuel-air ratio
\( f \)  fuel consumption, (lb/hr)
\( f_i \)  initial fuel rate, (lb/hr)
\( g \)  acceleration of gravity, 32.2 (ft/sec²)
\( h \)  hovering time, (hr)
\( hp \)  horsepower
\( J \)  mechanical equivalent of heat, 778 (ft-lb/Btu)
\( K = \frac{f_h}{W_e + W_{f,h}} \)
\( M \)  Mach number
\( P \)  total pressure, (lb/sq ft absolute)
\( p \)  static pressure, (lb/sq ft absolute)
\( \Delta P_f \)  static-pressure drop due to friction, (lb/sq ft)
\( q \)  dynamic pressure, (lb/sq ft)
\( R \)  gas constant, 53.3 (ft-lb/(°F-lb))
\( r \)  radius of rotor, (ft)
\( S \)  rotor-disk area, (sq ft)
\( T \)  total temperature, (°R)
\( t \)  static temperature, (°R)
\( V \)  velocity, (ft/sec)
\( W \)  gross weight of helicopter, 2560 (lb)
\( W_e \)  weight of helicopter when empty, (lb)
\( W_f \)  fuel flow, (lb/sec)
\( W_{f,1} \)  weight of initial fuel load, (lb)
\[ W_{f,h} \quad \text{weight of fuel at hovering time \( h \), (lb)} \]
\[ W_g \quad \text{gas flow, (lb/sec)} \]
\[ w \quad \text{drag power of rotor, (ft-lb/sec)} \]
\[ X \quad \text{ratio of cross-sectional area of internal air passage to cross-sectional area of rotor blades, (0.75)} \]
\[ Z = \frac{\frac{\gamma}{\gamma-1}}{\left[1 + \frac{(\gamma-1) M^2}{2}\right]} \]
\[ \gamma \quad \text{ratio of specific heat at constant pressure to specific heat at constant volume} \]
\[ \rho \quad \text{air density, (slugs/cu ft)} \]

Subscripts:
- \( 0 \): equivalent free-stream condition
- \( 2 \): station 2, burner entrance
- \( 3 \): station 3, burner exit
- \( 4 \): station 4, exhaust-nozzle exit
- \( D \): rotor drag, based upon plan area
- \( D,e \): drag of jet-engine bodies
- \( e \): reference engine (except in \( W_e \))
- \( h \): hovering
- \( j \): jet engine
- \( L \): lift
- \( m \): maximum power
- \( R \): rotor
- \( re \): reciprocating engine
- \( t \): rotor tip
ASSUMPTIONS AND BASIS OF CALCULATIONS

The rotors powered by each of the jet engines were assumed to replace the conventional rotor and reciprocating engine in an existing helicopter. Sketches of the engine installations investigated are presented in figure 1. The methods of calculating fuel consumption and hovering time are outlined in appendices A to C.

The Merrot turbine utilized hollow rotor blades, whereas the ram jets and pulse jets were assumed to be installed tangentially on the tips of the blades. The same airframe weight and the same gross helicopter weight were used for each installation. The value of disposable load was different for various weights of engine, gearing, and rotor.

Some of the simplifying assumptions infer idealistic conditions that will not be realized in actual performance, but it is believed that these assumptions do not preclude making a fair comparison of engines.

Method of Computing Rotor Performance

The horsepower required to hover was taken as the sum of the calculated values of power dissipated in the momentum of the downwash from the rotor and in the windage drag of the rotor blades, with the assumption that there were no mechanical losses. The power dissipated in momentum of the downwash was multiplied by 1.15 to account empirically for additional minor losses. (See reference 2.)

Computation of the windage drag of the rotor included the drags of the nacelles housing the ram-jet and pulse-jet engines at the tips of the rotor blades. The value of 0.04 that was chosen for the drag coefficient of the engine nacelles was based upon the frontal areas; this value was taken from a figure derived from ballistic-drag data from reference 3 and from data on low-drag wing-nacelle combinations with internal air flow presented in reference 4. A blade-profile drag coefficient of 0.011 (reference 5) was used in computing the windage drag of the rotor blades.

The horsepower required for hovering operation of the helicopter when powered by either ram jets, pulse jets, or the reciprocating engine was calculated as

\[
(hp)_h = \frac{1.15 W^{3/2}}{550 \ (28\rho)^{1/2}} + B \rho V_t^3 C_D c r t + B \rho V_t^3 C_{D,\text{e}} A_r
\]  

(1)
where the first term is the corrected horsepower required to lift the helicopter, the second term is the horsepower required for rotor drag, and the third term is the horsepower for jet-engine bodies. The derivation of this equation is given in references 1 and 2. Because the reciprocating engine in the existing helicopter developed 40 horsepower more than the power required for hovering, the horsepower calculated in equation (1) was increased by 40 for the ram-jet and pulse-jet engines. Further details of the horsepower calculations are presented in appendix B.

The maximum allowable hovering times were calculated by assuming that the total disposable loads consisted only of fuel, and by taking into consideration the decreasing weight of the helicopter.

Propulsion Systems

Reciprocating engine. - The helicopter powered by a reciprocating engine selected for this analysis is described in reference 6.

Nernst turbine. - The power and fuel consumption of the Nernst turbine as a reaction motor were computed because no experimental data on this type of engine are available. These computations consisted of a simple cycle analysis accounting for momentum-pressure loss in the burners. The calculations are described in appendix A and reference 7.

The weight of the rotor was estimated by assuming the blades to be aluminum shells with a wall thickness of 0.0234 inch. The maximum cycle temperature was selected by trial calculation to give a low fuel consumption. The temperature calculations were made for a rotor, 30 feet in diameter, operating at a rotor-tip Mach number of 0.8; each blade was assumed to contain a burner with a cross-sectional area of 0.5 square foot. After the maximum cycle temperature was selected, the rotor diameter and burner cross-sectional area were so varied that the lowest fuel consumption was obtained at the power required for the desired lift.

Ram jet. - Thrust and fuel consumption of the ram-jet engines, one on each of the three rotor blades, were obtained from experimental data presented in reference 9. The thrust and fuel consumption of the reference engine are shown in figure 2.

Pulse jet. - The thrust and fuel-consumption characteristics of the pulse-jet engine presented in figure 3 were obtained from
experimental data presented in reference 9 and were used to determine thrust and fuel consumption of the three pulse-jet engines on the helicopter rotor blades.

The sizes of both the ram-jet and pulse-jet engines were so selected as to permit an excess of 40 horsepower above that required for hovering. Calculations were made to select the rotor diameter and the tip speed giving minimum fuel consumption. Equations used in these calculations are given in appendix B.

Table of assumptions. - In all of the cases except that of the Nernst turbine, two methods of analysis were employed. One method was the calculation of optimum rotor dimensions by varying the section lift coefficient; the other method was the selection of the rotor diameter and the rotor tip speed that would give low fuel consumption while the rotor-blade chord was held constant and equal to the equivalent chord of the existing helicopter powered by a reciprocating engine. For the Nernst turbine, the rotor dimensions were selected at the values that gave the desired thrust with a minimum fuel consumption. Because the optimum dimensions were structurally impractical, the pertinent data listed in the following table on the engine, rotor, and airframe of the helicopter as powered by a reciprocating engine are for constant-chord analysis only; the table also includes the assumptions necessary for computing the performance of a helicopter as powered by any one of the four propulsion systems:
<table>
<thead>
<tr>
<th>Propulsion system</th>
<th>Reciprocating</th>
<th>Nernst Turbine</th>
<th>Ram Jet</th>
<th>Pulse Jet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross weight of helicopter fully loaded, lb</td>
<td>2560</td>
<td>2560</td>
<td>2560</td>
<td>2560</td>
</tr>
<tr>
<td>Weight of airframe and pilot, lb</td>
<td>1336</td>
<td>1336</td>
<td>1336</td>
<td>1336</td>
</tr>
<tr>
<td>Rotor airfoil section, NACA</td>
<td>0012</td>
<td>0015</td>
<td>0012</td>
<td>0012</td>
</tr>
<tr>
<td>Number of rotor blades</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Diameter of rotor, ft</td>
<td>38</td>
<td>——</td>
<td>——</td>
<td>——</td>
</tr>
<tr>
<td>Total plan area of three rotor blades, sq ft</td>
<td>65.5</td>
<td>——</td>
<td>——</td>
<td>——</td>
</tr>
<tr>
<td>Weight of rotor, lb</td>
<td>159</td>
<td>——</td>
<td>(a)</td>
<td>(a)</td>
</tr>
<tr>
<td>Rotor tip speed, ft/sec</td>
<td>448</td>
<td>b0.8</td>
<td>——</td>
<td>——</td>
</tr>
<tr>
<td>Windage drag coefficient of rotor blade</td>
<td>0.011</td>
<td>——</td>
<td>0.011</td>
<td>0.011</td>
</tr>
<tr>
<td>Nacelle drag coefficient based on frontal area</td>
<td>——</td>
<td>——</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Weight of engine, lb</td>
<td>362</td>
<td>——</td>
<td>d225</td>
<td>d129</td>
</tr>
<tr>
<td>Weight of shafting and gearing, lb</td>
<td>50</td>
<td>——</td>
<td>——</td>
<td>——</td>
</tr>
<tr>
<td>Maximum engine power, bhp</td>
<td>192</td>
<td>——</td>
<td>——</td>
<td>——</td>
</tr>
<tr>
<td>Power required to hover, bhp</td>
<td>152</td>
<td>——</td>
<td>——</td>
<td>——</td>
</tr>
<tr>
<td>Brake specific fuel consumption, lb/bhp-hr</td>
<td>0.45</td>
<td>——</td>
<td>——</td>
<td>——</td>
</tr>
<tr>
<td>Percentage of airfoil cross-sectional area providing free passage for combustion air</td>
<td>——</td>
<td>75</td>
<td>——</td>
<td>——</td>
</tr>
<tr>
<td>Velocity of air entering burner, ft/sec</td>
<td>——</td>
<td>150</td>
<td>——</td>
<td>——</td>
</tr>
<tr>
<td>Ratio of friction-pressure loss in burner to dynamic pressure at entrance to burner</td>
<td>——</td>
<td>0.5</td>
<td>——</td>
<td>——</td>
</tr>
<tr>
<td>Temperature ratio</td>
<td>——</td>
<td>——</td>
<td>5</td>
<td>——</td>
</tr>
<tr>
<td>Combustion efficiency</td>
<td>——</td>
<td>——</td>
<td>1.00</td>
<td>0.70</td>
</tr>
</tbody>
</table>

*aWeight of rotor blades proportional to their plan areas, based upon weight of rotor with reciprocating engine.

*bMach number.

*cFor optimum rotor dimensions, value varied with $C_L$ (reference 5).

*dIn lb/sq ft of frontal area.

Equations used in the calculation of weights and pay loads are given in appendix C.
FUEL-CONSUMPTION ESTIMATES

Except for the Nernst turbine, two methods were used to determine rotor and engine dimensions to give minimum fuel consumption: (1) Optimum rotor dimensions were determined by varying the lift coefficient and rotor diameter, which resulted in a variation of rotor-blade chord; and (2) the rotor diameter was varied as the chord was held constant at the value of that for the existing helicopter powered by the reciprocating engine. The optimum values are given as a matter of interest but the difficulty of making slender rotor blades safe and rigid makes impractical the use of blades of optimum proportions.

The Nernst turbine rotor dimensions determined the value of thrust and fuel consumption obtainable so that the chord and the diameter were selected at the values that gave the desired thrust with minimum fuel consumption.

Reciprocating Engine

When the lift coefficient was varied, the optimum value was calculated to be 0.80, the same optimum value that was found for the ram jet and the pulse jet. This value was obtained for a rotor tip speed of 448 feet per second. The optimum rotor diameter was the largest considered, 60 feet, and the corresponding chord was 0.447 foot. The minimum fuel consumption was 0.0178 pound per hour per pound of rotor thrust for a hovering horsepower of 101.

When the existing helicopter rotor-blade chord and diameter were used at the assumed tip speed of 448 feet per second, the minimum fuel consumption was 0.0267 pound per hour per pound of rotor thrust for a hovering horsepower of 152, or one and one-half times the value for the optimum rotor.

Nernst Turbine

The results of the sample computations of the effects of change in maximum cycle temperature on the fuel consumption of the Nernst turbine are shown in figure 4. The minimum fuel consumption in this case is attained at temperatures between 2000° and 2200° R. A temperature of 2000° R was chosen for subsequent studies.

The effects of dimensions of the rotor blades on fuel consumption and thrust are shown in figure 5. Change in the diameter of the rotor had little effect on minimum fuel consumption provided
that the cross-sectional area of the burner was properly selected. The required lift of 2560 pounds was achieved with the lowest fuel consumption with a rotor diameter of 20 feet and a burner cross-sectional area of 0.288 square foot, corresponding to a chord of 1.93 feet. The calculated section lift coefficient for these dimensions was 0.139 and the fuel consumption was 0.376 pound per hour per pound of rotor thrust.

Ram Jet

The effects of changes in rotor diameter and in lift coefficient (which determine the blade chord) on the fuel consumption of a rotor powered by ram-jet engines are shown in figure 6. The optimum rotor tip speed was greater than the 900 feet per second chosen for this fuel-consumption analysis. No rotor tip speeds in excess of 900 feet per second were considered because stresses and aerodynamic losses become high. The lowest fuel consumption was obtained with a lift coefficient of 0.80 and the largest rotor diameter considered. The chords of the rotor blades for the lift coefficient of 0.80 are shown to vary from 0.67 foot with a rotor diameter of 10 feet to 0.11 foot at a diameter of 60 feet. The lowest fuel consumption was 0.27 pound per hour per pound of rotor thrust.

When the blade chord was held constant and equal to the equivalent chord of the blades with the reciprocating engine, the diameter of the rotor affected minimum fuel consumption as shown in figure 7. The minimum fuel consumption was 0.73 pound per hour per pound of rotor thrust, or nearly three times the value for the optimum rotor.

The sacrifice in fuel economy shown to result from the use of the more practical rotor dimensions indicates that the selection of dependable structural dimensions is also a critical factor in performance considerations.

Pulse Jet

Fuel consumption of the pulse jet was computed for various rotor tip speeds and section lift coefficients to permit selection of the lowest specific fuel consumption as shown in figure 8. The lowest fuel consumption was found to occur with the largest rotor diameter considered. The lowest specific fuel consumption was 0.17 pound per hour per pound of rotor thrust, and occurred at a tip speed of 500 feet per second and a section lift coefficient
The corresponding chord of the blades was 0.36 foot. The dimensions of the rotor for lowest fuel consumption result in very slender blades as in the case of the rotor powered by the ram-jet engines.

When the blade chord was held constant and equal to the equivalent chord of the blades used with the reciprocating engine, the diameter of the rotor affected minimum fuel consumption as shown in figure 9. Although the fuel consumption continued to decrease with increasing rotor diameter, a diameter of 38 feet was selected for the rotor blade used with the pulse-jet engine in order to avoid the unwieldiness and increased weight of larger rotor blades. Furthermore, the fuel consumption corresponding to a 38-foot diameter was not excessively high compared to the fuel consumption obtained with rotors of larger diameter. The fuel consumption in this case was 0.26 pound per hour per pound of rotor thrust or 1.5 times the value obtained with the optimum rotor.

### Hovering Time and Disposable Load

Because installations comprising jet engines were much lighter than the installation for the reciprocating engine, the jet-propelled helicopters could lift greater pay loads for shorter periods of hovering. The combination of pay load and fuel load was called disposable load.

### Optimum Rotor Dimensions

In the comparison made with the selection of the optimum blade chord and a large rotor diameter, the ram-jet and pulse-jet installations lifted disposable loads (pay loads at zero hovering time) 41 and 33 percent greater, respectively, than the disposable load of the reciprocating engine installation, whereas the Nernst turbine lifted 3 percent less. (See fig. 10(a) and table I for values.) As hovering time was increased, the high fuel consumption of the jet engines caused a rapid decrease in pay loads of the jet-propelled helicopters. For the limiting condition of no pay load (all fuel load), the maximum hovering times of the Nernst-turbine, ram-jet, and pulse-jet installations were 4.6, 10.3, and 14.8 percent, respectively, of that of the reciprocating-engine installation. For hovering times greater than 0.30 hour, the pulse-jet helicopter afforded greater pay load than the other jet engines and could hover with a greater pay load than the reciprocating engine for hovering periods less than 0.88 hour (fig. 10(a)).
Constant Rotor-Blade Chord

The hovering time and pay-load characteristics computed with the assumptions of blade chord equal to that of the original rotor used with the reciprocating engine and a maximum rotor diameter of 35 feet are shown in figure 10(b). (See table I for values.) Although the pay loads and ranges of each installation except the Nernst turbine were less than those presented in figure 10(a) for optimum rotor dimensions, the relative positions of the various engines remained the same except that the ram jet instead of the Nernst turbine gave the least favorable hovering performance. A comparison of the maximum disposable loads and hovering times is made in the following table:

<table>
<thead>
<tr>
<th>Engine</th>
<th>Maximum disposable load (percent)</th>
<th>Maximum hovering time (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reciprocating</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Nernst turbine</td>
<td>119</td>
<td>8.7</td>
</tr>
<tr>
<td>Ram jet</td>
<td>148</td>
<td>5.9</td>
</tr>
<tr>
<td>Pulse jet</td>
<td>143</td>
<td>15.8</td>
</tr>
</tbody>
</table>

For hovering times greater than the short period of 0.05 hour, the pulse-jet engine afforded greater pay load than the other jet engines and could hover with a greater pay load than could the reciprocating engine for hovering periods less than 0.58 hour. The selected dimensions of the rotor blade and the rotor tip speed for the pulse-jet driven rotor are closer to the values of extant rotors. The calculated performance could therefore be more easily achieved in practice with the pulse-jet engine than with the other jet engines considered in this analysis.

SUMMARY OF RESULTS

Comparison of the calculated performance of three jet-propelled helicopters with that of a conventional helicopter powered by a reciprocating engine led to the following observations:

1. The maximum hovering time, obtained with the pay load displaced by fuel, was much greater for the conventional helicopter powered by a reciprocating engine than for any of the jet-propelled helicopters. The helicopter powered by a pulse-jet engine with a
maximum hovering time equal to approximately 15 percent of the maximum hovering time of the helicopter powered by a reciprocating engine showed better hovering performance than the other jet-engine installations.

2. For short periods of hovering, the jet-propelled helicopters (except the Normst turbine in the optimum-dimension comparison) could carry greater pay loads than the helicopter powered by the reciprocating engine. The reciprocating-engine installation could carry greater pay loads than any of the jet-engine installations for hovering times greater than 0.88 hour.

3. Among the helicopters dimensioned by maintaining a constant chord, for hovering times greater than the short period of 0.05 hour the helicopter powered by a pulse-jet engine afforded greater pay load than helicopters powered by the other jet engines.

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Several variables were investigated in order to obtain optimum performance of the Nernst turbine for the thrust desired. A further development of the theory outlined in this appendix can be found in reference 7.

The velocity of the gas entering each burner (one in each of the three blades) was considered to be fixed at 150 feet per second. The temperature before the burner was calculated from the general energy equation

\[ t_2 = \left( \frac{v_2^2 - v_0^2}{2g\Delta c_p} \right) + t_0 \]  

and the following values were substituted: \( v_2 \), 150 feet per second; \( c_p \), 0.240 Btu/(lb-°F); and \( t_0 \), 519° R. When adiabatic compression was assumed, the pressure before the burner was

\[ p_2 = p_0 \left( \frac{t_2}{t_0} \right)^{\gamma-1} \]  

and the density before the burner was

\[ \rho_2 = \frac{p_2}{g\gamma t_2} \]

where \( p_0 \) was taken as 2117 pounds per square foot absolute.

The total temperature before the burner was calculated as

\[ T_2 = t_2 + \frac{v_2^2}{2g\Delta c_p} \]

and the weight of gas flow as
where the fuel-air ratio was determined from figure 6 of reference 10. This figure is a chart for determining the fuel-air ratio for various values of rise in total temperature across the combustion chamber and combustion-chamber-outlet total temperature.

The dynamic pressure at the entrance to the burner was

\[ q_2 = \frac{V_2^2 \rho_2}{2} \]  

(A6)

of which one-half was assumed to be the pressure loss due to friction.

\[ \Delta P_r = \frac{1}{2} q_2 \]  

(A7)

The Mach number of the air entering the burner was calculated as

\[ M_2 = \frac{V_2}{\sqrt{\gamma R T_2}} \]  

(A8)

and the total pressure at the burner exit was calculated from

\[ P_3 = (P_2 + \Delta P_r) \frac{Z_3}{Z_2} \]  

(A9)

where

\[ Z = \left[ 1 + \frac{(\gamma-1)}{2} M^2 \right]^{\frac{\gamma}{\gamma-1}} \left[ 1 + \gamma M^2 \right]^{-1} \]  

(A10)

The values of \( Z \) were obtained from equations that are given in reference 11 and have been prepared in chart form. A value of 1.4 was used for \( \gamma \) at station 2; the value of \( \gamma \) at station 3 was read from table 2 in reference 12 for the value of the total temperature after the burner.
The velocity at the nozzle exit was calculated as

\[ V_4 = \sqrt{2gJ_{0,P} P_3 \left[ 1 - \left( \frac{P_0}{P_3} \right)^{\gamma-1} \right]} \]  

(A11)

with the value of \( c_p \) read from table 2 in reference 12.

By measurement, the chord of the NACA 0015 airfoil assumed was found to be

\[ c = \sqrt{\frac{A_2}{0.1032 X}} \]  

(A12)

where \( X \), the ratio of the cross-sectional area of the air passage inside the blade to the total cross-sectional area of the blade, was assumed to be 0.75.

The equation used for the drag power of the rotor was

\[ \omega = \frac{B \rho \, C_D \, d \, c \, V_t^3}{16} \]  

(A13)

When it was assumed that the rotor inflow was uniform over the rotor-disk area and that there were no rotational losses for hovering, the rotor thrust was calculated to be

\[ F_R = \frac{\left\{ \frac{B \rho \, V_t}{g} \left( V_4 - V_t \right) - \omega \right\} \, d \, \sqrt{\frac{\rho \pi}{2}}} {1.15} \]  

(A14)

where the factor 1.15 was used to allow for a difference between calculated and actual experimental thrust.

The fuel flow was

\[ W_f = B \, V_{2g} \, \rho_2 \, A_2 \, f/a \]  

(A15)
The corrected thrust desired was 2560 pounds, the gross weight of the helicopter. A temperature study was made first in order to select a total temperature at station 3 that would give approximately the desired thrust. This study was made at rotor-tip Mach numbers of 0.5 and 0.8, but 0.8 was selected for further study because the specific fuel consumption was lower. As a result of the temperature study, it was decided to use 2000°R as the total temperature at the burner exit in the investigation of the other variables, the cross-sectional burner area and the rotor diameter. Cross-sectional burner areas of 0.10, 0.25, 0.50, 0.75, 1.00 and 1.50 square feet, and rotor diameters of 10, 20, 30, and 40 feet were investigated in order to obtain desired thrust at a low specific fuel consumption. Detailed results of these calculations are given in table II.
APPENDIX B

METHOD OF CALCULATING HORSEPOWER AND FUEL CONSUMPTION FOR HELICOPTERS POWERED BY RECIPROCATING, RAM-JET, OR PULSE-JET ENGINES

The horsepower required for lifting the helicopter was computed as

\[(hp)_L = \frac{W^{3/2}}{550(2Sp)^{1/2}} \quad (B1)\]

The horsepower required for the rotor drag was

\[(hp)_D = \frac{C_D cr_t B \rho V_t^3}{8 \times 550} \quad (B2)\]

The horsepower required for the drag of the jet-engine bodies was

\[(hp)_{D,e} = \frac{C_D e Ar B \rho V_t^3}{2 \times 550} \quad (B3)\]

See references 1 and 2 for derivations.

The total horsepower for the reciprocating engine was calculated by adding the values obtained from equations (B1) and (B2) and allowing for calculated lift and excess power as shown in the following equation:

\[(hp)_{re} = \left[ \frac{1.15 W^{3/2} + C_D cr_t B \rho V_t^3}{550(2Sp)^{1/2}} \right] + 40 \quad (B4)\]
For the reciprocating engine, the fuel consumption was assumed to be 0.45 pound per horsepower-hour for hovering horsepower only. Table III presents values of horsepower and fuel consumption with an assumed rotor tip speed of 448 feet per second.

The horsepower required for the helicopter to hover when powered by either the ram jets or the pulse jets was calculated as the sum of the values obtained from equations (B1), (B2), and (B3), allowing for the extra lift horsepower as shown in the following equation:

\[
(hp)_h = \frac{1.15 W^{3/2}}{550(2S\rho)^{1/2}} + \frac{B \rho V_t^3}{1100} \left( \frac{C_D c r_t}{d} + C_{D,0} A_r \right)
\]  

(B5)

The total maximum engine horsepower was

\[
(hp)_m = (hp)_h + 40
\]  

(B6)

The jet-engine thrust was

\[
F_j = 550 \frac{(hp)_h}{V_t}
\]  

(B7)

and the fuel consumption was considered to be proportional to thrust delivered

\[
f = F_j \frac{f_e}{F_e}
\]  

(B8)

The simultaneous equations involving \(A_r\) and \(F\) were solved by trial and error by first assuming values of \(A_r\) and solving for thrust, and then computing the required \(A_r\) by assuming that the cross-sectional-engine area was proportional to the total horsepower.

\[
A_r = \frac{550 A_r}{(hp)_m V_t F_e} \quad \text{(for reference engine)}
\]  

(B9)

After the correct \(A_r\) was determined, equations (B5) to (B8) were recalculated.
For the ram-jet and pulse-jet engines, the rotor diameters, the chord of the rotor blades, the cross-sectional-engine area, and the rotor tip speeds were investigated in order to obtain the lowest possible fuel consumption. For the ram jet, calculations were made for rotor diameters of 10, 20, 30, 40, 50, and 60 feet, and tip speeds of 400, 600, 750, and 900 feet per second. For the pulse jet, rotor diameters of 10, 20, 30, 38, 40, 50, and 60 feet were investigated, as well as tip speeds of 300, 400, 500, and 600 feet per second. The minimum fuel-consumption values at the various rotor diameters are presented in table III.
Most of the assumptions for calculating weights and pay loads of the different helicopters are given in ASSUMPTIONS AND BASIS OF CALCULATIONS.

The weight of the rotor for the reciprocating and pulse-jet engines was 159 pounds, as given in reference 6. This rotor weight was used as the basis for calculations of rotor weights for diameters other than the standard of 38 feet.

The airframe weight was determined from calculations for the reciprocating-engine helicopter by subtracting the sum of the weights of the engine, the rotor, the gearing, and the useful load from the gross helicopter weight. The same airframe weight and the same gross helicopter weight were used for each installation. The value of disposable load was obtained by subtracting from the gross helicopter weight the sum of the weights of the airframe, the rotor, the engine, and the gearing.

The calculated values of weights and disposable loads for the four helicopters are given in table I.

The following equations were developed to determine the allowable hovering times of the different engines, taking into consideration the fact that the weight of the helicopter was steadily decreasing because of fuel consumption.

\[ \frac{dW_f}{dh} = K(W_e + W_f, h) \]  
(01)

Transposing and integrating between the limits of 0 and h results in

\[ - \log_e(W_e) + \log_e(W_e + W_f, l) = Kh \]  
(02)

Transposing again gives

\[ h = \frac{1}{K} \log_e \left( \frac{W_e + W_f, l}{W_e} \right) \]  
(03)
or

\[ h = \left( \frac{W_0 + W_f}{W_f} \right) \log_e \left( \frac{W_0 + W_f}{W_0} \right) \]  

(C4)

REFERENCES


### TABLE I - SELECTED PROPORTIONS FOR HELICOPTERS

POWERED BY VARIOUS ENGINES

<table>
<thead>
<tr>
<th>Proportions</th>
<th>Reciprocating</th>
<th>Nernst turbine</th>
<th>Ram jet</th>
<th>Pulse jet</th>
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<td>20</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Rotor tip speed, ft/sec</td>
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<td>896</td>
<td>900</td>
<td>500</td>
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<td>Specific fuel consumption, 1b/(hr)(lb rotor thrust)</td>
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<td>1336</td>
<td>1336</td>
<td>1336</td>
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### Constant rotor-blade chord

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<th>Ram jet</th>
<th>Pulse jet</th>
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<td>Rotor tip speed, ft/sec</td>
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<td>896</td>
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### Table II - Summary of Data for Turbine Showing Variation in Thrust and in Fuel Consumption with Change in Rotor Diameter and Cross-Sectional Burner Area

- Mass number 0.8; $T_p$, 5860 ft; $T_s$, 20000 ft; $V_e$, 150 ft/sec

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<thead>
<tr>
<th>Rotor Diameter (ft)</th>
<th>Cross-sectional Burner Area (sq ft)</th>
<th>Rotor-blade chord (ft)</th>
<th>Drag Power (ft-lb/sec)</th>
<th>Corrected Rotor Thrust (lb)</th>
<th>Fuel Consumption (lb/hr)</th>
<th>Corrected Rotor Thrust (lb/hr)</th>
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TABLE III - SUMMARY OF DATA FOR RECIPROCATING. RAM-JET, AND PULSE-JET ENGINES SHOWING 
VARIATION OF ROTOR DIAMETER AND ROTOR-TIP SPEED

| Engine | Rotor diameter | Effective chord | Rotor tip speed | Required cross-sectional area of one engine | Calculated lift horsepower | Drag horsepower required for hovering | Total horsepower available | Total horse- | Fuel consumption | Fuel consumption 
<table>
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<tr>
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<th></th>
<th></th>
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<th></th>
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<tbody>
<tr>
<td></td>
<td>(ft)</td>
<td>(ft)</td>
<td>(ft/sec)</td>
<td>(sq ft)</td>
<td>(hp)</td>
<td>(hp)</td>
<td>(hp)</td>
<td>power</td>
<td>(lb/hr)</td>
<td>Rotor thrust</td>
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<tr>
<td>Reciprocating</td>
<td>60</td>
<td>0.447</td>
<td>448</td>
<td>---</td>
<td>64.21 x 1.16</td>
<td>27.56</td>
<td>101.4</td>
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<td>1054</td>
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<td>Constant rotor-blade chord</td>
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<td>448</td>
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<td>191.6</td>
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Figure 1. Diagram of helicopter rotors powered by various engines:

(a) Rotor with reciprocating engine.
(b) Rotor with Nernst turbine.

Figure 1. - Continued. Diagram of helicopter rotors powered by various engines.
Figure 1. - Continued. Diagram of helicopter rotors powered by various engines.

(c) Rotor with ram jet.
Section A-A

Id) Rotor with pulse jet.

Figure 1. - Concluded. Diagram of helicopter rotors powered by various engines.
Figure 2. Variation of thrust and fuel consumption with Mach number for 20-inch ram jet. (Curves taken from figs. 14(b) and 23(b) of reference 8.)
Figure 3. - Variation of thrust and thrust specific fuel consumption with airspeed for reference pulse-jet engine. (Data taken from reference 9.)
Figure 4. Effect of maximum cycle temperature on fuel consumption for Nernst turbine. Mach number, 0.8; rotor diameter, 30 feet; cross-sectional burner area, 0.5 square foot.
Figure 5. - Variation of fuel consumption and thrust with cross-sectional burner area and rotor diameter for Nernst turbine. Mach number, 0.8; maximum cycle temperature, 2000° R.
Figure 6. Variation of fuel consumption with rotor dimensions for ram-jet engine. Rotor tip speed, 900 feet per second.
Figure 7. - Variation of minimum fuel consumption with rotor diameter for ram-jet engine. Rotor-blade chord, 1.15 feet; rotor tip speed, 900 feet per second.
Figure 8. - Variation of fuel consumption with rotor dimensions and rotor-tip speed for pulse-jet engine. Rotor diameter, 60 feet.

Figure 9. - Variation of minimum fuel consumption with rotor diameter for pulse-jet engine. Rotor-blade chord, 1.15 feet.
Figure 10. - Variation of pay load with hovering time for helicopters powered by various engines.
Figure 10. - Concluded. Variation of pay load with hovering time for helicopters powered by various engines.

(u) Constant rotor-blade chord.