RESEARCH MEMORANDUM

CONTROL REQUIREMENTS AND CONTROL PARAMETERS FOR A RAM JET

WITH VARIABLE-AREA EXHAUST NOZZLE

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SUMMARY

Control requirements and control parameters for a ram jet with a variable-area exhaust nozzle have been analyzed from calculated performance charts covering flight Mach numbers from 0.6 to 3.0 and altitudes from sea level to 30,000 feet. Requirements for maximum efficiency, safe and stable operation, maximum range of thrust at a given flight Mach number, and control of fuel flow are discussed and corresponding control parameters are selected. A hypothetical control system is described to illustrate the application of the control parameters.

Maximum efficiency was found to be attainable at any flight Mach number by application of a single control relation between combustion-chamber-inlet Mach number and flight Mach number that is dependent on diffuser characteristics. This relation could be maintained by regulation of the exhaust-nozzle area and is desirable in that stable shock location would simultaneously be achieved.

Airframe drag characteristics may preclude engine operation at maximum efficiency and necessitate provision for engine operation at minimum allowable fuel-air ratio. Such a situation would occur if steady-state operation were desired at a flight Mach number that requires less thrust than the thrust attainable at maximum efficiency.

Two methods of fuel control appear possible: The fuel flow may be directly controlled by manual operation of a fuel-throttle valve, or the fuel flow may be indirectly controlled from a flight Mach number setting.

INTRODUCTION

Although the ram jet has valuable potentialities derived from uniqueness in both design and flight application, its practicability
as a power plant for guided missiles or piloted supersonic aircraft will depend to a great extent upon the development of control systems that will permit these potentialities to be fully exploited. Some of the fundamental control requirements of the ram jet were therefore analyzed at the NACA Cleveland laboratory in order to present a rational basis for the development of ram-jet control systems. This analysis is part of a general research program dealing with the control problems of ram jets.

Preliminary analysis indicated that a fixed-configuration ram jet cannot operate efficiently at flight Mach numbers other than the design condition. In order to extend the analysis reported to include requirements for a more comprehensive latitude of control, the exhaust-nozzle area of the engine is assumed to be variable.

The ram-jet control requirements were analyzed for flight Mach numbers from 0.6 to 3.0 and altitudes from sea level to 30,000 feet for an engine configuration with a variable-area exhaust nozzle. Calculations were based on the theory of one-dimensional flow together with available ram-jet-component data (reference 1). The resultant operating performance of the ram jet is presented in terms of controllable variables. The control parameters that should be incorporated into an automatic control system are readily selected from calculated performance charts, and their required functions within control systems are deduced. A typical control system is schematically presented that would automatically satisfy all the principal control requirements of the ram jet. It is assumed that required instrumentation will be developed to meet the needs of such a control system.

METHOD OF ANALYSIS

Calculations were made for a typical ram jet, schematically shown in figure 1. The ratio of diffuser-inlet area to combustion-chamber area was fixed at 0.228. This value was selected to permit reasonable combustion-chamber-inlet velocities. (At a flight Mach number of 1.8, the combustion-chamber-inlet velocities would be below 150 ft/sec.) A typical maximum diffuser pressure-recovery relation with flight Mach number was used that could apply to a convergent-divergent, perforated, or spike (shock-cone) diffuser. The normal and oblique shocks were assumed always to be included within the lip of the diffuser. All calculations were made for a convergent variable-area exhaust nozzle.

Typical performance calculations (using the symbols defined in appendix A) are presented in appendix B, and a basic equation is
derived relating gas flow, temperature, area, pressure, and Mach number at any two stations.

For supersonic flight velocities, the diffuser pressure recovery \( P_2/P_0 \) determines the combustion-chamber-inlet Mach number \( M_2 \), inasmuch as the diffuser area is considered fixed. The flight Mach number \( M_0 \) and altitude determine the inlet-air flow \( W_a \). For any flight Mach number and altitude, the fuel-air ratio determines the fuel flow \( W_f \), the engine temperature ratio \( T \), and the combustion temperature \( T_4 \). The net thrust \( F_n \) and the required exhaust-nozzle area are calculated for any condition of flight Mach number, altitude, pressure recovery, and fuel flow.

Combined combustion-chamber and exhaust-nozzle pressure loss was obtained from ram-jet data (reference 1), which are replotted to give a general relation for pressure loss as a function of the combustion-chamber-inlet Mach number parameter \( M_2 N/T \). Any fuel similar to gasoline may be expected to result in similar pressure-loss characteristics.

Effective values of fuel flow and fuel-air ratio are used throughout, rather than absolute values, which would be affected by combustion efficiency. The effective fuel flow is equivalent to the product of the true fuel flow and the combustion efficiency. Similarly, stoichiometric fuel-air ratio refers to the fuel-air ratio at which maximum combustion temperature occurs and is therefore to be considered as an effective stoichiometric ratio, rather than a chemical stoichiometric ratio.

For subsonic flight velocities, a constant diffuser pressure recovery of 0.95 is used. For any flight Mach number and altitude, the air flow is determined by the combustion-chamber-inlet Mach number, and the fuel-air ratio determines the engine temperature ratio and the combustion temperature. The fuel flow depends on the fuel-air ratio and combustion-chamber-inlet Mach number. The net thrust and required exhaust-nozzle areas were calculated for any condition of flight Mach number, altitude, combustion-chamber-inlet Mach number, and fuel flow.
RESULTS AND DISCUSSION

Performance Charts

Calculation of the ram-jet variables, as indicated in METHOD OF ANALYSIS, provides sufficient data for performance charts such as those shown in figures 2 and 3 for supersonic and subsonic flight Mach numbers, respectively. These charts show the relation of all the important engine variables, including exhaust-nozzle area, to the thrust output of the engine. The values of thrust and exhaust-nozzle area have been corrected to the combustion-chamber area (which remains constant) in order that the charts may be applied to a ram jet of any diameter. The additional correction of thrust by altitude pressure ratio & brings values of thrust at any altitude into close proximity for each flight Mach number. The fuel-flow parameter $\frac{nW_f}{A_2^3}$ is used in order to simplify calculations when obtaining fuel flow from the air-flow parameter $\frac{W_a}{A_2^3}$ and the fuel-air ratio.

Supersonic conditions: For the supersonic-flight conditions, lines of constant diffuser pressure recovery, constant combustion-chamber-inlet Mach number, and constant combustion-chamber-inlet velocity coincide, and lines of constant fuel-air ratio, constant combustion temperature, constant temperature ratio, and constant fuel flow coincide for each condition of flight Mach number and altitude (fig. 2). Each condition of supersonic flight Mach number and altitude corresponds to a fixed value of air flow.

The charts indicate that operation along a line of maximum attainable diffuser pressure recovery $P_2/P_0$ results in maximum over-all engine efficiency because each point on such a line represents the minimum fuel flow at which the corresponding thrust can be obtained. The charts also indicate the limits of ram-jet operation at each flight Mach number and altitude in that each chart may be bounded on all four sides by a limiting condition of operation. Maximum attainable diffuser pressure recovery (line AB) represents one limitation and, as the recoveries are reduced, a line of minimum allowable recovery is reached that is determined by the maximum allowable combustion-chamber-inlet Mach number or velocity beyond which burner operation would be impossible. The other two limitations consist of a maximum allowable-temperature or fuel-air-ratio line at one end and a minimum fuel-air-ratio line at the other end (BC). The drag of the airframe in which the ram jet is to be installed cannot require
a thrust that is outside the area bounded by the four limiting lines, or steady flight at the desired Mach number is impossible. The maximum value of the ratio \(A_4/A_2\), as shown on several of the charts, is unity when the exhaust-nozzle area becomes equal to the combustion-chamber area.

Subsonic conditions. - Charts similar to those for the supersonic-flight conditions were obtained for the subsonic values of flight Mach number and are shown in figure 3. The chief differences in the subsonic charts, compared with those obtained for supersonic conditions, are that the air flow is no longer constant for each condition of flight Mach number and altitude, and that the pressure recovery of the diffuser was taken as substantially constant throughout the subsonic regime. For subsonic conditions, lines of constant air flow, combustion-chamber-inlet Mach number, and combustion-chamber-inlet velocity coincide, and lines of constant temperature, temperature ratio, and fuel-air ratio coincide. Lines of constant fuel flow (corrected to sea-level conditions and combustion-chamber area) are also drawn.

The subsonic charts indicate that operation at maximum efficiency would occur along a line drawn through the peaks of the fuel-flow curves because these peaks represent points of maximum thrust for a given fuel flow. Engine operation is limited on only three sides of the subsonic charts inasmuch as exhaust-nozzle area no longer has any effect on diffuser-pressure recovery, in contrast with supersonic conditions. In lieu of a pressure-recovery limit, a practical limit would, of course, exist because of the low values of thrust obtained as the constant-fuel-flow lines slope downward from their peak values. The limiting conditions of maximum fuel-air ratio or temperature, minimum fuel-air ratio, and maximum combustion-chamber-inlet velocity or Mach number are the same as for supersonic flight conditions.

Control Relations

The performance charts presented in figures 2 and 3 indicate that variation in exhaust-nozzle area is required in order to permit steady-state operation at all the flight conditions investigated. In general, the trend indicates that decreasing exhaust-nozzle areas are required for increasing flight Mach numbers.

For control application, the performance charts of figures 2 and 3 also indicate that for supersonic flight Mach numbers an automatic engine-control system should maintain maximum pressure.
recovery and that at subsonic flight Mach numbers operation along a line drawn through the maximum-thrust values of the constant fuel-flow lines should be maintained (lines AB on figs. 2 and 3). It is to be noted that for supersonic flight, the line of combustion-chamber-inlet Mach number coincides with that of diffuser pressure recovery, and that for subsonic flight the line through the peaks of the constant-fuel-flow curves may be approximated bys line of constant-combustion-chamber Mach number.

Control of the combustion-chamber-inlet Mach number in accordance with flight Mach number therefore appears to be an effective method of engine control for maximum efficiency and seems desirable because it can be utilized for both the subsonic and supersonic flight Mach numbers. (Other possible means of control for maximum efficiency, such as maintenance of maximum-pressure recovery or location and maintenance of optimum shock position, would be effective only for supersonic flight conditions.) Such a control system would control combustion-chamber-inlet Mach number by variation of exhaust-nozzle area in order to maintain the desired combustion-chamber-inlet Mach number at any flight Mach number. The desired relation between combustion-chamber-inlet Mach number and flight Mach number remains to be shown.

Control for maximum efficiency. - In order to establish a control relation between combustion-chamber-inlet Mach number and flight Mach number it must be realized that at each supersonic flight Mach number a maximum pressure recovery is obtainable from a given diffuser. Should a control system attempt to set a combustion-chamber Mach number corresponding to a pressure recovery greater than that obtainable with the given diffuser, the shock would travel to a position in front of the diffuser. The desired control relation between combustion-chamber Mach number and flight Mach number must therefore be based on the relation between maximum attainable pressure recovery for a given diffuser and flight Mach number. A typical diffuser-recovery curve of approximately maximum attainable recoveries is shown in figure 4 over a range of flight Mach numbers. For maximum engine efficiency, the exhaust-nozzle area should be varied in order to bring the pressure recovery as close to the curve as possible at each flight Mach number.

Because control of combustion-chamber-inlet Mach number has already been shown to be preferable to control of pressure recovery, the diffuser curve of figure 4 may be converted into a curve showing the relation between combustion-chamber Mach number and flight Mach number. Each value of pressure recovery has a corresponding value of combustion-chamber Mach number at a given flight Mach number, as
indicated by equation (B8) of appendix B. Values of combustion-chamber-inlet Mach number obtained from equation (B8) are shown in the supersonic portion of Figure 5, which is the desired control curve for the entire region of flight Mach numbers investigated. The subsonic portion of this control curve was obtained from the constant $M_2$ lines designated by AB in Figure 3. Figure 3 shows that these values of combustion-chamber-inlet Mach number remained substantially the same at a given flight Mach number, regardless of altitude, so that a single value of $M_2$ is obtained at each subsonic flight Mach number.

Figure 5 is a control curve in that it represents the relation to be maintained by variation of the exhaust-nozzle area. For a value of combustion-chamber-inlet Mach number above the value indicated by the curve for a given flight Mach number, the function of the exhaust-nozzle area is to decrease in order to reduce the value of $M_2$. Conversely, the function of the exhaust-nozzle area is to increase the value of combustion-chamber Mach number, should $M_2$ fall below the curve. Figure 5 does not represent the entire ram-jet-control solution, however, inasmuch as limiting conditions of engine operation and control of fuel flow must be considered.

Control for limiting conditions of engine operation. — Limiting conditions of ram-jet operation are not yet clearly defined, owing to the lack of any preponderance of engine data and because of the many variables that influence burner operation. Effective stoichiometric fuel-air ratio may be accepted as one specific limiting condition of operation because fuel-air ratios greater than the effective stoichiometric result in decreased values of thrust. Limiting material temperatures might possibly be exceeded before stoichiometric fuel-air ratio is obtained, so that temperature-limiting control may be considered an alternative to maximum-fuel-air-ratio control.

Other limiting control requirements, as indicated from existing ram-jet data, are those for maintenance of stable burner operation. Existing data have indicated that combustion-chamber-inlet Mach number (or combustion-chamber velocity) and fuel-air ratio affect burner operation. In this analysis the minimum allowable fuel-air ratio is assumed to be 0.03 and the maximum allowable combustion-chamber Mach number, 0.2.

Limiting control requirements that must be considered for overall ram-jet control are therefore assumed to consist of maximum fuel-air ratio, maximum temperature, minimum fuel-air ratio, and maximum combustion-chamber-inlet Mach number control. Coincident with the attainment of maximum pressure recovery through variation of the
Such concurrence indicates that the control relation of figure 5 has a favorable trend with respect to limitation of maximum allowable combustion-chamber Mach number.

Control for maximum range of thrust. - Control along a line of optimum combustion-chamber Mach number has previously been shown to be desirable (fig. 5). The drag of the airframe at a given flight Mach number, however, could conceivably be less than the lowest value of thrust obtainable at the optimum combustion-chamber Mach number so that steady-state operation at the given flight Mach number would be impossible. Such a situation would occur when the value of airframe drag corresponded to a point below the intersection of the optimum line of combustion-chamber Mach number (line AB) and the limiting line BC of minimum allowable fuel-air ratio (figs. 2 and 3). In order to permit the greatest latitude of engine operation, the control system must therefore permit engine operation along the line of minimum fuel-air ratio BC. The thrust range of the engine may, in this manner, be extended to the point at which line BC intersects the maximum allowable Mach number line corresponding to AB at an $M_2$ of 0.2, or to wide-open exhaust-nozzle area. Should the value of drag be below even this minimum point of engine thrust, steady-state operation at the given flight Mach number would, of course, be impossible.

An assumed drag curve at an altitude of 30,000 feet, typical of the drag obtained through supersonic flight Mach numbers, is shown by the dashed curve in figure 6 and was taken from reference 2. Superimposed on figure 6 are also curves showing the maximum thrust attainable for maximum efficiency, the minimum thrust attainable at maximum efficiency, and the minimum thrust attainable while maintaining a minimum fuel-air ratio of 0.03. The values of maximum thrust were obtained from figures 2 and 3 for values of $M_2$ from figure 5 at either stoichiometric fuel-air ratio or a $T_4$ of 4000° R; values of minimum thrust attainable at maximum efficiency correspond to the intersections of the optimum value8 of $M_2$ (fig. 5) with the fuel-air-ratio lines of 0.03 (figs. 2 and 3); and values8 of minimum thrust at limiting minimum fuel-air ratio correspond to the intersections of fuel-air-ratio lines of 0.03 with estimated maximum-allowable $M_2$ lines of 0.2. Figure 6 indicates the necessity of permitting engine operation below the values of thrust obtainable at maximum efficiency because the drag curve fails below these values8 in two places. For example, at a flight Mach number of 1.8, the maximum value of corrected thrust that could be attained would be 3240 pounds; and, if the engine were allowed to operate only along a line of maximum efficiency, the minimum value
of corrected thrust that could be attained would be 2070 pounds, which corresponds to the point of minimum allowable fuel-air ratio. The thrust required for steady-state operation, however, is only 1800 pounds according to the drag curve, and this value is below the value of 2070 pounds attainable at maximum efficiency. Steady-state operation at \( M_0 = 1.8 \) could not be attained under such conditions, and engine operation along the line of minimum allowable fuel-air ratio (line BC of fig. 2) would be required in order to permit attainment of the low value of thrust necessary. The engine-control system should therefore provide for variation in exhaust-nozzle area at the minimum allowable fuel-air ratio in addition to attempting to maintain engine operation along the control curve of figure 5. This requirement corresponds to operation along the line ABC of figure 2 such that the exhaust nozzle opens in order to permit reduced values of thrust.

From the drag curve of figure 6, the control curve of figure 5, and the performance charts of figures 2 and 3, the required variation in exhaust-nozzle area was obtained and is shown in figure 7. For the specific-drag curve used and with the assumed minimum fuel-air ratio of 0.03, the required exhaust-nozzle-area ratio \( A_4/A_2 \) is seen to exceed unity at a flight Mach number of 0.8. This discrepancy in exhaust-nozzle-area requirements occurs where both the thrust attainable at maximum efficiency and the thrust attainable at wide-open exhaust-nozzle area \( (A_4/A_2=1) \) exceed the thrust required by the drag curve. Figure 3 indicates that, if the minimum value of fuel-air ratio was assumed to be somewhat below 0.03, the required value of thrust would be obtained at an area ratio less than unity. The minimum allowable fuel-air ratio may therefore be critical in determining the controllable range of operation.

The possibility of closing the exhaust nozzle in order to reduce thrust at constant minimum fuel-air ratio is not considered in this investigation. In the subsonic region, reduction of exhaust-nozzle area would result in decreased air and fuel consumption, so that reduction of thrust by this method could be considered advantageous. In the supersonic region, reduction of exhaust-nozzle area at constant minimum fuel-air ratio would first regurgitate the shock to the front of the diffuser, thereby increasing external drag and effectively decreasing the net thrust. Further reduction of exhaust-nozzle area would restrict the air flow because subsonic conditions would now exist in front of the inlet diffuser, so that thrust could be further decreased.

Fuel control. - Previous discussion has dealt only with control problems arising at a given flight Mach number, so that a means of
controlling the fuel flow in order that any desired flight Mach number may be attained still is necessary. The following three general methods of fuel control are possible:

1. The flight Mach number may be maintained constant at a desired value regardless of altitude, but only in level flight, by means of a control which is so calibrated to the level-flight relation between fuel flow and flight Mach number over a range of altitudes (barometric control) that, if a certain flight Mach number is desired, the fuel flow would be manually set to the value at which the desired Mach number would be attained.

2. An engine parameter (T₄, T, or wₑ/ₑₒ) that maintains a satisfactory relation with both flight Mach number and fuel flow may be so controlled that a given setting of the parameter would result in a given fuel flow and Mach number.

3. The flight Mach number may be maintained constant at a desired value, regardless of altitude or attitude, by means of a variable set device sensitive to flight Mach number that controls the fuel flow.

As indicated in figures 2 and 3, fuel flow could be directly controlled in order to obtain a desired flight Mach number because an increase in fuel flow results in an increase in thrust. Similarly, increasing values of fuel-air ratio, combustion-chamber temperature, and engine temperature ratio also result in increased thrust. The variation of fuel flow, fuel-air ratio, combustion temperature, and engine temperature ratio with flight Mach number (obtained from the drag curve of fig. 6, supplemented by the control curve of fig. 5 and the performance charts of figs. 2 and 3) is shown in figure 8. The fuel-air ratio is assumed to be maintained at 0.03 in the regions where the drag curve is below the thrust obtainable at maximum efficiency. Figure 8 indicates that 8 method of control involving the setting of fuel flow in order to obtain a corresponding flight Mach number (method 1) would result in a region of unstable operation between points B and D. For instance, at any point along the negative slope such as point C, a small increase in fuel flow would cause the airframe to accelerate to a flight Mach number corresponding to point E, which is in a stable region. Similarly, a slight decrease in the fuel flow at point C would result in 8 flight Mach number at point A. The advantages of a simple fuel-control system requiring only a throttle valve in the fuel line would therefore be counterbalanced by the omission of a range of flight Mach numbers from the possible steady-state operating points. The extent of the region of negative slope is dependent upon the drag of the airframe and the variation of engine specific fuel consumption with flight Mach number.
In spite of the region of instability defined by points B and D, direct fuel-flow control is nevertheless more satisfactory than control of either engine-temperature ratio, fuel-air ratio, or combustion temperature. Engine temperature ratio \( T \) may be eliminated immediately as a possible control parameter because the slope of the curve is negative after a flight Mach number of 1.2. As has been indicated, the negative-slope part of the fuel-flow curve would result in unstable control characteristics, and the same reasoning may be applied in considering \( T \) as a control parameter.

The utility of fuel-air ratio and combustion temperature as fuel-flow-control parameters may be considered on the following basis: Assume engine operation at a point corresponding to point B in figure 8. Any slight increase in fuel flow would result in an airframe acceleration to point F. Thus, for a pilotless missile in which there is no outside influence to reduce the fuel flow, steady-state flight in the entire range of flight Mach numbers between points B and F would be eliminated. If a choice of control is to be made between direct fuel-flow control and either fuel-air ratio or combustion temperature, it is evident from figure 8 that direct fuel-flow control is more satisfactory because the region of flight Mach numbers encompassed by points B and F is less for the fuel-flow curve than for any of the other curves shown. Because all three engine parameters have been found to be less satisfactory than direct fuel-flow control, the second control possibility suggested may therefore be eliminated.

The remaining possibility for fuel control is method 3, that of maintaining constant flight Mach number by means of a variably set device sensitive to flight Mach number which controls the fuel flow. This method of control would be advantageous in coping with the unstable region in figure 8 denoted by points B and D, because stability can be obtained by means of proportional plus reset control action (reference 3). (A discussion of stability and control action is beyond the scope of this report. A comprehensive treatment of control fundamentals may be obtained from reference 4 and its associated bibliography.) The inherent disadvantages of fuel control by a device sensitive to flight Mach number include the complication of the device itself and the characteristic of the device to seek maximum power during a climb and minimum power during a dive in attempting to maintain the flight Mach number setting.

Consideration of the methods of fuel control discussed indicates that positive choice between direct fuel control and flight Mach number control should not be made without reference to the specific installation and the utilization of the engine. For instance, the simplicity of a fuel-throttle valve in a direct fuel-control system
might conceivably outweigh the disadvantage of a flight schedule in which steady-state operation over a certain range of flight Mach numbers would be impossible. To a great extent, the choice would depend upon the airframe drag because the negative-slope part of figure 8 and the range of Mach numbers included therein is a function of the airframe drag.

**DESCRIPTION OF HYPOTHETICAL CONTROL**

In order to clarify and expand the previous discussion of ramjet control requirements and parameters, a schematic diagram of a hypothetical control system is shown in figure 9. The control system is reduced to its individual control components as indicated by the dashed-line enclosures, and operation of the complete control system may be visualized from a description of the individual and related action of each component. For convenience of illustration, a hydraulic control system has been shown.

**Fuel-flow control.** - The fuel-flow control operates from a manual setting of the control lever, which in turn sets a desired flight Mach number \( M_0 \). When a differential exists between the setting of \( M_0 \) and the measured \( M_0 \), this differential is utilized to regulate valve A in the fuel-bypass line.

Should the differential set up by the fuel control be excessive, as in a rapid acceleration, the high values of fuel flow could result in either excessively high combustion temperatures or in fuel-air ratio beyond stoichiometric. At this point, the maximum-temperature-limiting control (or maximum-fuel-air-ratio control) becomes effective. Valve B installed in another fuel-bypass line is normally closed but opens when the combustion temperature exceeds a predetermined maximum. Fuel is diverted from the burner despite the setting of valve A and combustion temperature is thereby reduced to the allowable value.

**Minimum-fuel-air-ratio control.** - During deceleration, bypass valve A may possibly be open to such an extent that the fuel flow to the burner results in a fuel-air ratio below the minimum allowable value. The minimum fuel-air-ratio control then functions to increase the fuel flow by closing valve C, which is in series with valve A. Less fuel is therefore bypassed so that the measured fuel-air ratio is maintained at the minimum allowable value.

**Maximum-efficiency control.** - The exhaust-nozzle-area control incorporates the relation between \( M_0 \) and combustion-chamber-inlet Mach number \( M_2 \) shown in figure 5, which gives maximum efficiency.
for all conditions of operation. The system is balanced as long as the values of \( M_2 \) and \( M_0 \) correspond according to the relation of figure 5. For values of \( M_2 \) in excess of this relation, the system is so unbalanced as to close the exhaust-nozzle area and thereby reduce \( M_2 \). Conversely, for values of \( M_2 \) below those required according to figure 5, the system is so unbalanced as to open the exhaust-nozzle area and thereby increase \( M_2 \).

In the event that \( M_2 \) should tend to exceed the maximum allowable value determined for maintenance of stable burner operation, the control limiting maximum \( M_2 \) overrides the exhaust-nozzle-area control. The control limiting maximum \( M_2 \) acts to close the exhaust-nozzle area when the measured \( M_2 \) exceeds the predetermined allowable value and thereby maintains \( M_2 \) at this maximum.

**Fuel-flow-limiting control.** - If required, a fuel-flow-limiting control acts to maintain the internal shock when the exhaust-nozzle area is wide open. This control would be required for the condition in which wide-open exhaust-nozzle area occurred within the span of line AB (fig. 2), because after wide-open area had been attained additional fuel would cause the shock to travel to the outside of the diffuser. The action of the fuel-flow-limiting control is to open valve D in a fuel-bypass line. The fuel flow to the combustion chamber then decreases, and therefore \( M_2 \) increases, which causes the exhaust-nozzle area to move toward the closed position while valve D is reset.

**Change-over control at minimum fuel-air ratio.** - Engine operation at maximum efficiency in accordance with figure 5 reduces the range of thrust available at a given \( M_0 \). The airframe drag might possibly require a thrust value below the thrust obtainable at point B (fig. 2). Steady-state operation along the line BC would therefore be unattainable and a control permitting operation at the minimum fuel-air ratio (line BC) would be required for the attainment of lower values of thrust. Such a control is designated the change-over control at minimum fuel-air ratio on the schematic diagram of figure 9. In conjunction with the exhaust-nozzle-area control, it permits continuous operation along the path ABC of figure 2.

The change-over control is so energized from the minimum fuel-air-ratio control that, when minimum fuel-air ratio is attained, the main control lever becomes directly linked with the exhaust-nozzle area. The variation in exhaust-nozzle area is here assumed to permit attainment of the lowest values of desired thrust.
CONCLUSIONS

From an analysis based on calculated performance charts, requirements and parameters for control of a ram jet with a variable-area exhaust nozzle may be summarized as follows:

1. For maximum efficiency the exhaust-nozzle area should be regulated to attain operation at maximum diffuser pressure recovery during supersonic flight. Maximum efficiency may be attained by regulation of exhaust-nozzle area in accordance with a predetermined schedule of combustion-chamber-inlet Mach number with flight Mach number.

2. Regulation of combustion-chamber-inlet Mach number for maximum efficiency would simultaneously result in stable shock location.

3. Steady-state drag characteristics may require a thrust at a given Mach number that is less than the thrust attainable at maximum efficiency. For maximum range of thrust at a given flight Mach number, down to values of thrust below those at which maximum pressure recovery can be attained, the engine should be permitted to operate along a line of minimum allowable fuel-air ratio, despite the fact that this would incur a loss of efficiency. Operation along a line of minimum fuel-air ratio may be accomplished by means of a direct linkage between throttle and exhaust-nozzle area such that, when minimum fuel-air ratio is reached, the throttle will control the area.

4. For control of fuel flow in order to attain a desired flight Mach number, two possibilities exist: The fuel may be directly controlled by manual operation of a valve in the fuel line at the expense of a sacrifice in the range of possible flight Mach numbers, or the fuel flow may be indirectly controlled from a flight Mach number setting that would call for changes in fuel flow according to the difference between measured and desired flight Mach number.

5. Combustion temperature, engine-temperature ratio, and fuel-air ratio are unsatisfactory as fuel-flow-control parameters in comparison with direct fuel-flow control.

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APPENDIX A

SYMBOLS

The following symbols are used in this report:

A  area, sq ft
A_c  area corresponding to Mach number of 1, sq ft
C  constant
f(γ), f'(γ) functions of γ
F_n  net thrust, lb
g  acceleration due to gravity, ft/sec^2
M  Mach number
P  total pressure, lb/sq ft
p  static pressure, lb/sq ft
R  universal gas constant, ft-lb/(lb)(°R)
T  total temperature, °R
t  static temperature, °R
V  gas velocity, ft/sec
W  weight rate of flow, lb/sec
W_a  weight rate of air flow, lb/sec
W_f  weight rate of fuel flow, lb/sec
γ  ratio of specific heat at constant pressure to specific heat at constant volume
δ  ratio of altitude pressure to sea-level pressure
η  combustion efficiency
\( \theta \) ratio of altitude temperature to sea-level temperature

\( \rho \) density, lb/cu ft

\( T \) temperature ratio across ram jet, \( T_4/T_0 \)

Subscripts:

0 - 4 stations on ram jet (fig. 1)
APPENDIX B

PERFORMANCE CALCULATIONS

The following equation, which is the basis for calculation of engine variables, expresses the general relation between gas flow, total temperature, area, total pressure, Mach number, and ratio of specific heats at any two stations, designated stations A and B:

\[ \frac{W_A}{W_B} = \left( \frac{T_A}{T_B} \right)^{\frac{1}{2}} \left( \frac{P_A}{P_B} \right)^{\frac{1}{2}} \left( \frac{A_{cr}}{A} \right)^{\frac{1}{2}} f(\gamma)_A \left( \frac{A_{cr}}{A} \right)^{\frac{1}{2}} f(\gamma)_B \]

This equation is derived as

\[ W = A \sqrt{v} \rho \]

and, because \( \rho = \frac{P}{RT} \),

\[ W = A \left( \frac{V}{\sqrt{\gamma gRt}} \right) \sqrt{\gamma} \frac{P}{Rt} \sqrt{\frac{g}{R}} \]

and, because

\[ M = \frac{V}{\sqrt{\gamma gRt}} \]

and

\[ \frac{T}{t} = \frac{P}{P_0} \gamma = 1 + \frac{\gamma-1}{2} M^2 \]

therefore

\[ W = \sqrt{\frac{g}{R}} \frac{AP}{RT} \left( \frac{M \sqrt{\gamma}}{1 + \frac{\gamma-1}{2} M^2} \right)^{\frac{\gamma+1}{\gamma-1}} \]

(B1)
The effects of $\gamma$ are separated from the Mach number by rearranging equation (B1):

$$W = \left[ \frac{1.4 \frac{R}{\gamma}}{1.4+1} \right] A \left[ \frac{P}{NT} \right] \left[ \frac{M(\gamma+1)^{\frac{1}{2}(\gamma-1)}}{(1+\frac{\gamma-1}{2}M^2)^{\frac{1}{2}(\gamma-1)}} \right] \left[ \frac{\gamma+1}{2(\gamma-1)} \right]$$

(B2)

Let the first bracketed term, which is constant, be C. The second bracketed term can be shown to be almost independent of $\gamma$ and thus is $f(M)$. The third bracketed term is a function of $\gamma$ and is labeled $f(\gamma)$ and plotted in figure 10. Equation (B2) thus becomes

$$W = CA \frac{P}{NT} f(M) f(\gamma)$$

(B3)

The term $f(M)$ may be expressed in terms of area according to the following concept: Assume a venturi, placed at the section being considered, the throat area ($A_{cr}$) of which is such that the Mach number is sonic. At this area $f(M) = 1$ and equation (B3) becomes

$$W = CA_{cr} \frac{P}{NT} f(\gamma)$$

(B4)

From equations (B3) and (B4) the following equation is obtained:

$$f(M) = \frac{A_{cr}}{A}$$

as plotted in figure 11, and equation (B3) becomes

$$W = CA \frac{P}{NT} \left( \frac{A_{cr}}{A} \right) f(\gamma)$$

(B5)

From equation (B5), for any two stations (A and B) in the ram jet, equation (B6) is obtained:
\[
\frac{W_A}{W_B} \sqrt{\frac{T_A}{T_B}} = \left( \frac{A_A}{A_B} \right) \left( \frac{P_A}{P_B} \right) \left( \frac{A_{cr}}{A} \right)_A f(\gamma)_A \left( \frac{A_{cr}}{A} \right)_B f(\gamma)_B
\]  

(B6)

When equation (B6) is used for the free-stream and burner-inlet stations, \( W_0 = W_2 \), \( T_0 = T_2 \), and \( f(\gamma)_0 = f(\gamma)_2 = 1 \). The following equation is then obtained:

\[
\frac{A_0}{A_2} = \left( \frac{A_{cr}}{A} \right)_2 \left( \frac{P_2}{P_0} \right)
\]  

(B7)

Thus for subsonic \( M_0 \), the free-stream area is determined from the assumed \( P_2/P_0 \) and \( M_2 \). For supersonic \( M_0 \), the free-stream area was assumed to be fixed at \( A_1 \), and equation (B7) becomes:

\[
\left( \frac{A_{cr}}{A} \right)_2 = \left( \frac{A_{cr}}{A} \right)_0 \left( \frac{P_2}{P_0} \right)
\]  

(B8)

and \( M_2 \) is determined by the assumed \( P_2/P_0 \).

When equation (B6) is used for the free-stream and exhaust-nozzle stations, \( W_4/W_0 = 1 + \frac{W_f}{W_0} \), \( T_4/T_0 = T \), and \( f(\gamma)_0 = 1 \). Thus,

\[
\frac{A_4}{A_2} = A_0 \left[ \left( 1 + \frac{W_f}{W_0} \right) \sqrt{T} \left( \frac{A_{cr}}{A} \right)_0 \right] \left( \frac{P_4}{P_0} \right) \left( \frac{A_{cr}}{A} \right)_4 f(\gamma)
\]  

(B9)

In the derivation of the air-flow equation, the following expression was obtained:

\[
W = AM \ \sqrt{\gamma} \ \frac{P}{\sqrt{t}} \ \sqrt{\frac{P}{R}}
\]
When this equation is rearranged for the free-stream station,

\[
W_0 \sqrt{\frac{\theta_0}{R}} = \left( \frac{A_0}{A_2} \right) M_0 \frac{144 \times 14.7 \sqrt{\gamma_0 \theta_0}}{520}
\]  

(B10)

For subsonic \( M_0 \), corrected air flow is determined by the free-stream area, and for supersonic \( M_0 \), where \( A_0/A_2 = A_1/A_2 = 0.228 \), the corrected air flow is fixed at any \( M_0 \).

A convenient expression for momentum is derived as follows:

\[
\frac{WV}{g} = \frac{AV^2 C}{g} = AY \frac{\gamma^2 p}{\gamma g R} = \gamma A p M^2
\]

The net thrust from the ram jet can be expressed as

\[
F_n = \frac{W_4 V_4}{g} + (p_4 - P_0) A_4 - \frac{W_0 V_0}{g}
\]

Using equation (B11) then gives

\[
F_n = \gamma A_4 p_4 M_4^2 + (p_4 - P_0) A_4 - \gamma_0 A_0 P_0 M_0^2
\]

(B12)

For subsonic flow in the exhaust nozzle (where \( p_4 = P_0 \)) equation (B12) becomes

\[
F_n = \gamma A_4 P_0 M_4^2 - \gamma_0 A_0 P_0 M_0^2
\]

Rearrangement gives

\[
\frac{F_n}{A_2^8} = \left( \gamma \frac{A_4}{A_2} M_4^2 - \gamma_0 \frac{A_0}{A_2} M_0^2 \right) 144 \times 14.7
\]

(B13)

For sonic flow in the exhaust nozzle where \( M_4 = 1 \), equation (B12) can be rearranged:

\[
\frac{F_n}{A_2 p_0} = \frac{A_4}{A_2} \left[ \frac{p_4}{P_0} (\gamma_4 + 1) - 1 \right] - \gamma_0 \frac{A_0}{A_2} M_0^2
\]
or

\[
\frac{F_n}{A_2P_0} = \frac{A_4}{A_2} \left[ \frac{P_4}{P_0} \left( \frac{\gamma_4 + 1}{\gamma_4} \right) - 1 \right] - \frac{A_0}{A_2} M_0^2
\]

Because

\[
\frac{P_4}{P_4} = \left( \frac{\gamma_4 + 1}{2} \right)^{\frac{\gamma_4}{\gamma_4 - 1}}
\]

\[
\frac{F_n}{A_2P_0} = \frac{A_4}{A_2} \left[ \frac{P_4}{P_0} \left( \frac{\gamma_4 + 1}{\gamma_4} \right) - 1 \right] - \frac{A_0}{A_2} M_0^2
\]

If

\[
\frac{\gamma_4 + 1}{\gamma_4} = f'(\gamma)
\]

\[
\left( \frac{\gamma_4 + 1}{2} \right)^{\frac{\gamma_4}{\gamma_4 - 1}} - \frac{\gamma_4}{\gamma_4 - 1}
\]

then

\[
\frac{F_n}{A_2P_0} = 144 \times 14.7 \frac{A_4}{A_2} \left[ \frac{P_4}{P_0} f'(\gamma) - 1 \right] = 144 \times 14.7 \frac{A_0}{A_2} M_0^2
\]

(B14)

The function \( f'(\gamma) \) is plotted in figure 12.

The methods used in this report for calculating engine operating conditions are described. For subsonic flight speeds, conditions of constant \( M_2 \) and \( W_f/W_a \) were taken and the required \( A_4 \) and the resultant thrust were determined. At any \( M_0 \) and \( M_2 \), the free-stream area was determined from equation (B7) where a \( P_2/P_0 \) of 0.95 was assumed. From \( W_f/W_a \) and \( T_2 \), the values of \( \tau \) and \( T_4 \) were obtained from figure 13, which is based on combustion for octane fuel
and includes dissociation effects. The value of $\gamma_4$ was obtained from figure 14 and of $f(\gamma_4)$ from figure 10. From $M_2 \sqrt{T}$, the combustion-chamber pressure loss was obtained from figure 15, which was obtained from ram-jet data (reference 1) replotted to give the general curve shown. The pressure ratio in the exhaust nozzle was obtained from the equation

$$\frac{P_4}{P_0} = \frac{P_4}{P_2} \frac{P_2}{P_0}$$

From $P_4/P_0$, the values of $M_4$ and $(A_{cr}/A)_4$ were obtained from figure 16 and 11, where figure 16 is a plot of the equation

$$\frac{P_4}{P_4} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma-1}$$

The exhaust-nozzle area was obtained from equation (B9); the air flow from equation (B10); and the net thrust from equation (B13). Fuel flow was found from the air flow and fuel-air ratio.

For supersonic flight speeds, the free-stream area was assumed to be the inlet area of the diffuser (shock swallowed). Conditions of constant $P_2/P_0$ and $W_f/W_a$ were assumed, and the required $A_4$ and resultant $F_n$ were determined. At any $M_0$ and $P_2/P_0$, $M_2$ was found from equation (B8). From $W_f/W_a$ and $T_2$, the values of $\tau$ and $T_4$ were obtained from figure 13, $\gamma_4$ from figure 14, and $f(\gamma_4)$, from figure 10. From $M_2 \sqrt{T}$, $P_4/P_2$ was found from figure 15. From

$$\frac{P_4}{P_0} = \frac{P_4}{P_2} \frac{P_2}{P_0}$$

it was determined whether the exhaust nozzle had sonic or subsonic flow.

For sonic flow in the exhaust nozzle, $A_4$ was found from equation (B9) where $(A_{cr}/A)_4 = 1$, and $F_n$ from equation (B14), where $f'(\gamma)$ is found from figure 12. For subsonic flow in the exhaust nozzle, $M_4$ and $(A_{cr}/A)_4$ were obtained from figures 16 and 11; $A_4$ was obtained from equation (B9); and $F_n$ from equation (B13).

The air flow was found from equation (B10), and the fuel flow from the air flow and fuel-air ratio.
REFERENCES


Figure 1. - Ram-jet configuration.
Figure 2. - Performance charts for ram jet with variable-area exhaust nozzle at supersonic flight Mach numbers.
Figure 2. - Continued, Performance charts from jet with variable-area exhaust nozzle at supersonic flight Mach numbers.
Figure 2. Continued. Performance charts for ram jet with variable-area exhaust nozzle at supersonic flight Mach numbers.
Figure 2. Concluded. Performance charts for ram jet with variable-area exhaust nozzle at supersonic flight Mach numbers.
Figure 5. Performance charts for jet with variable area exhaust nozzle at subsonic flight Mach numbers.
Figure 3. - Computed performance charts for ram jet with variable-area exhaust nozzle at subsonic flight Mach number.
Figure 4. - Typical curve of maximum diffuser pressure recovery.

Figure 5. - Control relation for variation of combustion-chamber-inlet Mach number with flight Mach number for ram jet with variable-area exhaust nozzle.
Figure 6. - Comparison of typical supersonic drag curve (from reference 2) with possible range of thrust for ram jet with variable-area exhaust nozzle. Altitude, 30,000 feet.
Figure 7. - Required variation of ratio of exhaust-nozzle area to combustion-chamber-inlet area with flight Mach number for ram jet. Altitude, 30,000 feet.
Figure 8. Variation of temperature ratio across engine, combustion-gas temperature, fuel-air ratio, and corrected fuel flow with flight Mach number for ram jet with variable-area exhaust nozzle. Altitude, 30,000 feet.
Figure 9. - Schematic diagram of hypothetical ram-jet control system.
Figure 10. — Variation of $f(\gamma)$ with $\gamma$. 

Ratio of specific heats, $\gamma$
Figure 11. Variation of \( \frac{A_{cr}}{A} \) with Mach number.
Figure 12. – Variation of $f'(\gamma)$ with $\gamma$. 

Ratio of specific heats, $\gamma$ 

$f'(\gamma)$
Figure 13. - Relation between fuel-air ratio, combustion-chamber-inlet temperature, and temperature ratio for octene.
Figure 14. Relation between ratio of specific heats and combustion temperature for various fuel-air ratios.
Combustion-chamber pressure loss, $P_4/P_2$

Combustion-chamber-inlet Mach number parameter, $M_2 \sqrt{\tau}$

Figure 15. Relation between combustion-chamber pressure loss and combustion-chamber-inlet Mach number parameter for ram jet. (Combustion-chamber loss includes exhaust-nozzle losses.) Data cross-plotted from reference 1.
Figure 16. Variation of exhaust-nozzle pressure ratio with exhaust-nozzle Mach number for various values of ratio of specific heats.