INITIAL EXPERIMENTS ON FLUTTER OF UNSWEPT CANTILEVER WINGS AT MACH NUMBER 1.3

By W. J. Tuovila, John E. Baker, and Arthur A. Regier

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SUMMARY

A supersonic tunnel designed to operate at Mach number 1.3 was used for a preliminary experimental flutter investigation of widely different unswept cantilever wings. Data for 12 wings with mass-density parameters \( \frac{1}{\kappa} \) ranging from 52 to 268, center-of-gravity positions ranging from 46 to 63 percent chord from the leading edge, and elastic-axis positions ranging from 34 to 52 percent chord from the leading edge are considered.

A comparison is made of the test results with calculations of bending-torsion flutter obtained by the theory of flutter in supersonic two-dimensional flow and it is concluded that the test data are in reasonable agreement with the calculated results. In general, the theoretical values are conservative. As shown by the theory, the flutter results are quite sensitive to the location of the center of gravity. Thick and thin, blunt and sharp airfoil-section shapes were used, but no very pronounced effect of the section shape on flutter characteristics was found. The experiments include a study of the effect of the addition of tip moments of inertia. With the center of gravity of the tip weights coincident with the center of gravity of the wing section, no detrimental effect on the flutter speed was found.

INTRODUCTION

The background and theory for the flutter of an airfoil in a two-dimensional flow at supersonic speeds is given in reference 1. The present investigation is a preliminary survey to determine the possibility of using the theory of reference 1 for flutter at supersonic speeds to predict the coupled bending-torsion flutter of widely different unswept

*Supersedes the recently declassified NACA RM 18J11, "Initial Experiments on Flutter of Unswept Cantilever Wings at Mach Number 1.3" by W. J. Tuovila, John E. Baker, and Arthur A. Regier, 1949.
cantilever wings at a low supersonic Mach number. This preliminary investigation is not intended as a critical test of the theory since the analysis does not consider the effect of mode shape, aspect ratio, section shape, tip Mach cone, or viscous effects.

A single-degree-of-freedom torsional instability which may occur in the Mach number range 1.0 to 1.58 is discussed in reference 1. In order also to investigate the possible occurrence of such single-degree flutter on cantilever wings, the test apparatus was designed to operate at a Mach number of 1.3.

A brief discussion is given of the effects of concentrated masses placed at the wing tip, the center of gravity of the masses coinciding with the center of gravity of the wing, and the effects of sharp and blunt leading edges on the wings.

SYMBOLS

\begin{itemize}
  \item $b$ \hspace{1cm} semichord, ft
  \item $c_w$ \hspace{1cm} chord, ft
  \item $GJ$ \hspace{1cm} torsional stiffness, in²-lb
  \item $\rho$ \hspace{1cm} mass density of air in test section, slugs/cu ft
  \item $m$ \hspace{1cm} mass of wing per unit span, slugs/ft
  \item $M$ \hspace{1cm} Mach number
  \item $l/\kappa$ \hspace{1cm} mass-density parameter, $m/\pi \rho b^2$
  \item $I_\alpha$ \hspace{1cm} mass moment of inertia of wing about elastic axis per unit span
  \item $r_\alpha^2$ \hspace{1cm} radius-of-gyration parameter, $I_\alpha/mb^2$
  \item $V$ \hspace{1cm} flutter velocity, ft/sec
  \item $f_f$ \hspace{1cm} flutter frequency, cps
  \item $f_h$ \hspace{1cm} first-bending frequency, cps
  \item $f_\alpha$ \hspace{1cm} first-torsion frequency, cps
  \item $\omega_f = 2\pi f_f$
\end{itemize}
\[ \omega_n = 2\pi f_n \]
\[ \omega_\alpha = 2\pi f_\alpha \]

\( \varepsilon_n \)  
first-bending damping coefficient

\( \varepsilon_\alpha \)  
first-torsion damping coefficient

The tests were made at a Mach number of 1.3 in an "intermittent" two-dimensional supersonic tunnel having a 9.24-inch by 18.23-inch test section (figs. 1 and 2). The apparatus operates from atmospheric pressure to a vacuum. A quick-operating valve allows a steady-flow condition to be reached in approximately 0.15 second and this condition of steady flow can be maintained for 5 to 7 seconds. To prevent condensation in the test section, a room was constructed at the tunnel entrance in which the air could be heated. Variation of the air temperature from 170°F to 200°F results in a velocity range at the test section from 950 miles per hour to 990 miles per hour at a Mach number of 1.3. The test-section Mach number determined by optical means varied from 1.29 to 1.31. The test-section Mach number determined by a pressure survey showed a variation from 1.31 to 1.33 (fig. 3).

The models were mounted cantilever fashion from the side of the tunnel (fig. 4). In order to avoid possible model failure during the tunnel transient conditions, and since supersonic flutter characteristics were being determined, it was desirable to withhold the model from the flow during the periods of acceleration and deceleration. A pneumatic-cylinder arrangement was installed for this purpose (fig. 2). With this device, the model could be held outside the tunnel wall until stable flow conditions were reached; then, by means of electrically operated valves controlled by an observer, the model could be projected into the tunnel slowly and withdrawn quickly if necessary.

The flutter models were of rectangular plan form and were constructed of laminated Sitka spruce or aluminum alloy. The wing dimensions and parameters are given in table I. The wing chords ranged from 3.03 to 4.22 inches and the lengths (semispan) from 6 to 9.12 inches. Both thick and thin sections with blunt and sharp leading edges were used. The airfoil sections used were 3-, 5-, and 8-percent-thick circular arcs, 3-percent-thick double wedge, NACA 16-010, and NACA 65-007. The mass-density parameter \( 1/\kappa \) ranged from 52 to 268, the chordwise positions of the center of gravity ranged from 46 to 63 percent chord, and the positions of the elastic axis ranged from 34 to 52 percent chord.
Before each model was installed in the tunnel, its weight, moment of inertia, and section center-of-gravity position were determined. After installation in the tunnel, the elastic axis was located by observing the axis of zero twist optically. The first-bending frequency and the damping were obtained from a free-vibration record of the model. Since the wings were structurally uniform, the stiffness parameter $GJ$ could be computed from a torsional-vibration record obtained with a mass of large known moment of inertia attached to the wing tip. The uncoupled first-torsion frequency could then be calculated. The torsional damping was determined from the torsional-vibration-decay curve.

The models were designed not to flutter on the first run. The models were progressively modified by adding lead to shift their centers of gravity and by slotting to shift their elastic axes and change their bending and torsion frequencies, until flutter occurred. If a model was saved, its parameters were changed to yield another flutter point. The values of the radius-of-gyration parameter $r_g^2$ and chordwise center of gravity were determined from the unmodified wing and the added masses.

During each test run, the following data were recorded simultaneously by means of a recording oscillograph: flutter frequency, position of the model in the tunnel, and static pressure.

A sample record of the flutter of model B-5 is given in figure 5.

**METHOD OF ANALYSIS**

The flutter data for the particular wings tested are put in coefficient form and compared with the analytic solution of the supersonic bending-torsion flutter problem in a two-dimensional flow given in reference 1. The data of this paper were obtained at a Mach number of 1.3 and, since aerodynamic coefficients at this Mach number are not included in reference 1, a linear interpolation was made between the computed values of the flutter-speed coefficient at Mach numbers of 1.25 and 1.43, for which the aerodynamic coefficients are available. First-bending and uncoupled first-torsion frequencies and damping coefficients $g_\theta$ and $g_\alpha$ were used in the theoretical analyses.

An examination of the possible errors introduced into the results by the method of interpolation is desirable. It is known that the error may be very large; for example, in the case of torsional instability in one degree of freedom for the elastic-axis position at 50 percent chord, the interpolation was made directly between the aerodynamic coefficients at Mach numbers 1.25 and 1.43. This interpolation was necessary since the wing was stable at a Mach number of 1.43 and the theory yields no solution for the flutter-speed coefficient under these conditions.
RESULTS AND DISCUSSION

The significant flutter parameters and the results of the calculations are given in table I. A large number of tests were made on wings which did not flutter, but table I gives only the results for the wings for which flutter was observed. Altering a model to obtain flutter consisted in moving the center of gravity back in increments of about 2 percent of the chord. Consequently, the chordwise position of the center of gravity that would produce flutter is known to within about 2 percent chord. The results are sensitive to center-of-gravity position and this sensitivity may account for some of the scatter of the data. Contributing also to the scatter of the data are the inaccuracies in obtaining the wing parameters, the effect of the degree of penetration of the model into the tunnel, and errors in the determination of the flutter-speed coefficients arising from the method of interpolation.

Some of the results listed in table I are presented in figures 6 and 7. In figure 6 the theoretical and experimental flutter-speed coefficients are compared. The fact that the experimental data fall above the 45° line, for the most part, indicates that the theory of reference 1 is generally conservative as far as application to cantilever wings is concerned. From consideration of the fact that a slight inaccuracy in the location of the center of gravity has a large effect and also that effects of section shape, aspect ratio, and Mach cone are not accounted for, the agreement is not unsatisfactory. The theoretical and experimental flutter frequencies are compared in figure 7; the experimental frequencies ranged from about 0.61 to 1.09 times the theoretical values. In all cases the mode at flutter appeared to consist mainly of a coupling of first-bending and first-torsion modes.

Since the present investigation is of a preliminary nature and covers a wide range of parameters, no attempt was made to isolate the effects of separate parameters such as the mass-density parameter, frequency ratio, elastic axis, and center of gravity, which are treated by the two-dimensional theory, or parameters such as aspect ratio not treated by the theory.

An attempt was made to investigate some of the possible effects of airfoil-section shape on flutter. The intermingling of the data for the various models (figs. 6 and 7) suggests that the section shape has no very pronounced effect on flutter at a Mach number of 1.3. However, more difficulty due to divergence was experienced with thick models having blunt leading edges. This result is in accord with the higher-order method of calculation (order higher than in the linear method) for pressure distribution at supersonic speeds in steady flow, which shows that the center of pressure may move ahead of the 50-percent-chord position for thick blunt-nosed airfoils, particularly at Mach numbers near unity.
It was observed in the tests that the thick airfoils tended to diverge even though the elastic axis was ahead of the 50-percent-chord position.

Since practical winged vehicles pass through the subsonic speed range in order to reach supersonic speeds, some discussion of and comparison with subsonic criteria are desirable. For this purpose, incompressible flutter-speed coefficients were computed by the method of reference 2 for which first bending and uncoupled first torsion frequencies and damping coefficients \( g_n \) and \( g_\alpha \) were utilized. That flutter-speed coefficients based on subsonic theory are conservative with respect to supersonic speeds has been suggested in reference 3 and, in fact, is indicated by some of the calculations in reference 1. This conclusion is also indicated by the present tests, as illustrated in figure 8, in which the experimental flutter-speed coefficients are plotted against the incompressible theoretical flutter-speed coefficients. The statement may not be true in general; for example, the condition when the frequency ratio \( \frac{g_\alpha}{g_n} = 1 \) may need further investigation and, in any case, the margin of safety is not established. Some of the models were permitted to encounter the tunnel transient speeds and, for example, model F-1, which had fluttered at Mach number 1.3, was held in the tunnel while the tunnel was brought up to speed. The wing fluttered and broke at a Mach number of about 0.7, a result which is in general agreement with the subsonic calculation. Flutter data obtained with bombs and rocket missiles (references 4 to 6) and other experience indicate that if flutter failures occur, they occur, in general, at speeds near sonic. For the practical purpose of making preliminary estimates of a wing flutter speed, such formulas as, for example, the approximate flutter formula in reference 2 or similar criteria thus appear useful over a wide range of speeds.

In reference 3, Smilg suggests a torsional frequency criterion \( g_\alpha c_w > 1,000 \) feet per second as sufficient to prevent one degree of torsional flutter at supersonic speeds. The criterion is based on the assumption that the first-bending frequency is very high with respect to the first-torsion frequency. In order to explore this criterion, a cantilever model was equipped with tip weights at both the leading and trailing edges so that the torsional frequency was reduced. The results of the tests are shown in table 2. In all cases a slight shift of the center of gravity ahead of the location at flutter stopped the flutter. The fact that flutter is extremely sensitive to the center-of-gravity position and that the values of the product \( g_\alpha c_w \) are far below 1,000 indicates that the criterion is overly conservative when applied to cantilever wings with normal bending-torsion frequency ratios. The data further suggest that for cantilever wings the bending degree of freedom may suppress the one-degree-of-freedom torsional flutter and that bending-torsion effects occur.
The data further indicate that no harmful effect on the flutter speed ensues when the center of gravity of the tip weights and the wing coincide. It is observed that the frequency ratio varies from 0.55 to 0.85 and that the torsional frequency has been reduced to as low as one-third of the value for the wing without tip weights. For the largest mass moment of inertia on the wing tip (last case in Table II), it was necessary to move the center of gravity farther toward the trailing edge to produce flutter.

In figure 9, the theoretical curves represent flutter-speed coefficients for one-degree-of-freedom flutter calculated according to reference 1. The experimental flutter-speed coefficients shown in this figure, however, correspond to the coupled bending-torsion values.

An effort was made to obtain some systematic aspect-ratio effects from the present tests, but the results were rather contradictory. Some models which spanned the tunnel (except for 1/16-inch tip clearance) were used so that, presumably, two-dimensional flow over the wing could be expected. Flutter of full-span models of NACA 16-010 section could be stopped by retracting the tip from the boundary layer; however, when the tip was retracted from the boundary layer for the 3-percent-thick double-wedge models, the flutter amplitude increased. The effect of the subsonic boundary layer at the tip of the models is not known. In one particular case the model fluttered at 99 percent of the theoretical frequency on entering the tunnel and the frequency changed to 128 percent at a smaller amplitude when the model spanned the tunnel. As the model was retracted, the flutter frequency dropped to 99 percent of the theoretical value and fluttered to destruction. A more systematic investigation of the aspect ratio and tip and shape effects is desirable to clarify various aspects of the problem.

CONCLUSIONS

The results of a preliminary flutter investigation of widely different unswept cantilever wing models at a Mach number of 1.3 indicated the following conclusions:

1. Agreement between experimental and calculated flutter-speed coefficients is fairly satisfactory. In general, the theoretical values are conservative.
2. No very pronounced effect of airfoil-section shape on the flutter characteristics was found in these preliminary experiments; however, significant divergence effects were observed on thick blunt-nosed airfoils.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., October 13, 1948.

REFERENCES


### Table I

**Experimental and Theoretical Results of Planer Investigatons**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>A-1</th>
<th>B-1</th>
<th>B-2</th>
<th>B-3</th>
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<th>B-5</th>
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<th>D-1</th>
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<td>4.03</td>
<td>4.03</td>
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<td>39.6</td>
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<td>0.37</td>
<td>0.40</td>
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<td>104.1</td>
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<td>2125</td>
<td>879</td>
<td>909.5</td>
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<td>1220</td>
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<td>$f_{ng}$, ose</td>
<td>133.4</td>
<td>95.0</td>
<td>100.9</td>
<td>96.0</td>
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<td>$\alpha \times 10^3$ (best section)</td>
<td>0.876</td>
<td>0.965</td>
<td>0.890</td>
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<td>0.886</td>
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<td>$\alpha_{d}$</td>
<td>0.15</td>
<td>0.583</td>
<td>0.585</td>
<td>0.533</td>
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<td>0.64</td>
<td>0.432</td>
<td>0.435</td>
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<td>$\alpha_{d}$</td>
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<td>0.585</td>
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<td>7.02</td>
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<td>0.882</td>
<td>0.847</td>
<td>0.707</td>
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<td>0.740</td>
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<td>0.803</td>
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<td>$a_{d}$, (theoretical)</td>
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<td>0.797</td>
<td>0.000</td>
<td>0.783</td>
<td>0.786</td>
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<td>4.612</td>
<td>3.10</td>
<td>3.71</td>
<td>2.811</td>
<td>3.401</td>
<td>2.965</td>
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TABLE II

Flutter data on a cantilever model with tip weights

Wing chord = 4 in.; elastic axis at 47 percent chord; \( r_a^2 = 0.23 \) (for wing with no tip weights); weight of wing with no tip weights = 0.0806 lb; length of wing (semispan) = \( 6\frac{1}{4} \) in.; tip-weight center of gravity coincides with the wing center of gravity.

<table>
<thead>
<tr>
<th>Frequency, cps</th>
<th>( \alpha_n/\alpha_b )</th>
<th>Center of gravity of wing and tip weights, percent chord</th>
<th>( \alpha_{c_w} ), fps</th>
<th>Tip weights, lb</th>
<th>Moment of inertia of tip weights about c.g., in-lb-sec(^2)</th>
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<tr>
<td>Torsion</td>
<td>Bending</td>
<td>Flutter</td>
<td>( \alpha_n/\alpha_b )</td>
<td>Center of gravity of wing and tip weights, percent chord</td>
<td>( \alpha_{c_w} ), fps</td>
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<tr>
<td>244</td>
<td>134</td>
<td>153</td>
<td>0.550</td>
<td>50</td>
<td>510</td>
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<td>133</td>
<td>102</td>
<td>105</td>
<td>0.767</td>
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<td>103</td>
<td>81</td>
<td>86</td>
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<td>80</td>
<td>68</td>
<td>74</td>
<td>0.850</td>
<td>53</td>
<td>167</td>
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Figure 1.- Diagram of the supersonic flutter apparatus.
Figure 2. - General view of the Langley supersonic flutter apparatus.
Figure 3.- Mach number survey of the test section.
Figure 4. - Flutter model installed in the test section.
Figure 5.- Sample oscillograph record of the flutter of model B-5.
Figure 6.- Comparison of experimental values of $V/\beta_{\alpha}$ to theoretical values of $V/\beta_{\alpha}$ at $M = 1.3$. 
Figure 7. - Comparison of experimental values of $\omega_f/\omega_\alpha$ with theoretical values of $\omega_f/\omega_\alpha$ at $M = 1.3$. 
Figure 8. - Comparison of experimental values of $V/b\alpha_x$ at $M = 1.3$ with theoretical incompressible values of $V/b\alpha_x$ at $M = 0$. 
Figure 9.- Comparison of experimental values of $V/\delta_{d\alpha}$ for coupled wing flutter with theoretical values for one-degree-of-freedom torsional flutter. $M = 1.3, \alpha = 0.04$. 