Preliminary Estimate of Performance of a Turbojet Engine When Inlet Pressure Is Reduced Below Exhaust Pressure

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Preliminary investigations of a turbojet engine were conducted with compressor-inlet total pressure at various values below that of the exhaust pressure to determine engine performance under conditions simulating operation with inlet-duct losses. The data are also applicable to the problems of thrust control and of use of the turbojet engine as a pump for removal of wing and fuselage boundary-layer air. Because the experimental range was extremely limited, the curves were extended by the use of equilibrium-operation-performance equations and the engine-component characteristics determined from the experimental investigations.

With decreasing inlet-to-exhaust pressure ratios, the static thrust and the air consumption of the engine rapidly decreased and operating temperatures and specific fuel consumption sharply increased. Calculations of the take-off-run requirements for a pursuit-type jet-propelled airplane indicated that an improvement in ducting that would increase the inlet-to-exhaust pressure ratio from 0.94 to 0.98 would allow approximately 14-percent decrease in the take-off distance.

Altitude operating conditions were more favorable than sea-level conditions to engine operation with reduced inlet-to-exhaust pressure ratio; therefore, use of the turbojet engine as a pump for boundary-layer removal appears to be more practical for altitude operation. Throttling the air supply was unsatisfactory as a primary means of thrust control at sea level because the present maximum allowable turbine-inlet temperature of 2000° R was reached by throttling to an inlet-to-exhaust pressure ratio of 0.89, which allowed only a 20-percent decrease in thrust. A variable-area jet nozzle was found more satisfactory than inlet throttling as a primary means of thrust control. A combination of a variable-area jet nozzle and inlet throttling was found to allow approximately 63-percent reduction in
static sea-level thrust. Extension of the data to altitude condi-
tions showed that because of the lower operating temperatures
throttling to a much lower inlet-to-exhaust pressure ratio than at
sea level was possible.

INTRODUCTION

The critical material temperatures of turbojet engines impose
a limit of low energy addition per pound of air consumed as compared
with that of conventional reciprocating engines. Because of this
low energy level, losses in the fluid-flow systems are magnified in
terms of reduced engine performance. It is therefore important
that the air-induction system of a turbojet-engine installation be
highly efficient if maximum airplane and engine performance are to
be realized. Because the turbojet engine requires from seven to
ten times as much air as a reciprocating engine of similar rating,
the difficulties involved in providing high-efficiency inlet ducting
are greatly increased. Various investigations of duct and inlet-
duct design (references 1 to 5) have been made. Because the size
and the arrangement of inlet ducting often result in a compromise
with other design factors, the effect of inlet-duct losses on
performance of the turbojet engine must be determined in order to
evaluate the importance of inlet-duct design to over-all airplane
performance.

An investigation of the effects of inlet losses on the perform-
ance of turbojet engines is being conducted at the NACA Cleveland
laboratory. The preliminary investigation reported herein was limited
to static tests with atmospheric exhaust pressure and with reduced
compressor-inlet pressures for two jet-nozzle diameters. The engine
was mounted in a static test cell in which pressure could be reduced
during engine operation by throttling the air inlet to the cell.
Strength of the cell structure, and in some cases the allowable tail-
pipe gas temperature, imposed limitations that resulted in a narrow
range of obtainable inlet pressures. The data were therefore
extrapolated using the performance of individual components determined
from the engine investigations. The results obtained are applied
to demonstrate the effect of change in inlet-duct losses on the
length of airplane take-off run.

The engine performance results, in terms of ratio of compressor-
inlet total pressure to jet-nozzle exhaust pressure, are also
applicable to the determination of (1) the performance and the
limitations of the turbojet engine as a pump for airplane boundary-
layer control, and (2) the possibilities of inlet-duct throttling
as a means of thrust control.
APPARATUS

The investigations were made in a static test cell using an I-16-6 turbojet engine with the usual 12.40-inch-diameter (0.838 sq ft) jet nozzle for the first series of runs and a nozzle increased to the tail-pipe diameter of 14.25 inches (1.107 sq ft) for the second series. The engine combustion air entered the air-tight cell through a standard A.S.M.E. air-measuring nozzle, as diagramatically shown in figure 1. Reduction in compressor-inlet pressure was obtained by attaching various orifices to the outlet of the diffuser, which reduced the cell pressure during engine operation. The construction of the cell allowed a pressure reduction of only 40 inches of water.

Engine thrust was measured by means of a calibrated strain-gage thrust meter. The thrust meter was attached through linkage to the frame supporting the engine. The thrust-meter calibration included determinations over a range of cell pressures; for the calibration, cell pressure was reduced by means of a positive-displacement blower. The calibration then automatically corrected for the differential pressure across the tail-pipe seal diaphragm including the tail-pipe cross-sectional area. The positive-displacement blower, together with an exhaust pipe and a standard A.S.M.E. orifice, was also used to determine cell leakage. Leakage was found to be negligible.

The location of engine instrumentation is shown in figure 2. Temperature and total pressure at the compressor inlet (station 1) were obtained from averaging the readings of ten thermocouples and of ten total-head tubes; five were equally spaced circumferentially on the front and five on the rear compressor-inlet screen. Rakes containing four total-head tubes and two thermocouples were installed in three compressor-exhaust elbows (station 2). On each of these elbows, four wall static-pressure taps were located circumferentially. Because of the unreliability of instrumentation in the burner outlet, no instrumentation was installed at the turbine inlet (station 3). The total pressure at station 3 was assumed to be 95 percent of the total pressure at station 2, and the total temperature was calculated by equating turbine and compressor work. Instrumentation of the tail pipe (station 4) consisted of four static-pressure taps and four thermocouples. Barometric pressure was used as the jet-exhaust pressure.

PRESENTATION OF DATA

Data were corrected to a jet-nozzle exhaust pressure of 14.7 pounds per square inch absolute and a compressor-inlet
temperature of 519° R by the conventional correction methods (reference 6). The results were plotted as a series of curves showing variations in thrust, air consumption, fuel consumption, tail-pipe total temperature, and compressor-inlet total pressure with change in engine speed. A set of performance curves was obtained for each orifice used to throttle the combustion-air supply. Cross plots of these curves were then made to obtain charts showing engine performance at reduced compressor-inlet pressures with inlet-to-exhaust pressure ratio as the independent variable. For simplicity, inlet-to-exhaust pressure ratio will hereinafter be referred to as "inlet pressure ratio". The symbols used in the figures and in the derivation of the equations are defined in appendix A.

Because the range of data was so greatly restricted, the performance charts were extended by the use of equations involving the component characteristics and efficiencies. These equations together with their derivations are included as appendix B. The data obtained were too meager to indicate the change in compressor performance with decrease in compressor-inlet pressure; compressor temperature rise and pressure ratio were therefore assumed to vary only with compressor rotor speed. Pressure drop through the burner was assumed to be 5 percent of the total pressure at the inlet to the burner. Burner efficiency obtained from the data was plotted against gas temperature rise across the burner for increments of burner-inlet total pressure. The burner performance was then extrapolated by use of data obtained by the General Electric Company. Turbine efficiency, jet-nozzle efficiency, over-all expansion efficiency, turbine-nozzle flow coefficient, and jet-nozzle flow coefficient were determined from plots against the pressure ratio taken across the particular component being considered.

In the determination of the component characteristics from the experimental data, constant average values of specific heat and specific-heat ratio were used for each change of state. The same values were used in the calculation of engine performance.

These simplifications result in negligible errors for small reductions in compressor-inlet pressure but are of increasing importance for larger reductions. The results are considered sufficiently accurate, however, to indicate general trends and to permit approximation of limits of engine operation.
RESULTS AND DISCUSSION

General Performance

The cross plots of experimental engine data, corrected gross thrust $F_g/\theta$ against inlet pressure ratio $P_1/P_2$ are presented in figure 3 as dashed curves for the engine with the 12.40-inch-diameter jet nozzle; the solid curves are the calculated data. These curves show a rapidly decreasing thrust with reduced inlet pressure ratio. At maximum engine speed, 16,500 rpm, reducing the inlet pressure ratio from 1.0 to 0.9 reduced gross thrust from 1770 to 1455 pounds, an 18-percent reduction in thrust for a 10-percent reduction in inlet pressure ratio. This percentage reduction in thrust appears to be constant for all engine speeds $N$ over this range of inlet pressure ratio. At the lower values of inlet pressure ratio, the calculated values of thrust decrease at a somewhat lower rate. In actual operation, the thrust would probably decrease at a rate slightly greater than that calculated because of the decrease in compressor efficiency and the increase in burner-momentum pressure loss over that used in the calculations.

The variation in corrected air consumption $W_a\sqrt{\theta}/\sigma$ with inlet pressure ratio is shown in figure 4. The air consumption decreases at an increasing rate at the lower values of inlet pressure ratio and at the lower speeds. Reduction of inlet pressure ratio from 1.0 to 0.9 is accompanied by a decrease in air consumption of 10 percent at an engine speed of 16,500 rpm, of 17 percent at 12,000 rpm, and of 20 percent at 10,000 rpm. Further reductions in inlet pressure ratio result in a more rapid decrease in air consumption.

The increase in corrected specific fuel consumption $W_f/F_g\sqrt{\theta}$ (lb fuel/(hr)(lb thrust)) with a decrease in inlet pressure ratio is shown in figure 5. The specific fuel consumption is seen to increase more rapidly at the lower inlet pressure ratios and at the lower engine speeds. At 16,500 rpm, a change in inlet pressure ratio from 1.0 to 0.9 results in approximately a 13-percent increase in specific fuel consumption.

The tail-pipe $T_5/\theta$ and the turbine-inlet $T_3/\theta$ total temperatures are shown in figures 6 and 7, respectively. These rates of temperature rise are seen to increase rapidly as the inlet pressure ratio is decreased and are extreme at low engine speeds. If the assumption is made that neither burner blow-out nor compressor surge is encountered, turbine-inlet total temperature is the factor that limits engine performance if a reasonable service life is to be expected. Figure 7 shows the limits of inlet-duct losses allowable
in the engine investigated if any particular turbine-inlet-temperature limit is not to be exceeded. If the current limit of approximately 2000° R is not to be exceeded, the minimum inlet pressure ratio obtainable is approximately 0.83 at an engine speed of 16,500 rpm and is about 0.80 at 13,000 to 15,000 rpm. If the temperature limit were raised to 2300° R, the higher engine speeds would then permit the greatest reduction in compressor-inlet pressure without exceeding the new limit. The turbine-inlet total temperatures indicated by figure 7 have been cross-plotted on the curves of corrected thrust, air consumption, and specific fuel consumption shown in figures 8, 9, and 10, respectively.

The sea-level data of figures 8, 9, and 10 have been extrapolated by conventional methods (see appendix B) to static operation at an altitude of 30,000 feet in figures 11, 12, and 13, respectively. These figures show the trends toward reduced thrust and air consumption and increased specific fuel consumption at altitude. Primarily it is shown, however, that at altitude, much lower inlet pressure ratios are possible without exceeding the maximum allowable turbine-inlet temperature. This predicted reduction exists because as the compressor-inlet temperature decreases with altitude, the temperatures throughout the operating cycle are proportionally decreased. The allowable inlet-pressure-ratio range is extended from a minimum of 0.89 at sea level to approximately 0.69 at an altitude of 30,000 feet.

Although the thrust of figure 11 and the specific fuel consumption of figure 13 are for static operation (zero flight velocity), they may easily be extended to any desired flight velocity by correcting the thrust for engine inlet-air approach velocity.

The calculated performance of the engine using a 14.25-inch-diameter jet nozzle is shown in figures 14, 15, and 16. A comparison of figures 8 and 14 show a substantial reduction in thrust with the larger jet nozzle. At 16,500 rpm and an inlet pressure ratio of 1.0, opening the jet nozzle from 12.40 to the full tail-pipe diameter of 14.25 inches results in a reduction in static thrust from approximately 1770 to 1120 pounds or approximately 37 percent. Also much larger reductions in inlet pressure ratio can be allowed without exceeding a turbine-inlet total temperature of 2000° R. The decrease in thrust with decrease in inlet pressure ratio, as with the 12.40-inch-diameter jet nozzle, is very rapid.

The effect of increased jet-nozzle area on air consumption is shown, by a comparison of figures 9 and 15, to be negligible. This phenomenon results from the restriction of air flow by critical flow in the turbine nozzle over most of the engine operating range. The
specific fuel consumption is shown by figures 10 and 16 to have increased appreciably with the larger jet-exhaust nozzle and is seen to have increased at the maximum engine speed and inlet pressure ratio of 1 from 1.15 to 1.42 pounds of fuel per hour per pound of thrust, an increase of 23.5 percent.

Application to Inlet-Duct Losses

At maximum engine speed and at higher inlet pressure ratios, a 1.8-percent loss in thrust existed for each 1.0-percent decrease in inlet pressure ratio (fig. 8). This reduction in thrust with decreasing inlet pressure ratio is of considerable importance during the critical period of the take-off run, which for existing jet-propelled airplanes is much greater than desirable.

In order to permit evaluation of an improvement in inlet ducting for a typical engine installation, it was assumed that the characteristics presented in figures 8, 9, and 10 applied and that the ducting used gave an effective inlet pressure ratio of 0.94. The improvement in airplane performance that could be obtained by an assumed improvement in inlet ducting, which would give an inlet pressure ratio of 0.98, was then calculated by the method of appendix C. The calculations indicated that the take-off distance could be reduced approximately 14 percent, from 3050 feet to 2616 feet.

The sea-level operation of the engine with an enlarged jet nozzle will allow considerably greater inlet-duct losses without exceeding limiting temperatures but at a sacrifice in thrust (figs. 8 and 14) and specific fuel-consumption (figs. 10 and 16).

The effect of inlet-duct losses on thrust at altitude is shown by a comparison of figures 8 and 11. The decreased compressor-inlet temperature resulting in decreased operating temperatures extends the allowable operation to inlet-pressure ratios as low as 0.69 at the maximum engine speed. The trend of rapidly decreasing thrust with decreasing inlet pressure ratio at sea level is also shown for the altitude condition. The data of figure 11 can be corrected to reasonable flight velocities by correcting the thrust for inlet-air approach momentum. Although the thrust decreases with increasing flight speed, at any particular flight speed the performance trends with decreasing inlet pressure ratio are generally the same as those shown in figure 11.

Similarly, the trends of increasing specific fuel consumption with decreasing inlet pressure ratio, such as shown in figure 13, will exist for any particular conditions of altitude and flight speed.
Because this investigation was limited to consideration of cases in which inlet total pressure is less than the exhaust-static pressure, the engine data are not directly applicable to flight conditions that result in positive ram at the compressor inlet. The general trends of rapidly decreasing thrust, higher operating temperatures, and increased specific fuel consumption with decreasing inlet pressure ratios show, however, the need for highly efficient inlet-duct systems in turbojet-engine installations.

**Application to Airplane Boundary-Layer Control**

Removal of boundary-layer air from wings and fuselage promises considerable improvement in airplane performance in the form of increased range or increased pay load, (references 7 and 8). The use of the turbojet engine as a pump to remove this air in order to use it for propulsion has frequently been suggested. The considerations involved in evaluating this type of installation are: (1) the possibility of the turbojet engine accomplishing the pressure rise required for pumping the boundary-layer air from its low pressure on wing and fuselage surfaces through the necessary ducting to the higher atmospheric pressure at the engine jet-nozzle exhaust; and (2) the possibility of decreased airplane drag more than compensating for the loss in engine thrust and increased engine specific fuel consumption.

The sea-level data of figures 8, 9, and 10 indicate a narrow range of inlet pressure ratio at maximum engine speed as limited by the turbine-inlet total temperature, a rapid decrease in thrust and air-handling capacity, and a rapid increase in specific fuel consumption with decreasing inlet pressure ratio, respectively. Use of an enlarged jet-nozzle area (figs. 14 to 16) limited by a turbine-inlet total temperature of 2000° R permits a much wider range of inlet pressure but at a sacrifice in engine thrust and specific fuel consumption. The air-handling capacity of the engine (fig. 15) is not noticeably affected by a change in jet-nozzle area. At least in this respect, the variable-area jet nozzle as a means of increasing the allowable pumping range of the engine would have considerable advantage over decreasing engine speed to increase the range of pumping-pressure control.

The altitude operating conditions are more favorable to engine operation with reduced inlet pressure ratios as shown by a comparison of the allowable temperature limits of figures 8 and 11. For this reason, use of the turbojet engine as a pump for boundary-layer-air removal appears to be more practical for altitude operation than for take-off and flight at or near sea level.
Although a more complete study is required, as outlined in aforementioned considerations (1) and (2), in order to determine definitely the feasibility of adopting the turbojet engine as a pump for boundary-layer removal, the engine characteristics of rapidly decreasing thrust, increasing specific fuel consumption, and increasing operating temperatures with decreasing inlet pressure ratios indicate the need for a more satisfactory method of removing boundary-layer air.

Application to Turbojet-Engine Thrust Control

Use of the turbojet engine with the characteristically slow development of full thrust has greatly increased the hazards of certain flight operations and maneuvers, such as aircraft-carrier landings. In the event of a "wave-off" during such a landing, the engine must recover thrust rapidly enough to allow the airplane to clear the barrier and any aircraft parked on the flight deck.

The slow recovery of thrust results from the high inertia forces of the compressor and the turbine, which limit the rate at which the engine can be accelerated. Furthermore, an attempt to rapidly accelerate the engine to higher speeds will result in extremely high operating temperatures. Apparently, therefore rapid thrust response should be obtained by eliminating the need for rapid recovery of engine speed through reducing thrust by some other means such as throttling the inlet air.

At maximum engine speed, throttling to the maximum allowable turbine-inlet total temperature of 2000° R (inlet pressure ratio, 0.89) reduces sea-level static thrust from 1770 to about 1430 pounds (fig. 8). At a landing approach speed of 100 miles per hour, the thrust, corrected for inlet-air approach momentum, would be approximately 1623 and 1300 pounds, respectively, a reduction of approximately 20 percent. A reduction of thrust much greater than that obtainable by throttling is required and it is necessary to use other means, or a combination of throttling and other means, for controlling thrust at or near sea-level conditions.

This greater reduction can be accomplished in part by a variable-area jet nozzle. At maximum engine speed, as previously noted, approximately 37-percent reduction in thrust is obtained by opening the jet from a diameter of 12.40 to 14.25 inches. By combination of the variable jet nozzle and inlet throttling, and by the assumption that the operation is limited only by turbine-inlet temperature, a reduction in static thrust to 660 pounds or 63 percent can be realized. A further reduction in thrust is possible by
use of an even larger jet nozzle. This use can be accomplished by having a larger tail pipe on the engine with a variable-area jet nozzle having a greater variable-area ratio, or by using a variable-jet nozzle that will allow opening the jet to a diffusing section.

Although an inlet throttle can be used under sea-level operating conditions as a supplement to other means of thrust control (such as variable-area jet nozzles), if the complications of a multiple-control system are warranted, inlet throttling alone is an unsatisfactory method of thrust control when large reductions in thrust are required for maneuvers such as aircraft-carrier landings.

Thrust control at altitude while maintaining maximum engine speed may be of importance for maneuvers such as combat-area patrol in which instantaneous recovery of thrust may be required in the event of attack by enemy planes. Throttling as a means of rapid thrust control at altitude has greater potentialities than at sea level because, as previously discussed, the lower compressor-inlet-air temperatures result in lower engine operating temperatures. A reduction of inlet pressure ratio to 0.69 is shown to be possible (fig. 11). This decrease represents a reduction in static thrust from 900 to 425 pounds, or 53 percent. As shown by figure 13, the trends of increasing engine specific fuel consumption and temperature with throttling also exist at altitude. The increasing specific fuel consumption and operating temperatures must be considered in evaluating the throttling of inlet air as a means of thrust control of a turbojet engine.

SUMMARY OF RESULTS

Preliminary investigations and calculated performance of a turbojet engine with a nominal thrust of 1600 pounds indicate that:

1. Inlet-duct losses, resulting in compressor-inlet pressures below that of the atmosphere into which the jet-nozzle discharged, resulted in marked decreases in static thrust and in air consumption and sharp increases in operating temperatures and in specific fuel consumption.

2. Comparatively small reductions in the ratio of compressor-inlet total pressure to exhaust static pressure at sea-level operating conditions and at high engine speeds resulted in exceeding the present allowable turbine-inlet total temperature of 2000° R. Enlargement of the jet-nozzle allowed appreciable reductions of inlet pressure without exceeding allowable operating temperatures but at a great sacrifice in thrust and in increased specific fuel consumption.
3. An improvement in inlet ducting that would increase the inlet pressure ratio from 0.94 to 0.98 was shown by calculations to allow a pursuit-type jet-propelled airplane to operate with approximately 14 percent decrease in take-off distance.

4. Use of the turbojet engine as a pump to remove boundary-layer air from wing and fuselage surfaces of an airplane would result in decreased engine thrust, increased engine specific fuel consumption, and increased engine operating temperatures. Altitude operating conditions were more favorable than sea-level conditions to engine operation with reduced inlet pressure; therefore, use of the turbojet engine as a pump for boundary-layer removal appeared to be more practical for altitude operation.

5. For flight operations at sea level such as aircraft-carrier landing approach, throttling the air supply to the engine was unsatisfactory as a primary means of thrust control because of the small range of control obtainable. At the limiting turbine-inlet temperature of 2000°F, at maximum engine speed, and an inlet-to-exhaust pressure ratio of 0.89, the thrust reduction obtainable was calculated as approximately 20 percent of the unthrottled engine thrust. A variable-area jet nozzle at maximum speed and without throttling will allow thrust reductions of approximately 37 percent. With increased jet-nozzle area, increased throttling was permitted; combination of these two methods of thrust control may allow thrust reduction of approximately 63 percent. At altitude, greater thrust reductions were obtainable by throttling because reduced inlet combustion-air temperatures resulted in generally lowered operating temperatures, which in turn allowed operation at lower inlet-to-exhaust pressure ratios.

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APPENDIX A

SYMBOLS

The following symbols are used in presentation of the experimental and calculated data and in the development of equations:

- **A** area normal to direction of flow, sq ft
- **C** nozzle area coefficient
- **C_{v}** nozzle velocity coefficient
- **c_{p}** specific heat at constant pressure, Btu/(lb)(°F)
- **F_{g}** gross thrust, lb
- **F_{n}** net thrust, lb
- **E** energy absorbed by auxiliary case and bearing friction, Btu/(lb fluid consumed by engine)
- **g** acceleration of gravity, 32.2 ft/sec^2
- **H** total (stagnation) enthalpy, Btu/lb
- **h** static enthalpy, Btu/lb
- **J** Joule's constant, 778 ft-lb/Btu
- **N** engine speed, rpm
- **P** total (stagnation) pressure, lb/sq in. absolute
- **p** static pressure, lb/sq in. absolute
- **Q** lower heating value of fuel, Btu/lb
- **R** gas constant, ft-lb/(lb)(°F)
- **R_{m}** effective ground resistance, lb
- **S** distance, ft
- **T** total (stagnation) temperature, °R
- **t** static temperature, °R
- **V** velocity, ft/sec
- **V_{s}** sonic velocity, ft/sec
\( W_t \) airplane take-off velocity, ft/sec
\( W_e \) engine air consumption, lb/sec
\( W_f \) engine fuel consumption, lb/sec
\( W_g \) \( W_f + W_e \), lb/sec
\( W_p \) airplane gross weight, lb
\( \gamma \) specific heat ratio
\( \eta_b \) burner efficiency
\( \eta_e \) expansion efficiency, efficiency of total expansion process (turbine expansion plus exhaust-jet-nozzle expansion)
\( \eta_t \) turbine efficiency
\( \rho \) fluid density, lb/ft³

A primo indicates isentropic or ideal process.

Subscripts:
0 free-stream conditions
1 compressor inlet
2 compressor outlet
3 turbine inlet
4 tail pipe
5 jet-nozzle inlet
6 jet-nozzle outlet
7 ambient conditions at jet-nozzle outlet

The following conventional factors used to correct engine data to standard sea-level conditions also provide a means of estimating altitude performance from sea-level data:

\( p_7 \) Jet-nozzle exhaust static pressure, lb/sq in. absolute
\( p_0 \) Pressure of NACA standard atmosphere at sea level, lb/sq in. absolute

\( T_1 \) Compressor-inlet total temperature, °R
\( T_0 \) Temperature of NACA standard atmosphere at sea-level, °R
APPENDIX B

GENERALIZED PERFORMANCE EQUATIONS

The following equations are similar to equations previously developed by the General Electric Company. Because turbojet-engine equilibrium-operation-performance equations are not generally available in the literature, the following were developed:

In order to determine the performance of a simple turbojet engine, it is necessary to establish the equilibrium operating temperature at the turbine inlet $T_3$ for each particular operating condition of interest. Because compressor performance cannot be accurately calculated, the compressor characteristics are assumed to be known or to have been estimated from the performance of a similar unit. Sonic velocity is also assumed to exist at the turbine-nozzle throat at all times. For the rare operating conditions where this condition was not true, the turbine nozzle area coefficient is so defined as to correct for this condition.

**Equation for equilibrium engine operation.** - From a component energy balance, it can be shown that:

\[
\text{Turbine enthalpy change} = \text{compressor enthalpy change} + \text{energy absorbed by auxiliary case and bearing friction} \tag{1}
\]

Also

\[
\text{Turbine enthalpy change} = \text{total expansion enthalpy change} - \text{jet-nozzle expansion enthalpy change} \tag{2}
\]

By equating the left-hand terms of equations (1) and (2),

\[
W_a(H_2 - H_1) + E = (W_a + W_r) \left[ (H_3 - h_6) - (H_4 - h_6) \right] \tag{3}
\]

in which the enthalpy at stations 1 to 4 is total (stagnation) enthalpy and the jet-nozzle static enthalpy $h_6$ is based on the
static temperature assuming total expansion to ambient-air pressure across the jet nozzle. From the known or assumed compressor performance characteristics and inlet-air conditions, $H_2 - H_1$ is known. The energy absorbed by the auxiliary case plus the bearing friction $E$ is generally small and frequently can be neglected. When the expansion efficiency is defined as

$$\eta_e = \frac{H_3 - h_6}{H_3 - h_6'}$$

$$H_3 - h_6 = \eta_e(H_3 - h_6')$$

$$= \eta_e \rho_3 A_3 v_3$$

It is now necessary only to evaluate the term of equation (3) containing $H_4$.  

$$v_6 = \sqrt{\frac{2gI}{\gamma} (H_4 - h_6)}$$

$$v_3 = \sqrt{\frac{2gI}{\gamma} (H_3 - h_3)}$$

$$w_g = \rho_6 A_6 v_6$$

$$\rho_3 A_3 v_3$$

Sonic velocity is assumed at the turbine-nozzle throat; therefore

$$\frac{P_3}{P_3} = \left(\frac{2}{\gamma_3 + 1}\right)$$

From the perfect gas law

$$P = \rho RT$$

The adiabatic process relation is

$$\frac{T}{t} = \left(\frac{P}{P}\right)^{\frac{\gamma - 1}{\gamma}}$$
From equation (6)
\[ H_4 - h_6 = \frac{v_6^2}{2\gamma_j} \]  
(12)

From equations (8) and (12)
\[ H_4 - h_6 = \left(\frac{c_3}{c_6}\right)^2 \left(\frac{A_3}{A_6}\right)^2 \left(\frac{\rho_3}{\rho_6}\right)^2 \left(\frac{v_3^2}{2\gamma_j}\right) \]  
(13)

By substituting the value of \( v_3 \) from equation (7)
\[ H_4 - h_6 = \left(\frac{c_3}{c_6}\right)^2 \left(\frac{A_3}{A_6}\right)^2 \left(\frac{\rho_3}{\rho_6}\right)^2 (R_j - h_3) \]  
(14)

From equations (9) and (10)
\[ \rho_3 = \frac{p_3}{Rt_3} = \frac{p_3 \left(\frac{2}{\gamma_j} + 1\right)}{Rt_3} \]  
(15)

From equations (9) and (11)
\[ \frac{t_3}{T_3} = \left(\frac{p_3}{p_3}\right)^{\frac{\gamma_j - 1}{\gamma_j}} = \frac{2}{\gamma_j + 1} \]  
(16)

From equations (15) and (16)
\[ \rho_3 = \frac{p_3}{Rt_3} \left(\frac{2}{\gamma_j + 1}\right) \]  
(17)

From equation (4) and by assuming a negligible difference between the actual and adiabatic specific heats
\[ t_6 = T_3 \left\{ 1 - \eta_e \left[ 1 - \left(\frac{p_6}{p_3}\right)^{\frac{\gamma - 1}{\gamma}} \right] \right\} \]  
(18)
For simplicity, the subscript notation indicating the effective $\gamma$ to be used between specific stations is omitted when the stations are obvious from the equation. Then from equations (10) and (18)

$$\rho_6 = \frac{p_6}{R_t_6} = \frac{p_6}{R_t_3} \left\{ 1 - \eta_e \left[ 1 - \left( \frac{p_6}{p_3} \right)^{\gamma - 1} \right] \right\}$$  (19)

Substituting equation (17) and (19) in equation (14)

$$H_4 - h_6 = \left( \frac{c_3}{c_6} \right)^2 \left( \frac{A_3}{A_6} \right)^2 \left\{ \frac{p_3}{p_6} \left( \frac{2}{\gamma_3 + 1} \right)^{\frac{1}{\gamma_3 - 1}} \left\{ 1 - \eta_e \left[ 1 - \left( \frac{p_6}{p_3} \right)^{\gamma - 1} \right] \right\} \right\}^2$$  (20)

but

$$H_3 - h_3 = c_{p,3} T_3 \left( \frac{\gamma - 1}{\gamma_3 + 1} \right)$$  (21)

Substituting the value of $\frac{p_3}{p_5}$ from equation (9)

$$H_3 - h_3 = c_{p,3} T_3 \left( \frac{\gamma_3 - 1}{\gamma_3 + 1} \right)$$  (22)

Then equation (20) becomes

$$H_4 - h_6 = \left( \frac{c_3}{c_6} \right)^2 \left( \frac{A_3}{A_6} \right)^2 \left\{ \frac{p_3}{p_6} \left( \frac{2}{\gamma_3 + 1} \right)^{\frac{1}{\gamma_3 - 1}} \left\{ 1 - \eta_e \left[ 1 - \left( \frac{p_6}{p_3} \right)^{\gamma - 1} \right] \right\} \right\}^{\frac{\gamma - 1}{\gamma_3 + 1}}$$  (23)
Substituting equations (5) and (23) in equation (3) and solving for \( T_3 \) gives an equation for equilibrium operation of the simple turbojet engine.

\[
T_3 = \frac{C_{p,1.2}(T_2 - T_1) + E}{\left(1 + \frac{W_o}{W_a}\right) \left[c_{p,3.6} \eta_e \left(l - \frac{P_6}{P_3} \right)^{-\frac{\gamma-1}{2}} \right] - \left(\frac{\gamma-1}{1+\gamma_3}\left(\frac{C_3 A_3}{C_6 A_6}\right)^2 \frac{P_3}{P_6} \frac{2}{\gamma_3+1}\right) C_{p,3} \left(1 - \eta_e \left(1 - \left(\frac{P_6}{P_3} \right)^{-\frac{\gamma-1}{2}}\right)\right)}
\]

(24)

Equation for total gas flow. - From equations (8) and (17) and the equation for sonic velocity

\[
v_s = \sqrt{\frac{g R T_3}{\gamma}}
\]

\[
W_g = W_a + W_f = C_3 \rho_3 A_3 V_3 = C_3 \rho_3 A_3 V_s
\]

\[
= \frac{C_3 P_3 A_3}{RT_3} \left(\frac{2}{\gamma + 1}\right) \frac{\gamma-1}{\gamma} \sqrt{g R T_3}
\]

(25)

From equation (16)

\[
t_3 = T_3 \frac{2}{\gamma_3 + 1}
\]

(26)
Substituting in equation (25) and combining terms

\[
W_g = c_3 A_3 \left( \frac{2}{\gamma_3 + 1} \right)^{2(\gamma_3 - 1)} P_3 \sqrt{\frac{\gamma_3 R T_3}{W_f}} \quad (27)
\]

Equation for fuel consumption. - From an energy balance and by assuming the fuel temperature is nearly equal to the air temperature at station 2.

Heat supplied by fuel = increase in heat content of fuel-air mixture

\[
\eta_b W_f Q = W_g c_{p,2-3}(T_3 - T_2) \quad (28)
\]

Solving for \( W_f \)

\[
W_f = \frac{W_g c_{p,2-3}(T_3 - T_2)}{\eta_b Q} \quad (29)
\]

Equation for gross thrust. - Under static conditions (zero flight velocity), the engine thrust is equal to the momentum of the jet issuing from the engine.

\[
F_g = \frac{W_g}{g} V_6 = \frac{W_g}{g} C_{V,6} V_6' \quad (30)
\]

Substituting the equation for \( V_6' \)

\[
F_g = \frac{W_g}{g} C_{V,6} \left( \sqrt{2 g J c_{p,4-6}(T_4 - T_6')} \right)
\]

\[
= \frac{W_g}{g} C_{V,6} \left[ 2g J c_{p,4-6} T_4 \left( 1 - \frac{P_6}{P_4} \right) ^{\gamma-1} \right] \quad (31)
\]

\( T_4 \) may be evaluated by equating the compressor and turbine work equation (1)

\[
c_{p,1-2}(T_2 - T_1) + E = c_{p,3-4}(T_3 - T_4) \left( 1 + \frac{W_f}{W_a} \right) \quad (32)
\]
\[ T_4 = T_3 - \frac{c_{p,1-2}(T_2 - T_1) + E}{c_{p,3-4} \left( 1 + \frac{W_f}{W_a} \right)} \]  

Substituting the adiabatic process relation

\[ P_4 = P_3 \left( \frac{T_{4'}}{T_3} \right)^{\frac{\gamma}{\gamma-1}} \]  

\[ T_{4'} = T_3 - \frac{c_{p,1-2}(T_2 - T_1) + E}{c_{p,3-4} \left( 1 + \frac{W_f}{W_a} \right) \eta_t} \]  

Substituting equation (36) in equation (34)

\[ P_4 = P_3 \left[ 1 - \frac{c_{p,1-2}(T_2 - T_1) + E}{c_{p,3-4} \left( 1 + \frac{W_f}{W_a} \right) \eta_t T_3} \right]^{\frac{\gamma}{\gamma-1}} \]
Substituting equations (33) and (37) in equation (31) and grouping terms

\[
F_g = \frac{W_e}{g} C_{v,6} \left\{ 2 \begin{align*}
&\begin{align*}
T_3 - c_p,3-4 \left( 1 + \frac{w_f}{w_a} \right) \\
&- c_p,1-2 \left( \frac{T_2 - T_1}{T_1} + \frac{w_f}{w_a} \right) \\
&\frac{1}{1 - \left[ \frac{c_p,1-2 \left( \frac{T_2 - T_1}{T_1} + \frac{w_f}{w_a} \right)}{c_p,3-4 \left( 1 + \frac{w_f}{w_a} \right) \eta_T T_3} \right]^{\frac{\gamma - 1}{\gamma}}} \\
&\end{align*}
\end{align*}
\right\}^{\frac{\gamma - 1}{\gamma}}
\]  

Equation for net thrust. - The net thrust of an engine in flight is equal to the change of momentum of the air in passing through the engine; that is, the gross thrust minus the approach momentum of the air consumed by the engine.

\[
F_n = F_g - \frac{W_a}{g} V_0
\]
APPENDIX C

CALCULATION OF EFFECT OF IMPROVEMENT IN INLET DUCTING ON LENGTH OF TAKE-OFF RUN

The method used in calculating the length of the take-off run is that described by Hartman in reference 9. The airplane data used were obtained from the manufacturer's tests of a single engine, jet-propelled, pursuit-type airplane.

The effective ground resistance (friction plus windage) is determined as follows:

Airplane gross weight

\[ W_p = 11,490 \text{ lb} \]

Airplane take-off velocity

\[ V_t = 161 \text{ ft/sec} \]

Measured airplane take-off distance

\[ S = 3050 \text{ ft} \]

Engine rated gross thrust

\[ F_g = 3750 \text{ lb} \]

Engine rated air consumption

\[ W_a = 80 \text{ lb} \]

Assumed effective ram-pressure ratio at 0.7 take-off velocity

\[ \frac{F_t}{p_7} = 0.94 \]

0.7 take-off velocity

\[ 0.7 V_t = 113 \text{ ft/sec} \]

Gross thrust at a ram-pressure ratio of 0.94, assuming that the performance of figure 8 applies proportionately to the larger engine

\[ F_g = 3750 \frac{1575}{1770} = 3337 \]
Air consumption at $P_1/p_7 = 0.94$, assuming that the performance of figure 9 applies proportionately to the larger engine

$$W_a = 80 \frac{30.2}{32.2} = 75.0$$

The fuel consumption at $P_1/p_7 = 0.94$, assuming figure 10 applies directly

$$W_f = \frac{1.25 \times 3337}{3600} = 1.16$$

$$W_g = W_a + W_f = 76.2$$

Net thrust at $P_1/p_7 = 0.94$ and at 0.7 take-off velocity

$$F_n = F_g - \frac{W_g}{g} 0.7 V_t$$

$$F_n = 3337 - \frac{76.2}{32.2} 113 = 3070$$

From reference 7

$$S \approx \frac{W_p V_t^2}{64(F_n - R_m)}$$

in which $(F_n - R_m)$ is the force available for accelerating the airplane. Effective ground resistance (windage + friction) at 0.7 take-off velocity

$$R_m \approx F_n - \frac{W_p V_t^2}{64 S}$$

$$\approx 3070 - \frac{11,490 \times 161^2}{64 \times 3050} = 1544$$

The length of ground roll with an assumed improvement in ducting to give an increase in inlet-to-exhaust pressure ratio to 0.98 is determined as follows:

$$F_g = 3750 \frac{1700}{1770} = 3602$$

$$W_a = 80 \frac{31.5}{32.2} = 78.3$$
\[ W_f = \frac{1.18 \times 3602}{3600} = 1.18 \]

At 0.7 take-off velocity

\[ F_n = 3602 - \frac{79.5}{32.2} \text{ ft} = 3323 \]

\[ S = \frac{11.490 \times 1612}{64(3323 - 1544)} \approx 2616 \]

The percentage decrease in length of take-off run with an assumed improvement in ducting to give an increase in inlet-to-exhaust pressure ratio is

\[ \frac{3050 - 2616}{3050} \approx 14.2 \]

REFERENCES


8. Freeman, Hugh B.: Boundary-Layer-Control Tests of Two Wings in

TN No. 557, 1936.
Figure 1. - Diagrammatic sketch of static-test-cell installation of turbojet engine with nominal thrust rating of 1600 pounds.
Figure 2. - Station notation used for instrumentation and analysis.
Figure 3. - Effect of reduced inlet-to-exhaust pressure ratio on corrected gross thrust. Turbojet engine with 12.40-inch-diameter jet nozzle; nominal thrust rating, 1600 pounds.
Figure 4. - Effect of reduced inlet-to-exhaust pressure ratio on corrected air consumption. Turbojet engine with 12.40-inch-diameter jet nozzle; nominal thrust rating, 1600 pounds.
Figure 5. - Effect of reduced inlet-to-exhaust pressure ratio on corrected specific fuel consumption. Turbojet engine with 12.40-inch-diameter jet nozzle; nominal thrust rating, 1600 pounds.
Figure 6. - Effect of reduced inlet-to-exhaust pressure ratio on corrected tail-pipe total temperature. Turbojet engine with 12.40-inch-diameter jet nozzle; nominal thrust rating, 1600 pounds.
Figure 7. - Effect of reduced inlet-to-exhaust pressure ratio on corrected turbine-inlet total temperature. Turbojet engine with 12.40-inch-diameter jet nozzle; nominal thrust rating, 1600 pounds.
Figure 8. - Corrected gross thrust and turbine-inlet total temperature at sea level and inlet-to-exhaust pressure ratios below 1. Turbojet engine with 12.40-inch-diameter jet nozzle; nominal thrust rating, 1600 pounds. (Data taken from figs. 3 and 7.)
Figure 9. – Corrected air consumption and turbine-inlet total temperature at sea level and inlet-to-exhaust pressure ratios below 1. Turbojet engine with 12.40-inch-diameter jet nozzle; nominal thrust rating, 1600 pounds. (Data taken from figs. 4 and 7.)
Figure 10. - Corrected specific fuel consumption and turbine-inlet total temperature at sea level and inlet-to-exhaust pressure ratios below 1. Turbojet engine with 12.40-inch-diameter jet nozzle; nominal thrust rating, 1600 pounds. (Data taken from figs. 5 and 7.)
Figure 11. - Approximate net thrust and turbine-inlet total temperature at altitude of 30,000 feet and inlet-to-exhaust pressure ratios below 1. Turbojet engine with 12.40-inch-diameter jet nozzle; nominal thrust rating, 1600 pounds. (Thrust extrapolated from sea-level data of fig. 8.)
Figure 12. - Approximate air consumption at altitude of 30,000 feet and inlet-to-exhaust pressure ratios below 1. Turbojet engine with 12.40-inch-diameter jet nozzle; nominal thrust rating, 1600 pounds. (Air consumption extrapolated from sea-level data of fig. 9.)
Inlet-to-exhaust pressure ratio, $P_1/\rho_7$

Turbine-inlet temperature, $T_3$ (°R)
- 2400
- 2200
- 2000
- 1800
- 1600
- 1400

Engine speed, $N$ (rpm)
- 10,000
- 11,000
- 12,000
- 13,000
- 14,000
- 15,000
- 16,000
- 16,500

Figure 13. Approximate specific fuel consumption at altitude of 30,000 feet and inlet-to-exhaust pressure ratios below 1. No correction made for change in combustion efficiency at altitude conditions. Turbojet engine with 12.40-inch-diameter jet nozzle; nominal thrust rating, 1600 pounds. (Fuel consumption extrapolated from sea-level data of fig. 10.)
Figure 14. - Corrected gross thrust and turbine-inlet total temperature at inlet-to-exhaust pressure ratios below 1. Turbojet engine with 14.25-inch-diameter jet nozzle; nominal thrust rating, 1600 pounds.
Figure 15. Corrected air consumption and turbine-inlet total temperature at inlet-to-exhaust pressure ratios below 1. Turbojet engine with 14.25-inch-diameter jet nozzle; nominal thrust rating, 1600 pounds.
Figure 16. - Corrected specific fuel consumption and turbine-inlet total temperature at inlet-to-exhaust pressure ratios below 1. Turbojet engine with 14.25-inch-diameter jet nozzle; nominal thrust rating, 1600 pounds.