ANALYTICAL DETERMINATION OF EFFECT OF WATER INJECTION
ON POWER OUTPUT OF TURBINE-PROPELLER ENGINE

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SUMMARY

The effect of water injection on power output of a turbine-propeller engine incorporating a centrifugal compressor is determined by computing the wet compression process as polytropic and using currently available data on the effect of water injection at the compressor inlet to determine empirical constants related to the rate of evaporation. The analytical investigation includes compressor tip speeds of 1200, 1500, and 1800 feet per second at a turbine-inlet temperature of 2000° R.

The calculations based on experimental data indicate that with the use of water injection the shaft power of the engine may be increased by more than 78 percent at a compressor tip speed of 1800 feet per second without increase in turbine-inlet temperature. The liquid consumption is increased 4.5 times in order to achieve this augmentation.

If sufficient water is injected within the compressor to saturate the air at the compressor outlet, power augmentation as great as 200 percent is possible at a compressor tip speed of 1800 feet per second.

INTRODUCTION

Power augmentation of gas-turbine engines for improved take-off, climb, and emergency use is of importance in increasing the effectiveness of the application of turbine-propeller engines to civilian and military aircraft. One method of increasing the power of gas-turbine engines is the use of evaporative cooling during the compression process in order to increase the compressor pressure ratio.

Water injection at the compressor inlet has been successfully used to increase the thrust of turbojet engines. References 1 and 2
indicate that the sea-level static thrust of a centrifugal-flow-type turbojet engine may be increased about 25 percent by use of liquid injection. Reference 3 indicates that greater thrust augmentation is possible at a compressor pressure ratio of 11 than of 4.

As part of a general research program being conducted at the NACA Lewis laboratory to investigate methods of improving power augmentation, the performance of a centrifugal-flow-type turbine-propeller engine with water injection at the compressor inlet was studied. The wet-compression process (with the evaporating liquid extracting heat from the working fluid) was considered as polytropic. Calculations were made with values of cooling effectiveness (ratio of actual cooling to possible cooling with all the injected liquid evaporated) of 1.0, 0.8, 0.6, and 0.4 and with values of cooling effectiveness computed from the experimental data of reference 1. The engine performance was calculated at compressor tip speeds of 1200, 1500, and 1800 feet per second (for compressor pressure ratios without water injection of approximately 2.6, 4.0, and 6.0, respectively) at sea-level static conditions with a turbine-inlet temperature of 2000° R.

SYMBOLS

The following symbols are used in this report:

A flow area in turbine stator, sq ft

$c_p$ specific heat at constant pressure of coolant - air mixture,
$\text{Btu/(lb)(°F)}$

$c_v$ specific heat at constant volume of coolant - air mixture,
$\text{Btu/(lb)(°F)}$

$g$ acceleration due to gravity, $32.2 \text{ ft/sec}^2$

$H$ enthalpy, Btu/lb

$H'$ enthalpy of coolant vapor, Btu/lb

$H_w$ enthalpy of liquid and vapor, Btu/lb

$h$ lower heating value of fuel, Btu/lb

hp horsepower
J  mechanical equivalent of heat, 778 ft-lb/Btu
K  ratio of actual cooling to possible cooling with all injected liquid evaporated, Q/Q
M  tip Mach number based on inlet stagnation temperature
n  polytropic exponent
P  stagnation pressure, lb/sq ft abs
Q  effective cooling parameter, QK
\( \bar{Q} \)  cooling parameter (or ratio of possible cooling to shaft work),
\[
\frac{W_w}{W_a} \left( \frac{H'_{w,2} - H_{w,1} - \frac{SU^2}{gJ}}{\frac{SU^2}{gJ}} \right)
\]
Qd  mechanical work dissipated in heat, Btu/lb
R  gas constant of coolant - air mixture, ft-lb/(lb)(°R)
S  slip factor,
\[
\frac{Q_d + \frac{1}{J} \int \frac{dP}{U^2 \rho}}{gJ}
\]
T  stagnation temperature, °R
U  tip speed, ft/sec
W  weight flow, lb/sec
\( \gamma \)  ratio of specific heats
\( \eta \)  adiabatic efficiency
\( \eta_p \)  polytropic efficiency
\( \rho \)  stagnation density, lb/cu ft
\psi \text{ compressor pressure coefficient, } \frac{\int \frac{dp}{\rho}}{U^2 G}

Subscripts:
1 compressor inlet
2 compressor outlet
3 turbine inlet
4 turbine outlet
a air
b burner
c compressor
f fuel
g gas
s shaft
t turbine
w liquid and vapor

ANALYSIS

When liquid coolant is injected at the compressor inlet, the inducted air is cooled by evaporation of the injected liquid from suspended droplets and from liquid on the coolant-wetted surfaces within the compressor.

If no heat is transferred through the compressor case, the general energy equation expressed in the symbol notation of this report is

\[ W_{w_h,1} + W_{a_h,1} + \frac{hp_c S 550}{J} = W_{w_h,2} + W_{a_h,2} \]  \( (1) \)
and from the definition of the compressor slip factor, the power required to drive the compressor is related to the compressor tip speed as

\[
\frac{hP_c}{(W_t+W_a)J} = \frac{SU^2}{gJ}
\]  

(2)

Also, from the second law of thermodynamics

\[
\frac{SU^2}{gJ} = Q_d + \frac{1}{J} \int \frac{dP}{\rho}
\]  

(3)

where \( \int \frac{dP}{\rho} \) is defined as the compression work per pound of gas compressed or the area of the pressure-volume diagram (fig. 1). Then

\[
\frac{W_w}{W_a} \left( \frac{H_w,2-H_w,1-SU^2}{gJ} \right) + H_a,2-H_a,1 = \frac{SU^2}{gJ}
\]  

(4)

The compression work per pound of gas may be related to the compressor tip speed by the following expression, which also defines the compressor pressure coefficient:

\[
\frac{1}{J} \int \frac{dP}{\rho} = \psi \frac{U^2}{gJ}
\]  

(5)

The compressor polytropic efficiency, defined as the ratio of compression work to shaft work, is

\[
\eta_{p,c} = \frac{1}{J} \int \frac{dP}{\rho} = \frac{\psi}{S} \frac{SU^2}{gJ}
\]  

(6)

Combining equations (3), (5), and (6) gives

\[
Q_d = \left(1-\eta_{p,c}\right)\frac{SU^2}{gJ}
\]  

(7)
The process of compressing air with evaporative cooling is considered polytropic. With this assumption, equation (5) may be integrated to yield

$$\frac{1}{\gamma} \int_{P_1}^{P_2} \frac{dP}{\rho} = \frac{n}{n-1} \frac{RT_1}{\gamma} \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

(6)

$$= \frac{n}{n-1} \frac{\gamma-1}{\gamma} (H_{a,2} - H_{a,1})$$

(9)

The polytropic exponent $n$ may be determined by equating equations (9) and (5), solving for $H_{a,2} - H_{a,1}$, substituting the value obtained in equation (4), and using equation (6). Then

$$n = \frac{\eta_{p,c} \frac{\gamma}{\gamma-1}}{\eta_{p,c} \frac{\gamma}{\gamma-1} - 1+Q}$$

(10)

where $Q$, the ratio of heat removed by evaporation from each pound of compressed air to shaft work, is

$$Q = \frac{W_a}{W_a} \left( H_{w,2} - H_{w,1} - \frac{SU^2}{gJ} \right)$$

(11)

$$Q = \frac{W_a}{W_a} \left( H_{w,2} - H_{w,1} - \frac{SU^2}{gJ} \right) \left( \frac{SU^2}{gJ} \right) \equiv \bar{Q}_{k}$$

(12)

But $H_{w,2}$ is generally indeterminate because of the unknown quantity of unevaporated liquid present in the air leaving the compressor, so that
Because the cooling effectiveness $K$ may vary with compressor-inlet temperature, inlet pressure, inlet relative humidity, and $\bar{\dot{m}}$, it must be experimentally determined. The manner in which the coolant is injected and the average droplet size at the compressor inlet may also affect $K$.

The variation of the polytropic exponent with $Q$ for various values of the compressor polytropic efficiency $\eta_{p,c}$ is shown in figure 2. The polytropic exponent decreases with an increase in compressor polytropic efficiency and with an increase in $Q$. When $Q$ is 1.0, the polytropic exponent is 1.0; that is, the process is isothermal.

When the polytropic exponent is determined, the compressor pressure ratio and compressor-outlet temperature ratio can be obtained from equations (6) and (8), after converting compressor tip speed to Mach number:

$$\frac{P_2}{P_1} = \left( 1 + \frac{n-1}{n} \Psi_2 M_c^2 \right)^n$$

And from the relation between pressure and temperature for a polytropic process

$$\frac{T_2}{T_1} = \left( 1 + \frac{n-1}{n} \Psi_2 M_c^2 \right)$$

**PROCEDURE**

The performance of a turbine-propeller engine incorporating a centrifugal compressor with water injection at the compressor inlet is computed for NACA standard sea-level static conditions at compressor tip speeds of 1200, 1500, and 1800 feet per second and the following constants:

- $H'_{w,c} - H_{w,1} - \frac{SU^2}{\rho g}$ possible cooling per pound of water injected, 1100 Btu per pound
- $K$ cooling effectiveness, 1.0, 0.8, 0.6, 0.4, and values from fig. 3
The compressure pressure and temperature ratios are computed using equations (13) and (14), respectively. Because the slip factor $S$ of a centrifugal compressor generally varies inappreciably with engine speed or with corrected air flow at any given speed, the evaporative cooling during compression is assumed to have a negligible effect on the slip factor. The pressure coefficient $\psi$ is also assumed to be unaffected by water injection.

The experimental data from reference 1, plotted in figure 3, are used to evaluate the effect of cooling parameter $Q$ on the cooling effectiveness $K$. The maximum value of effective cooling parameter $Q$ obtained from the data of reference 1 was 0.43, whereas calculations indicate that a value of $Q$ of 0.685 is necessary for saturation of air with water at the compressor outlet when dry air at standard conditions enters the compressor.

From equation (2), the horsepower required to drive the compressor per pound of air per second is

$$ \frac{\text{hp}_C}{\text{lb}_a} = \frac{\text{SU}^2}{8550} \left( 1 + \frac{W_w}{W_a} \right) $$

(15)

The required fuel-air ratio is

$$ \frac{W_f}{W_a} = \frac{(1 + \frac{W_w}{W_a})^{c_{pb}(T_3-T_2)}}{\eta_b^{c_{pb}(T_3-T_2)}} $$

(16)
where $T_2$ for any values of $Q$ and $U$ is computed with $K$ equal to 1. The fuel-air ratio $W_f/W_a$ for a given $U$ is the same for any value of $K$ because it has little effect on $W_f/W_a$ whether water is evaporated in the compressor or in the burner.

The power of the turbine is computed assuming complete expansion through the turbine. That is, the turbine-outlet stagnation pressure is assumed equal to the compressor-inlet stagnation pressure, so that

$$\frac{P_3}{P_4} = \frac{P_3}{P_2} \frac{P_2}{P_1}$$  \hspace{1cm} (17)

With the pressures and the temperatures at stations 2 and 3 known, the gas flow through the engine, assuming the turbine stator choked, is determined by

$$W_g = \sqrt{\frac{\gamma + 1}{\gamma - 1} \left( \frac{\gamma}{\gamma + 1} \right)} \frac{A_3 P_3}{R} \left( \frac{2}{\gamma} \right)^{\frac{\gamma}{\gamma - 1}} \left( \frac{T_3}{T_3} \right)$$  \hspace{1cm} (18)

Accounting for the coolant vapor and combustion products in the gas at station 3 gives the air flow as

$$W_a = \frac{W_g}{1 + \frac{W_f}{W_a} + \frac{W_w}{W_a}}$$

The horsepower developed by the turbine per pound of air flow per second is

$$\frac{hpt}{W_a} = c_p T_3 \left( 1 - \frac{P_4}{P_3} \right) \eta_t \left( 1 + \frac{W_f}{W_a} + \frac{W_w}{W_a} \right) \frac{778}{550}$$  \hspace{1cm} (19)

where $P_4/P_3$ is obtained from equation (17).
Shaft horsepower is the turbine power minus the compressor power

\[ h_{p_s} = h_{p_t} - h_{p_c} \]

The specific heats at constant pressure and constant volume for the mixture of coolant and air at the burner outlet (neglecting the effect of the combustion products) are, respectively,

\[ c_p = \frac{c_{p,a} W_a + c_{p,w} W_w}{W_a + W_w} \]

and

\[ c_v = \frac{c_{v,a} W_a + c_{v,w} W_w}{W_a + W_w} \]

The ratio of specific heats is

\[ \gamma = \frac{c_p}{c_v} \]

The gas constant for the mixture of coolant and air is

\[ R = \frac{R_w W_w + R_a W_a}{W_a + W_w} \]

The properties of air and water vapor used in the foregoing equations are from standard tables.

RESULTS AND DISCUSSION

Engine-performance characteristics at sea-level static conditions presented herein are based on a turbine-inlet stagnation temperature of 2000° R with compressor tip speeds and water injection rates as operating variables. The relative humidity of the air before entering the compressor inlet is also considered a variable; if the incoming air is saturated, the amount of water that must be injected at the compressor inlet to saturate the air at the compressor outlet is only slightly less than that if dry air at the same temperature and pressure had entered the compressor.
When engine performance with water injection is calculated, the relative humidity can therefore be disregarded except for possible effects of inlet humidity on cooling effectiveness \( K \).

The variation of compressor pressure ratio, compressor horsepower per pound of air per second, turbine horsepower per pound of air per second, and shaft horsepower per pound of air per second with the cooling parameter \( Q \) for various compressor tip speeds and various values of cooling effectiveness \( K \) are shown in figures 4 to 7, respectively. The greatest percentage increase in compressor pressure ratio and horsepower per pound of air for a given \( Q \) occurs at \( K \) of 1.0. The experimental values of \( K \) (from fig. 3) indicate that compressor pressure ratio and horsepower per pound of air rapidly increase with \( Q \) when low water rates are used; then as \( Q \) increases, the compressor pressure ratio and horsepower per pound of air become nearly constant. When better means of evaporating the injected water are applied, this operating line approaches the line where \( K \) is equal to 1.0. Saturation at the compressor outlet occurs at \( Q \) of 0.685.

The variation of shaft horsepower per pound of air per second with the total-liquid - air ratio for various compressor tip speeds and theoretical and experimental values of cooling effectiveness are shown in figure 8. The total liquid flow is the total fuel flow plus the total amount of water injected at the compressor inlet. The shaft horsepower per pound of air increases with the total-liquid - air ratio. The greatest percentage increase in shaft horsepower per pound of air occurs at the higher compressor tip speeds.

The variation of specific liquid consumption with percentage power augmentation for various compressor tip speeds and various values of cooling effectiveness is shown in figure 9. Power augmentation is the augmented power minus the power without water injection divided by the power without water injection. At a compressor tip speed of 1200 feet per second and a value of \( K \) of 1.0, the specific liquid consumption is increased 0.06 pound per horsepower-hour per 1.0-percent increase in shaft horsepower for low water injection rates. At the higher water injection rates, the specific liquid consumption increases 0.01 pound per horsepower-hour per 1.0-percent increase in shaft power. When the compressor tip speed is increased to 1500 feet per second and \( K \) equals 1.0, the specific liquid consumption is increased 0.07 pound per horsepower-hour per 1.0-percent increase in power for low water injection rates. As the water injection rate is increased for the compressor tip speed of 1800 feet per second, the specific liquid
consumption increases 0.005 pound per horsepower-hour per 1.0-percent increase in power. At the low water injection rates, the experimental K curve follows closely to the curve of K equal to 1.0 for all compressor tip speeds. At the higher rates of water injection for all compressor tip speeds, the specific liquid consumption increases approximately 0.1 pound per horsepower-hour per 1.0-percent increase in power.

The variation of the total-liquid - initial-fuel ratio with percentage power augmentation is shown in figure 10. A power augmentation of 25 percent is possible when the total liquid flow (water plus fuel) is twice the initial fuel flow for any compressor tip speed above 1200 feet per second and K is equal to 1.0. For K equal to 1.0, a power augmentation of 100 percent is possible when the total liquid flow is 4.5 times the initial fuel flow for compressor tip speeds of 1500 feet per second or higher. The experimental K values (from fig. 3) show that a power augmentation of 20 percent can be obtained for a compressor tip speed of 1200 feet per second when the total liquid flow is twice the initial fuel flow and a power augmentation of 22 percent can be obtained for a compressor tip speed of 1800 feet per second for the same total-liquid - initial-fuel ratio. When the total liquid flow is 4.5 times the initial fuel flow, a power augmentation of 78 percent can be obtained for a compressor tip speed of 1800 feet per second when the experimental K values are used.

If sufficient water is injected within the compressor to saturate the air at the compressor outlet, power augmentation of 200 percent is possible at a compressor tip speed of 1800 feet per second.

SUMMARY OF RESULTS

The calculated results obtained from a theoretical investigation of water injection treated as a polytropic process in a turbine-propeller engine with a centrifugal compressor for various values of evaporative cooling and compressor tip speeds of 1200, 1500, and 1800 feet per second at sea-level static conditions and constant turbine-inlet temperature of 2000° R indicate that:

1. A power augmentation of 25 percent is possible when the total liquid flow is approximately twice the initial fuel flow for complete evaporation at any compressor tip speed. A power augmentation of 100 percent is possible when the total liquid flow is approximately 4.5 times the initial fuel flow for complete evaporation and compressor tip speeds of 1500 feet per second or higher.
2. A power augmentation of 78 percent is possible when 78 percent of the injected liquid is evaporated and the total liquid flow is 4.5 times the initial fuel flow for a compressor tip speed of 1800 feet per second.

3. A power augmentation of 200 percent is possible if sufficient water is injected within the compressor to saturate the air at the compressor outlet for a compressor tip speed of 1800 feet per second.

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REFERENCES


Figure 1. - Pressure-volume diagram showing effect of coolant injection on compression process for constant shaft power.
Figure 2. - Variation of polytropic exponent with effective cooling parameter for various compressor polytropic efficiencies.
Figure 3. - Variation of cooling effectiveness for various compressor tip speeds with cooling parameter. (Data taken from reference 1.)
Figure 4. - Variation of compressor pressure ratio with cooling parameter for various compressor tip speeds and values of cooling effectiveness. Compressor slip factor, 0.92; compressor pressure coefficient, 0.70; compressor-inlet temperature, 518.6 \degree R; compressor-inlet pressure, 14.7 pounds per square inch absolute.
Figure 4. - Concluded. Variation of compressor pressure ratio with cooling parameter for various compressor tip speeds and values of cooling effectiveness. Compressor slip factor, 0.92; compressor pressure coefficient, 0.70; compressor-inlet temperature, 518.6° R; compressor-inlet pressure, 14.7 pounds per square inch absolute.
Figure 5. - Variation of power required to drive compressor per pound of air per second with cooling parameter for various compressor tip speeds. Compressor slip factor, 0.92; compressor-inlet temperature, 518.6 °R; compressor-inlet pressure, 14.7 pounds per square inch absolute.
Figure 6. - Variation of power developed by turbine per pound of air per second with cooling parameter for various compressor tip speeds and cooling effectiveness. Compressor slip factor, 0.92; compressor pressure coefficient, 0.70; turbine efficiency, 0.825; compressor-inlet temperature, 518.6° R; compressor-inlet pressure, 14.7 pounds per square inch absolute; turbine-inlet temperature, 2000° R.
(c) Compressor tip speed, 1800 feet per second.

Figure 6. - Concluded. Variation of power developed by turbine per pound of air per second with cooling parameter for various compressor tip speeds and cooling effectiveness. Compressor slip factor, 0.92; compressor pressure coefficient, 0.70; turbine efficiency, 0.825; compressor-inlet temperature, 518.6° R; compressor-inlet pressure, 14.7 pounds per square inch absolute; turbine-inlet temperature, 2000° R.
(a) Compressor tip speed, 1200 feet per second.

(b) Compressor tip speed, 1500 feet per second.

Figure 7. - Variation of shaft horsepower per pound of air per second with cooling parameter for various compressor tip speeds and cooling effectiveness. Compressor slip factor, 0.92; compressor pressure coefficient, 0.70; turbine efficiency, 0.825; compressor-inlet temperature, 518.6° R; compressor-inlet pressure, 14.7 pounds per square inch absolute; turbine-inlet temperature, 2000° R.
Figure 7.— Concluded. Variation of shaft horsepower per pound of air per second with cooling parameter for various compressor tip speeds and cooling effectiveness. Compressor slip factor, 0.92; compressor pressure coefficient, 0.70; turbine efficiency, 0.825; compressor-inlet temperature, 518.6° R; compressor-inlet pressure, 14.7 pounds per square inch absolute; turbine-inlet temperature, 2000° R.
(a) Compressor tip speed, 1200 feet per second.

(b) Compressor tip speed, 1500 feet per second.

Figure 8. - Variation of shaft horsepower per pound of air per second with total-liquid-air ratio for various compressor tip speeds and cooling effectiveness. Compressor slip factor, 0.92; compressor pressure coefficient, 0.70; turbine efficiency, 0.825; compressor-inlet temperature, 518.6° R; compressor-inlet pressure, 14.7 pounds per square inch absolute; turbine-inlet temperature, 2090° R.
Figure 8. - Concluded. Variation of shaft horsepower per pound of air per second with total-liquid-air ratio for various compressor tip speeds and cooling effectiveness. Compressor slip factor, 0.92; compressor pressure coefficient, 0.70; turbine efficiency, 0.825; compressor-inlet temperature, 518.6° R; compressor-inlet pressure, 14.7 pounds per square inch absolute; turbine-inlet temperature, 2000° R.
Figure 9. - Variation of specific liquid consumption with power augmentation for various compressor tip speeds and cooling effectiveness. Compressor slip factor, 0.92; compressor pressure coefficient, 0.70; turbine efficiency, 0.825; compressor-inlet temperature, 518.6°F; compressor-inlet pressure, 14.7 pounds per square inch absolute; turbine-inlet temperature, 2000°F.
Figure 9. - Concluded. Variation of specific liquid consumption with power augmentation for various compressor tip speeds and cooling effectiveness. Compressor slip factor, 0.92; compressor pressure coefficient, 0.70; turbine efficiency, 0.825; compressor-inlet temperature, 518.6°F; compressor-inlet pressure, 14.7 pounds per square inch absolute; turbine-inlet temperature, 2000°F.
Figure 10. - Variation of total-liquid - initial-fuel ratio with power augmentation. Compressor slip factor, 0.92; compressor pressure coefficient, 0.70; turbine efficiency, 0.825; compressor-inlet temperature, 518.60°F; compressor-inlet pressure, 14.7 pounds per square inch absolute; turbine-inlet temperature, 2000°F.
Figure 10. - Concluded. Variation of total-liquid - initial-fuel ratio with power augmentation. Compressor slip factor, 0.92; compressor pressure coefficient, 0.70; turbine efficiency, 0.825; compressor-inlet temperature, 518.6\(^\circ\) R; compressor-inlet pressure, 14.7 pounds per square inch absolute; turbine-inlet temperature, 2000\(^\circ\) R.