NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2579

APPROXIMATE METHODS FOR CALCULATING THE FLOW ABOUT NONLIFTING BODIES OF REVOLUTION AT HIGH SUPERSONIC AIRSPEEDS

By A. J. Eggers, Jr., and Raymond C. Savin

Ames Aeronautical Laboratory
Moffett Field, Calif.

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SUMMARY

Flow at high supersonic speeds about a body of revolution is investigated analytically. With the assumption that the flow at the vertex is conical, it is found that algebraic solutions can be obtained which yield the Mach numbers and pressures at the surface over a considerable range of free-stream Mach numbers and apex angles. In the special case of cones, these solutions define the entire flow field with good accuracy, and may therefore provide a useful adjunct to the well-known M.I.T. tables.

The investigation of flow downstream of the vertex reveals that when the value of the hypersonic similarity parameter for the flow (i.e., the ratio of the free-stream Mach number to the slenderness ratio of the body) is large compared to 1, the Mach number along a streamline (downstream of the nose shock) varies with flow inclination angle in approximately the same manner as for two-dimensional (Prandtl-Meyer) flow. In the special case of streamlines near the surface, it is suggested that this parameter may approach 1. This result and the solutions obtained for flow at the vertex are combined to yield what might be called a conical-shock-expansion method for calculating the Mach number and pressure distributions at the surface of a body. These calculations are shown to be particularly simple in the case of slender bodies.

Surface Mach number and pressure distributions calculated with the simplified methods of this paper for a number of ogives are found to be in good agreement with those obtained with the method of characteristics at values of the hypersonic similarity parameter greater than 1. In the case of the conical-shock-expansion calculations, the agreement is within the order of accuracy of the characteristics solutions when the hypersonic similarity parameter has a value of only 2. Because of the relative simplicity of these calculations, the methods for determining the flow at the surface of a body may prove useful for engineering purposes.

INTRODUCTION

Determination of supersonic flow fields about nonlifting bodies of revolution by means of the method of characteristics (see, e.g., reference 1) is generally accepted as an accurate but tedious and time-consuming operation. Because of the latter features of this method, recourse is often made to simpler methods which, although less accurate, can be applied with relative simplicity and rapidity.

Perhaps the most widely used simplified theory of axially symmetric supersonic flow is the linear theory, which was first employed by Kármán and Moore (reference 2) to study supersonic flow about a body of revolution. This theory has a limited range of applicability, however, due to the assumption in its development of potential flow with infinitesimal disturbances. In particular, it is only applicable with good accuracy to bodies of practical slenderness ratios operating at low supersonic airspeeds.

More recently, Van Dyke (reference 3) has developed a second-order supersonic flow theory which yields results which are generally more accurate than those obtained with the linear or first-order theory, although the calculations are, of course, more lengthy. The range of applicability of Van Dyke's second-order solution is specifically limited, however, to cases for which the product of the free-stream Mach number and maximum slope of the body is less than 1; hence, it is not generally suitable for calculating the flow about bodies of usual proportions at high supersonic, or hypersonic, airspeeds.

Flow at hypersonic airspeeds in the limit as the free-stream Mach number approaches infinity and the ratio of specific heats of the gas downstream of a shock wave approaches 1 has been studied by Busemann (reference 4). It was found that under these circumstances a very simple expression is obtained for the pressures acting on a nonlifting body of revolution. This expression provides in general, however, only qualitatively accurate pressure distributions at high but finite free-stream Mach numbers where the ratio of specific heats is greater than 1 and is usually closer to the ideal diatomic gas value of 1.4.

Perhaps the first step in the direction toward providing simplified methods for determining axially symmetric flows in the high supersonic speed range was made by Tsien (reference 5). Tsien demonstrated that a similarity law (which is analogous to the well-known law applicable at low supersonic speeds) exists for slender pointed bodies in this range. Thus, with the aid of this law, the flow about a family of shapes all having the same thickness distribution can be easily determined, provided the ratio of the free-stream Mach number to the slenderness ratio is the same for all shapes, and provided the flow about one of these shapes is known. This result materially reduces the net labor associated with calculating flows about a number of related bodies of revolution operating at high supersonic airspeeds; however, the problem remains of calculating the flow about a representative body for each value of the similarity parameter. At present, this problem can only be solved with good accuracy by means of the method of characteristics.

There appears, then, to be a need for a simplified theory which provides, with engineering accuracy, the Mach number and pressure distributions about pointed nonlifting bodies of revolution operating at high supersonic airspeeds. The development of such a theory is undertaken in this report, and it is found that by treating the flow in two parts, namely, the flow at the vertex and the flow downstream of the vertex, a theory of the desired simplicity and accuracy can, in fact, be obtained. In the special case of flow about cones, the theory demonstrates surprising accuracy over a considerable range of supersonic airspeeds.

SYMBOLS

c_1 first family characteristic line

c_2 second family characteristic line

d maximum diameter of a body of revolution

E entropy

K hypersonic similarity parameter $\left(M_0\frac{d}{l}\right)$

l characteristic body length (measured from vertex to most forward point of maximum diameter)

M Mach number (ratio of local velocity to local velocity of sound)

n distance measured normal to ray passing through vertex of cone

p static pressure

q dynamic pressure

P pressure coefficient $\left(\frac{p-p_0}{q_0}\right)$

R gas constant

r radial distance from vertex of the cone

S distance along a streamline

u,v velocity components parallel and normal, respectively, to ray passing through vertex of cone

V resultant velocity $\left(\sqrt{u^2 + v^2}\right)$

$\hat{V}$ maximum velocity obtainable by expanding to zero temperature

x,y rectangular coordinates

β Mach angle $\left(\text{arc sin } \frac{1}{M}\right)$

γ ratio of specific heat at constant pressure to specific heat at constant volume

δ angle of flow inclination with respect to the body axis

ρ mass density

ω angle between axis of cone and ray passing through vertex of cone

Subscripts

o free-stream conditions

c conditions on the surface of a cone

N conditions on the surface at the vertex of a body

s conditions immediately behind the shock wave at the vertex of a body

ANALYSIS OF FLOW ABOUT A NONLIFTING BODY OF REVOLUTION

This investigation is concerned with a simplified method of calculating axially symmetric flow about a body of revolution traveling at high supersonic airspeeds. It is assumed throughout the analysis that the disturbed flow is everywhere supersonic, and thus, of course, that the body has a sharp nose or vertex. With these restrictions on the free-stream Mach number and body shape, it is evident that a typical flow field will be characterized by the bow shock wave lying close to the surface of the body. For the purpose of analysis, it is convenient to study the flow field in two parts, namely, the flow at the vertex and the flow downstream of the vertex. The results of these two phases of the investigation will then be applied to the determination of the Mach number and pressure distributions over the body surface.

Flow at the Vertex of a Body

It follows from the assumptions basic to this analysis that the nose shock is attached, and consequently that the initial conditions of the flow at the vertex will be the same as for a cone tangent to the body at the vertex and operating in the same free stream. In the classical paper of Taylor and Maccoll (reference 6) the basic differential equations were developed for steady, axially symmetric supersonic flow about cones.[1] The essential assumption in their analysis was that fluid properties were constant along radial lines passing through the vertex; hence the flow was irrotational. This assumption will be employed here to redevelop these differential equations in a form more suitable for this analysis.

A schematic diagram of axially symmetric supersonic flow about a cone is shown in figure 1. Considering first the flow downstream of the shock wave, the equations of motion and continuity are written (after reference 6) in the forms

$$V \frac{dV}{d\omega} + \frac{\gamma-1}{2} (\hat{V}^2 - V^2) \frac{1}{\rho} \frac{d\rho}{d\omega} = 0 \tag{1}$$

and

$$\frac{d}{d\omega} (\rho\ v \sin \omega) + 2\ \rho\ u \sin \omega = 0 \tag{2}$$

respectively. The condition of irrotationality is given by the expression

$$v = \frac{du}{d\omega} \tag{3}$$

Now it is evident from figure 1 that

$$u = V \cos (\omega - \delta) \tag{4}$$

and

$$v = -V \sin (\omega - \delta) \tag{5}$$

Therefore, equations (1) and (2) may be combined to obtain the relation

[1]As pointed out in reference 6, Busemann had previously suggested a graphical solution of the problem of supersonic flow past a cone.

$$\frac{\gamma-1}{2}\left(\hat{V}^2 - V^2\right)\left[V \cos (\omega-\delta)\left(1 + \frac{d\delta}{d\omega}\right) - \sin (\omega-\delta)\left(V \cot \omega + \frac{dV}{d\omega}\right)\right] + V^2 \frac{dV}{d\omega} \sin (\omega-\delta) = 0 \tag{6}$$

Similarly, the irrotationality equation may be written

$$\frac{dV}{V} = -\tan (\omega-\delta)\, d\delta \tag{7}$$

Combining equations (6) and (7), there results, then, the equation of motion

$$\frac{\gamma-1}{2}\left[\left(\frac{\hat{V}}{V}\right)^2 - 1\right]\left[\cot (\omega-\delta)\left(1 + \frac{d\delta}{d\omega}\right) - \cot \omega + \tan (\omega-\delta)\frac{d\delta}{d\omega}\right] - \tan (\omega-\delta)\frac{d\delta}{d\omega} = 0 \tag{8}$$

which will be employed in the following development.

Equations (7) and (8) are amenable to numerical integration following methods similar to those employed earlier in reference 6. In order to obtain algebraic solutions to flows at the vertex, however, it is necessary to simplify these equations. To this end, since high Mach number flows are of principal interest, an assumption which has proved useful in studying other aspects of these flows (see, e.g., reference 7) is employed, namely, the magnitude of the resultant velocity vector is essentially constant, the change in velocity being primarily one of change in direction. With the aid of this assumption it is possible to first determine a relation between δ and ω for a given V using equation (8), and then to determine more accurately the nature of the dependence of V on δ and ω using equation (7). This procedure will be employed for the following two cases: (1) slender cones for which δ, the angle of flow inclination, is small compared to 1 radian throughout the flow field, and (2) cones for which $\omega-\delta$, the difference between the ray angle and the inclination angle, is small compared to 1 radian throughout the conical part of the flow field.[2]

Analysis of cones for which $\delta \ll 1$ (slender cones).- With the restriction imposed upon δ in this study, equation (8) may be reduced to the form

[2]In general, $\omega-\delta$ will be small for blunt cones; however, at Mach numbers very large compared to 1, $\omega-\delta$ will be small for relatively slender cones as well.

$$\delta \cot\omega + \frac{d\delta}{d\omega}\left[1-M^2\sin^2\omega\,(1-\delta \cot\omega)^2\right] = 0 \tag{9}$$

where M is considered constant. This relation is still nonlinear in both δ and ω, and is not readily amenable to exact algebraic solution. An approximate solution may be obtained, however, in the following manner. Near the surface of the cone, equation (9) reduces to the linear equation[3]

$$\delta \cot\omega + \frac{d\delta}{d\omega} = 0$$

which has the solution

$$\delta = \frac{k_1}{\sin\omega} \tag{10}$$

Now $\delta \cot\omega$ decreases rapidly with increasing ω away from the cone surface; hence it is suggested that without appreciable loss of accuracy, the solution for δ (near the surface) given by this equation can be substituted into the coefficient of $d\delta/d\omega$ in equation (9). Performing this operation, equation (9) becomes

$$\delta \cot\omega + \frac{d\delta}{d\omega}\left[1 - M^2\sin^2\omega\,(1-k_1 \csc\omega \cot\omega)^2\right] = 0 \tag{11}$$

which is linear in δ and can be integrated to yield (to the order of accuracy of this analysis)

$$\frac{\delta}{\delta_c} = \left[\frac{1 + \dfrac{2M^2(\sin^2\omega_c - \sin^2\omega)}{1+\sqrt{1+4M^2\sin^2\omega_c}}}{1 + \dfrac{2M^2(\sin^2\omega_c - \sin^2\omega)}{1-\sqrt{1+4M^2\sin^2\omega_c}}}\right]^{\frac{1}{2\sqrt{1+4M^2\sin^2\omega_c}}} \tag{12}$$

which satisfies the boundary condition at the surface of the cone (i.e., $\delta = \delta_c$ at $\omega = \omega_c$).

Since M was assumed constant throughout the conical part of the flow field, it can conveniently be taken as the Mach number just downstream of the shock. In this case, flow conditions at the shock (i.e., M_s, ω_s, and δ_s) are obtained by the simultaneous solution of equation (12) and the oblique shock-wave equations

[3] It is clear that cot ω could be replaced with $1/\omega$ in this equation; however, the trigonometric operator is retained for consistency with the rest of the analysis in which such operators must, in general, be retained (ω is restricted to be small only near the surface of the cone).

$$M_s^2 = \frac{(\gamma+1)^2 M_0^4 \sin^2\omega_s - 4(M_0^2\sin^2\omega_s - 1)(\gamma M_0^2\sin^2\omega_s + 1)}{[2\gamma\ M_0^2\sin^2\omega_s - (\gamma-1)]\ [(\gamma-1)\ M_0^2\sin^2\omega_s + 2]} \tag{13}$$

and

$$\delta_s = \tan^{-1}\left[\frac{\cot\omega_s\ (M_0^2\sin^2\omega_s - 1)}{\frac{\gamma+1}{2} M_0^2 - (M_0^2\sin^2\omega_s - 1)}\right] \tag{14}$$

Having determined the flow conditions at the shock, the variation of δ with ω throughout the entire flow field about a cone is known from equation (12).

It now remains to more accurately determine the magnitude of the velocity throughout the conical part of the flow field. For this purpose, equation (7) must be employed in combination with an expression relating ω and δ. Equation (12) provides such a relationship; however, it is unnecessarily complicated for determining the small changes in V. A much simpler expression of acceptable accuracy is obtained by neglecting $\delta \cot \omega$ in the coefficient of $d\delta/d\omega$ appearing in equation (9),[4] thus yielding

$$\frac{d\delta}{d\omega} = \frac{-\delta \cot\omega}{1 - M^2\sin^2\omega} \tag{15}$$

or

$$\delta = k_2\sqrt{\csc^2\omega - M^2} \tag{16}$$

In this equation, k_2 and M^2 are determined by the requirements that

$$\delta = \delta_c \text{ at } \omega = \omega_c$$

and

$$\delta = \delta_s \text{ at } \omega = \omega_s$$

where the latter quantities are known from the previous analysis. Substitution of equation (16) into equation (7) provides the following relation for V; namely,

[4]As will be shown later, in the cases where this simplifying assumption introduces significant error in the variation of V with ω (viz., when $M^2 \sin^2\omega$ becomes of the order of 1) the results of the subsequent analysis of cones for which $\omega - \delta \ll 1$ may be applied with good accuracy.

$$\frac{dV}{V} = \frac{-k_2 d\delta}{\sqrt{\delta^2-k_2{}^2(1-M^2)}} + \left[1 + \frac{k_2{}^2}{\delta^2-k_2{}^2(1-M^2)}\right]\delta d\delta \tag{17}$$

which can be integrated to yield

$$l n\ V = -k_2 l n\left[\delta+\sqrt{\delta^2-k_2{}^2(1-M^2)}\right] + \frac{\delta^2}{2} + \frac{k_2{}^2}{2}\, l n\ [\delta^2-k_2{}^2(1-M^2)] + k_3 \tag{18}$$

where k_3 is determined by the requirement that $V = V_s$ when $\delta = \delta_s$. Replacing the constants in this expression with their values given by the imposed boundary conditions, there is then obtained the relation

$$\left(\frac{V}{V_s}\right)^2 = e^{(\delta^2-\delta_s{}^2)}\left[\frac{(\delta^2-\delta_s{}^2)\cot^2\omega_c + (\delta_c{}^2-\delta^2)\cot^2\omega_s}{(\delta_c{}^2-\delta_s{}^2)\cot^2\omega_s}\right]^{\frac{\delta_c{}^2-\delta_s{}^2}{\cot^2\omega_c-\cot^2\omega_s}}$$

$$\left[\frac{\delta_s\sqrt{\cot^2\omega_c-\cot^2\omega_s}+\sqrt{(\delta_c{}^2-\delta_s{}^2)\cot^2\omega_s}}{\delta\sqrt{\cot^2\omega_c-\cot^2\omega_s}+\sqrt{(\delta^2-\delta_s{}^2)\cot^2\omega_c+(\delta_c{}^2-\delta^2)\cot^2\omega_s}}\right]^{2\sqrt{\frac{\delta_c{}^2-\delta_s{}^2}{\cot^2\omega_c-\cot^2\omega_s}}} \tag{19}$$

Knowing the velocity, the Mach number may, of course, be determined from the relation

$$M^2 = \frac{M_s{}^2\left(\frac{V}{V_s}\right)^2}{1-\frac{\gamma-1}{2}M_s{}^2\left[\left(\frac{V}{V_s}\right)^2-1\right]} \tag{20}$$

and the pressure coefficient anywhere in the flow field may be obtained with the aid of the expression

$$P = \frac{2}{\gamma M_o{}^2}\left(\frac{p_s}{p_o}\,\frac{p}{p_s} - 1\right) \tag{21}$$

where

$$\frac{p_s}{p_o} = \frac{2\gamma\, M_o{}^2\, \sin^2\omega_s\, -(\gamma-1)}{\gamma+1} \tag{22}$$

and

$$\frac{p}{p_s} = \left(\frac{1 + \frac{\gamma-1}{2} M_s{}^2}{1 + \frac{\gamma-1}{2} M^2}\right)^{\frac{\gamma}{\gamma-1}} \tag{23}$$

The Mach number and pressure distributions (as well as the orientation of the conical shock) throughout the flow field about a slender cone are now known. The flow field about a cone for which ω-δ is small compared to 1 will be considered next.

<u>Analysis of cones for which ω-$\delta \ll 1$.</u>- In this case, equation (8) may be reduced to the form

$$1 - (\omega\text{-}\delta)\ \cot\ \omega + \frac{d\delta}{d\omega}\,[1 - M^2(\omega\text{-}\delta)^2] = 0 \tag{24}$$

where M is again considered constant. This nonlinear relation will be solved in a manner analagous to that employed in the solution of equation (9). Thus, near the surface of the cone, equation (24) reduces to[5]

$$\frac{d\delta}{d\omega} = -1$$

or

$$\delta + \omega = 2\delta_c \tag{25}$$

Combining equations (24) and (25) yields

$$d\delta = \frac{2\ (\omega - \omega_c)\ \cot\omega\ -1}{1 - 4M^2\ (\omega\text{-}\omega_c)^2}\,d\omega \tag{26}$$

which can be integrated to yield (substituting in the boundary condition at the cone surface)

[5]It should be noted that -1 is the exact value of $d\delta/d\omega$ at the surface, so long as the shock is attached. Consequently, in this analysis and the slender cone analysis both the magnitude and rate of change of δ with ω are satisfied at the surface.

$$\delta-\delta_c=\ln\left\{\left[\frac{\tan\omega_c+(\omega-\omega_c)}{\tan\omega_c}\right]^{\frac{2\tan\omega_c\sec^2\omega_c}{4M^2\tan^2\omega_c-1}}\left[1+2M(\omega-\omega_c)\right]^{\frac{\tan\omega_c+M(1+2M\tan\omega_c)}{4M^2(1-2M\tan\omega_c)}}\right.$$

$$\left.\left[1-2M(\omega-\omega_c)\right]^{\frac{\tan\omega_c+M(2M\tan\omega_c-1)}{4M^2(1+2M\tan\omega_c)}}\right\} \tag{27}$$

Again M is chosen as the Mach number just downstream of the shock, thus enabling M (or M_s), ω_s, and δ_s to be determined by solving equations (13), (14), and (27) simultaneously. Knowing these quantities, the relation between δ and ω throughout the remainder of the flow field may be obtained with equation (27).

In order to determine the small variations in V throughout the conical flow field, it is sufficiently accurate (since $\omega-\delta \ll 1$) to assume a linear variation of δ with ω, namely,

$$\delta = k_4\omega + k_5 \tag{28}$$

where k_4 and k_5 are fixed by the requirements that

$$\delta = \delta_c \text{ at } \omega = \omega_c$$

and

$$\delta = \delta_s \text{ at } \omega = \omega_s$$

Combining equations (28) and (7) there is then obtained the relation[6]

$$\frac{dV}{V} = -\tan\left[(\delta_c-\delta)\left(\frac{\omega_s-\delta_s}{\delta_c-\delta_s}\right)\right]d\delta \tag{29}$$

[6]The retention of the tangent operator in this relation is, to the accuracy of this analysis, optional.

which is easily integrated to yield

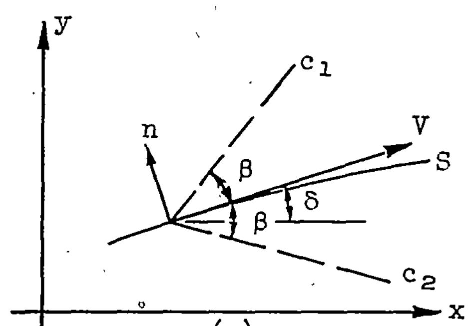
$$\frac{V}{V_s} = \left\{ \frac{\cos\ (\omega_s - \delta_s)}{\cos\left[\frac{(\delta_c - \delta)\ (\omega_s - \delta_s)}{\delta_c - \delta_s}\right]} \right\}^{\frac{\delta_c - \delta_s}{\omega_s - \delta_s}} \tag{30}$$

from which the Mach number and pressure coefficient anywhere in the flow field can be obtained by employing equations (20) through (23).

There is a well-defined restriction on the range of applicability of the results of this analysis. In particular, when $M_s(\omega_s - \omega_c) > 1/2$, an imaginary value of δ_s is obtained from equation (27). This result permits the establishment of a boundary in the (M_o, δ_c) plane, given by $M_s(\omega_s - \omega_c) = 1/2$, separating the area in which equation (27) applies from the area in which it obviously does not apply. This boundary is shown in figure 2,[7] and it is clear that there is a minimum value of δ_c for any M_o, below which the cone solutions just presented do not apply. In the area below this boundary, the slender cone solutions must be employed.

Flow Downstream of the Vertex of a Body

Simplified expressions have been obtained for calculating the flows about cones operating at high supersonic airspeeds, and as was pointed out previously, these expressions can be employed to determine the fluid properties at the vertices of pointed bodies of revolution other than cones. The investigation of the flow downstream of the vertices of such bodies is now undertaken. For this purpose it is convenient to first obtain an exact expression describing the fluid motion along a typical streamline in axially symmetric flow. To this end, consider sketch (a) showing the first and second family characteristic lines passing through a point on such a streamline. The compatibility equations defining the variation of fluid properties along these characteristic or Mach



(a)

[7]The boundary curve was obtained by using the results of reference 8. If the equations just developed were used, a boundary curve slightly below the one shown would have been obtained. Consequently, the indicated area of applicability of equation (27) is slightly conservative.

lines may be written (see, e.g., reference 1)

$$\frac{-\cos^2\beta}{\frac{\gamma-1}{2}+\sin^2\beta}\frac{\partial\beta}{\partial c_1}=\frac{\partial\delta}{\partial c_1}+\frac{\sin\beta\sin\delta}{y}+\frac{1}{\gamma R}\left(\frac{dE}{dn}\right)\sin^3\beta \tag{31}$$

along C_1, and

$$\frac{-\cos^2\beta}{\frac{\gamma-1}{2}+\sin^2\beta}\frac{\partial\beta}{\partial c_2}=-\frac{\partial\delta}{\partial c_2}+\frac{\sin\beta\sin\delta}{y}-\frac{1}{\gamma R}\left(\frac{dE}{dn}\right)\sin^3\beta \tag{32}$$

along C_2. Combining these equations, and noting that

$$\frac{\partial\beta}{\partial S}=\frac{\partial\beta}{\partial c_1}\frac{\partial c_1}{\partial S}+\frac{\partial\beta}{\partial c_2}\frac{\partial c_2}{\partial S}$$

and

$$\frac{\partial\delta}{\partial S}=\frac{\partial\delta}{\partial c_1}\frac{\partial c_1}{\partial S}+\frac{\partial\delta}{\partial c_2}\frac{\partial c_2}{\partial S}$$

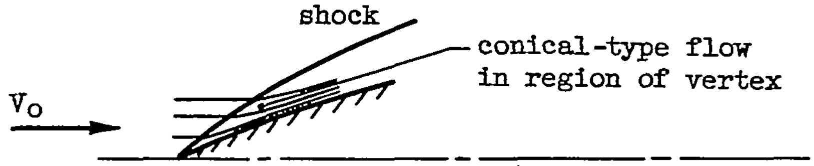
where

$$\frac{\partial c_1}{\partial S}=\frac{\partial c_2}{\partial S}=\frac{1}{2\cos\beta}$$

there is obtained the relation

$$\frac{\cos^3\beta}{\frac{\gamma-1}{2}+\sin^2\beta}\,d\beta=\cos\beta\,d\delta-\left(\frac{\partial\delta}{\partial c_1}+\frac{\sin\beta\sin\delta}{y}\right)dS \tag{33}$$

It is evident that the troublesome quantity in this equation is $\partial\delta/\partial c_1$, the rate of change of flow inclination along a first family Mach line. An insight into the behavior of this quantity in the flow field about a body of revolution operating at high supersonic airspeeds may be obtained, however, when the value of the similarity parameter K (the ratio of the free-stream Mach number to the slenderness ratio) for the body is large compared to 1.[8] To illustrate, consider first the flow between the shock wave and the surface in the region of the vertex of the body. Since the flow is conical in the limit as the vertex is approached, it seems reasonable to assume that the flow remains predominantly conical in type (see, e.g., the streamline pattern shown in sketch (b) for some distance downstream of the vertex.



(b)

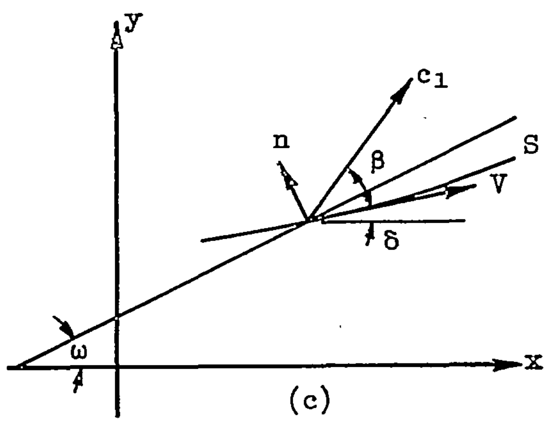
With this assumption[9] we have approximately, in the region under consideration (see analysis of cone for which $\omega-\delta \ll 1$),

$$\frac{d\delta}{d\omega} = -1$$

where

$$d\omega = \frac{\sin\omega \, dn}{y}$$

and n is measured normal to the ray making the angle ω with the x axis as shown in sketch (c). However, dn may be related to dc_1 by the following expression



(c)

$$dn = dc_1 \sin(\beta + \delta - \omega)$$

[8]When K is less than 1 the second-order theory may ordinarily be employed to analyze the flow field about a body.

[9]It is tacitly assumed throughout this and subsequent analyses that γ of the fluid is greater than 1. At the extreme hypersonic speeds where, for example, γ of the air flow downstream of the bow shock approaches 1, the streamline pattern must, as pointed out by Busemann in reference 4, have the shape of the body in the infinitesimally thin region between the shock and the surface. In this limiting case, Busemann's analysis will apply.

hence, combining these equations yields (in the notation of equation (33))

$$\frac{\partial \delta}{\partial c_1} = \frac{-\sin\,(\beta + \delta - \omega)\sin\omega}{y} \tag{34}$$

To the accuracy of this analysis, however (see equation (25)),

$$\omega = 2\delta_c - \delta$$

Introducing this relation into the above expression for $\partial\delta/\partial c_1$, and recalling that $2(\delta_c - \delta) = \omega - \delta \ll 1$, there is then obtained

$$\frac{\partial \delta}{\partial c_1} = \frac{-[\sin\beta\,\sin\delta - 2(\delta_c - \delta)\,\sin\,(\delta - \beta) - 4(\delta_c - \delta)^2\,\cos\beta\,\cos\delta]}{y} \tag{35}$$

Consistent with the assumption that K is large compared to 1, this equation may be further simplified, since the last two terms on the right in the numerator are negligibly small compared to the first term. In this case equation (35) reduces to the form

$$\frac{\partial \delta}{\partial c_1} = \frac{-\sin\beta\,\sin\delta}{y} \tag{36}$$

With the aid of this expression, equation (33) readily reduces to the more tractable form

$$\frac{\cos^2\beta}{\frac{\gamma - 1}{2} + \sin^2\beta}\,d\beta = d\delta \tag{37}$$

relating the change in Mach number with flow inclination along a streamline in the region of the vertex. Equation (37) is recognized, of course, as the differential equation for Prandtl-Meyer flow.

Consider now the flow downstream of the region of the vertex (just considered) where the slopes of the streamlines are relatively small[10] and the ordinates are relatively large. Flow in this region is, in general, certainly not predominantly of the conical type, and hence the previous analysis cannot be expected to apply; however, for the values of K under consideration, certain observations can be made regarding the relative order of magnitude of the terms in equation (33). For example, it is noted that as K is increased for a given body (which is tantamount to increasing M_o), the term $\frac{\sin\beta\sin\delta}{y}\,dS$ decreases while the term $\cos\beta d\delta$ increases. As K takes on large values compared to 1, the former term must become, in fact, small compared to the latter term. It is thus indicated that for flow of the type under consideration equation (33) may be simplified to the two-dimensional form

$$\frac{\cos^3\beta}{\frac{\gamma-1}{2}+\sin^2\beta}\,d\beta = \cos\beta d\delta - \frac{\partial\delta}{\partial c_1}\,dS$$

Similarly, the compatability equations reduce to their two-dimensional forms, which implies, of course, that the flow field itself is approximately two-dimensional. This being the case, however, it may be shown (from the results presented in reference 9) that $\partial\delta/\partial c_1$ is generally small compared to $\partial\delta/\partial c_2$ and hence that $(\partial\delta/\partial c_1)dS$ is likewise small compared to $\cos\beta d\delta$. Thus essentially two-dimensional (Prandtl-Meyer) flow is found to prevail downstream of the region of the vertex. This result, although remarkably simple, is not entirely surprising since it would be anticipated that three-dimensional effects would be reduced as the local Mach numbers become large and hence the local Mach angles become small. In any case, we have the result that when K is large compared to 1, the Mach number and flow-inclination angle along a streamline in the flow field about a body of revolution are related by the familiar isentropic expansion relation (obtained by integrating equation (37)).

$$\delta_B - \delta_A = \sin^{-1}\frac{1}{M_A} - \sin^{-1}\frac{1}{M_B} + \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma+1}{(\gamma-1)(M_B^2-1)}} - \tan^{-1}\sqrt{\frac{\gamma+1}{(\gamma-1)(M_A^2-1)}} \tag{38}$$

[10]In this discussion it is assumed that $\partial\delta/\partial S<0$ on the body surface. In cases where $\partial\delta/\partial S>0$ on the surface, it is not evident that the argument will apply. It is suggested, however, that in these cases the previous arguments concerning flow in the region of the vertex will apply over a larger region downstream of the vertex.

where A and B are different points on the same streamline.

If the streamline flow pattern were known throughout the flow field, then the Mach number distribution could be readily obtained with equation (38). Perhaps the most useful application of this and previous results of the analysis is, however, to the determination of Mach number and pressure distributions at the surface of a body. Attention is therefore turned to this calculation.

Flow at the Surface of a Body

The procedure for determining flow conditions at the surface is entirely analogous to that employed in the application of the so-called shock-expansion method to airfoils, and hence might be called a conical-shock-expansion method. For example, the Mach number on the surface at the vertex is obtained with equations (13), (14), and (20) in combination with equations (12) and (19), or (27) and (30) depending on whether $\delta_c \ll 1$, or the values of M_o and δ_c are such that $\omega-\delta \ll 1$, respectively.[11] The variation of Mach number downstream of the vertex is then obtained with equation (38). Knowing the Mach number distribution, the pressure distribution (in coefficient form) on the surface is readily obtained with equations (21) through (23).

Simplified Expressions for Slender Bodies

In the case of slender bodies the above described calculations become so simple as to warrant special attention. This additional simplification arises from the fact that now, not only is ω-δ small throughout the flow field, but also both ω and δ are small throughout the field. Thus, too, M is everywhere large compared to 1, and it results, as shown in appendix A, that explicit solutions can be obtained for the Mach number and pressure at any point on the surface of a body. These solutions may be summarized (from appendix A) as follows. The local Mach number at any point is given by the relation

$$M = \frac{M_N}{1 - \frac{\gamma-1}{2}\left(M_N \delta_N\right)\left(1 - \frac{\delta}{\delta_N}\right)} \tag{39}$$

[11] It is clear, of course, that the results of reference 8 may also be used for this purpose. It is preferable, in fact, to use these results when interpolation is not necessary.

where

$$\left.\begin{aligned} M_N^2 &= M_s^2\left\{1+\frac{\gamma-1}{2}(M_s\,\delta_N)^2\left[1+\ln\left(\frac{\delta_s}{\delta_N}\right)^2-\left(\frac{\delta_s}{\delta_N}\right)^2\right]\right\} \\ M_s^2 &= \frac{M_o^2\left[1+\frac{\gamma+1}{2}(M_o\delta_N)^2\right]}{\left[1+\gamma\,(M_o\delta_N)^2\right]\left[1+\frac{\gamma-1}{2}(M_o\delta_N)^2\right]} \\ \text{and} \quad \left(\frac{\delta_s}{\delta_N}\right)^2 &= \frac{(M_o\delta_N)^2}{1+\frac{\gamma+1}{2}(M_o\delta_N)^2} \end{aligned}\right\} \qquad (40)$$

The pressure coefficient at any point on the surface may be obtained from the expression

$$P = \frac{2}{\gamma M_o^2}\left\{\left[1+\gamma(M_o\delta_N)^2\right]\left(\frac{M_s}{M}\right)^{\left(\frac{2\gamma}{\gamma-1}\right)}-1\right\} \qquad (41)$$

It is interesting to note that these expressions predict the ratios of local to free-stream Mach numbers, and local to free-stream static pressures to be the same at corresponding points on related bodies, provided the flow fields about these bodies are related by the same value of the hypersonic similarity parameter. These predictions are identical to those of the hypersonic similarity law (reference 7) and consequently they provide a necessary check on the validity of the assumptions underlying the development of the simplified methods of this paper. It remains now to evaluate the accuracy of these methods by comparing their predictions with those of more exact theories.

Comparison of Approximate and Exact Calculations of Flow About a Body of Revolution and Discussion of Results

The flow fields about a number of nonlifting cones operating at supersonic airspeeds have been computed numerically by Kopal (reference 8) following a procedure similar to that first employed by Taylor and Maccoll. The results of these calculations provide an accurate check on the approximate solutions developed in this paper for cones.

In figures 3 through 6 are shown the variations of δ with ω for cones having semiapex angles of 5°, 10°, 20° and 40°. The predictions of the slender-cone solutions are observed to be in good agreement with the results of reference 8 for both the 5° and 10° cones (figs. 3 and 4) even at Mach numbers as low as 1.5, and in from fair to good agreement for the 20° cone. The agreement improves with increasing Mach number, as would be expected. At the higher Mach numbers, where the cone solution for which $\omega-\delta \ll 1$ applies, δ is accurately predicted as a function of ω for all four cones, the range of applicability (in terms of Mach number) being the largest, of course, for the 40° cone. It is interesting to note that fair agreement is obtained for the latter cone down to a free-stream Mach number as low as 2, which is only slightly above the Mach number for shock detachment.

The variations of velocity with ray angle throughout the flow fields about the above described cones are shown in figures 7 through 10. The agreement between the cone solutions of this paper and the results of reference 8 for these variations parallels that found for the flow inclination angle as a function of the ray angle. In general the velocity variations are small, decreasing percentagewise with increasing Mach number, as was assumed in the analysis. It is encouraging to note, too, that either or both of the two approximate solutions generally provide an accurate prediction of the variation of δ and V with ω throughout the flow field (e.g., figs. 4 and 8). It appears that a rule of thumb for choosing the preferable solution is to employ the cone solution for which $\omega-\delta \ll 1$ whenever it is applicable (fig. 2).

The pressure coefficients on the surfaces of the four cones have been calculated using the two approximate solutions, and the results of these calculations, along with the predictions of second-order theory (taken from reference 3) and the results obtained from reference 8 are presented in figure 11. It is observed that the two approximate solutions overlap to predict pressure coefficients for the 5°, 10°, and 20° cones that are in good agreement with those of reference 8 at Mach numbers from approximately 1.5 to infinity. Comparable accuracy is obtained from the cone solutions for which $\omega-\delta \ll 1$ for the 40° cone at Mach numbers above about 3. In general, of course, the agreement improves with increasing Mach number. The second-order theory yields more accurate results than the slender cone theory for the 5° and 10° cones at the lower Mach numbers, although the reverse seems to be the case for the 20° cone. Neither second-order nor slender-cone theory is applicable to the 40° cone.

From the preceding comparison of the conical flow calculations of reference 8 with the predictions of the simplified solutions of this paper, it is indicated that the latter solutions may be employed to predict the properties in the flow field about a cone with from good to excellent accuracy, depending on whether the Mach number of the free stream corresponds to intermediate or high supersonic airspeeds. It is therefore suggested that these solutions may, for example, be particularly useful at high supersonic speeds for accurately determining the conical flow fields about cones at Mach numbers not treated in the M.I.T. tables (reference 8).

It remains now to determine the accuracy with which the solutions for flow about cones in combination with the isentropic expansion equations predict the flow at the surface of bodies of revolution other than cones. The Mach number and pressure distributions at the surface of a family of ogives operating at a free-stream Mach number of 6 were calculated using the simplified methods of this paper in the manner described in the analysis. These distributions are presented in figures 12 and 13 for values of K varying from 1/2 to 2. Also shown are the results presented in reference 10,[12] obtained with the method of characteristics including effects of rotation. In general it is observed that with increasing K, the conical-shock-expansion theory of this paper yields Mach numbers and pressures that are in better agreement with the predictions of characteristics theory. Indeed at K as low as 2, the predictions of the two theories agree within the accuracy of the characteristics solution on the ogival nose.[13] The several assumptions made in the development of the conical-shock-expansion method are thus justified in part. It is somewhat surprising, however, that the method works as well as it does at values of K near 1. It is suggested that this result may be peculiar to streamlines near the surface of a body, for it may easily be shown (with an argument analogous to that presented in the analysis) that in the region of the vertex where three-dimensional effects are perhaps most pronounced, flow in stream tubes adjacent to the surface nevertheless shows agreement with the two-dimensional (Prandtl-Meyer) relation between stream inclination and Mach number or pressure for values of K approaching 1.

The slender body solution also displays increasing accuracy with increasing K, although it consistently predicts too high Mach numbers and too high pressures on the surface near the vertex. This result is

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[12]The Mach number distributions presented here were calculated from the pressure distributions given in reference 10.

[13]The frequently suggested method of determining surface pressures on a body of revolution by assuming the pressure at a point will be the same as on a cone tangent to the body at that point was also tried and, in general, the pressures were too high. The error was less at larger K, however, although it was greater than for the conical-shock-expansion theory. As would be expected, the Mach number distributions were generally in considerable error.

primarily a consequence of the approximate nature of the conical flow solution employed in the development of the theory. In any case even at a K of 1, the slender body theory displays sufficient accuracy for many engineering purposes. Indeed, upon inspection of the pressure distributions, it is evident that, although doubtless fortuitously, the slender body theory will yield more accurate drag coefficients than the conical-shock-expansion theory at the lower values of K.

Comparisons similar to those just discussed were made for the other ogives considered in reference 10, and in general the same results were obtained; namely, for $K>1$, the simplified theories of this paper were in good agreement with the predictions of the characteristics solutions, the difference between the predictions of the latter theory and the conical-shock-expansion theory being of the order of accuracy of the characteristics solutions at K as low as 2.

CONCLUDING REMARKS

With the assumption that the flow at the vertex of a body is conical, it was found that simple approximate solutions can be obtained which yield the Mach number and pressure at the surface over a considerable range of free-stream Mach numbers and apex angles. In the special case of cones, these solutions define the entire flow field with good accuracy, and may therefore provide a useful adjunct to the well-known M.I.T. tables for flow about cones.

The investigation of flow downstream of the vertex revealed that when the hypersonic similarity parameter K (the ratio of free-stream Mach number to slenderness ratio) is large compared to 1, the Mach number varies with flow inclination angle along a streamline in approximately the same manner as for two-dimensional (Prandtl-Meyer) flow. In the special case of streamlines near the surface, it was suggested that this parameter may approach 1. This and preceding results obtained for flow at the vertex were combined to yield what might be termed a conical-shock-expansion method for calculating the Mach number and pressure distributions at the surface of a body. In the special case of slender bodies, exceedingly simple explicit expressions were obtained for these quantities.

Surface Mach number and pressure distributions were calculated with the simplified methods of this paper for a family of ogives traveling at a Mach number of 6. These distributions were in good agreement with those obtained with the method of characteristics at values of K greater than 1. In the case of the conical-shock-expansion calculations, the agreement was within the order of accuracy of the characteristics solutions at K as low as 2.

Because of the relative simplicity of the proposed methods of determining the flow at the surface of a body operating at high supersonic airspeeds ($K > 1$), these methods should prove useful for engineering purposes. It is also suggested that the same general approach may be applicable (again for $K > 1$) in developing methods for calculating flow at the surface of pointed bodies of revolution at small angles of attack.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif., Sept. 14, 1951

APPENDIX A

FLOW AT THE SURFACE OF A SLENDER BODY OPERATING AT HIGH SUPERSONIC AIRSPEEDS

If a slender body (i.e., a body on the surface of which the slopes are everywhere small compared to 1) is operating at free-stream Mach numbers very large compared to 1 (again, of course, $K>1$), the local Mach numbers will likewise be large compared to 1. It follows, then, that the inclination of the nose shock wave will be small and, consequently, that ω will always be small. In this case the relation between δ and ω at the vertex is extremely simple;[14] namely, (see equation (10))

$$\delta = \frac{\delta_N^2}{\omega} \tag{A1}$$

Combining this expression with equation (7), the relation

$$\frac{V}{V_s} = \left(\frac{\delta_s}{\delta}\right)^{\delta_N^2} e^{\frac{1}{2}(\delta^2-\delta_s^2)} \tag{A2}$$

defining the velocities in the flow field is easily obtained. Hence, the Mach number on the surface at the vertex, M_N, may (to the order of accuracy of this analysis) be related to M_s by combining equation (20) with this expression to yield

$$M_N^2 = M_s^2\left\{1 + \frac{\gamma-1}{2}(M_s\delta_N)^2\left[1 + ln\left(\frac{\delta_s}{\delta_N}\right)^2 - \left(\frac{\delta_s}{\delta_N}\right)^2\right]\right\} \tag{A3}$$

Now the oblique shock-wave equations for flow of the type under consideration reduce to

[14] The equations developed in the following analysis could be obtained from the final expressions previously developed by reducing them to conform to the assumptions of this slender-body theory. It seems simpler, however, to proceed, as indicated, from the basic-flow equations.

$$M_s^2 = \frac{(\gamma+1)^2 M_o^2 (M_o\omega_s)^2}{[2\gamma\ (M_o\omega_s)^2 - (\gamma-1)]\ [(\gamma-1)(M_o\omega_s)^2 + 2]} \tag{A4}$$

and

$$\frac{p_s}{p_o} = \frac{2\gamma}{\gamma+1}\ (M_o\omega_s)^2 - \frac{\gamma-1}{\gamma+1} \tag{A5}$$

where

$$M_o\omega_s = M_o\delta_s\ \frac{(\gamma+1)}{4} \left\{1 + \frac{\sqrt{[(\gamma+1)M_o\delta_s]^2 + 16}}{(\gamma+1)\ M_o\delta_s}\right\} \tag{A6}$$

Combining equations (A1) and (A6) there then results

$$M_o\omega_s = \sqrt{1 + \frac{\gamma+1}{2}\ (M_o\delta_N)^2} \tag{A7}$$

and

$$\left(\frac{\delta_s}{\delta_N}\right)^2 = \frac{(M_o\delta_N)^2}{1 + \frac{\gamma+1}{2}(M_o\delta_N)^2} \tag{A8}$$

Equation (A4) can now be written (considering equation (A7))

$$M_s^2 = \frac{M_o^2\left[1 + \frac{\gamma+1}{2}\ (M_o\delta_N)^2\right]}{\left[1 + \gamma\ (M_o\delta_N)^2\right]\ \left[1 + \frac{\gamma-1}{2}\ (M_o\delta_N)^2\right]} \tag{A9}$$

Equations (A3), (A8), and (A9) provide the Mach number on the surface at the vertex. Knowing M_N, a simplified expression for the Mach number anywhere on the surface of the body may be obtained from equation (37). Since the local Mach number is assumed very large, this equation may be reduced to the form

$$\frac{2}{\gamma-1}\,d\beta = d\delta$$

which, in turn, can be written, upon integration, to yield the local Mach number

$$M = \frac{M_N}{1 - \frac{\gamma-1}{2} (M_N\delta_N) \left(1 - \frac{\delta}{\delta_N}\right)} \tag{A10}$$

Now the pressure coefficient is given by the expression

$$P = \frac{2}{\gamma M_o{}^2} \left(\frac{p_s}{p_o} \frac{p}{p_s} - 1 \right) \tag{A11}$$

The pressure rise across the shock may be obtained by combining equations (A5) and (A7) to yield

$$\frac{p_s}{p_o} = 1 + \gamma \, (M_o\delta_N)^2$$

Similarly, the ratio of the pressure at the surface to the pressure at the shock can be expressed (to the order of accuracy of this analysis) in the form

$$\frac{p}{p_s} = \left(\frac{M_s}{M}\right)^{\frac{2\gamma}{\gamma-1}}$$

With the aid of these expressions, equation (A11) can now be written

$$P = \frac{2}{\gamma M_o{}^2} \left\{ \left[1 + \gamma \, (M_o\delta_N)^2\right] \left(\frac{M_s}{M}\right)^{\frac{2\gamma}{\gamma-1}} - 1 \right\} \tag{A12}$$

yielding the pressure coefficient at any point on the surface of the body.

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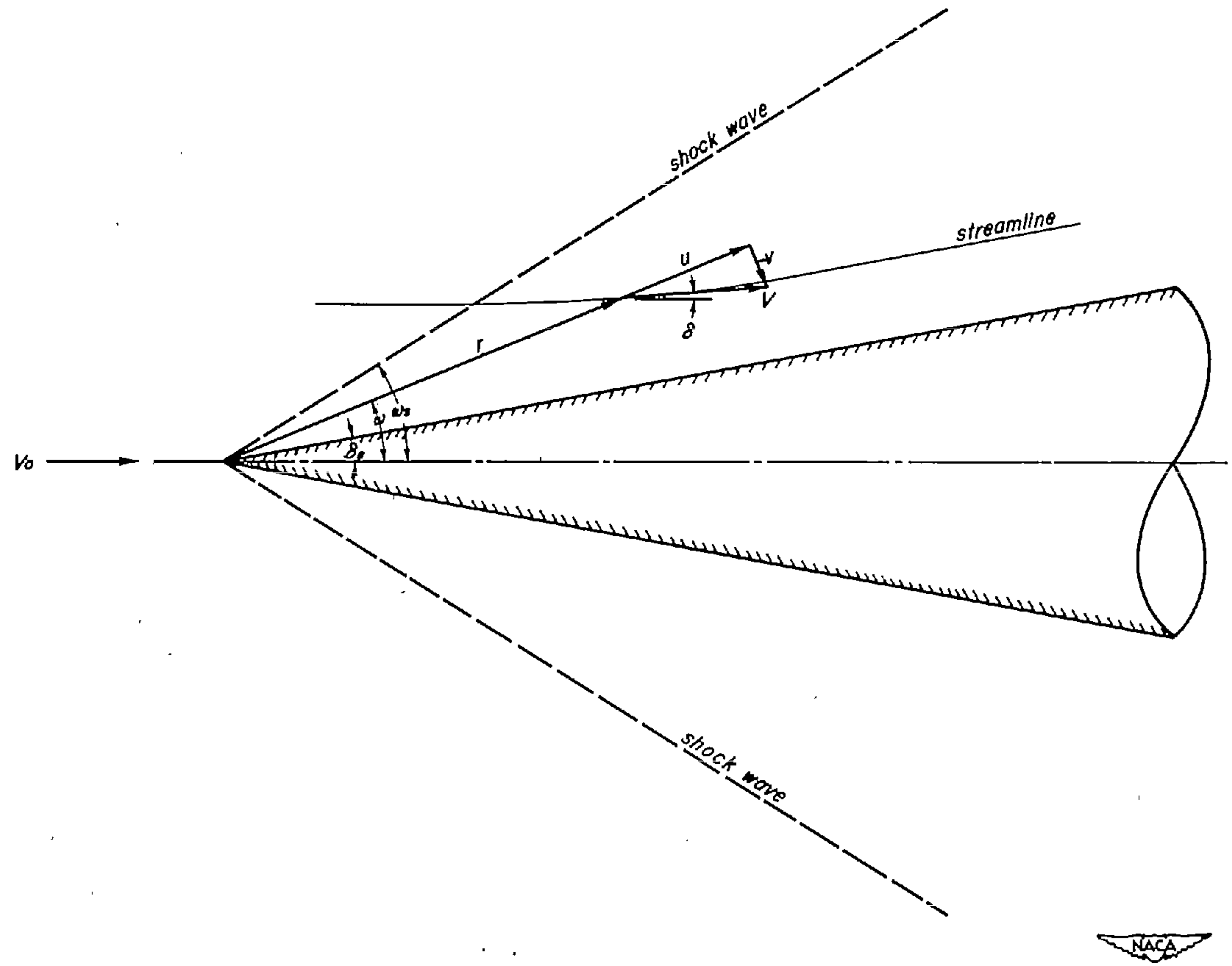

Figure 1.—Schematic diagram of supersonic flow about a nonlifting cone.

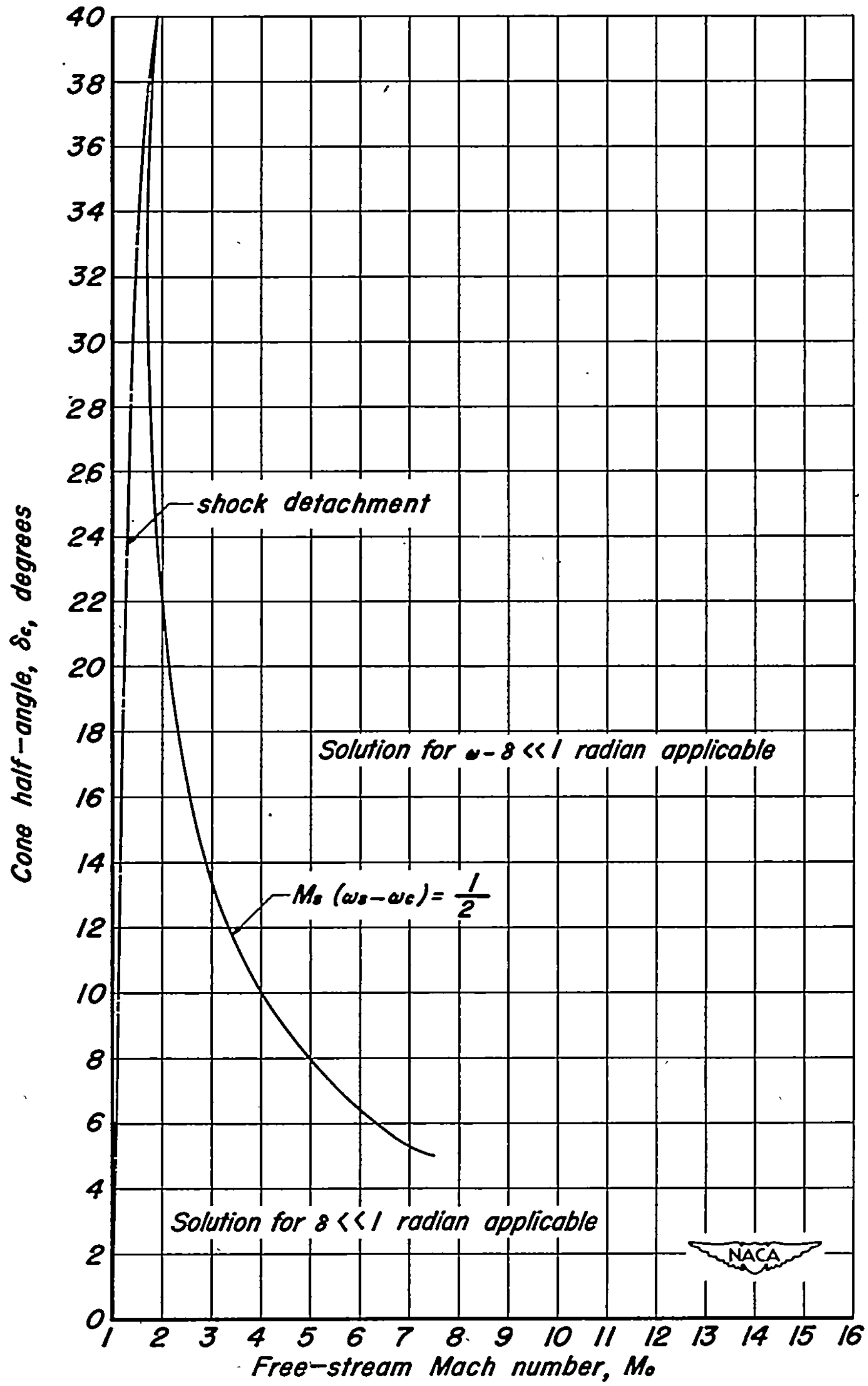

Figure 2.—Variation of cone half-angle with free-stream Mach number showing range of applicability of cone solution for $\omega - \delta \ll 1$.

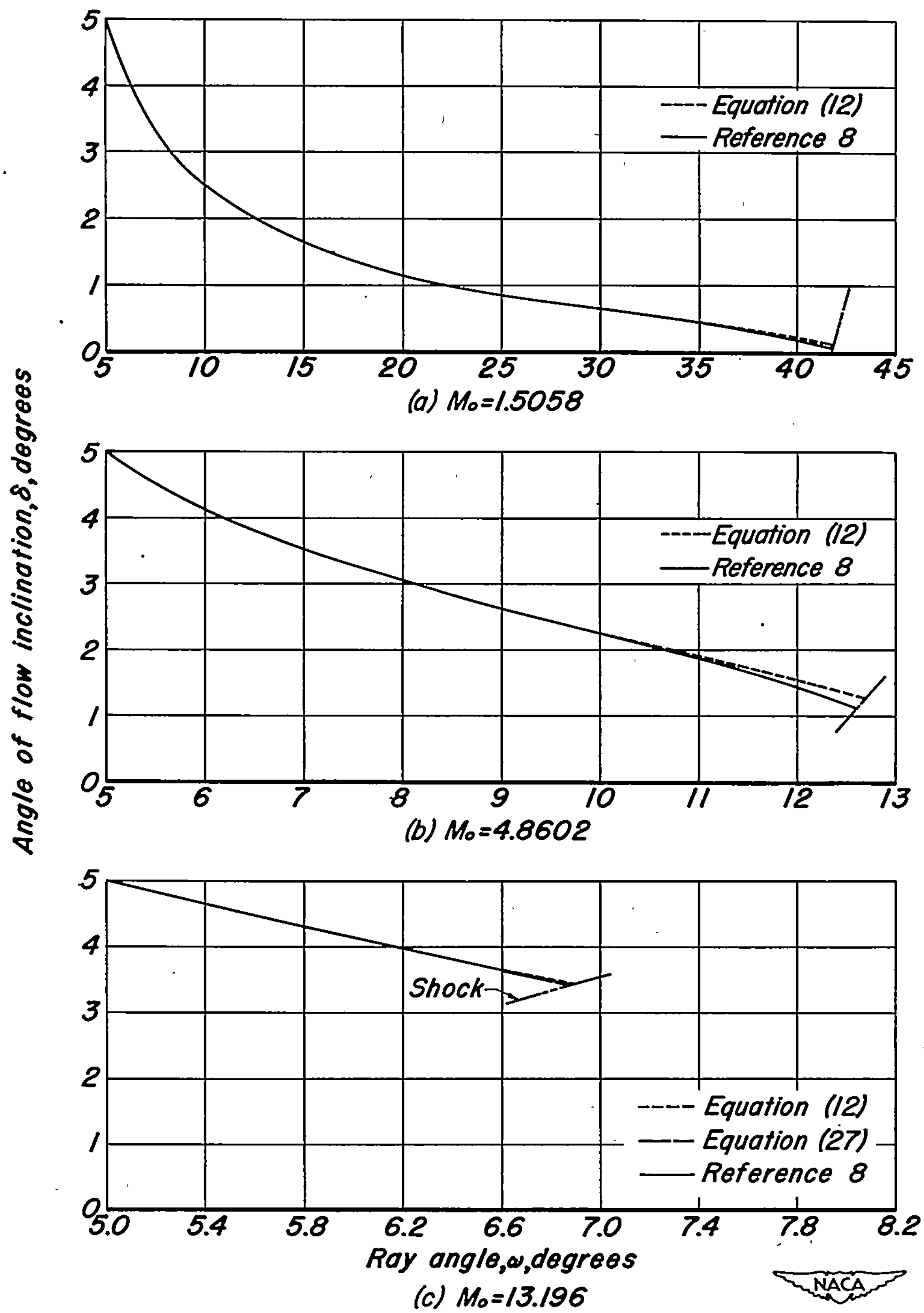

Figure 3.—Variation of flow inclination angle with ray angle for a 5° half-angle cone operating at Mach numbers of 1.5058, 4.8602, and 13.196.

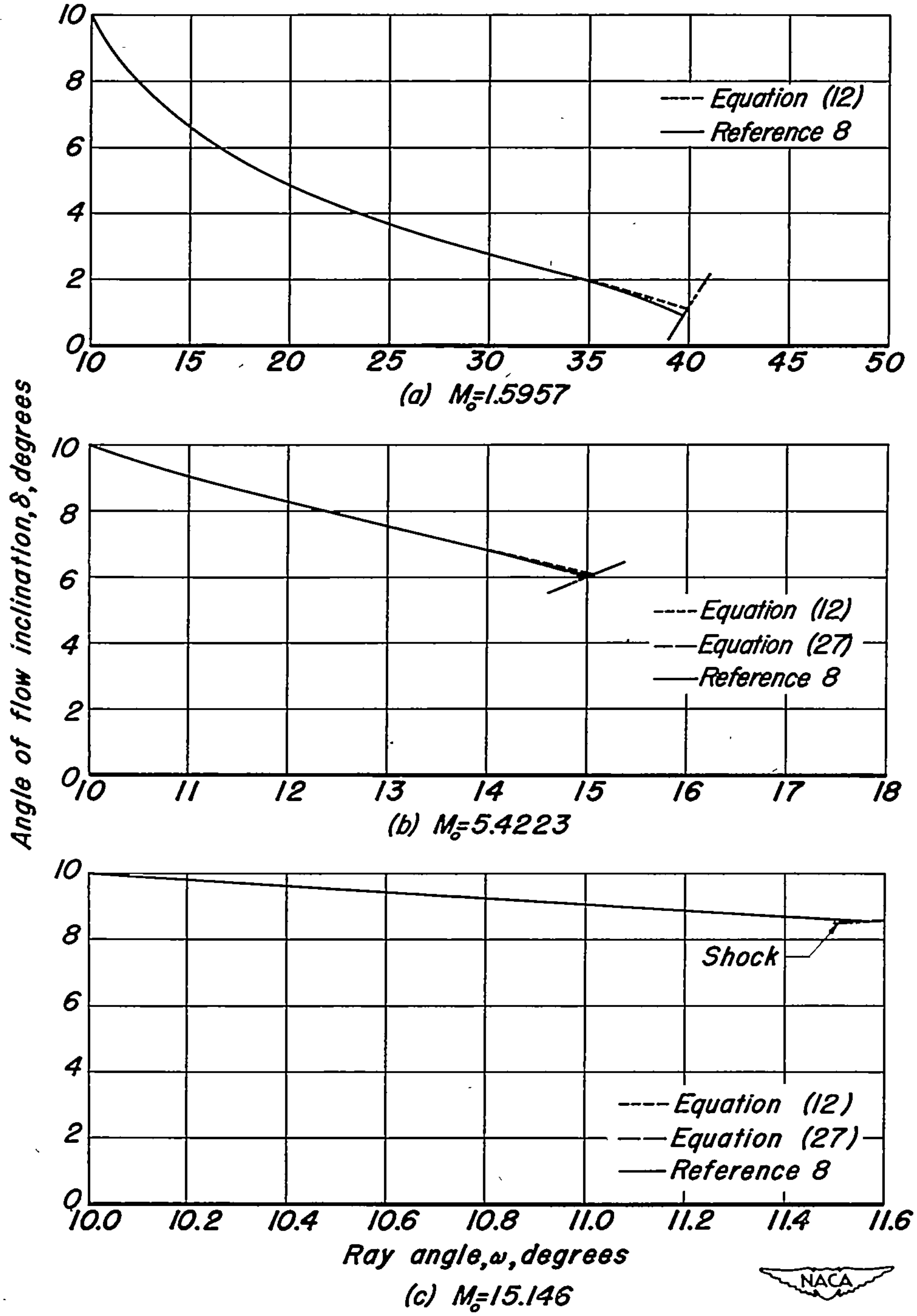

Figure 4.—Variation of flow inclination angle with ray angle for a 10° half-angle cone operating at Mach numbers of 1.5957, 5.4223, and 15.146.

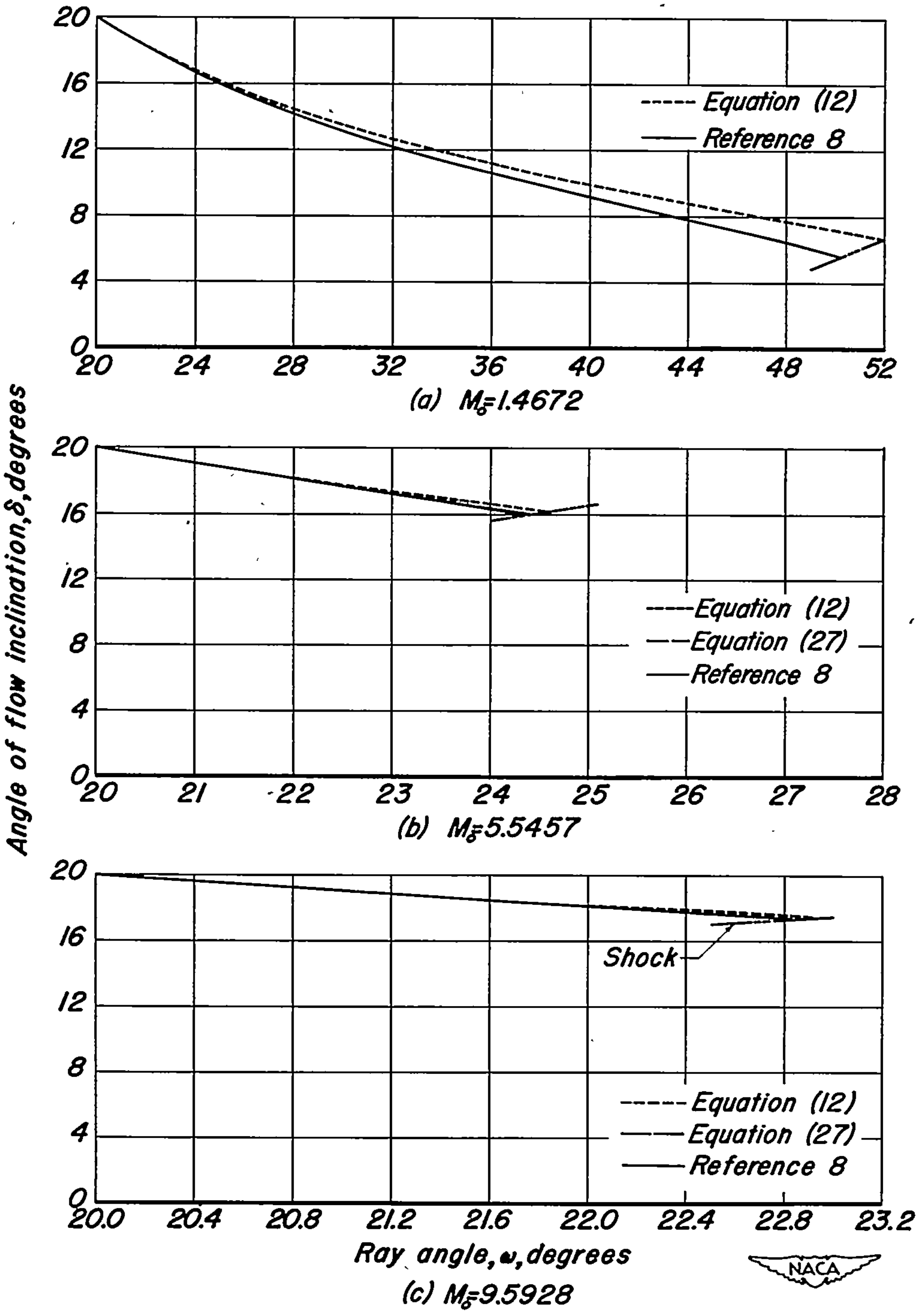

Figure 5.—Variation of flow inclination angle with ray angle for a 20° half-angle cone operating at Mach numbers of 1.4672, 5.5457, and 9.5928.

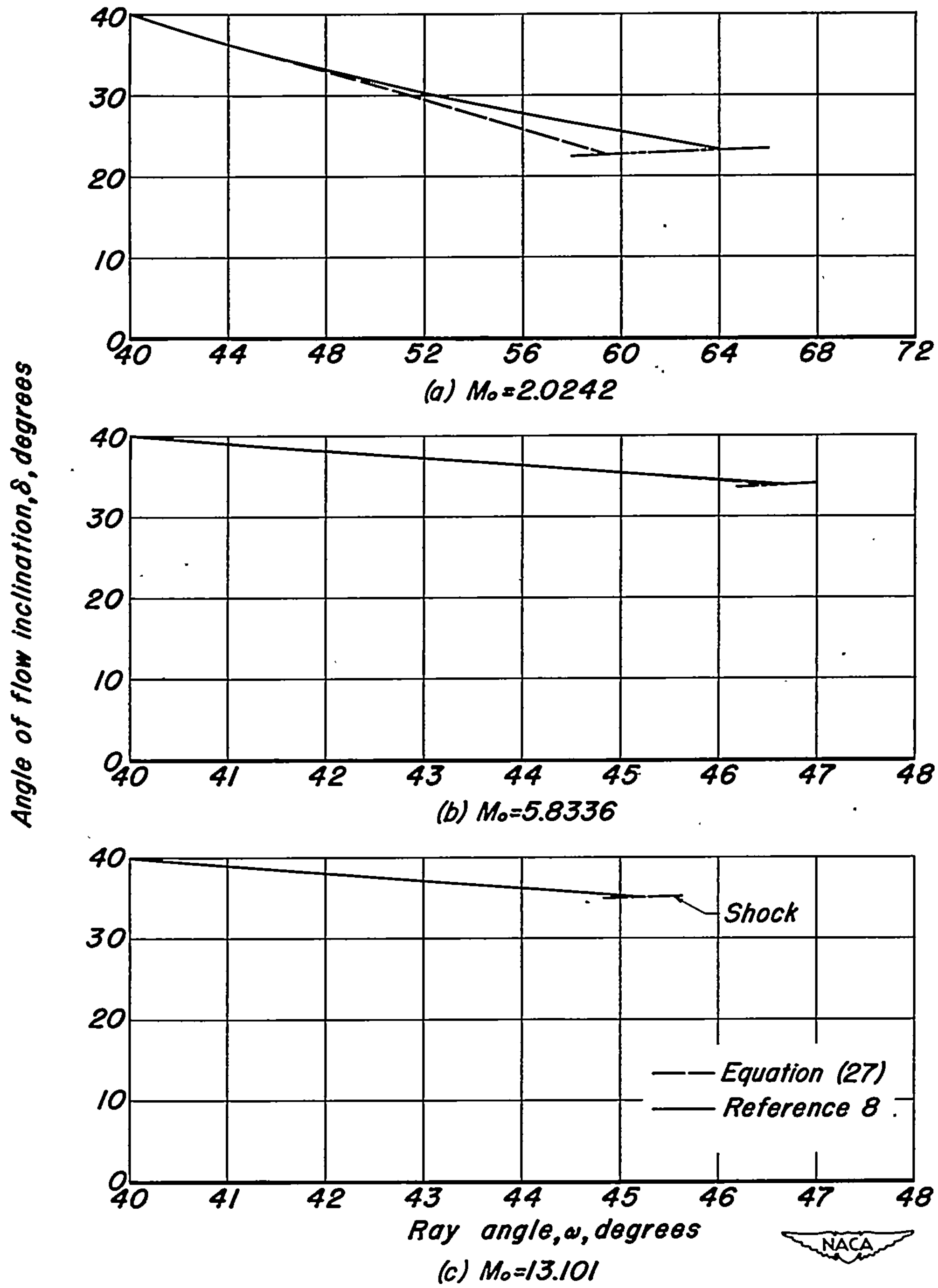

Figure 6.—Variation of flow inclination angle with ray angle for a 40° half-angle cone operating at Mach numbers of 2.0242, 5.8336, and 13.101.

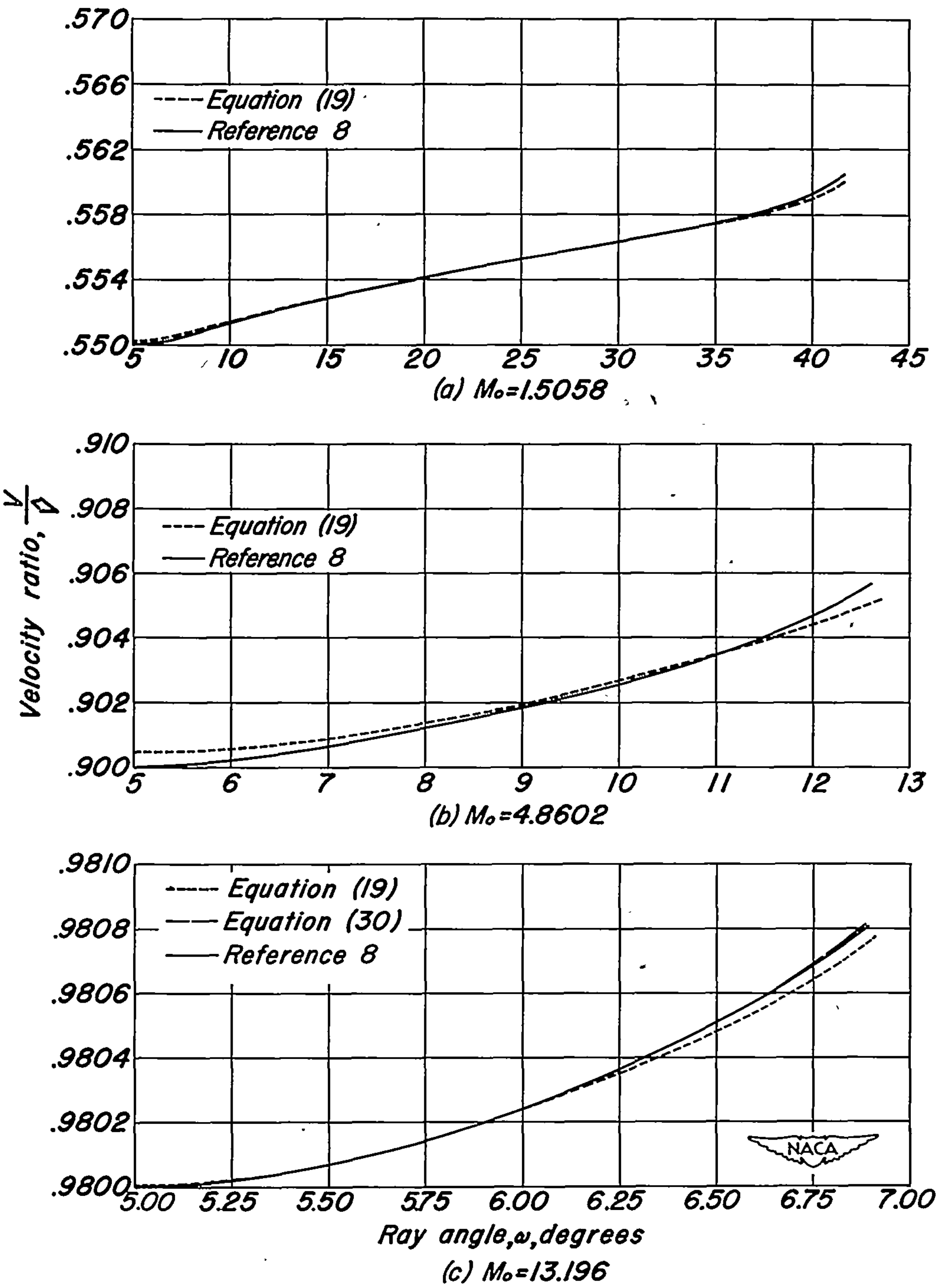

Figure 7 .—Variation of the ratio of local velocity to limiting velocity with ray angle for a 5° half-angle cone operating at Mach numbers of 1.5058, 4.8602, and 13.196.

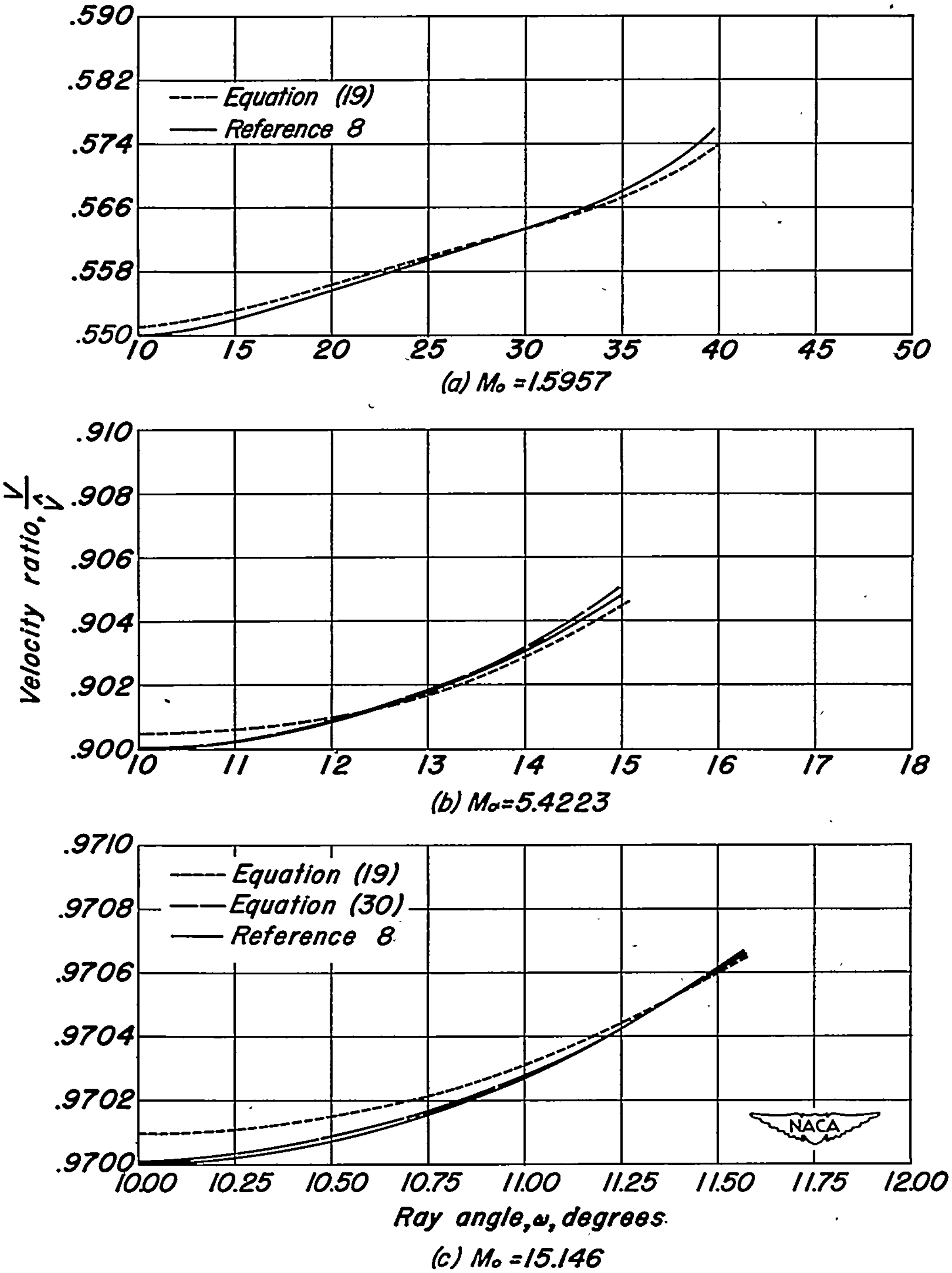

Figure 8.—Variation of the ratio of local velocity to limiting velocity with ray angle for a 10° half-angle cone operating at Mach numbers of 1.5957, 5.4223, and 15.146.

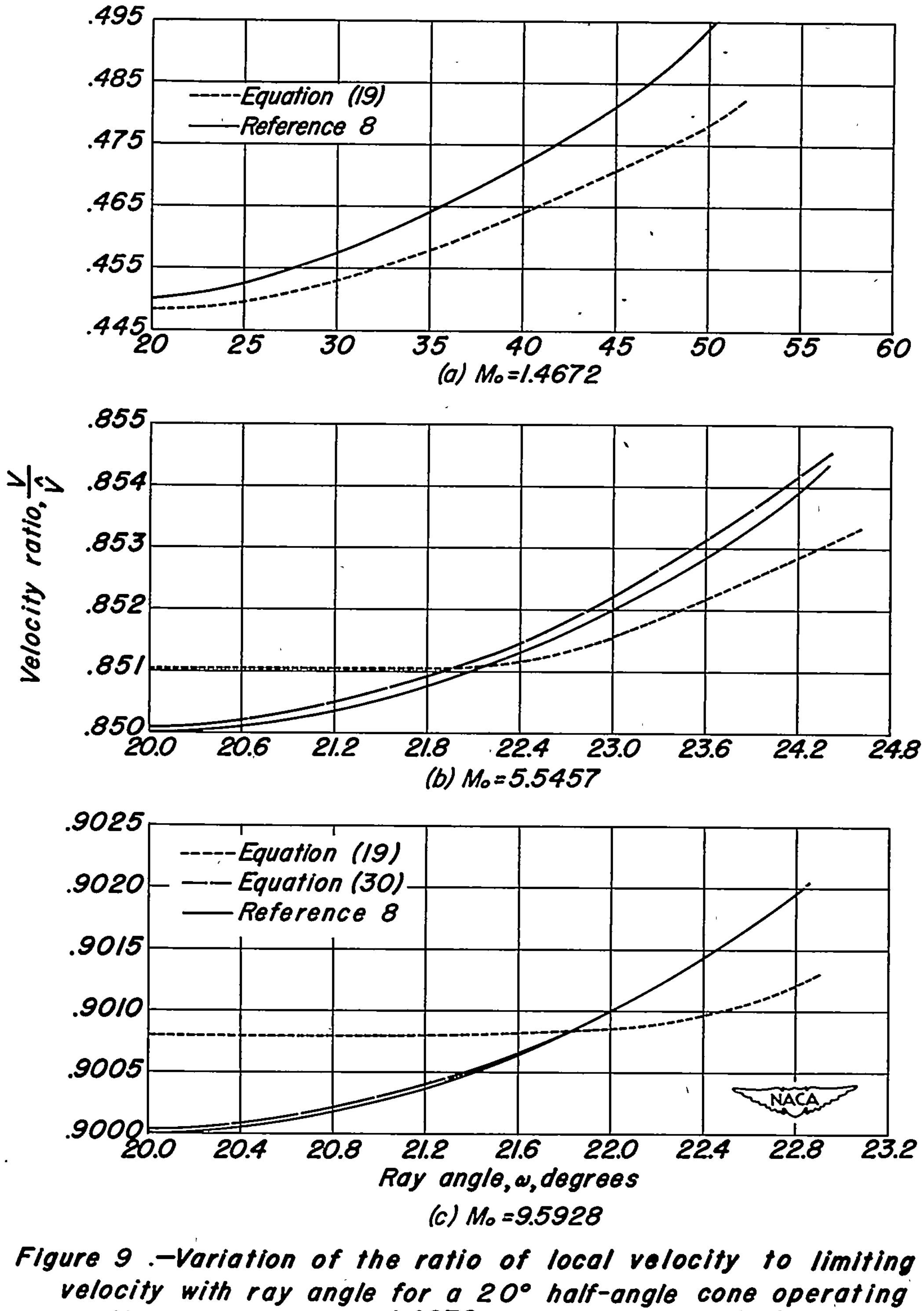

Figure 9 .—Variation of the ratio of local velocity to limiting velocity with ray angle for a 20° half-angle cone operating at Mach numbers of 1.4672, 5.5457, and 9.5928.

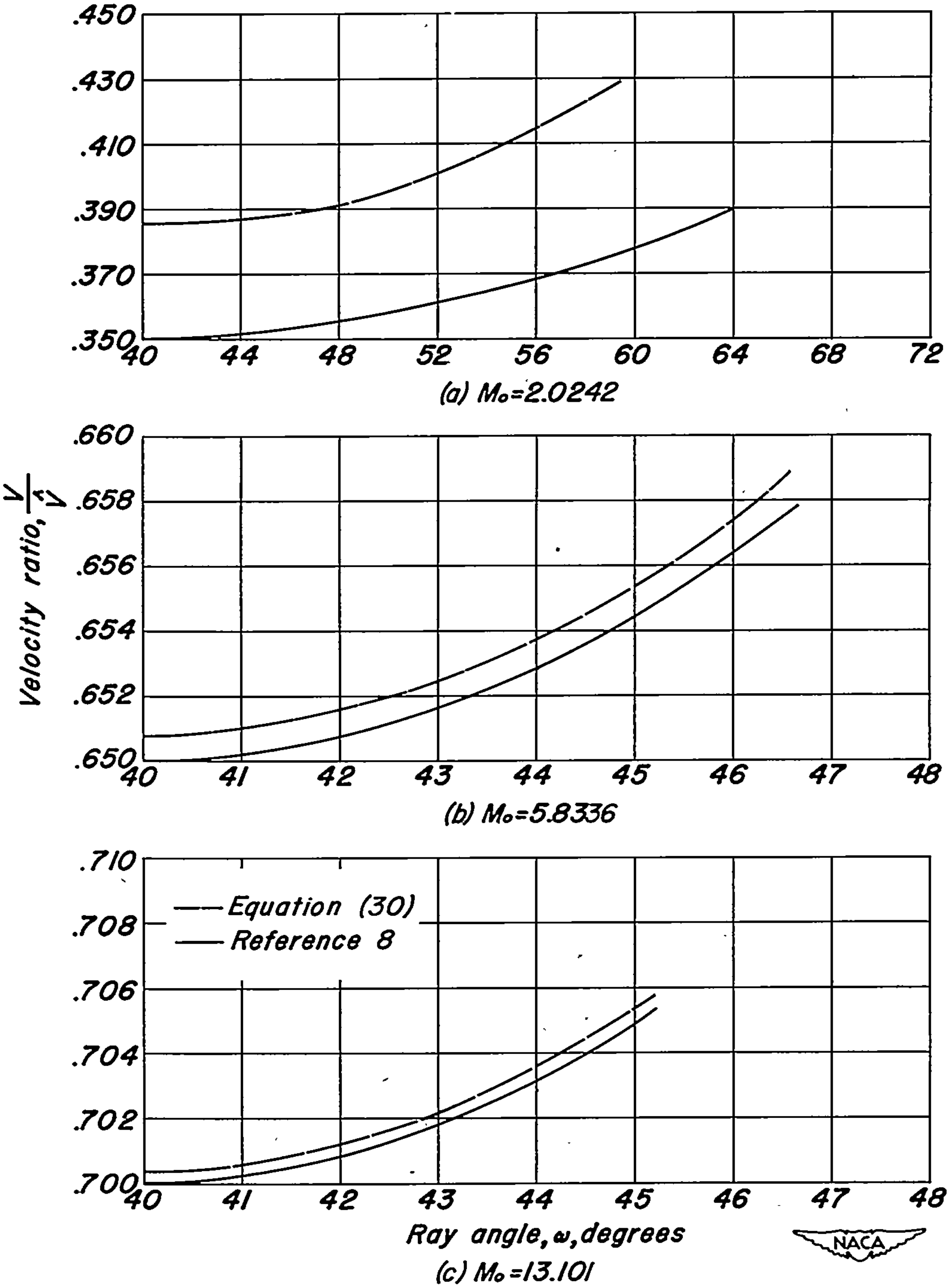

Figure 10.—Variation of the ratio of local velocity to limiting velocity with ray angle for a 40° half-angle cone operating at Mach numbers of 2.0242, 5.8336, and 13.101.

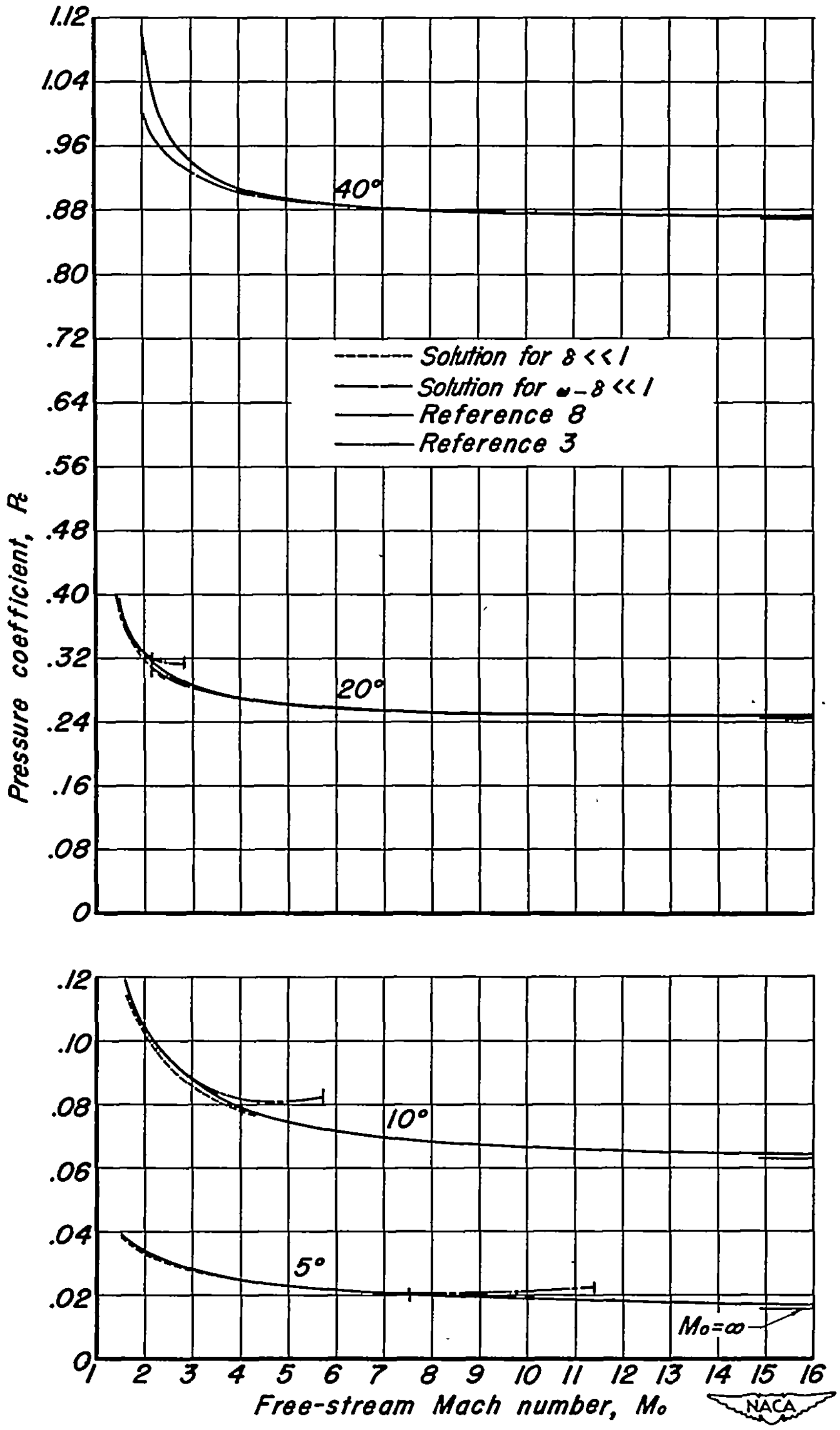

Figure 11.—Variation of pressure coefficient with free-stream Mach number for 5°, 10°, 20°, and 40° half-angle cones.

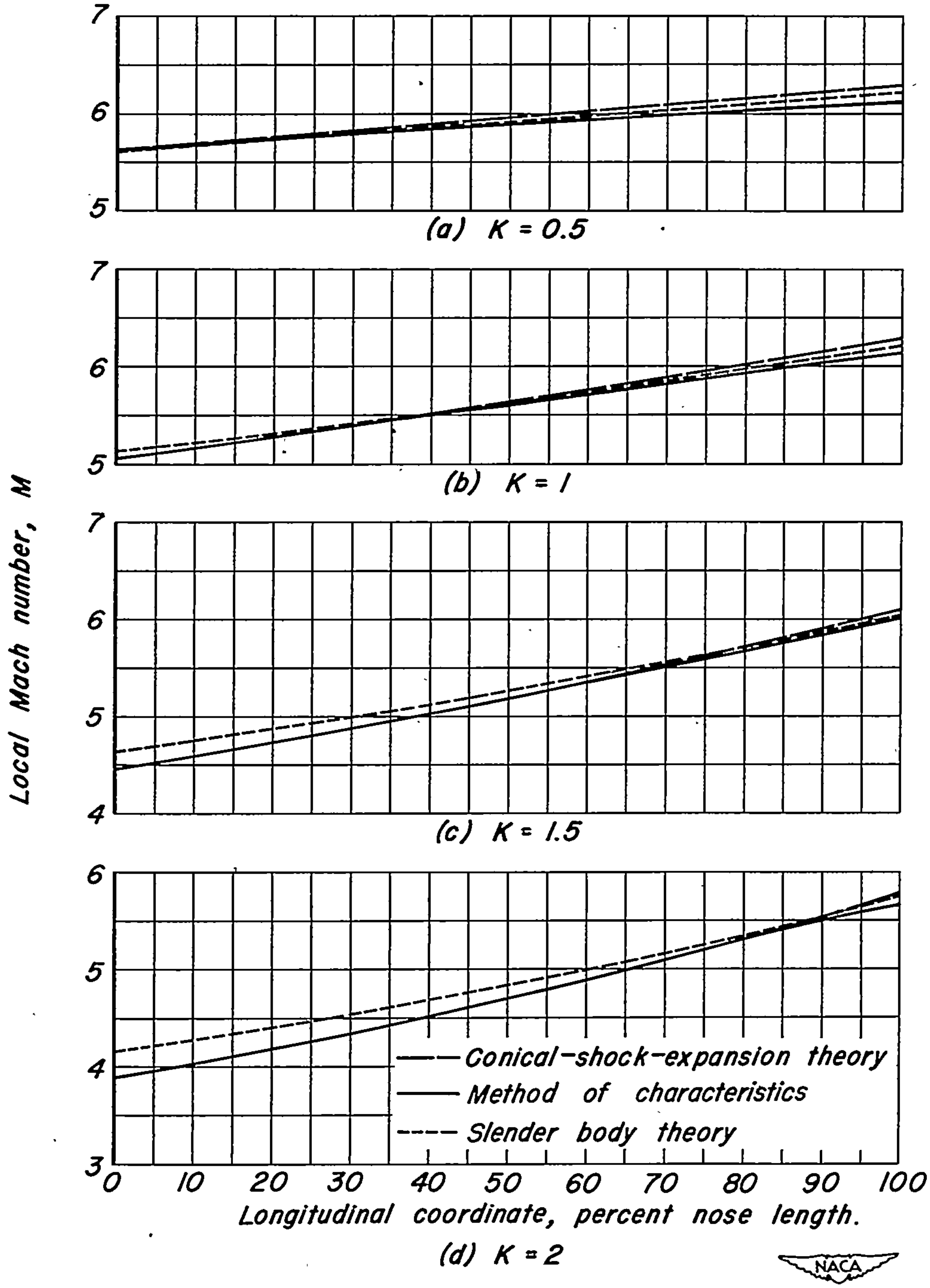

Figure 12.—Variation of local Mach number along ogives corresponding to four values of the similarity parameter, K, for a free-stream Mach number of 6.

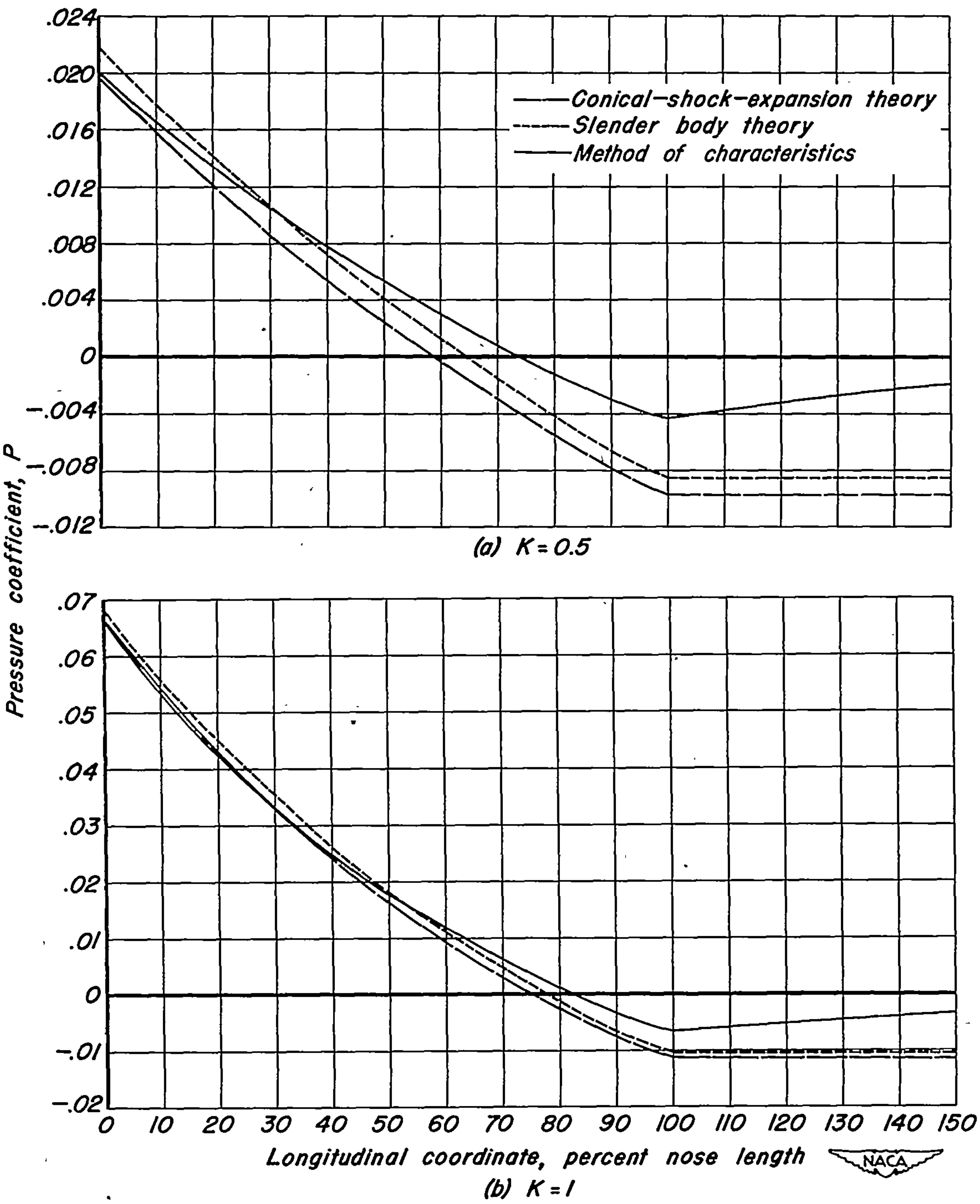

Figure 13.—Variation of pressure coefficient along ogives corresponding to four values of the similarity parameter, K, for a free-stream Mach number of 6.

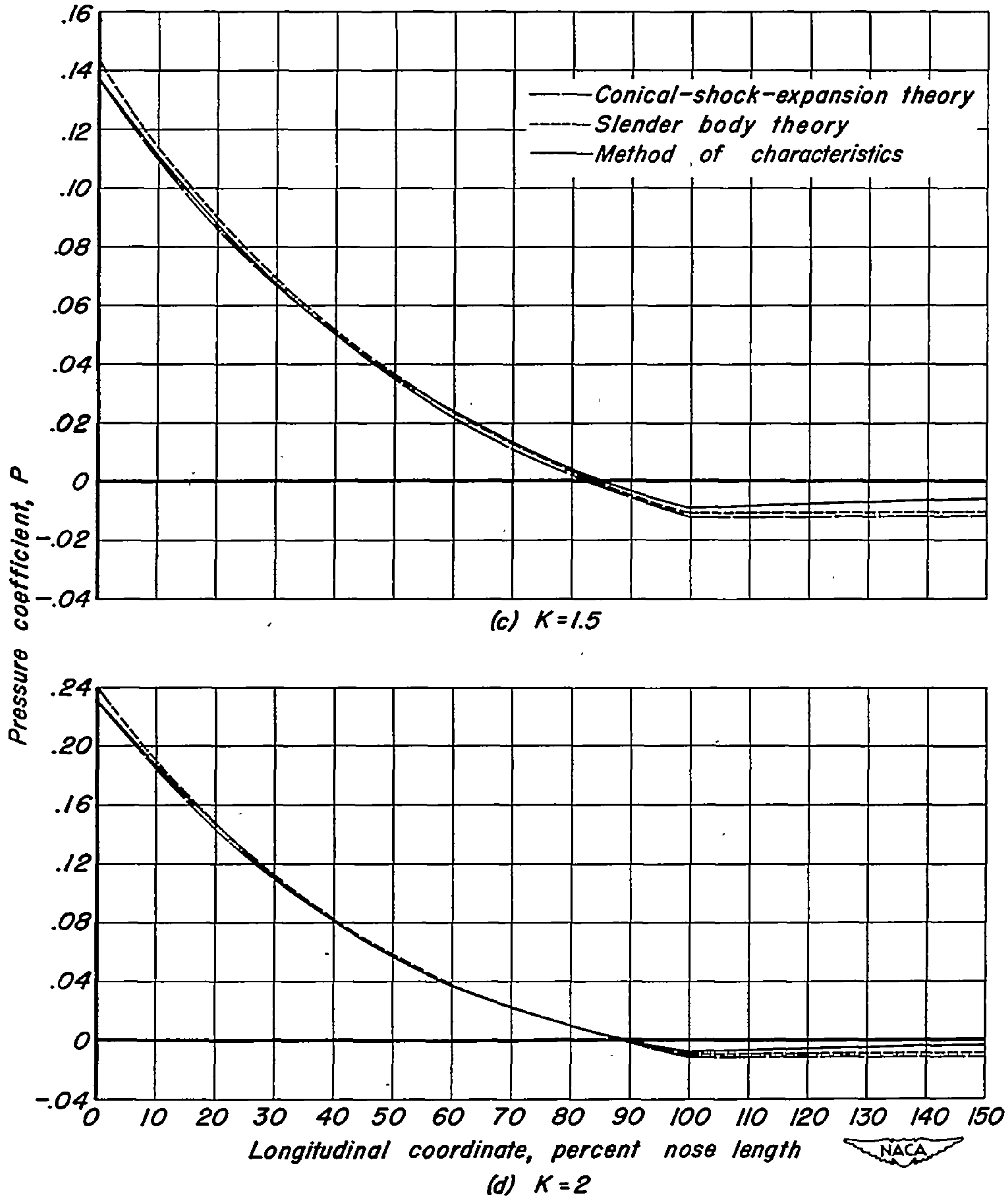

Figure 13.—Concluded.

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