NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2910

AN APPLICATION OF THE METHOD OF CHARACTERISTICS TO TWO-DIMENSIONAL TRANSONIC FLOWS WITH DETACHED SHOCK WAVES

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Washington
March 1953

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SUMMARY

An application of the method of characteristics is presented which affords a means for determining the surface pressures for a class of two-dimensional airfoils of given nose shape and arbitrary rear part in a sonic or supersonic stream if surface pressure data are given for one member of the class. For engineering purposes, the method of characteristics may be replaced by a simple application of Prandtl-Meyer flow concepts. An explanation of the nonlinear force characteristics of two-dimensional airfoils at transonic speeds is presented on the basis of sensitivity of these flows to changes in geometry and angle of attack.

INTRODUCTION

Although considerable progress has been made during the past few years in the development of experimental techniques for testing at transonic speeds, it is still difficult and time consuming to obtain reliable data. Analytic developments have similarily proved difficult - indeed, relevant transonic-flow solutions have been found only for wedge profiles (refs. 1 to 3). Even these solutions do not prove as useful as may be desired because numerical methods dominate in the course of the solution. In one sense, these analytic solutions are similar to experimental data in that the results are confined to the specific profile and stream Mach number chosen. In order to obtain the flow properties for other conditions, additional tests or additional solutions are required. Thus, an inordinate amount of labor is required to discover the important features of transonic flows with these essentially numerical approaches.

One of the most useful contributions of theory has been the transonic similarity rules (refs. 4 to 8). These similarity rules are used to relate the aerodynamic characteristics of airfoils in similar flows.

Flow similarity at transonic speeds requires that a function of the stream Mach number and some geometric parameter of the airfoil remain constant; that is, for similar flows, the stream Mach number and airfoil geometry cannot, in general, be varied independently. This requirement, however, causes some difficulty in obtaining an understanding of transonic flows from the similarity rules alone.

An inherently more useful relation is one which permits the influence of stream Mach number and airfoil geometry to be considered separately. Such relations are usually called velocity-correction formulas. A familiar example is the Kármán-Tsien velocity-correction formula (ref. 9) which relates the pressure coefficient in compressible flow to that for incompressible flow past the same airfoil. A velocity-correction formula applicable at transonic speeds has been found only for two-dimensional airfoils with a fixed sonic point (ref. 10). Consequently, transonic data cannot be extended to the degree that is possible at subsonic speeds.

Similarity rules and velocity-correction formulas have been used to estimate the transonic aerodynamic characteristics of a family of airfoils from the known characteristics of one member of the family. In addition, for sonic or supersonic stream Mach numbers, a method is available for extending surface pressure data to airfoils which are not geometrically similar; that is, to airfoils of a basically different shape. This method, which has received little attention at transonic speeds although it has been indicated by Guderley and Yoshihara (ref. 1), makes use of the familiar characteristic theory of supersonic flow. In reference 11 a comparison has been made of the flow fields determined by the method of characteristics with those determined experimentally by means of an interferometer. The supersonic flow field was calculated by starting from a sonic line determined from an interferogram. This application of the method of characteristics is extremely tedious because the Mach angle changes rapidly in the neighborhood of the sonic line.

The primary purpose of the present paper is to discuss the application and limitations of the method of characteristics as a means for extending transonic data. A secondary purpose is to present an explanation of the occurrence of linear and nonlinear pressure variations for two-dimensional airfoils at transonic speeds. This explanation is based in part upon transonic similarity rules and in part upon information obtained from an application of the method of characteristics.

SOME GENERAL FEATURES OF TRANSONIC FLOW FIELDS

A review of some of the general features of transonic flow fields is presented prior to a discussion of the application of the method of characteristics. Consider a sharp-nosed airfoil in a supersonic stream. For a sufficiently high Mach number, a shock wave is attached to the leading edge and the flow is completely supersonic. Behind the trailing-edge shock, the Mach number is approximately the stream value. With a reduction in the stream Mach number, the shock wave detaches from the nose and the flow over the leading part of the airfoil is subsonic. At some point on the airfoil the flow attains sonic velocity. Downstream of this point the flow is supersonic and the Mach number behind the trailing-edge shock is usually somewhat higher than the stream value.

Some general features of the slightly supersonic flow past a symmetrical blunt-nosed airfoil are illustrated in figure 1. The leading shock is detached and the flow is subsonic in the region bounded by the shock and the sonic line. The presence of the sonic line separating the regions of subsonic and supersonic flow distinguishes the supersonic expansion from that of a pure Prandtl-Meyer expansion. Some of the expansion waves originating at the surface propagate to the sonic line where they are reflected as compression waves. Some of these reflected compression waves meet the airfoil and thereby decrease the effect of the expansion caused by the convex curvature of the surface. These reflected compression waves from the sonic line may be considered as a subsonic influence on the downstream region of supersonic flow.

Limiting characteristic.- The last expansion wave to intersect the sonic line is called the limiting characteristic. Waves downstream of the limiting characteristic do not intersect the sonic line - they interact with the leading and trailing shock waves.

Figure 1 also shows some features of the flow in the hodograph plane. Typical characteristics are shown in both the physical and the hodograph plane, where the letters denote corresponding points. Along the limiting characteristic the flow deflection is positive and is greatest at the surface of the body; consequently, the location of the limiting characteristic on the surface of the airfoil is upstream of the position of maximum thickness. This point will occur far forward on airfoils with a blunt nose.

The limiting characteristic divides the flow field into two distinct regions. Only those disturbances propagated along the expansion waves upstream of the limiting characteristic can influence the region of subsonic flow. Disturbances propagated along the expansion waves downstream of the limiting characteristic do not intersect the sonic line and, therefore, cannot influence the subsonic field. Thus, the subsonic flow field is dependent upon only that portion of the airfoil which lies ahead of the limiting characteristic. The flow downstream of the limiting characteristic is dependent in part upon the shape of the rear of the airfoil and in part upon the flow ahead of the limiting characteristic through the reflected compression waves from the sonic line. Consequently, the flow past an arbitrarily shaped rear part (that is, the part of the airfoil downstream of the limiting characteristic) may be determined by the method of characteristics if the flow properties along the limiting characteristic are known. For sufficiently high stream Mach numbers or extremely thin airfoils, the last reflected compression wave may intersect the airfoil surface. In this case there is no subsonic influence downstream of this characteristic. By neglecting the small influence of the rotation due to the shock curvature downstream of the intersection of the limiting characteristic with the detached bow shock wave, the surface pressures are given by Prandtl-Meyer flow concepts. The flow properties along the limiting characteristic may be determined from experimental surface pressure data and remain fixed provided changes in the airfoil geometry are restricted to the rear part of the airfoil; therefore, the aerodynamic characteristics of a class of airfoils with a fixed nose shape but arbitrary rear part may be determined from data for one member of the class.

The essential restriction of the proposed application of the method of characteristics is that the reflected compression waves from the sonic line must be unaffected by changes in geometry of the airfoil: that is, no changes in airfoil geometry are permitted which alter the subsonic region of flow. The following section contains a discussion of permissible changes in airfoil geometry.

Streamline of no reflection.- For any given nose contour, one shape of the rear part has particular significance for the discussion of permissible changes in geometry: namely, the rear part for which no waves are propagated from the airfoil to the flow field. This rear part of no reflection is easily constructed by the method of characteristics by deflecting the surface streamline the required amount to cancel the incident compression waves. This construction is the same as that employed in the design of supersonic nozzles where the walls are shaped to produce a uniform flow.

Figure 2 illustrates the streamline of no reflection for a given nose shape. The shape of the rear part of no reflection differs for each nose shape and stream Mach number. In any case, however, this rear part is infinitely long and the flow downstream of the last compression wave is uniform at a Mach number of 1. The importance of the rear part of no reflection is that only expansion waves arise from any rear part of convex curvature which lies within the no-reflection streamline. Since these expansion waves cannot influence the region of subsonic flow, the proposed application of the method of characteristics may be used for any rear part of convex curvature lying within the streamline of no reflection.

For any rear part which lies outside the no-reflection streamline, compression waves arise even if the curvature is convex. The possibility exists, therefore, that shocks would be formed which would propagate to the sonic line, because shock waves lie ahead of the characteristics. Since most airfoil rear parts of practical interest lie within the no-reflection streamline, however, this possibility is of little importance and is not considered further.

A concave curvature or the deflection of a flap will cause compression waves to arise from airfoil rear parts which lie within the no-reflection streamline. These compression waves could conceivably influence the subsonic region of flow; however, calculation of the flow field by the method of characteristics indicates that the flow becomes sonic somewhere on the flap before this condition occurs. Further deflection of the flap would cause the flow to become subsonic, thus creating a mixed flow which cannot be treated by the method of characteristics. The practical limit for calculations involving compression waves appears to be the condition where the flow becomes sonic at the surface. In general, large changes in airfoil geometry are permitted far downstream of the limiting characteristic where the local Mach numbers are not near 1 and the distance separating the limiting characteristic and downstream shock waves is large.

APPLICATION OF METHOD OF CHARACTERISTICS

The flow past an arbitrary contour downstream of the limiting characteristic may be calculated provided the flow properties along the limiting characteristic are given. These quantities can be determined from an experimental pressure distribution over the supersonic region of the airfoil. If the surface pressure or Mach number distribution over an airfoil is known, the value of the Prandtl-Meyer property angle ν at the surface may be determined from Prandtl-Meyer flow tables. Since the flow deflection angle θ at the surface is known from the geometry of the airfoil, the characteristic net for the region of supersonic flow may be calculated. In particular, the values of ν and θ may be determined along the limiting characteristic and the flow field for any airfoil rear part may be calculated by characteristic methods. For nonsymmetric flows (flows over cambered airfoils or airfoils at an angle of attack), the characteristic net for each surface must be determined for each test condition. For these nonsymmetric conditions, the limiting characteristic will not, in general, occur at the same chordwise location on each surface. Thus, the geometric alterations to each surface of the airfoil must be confined to regions downstream of the respective limiting characteristics.

Along the family of characteristics originating at the upper surface, the quantity $\nu - \theta$ is constant. Since ν is zero along the sonic line, it may be seen from figure 1 that the equation of the limiting characteristic for the upper surface is $\nu - \theta = -\theta_0$ where θ_0 is the flow deflection which gives sonic velocity behind the shock. This quantity is given as a function of stream Mach number in figure 3. This figure, together with the surface distribution of $\nu - \theta$ evaluated from the airfoil geometry and the known pressure distribution, may be used to determine the location of the limiting characteristic on the airfoil. Since the limiting characteristic is far forward on airfoils with a blunt nose, a large part of these airfoils may be modified without disturbing the subsonic region of flow.

For engineering purposes, it is not necessary to calculate the characteristic net in order to determine the pressure distribution for an arbitrary contour. From the known pressure distribution, the Prandtl-Meyer property angle ν, which is a function only of the local Mach number, can be determined at the surface. The pressure distribution over an airfoil modified by an amount $\Delta\theta$, which may be a function of the chordwise location, can be determined by use of the equation $\Delta\nu = -\Delta\theta$. This procedure neglects the change in distribution of the reflected compression waves at the surface due to a change in the airfoil contour. The error is small, however, in regions where the density of the reflected compression waves is small. Since Prandtl-Meyer flow satisfies the condition $\Delta\nu = -\Delta\theta$, this approximate procedure may be considered an application of Prandtl-Meyer flow concepts.

Calculations of this nature, which start from an experimentally determined pressure distribution, necessarily contain the influence of the boundary layer. Since small changes in the rear part of an airfoil do not change the boundary layer appreciably, the included boundary-layer effect should approximate the real flow except in regions where the flow is separated.

The method based on Prandtl-Meyer flow concepts has been utilized to calculate some flap characteristics for a double-wedge diamond-shaped airfoil of 10 percent thickness from the known pressure distribution over the basic profile. (See ref. 10.) In figure 4 the local Mach number distribution over both the upper and lower surfaces is presented for several flap angles at a stream Mach number of 1.20 for a 25-percent-chord flap. The section lift coefficient due to flap deflection, shown in the same figure, is linear for small deflections where the velocities on the flap differ appreciably from sonic velocity and is nonlinear for the larger flap angles where the velocity on the upper surface of the flap is everywhere near sonic velocity. These linear and nonlinear characteristics may be predicted by a consideration of Prandtl-Meyer flow together with the transonic similarity rules.

LINEAR AND NONLINEAR FORCE AND PRESSURE CHARACTERISTICS

Nonlinear characteristics, as exemplified by the preceding calculation of lift due to flap deflection, may be explained on the basis of the sensitivity of local regions of near-sonic flow to a small disturbance. The sensitivity of a flow is defined as the change in some flow quantity, such as the pressure coefficient P or local Mach number M, due to a small disturbance such as a flap deflection δ or a change in airfoil thickness ratio t/c. The sensitivity of a flow may be denoted by a derivative such as $\frac{dP}{d\delta}$ or $\frac{dP}{d(t/c)}$. The relation between the flow sensitivity and nonlinear characteristics is contained in the following statement:

> Local regions of near-sonic flow are very sensitive to small disturbances - a circumstance which gives rise to nonlinear pressure variations; or stated another way, the introduction of a small disturbance into a region of near-sonic flow creates large changes in the flow and these changes vary nonlinearly with the disturbance strength.

The validity of this statement may be established by considering separately the effects of changes in angle of attack, airfoil thickness ratio, and airfoil geometry by means of the transonic similarity rules and considerations of Prandtl-Meyer flow.

The similarity rule for lifting wings with thickness (ref. 8) gives the result that, for a stream Mach number of 1, the lift coefficient is proportional to the angle of attack α for thick airfoils at small angles of attack and is proportional to the angle of attack to the two-thirds power for thin airfoils at small angles of attack. Experimental data indicate that thick airfoils possess no large regions of near-sonic flow at the surface, whereas thin airfoils possess large regions of near-sonic flow at the surface. Linear lift characteristics are indicated if no large regions of near-sonic flow exist at the surface; nonlinear lift characteristics are indicated if there are large regions of near-sonic flow at the surface.

Similar results are obtained from the similarity rule for airfoils with thickness at zero angle of attack for a stream Mach number of 1 since, from the similarity rule,

$$\left[\frac{dP}{d(t/c)}\right]_{M_\infty=1} \propto \frac{1}{(t/c)^{1/3}}$$

As the thickness ratio is decreased, the local Mach numbers approach 1 and the sensitivity of the flow increases.

Since information was obtained from the similarity rules for lifting and nonlifting wings with thickness only for a stream Mach number of 1, it is not possible to decide from these rules alone whether the flow sensitivity arises because the local Mach numbers are near 1 or because the stream is sonic. This question may be answered, however, by considering the sensitivity of a known flow which obeys the transonic similarity rule. Prandtl-Meyer flow proves useful in this regard and, in addition, may be used to show the effect of changes in geometry on the sensitivity. As discussed in the preceding section, changes in the surface pressures due to a change in geometry may be approximated by an application of Prandtl-Meyer flow concepts. Conclusions regarding the sensitivity of Prandtl-Meyer flow will also apply to those regions which can be determined by the method of characteristics.

The exact pressure coefficients for Prandtl-Meyer flow together with the transonic approximation for a stream Mach number of 1 are presented in figure 5. (Some supplementary remarks concerning Prandtl-Meyer flow are presented in the appendix.) An examination of figure 5 shows that a small change in θ creates a large change in the pressure coefficient when the local Mach numbers are near 1 regardless of the stream Mach number.

Since the flow sensitivity as indicated by the transonic similarity rules becomes infinite as the local Mach number approaches 1 for a stream Mach number of 1, and since Prandtl-Meyer flow, which can be expressed in the form of the similarity rule, becomes infinitely sensitive as the local Mach number approaches 1 for all stream Mach numbers, the sensitivity arises because the local Mach numbers are near 1.

Shock waves at transonic speeds are necessarily weak and may be approximated by an isentropic compression. An essential difference between isentropic and nonisentropic compressions is that an isentropic compression is infinitely sensitive for a local Mach number of 1, whereas a nonisentropic compression is infinitely sensitive for the slightly subsonic local Mach number corresponding to shock detachment. Since the change in surface pressures due to a change in airfoil geometry downstream of the limiting characteristic is approximately the same as that for the corresponding Prandtl-Meyer flow (provided the density of reflected compression waves is small), the sensitivity of these flows may be determined from a consideration of Prandtl-Meyer flow alone for both expansions and compressions. Thus, changes in geometry in regions of near-sonic flow cause large changes in the flow field which in turn give rise to nonlinear characteristics; linear characteristics are to be expected if the local Mach numbers are not near 1. Figure 4 presents a typical example of linear and nonlinear lift characteristics.

CONCLUDING REMARKS

An application of the method of characteristics is presented which affords a means for determining the surface pressures for a class of two-dimensional airfoils of given nose shape and arbitrary rear part in a sonic or supersonic stream if surface pressure data are given for one member of the class. For engineering purposes, the method of characteristics may be replaced by a simple application of Prandtl-Meyer flow concepts. The method appears to be applicable to airfoil rear parts lying within the no-reflection streamline provided the flow downstream of the limiting characteristic is everywhere supersonic.

A consideration of some results obtained from Prandtl-Meyer flow concepts together with the transonic similarity rules indicates that regions of near-sonic flow are very sensitive to small disturbances and that airfoils which possess such regions at the surface will exhibit nonlinear characteristics. Also, airfoils with no large regions of near-sonic flow at the surface are expected to have linear characteristics since these flows are not nearly so sensitive.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., September 22, 1952.

APPENDIX

SUPPLEMENTARY REMARKS CONCERNING PRANDTL-MEYER FLOW

The approximation for Prandtl-Meyer flow in the transonic region (ref. 8) may be written

$$P = \frac{2}{(\gamma + 1)M_\infty^2}\left\{M_\infty^2 - 1 - \left[\left(M_\infty^2 - 1\right)^{3/2} - \frac{3}{2}(\gamma + 1)M_\infty^2\theta\right]^{2/3}\right\} \tag{1}$$

which may be expressed in the form

$$P = \frac{\theta^{2/3}}{\left[(\gamma + 1)M_\infty^2\right]^{1/3}} f\left(\frac{M_\infty^2 - 1}{\left[(\gamma + 1)M_\infty^2\theta\right]^{2/3}}\right)$$

where

$$f(K) = -K + \left(K^{3/2} + \frac{3}{2}\right)^{2/3}$$

in agreement with the transonic similarity rule.

From equation (1) together with the relation

$$\frac{3}{2}(\gamma + 1)M_\infty^2\theta = \left(M^2 - 1\right)^{3/2} - \left(M_\infty^2 - 1\right)^{3/2}$$

the sensitivity of Prandtl-Meyer flow may be written

$$\frac{dP}{d\theta} = \frac{2}{\sqrt{M^2 - 1}}$$

which depends only upon the local Mach number and becomes infinite for a local Mach number of 1. Although equation (1) contains an abundance of fractional exponents, the pressure coefficient varies approximately linearly with θ for local Mach numbers not near 1. For local Mach numbers near 1, the nonlinear form must be used since the slope $dP/d\theta$ varies rapidly with θ. This result illustrates the reason for the failure of linearized theory at transonic speeds.

REFERENCES

1. Guderley, G., and Yoshihara, H.: The Flow Over a Wedge Profile at Mach Number 1. Jour. Aero. Sci., vol. 17, no. 11, Nov. 1950, pp. 723-735.

2. Vincenti, Walter G., and Wagoner, Cleo B.: Transonic Flow Past a Wedge Profile With Detached Bow Wave - General Analytical Method and Final Calculated Results. NACA TN 2339, 1951.

3. Cole, Julian D.: Drag of a Finite Wedge at High Subsonic Speeds. Jour. Math. and Phys., vol. XXX, no. 2, July 1951, pp. 79-93.

4. Von Kármán, Theodore: The Similarity Law of Transonic Flow. Jour. Math. and Phys., vol. XXVI, no. 3, Oct. 1947, pp. 182-190.

5. Kaplan, Carl: On Similarity Rules for Transonic Flows. NACA Rep. 894, 1948. (Supersedes NACA TN 1527.)

6. Busemann, Adolf: Application of Transonic Similarity. NACA TN 2687, 1952.

7. Spreiter, John R.: Similarity Laws for Transonic Flow About Wings of Finite Span. NACA TN 2273, 1951.

8. Harder, Keith C.: Transonic Similarity Rules for Lifting Wings. NACA TN 2724, 1952.

9. Von Kármán, Th.: Compressibility Effects in Aerodynamics. Jour. Aero. Sci., vol. 8, no. 9, July 1941, pp. 337-356.

10. Ivey, H. Reese, and Harder, Keith C.: A Velocity-Correction Formula for the Calculation of Transonic Mach Number Distributions Over Diamond-Shaped Airfoils. NACA TN 2527, 1951.

11. Wood, George P.: Experiments on Transonic Flow Around Wedges. NACA TN 2829, 1952.

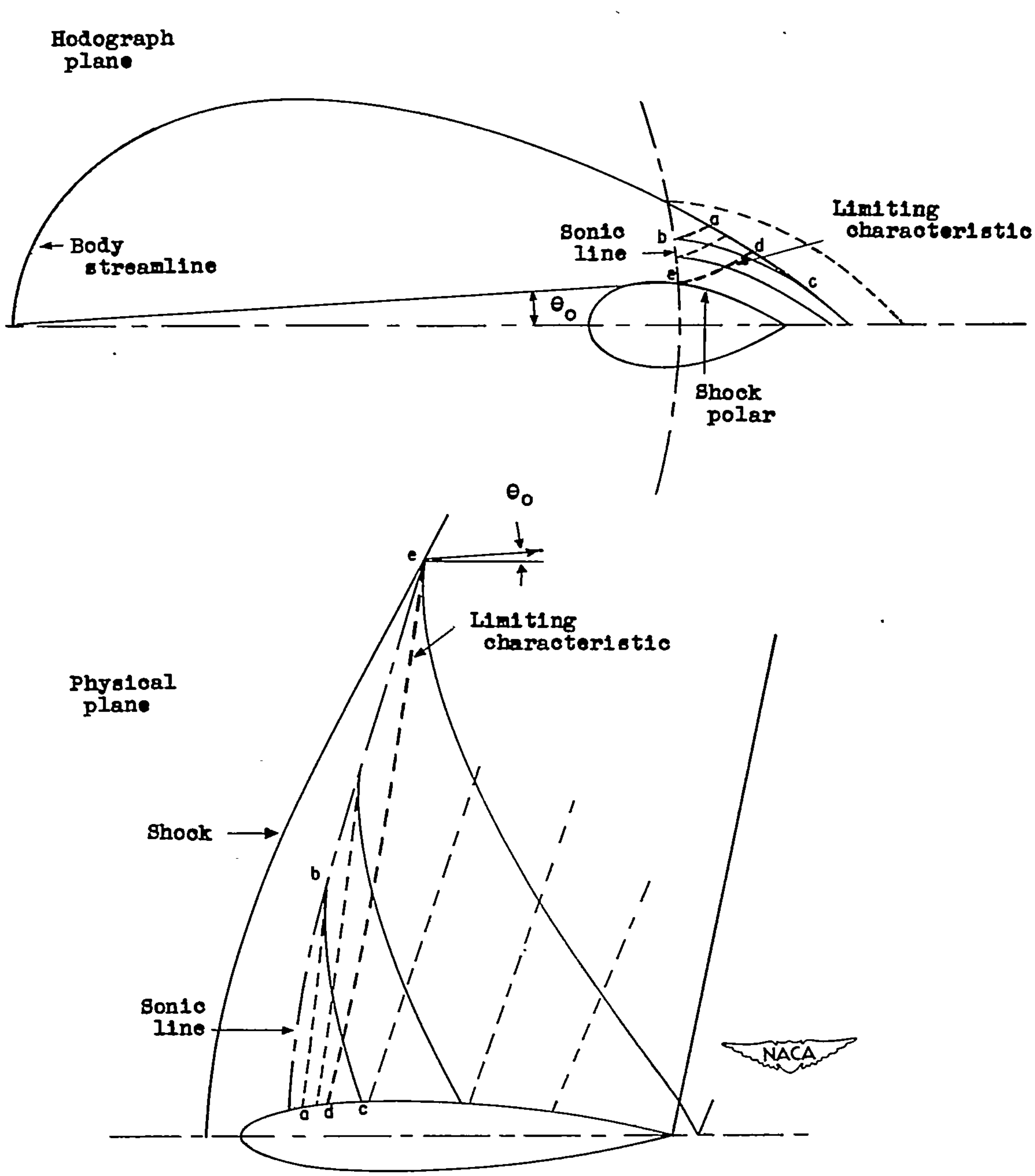

Figure 1.- Sketch illustrating some features of the flow past a symmetrical blunt-nosed airfoil in a slightly supersonic stream.

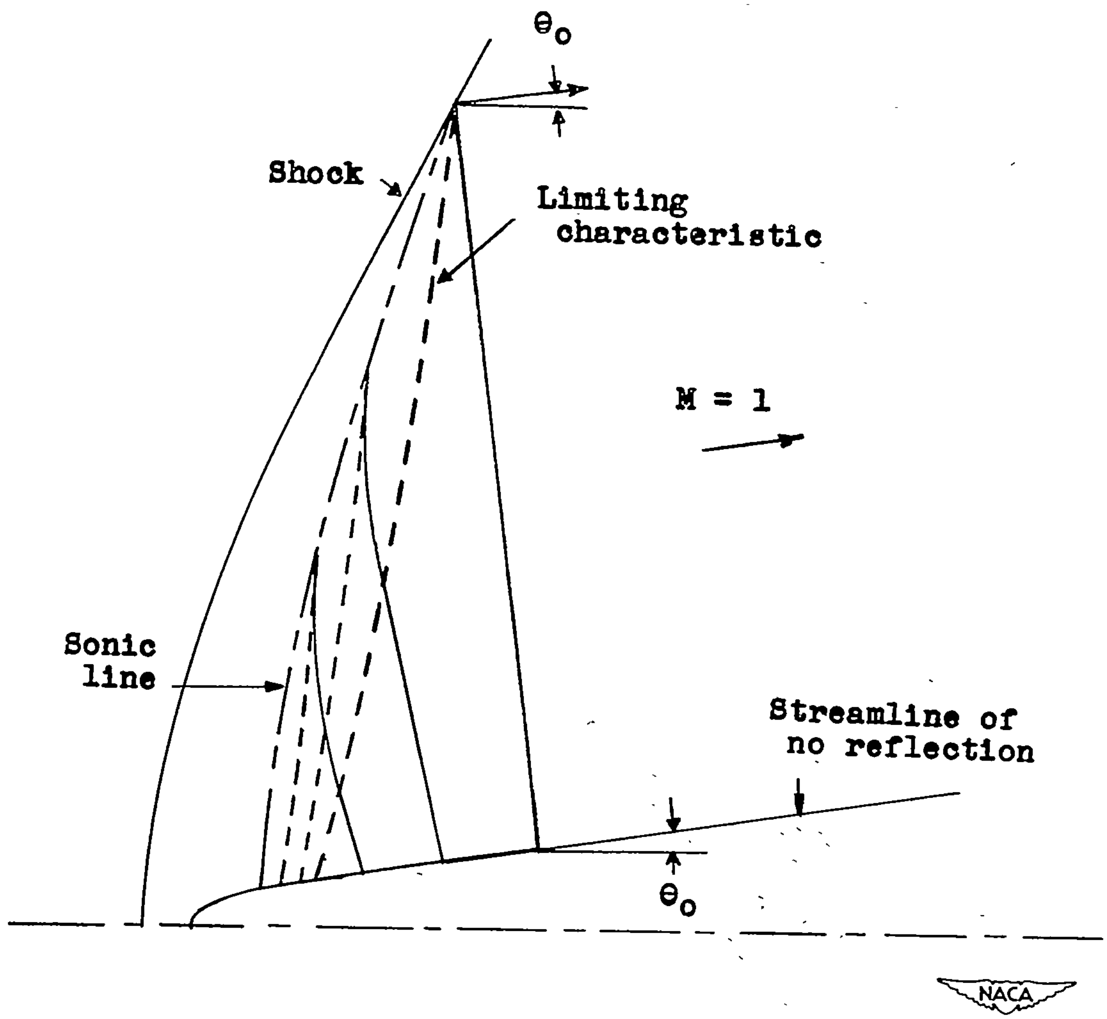

Figure 2.- Sketch illustrating the streamline of no reflection for a given nose shape.

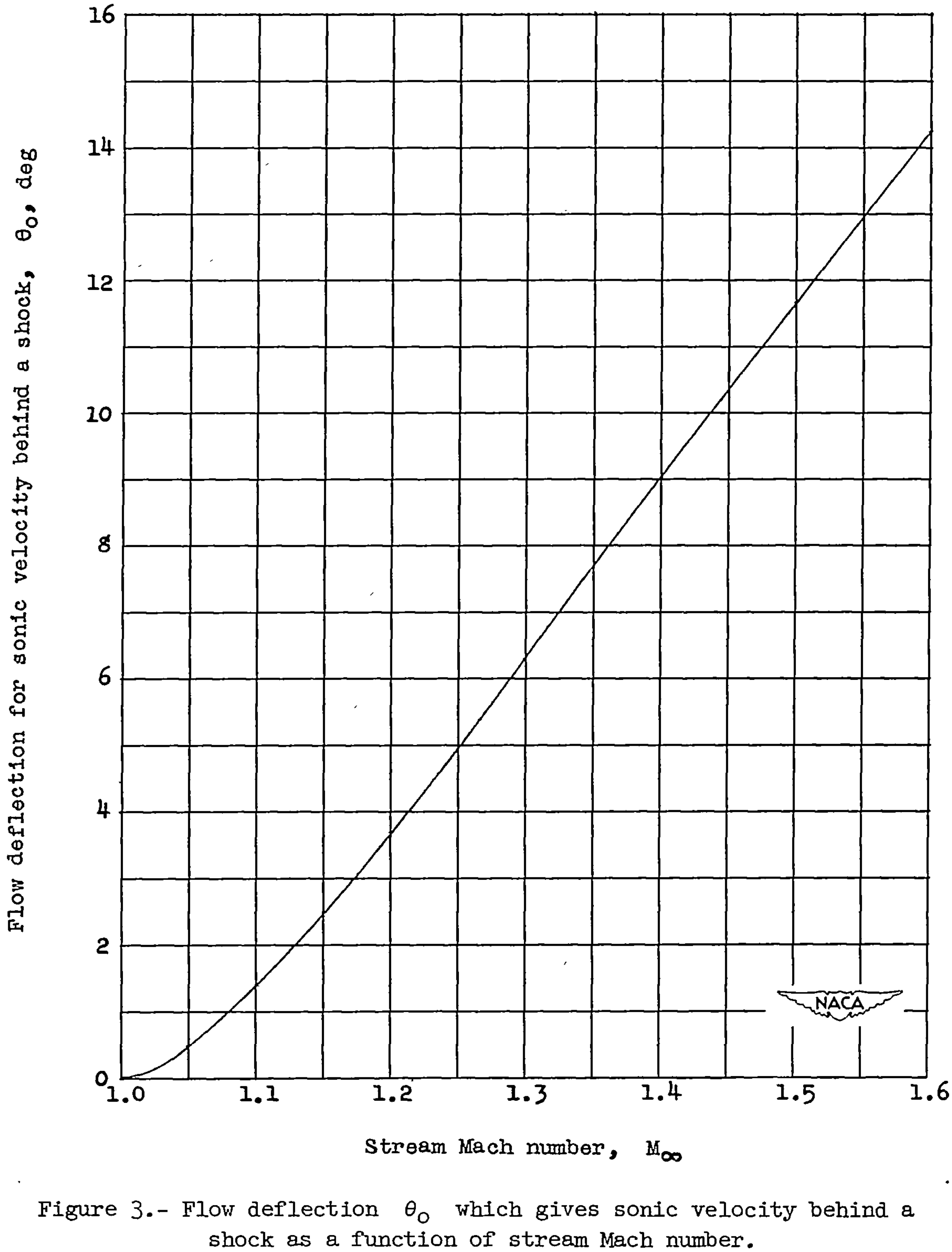

Figure 3.- Flow deflection θ_o which gives sonic velocity behind a shock as a function of stream Mach number.

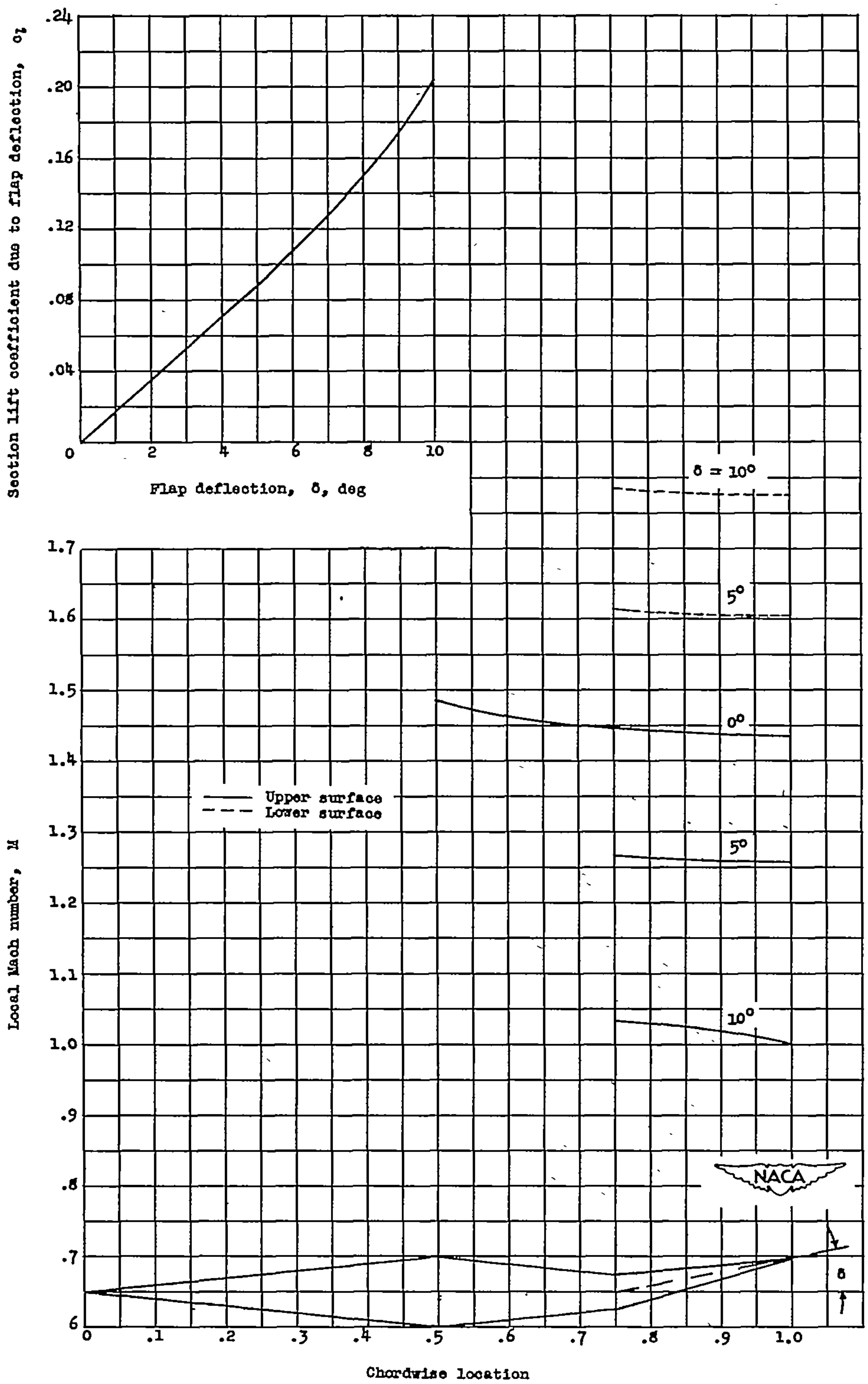

Figure 4.- Local Mach number distribution and section lift coefficient due to flap deflection for a diamond-shaped airfoil. $M_\infty = 1.20$.

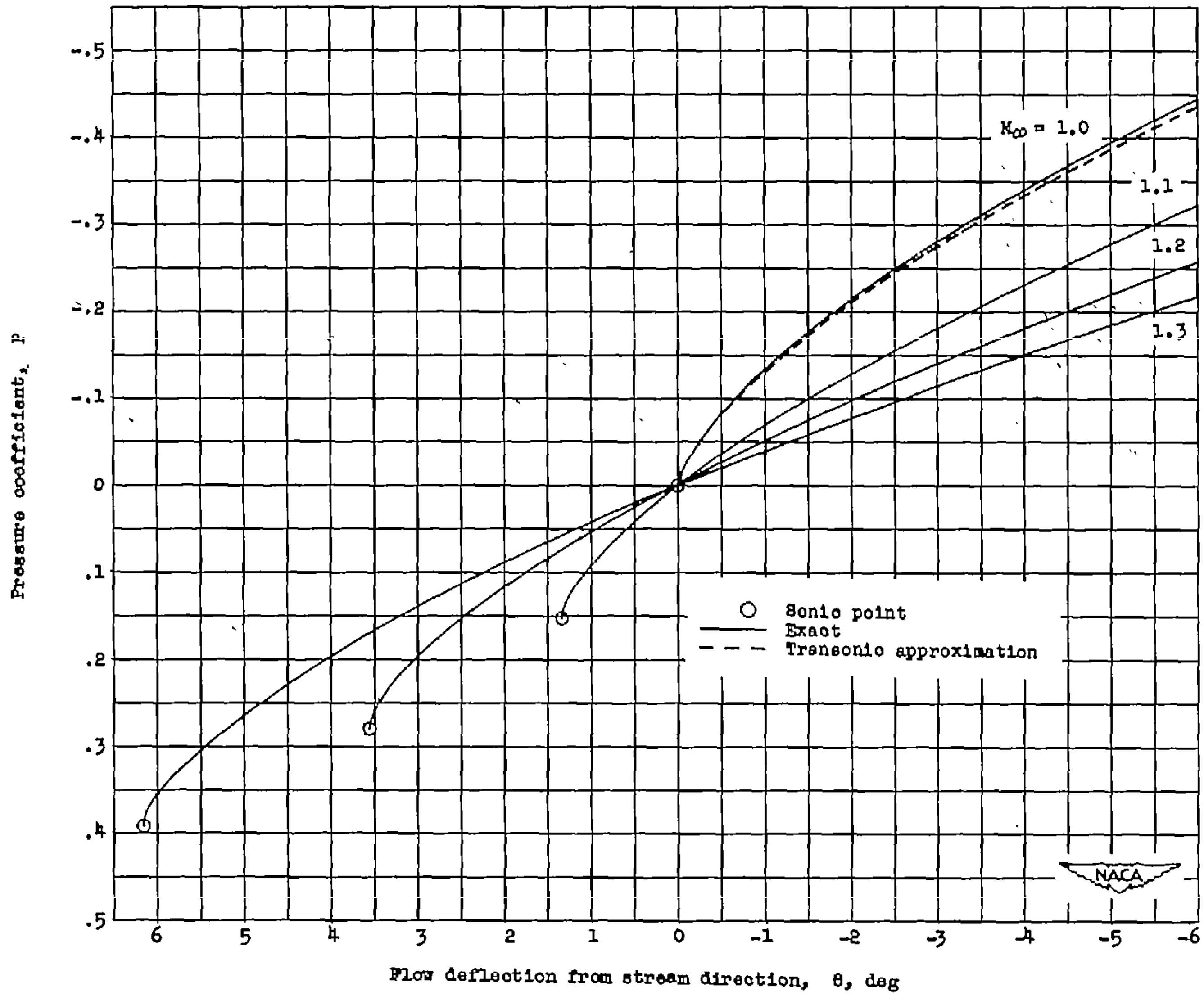

Figure 5.- Pressure coefficient for Prandtl-Meyer flow for several values of the stream Mach number.

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