STATISTICAL STUDY OF OVERSTRESSING IN STEEL

By G. E. Dieter, G. T. Horne, and R. F. Mehl

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SUMMARY

The effect of overstressing on the fatigue properties of SAE 4340 steel has been studied statistically. The effect of microstructure on the susceptibility to reduction in fatigue life due to cycles of over-stress was investigated. When tested at equivalent percent stress, the quenched and spheroidized structure was found to be more susceptible to fatigue damage.

The effect of overstress on the endurance-limit statistics was studied for the quenched and spheroidized structure. Enough specimens were tested to determine the endurance-limit statistics of damaged specimens by the probit method. The decrease in the mean endurance limit due to cycles of overstressing was much greater than would be expected from nonstatistical investigations which are reported in the literature. The effect is interpreted as support for the belief that the bulk of the fatigue damage takes place before the first 50 percent of the total fatigue life.

INTRODUCTION

One object of the statistical studies of the fatigue of metals carried out in this laboratory over the past 4 years has been to investigate various fatigue phenomena from a statistical viewpoint to see whether they are truly "facts of fatigue." An example of a case where a new interpretation arose from statistical study was the work of Epremian and Mehl (ref. 1) on the understressing phenomenon. Their investigation showed that improvement in fatigue life due to understressing, commonly attributed to the effects of beneficial cold-work, could be explained by statistical selectivity in materials which did not undergo strain-induced reactions.

Another important phenomenon in need of statistical study was the effect of cycles of overstressing on the fatigue life and the endurance limit. This represents a problem of extreme importance in the aircraft industry where sudden repeated overloading due to gust loads may be expected. An excellent survey of this and other fatigue problems
associated with aircraft structures has recently been presented by Dryden, Rhode, and Kuhn (ref. 2). This is a problem of extreme complexity which must be understood in simplified laboratory tests before the practical problem can be attacked. Accordingly, the purpose of this investigation was to study overstressing from a statistical viewpoint with the object of assessing the reliability of conclusions drawn from nonstatistical investigations and with the hope that the statistical approach would provide some new information which would aid in explaining this phenomenon.

Before proceeding further it is important to define the terminology which will be used throughout this report. Overstressing refers to testing a virgin specimen for some number of cycles less than that required for failure at a stress above the endurance limit and then subsequently running the specimen to failure at another stress. The initial stress to which the specimen is subjected is called the prestress; the final stress is called the test stress. The ratio of the number of cycles of overstress at the prestress to the mean virgin fatigue life at this same stress is called the cycle ratio \( R \).

Fatigue damage, which ultimately results in a crack and subsequent failure, occurs at some stage in the fatigue test. The nature of this damage has never been precisely described and all attempts to measure it by supplementary methods such as X-ray and damping measurements have met with little success. The only method available at the present time for obtaining information on fatigue damage is actually to carry out fatigue tests. The literature on such tests is extensive and has been reviewed in the appendix to this report. The various methods used by earlier investigators for measuring damage are discussed in view of more recent knowledge of the statistical nature of fatigue.

This statistical investigation of overstressing consisted of two separate parts. The first was an investigation of the effect of metallurgical structure on the fatigue damage produced by overstressing. Fatigue damage was measured by the percentage decrease in fatigue life at the test stress for specimens subjected to varying cycle ratios of fatigue damage at the prestress. The second part of this investigation was a study of the effect of a certain amount of overstressing on the endurance limit and its statistics.

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EXPERIMENTAL WORK

Materials and Preparation of Specimens

This series of investigations was carried out with SAE 4340 steel from the same heat as that used by Epremian and Mehl (ref. 1). This was an aluminum-killed, basic electric-furnace heat with the following chemical composition:

<table>
<thead>
<tr>
<th>C</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Si</th>
<th>Cr</th>
<th>Ni</th>
<th>Mo</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>0.83</td>
<td>---</td>
<td>0.27</td>
<td>0.77</td>
<td>1.82</td>
<td>0.37</td>
<td></td>
</tr>
</tbody>
</table>

Extensive tests by Epremian and Mehl showed that the steel had a low susceptibility toward quench-cracking. Specimen blanks were austenitized for 2 hours at 845°C and then quenched in oil. They were then tempered to the same strength levels as those used by Epremian and Mehl. One group of blanks designated quenched and tempered (Q & T) was tempered to approximately 170,000-psi tensile strength by heating for 16 hours at 525°C. The second group, quenched and spheroidized (Q & S), was tempered to a tensile strength of roughly 110,000 psi by heating for 16 hours at 675°C. The microstructures obtained from the two heat treatments are shown in figures 1 and 2. The mechanical properties corresponding to these structures are listed below:

<table>
<thead>
<tr>
<th>Structure</th>
<th>Ultimate strength, psi</th>
<th>Yield strength, psi</th>
<th>Elongation, percent</th>
<th>Reduction in area, percent</th>
<th>Rockwell hardness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quenched and tempered</td>
<td>171,000</td>
<td>150,500</td>
<td>17</td>
<td>54.6</td>
<td>C-39</td>
</tr>
<tr>
<td>Quenched and spheroidized</td>
<td>109,900</td>
<td>94,700</td>
<td>26</td>
<td>50</td>
<td>B-98</td>
</tr>
</tbody>
</table>

Cantilever-beam fatigue specimens of rectangular cross section of the same type and dimensions as those used by Epremian and Mehl were machined from the heat-treated blanks. The maximum stress section near the fillet was 0.500 by 0.250 inch. The specimens were polished on No. 100, 240, and 600 abrasive belts. They were then stress-relieved in vacuum for 1 hour at a temperature 50°C below the tempering temperature. The final operation was a light polish with 3/0 metallographic paper in the longitudinal direction.
Testing Equipment and Procedure

General experimental procedure. - All specimens were tested in pneumatic vibrating cantilever fatigue machines. These machines have been previously described by Quinlan (ref. 3) and Epremian and Mehl (ref. 1). This machine pneumatically vibrates a cantilever specimen at its natural frequency of vibration. The criterion for failure is the formation of a small crack, corresponding to a drop in the natural frequency of the specimen of about 2 percent which actuates the automatic shutdown device. This corresponds to a crack length of from 0.1 to 0.2 inch, measured across the width of the specimen, for the stresses used in this investigation.

It was felt that the scatter in results could be minimized by using a finer criterion for failure. Further, in overstressing tests where part of the test is run at one stress and then completed at another stress, the possibility of a crack forming during the prestress must be eliminated. The procedure adopted was to observe the critically stressed section of the specimen with oblique light during the entire course of the test. The first small crack was readily detectable with the unaided eye. Measurements of crack length showed that it varied from 0.015 to 0.030 inch. Simultaneous readings of frequency of vibration showed that the appearance of this crack corresponded to a drop in frequency of 50 to 75 cpm. This drop in frequency could not be used as a criterion of failure owing to small unpredictable changes in frequency due to relaxation in the clamping grips which often occurred at the outset of a test. This method of detecting a fatigue crack was possible only because the duration of the fatigue test at the high stresses used in the overstressing investigation was relatively small. The significance of this criterion of failure will be discussed later under "Experimental Results and Analysis."

The stress applied to a fatigue specimen was calculated from equation (1) (see ref. 1 for derivation):

\[ S = K A_m f^2 \]  \hspace{1cm} (1)

where

- **K** constant, determined from dimensions of specimen and its experimentally determined deflection curve
- **A_m** semiamplitude of vibration at free end of specimen, in.
- **f** natural frequency of vibration, cps
The stress for both quenched and tempered and quenched and spheroidized specimens was given by equation (2):

\[ S = 30.69 \ A_m f^2 \] (2)

It will be noted that frequency enters this equation as a squared term and therefore has an important contribution in determining the stress. The frequency of vibration was measured with a Frahm vibrating-reed tachometer, accurate to 0.5 percent of the frequency reading and with a sensitivity of 25 cpm. The tachometer was checked against a standard precision tuning fork at the frequency of vibration of the specimens. The natural frequency of vibration of a cantilever-beam specimen with a piston clamped at one end can be approximated rather well by equation (3) (ref. 4):

\[ f = \frac{1}{2\pi} \sqrt{\frac{3EI}{L^3(M + 0.23m)}} \] (3)

where

- \( f \) natural frequency, cps
- \( E \) modulus of elasticity, psi
- \( I \) moment of inertia, in.\(^4\)
- \( L \) length of specimen, in.
- \( M \) mass of piston, slugs
- \( m \) mass of specimen, slugs

Equation (4) gives the change in frequency due to changes in the factors influencing the frequency:

\[ \text{Percentage change in frequency} = 100 \left( \frac{df}{f} \right) = \frac{100}{2} \left( \frac{dE}{E} + \frac{db}{b} + \frac{3dh}{h} + \frac{3dL}{L} + \frac{dM}{M + 0.23m} + \frac{0.23dm}{M + 0.23m} \right) \] (4)

where

- \( b \) width of specimen, in.
- \( h \) thickness of specimen, in.
Consideration of this problem shows that changes in thickness and length of the specimen, and slight differences between the weights of the pistons, would cause the greatest variation in frequency from specimen to specimen. In addition, variations in clamping pressure are thought to influence the frequency. A maximum variation in frequency of about 3.5 percent was calculated from equation (4) on the basis of the dimensions of the specimen and their estimated variation. The actual measured variation in frequency was about 4.5 percent. The mean frequency was 7,650 cpm. At a stress of 68,000 psi a variation in frequency of 4.5 percent would result in a change in stress of about 6,000 psi. Such a condition cannot be tolerated. Therefore, the procedure adopted was to change the amplitude depending on the measured frequency to give a constant stress. This same procedure was used in the previous work on aluminum alloys with these machines (ref. 5), but was not used by Epremian and Mehl (ref. 1).

Further, tests of surplus SAE 4340 steel specimens remaining from the work of Epremian and Mehl (ref. 1) showed that they vibrated with a frequency of 127.5 cps (7,650 cpm). A check of their calculations showed that all of their stresses were calculated on a basis of a natural frequency of 133 cps. A sufficient number of these specimens were tested to verify the fact that this difference was real. Accordingly, the stresses shown on their curves for SAE 4340 steel should be multiplied by a factor of 0.919, or \((127.5)^2/(133)^2\). Their fatigue lives should be multiplied by a factor of 0.958. Such a correction will not influence the position of log \(N\) on the plotted S-N curves. However, the fact that all specimens were assumed to have a constant frequency may be of more consequence, since it is possible that the specimens were tested over a narrow spectrum of stress rather than at a constant stress. Unfortunately, there is no method of correcting for this. No change was found in the conclusions made by Epremian and Mehl when their stresses were corrected as indicated above.

Effect of microstructure on overstressing damage. - It was originally planned to utilize the fatigue statistics of this steel determined by Epremian and Mehl (ref. 1). However, the factors mentioned above made it desirable to determine anew the virgin fatigue life at the stresses used in this investigation. This part of the program was to investigate the effect of metallurgical structure on the damage produced by overstressing using a statistical size sample so that conclusions could be drawn with some degree of confidence. To do this it was necessary to use some reference level upon which to base a comparison between the two structures.

It seems logical that the endurance limit (specifically, the mean endurance limit \(S\)) should be related in some way to the level of reference since experience shows that the damage increases as the stress is increased above the endurance limit. Previous workers have simply tested two materials at equal percentages of their endurance limits. However,
it seemed desirable to include the ultimate tensile strength in this parameter of equivalence since the overstressing damage most certainly increases rapidly as the stress approaches the tensile strength; further, the mean endurance limit is not always a fixed percentage of the ultimate tensile strength. The prestress and test stress for the two structures were taken at approximately equal values of a parameter which will be called the percent stress. It will be noted that this is the same parameter which was suggested by Epremian and Mehl (ref. 1) as a new method for plotting fatigue data. This percent stress is given by:

\[
S^* = \left( \frac{S - \bar{S}}{S_u - \bar{S}} \right) \times 100
\]  

where

- \( S \)  test stress or prestress, psi
- \( \bar{S} \)  mean endurance limit, psi
- \( S_u \)  ultimate tensile strength, psi

The values of prestress and test stress used for the two structures are shown in table I. For the materials used in this investigation there was no real difference between equivalent percent stress and equivalent percentages of the mean endurance limit.

The stress levels used in this work were selected to correspond with the stresses at which the fatigue statistics for these materials had been determined by Epremian and Mehl. The lower stress was chosen to be above the 2\( \sigma \) limit of the mean endurance limit, while the limiting condition on the prestress was the maximum amplitude attainable for the quenched and tempered structure.

Approximately 20 specimens of each structure were tested at the prestress for cycle ratios of 0.10, 0.30, and 0.60. The cycle ratio was based on the mean fatigue life of virgin specimens tested at the same stress where the criterion of failure was the appearance of a crack about 0.020 inch long. At the completion of the cycle ratio the amplitude was decreased rapidly to the test stress and the specimen run to failure, as indicated by the first visible small crack. The mean life of the prestressed specimens was determined and used in calculating the percent damage by equation (6). The test procedure is illustrated by figure 3, where AC represents the prestress and DF the test stress. The cycle ratio at the prestress is AB/AC while the damage produced by this amount of overstress is EF/DF. The percent damage \( D \) is given by the following equation:
\[ D = \left( \frac{N_{OT} - N_{P}}{N_{OT}} \right) \times 100 \]  

where

\( N_{OT} \)  \text{ antilogarithm of log } N \text{ for virgin specimens tested at test stress} 

\( N_{P} \)  \text{ antilogarithm of log } N \text{ for specimens prestressed and tested to failure at test stress} 

**Effect of overstressing on endurance limit.** - In the second part of this investigation the influence of a certain amount of overstress on the endurance-limit statistics was studied for the quenched and spheroidized structure. Enough specimens were prestressed at 68,600 psi for cycle ratios of 0.30 and 0.60 to determine the endurance-limit statistics of these damaged specimens by the probit method. The probit method has been discussed in a previous report (ref. 1). Complete details for making a probit analysis are given by Finney (ref. 6).

**EXPERIMENTAL RESULTS AND ANALYSIS**

**Effect of Microstructure on Overstressing Damage**

The fatigue statistics based on 20 specimens tested to failure at the prestress and the test stress for each structure are given in table II. These data are based on the "first crack" criterion of failure. The antilogarithm of log \( N_{O} \), \( N_{O} \), was used in each case to calculate the cycle ratio at the prestress and the damage at the test stress.

The mean fatigue life and its standard deviation for groups of specimens given different cycle ratios at the prestress and tested to the appearance of the first visible crack at the test stress are listed in table III. The percent of damage due to the specific cycle ratios of overstress expressed as percent reduction in the virgin life at this stress (log \( N_{O} \)) is also tabulated.

The \( t \) test (ref. 7) for a significant difference between the means of two samples was applied to log \( N_{OT} \) and log \( N_{P} \) at each cycle ratio. The modified \( t \) test for the case where the variances of the two samples are not equal was used where applicable. For both structures it was found that the difference between log \( N_{OT} \) and log \( N_{P} \) at \( R = 0.30 \)
and 0.60 was significant at the 5-percent level of significance. In both cases the difference was not significant at $R = 0.10$. Table III also shows that the variability in $\log N$ was greater for the specimens which were given cycles of overstress. There is also a trend toward smaller scatter in the life of overstressed specimens with increasing cycle ratio.

The effect of these prestressing sequences on the resulting damage expressed as reduction in life at the test stress is shown in figure 4. The curves show the damage at the test stress plotted against the cycle ratio at the prestress. The curves for both structures lie above the 45° line, indicating that the cycle ratio of prestress produced greater than proportionate damage to the fatigue life at the test stress. The curve for the quenched and spheroidized structure lies above that of the quenched and tempered structure, indicating that it was more susceptible to damage resulting from overstressing.

Influence of Criterion of Failure on Fatigue Life

As was indicated previously, it was found early in this investigation that the scatter in fatigue life could be reduced considerably at the high stresses for the overstressing tests by refining the end point by which failure was defined. Table IV compares the fatigue-life statistics based on the appearance of the first visual crack of from 0.015 to 0.030 inch in length with those based on automatic shutdown of the machines for cracks of from 0.1 to 0.2 inch in length.

The difference in scatter due to the two criteria of failure was analyzed with the F test of significance at the 5-percent level of confidence. The difference between $\log N$ obtained with the two end points was investigated with the t test at the 5-percent level of significance. The results are summarized in tables V and VI.

These tables show that the finer criterion of failure reduced the scatter in fatigue life, although the mean fatigue life was not significantly affected.

Effect of Overstressing on Endurance-Limit Statistics

For this part of the investigation quenched and spheroidized specimens of SAE 4340 were prestressed at 68,600 psi and then tested at lower stresses within the statistical range of the endurance limit. The endurance-limit statistics of these damaged specimens were determined by the probit method.
The virgin endurance-limit statistics determined by Epremian and Mehl (ref. 1) for specimens of identical shape, composition, and mechanical properties were used for comparison with the endurance-limit statistics of the damaged specimens. In using these data the values of stress were multiplied by 0.919 to correct for the error in frequency which was discussed earlier. The virgin endurance-limit statistics are listed in table VII together with the values obtained for specimens subjected to various prestress treatments.

The first group of specimens was given a cycle ratio of 60 percent at 68,600 psi in order to produce the greatest amount of damage at the prestress without running the risk of specimens failing prematurely because of overlapping with the distribution of fatigue life for specimens tested to failure at this stress. The percentage of failures for groups of 20 specimens tested at each stress is indicated in table VII. The very pronounced decrease in the mean endurance limit and the large standard deviation were not expected. At face value these statistics indicate that there is no endurance limit in the statistical sense. Substituting the values of $S$ and $\sigma$ from table VII into the equation of the probit regression line, it is found that there is about a 25-percent probability of failure at zero stress. Two facts may account for this situation. The determination of the probit regression line from these data is quite uncertain since at all of the stresses the percentage of failures was greater than 50 percent. Therefore, extrapolation of the probit regression line to zero stress is of doubtful validity. Secondly, the possibility that at a cycle ratio of 60 percent small cracks, not readily visible, might have been produced cannot be overlooked. These cracks would open up during testing at a lower stress and cause a greater percentage of failures than would be found otherwise. Therefore, it was concluded that the decrease in the endurance limit produced by prestressing was marked, but it could not be determined whether the effect could be attributed to fatigue damage alone.

A second group of specimens was prestressed at 68,600 psi for a cycle ratio of 30 percent. The probability of a specimen containing a visible crack after this number of cycles at 68,600 psi was about 1 in 1,000,000. These specimens were then tested at the same low stresses used with the 60-percent cycle-ratio specimens. The percentage failures at each stress are indicated in table VII. The mean endurance limit for this case is also markedly decreased below that of virgin specimens given no prestressing treatment. The standard deviation of $S$ is also increased, but not to the degree found for the specimens given a 60-percent cycle ratio. Thus it appears that the endurance limit is more sensitive to overstressing damage than has been reported heretofore in the literature.
DISCUSSION

The results of the statistical study of the effect of cycles of over-stress on the fatigue life showed no unusual effects not previously observed in nonstatistical investigations. This investigation resulted in the same type of curve of damage versus cycle ratio as found by Kommers (ref. 8) and Bennett (ref. 9). Their conclusion that cycles of over-stressing produced greater than proportionate damage to fatigue life at a lower test stress has been confirmed statistically.

None of these investigators, however, made a direct study of the effect of metallurgical structure on the sensitivity to fatigue damage. The statement is found in the literature that harder, stronger steels are much more sensitive to damage than very ductile, low-strength steels. Such conclusions have been reached from the comparison of overstressing tests on normalized steels with tensile strengths under 100,000 psi. The comparisons were not made on the same steel and on a common strength basis, such as equal percentages above the endurance limits.

The present investigation shows that when the same steel is heat-treated to two structures with different tensile strengths the weaker, more ductile structure is more susceptible to fatigue damage. It may be argued that there is some effect due to the fact that the difference between the prestress and the test stress is smaller for the quenched and spheroidized structure. However, it has been shown several times in non-statistical overstressing tests, and more recently in the statistical investigation of Dolan and Brown (ref. 10) on aluminum, that the curve of damage versus cycle ratio approaches the 45° line more closely the smaller the difference between these two stresses. Thus, it would be expected that increasing the spread between prestress and test stress for the spheroidized structure or decreasing the spread for the tempered structure would only accentuate the difference between the two damage curves. The use of a finer criterion for failure has been shown to reduce the scatter in fatigue life, without, in general, significantly affecting the mean fatigue life.

The investigation of the effect of overstressing on the endurance-limit statistics shows a far greater reduction of the endurance limit due to fatigue damage than has been reported in nonstatistical investigations. In this investigation a 30-percent cycle ratio of fatigue damage at 68,600 psi (19.5 percent above the mean endurance limit) resulted in a decrease of 28 percent in the mean virgin endurance limit. Bennett's results (ref. 9), obtained by conventional methods for SAE 4130 steel heat-treated to about the same mechanical properties, showed that a cycle ratio of 33.3 percent at 54,000 psi (15 percent above the virgin endurance limit) resulted in a decrease in the endurance limit of only 5 percent.
Bennett's curves showing the effect of various cycle ratios of overstressing on the endurance limit are given in figure 5.

The smaller decrease in the endurance limit of damaged specimens found in nonstatistical investigations can perhaps be attributed to the poor ability of the conventional fatigue test in accurately predicting the true endurance limit. It is quite possible at a high stress not too far below the virgin endurance limit, where only a few run-outs would be obtained out of a sample of 20 damaged specimens, that one of the run-outs would be found on the first or second specimen tested. Thus, if a single specimen tested at this stress happened to be such a superior specimen and run out, then this stress would be taken as the endurance limit. Support for this idea is found in the fact that in all of the groups of 20 prestressed specimens a run-out was found among the first three specimens tested at each stress. It is also possible that a certain amount of bias is introduced in the determination of the endurance limit of prestressed specimens when the statistical nature of the endurance limit is not recognized. For example, if a specimen subjected to a cycle ratio of prestress of 50 percent runs out at a higher stress than a specimen given only a 30-percent cycle ratio, as is possible when only a few specimens are tested, then the data for the latter specimen would likely become suspect.

Thus, it is felt that the marked decrease in the endurance limit due to previous cycles of overstressing is a real effect. The fact that a cycle ratio of only 30 percent produced such a noticeable decrease in the mean endurance limit is taken as evidence that the nucleus of the fatigue crack occurs early in the fatigue process, at least within the first 30 percent of the total fatigue life. The fatigue damage produced during this relatively short time is of such a nature that it becomes a visible crack at the lower stress.

There has been growing support for the idea that the fatigue crack forms early in the life of a fatigue specimen. Ferguson (ref. 11) reported that fatigue cracks could be clearly seen by the eye when lead-alloy fatigue specimens which had been stopped after about 10 percent of their total fatigue life were stretched less than 5 percent in tension. Love (ref. 12), using the electron microscope, observed small cracks at both the grain boundaries and along slip bands in ingot iron fatigue specimens after 100 to 1,000 cycles at a stress where the specimens broke after about $6 \times 10^5$ cycles. Fenner, Owen, and Phillips (ref. 13) reported that fine cracks could be detected at the root of a very sharp notch after about 1,000 to 2,000 cycles. Failure did not occur until $2 \times 10^7$ cycles.

Sinclair and Dolan (ref. 14) used a recrystallization technique for studying the progress of fatigue damage. They postulated three stages for the fatigue process. The first stage consists of slip and fragmentation of the grains, accompanied by work-hardening. The second stage was
considered to be produced by a "statistical disruption of high-energy bonds in severely cold-worked metal" which results in the formation of submicroscopic cracks. The third stage consisted of a coalescence of the submicroscopic cracks to form visible spreading cracks which result ultimately in failure. It was felt that fatigue damage occurred only in the second and third stages and that the latter was of relatively short duration compared with the total number of cycles required for failure. However, very little was known about the relative duration of the first and second stages.

It was reasoned that a fatigue specimen would never fail if it were given recrystallization treatments at regular intervals less than the interval required for the completion of the work-hardening stage, the first stage. Alpha brass fatigue specimens were given an initial anneal at 750° F for 4 hours. Fifteen specimens were then tested at a constant stress to determine the virgin fatigue-life statistics. Other groups of 10 specimens tested at the same stress were removed from the fatigue machines at intervals of 20 and 50 percent of the mean virgin fatigue life and given an anneal for 1/2 hour at 750° F. These recrystallization treatments were repeated at these regular intervals until failure occurred. The mean fatigue lives of the specimens given the recrystallization treatment were not significantly different from those for the virgin specimens. Thus, Sinclair and Dolan concluded that the first stage of fatigue during which only work-hardening is believed to occur is of short duration. Fatigue damage is initiated at a relatively early stage of the total fatigue life. They further concluded that the only kind of fatigue damage which would not be repaired by recrystallization is a very fine crack.

Thus, in the face of this evidence the large decrease in the endurance limit due to overstressing is not too surprising a result. It is proposed that the nuclei of fatigue damage form early in the fatigue process. There is evidently considerable statistical variation associated with their formation. Those nuclei which have reached some critical size during the cycles of prestress grow into visible cracks upon subsequent testing at a lower stress. Those specimens for which the nuclei have not reached the critical size at the prestress become run-outs upon testing at the lower stress. Thus, fatigue can be considered as a nucleation and growth process.

CONCLUSIONS

A statistical study of the effect of overstressing on the fatigue properties of SAE 4340 steel indicated the following conclusions:
1. Statistical investigation has shown that a quenched and spheroidized structure is more susceptible to overstressing damage than a quenched and tempered structure.

2. The curve of damage versus cycle ratio obtained by statistical tests agrees with those reported in nonstatistical investigations. Therefore, nonstatistical investigations of the effect of fatigue damage on fatigue life are, in general, believed to be reliable.

3. The scatter in fatigue life has been shown to be reduced significantly by using a more precise criterion of failure.

4. The effect of cycles of overstress on the endurance limit has been shown by statistical tests to be far greater than heretofore reported. The endurance limit is markedly reduced by a small number of cycles of overstress. This is interpreted as further evidence that fatigue damage, and probably the fatigue crack, forms early in the total life of a fatigue specimen.

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SUPPLEMENTARY METHODS OF MEASURING FATIGUE DAMAGE

It is presumed that fatigue damage occurs during overstressing at the prestress. The nature of this damage has never been precisely described and all attempts to measure it by some supplementary method have met with little success. Cycles of overstress produce little change in the static tensile properties, the only effects appearing to be a slight increase in the yield point and a decrease in the elastic limit (ref. 15; see ref. 16 for a detailed summary). Kies and Holhauser (ref. 17) attempted to measure the progress of fatigue damage in overstressing by supplementary tension-impact tests. However, no loss in tension-impact resistance could be detected until a crack was formed in the specimen.

With the advent of the widespread use of X-ray methods for studying the internal structure of metals numerous attempts were made to adapt these techniques to the detection of fatigue damage. Conflicting reports of success and failure in detecting damage with X-rays have been made. Barrett (ref. 18), in summarizing this work, states that X-ray methods do not give a reliable prediction of fatigue failure. The blurring of spots with increasing number of cycles is associated with plastic deformation within the crystal which is of the same nature whether the deformation is the result of static or repeated loading. Further, this evidence of plastic deformation cannot differentiate between safe and unsafe stress amplitudes. Norton (ref. 19), in a recent survey of fatigue damage, confirms this opinion.

Damping capacity, the amount of work dissipated into heat by a unit volume of material during a completely reversed cycle of unit stress, should be in some way closely connected with fatigue since it is a property which is sensitive to localized plastic deformation within the metal. In general, no reliable correlation between damping capacity and fatigue has been observed. However, Hanstock (ref. 20), using a very sensitive damping apparatus, has reported an increase in damping capacity prior to fatigue failure. Damping-capacity studies appear to be a promising method of obtaining information on the effect of overstress damage in fatigue. Such work would require very careful instrumentation.

The only method available, therefore, for obtaining information on fatigue damage is actually to carry out fatigue tests. Unfortunately, this provides little information about the basic nature of fatigue damage. Further, the usual scatter in fatigue tests is usually compounded by running tests at more than one stress level. Several methods have been used for evaluating fatigue damage, each with its own criterion for measuring.
damage. These will be discussed below with respect to their applicability in view of the statistical nature of fatigue.

Overstressing Tests on Steel

French (ref. 21) made one of the first investigations of the effect of overstress. He defined a "damage line" which was obtained by running several specimens at a given stress for different numbers of cycles. The stress was then reduced to the virgin endurance limit \( S_0 \) and the specimen tested to see if it would break in \( 1 \times 10^7 \) cycles. Specimens which broke upon subsequent testing at \( S_0 \) were said to have undergone damage at the prestress. The procedure was repeated for several stresses and a damage curve drawn to delineate the region of prestress for which specimens had suffered damage. Figure 6 illustrates the method and the type of curve presented by French and his followers.

Such a procedure tells nothing about the degree of damage produced by varying overstress treatments. This led subsequent investigators to adopt other methods for evaluating the effect of cycles of overstress on the fatigue properties. A more serious objection to French's damage line can be raised in light of the statistical nature of the endurance limit. There is no assurance that a specimen which failed at \( S_0 \) after prior cycles at some prestress would not have broken at \( S_0 \) even if it had not been subjected to overstress. To use this method with full recognition of the statistical nature of fatigue the endurance-limit statistics should be known for the virgin material and at least 20 specimens tested at each combination of prestress and cycle ratio. The number of specimens required makes this method impractical for statistical tests.

Kommers (ref. 16) presented a logical extension to French's method. His method of evaluating damage due to overstressing was to subject enough specimens to a given cycle ratio at a certain prestress to determine the new endurance limit of the damaged specimens. The percentage decrease or increase in the endurance limit was used as the measure of damage. Kommers tested three plain carbon steels and a cast iron by this method. His results were plotted as curves of percent damage versus cycle ratio. In most instances he obtained a nearly linear relation between the two variables. In a later paper (ref. 22) Kommers extended his work to ingot iron and SAE 1030 steel. For annealed ingot iron he found the curious fact that for a 10-percent prestress the damage showed progressive recovery with increasing cycle ratio up to \( R = 58 \) percent. This was the only definite case of recovery of damage observed by Kommers. This method is also suspect because of the statistical nature of the endurance limit. It is quite probable that errors were introduced in the cycle ratio of prestress because of ignorance of the mean life of the virgin material and more serious errors exist in determining an endurance limit with only a few specimens.
In this same paper Kommers reported on understressing tests on annealed ingot iron with an endurance limit of 26,200 psi. Specimens were understressed at 26,000 psi up to $6 \times 10^7$ cycles and then tested at higher stresses. The increase in the endurance limit with increase in the number of cycles of understress was definite, but not regular. It was noted that annealed ingot iron, as compared with cast iron, required comparatively long runs of understressing to obtain substantial increases in fatigue strength. No investigation was made of the effect of initial degree of understressing on the increase in endurance limit. Kommers also used a coaxing procedure in his study of understressing. Specimens were run for some number of cycles of understress and the stress was then increased by some percentage of the virgin endurance limit and the specimens were run for an additional number of cycles. This procedure was repeated, increasing the stress in uniform increments at periods of 1 or 2 days. After this coaxing treatment the stress at which fracture finally occurred was considerably above the original endurance limit.

Müller-Stock (ref. 23) presented a method of evaluating fatigue damage which has been used frequently by subsequent investigators. He subjected specimens to different cycle ratios at a prestress and then tested the specimens to failure at a higher test stress. The decrease in fatigue life of prestressed specimens at this higher test stress from that expected for virgin specimens at the test stress was taken as the measure of fatigue damage. Müller-Stock, recognizing that the scatter in number of cycles to failure was very great, tested 200 specimens at his test stress to determine the extent of the scatter and fix accurately the location of the virgin life at that stress. He also determined the effect of over-stress on the entire S-N curve by testing a number of specimens for fixed cycle ratios at the prestress and then using these specimens to determine the entire S-N curve. He thus obtained a family of S-N curves for each different cycle ratio at a certain prestress.

Kommers (ref. 8) adopted the method first used by Müller-Stock for further study on over-stressing. He presented his results as curves of percent damage versus cycle ratio. He reported a large amount of scatter in the data, making it necessary for him to draw average curves for the results. His tests indicated that for a prestress higher than the test stress the damage at the test stress may in some cases be equal to the damage at the prestress, while in other cases the higher prestress may cause a greater percentage of damage at the lower test stress. A low prestress for a given cycle ratio followed by test to failure at a higher stress generally showed that the damage at the lower stress caused a smaller percentage at the higher stress. For both combinations of over-stress mentioned above, Kommers reported that steels of higher tensile strength and lower ductility were more sensitive to over-stress than weaker steels with greater ductility.
Bennett (ref. 9) made a very thorough study of overstressing, using both notched and smooth specimens of SAE X4130 steel tested on R. R. Moore machines. Recognizing the appreciable amount of scatter found in previous work of this kind, he used a semistatistical approach by testing 6 to 19 specimens at each prestress condition and reporting the median value. Damage tests were conducted on the notched specimens at four prestress levels and two test stresses for cycle ratios of 10, 25, 50, 75, and 90 percent. The criterion of failure was a deflection of the specimen of 0.005 millimeter. The method of Miller-Stock was used to determine the damage and the results were plotted as curves of percent damage versus cycle ratio. In all cases it was found that if the prestress was lower than the test stress the damage curve fell below the 45° line, indicating that the damage at the test stress was less than that produced by the cycle ratio at the prestress. For prestresses greater than the test stress the damage curves fell above the 45° line. The higher the value of the test stress, for a constant difference between prestress and test stress, the more nearly the damage curves approached the 45° line, indicating the condition where the damage at the prestress equaled that produced at the test stress.

To minimize further the scatter in the tests of smooth specimens, Bennett attempted to separate the number of cycles for crack initiation, during which damage occurs, from the cycles necessary to propagate the crack. The increase in length of fatigue cracks was measured with a microscope using stroboscopic light. It was found that the number of cracks in the 4-millimeter central zone of the specimen increased linearly with increasing test stress on a semilogarithmic plot. Measurements of the rate of growth of the cracks showed the surprising result that the crack seemed to start from a finite value of crack length, rather than at zero. The crack length could be expressed by the following equation:

\[ \log (L - C) = \alpha N \]

where

- \( L \) crack length, deg of arc
- \( N \) number of cycles
- \( C, \alpha \) constants

The number of cycles to failure was calculated from the above relation assuming that cracks grew exponentially from an initial arc length of 6°. The cycle ratios were based on this number of cycles to initiate a crack. Damage tests were conducted by prestressing a large number of smooth specimens at two stress levels for cycle ratios of 10, 331/3, 662/3, and 90 percent. These prestressed specimens were then used to determine conventional S-N curves, a family of S-N curves with parameters of constant cycle ratio being obtained for each prestress (fig. 5). The decrease
in endurance limit due to overstressing was relatively small. A 90-percent cycle ratio at 54,000 psi reduced the virgin endurance limit from 46,000 psi to 42,000 psi.

Overstressing on Aluminum Alloys

Bennett and Baker (ref. 24) made a comprehensive study of the effect of overstressing on aluminum alloys. Bare and alclad 24S-T sheet were tested in a Krouse sheet-bending fatigue machine. The effect of prior static loads on the fatigue properties of alclad 24S-T was studied briefly. It was found that prior static loads have little or no effect on the fatigue behavior except at the lowest fatigue test stress of 20,000 psi. At this stress, with a prior static stress of 40,000 psi, the specimens loaded in the same direction as the fatigue test load endured 10 times as many cycles before fracture as those loaded in the opposite direction.

In the fatigue damage tests the specimens were stressed for a predetermined number of cycles at one amplitude and tested to fracture at a second amplitude. Damage due to overstretch was measured as the percentage decrease in number of cycles required to form a crack in the prestressed specimen compared with the original material. The formation of a small crack was determined by the fracture of a fine wire cemented to the specimen at the region of maximum stress. A straight-line relationship was found between the logarithm of the number of cycles to produce a crack and the reciprocal of the stress.

The data for aluminum were less reproducible and more widely scattered than in Bennett's previous work with steel. Thus, the results could not be readily analyzed and only limited conclusions could be drawn. For prestresses at 32,500 and 22,500 psi the precision of the data did not warrant any further conclusions other than that the assumption of a linear relation between damage and cycle ratio was within experimental error. Prestressing at 17,000 psi, in effect an understressing treatment, resulted in improvement in fatigue life (negative damage). For specimens subsequently tested at 20,000 psi the improvement in fatigue life after either 2,000 or 2,000,000 cycles was more than 400 percent.

Grover, Bishop, and Jackson (ref. 25) included some fatigue damage work in their investigation of the fatigue strength of aircraft materials. Specimens were tested in direct repeated stress with a constant mean stress of one-fourth the ultimate tensile strength of the material. This unusual procedure was used because it seemed useful for calculations regarding gust loading of aircraft structures. Tests were made for each material at two levels of maximum stress. Damage was evaluated by the method of Miller-Stock and Bennett (see above) so that curves of damage against cycle ratio could be drawn. They concluded that for all three materials (24S and 7S3 aluminum and SAE 4130 steel) low initial prestress
gave less than proportionate damage at a higher test stress. For steel
the application of a high prestress produced greater than proportionate
damage at a lower test stress. These results are in agreement with the
work of previous investigators. For aluminum alloys considerable
strengthening at a low test stress was produced by small cycle ratios at
the high prestress. The authors ascribed this result to a combination of
local cold-work and local stress relief due to exceeding the yield
strength at the high stress.

Attempts at Predicting Cumulative Failure

Miner (ref. 26) proposed that the cumulative damage under repeated
loads was related to the net work absorbed by the specimen. He postulated
that when the total damage, expressed as the summation of the cycle ratios
at the various stress levels $\sum \frac{N_i}{N}$, reached unity failure would occur.
This means that the curve of damage versus cycle ratio determined by
Müller-Stock's method should fall on a 45° line. Miner's experimental
work on 24S-T alclad sheet verified his hypothesis, but it has been repu-
diated by more recent work.

Richart and Newmark (ref. 27) proposed that it was reasonable to
assume the existence of a damage - cycle-ratio relation during the process
of forming a fatigue crack at an overstress, but that this relation was
different at different stress levels. They defined the term "block of
cycle ratio," which was the number of cycles at a given stress which rep-
resents a given increment in value of cycle ratio. They proposed that
lines of constant degree of damage can be constructed on an S-N diagram
and, from these, the equivalent cycle ratio for any subsequent stress can
be determined. These curves may be used in groups of two or more to
determine the value of cumulative cycle ratio available when various
amounts of cycle ratio are applied at each stress.

Dolan, Richart, and Work (ref. 28) studied the fatigue life of
SAE 4340, 1045, and 2340 steel and 7075 aluminum subjected to repeated
stress fluctuating between two different amplitudes. Three stress his-
tories were used:

(a) Amplitude of stress oscillating gradually between a minimum and
maximum value, with a period of 10,000 cycles.

(b) Stress amplitude changing abruptly from maximum to minimum
stress, with intervals of 5,000 cycles at each stress.

(c) Same as (b) except that the interval at the maximum stress was
only 1,000 cycles, while that of the minimum stress was 9,000 cycles.

A marked increase in fatigue life was noted for the unnotched specimens
when the maximum stress was just above the original endurance limit. For
conditions where both the maximum and minimum stresses represented overstressing, fatigue failure always occurred before the cumulative number of cycles at either the maximum or minimum stress reached the life for the virgin S-N curve. For conditions of overstressing in which there was only a small difference between the magnitudes of the maximum and minimum stresses, the fatigue life was as long as if practically all of the cycles of stress had been applied at the minimum stress.

Luthander and Wållgren (ref. 29) attempted to predict the life of a structure subjected to repeated load with variable stress limits corresponding to load variations encountered in aircraft. Preliminary tests indicated that the duration of cycles, frequency, and pauses between loads did not have a significant effect. Tests were carried out on alclad sheet. Fairly good agreement was reported between their data and Miner's hypothesis that failure will occur when the summation of the ratios of the number of cycles at a given stress level to the number of cycles at that level causing failure is equal to unity.

Statistical Investigations

Only one statistical investigation of overstressing has been reported in the literature. Dolan and Brown (ref. 10), recognizing the large amount of scatter usually found in overstressing investigations, made tests on 75S-T aluminum with enough specimens to estimate the scatter. Small rotating cantilever-beam specimens were prestressed for predetermined cycle ratios at one stress level and then run to failure at a second test stress. A test stress of 35,000 psi was used in all cases. The prestress levels were 45,000, 40,000, and 30,000 psi. The fatigue damage was expressed as the percentage reduction in life at the test stress due to prestressing. A sufficient number of specimens were run to failure at each of the four stresses to determine the statistical mean life of the virgin specimens. These values were used in determining the cycle ratios and the damage. From 5 to 25 specimens were subjected to each combination of prestress and cycle ratio.

The authors plotted their results in the conventional manner as percent damage versus cycle ratio. Curves were drawn through the mean values of damage for all tests at a given cycle ratio. It is somewhat illuminating to observe the large scatter of the individual test points which they plotted (a rare event in overstressing investigations where smoothed curves are usually drawn to minimize the scatter). However, the curves through the mean values are fairly smooth except at cycle ratios of 85 percent where premature failures occurred at the prestress because of the number of cycles cutting into the tail of the distribution of fatigue life for the virgin specimens.
The results of Bennett's work with steel were confirmed in this work for an aluminum alloy. When the magnitude of the prestress was greater than that of the test stress, the damage was proportionately greater than the cycle ratio of the prestressing. The damage was less than the cycle ratio for a prestress lower than the test stress. For the same level of test stress the smaller the difference between prestress and test stress, the more closely the damage curve approached the 45° line.
REFERENCES


### TABLE I

**COMPARISON OF $S^*$ AND PERCENT $\bar{S}$**

<table>
<thead>
<tr>
<th>Structure</th>
<th>Stress, psi</th>
<th>$S_u$</th>
<th>$\bar{S}$ (a)</th>
<th>$S^*$</th>
<th>Percent $\bar{S}$, $(S - \bar{S})/\bar{S}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q &amp; T</td>
<td>108,700 (prestress) 102,200 (test stress)</td>
<td>171,000</td>
<td>89,700</td>
<td>23.4</td>
<td>21.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15.4</td>
<td>14.0</td>
</tr>
<tr>
<td>Q &amp; S</td>
<td>68,600 (prestress) 64,800 (test stress)</td>
<td>109,900</td>
<td>57,400</td>
<td>21.3</td>
<td>19.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14.1</td>
<td>12.9</td>
</tr>
</tbody>
</table>

*Value from ref. 1 corrected for error in frequency.

### TABLE II

**VIRGIN FATIGUE STATISTICS**

<table>
<thead>
<tr>
<th>Stress</th>
<th>$\log N_0$</th>
<th>$N_0$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SAE 4340 - Quenched and tempered</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>108,700</td>
<td>4.5975</td>
<td>$3.958 \times 10^4$</td>
<td>0.0982</td>
</tr>
<tr>
<td>102,200</td>
<td>4.7513</td>
<td>5.640</td>
<td>.1186</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stress</th>
<th>$\log N_0$</th>
<th>$N_0$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SAE 4340 - Quenched and spheroidized</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>68,600</td>
<td>4.9921</td>
<td>$9.820 \times 10^4$</td>
<td>0.0873</td>
</tr>
<tr>
<td>64,800</td>
<td>5.2022</td>
<td>$1.593 \times 10^5$</td>
<td>.1327</td>
</tr>
</tbody>
</table>
### TABLE III

**FATIGUE STATISTICS FOR DAMAGED SPECIMENS**

<table>
<thead>
<tr>
<th>Cycle ratio, R</th>
<th>( \frac{\log N_p}{\text{at test stress}} )</th>
<th>( \bar{N}_p ) at test stress</th>
<th>( \sigma )</th>
<th>( \frac{\log N_{OT}}{\text{at test stress}} )</th>
<th>( \bar{N}_{OT} ) at test stress</th>
<th>( \sigma_{OT} )</th>
<th>Damage, ( D ), percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>4.7118</td>
<td>5.150 \times 10^4</td>
<td>0.2979</td>
<td>4.7513</td>
<td>5.640 \times 10^4</td>
<td>0.1186</td>
<td>9</td>
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<tr>
<td>0.30</td>
<td>4.5383</td>
<td>3.454</td>
<td>0.1515</td>
<td>4.7513</td>
<td>5.640</td>
<td>0.1186</td>
<td>39</td>
</tr>
<tr>
<td>0.60</td>
<td>4.3043</td>
<td>2.015</td>
<td>0.1414</td>
<td>4.7513</td>
<td>5.640</td>
<td>0.1186</td>
<td>64</td>
</tr>
<tr>
<td>SAE 4340 - Quenched and tempered</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 0.10           | 5.1217                          | 13.23                          | 0.2360 | 5.2022                          | 15.93                          | 0.1327 | 17              |
| 0.30           | 4.9120                          | 8.166                          | 0.2116 | 5.2022                          | 15.93                          | 0.1327 | 49              |
| 0.60           | 4.5869                          | 3.862                          | 0.2088 | 5.2022                          | 15.93                          | 0.1327 | 76              |
| SAE 4340 - Quenched and spheroidized |

### TABLE IV

**INFLUENCE OF CRITERION OF FAILURE ON FATIGUE-LIFE STATISTICS**

<table>
<thead>
<tr>
<th>Stress, psi</th>
<th>First visual crack</th>
<th>Machine shutdown</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{\log N_0}{\sigma_0} )</td>
<td>( \frac{\log N_f}{\sigma_f} )</td>
</tr>
<tr>
<td>SAE 4340 - Quenched and tempered</td>
<td></td>
<td></td>
</tr>
<tr>
<td>108,700</td>
<td>4.5975</td>
<td>0.0982</td>
</tr>
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<td>4.7513</td>
<td>0.1186</td>
</tr>
<tr>
<td>SAE 4340 - Quenched and spheroidized</td>
<td></td>
<td></td>
</tr>
<tr>
<td>68,600</td>
<td>4.9921</td>
<td>0.0873</td>
</tr>
<tr>
<td>64,800</td>
<td>5.2022</td>
<td>0.1327</td>
</tr>
</tbody>
</table>
### TABLE V

**DIFFERENCE IN $\sigma$ DUE TO END POINT**

<table>
<thead>
<tr>
<th>Material</th>
<th>Stress, psi</th>
<th>Results of $F$ test at 5-percent level</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAE 4340 (Q &amp; T)</td>
<td>102,200</td>
<td>Significant</td>
</tr>
<tr>
<td>SAE 4340 (Q &amp; S)</td>
<td>68,500</td>
<td>Significant</td>
</tr>
<tr>
<td>SAE 4340 (Q &amp; S)</td>
<td>64,800</td>
<td>Not significant</td>
</tr>
</tbody>
</table>

### TABLE VI

**DIFFERENCE IN $\log N$ DUE TO END POINT**

<table>
<thead>
<tr>
<th>Material</th>
<th>Stress, psi</th>
<th>Results of $t$ test at 5-percent level</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAE 4340 (Q &amp; T)</td>
<td>102,200</td>
<td>Not significant</td>
</tr>
<tr>
<td>SAE 4340 (Q &amp; S)</td>
<td>68,500</td>
<td>Not significant</td>
</tr>
<tr>
<td>SAE 4340 (Q &amp; S)</td>
<td>64,800</td>
<td>Significant</td>
</tr>
</tbody>
</table>
TABLE VII

ENDURANCE-LIMIT STATISTICS FOR VIRGIN AND DAMAGED QUENCHED AND SPHEROIDIZED SPECIMENS OF SAE 4340 STEEL

<table>
<thead>
<tr>
<th>Virgin specimens, no prestress</th>
<th>Damaged specimens, prestressed at 68,600 psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test stress, psi</td>
<td>Test stress, psi</td>
</tr>
<tr>
<td>Failures, percent</td>
<td>Failures, percent R = 0.30</td>
</tr>
<tr>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td>59,860</td>
<td>54,800</td>
</tr>
<tr>
<td>57,360</td>
<td>50,000</td>
</tr>
<tr>
<td>54,870</td>
<td>38,000</td>
</tr>
<tr>
<td>80</td>
<td>85</td>
</tr>
<tr>
<td>50</td>
<td>79</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
</tr>
</tbody>
</table>

\[ \bar{S} = 57,400 \text{ psi}; \text{ standard deviation } \sigma = 2,490 \text{ psi.} \]

\[ \bar{S} = 40,900 \text{ psi}; \text{ standard deviation } \sigma = 12,500 \text{ psi.} \]

\[ \bar{S} = 20,100 \text{ psi}; \text{ standard deviation } \sigma = 30,000 \text{ psi.} \]
Figure 1.- Microstructure of SAE 4340 steel, quenched and tempered. 
X1,000; nital etch.

Figure 2.- Microstructure of SAE 4340 steel, quenched and spheroidized. 
X1,000; nital etch.
Figure 3: Schematic representation of test procedure for overstressing.
Figure 4.—Damage at test stress versus cycle ratio at prestress. Prestress, 68,600 psi, and test stress, 64,800 psi, for quenched and spheroidized (Q & S) structure; prestress, 108,700 psi, and test stress, 102,200 psi, for quenched and tempered (Q & T) structure.
(a) Prestress, ±54,000 psi.

(b) Prestress, ±48,000 psi.

Figure 5.- S-N curves for specimens given cycle ratios of overstress at 54,000 and 48,000 psi. (From ref. 9 through courtesy of the A.S.T.M.)
Figure 6.- Schematic representation of French's damage line.