NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2616

ACHIEVEMENT OF CONTINUOUS WALL CURVATURE IN DESIGN OF TWO-DIMENSIONAL SYMMETRICAL SUPERSONIC NOZZLES

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ACHIEVEMENT OF CONTINUOUS WALL CURVATURE IN DESIGN OF TWO-DIMENSIONAL SYMMETRICAL SUPERSONIC NOZZLES

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SUMMARY

Auxiliary boundary conditions are derived to assure continuity of wall curvature in applying the method of characteristics to the design of two-dimensional symmetrical supersonic nozzles. An illustrative example is included.

INTRODUCTION

The method of characteristics was first applied by Prandtl and Busemann to the evaluation of supersonic flows in 1929 (reference 1). Since then, a number of expositions (for example, see references 2, 3, and 4) have described various applications of characteristics to the design of two-dimensional supersonic nozzles. The problem considered was to determine wall contours that would transform a uniform or source flow usually at Mach number of unity to a uniform shock-free flow at some higher Mach number.

The wall contours so determined usually contain discontinuities in curvature which are tolerable for fixed geometry designs. In variable Mach number flexible-walled nozzles, however, a discontinuity in curvature implies a multivalued stress at a point, which in turn can only occur if there is a restraining moment. The design of flexible-walled supersonic wind tunnels usually does not include provision for such a restraining moment and hence the wall fairs the curvature across the discontinuity. If the characteristic diagram for the tunnel requires a discontinuity, then imperfections in the actual flow must appear.

Even if a rigid-walled nozzle is used, the probability that the theoretical gas flow will occur is slight. Changes in the ratio of specific heats, or any condensation of the gas components, or imperfections in the corrections for boundary layer can shift the characteristic pattern along the nozzle wall. The seriousness of such a shift

is minimized if the curvature is continuous and small. Here again nozzles with inherent continuity of curvature from the characteristic pattern are desired.

A plot of wall angles for a conventional supersonic nozzle including a discontinuity of curvature at the point of inflection is presented in figure 1. Also shown is a schematic plot on which the discontinuity in curvature has been eliminated. The auxiliary restrictions which must be imposed in the design of two-dimensional supersonic nozzles to eliminate discontinuity of wall curvature are presented herein. The work was completed during May, 1951, at the NACA Lewis laboratory. 2325

ANALYSIS AND DISCUSSION

The problem is to determine the auxiliary conditions which must be satisfied to eliminate discontinuities of wall curvature in the design of supersonic nozzles. In conventional designs, a discontinuity occurs at the geometric point of inflection and also at the start of the test section.

For convenience, two sets of characteristics ψ^+ and ψ^- will be utilized. In brief,

$$\psi^+ + \psi^- = \nu \tag{1}$$

is the Prandtl-Meyer expansion angle tabulated as a function of Mach number in reference 5. The quantity

$$\psi^+ - \psi^- = \theta \tag{2}$$

is the local flow deflection angle with respect to the axis. Either ψ^+ or ψ^- characteristics represent Mach lines in the flow. The ψ^+ set (fig. 2) originates on the contour wall and moves from left to right as the characteristic is traced from the wall to the tunnel center line. The ψ^- set is traced from left to right in projecting from the axis to the contour streamline. Thus, at the wall, ψ^+ is always larger than or equal to ψ^-.

The characteristic point is that place on the wall where ψ^+ first assumes the test-section value. The point of inflection occurs at the position of maximum wall angle.

The curvature along a streamline may be expressed mathematically by

$$C = \frac{d\theta}{ds}$$

where C is the curvature and s is the arc length. Continuity of curvature requires that

$$\frac{d\theta}{ds} = \frac{d\psi^+}{ds} - \frac{d\psi^-}{ds} \tag{2a}$$

be continuous.

2325 At the characteristic point and downstream therefrom, ψ^+ is constant at the test-section value. The curvature is then

$$\frac{d\theta}{ds} = -\frac{d\psi^-}{ds} \tag{3}$$

downstream of the characteristic point. The point of inflection, on the other hand, has zero curvature so that at the point of inflection

$$\frac{d\psi^+}{ds} = \frac{d\psi^-}{ds}$$

Thus, if the characteristic point coincided with the point of inflection, continuity of curvature would require that the local values of $\frac{d\psi^+}{ds} = \frac{d\psi^-}{ds} = -\frac{d\theta}{ds} = 0$. A zero value of $d\psi^-/ds$ at the point of inflection results only if $d\psi^+/ds$ vanishes at some point on the wall farther upstream. Hence, at that upstream point, $d\theta/ds = -d\psi^-/ds$. Because $d\psi^-/ds$ must be positive to avoid compression waves, $d\theta/ds$ would then be negative and the nozzle would contain at least two inflection points. Rather odd-shaped walls therefore result which are undesirable for flexible nozzles. Thus the characteristic point should not coincide with the point of inflection if the wall curvature is to be continuous. (This coincidence does occur on most conventional nozzle designs.) Likewise, by similar arguments, the point of inflection should not be downstream of the characteristic point.

The remaining possibility is for the point of inflection to be upstream of the characteristic point. No difficulty is experienced in achieving continuity of curvature at the point of inflection so long as $\frac{d\psi^+}{ds}$ and $\frac{d\psi^-}{ds}$ are continuous functions. Continuity can be achieved at the characteristic point also if $\frac{d\psi^+}{ds}$ approaches zero from the upstream side. The curvature would then blend smoothly with its downstream value given by equation (3). Thus ψ^+ is tangent at the characteristic point to its final value.

Vanishing of $\frac{d\psi^+}{ds}$ at the characteristic point infers a vanishing $\frac{d\psi^-}{ds}$ at the start of the test section (fig. 2). Hence the discontinuity of curvature at the test-section wall junction is likewise eliminated.

Thus the auxiliary conditions to assure continuity of wall curvature are:

(1) The geometric point of inflection $\left(\frac{d\theta}{ds} = 0\right)$ must be upstream of the characteristic point.

(2) The derivatives $\frac{d\psi^+}{ds}$ and $\frac{d\psi^-}{ds}$ must be continuous functions, and as a consequence ψ^+ must be tangent to its ultimate test-section value at the characteristic point. The continuity of $\frac{d\psi^+}{ds}$ and $\frac{d\psi^-}{ds}$ also infers continuity of Mach number gradient along the axis.

Nozzles with continuous wall curvature will always be longer than the shortest possible nozzle for a given Mach number and test-section size.

An example follows for which the wall curvature is continuous. Plots of ψ^+, ψ^-, θ, and Mach number distributions for an $M = 4$ nozzle are presented in figure 3. The flow was expanded from a uniform Mach number of unity at the throat in the usual manner (reference 2) to the wall point of inflection, where the curves of ψ^+ and ψ^- have the same slope. The ψ^+ curve was then faired from tangency at the point of inflection to the constant-test-section value in a horizontal tangent at the characteristic point. The remaining values of ψ^- and θ were determined by the characteristic plots for the nozzle. As a consequence of the ψ^+ fairing at the characteristic point, θ and ψ^- approach horizontal tangents at the test-section junction as does the wall Mach number.

In practice, nozzles of predetermined length are often desired. Precise estimation of the nozzle length for a given test-section size is difficult when only the characteristic point and final Mach number are specified. An alternative design procedure is therefore useful. The flow in the vicinity of the nozzle throat can be obtained by characteristics in the usual manner. A curve of either ψ^+ or Mach number is thus obtained along the axis which properly matches the initial supersonic flow to the sonic flow at the throat. From the design Mach number and test-section size, the upstream vertex of the test diamond is then located. Continuity of $d\psi^+/ds$ and $d\psi^-/ds$ along the nozzle contour implies continuity of $d\psi^+/ds$ and also dM/dx along the axis. Either the ψ^+ value or the Mach number distribution along the axis should then be faired in a smooth curve to a horizontal tangent at the upstream vertex of the test diamond. Working from the axis the remaining wall contours are determined by the characteristic plots to give a nozzle of the required length and size.

CONCLUSIONS

Auxiliary boundary conditions were derived to assure continuity of wall curvature in the application of the method of characteristics to two-dimensional symmetrical supersonic nozzles. These conditions are:

1. The point of inflection must be upstream of the characteristic point.

2. The first derivatives of the characteristic values $\frac{d\psi^+}{ds}$ and $\frac{d\psi^-}{ds}$ must be continuous functions. Hence ψ^+ must be tangent to its test-section value at the characteristic point. This latter restriction infers that the wall slope approaches a minimum or horizontal tangent (that is, zero curvature) at the test-section junction. Also the axial Mach number gradient must be continuous and approach zero at the upstream vertex of the test diamond.

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National Advisory Committee for Aeronautics
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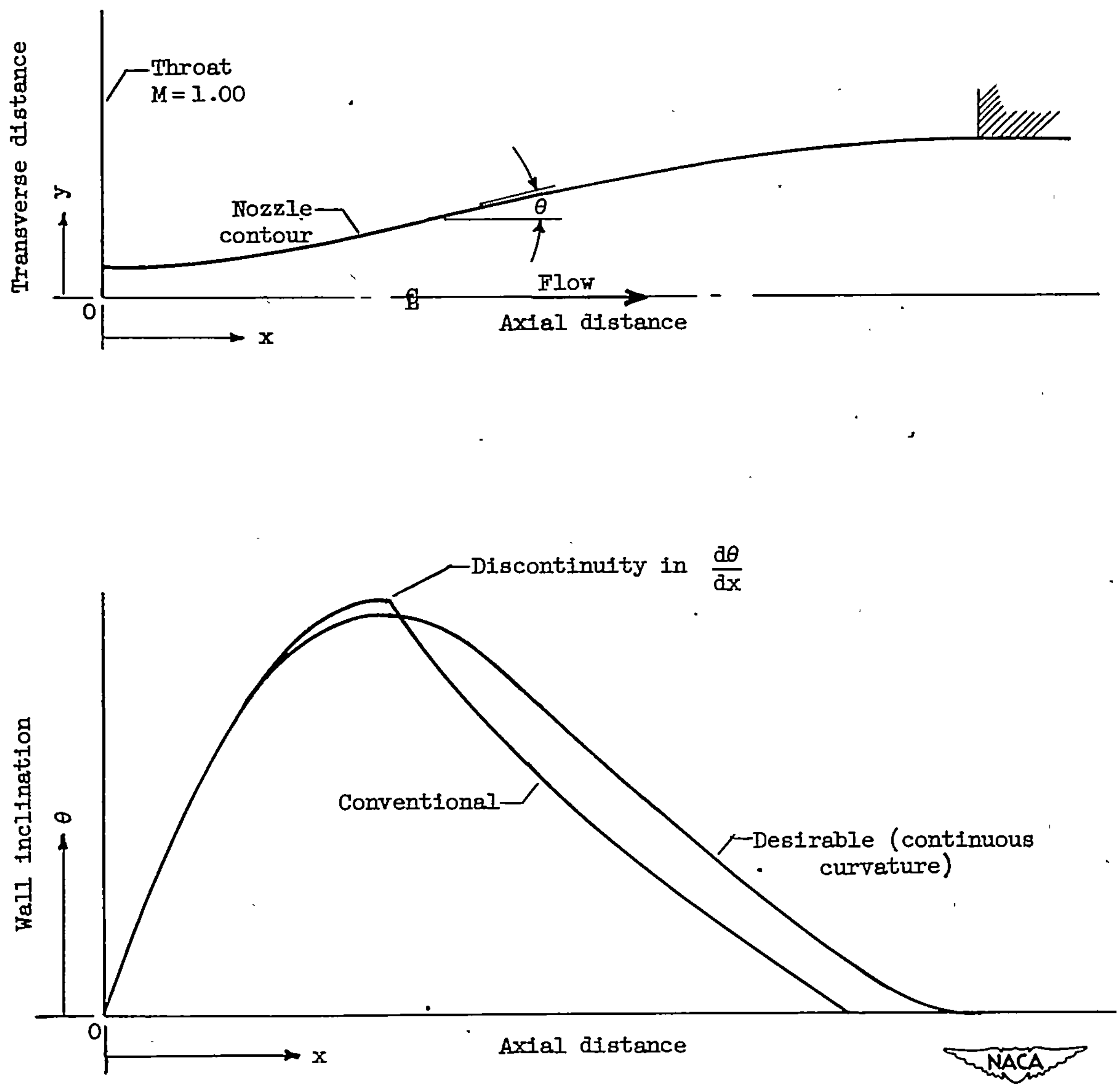

Figure 1. - Comparison of nozzle wall inclination for conventional and desirable nozzles.

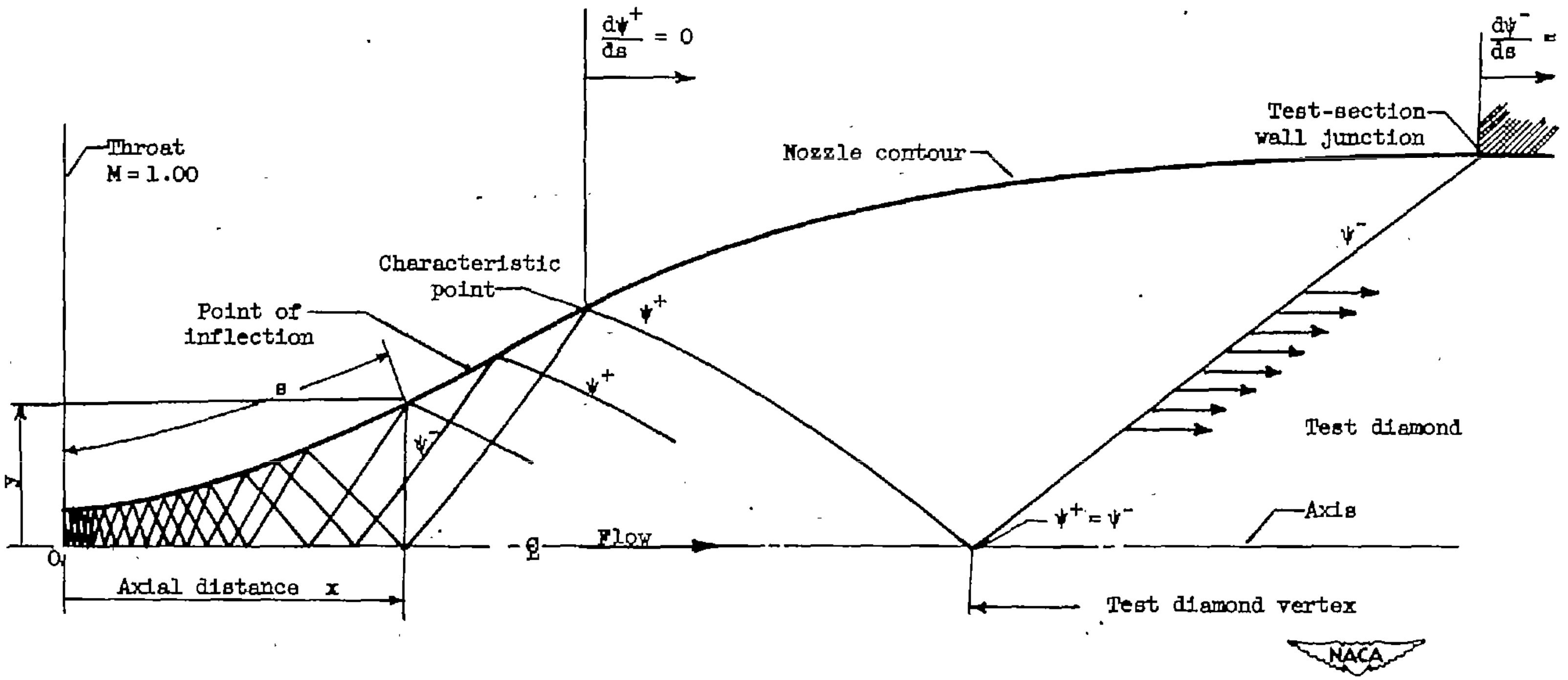

Figure 2. - Supersonic nozzle showing characteristic rays and coordinates.

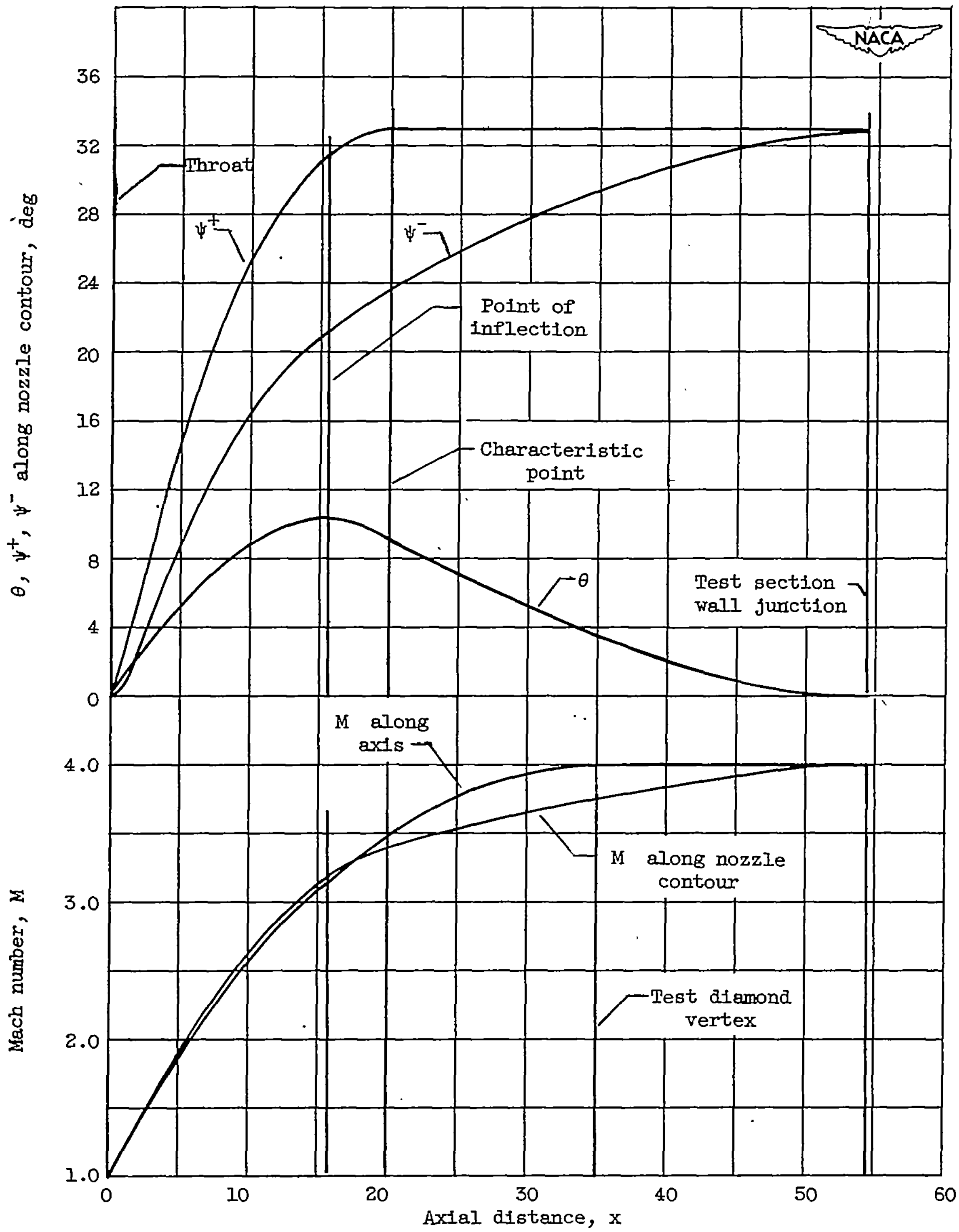

Figure 3. - Variation of flow parameters for a Mach number 4 two-dimensional supersonic nozzle.

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