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## EQUAL-STRENGTH DESIGN OF TENSION-FIELD WEBS AND UPRIGHTS

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## SUMMARY

A method is hereby presented for proportioning thin-web beams to attain equal strength of web and uprights which may in turn be employed toward optimum design of these components.

Improved empirical formulas for this purpose are developed and the results checked by experimental loading of six beams. The empirical formulas developed are subject to the limitations of the imposed conditions of this investigation and proportions of uprights as brought out in the experimental results and conclusions.

## INTRODUCTION

The strength analysis of incomplete diagonal tension-field beams has been greatly aided by the development of a modified engineering theory summarized in reference 1. With the simplified procedure supplied by such an analysis, the problem at once is presented of how such beams may be proportioned for best design.

In aircraft structures especially, the problem of "optimum" design of any given part is of major importance, that is, the problem of how the lightest possible structure consistent with safety may be designed and built for a given combination of loads. It is with this idea in mind that the following method of determining the proportions of an "equal-strength" beam is advanced which is the first step toward the attainment of an optimum design. An equal-strength design is defined as being one in which the uprights and web of a beam approach their individual maximum allowable stresses at the same value of beam load, thus resulting in maximum utilization of the strength of each part.

In order that various designs may be compared as to their "efficiency," an index of comparison has been developed which has as its basis the load carried in shear per square inch of effective web section. On the basis of this index a comparison can be made between various beams to ascertain which of several designs is the best for given conditions of loading.

Several beams were designed by the methods of this report and tested to determine the reliability of the basic theory in the analysis of equal-strength beams.

This investigation was carried out at the University of Minnesota under the sponsorship and with the financial assistance of the National Advisory Committee for Aeronautics.

## SYMBOLS

| | |
|---|---|
| A | cross-sectional area, square inches |
| E | Young's modulus, ksi |
| I | moment of inertia, inches$^4$ |
| P | web shear force, kips |
| R | coefficient of edge restraint (see formula (7)) |
| d | spacing of uprights, inches |
| e | distance from median plane of web to centroid of (single) upright, inches |
| h | depth of beam, inches (see "Special combinations") |
| k | diagonal-tension factor (see formula (8)) |
| t | thickness, inches (use without subscript signifies thickness of web) |
| $\alpha$ | angle between neutral axis of beam and direction of diagonal tension, degrees |
| $\rho$ | centroidal radius of gyration of cross section of upright about axis parallel to web, inches (no sheet should be included) |
| $\sigma$ | normal stress, ksi |
| $\tau$ | shear stress, ksi |

Subscripts

| | |
|---|---|
| u | upright |
| cr | critical |
| ult | at failure |

all allowable

max maximum

ss simple support

e effective

Special combinations

$d_c$ clear width between uprights (measured between rivet lines on single uprights), inches

$h_c$ clear depth between flanges, inches

$h_e$ depth of beam measured between centroids of flanges, inches

$h_u$ length of upright measured between centroids of upright-to-flange rivet patterns (see condition (6) under "Limits of Investigation"), inches

$K_{ss}$ theoretical buckling coefficient for plates with simply supported edges (fig. 5(a), reference 1)

$\sigma_o$ "basic" allowable stress for forced crippling of uprights (valid for stresses below proportional limit in compression of upright material), ksi

$\omega d$ flange flexibility factor $\left(0.7d\left[\frac{1}{\left(I_c + I_T\right)\left(h_e/t\right)}\right]^{1/4}\right)$

$I_c$ moment of inertia of compression flange about its own axis perpendicular to web

$I_T$ moment of inertia of tension flange about its own axis perpendicular to web

$R_d$ restraint coefficient for edges of sheet along flanges, equal to 1.62 for the conditions of this investigation

$R_h$ restraint coefficient for edges of sheet along upright (fig. 5(b), reference 1)

$C_2$, $C_1$ see equation (1)

## LIMITS OF INVESTIGATION

As shown in reference 1, a beam design may be expressed largely as a function of the dimensionless ratios $A_{ue}/td$, $t_u/t$, $\tau/\tau_{cr}$, $(d/h)_c$, and $h_c/t$. Because of the limited applicability of the data in reference 1 and the need to reduce the number of variables as much as possible, a number of limitations and assumptions were made as to the extent and conditions of the investigation of equal-strength designs as follows:

(1) $0.3 < (d/h)_c < 0.8$.

(2) $250 < h_c/t < 800$. (The lower limit is later raised to 400.)

(3) $1.0 < t_u/t$.

(4) $0.1 < A_{ue}/td < 0.8$.

(5) $3.0 < t_f/t$.

(6) $h_u = h_c = h_e/1.05$ (preliminary assumption).

(7) Flange-to-web riveting:

(a) Web rivet between cap angles

(b) Web rivet to outside of leg of cap angles with at least a double row of rivets, with heavy washers between the rivet head and the web

(8) Single uprights, normal to beam axis, riveted directly to web.

(9) Web material of alclad 75S-T6 aluminum alloy with $E = 10.6 \times 10^6$ psi.

(10) Upright material of aluminum alloy with $E = 10.6 \times 10^6$ psi; in beam tests, 24S-T4 aluminum alloy was used.

(11) Flanges were stiff enough to avoid appreciable concentration of web stress; that is, $C_2 = 0$. (In the beams tested, 61S-T6 aluminum-alloy angles were used but no special material is implicit in the formulas.)

Limitations (1), (2), (4), (6), and (8) were necessary to reduce the number of variables and to restrict investigations to dimensions consistent with the limitations imposed on the data in reference 1. Limitation (3) was necessitated by the fact that the type of support given the web by single uprights with ratios of $t_u/t$ less than 1.2, as determined empirically and presented in reference 1, page 18, is extremely doubtful.

Limitations (5), (7), and (8) were made primarily to reduce the number of variables, and in so doing gain a fixed known type of support at the edges of the web in a panel.

Limitation (9) determines the shear strength properties of the web which must be known in order that the proper relationships between the design variables may be determined.

The two limitations (10) and (11), in common with most of the conditions, are in keeping with general aircraft practices and known good results. The latter limitation was adopted after an investigation of several practical beam designs in all of which the value of $\omega d$ was such that the factor $C_2$ was negligible. It was decided that this limitation would not impose a restriction of any consequence on the proposed designs.

## THEORY

### Summary of Analytical Equations (Reference 1)

The basic requirement of the equal-strength design is that the upright and web of the beam approach their individual values of allowable stress at the same value of beam shear load. The size of the beam caps is primarily determined by the bending moment which must be resisted by the beam and is related to the web design only by the assumption here made that the bending of the caps will be negligible in its effect on the web strength (see reference 1, fig. 13). Hence the failure of the caps will not be considered in the investigation. From the equations and data in reference 1, the basic relationships between equal-strength design parameters will now be determined.

The allowable values of shear stress for beam webs made of alclad 75S-T6 and 24S-T4 aluminum alloy are given in reference 1, figure 14, (modified by reference 2) as a function of the diagonal-tension factor k and of the edge support provided by the beam caps.

The maximum shear stress in the beam web is given by the formula (reference 1, equation (14))

$$\tau_{max} = \tau\left(1 + kC_1\right)\left(1 + kC_2\right) \tag{1}$$

where $C_1$ is an internal stress factor which allows for the angle of diagonal tension in the web being different from 45°, and $C_2$ is a stress factor which allows for increased stress in the web because of bending of the flanges between uprights. For the type and size of beam flanges here considered, the maximum effect of $C_2$ is about 1.3 percent and is neglected as already noted.

By setting the maximum allowable stress of the web equal to $\tau_{all}$, and putting $C_2$ equal to zero, equation (1) is modified to

$$\tau_{all} = \tau\left(1 + kC_1\right) \tag{2}$$

At a maximum value of beam load it is further assumed that the maximum stress in the uprights is approaching its maximum allowable value at which failure occurs. In the case of beams with single uprights, the "basic" allowable stress for forced crippling of the stiffener, assuming perfectly elastic upright material, is given by the empirical equation (reference 1, formula (13 a))

$$\sigma_o = 28k\sqrt{t_u/t} \tag{3}$$

for values of $k$ less than 0.5, an effective value of $k$ must be used in formula (3) as determined by the expression (reference 1, formula (13 c))

$$k_e = 0.15 + 0.7k \tag{4}$$

The maximum value of stress which occurs in the upright at (or near) the neutral axis of the beam is given by the formula (reference 1, formula (11))

$$\frac{\sigma_{u_{max}}}{\sigma_u} = 1 + \left[\left(\frac{\sigma_{u_{max}}}{\sigma_u}\right)_o - 1\right](1 - k) \tag{5}$$

An empirical formula giving the same ratio in terms of beam design parameters is

$$\frac{\sigma_{u_{max}}}{\sigma_u} = 1.775 - 0.646(1 - k)(d/h)_u - 0.775k \qquad (6)$$

where $\sigma_u$ is the average upright stress. For equal-strength design the value of $\sigma_o$ is set equal to $\sigma_{u_{max}}$, thus relating the ratio $t_u/t$ with the other design variables.

The variation of $\sigma_u/\tau$, k, $A_{ue}/td$, and $\tau/\tau_{cr}$, which determines the value of k, the diagonal tension factor, is given in figure 8, reference 1.

The critical buckling stress of the web in shear may be determined by the following formula found in reference 1 (formula (7)).

$$\tau_{cr} = K_{ss}E\left(t/d_c\right)^2\left[R_h + \frac{1}{2}\left(R_d - R_h\right)\left(d_c/h_c\right)^3\right] \qquad (7)$$

The empirical expression for k given as formula (5) in reference 1 can be reduced to the following form, readily calculated by use of a log-log slide rule, and is identical to the original expression for k,

$$k = \frac{\left(\tau/\tau_{cr}\right)^{0.434} - 1.0}{\left(\tau/\tau_{cr}\right)^{0.434} + 1.0} \qquad (8)$$

It is now possible to determine the relationships between the various thin-web-beam design parameters by utilizing the formulas just determined and the empirical data presented in the graphs of reference 1.

## Criterion of Beam Efficiency

The index of comparison used herein is based on the load carried in shear per square inch of effective web section. The effective web section in shear is defined as

$$A_e = h_e t + \frac{h_u A_u}{d_c} \qquad (9)$$

The load at failure of the beam is

$$P_{ult} = \tau_{ult} h_e t$$

The shear stress at failure based on the "effective area" of the web in shear is now

$$\tau_e = \frac{P_{ult}}{A_e} = \tau_{ult}\left[\frac{1}{1 + \left(\frac{h_u}{h_e}\right)\left(\frac{A_u}{td}\right)}\right] \tag{10}$$

A brief study of formula (10) indicates that a high value of $\tau_e$ is indicative of a high value of beam efficiency in the sense of this report, on the assumption that the material throughout is aluminum alloy of standard density. Should the uprights of a beam be made of different density material, the term $\left(\frac{h_u}{h_e}\right)\left(\frac{A_u}{td}\right)$ in the denominator must be multiplied by the ratio $w_u/w_o$, where $w_u$ is the density of the upright material and $w = w_o$ is the density of the web material (standard). If the web material is also nonstandard, the 1 in the denominator must be replaced by $w/w_o$ for any comparisons with beams of other material.

## RESULTS AND DISCUSSION OF CALCULATIONS FROM THEORY

A set of equal-strength proportions for thin-web beams was determined by following the procedure given in the appendix while at the same time keeping within the limitations previously set.

By plotting $A_{ue}/td$ against $t_u/t$ it was found that one simple empirical equation could represent the average curve of these variables between limits of $400 \leqq h_c/t \leqq 800$ and $0.4 \leqq (d/h)_c \leqq 0.8$. Equation (11) given below is the final form of the average curve referred to above.

$$\frac{A_{ue}}{td} = \frac{1.16}{\sqrt{t_u/t}} - 0.28 \tag{11}$$

Figure 1 presents a plot of equation (11) together with two curves representing the maximum and minimum calculated values of the design variables as given in table I. Values of $A_{ue}/td$ and $t_u/t$ for $h_c t < 400$ and $(d/h)_c < 0.4$ were omitted from consideration in the determination of the curves in figure 1 because of their scatter and also because they were beyond the range of probable best design.

A graphic summary of the computations in table I is given in figure 2 for the dimensionless design parameters $A_{ue}/td$ and $t_u/t$ which clearly shows the scatter for values of $h_c/t < 400$ and $(d/h)_c < 0.4$.

The uniform variation of these curves is interesting to note but the data in them were excluded from further consideration for the reason previously stated and also because of the doubt concerning the application of data contained in reference 1 to these lower limits of design parameters.

It is believed that the application of the more-complete data contained in reference 2 to the design of equal-strength-web beams will result in the establishment of improved curves for values of $h_c/t < 400$ and $(d/h)_c < 0.4$.

It was also found that the nominal value of web shear stress at failure was relatively constant with varying $A_{ue}/td$ for a given value of $(d/h)_c$ and $h_c/t$ in the range of variables investigated. The average values of web shear stress at failure are plotted in figures 3(a) and 3(b) and may be represented by the following empirical equation

$$\tau_{ult} = 2.15 + 6.6\left(\frac{t}{h_c}\right)^{1/3}\left[6 + \left(\frac{h}{d}\right)_c\right] \qquad (12)$$

The effect of an arbitrary value of $C_2 = 0.04$ upon the equal-strength proportions previously given was investigated and found to be negligible. The only appreciable effect was to reduce the computed upright stress $\sigma_u$ approximately 2 to 4 percent (see table I and fig. 4).

As noted in reference 1, the problem of "column" failures in single uprights has not been investigated to any extent, and test results are greatly at variance with theoretical results. The following two criterions are suggested for strength design in reference 1:

(a) The stress $\sigma_u$ should be no greater than the column yield stress for the upright material

(b) The stress at the centroid of the upright (which is the average stress over the cross section) should be no greater than the allowable column stress for the slenderness ratio $h_u/2\rho$

In compliance with the first criterion which accounts for the upright acting as an eccentrically loaded compression member, a proportional limit stress of 43 ksi was chosen as the limiting value of $\sigma_o$ in the uprights. This corresponds closely to the proportional limit stress of 24S-T4 aluminum alloy. The lower limits of application of formula (11) determined by the above limitations are given below in the table. If the upright stress is greater than the proportional limit, the procedure given in reference 1, page 13, must be utilized.

| $(d/h)_c$ | $A_{ue}/td$ | | |
|---|---|---|---|
| | $h_c/t = 400$ | $h_c/t = 600$ | $h_c/t = 800$ |
| 0.4 | 0.10 | 0.16 | 0.22 |
| .5 | .10 | .19 | .24 |
| .6 | .12 | .21 | .26 |
| .7 | .14 | .22 | .27 |
| .8 | .17 | .24 | .29 |

The second criterion given above is an attempt to take into account a full-wave type of buckling failure that has been observed in very slender uprights by NACA.

## PROCEDURE FOR EQUAL-STRENGTH DESIGN OF A THIN-WEB BEAM

The design of a typical stiffened thin-web beam may be divided into four parts:

(1) Web design

(2) Upright design

(3) Cap design

(4) Rivet attachment design

Each part will be briefly discussed as it relates to equal-strength design. See the appendix for more detailed steps.

## Web Design

A method is presented which will give an approximation to the web thickness very quickly. First the data contained in figure 3(a) are approximated by a simple straight-line equation as

$$\tau_{ult} = 32.00 - 0.0045\left(h_c/t\right) \qquad (13)$$

A random choice of reasonable values of $\tau_{ult}$ and $h_c/t$ from figure 3(a) was made to obtain the constants in formula (13). By utilizing condition (6) under "Limits of Investigation" and also the definition of $\tau_{ult}$ as

$$\tau_{ult} = \frac{P_{ult}}{h_e t}$$

formula (13) may be manipulated into the following form

$$t = 0.031\left(\frac{P_{ult}}{h_e} + 0.0043\ h_e\right) \qquad (14)$$

After a preliminary estimate of $t$ has been made by utilizing formula (14), the curves of figure 3 or formula (12) should be used to obtain final estimates of the web thickness.

It should be noted that $P_{ult}$ in a tapered beam represents not the total shear but the net shear carried by the web. When a value of $(d/h)_c$ is finally chosen, the strength of the web should be checked on figure 3 to be sure that it is sufficiently high. As stated previously, the ultimate shear strength of the web was found to be relatively independent of a variation in $A_{ue}/td$ at a given value of $(d/h)_c$ and $h_c/t$.

## Upright Design

The problem of upright design lies in the selection of an upright form, size, and spacing which will fulfill the requirements as set forth by formula (11). Since several types of uprights are available to a designer, it is simple to design several beams and compare them on the basis of the criterion set up by formula (10).

When a particular upright has been tentatively chosen, say an extruded angle, and the web thickness determined, the only variable remaining unknown in formula (11) is the upright spacing which may now be determined. After choosing several uprights and determining their spacing (and also checking the strength of the web for each panel aspect ratio), formula (10) may be utilized to determine the best combination of web and upright.

In beams in which the web is overstrength the designer will want to reduce the size of the upright in order that weight may be saved. An approach to this type of design is suggested below.

It will be initially assumed that a web has been selected which is overstrength; that is, the maximum shear which the web may resist in an equal-strength design is greater than the maximum load to which the web will be subjected. The problem is to design the upright so that it will fail as the maximum design load of the beam is reached. The method given below first determines the upright which is necessary for the given web as an equal-strength design, and then reduces the upright area while maintaining the same upright thickness and spacing.

(1) Determine an upright spacing and size which will be of sufficient size to form an equal-strength design in conjunction with the web previously assumed to have been designed.

(2) From the calculations given in table I it is possible to determine the critical buckling stress $\tau_{cr}$ for the web. Since only the area of the upright is going to be changed, the value of $\tau_{cr}$ will not be affected, provided the ratios $t_u/t$ and $(d/h)_c$ from step (1) are maintained.

(3) Determine the ratio $\tau/\tau_{cr}$, where $\tau$ will now be calculated on the basis of the design web shear and will be less than $\tau_{ult}$ for the web as given in figure 3(a). The quantity $\tau_{cr}$ is obtained from step (2).

(4) Calculate the diagonal tension factor k using the value of of $\tau/\tau_{cr}$ from step (3).

(5) Calculate the stress $\sigma_o$ from formulas (3) and (4). Use the values of $t_u/t$ and k from steps (1) and (4), respectively

(6) Calculate the ratio $\sigma_{u_{max}}/\sigma_u$ from formula (6). Use $(d/h)_c$ and k from steps (1) and (4), respectively.

(7) The values $\sigma_o$ and $\sigma_{u_{max}}$ are assumed to be equal at failure of the upright, hence the upright stress $\sigma_u$ may be calculated as

$$\sigma_u = \frac{\sigma_o}{\sigma_{u_{max}}/\sigma_u}$$

(8) The value of $\sigma_u/\tau$ is calculated from the values obtained in steps (3) and (7).

(9) The values of $\tau/\tau_{cr}$ and $\sigma_u/\tau$ are now known for the final design of the beam from steps (3) and (8). The value of $A_{ue}/td$ may be determined from the diagonal-tension analysis charts, figures 8 and 9 of reference 1, or from formula (11).

(10) The upright must now be designed so that the above value of $A_{ue}/td$ is satisfied while retaining the values of $(d/h)_c$ and $t_u/t$ from step (1). See also the limitations in the section entitled "Results and Discussion of Calculations from Theory."

This procedure may be repeated several times and the various designs so obtained may then be compared through the use of formula (10) to determine the best design.

A brief investigation of beams 4 and 5 indicates that savings in total beam weight (exclusive of flanges or caps) of approximately the same order of magnitude as the decrease in beam strength may be expected from the use of the method given above; that is, for a decrease in ultimate load of (say) 20 percent from the maximum value for a given web, an upright may be designed which will result in a reduction of beam weight of approximately 20 percent also. The results of the investigation of beams 4 and 5 are summarized in table II.

## Cap Design

As previously noted it is believed that the size of the beam caps will be determined primarily by the bending moment which must be resisted by the beam. A lower limit of the size of the caps may be established tentatively as a result of the investigation of the effect of the stress concentration factor $C_2$ upon equal-strength design proportions. Since a value of $C_2 = 0.04$ was found to have negligible effect upon computed design proportions, it was possible to select a maximum value of $\omega d$ of 1.68 tentatively (see fig. 13, reference 1). If equal moments of inertia of top and bottom beam caps can be assumed, then the value of the moment of inertia of one of the caps about its lateral axis normal to the web may be expressed from figure 13 in reference 1 as

$$I_{min} = 0.01507 \frac{d_c^4}{h_e/t} \text{ inches}^4 \qquad (15)$$

## Rivet Design

The various rivet designs must, of course, agree with the limitations imposed on this investigation. A suggested method which appears to be satisfactory for determining spacing and similar dimensions is given in reference 1.

## TEST SPECIMENS

The test specimens were constructed of aluminum alloy using 75S-T6 alclad for the webs, 24S-T4 for the uprights, and 61S-T6 angles for the caps. The ratio of web depth to web thickness was approximately 405 for all beams, and the ratios of upright spacing to web depth were 0.8, 0.6, and 0.4. Only single-upright beams were tested. Beam 6 was identical to beam 2 with the exception that the edge of the attached leg of the upright was bent up (see fig. 5). The purpose in doing this was to provide better support for the web through the use of an upright which would better resist the action of the web wrinkles in forcing the buckling of the attached leg of the upright.

All uprights were angles formed from 24S-T4 aluminum having thicknesses of 0.091 and 0.051 inch and had equal-length legs. The radius of curvature of all bends was approximately five times the upright thickness. For the simple case of an equal-leg 90° angle formed from aluminum sheet which was used in this series of tests, it was possible to express the physical properties of the cross section (area and moment of inertia) in relatively simple analytical formulas from which were determined the dimensions of an upright necessary to fulfill a set of given conditions previously determined $\left(\text{upright thickness, } A_{u_e}, \text{ and } d_c\right)$.

The test beams were designed to meet the specifications developed in the previous section of this report for equal-strength beams. In determining the above-mentioned ratios of upright spacing to web depth (panel aspect ratio), the value of $h_c$ was empirically computed as

$$h_c = h_e/1.05$$

where for all beams $h_e$ is 15 inches.

During construction of the beams, the value of $h_c$ was found to be 12.96 inches, instead of 14.28 inches as indicated by the above empirical formula. To maintain the design values of panel aspect ratio, it was decided to decrease the upright spacing while retaining the upright dimensions already calculated for the larger upright spacings based on $h_c$ of 14.28 inches, thus increasing the values of $A_u/td$ and $A_{ue}/td$ above the design values. The increase in $A_{ue}/td$ varied between 4 and 9 percent for the different beams. This change resulted in the calculated strength of the uprights being slightly greater than that of the web excepting in beam 6. However, the predicted strengths of a beam as determined by web and upright failure did not differ by appreciable amounts despite the change in design noted above. The maximum difference in the two computed values of failing load for any one beam was approximately 7 percent and went as low as approximately 1 percent, for specimens 1 to 5.

Nominal dimensions of the beams and uprights are shown in figures 5 and 6. The properties of each beam are given in table III. Nominal dimensions of web and upright thickness were used in the analysis of all beams.

The specimens were tested as simply supported beams with no lateral flange support, as shown in figure 7. In effect, there were two shear test panels in each beam, each a rectangle about 31 inches long located midway between the center and tip (see figs. 6 and 7).

## TEST PROCEDURE

Stresses in the uprights were determined by measuring the strains with resistance-type wire strain gages mounted in pairs at several stations on the outstanding legs of the uprights (fig. 6). Local strains were measured to an accuracy of ±1 percent by the strain gages, and loads were measured to an accuracy of approximately ±1 percent by the manually operated beam balance of the testing machine.

Several test runs were made on each beam until repeatable strain measurements were obtained. Care was taken to keep all design stresses below the proportional limit stress of the beam material. Beams were tested to failure using load increments of 6000 pounds.

## RESULTS AND DISCUSSION OF TESTS

The results of the investigation are shown in table IV. Experimental and predicted loads at failure are recorded. The failures in all cases ultimately consisted of ruptured webs and resultant distortion of the uprights. In beams 2 and 4 a distinct "waving" of the attached leg of the upright was apparent before failure of the web of the beam, and it is possible that the forced crippling of the attached leg of the upright resulted in a concentration of load in the web, with resultant rupturing of the web. The order of events is impossible to determine, however, because of the suddenness of failure. After failure of the beam, the wrinkling of the web through the uprights was obvious (figs. 8 to 10). However, the thicker uprights were only slightly deformed by the wrinkles in the web, although after failure of the web the uprights tended to rotate and bend (figs. 11 and 12).

In many cases there was evidence of shear failure along the rivet lines of web-to-flange and web-to-upright attachment (figs. 11 and 12) but it is believed that this was a result rather than a cause of the initial failure of the web.

The actual and predicted variation of upright stress $\sigma_u$ may be found in figure 13. The predicted stresses agreed quite well with the values determined from the tests. In beam 2 the measured local stresses in the uprights show a net tension value instead of a net compression value as would be expected (fig. 5). It is believed that this is due to an insufficient number of strain gages on the upright. A similar tendency may be noted in the upright stress curve for beam 1, figure 5. Following the testing of beams 1 and 2 the number of strain gages on each upright was increased and consequently more consistent data were obtained as evidenced by the upright stress curves for the last four beams tested and shown in figure 5.

Beam 6 was constructed to show the effect of a buckle-resistant attached upright leg in a beam otherwise similar to beam 2. The results given in table IV show that while the predicted strength of beam 6 decreased, the actual strength of the beam increased considerably. The ultimate failure of this beam occurred as a web rupture with distinct waving of the attached leg of the upright, but the waving was not nearly so severe as the outright buckling of the attached leg of the plain equal-angle upright used in beam 2 (figs. 8 and 10).

From the results of the tests conducted it appears that the method of analysis of reference 1 is conservative when applied to equal-strength designs provided the uprights do not have long attached legs which may be outside the range covered by available test data. That such extreme proportions may be unduly susceptible to edge failure by forced crippling due to the action of the wrinkles of the web on the attached leg of the upright is suggested by the tests on beams 2 and 6. This effect is probably more noticeable in cases where the upright thickness is only slightly greater than the web thickness. Also, the low ratio of ultimate load to predicted load for beam 4 (see table IV) seems to strengthen the above supposition inasmuch as beam 4 has the highest value of the ratio of upright attached leg length to thickness. The characteristics of beam 2 were considerably improved by simply turning up a small portion of the edge of the attached leg of the stiffener and thus providing the attached leg with more support to resist the action of the web in forming wrinkles (see beam 6 in table IV).

## CONCLUSIONS

The method of analysis of NACA TN 1364 is applicable to the design of beams of approximately equal strength in the uprights and webs, provided the uprights do not have long attached legs which may be outside the range covered by available test data.

The empirical equations developed in the first part of the present report are conservative when used in the design of thin-web beams within the limits noted above.

University of Minnesota
Minneapolis, Minn., June 1, 1950

# APPENDIX

## CALCULATION OF EQUAL-STRENGTH BEAM DESIGN PROPORTIONS

The procedure utilized to calculate the parameter ratios of various equal-strength beam designs involves numerous trial-and-error calculations. The various steps will be enumerated below. It should be noted that as a result of limitation (5) under "Limits of Investigation" the value of $R_d$ is constant at 1.62. The steps in the design procedure are as follows:

(1) Select an initial set of values for $(d/h)_c$, $h_c/t$, and $A_{ue}/td$. These will be constant for one complete set of calculations.

(2) Determine $K_{ss}$ from figure 5(a), reference 1, and calculate the value of $\tau_{ss}$, where

$$\tau_{ss} = K_{ss} E \left(t/d_c\right)^2$$

(3) Calculate $\tau_{cr}$ from formula (7) after assuming an initial value for $R_h$. For a first choice, usually assume that $t_u/t$ is greater than 3.0, when $R_h = 1.31$ (fig. 5(b), reference 1). As stated previously, $R_d$ has a constant value of 1.62 for the conditions of this investigation.

(4) Estimate a preliminary value of $\tau/\tau_{cr}$ and utilize either figure 8 or 9, reference 1, to determine the ratio of $\sigma_u/\tau$. An approximate value of $\tau/\tau_{cr}$ may be obtained by dividing 31,000 by the value of $\tau_{cr}$ calculated in step (3).

(5) Calculate the diagonal tension factor $k$ using formula (8) and the value of $\tau/\tau_{cr}$ assumed in step (4).

(6) From figures 11 and 12, reference 1, the value of $\tan \alpha$ and subsequently the shear-stress concentration factor $C_1$ may be determined.

(7) Calculate the value of $\tau_{max}$ as

$$\tau_{max} = \left(\tau/\tau_{cr}\right)\left(\tau_{cr}\right)\left(1 + kC_1\right)$$

(8) Determine the value of $\tau_{all}$ as a function of the diagonal tension factor k from figure 14(b), reference 1. If the value of $\tau_{max}$ from step (7) does not agree with $\tau_{all}$ thus found, then the above process must be repeated beginning with step (4) and using a suitably revised estimate of the ratio $\tau/\tau_{cr}$ until a good agreement is finally obtained between the stresses $\tau_{all}$ and $\tau_{max}$. Agreement to within 100 psi is as close as is warranted by the figures being utilized.

(9) Assuming that step (8) is completed satisfactorily, the upright stress $\sigma_u$ may now be computed from data in steps (3) and (4) as

$$\sigma_u = \left(\sigma_u/\tau\right)\left(\tau/\tau_{cr}\right)\tau_{cr}$$

(10) Compute the ratio $\sigma_{u_{max}}/\sigma_u$ from formula (6), and then determine $\sigma_{u_{max}}$.

$$\sigma_{u_{max}} = \left(\sigma_{u_{max}}/\sigma_u\right)\sigma_u$$

(11) To determine the ratio $t_u/t$ let $\sigma_{u_{max}}$ be equal to $\sigma_o$. Then utilizing formula (3), the following may be obtained:

$$\begin{aligned}\sigma_{u_{max}} &= \sigma_o \\ &= 28k\sqrt{t_u/t}\end{aligned}$$

then

$$\left(t_u/t\right) = \left(\sigma_{u_{max}}/28k\right)^2$$

Use $k_e$, formula (4), if k is less than 0.5.

(12) The value of $t_u/t$ obtained from step (11) must now agree with the value of $t_u/t$ corresponding to the value of $R_h$ assumed in step (3). If this is not true, then the entire previous procedure must be repeated beginning with step (3) with a suitably revised estimate of the value of $R_h$.

It was found that after some experience two or three estimates resulted in answers which were as accurate as could be expected from the graphs employed.

These steps were repeated for eight values of $A_{ue}/td$ varying from 0.1 to 0.8 in steps of 0.1 while maintaining $(d/h)_c$ and $h_c/t$ at constant values. Eventually $(d/h)_c$ and $h_c/t$ were also varied separately and in all combinations for the following values as tabulated in table I:

$$(d/h)_c = 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$$

$$h_c/t = 250, 400, 600, 800$$

## REFERENCES

1. Kuhn, Paul, and Peterson, James P.: Strength Analysis of Stiffened Beam Webs. NACA TN 1364, 1947.

2. Levin, Ross L., and Sandlin, Charles W., Jr.: Strength Analysis of Stiffened Thick Beam Webs. NACA TN 1820, 1949.

TABLE I.- THIN-WEB-BEAM CALCULATIONS FOR

EQUAL-STRENGTH DESIGNS

(a) $C_2 = 0$

| $\left(\frac{d}{h}\right)_c$ | $\frac{h_c}{t}$ | $\frac{A_{ue}}{td}$ | $\tau_{cr}$ (ksi) | $\frac{\tau}{\tau_{cr}}$ | $\frac{\sigma_u}{\tau}$ | k | $\sigma_o$ (ksi) | $\frac{t_u}{t}$ |
|---|---|---|---|---|---|---|---|---|
| 0.3 | 250 | 0.1 | 13.700 | 2.45 | 0.350 | 0.196 | 17.230 | 4.60 |
| | | .2 | 13.720 | 2.45 | .290 | .196 | 14.310 | 3.18 |
| | | .3 | 13.400 | 2.54 | .264 | .199 | 13.180 | 2.46 |
| | | .4 | 13.200 | 2.58 | .230 | .204 | 11.460 | 1.95 |
| | | .5 | 12.600 | 2.67 | .220 | .210 | 10.800 | 1.69 |
| | | .6 | 12.000 | 2.80 | .207 | .220 | 10.120 | 1.41 |
| 0.3 | 400 | 0.1 | 5.360 | 5.90 | 0.730 | 0.367 | 31.600 | 7.74 |
| | | .2 | 5.360 | 5.90 | .600 | .367 | 26.000 | 5.25 |
| | | .3 | 5.360 | 5.90 | .515 | .367 | 22.300 | 3.86 |
| | | .4 | 5.320 | 5.96 | .455 | .368 | 19.750 | 3.00 |
| | | .5 | 5.240 | 6.08 | .410 | .372 | 17.820 | 2.41 |
| | | .6 | 5.040 | 6.30 | .370 | .379 | 15.980 | 1.89 |
| | | .7 | 4.840 | 6.53 | .345 | .386 | 14.800 | 1.58 |
| | | .8 | 4.670 | 6.78 | .325 | .393 | 13.920 | 1.38 |
| 0.3 | 600 | 0.1 | 2.380 | 12.60 | 1.120 | 0.500 | 43.400 | 9.60 |
| | | .2 | 2.380 | 12.70 | .900 | .501 | 35.100 | 6.30 |
| | | .3 | 2.380 | 12.78 | .760 | .502 | 29.800 | 4.49 |
| | | .4 | 2.380 | 12.82 | .657 | .502 | 25.870 | 3.38 |
| | | .5 | 2.355 | 13.00 | .580 | .505 | 22.850 | 2.61 |
| | | .6 | 2.295 | 13.25 | .520 | .509 | 20.350 | 2.04 |
| | | .7 | 2.185 | 13.93 | .480 | .516 | 18.720 | 1.68 |
| | | .8 | 2.125 | 14.29 | .446 | .520 | 17.370 | 1.42 |
| 0.3 | 800 | 0.1 | 1.340 | 21.90 | 1.407 | 0.585 | 51.100 | 9.74 |
| | | .2 | 1.340 | 22.00 | 1.115 | .585 | 40.700 | 6.18 |
| | | .3 | 1.340 | 22.10 | .932 | .585 | 34.200 | 4.36 |
| | | .4 | 1.340 | 22.15 | .800 | .588 | 29.400 | 3.20 |
| | | .5 | 1.310 | 22.70 | .710 | .590 | 26.100 | 2.50 |
| | | .6 | 1.275 | 23.30 | .630 | .594 | 23.150 | 1.94 |
| | | .7 | 1.212 | 24.55 | .577 | .599 | 21.200 | 1.60 |
| | | .8 | 1.158 | 25.70 | .535 | .608 | 19.550 | 1.32 |

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TABLE I.- THIN-WEB-BEAM CALCULATIONS FOR

EQUAL-STRENGTH DESIGNS - Continued

(a) $C_2 = 0$ - Continued

| $\left(\frac{d}{h}\right)_c$ | $\frac{h_c}{t}$ | $\frac{A_{ue}}{td}$ | $\tau_{cr}$ (ksi) | $\frac{\tau}{\tau_{cr}}$ | $\frac{\sigma_u}{\tau}$ | k | $\sigma_o$ (ksi) | $\frac{t_u}{t}$ |
|---|---|---|---|---|---|---|---|---|
| 0.4 | 250 | 0.1 | 8.100 | 4.02 | 0.556 | 0.293 | 24.800 | 6.99 |
| | | .2 | 8.100 | 4.02 | .462 | .293 | 20.600 | 4.64 |
| | | .3 | 8.100 | 4.02 | .400 | .293 | 17.800 | 3.41 |
| | | .4 | 8.000 | 4.07 | .355 | .296 | 16.000 | 2.60 |
| | | .5 | 7.710 | 4.22 | .325 | .303 | 14.400 | 2.08 |
| | | .6 | 7.390 | 4.40 | .300 | .311 | 13.220 | 1.66 |
| | | .7 | 7.150 | 4.53 | .280 | .316 | 12.300 | 1.40 |
| | | .8 | 6.830 | 4.71 | .265 | .324 | 11.500 | 1.18 |
| 0.4 | 400 | 0.1 | 3.160 | 9.67 | 0.978 | 0.456 | 38.250 | 8.50 |
| | | .2 | 3.160 | 9.70 | .800 | .457 | 31.400 | 5.69 |
| | | .3 | 3.160 | 9.76 | .675 | .457 | 26.600 | 4.08 |
| | | .4 | 3.160 | 9.78 | .581 | .459 | 22.950 | 3.03 |
| | | .5 | 3.090 | 9.98 | .522 | .462 | 20.600 | 2.41 |
| | | .6 | 2.990 | 10.32 | .470 | .466 | 18.480 | 1.93 |
| | | .7 | 2.875 | 10.71 | .436 | .473 | 17.080 | 1.61 |
| | | .8 | 2.760 | 11.20 | .405 | .480 | 15.900 | 1.36 |
| 0.4 | 600 | 0.1 | 1.407 | 20.83 | 1.382 | 0.577 | 49.300 | 9.30 |
| | | .2 | 1.407 | 20.97 | 1.095 | .578 | 39.300 | 5.90 |
| | | .3 | 1.407 | 21.05 | .920 | .579 | 33.150 | 4.18 |
| | | .4 | 1.407 | 21.10 | .790 | .580 | 28.550 | 3.08 |
| | | .5 | 1.375 | 21.60 | .695 | .583 | 25.100 | 2.36 |
| | | .6 | 1.318 | 22.60 | .622 | .588 | 22.500 | 1.87 |
| | | .7 | 1.268 | 23.45 | .570 | .595 | 20.500 | 1.52 |
| | | .8 | 1.210 | 24.60 | .530 | .600 | 19.070 | 1.29 |
| 0.4 | 800 | 0.1 | 0.791 | 36.10 | 1.715 | 0.651 | 57.900 | 10.10 |
| | | .2 | .791 | 36.40 | 1.322 | .652 | 45.000 | 6.06 |
| | | .3 | .791 | 36.70 | 1.092 | .654 | 37.400 | 4.18 |
| | | .4 | .791 | 36.80 | .925 | .655 | 31.750 | 3.00 |
| | | .5 | .775 | 37.60 | .820 | .655 | 28.150 | 2.36 |
| | | .6 | .743 | 39.40 | .730 | .663 | 25.000 | 1.82 |
| | | .7 | .702 | 41.60 | .658 | .669 | 22.500 | 1.44 |
| | | .8 | .669 | 43.70 | .608 | .675 | 20.750 | 1.21 |

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TABLE I.- THIN-WEB-BEAM CALCULATIONS FOR

EQUAL-STRENGTH DESIGNS - Continued

(a) $C_2 = 0$ - Continued

| $\left(\frac{d}{h}\right)_c$ | $\frac{h_c}{t}$ | $\frac{A_{ue}}{td}$ | $\tau_{cr}$ (ksi) | $\frac{\tau}{\tau_{cr}}$ | $\frac{\sigma_u}{\tau}$ | k | $\sigma_o$ (ksi) | $\frac{t_u}{t}$ |
|---|---|---|---|---|---|---|---|---|
| 0.5 | 250 | 0.1 | 5.480 | 5.79 | 0.723 | 0.364 | 29.500 | 6.80 |
| | | .2 | 5.480 | 5.80 | .580 | .364 | 23.700 | 4.37 |
| | | .3 | 5.480 | 5.80 | .512 | .364 | 20.950 | 3.41 |
| | | .4 | 5.420 | 5.85 | .452 | .365 | 18.470 | 2.65 |
| | | .5 | 5.290 | 6.03 | .406 | .372 | 16.650 | 2.10 |
| | | .6 | 5.030 | 6.29 | .370 | .379 | 15.000 | 1.66 |
| | | .7 | 4.860 | 6.49 | .346 | .385 | 13.940 | 1.41 |
| | | .8 | 4.660 | 6.76 | .327 | .392 | 13.130 | 1.23 |
| 0.5 | 400 | 0.1 | 2.140 | 14.00 | 1.168 | 0.516 | 42.600 | 8.66 |
| | | .2 | 2.140 | 14.05 | .942 | .518 | 34.500 | 5.66 |
| | | .3 | 2.140 | 14.10 | .792 | .518 | 29.100 | 4.02 |
| | | .4 | 2.140 | 14.15 | .680 | .518 | 25.100 | 3.00 |
| | | .5 | 2.100 | 14.48 | .608 | .523 | 22.400 | 2.34 |
| | | .6 | 2.015 | 15.02 | .545 | .529 | 20.000 | 1.83 |
| | | .7 | 1.923 | 15.74 | .504 | .535 | 18.480 | 1.52 |
| | | .8 | 1.840 | 16.45 | .468 | .541 | 17.100 | 1.27 |
| 0.5 | 600 | 0.1 | 0.952 | 30.20 | 1.605 | 0.630 | 53.900 | 9.30 |
| | | .2 | .952 | 30.45 | 1.250 | .630 | 42.300 | 5.75 |
| | | .3 | .952 | 30.75 | 1.038 | .630 | 35.500 | 4.05 |
| | | .4 | .952 | 30.80 | .892 | .632 | 30.750 | 2.98 |
| | | .5 | .928 | 31.70 | .780 | .632 | 27.000 | 2.33 |
| | | .6 | .892 | 32.90 | .700 | .640 | 23.900 | 1.78 |
| | | .7 | .850 | 34.55 | .632 | .646 | 21.700 | 1.44 |
| | | .8 | .794 | 37.00 | .581 | .655 | 19.730 | 1.16 |
| 0.5 | 800 | 0.1 | 0.535 | 52.40 | 1.910 | 0.696 | 60.900 | 9.76 |
| | | .2 | .535 | 53.00 | 1.470 | .697 | 47.400 | 5.88 |
| | | .3 | .535 | 53.20 | 1.196 | .697 | 38.700 | 3.92 |
| | | .4 | .534 | 53.70 | 1.017 | .700 | 33.150 | 2.86 |
| | | .5 | .519 | 55.50 | .885 | .702 | 28.950 | 2.16 |
| | | .6 | .496 | 58.00 | .795 | .708 | 25.900 | 1.71 |
| | | .7 | .470 | 61.40 | .716 | .714 | 23.300 | 1.36 |
| | | .8 | .441 | 65.40 | .658 | .720 | 21.400 | 1.13 |

TABLE I.- THIN-WEB-BEAM CALCULATIONS FOR

EQUAL-STRENGTH DESIGNS - Continued

(a) $C_2 = 0$ - Continued

| $\left(\frac{d}{h}\right)_c$ | $\frac{h_c}{t}$ | $\frac{A_{ue}}{td}$ | $\tau_{cr}$ (ksi) | $\frac{\tau}{\tau_{cr}}$ | $\frac{\sigma_u}{\tau}$ | k | $\sigma_o$ (ksi) | $\frac{t_u}{t}$ |
|---|---|---|---|---|---|---|---|---|
| 0.6 | 250 | 0.1 | 4.000 | 7.77 | 0.870 | 0.417 | 33.100 | 7.20 |
| | | .2 | 4.000 | 7.77 | .710 | .417 | 27.100 | 4.80 |
| | | .3 | 4.000 | 7.77 | .600 | .417 | 22.900 | 3.44 |
| | | .4 | 3.950 | 7.90 | .520 | .420 | 19.900 | 2.56 |
| | | .5 | 3.855 | 8.10 | .480 | .425 | 18.300 | 2.14 |
| | | .6 | 3.710 | 8.40 | .430 | .429 | 16.400 | 1.70 |
| | | .7 | 3.580 | 8.70 | .400 | .437 | 15.200 | 1.44 |
| | | .8 | 3.390 | 9.12 | .373 | .446 | 14.050 | 1.18 |
| 0.6 | 400 | 0.1 | 2.200 | 18.65 | 1.320 | 0.561 | 45.400 | 8.31 |
| | | .2 | 2.200 | 18.85 | 1.053 | .562 | 36.800 | 5.40 |
| | | .3 | 2.200 | 18.90 | .883 | .564 | 30.700 | 3.78 |
| | | .4 | 1.565 | 19.04 | .760 | .564 | 26.450 | 2.82 |
| | | .5 | 1.528 | 19.55 | .676 | .569 | 23.550 | 2.18 |
| | | .6 | 1.465 | 20.40 | .603 | .574 | 21.000 | 1.71 |
| | | .7 | 1.409 | 21.20 | .555 | .580 | 19.300 | 1.41 |
| | | .8 | 1.340 | 22.30 | .514 | .587 | 17.820 | 1.17 |
| 0.6 | 600 | 0.1 | 0.700 | 40.50 | 1.785 | 0.665 | 57.150 | 9.43 |
| | | .2 | .700 | 41.00 | 1.364 | .666 | 44.300 | 5.64 |
| | | .3 | .700 | 41.30 | 1.130 | .669 | 36.800 | 3.86 |
| | | .4 | .695 | 41.60 | .953 | .670 | 31.050 | 2.74 |
| | | .5 | .681 | 42.50 | .857 | .672 | 28.000 | 2.21 |
| | | .6 | .651 | 44.70 | .752 | .677 | 24.600 | 1.69 |
| | | .7 | .618 | 47.20 | .679 | .683 | 22.250 | 1.36 |
| | | .8 | .584 | 50.00 | .626 | .690 | 20.450 | 1.13 |
| 0.6 | 800 | 0.1 | 0.394 | 70.50 | 2.075 | 0.728 | 63.600 | 9.76 |
| | | .2 | .394 | 71.50 | 1.584 | .728 | 49.300 | 5.85 |
| | | .3 | .394 | 72.00 | 1.280 | .730 | 40.000 | 3.84 |
| | | .4 | .400 | 72.80 | 1.085 | .731 | 34.100 | 2.78 |
| | | .5 | .379 | 75.40 | .940 | .732 | 29.600 | 2.09 |
| | | .6 | .362 | 79.10 | .837 | .739 | 26.400 | 1.63 |
| | | .7 | .347 | 82.60 | .752 | .743 | 23.700 | 1.30 |
| | | .8 | .324 | 88.40 | .690 | .750 | 21.700 | 1.07 |

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TABLE I.- THIN-WEB-BEAM CALCULATIONS FOR

EQUAL-STRENGTH DESIGNS - Continued

(a) $C_2 = 0$ - Continued

| $\left(\frac{d}{h}\right)_c$ | $\frac{h_c}{t}$ | $\frac{A_{ue}}{td}$ | $\tau_{cr}$ (ksi) | $\frac{\tau}{\tau_{cr}}$ | $\frac{\sigma_u}{\tau}$ | k | $\sigma_o$ (ksi) | $\frac{t_u}{t}$ |
|---|---|---|---|---|---|---|---|---|
| 0.7 | 250 | 0.1 | 3.160 | 9.70 | 0.980 | 0.457 | 35.300 | 7.19 |
| | | .2 | 3.160 | 9.70 | .800 | .457 | 28.800 | 4.76 |
| | | .3 | 3.160 | 9.74 | .675 | .458 | 24.400 | 3.43 |
| | | .4 | 3.125 | 9.87 | .582 | .459 | 21.100 | 2.56 |
| | | .5 | 3.040 | 10.11 | .525 | .464 | 18.930 | 2.03 |
| | | .6 | 2.930 | 10.50 | .473 | .470 | 17.050 | 1.61 |
| | | .7 | 2.820 | 10.89 | .438 | .476 | 15.720 | 1.35 |
| | | .8 | 2.700 | 11.38 | .409 | .484 | 14.650 | 1.15 |
| 0.7 | 400 | 0.1 | 1.233 | 23.65 | 1.455 | 0.596 | 48.000 | 8.28 |
| | | .2 | 1.233 | 23.80 | 1.147 | .596 | 38.000 | 5.20 |
| | | .3 | 1.233 | 24.00 | .958 | .597 | 32.050 | 3.68 |
| | | .4 | 1.233 | 24.20 | .824 | .600 | 27.600 | 2.70 |
| | | .5 | 1.190 | 24.90 | .725 | .603 | 24.200 | 2.06 |
| | | .6 | 1.150 | 25.80 | .650 | .608 | 21.720 | 1.63 |
| | | .7 | 1.102 | 26.90 | .592 | .612 | 19.740 | 1.33 |
| | | .8 | 1.022 | 28.95 | .545 | .623 | 18.000 | 1.08 |
| 0.7 | 600 | 0.1 | 0.548 | 51.20 | 1.898 | 0.693 | 58.500 | 9.10 |
| | | .2 | .548 | 51.95 | 1.457 | .694 | 45.600 | 5.50 |
| | | .3 | .548 | 52.21 | 1.190 | .695 | 37.400 | 3.69 |
| | | .4 | .545 | 52.85 | 1.012 | .697 | 32.050 | 2.70 |
| | | .5 | .528 | 54.50 | .883 | .700 | 27.830 | 2.02 |
| | | .6 | .507 | 56.90 | .790 | .705 | 25.000 | 1.60 |
| | | .7 | .484 | 59.59 | .712 | .710 | 22.500 | 1.28 |
| | | .8 | .451 | 64.02 | .653 | .718 | 20.600 | 1.05 |
| 0.7 | 800 | 0.1 | 0.308 | 89.28 | 2.200 | 0.750 | 65.400 | 9.70 |
| | | .2 | .308 | 90.33 | 1.680 | .753 | 50.500 | 5.75 |
| | | .3 | .308 | 91.00 | 1.350 | .753 | 40.800 | 3.76 |
| | | .4 | .307 | 91.70 | 1.140 | .753 | 34.650 | 2.71 |
| | | .5 | .298 | 95.40 | .980 | .756 | 30.060 | 2.03 |
| | | .6 | .285 | 99.50 | .870 | .760 | 26.800 | 1.59 |
| | | .7 | .271 | 104.9 | .780 | .766 | 22.820 | 1.24 |
| | | .8 | .256 | 111.0 | .715 | .771 | 21.800 | 1.02 |

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TABLE I.- THIN-WEB-BEAM CALCULATIONS FOR

EQUAL-STRENGTH DESIGNS - Continued

(a) $C_2$ = 0 - Concluded

| $\left(\frac{d}{h}\right)_c$ | $\frac{h_c}{t}$ | $\frac{A_{ue}}{td}$ | $\tau_{cr}$ (ksi) | $\frac{\tau}{\tau_{cr}}$ | $\frac{\sigma_u}{\tau}$ | k | $\sigma_o$ (ksi) | $\frac{t_u}{t}$ |
|---|---|---|---|---|---|---|---|---|
| 0.8 | 250 | 0.1 | 2.608 | 11.60 | 1.070 | 0.489 | 37.800 | 7.58 |
| | | .2 | 2.608 | 11.65 | .870 | .489 | 30.850 | 5.00 |
| | | .3 | 2.608 | 11.70 | .733 | .489 | 26.000 | 3.58 |
| | | .4 | 2.600 | 11.75 | .633 | .490 | 22.550 | 2.68 |
| | | .5 | 2.522 | 12.10 | .565 | .494 | 20.100 | 2.10 |
| | | .6 | 2.450 | 12.45 | .510 | .499 | 18.100 | 1.68 |
| | | .7 | 2.380 | 12.80 | .470 | .501 | 16.650 | 1.42 |
| | | .8 | 2.270 | 13.40 | .435 | .510 | 15.300 | 1.16 |
| 0.8 | 400 | 0.1 | 1.021 | 28.30 | 1.560 | 0.620 | 50.500 | 8.50 |
| | | .2 | 1.021 | 28.50 | 1.225 | .621 | 40.000 | 5.30 |
| | | .3 | 1.021 | 28.80 | 1.015 | .622 | 33.500 | 3.70 |
| | | .4 | 1.019 | 28.80 | .870 | .623 | 28.700 | 2.72 |
| | | .5 | .990 | 29.80 | .760 | .626 | 25.150 | 2.06 |
| | | .6 | .956 | 30.90 | .680 | .630 | 22.300 | 1.60 |
| | | .7 | .925 | 31.90 | .620 | .636 | 20.400 | 1.32 |
| | | .8 | .890 | 33.10 | .580 | .640 | 19.050 | 1.13 |
| 0.8 | 600 | 0.1 | 0.454 | 61.5 | 2.000 | 0.712 | 61.000 | 9.40 |
| | | .2 | .454 | 62.2 | 1.530 | .712 | 47.200 | 5.60 |
| | | .3 | .454 | 62.7 | 1.240 | .715 | 38.600 | 3.72 |
| | | .4 | .452 | 63.2 | 1.050 | .716 | 32.800 | 2.67 |
| | | .5 | .440 | 65.0 | .915 | .722 | 28.500 | 2.00 |
| | | .6 | .425 | 67.5 | .820 | .723 | 25.600 | 1.60 |
| | | .7 | .405 | 71.0 | .730 | .729 | 22.800 | 1.25 |
| | | .8 | .385 | 74.8 | .670 | .733 | 20.900 | 1.06 |
| 0.8 | 800 | 0.1 | 0.255 | 106.5 | 2.30 | 0.767 | 67.100 | 9.71 |
| | | .2 | .255 | 108.0 | 1.75 | .767 | 51.900 | 5.80 |
| | | .3 | .255 | 109.5 | 1.40 | .768 | 42.000 | 3.82 |
| | | .4 | .254 | 110.5 | 1.17 | .768 | 35.300 | 2.70 |
| | | .5 | .247 | 115.0 | 1.01 | .772 | 30.800 | 2.03 |
| | | .6 | .238 | 119.5 | .89 | .777 | 27.100 | 1.56 |
| | | .7 | .227 | 126.0 | .80 | .782 | 24.500 | 1.25 |
| | | .8 | .214 | 133.0 | .73 | .786 | 22.300 | 1.02 |

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TABLE I.- THIN-WEB-BEAM CALCULATIONS FOR

EQUAL-STRENGTH DESIGNS - Continued

(b) $C_2 = 0.04$

| $\left(\frac{d}{h}\right)_c$ | $\frac{h_c}{t}$ | $\frac{A_{ue}}{td}$ | $\tau_{cr}$ (ksi) | $\frac{\tau}{\tau_{cr}}$ | $\frac{\sigma_u}{\tau}$ | k | $\sigma_o$ (ksi) | $\frac{t_u}{t}$ |
|---|---|---|---|---|---|---|---|---|
| 0.3 | 250 | 0.1 | 13.700 | 2.465 | 0.350 | 0.192 | 17.370 | 4.75 |
| | | .2 | 13.700 | 2.47 | .290 | .195 | 14.400 | 3.25 |
| | | .3 | 13.680 | 2.47 | .260 | .195 | 12.900 | 2.56 |
| | | .4 | 13.200 | 2.55 | .230 | .200 | 11.350 | 1.96 |
| | | .5 | 12.600 | 2.66 | .220 | .210 | 10.790 | 1.68 |
| | | .6 | 12.000 | 2.80 | .207 | .220 | 10.120 | 1.41 |
| 0.3 | 400 | 0.1 | 5.360 | 5.80 | 0.730 | 0.366 | 31.400 | 7.63 |
| | | .2 | 5.360 | 5.80 | .600 | .366 | 25.600 | 5.12 |
| | | .3 | 5.360 | 5.80 | .510 | .366 | 21.700 | 3.65 |
| | | .4 | 5.320 | 5.86 | .450 | .367 | 19.200 | 2.88 |
| | | .5 | 5.220 | 6.00 | .405 | .371 | 17.300 | 2.30 |
| | | .6 | 5.000 | 6.26 | .370 | .378 | 15.800 | 1.82 |
| | | .7 | 4.800 | 6.50 | .340 | .385 | 14.400 | 1.50 |
| | | .8 | 4.600 | 6.67 | .320 | .389 | 13.500 | 1.31 |
| 0.3 | 600 | 0.1 | 2.380 | 12.40 | 1.110 | 0.498 | 42.400 | 9.20 |
| | | .2 | 2.380 | 12.50 | .900 | .500 | 34.450 | 6.10 |
| | | .3 | 2.380 | 12.51 | .755 | .500 | 29.000 | 4.30 |
| | | .4 | 2.380 | 12.52 | .650 | .500 | 25.000 | 3.20 |
| | | .5 | 2.350 | 12.70 | .580 | .501 | 22.350 | 2.54 |
| | | .6 | 2.260 | 13.20 | .520 | .508 | 19.900 | 1.97 |
| | | .7 | 2.165 | 13.70 | .480 | .514 | 18.300 | 1.62 |
| | | .8 | 2.080 | 14.33 | .445 | .520 | 17.000 | 1.36 |
| 0.3 | 800 | 0.1 | 1.340 | 21.30 | 1.390 | 0.580 | 49.450 | 9.25 |
| | | .2 | 1.340 | 21.45 | 1.100 | .582 | 39.350 | 5.85 |
| | | .3 | 1.340 | 21.60 | .930 | .585 | 33.400 | 4.20 |
| | | .4 | 1.340 | 21.70 | .800 | .585 | 28.900 | 3.12 |
| | | .5 | 1.310 | 22.20 | .700 | .587 | 25.200 | 2.35 |
| | | .6 | 1.260 | 23.10 | .630 | .593 | 22.650 | 1.87 |
| | | .7 | 1.200 | 24.30 | .570 | .598 | 20.500 | 1.50 |
| | | .8 | 1.148 | 25.40 | .535 | .607 | 19.200 | 1.28 |

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TABLE I.- THIN-WEB-BEAM CALCULATIONS FOR

EQUAL-STRENGTH DESIGNS - Continued

(b) $C_2$ = 0.04 - Continued

| $\left(\frac{d}{h}\right)_c$ | $\frac{h_c}{t}$ | $\frac{A_{ue}}{td}$ | $\tau_{cr}$ (ksi) | $\frac{\tau}{\tau_{cr}}$ | $\frac{\sigma_u}{\tau}$ | k | $\sigma_o$ (ksi) | $\frac{t_u}{t}$ |
|---|---|---|---|---|---|---|---|---|
| 0.5 | 250 | 0.1 | 5.480 | 5.71 | 0.710 | 0.363 | 28.600 | 6.40 |
| | | .2 | 5.480 | 5.71 | .590 | .363 | 23.750 | 4.44 |
| | | .3 | 5.480 | 5.71 | .508 | .363 | 20.420 | 3.27 |
| | | .4 | 5.400 | 5.79 | .450 | .364 | 18.100 | 2.55 |
| | | .5 | 5.250 | 5.96 | .405 | .370 | 16.300 | 2.01 |
| | | .6 | 5.000 | 6.25 | .370 | .380 | 14.880 | 1.62 |
| | | .7 | 4.820 | 6.46 | .345 | .385 | 13.700 | 1.36 |
| 0.5 | 400 | 0.1 | 2.140 | 13.75 | 1.160 | 0.515 | 41.600 | 8.33 |
| | | .2 | 2.140 | 13.80 | .940 | .515 | 33.800 | 5.50 |
| | | .3 | 2.140 | 13.82 | .785 | .515 | 28.300 | 3.85 |
| | | .4 | 2.123 | 14.00 | .680 | .518 | 24.600 | 2.90 |
| | | .5 | 2.090 | 14.21 | .600 | .520 | 21.700 | 2.25 |
| | | .6 | 1.990 | 14.95 | .540 | .528 | 19.500 | 1.75 |
| | | .7 | 1.912 | 15.50 | .500 | .533 | 18.000 | 1.45 |
| | | .8 | 1.825 | 16.25 | .465 | .540 | 16.650 | 1.22 |
| 0.5 | 600 | 0.1 | 0.952 | 29.40 | 1.590 | 0.625 | 52.000 | 8.85 |
| | | .2 | .952 | 29.80 | 1.240 | .630 | 41.000 | 5.45 |
| | | .3 | .952 | 29.90 | 1.030 | .630 | 34.200 | 3.77 |
| | | .4 | .945 | 30.30 | .880 | .630 | 29.400 | 2.80 |
| | | .5 | .922 | 31.10 | .780 | .631 | 26.100 | 2.19 |
| | | .6 | .885 | 32.30 | .700 | .638 | 23.300 | 1.70 |
| | | .7 | .838 | 34.10 | .630 | .644 | 20.750 | 1.33 |
| | | .8 | .778 | 36.70 | .580 | .655 | 19.100 | 1.09 |
| 0.5 | 800 | 0.1 | 0.535 | 51.0 | 1.900 | 0.692 | 59.000 | 9.20 |
| | | .2 | .535 | 51.7 | 1.450 | .693 | 45.600 | 5.50 |
| | | .3 | .535 | 52.0 | 1.185 | .694 | 37.400 | 3.71 |
| | | .4 | .531 | 53.0 | 1.010 | .697 | 32.100 | 2.70 |
| | | .5 | .514 | 54.6 | .880 | .700 | 28.000 | 2.04 |
| | | .6 | .490 | 57.5 | .790 | .706 | 25.200 | 1.63 |
| | | .7 | .460 | 61.1 | .712 | .715 | 22.650 | 1.28 |
| | | .8 | .436 | 64.4 | .650 | .720 | 20.550 | 1.04 |

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TABLE I.- THIN-WEB-BEAM CALCULATIONS FOR

EQUAL-STRENGTH DESIGNS - Concluded

(b) $C_2 = 0.04$ - Concluded

| $\left(\frac{d}{h}\right)_c$ | $\frac{h_c}{t}$ | $\frac{A_{ue}}{td}$ | $\tau_{cr}$ (ksi) | $\frac{\tau}{\tau_{cr}}$ | $\frac{\sigma_u}{\tau}$ | k | $\sigma_o$ (ksi) | $\frac{t_u}{t}$ |
|---|---|---|---|---|---|---|---|---|
| 0.8 | 250 | 0.1 | 2.608 | 11.40 | 1.070 | 0.483 | 37.100 | 7.30 |
| | | .2 | 2.608 | 11.45 | .870 | .483 | 30.300 | 4.85 |
| | | .3 | 2.608 | 11.50 | .730 | .485 | 25.500 | 3.44 |
| | | .4 | 2.584 | 11.60 | .630 | .487 | 22.000 | 2.55 |
| | | .5 | 2.522 | 11.89 | .560 | .490 | 19.600 | 2.03 |
| | | .6 | 2.450 | 12.20 | .510 | .495 | 17.800 | 1.64 |
| | | .7 | 2.365 | 12.60 | .470 | .500 | 16.300 | 1.36 |
| | | .8 | 2.256 | 13.26 | .430 | .506 | 14.950 | 1.12 |
| 0.8 | 400 | 0.1 | 1.210 | 27.60 | 1.550 | 0.616 | 49.100 | 8.10 |
| | | .2 | 1.210 | 27.90 | 1.220 | .618 | 39.000 | 5.10 |
| | | .3 | 1.210 | 28.00 | 1.015 | .620 | 32.700 | 3.55 |
| | | .4 | 1.015 | 28.20 | .870 | .620 | 28.000 | 2.61 |
| | | .5 | .985 | 29.20 | .760 | .623 | 24.550 | 1.99 |
| | | .6 | .950 | 30.25 | .680 | .628 | 21.900 | 1.66 |
| | | .7 | .920 | 31.30 | .620 | .632 | 19.950 | 1.28 |
| | | .8 | .880 | 32.70 | .580 | .636 | 18.600 | 1.09 |
| 0.8 | 600 | 0.1 | 0.454 | 60.00 | 1.980 | 0.710 | 59.000 | 8.83 |
| | | .2 | .454 | 60.60 | 1.520 | .710 | 45.700 | 5.30 |
| | | .3 | .454 | 61.20 | 1.240 | .710 | 37.700 | 3.60 |
| | | .4 | .451 | 61.60 | 1.050 | .712 | 32.000 | 2.59 |
| | | .5 | .436 | 63.80 | .915 | .718 | 27.700 | 1.91 |
| | | .6 | .417 | 67.20 | .820 | .723 | 25.000 | 1.50 |
| | | .7 | .400 | 69.80 | .730 | .728 | 22.300 | 1.20 |
| | | .8 | .381 | 73.40 | .670 | .731 | 20.700 | 1.03 |
| 0.8 | 800 | 0.1 | 0.255 | 103.5 | 2.280 | 0.763 | 65.000 | 9.25 |
| | | .2 | .255 | 105.0 | 1.740 | .763 | 50.100 | 5.51 |
| | | .3 | .255 | 106.5 | 1.400 | .767 | 40.900 | 3.62 |
| | | .4 | .252 | 108.1 | 1.170 | .767 | 34.300 | 2.55 |
| | | .5 | .245 | 112.2 | 1.010 | .770 | 29.900 | 1.93 |
| | | .6 | .238 | 116.0 | .890 | .775 | 26.400 | 1.49 |
| | | .7 | .224 | 122.9 | .800 | .780 | 23.600 | 1.17 |
| | | .8 | .212 | 130.0 | .735 | .782 | 21.700 | 0.99 |

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TABLE II.- SUMMARY OF OVERSTRENGTH-WEB DESIGNS

| Beam (1) | Decrease in web shear (percent) | $\tau_{max}$ | $\frac{\tau_{max}}{\tau_{cr}}$ | k | $\frac{\sigma_u}{\tau}$ | $\frac{A_{ue}}{td}$ | $(b/t)_u$ | $t_u$ | $\frac{A_u}{td}$ | Decrease in upright weight (percent) | Decrease in beam weight (percent) (2) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 0 | 29.75 | 20.85 | 0.577 | 0.589 | 0.678 | 56.5 | 0.051 | 1.15 | 0 | 0 |
| 4a | 12 | 26.20 | 18.36 | .558 | .641 | .53 | 44.7 | .051 | .913 | 21 | 11 |
| 4b | 20 | 23.80 | 16.80 | .545 | .688 | .43 | 37.1 | .051 | .752 | 34 | 18 |
| 4c | 30 | 20.83 | 14.6 | .524 | .750 | .35 | 30.8 | .051 | .623 | 46 | 24 |
| 4d | 40 | 17.85 | 12.52 | .500 | .827 | .25 | 23.2 | .051 | .462 | 60 | 32 |
| 5 | 0 | 29.75 | 19.3 | .566 | .852 | .414 | 14.08 | .091 | .832 | 0 | 0 |
| 5a | 12 | 26.20 | 17.0 | .547 | .84 | .31 | 11.03 | .091 | .66 | 21 | 9 |

[1]The unlettered numbers represent equal-strength beams as originally designed and tested (see tables III and IV). The letters represent design modifications in which only part of the total web strength is assumed utilized as represented by figures in the second and third columns.

[2]Exclusive of flanges or caps.

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TABLE III.- PROPERTIES OF TEST BEAMS

[Beam webs were 0.032-in. alclad 75S-T6 aluminum alloy; beam flanges were two 61S-T6 aluminum-alloy extruded angles, 2 by 3 by 1/4 in., beam uprights were formed 24S-T4 aluminum alloy with equal-leg angles]

| Beam | $h_e$ (in.) | $h_u$ (in.) | $h_c$ (in.) | $d_c$ (in.) | Uprights (in.) | $A_u$ (sq in.) | $A_{ue}$ (sq in.) | $\frac{A_{ue}}{td}$ | $(\omega d)$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 14.96 | 13.57 | 12.96 | 10.36 | 1.57 by 0.091 | 0.260 | 0.136 | 0.409 | 1.41 |
| 2 | 14.96 | 13.57 | 12.96 | 5.18 | 2.17 by 0.051 | .213 | .123 | .744 | .70 |
| 3 | 14.96 | 13.57 | 12.96 | 5.18 | 1.03 by 0.091 | .162 | .0754 | .455 | .70 |
| 4 | 14.96 | 13.57 | 12.96 | 7.78 | 2.88 by 0.051 | .286 | .169 | .678 | 1.06 |
| 5 | 14.96 | 13.57 | 12.96 | 7.78 | 1.28 by 0.091 | .207 | .103 | .414 | 1.06 |
| 6 | 14.96 | 13.57 | 12.96 | 5.18 | (See fig. 5) | .213 | .108 | .654 | .70 |

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TABLE IV.- TEST DATA AND RESULTS

| Beam | $(\tau_{cr})_{calc}$ (ksi) | $P_{ult}$ (kips) | $\tau_{ult} = \frac{P_{ult}}{2 h_e t}$ (ksi) | $\frac{\tau_{ult}}{(\tau_{cr})_{calc}}$ | k | Predicted $P_{ult}$ | | | $\frac{P_{ult}}{P'}$ (4) |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | $P_1$ (kips) (1) | $P_2$ (kips) (2) | $P_3$ (kips) (3) | |
| 1 | 1.005 | 32.63 | 33.90 | 33.80 | 0.644 | 30.20 | 28.20 | 28.15 | 1.16 |
| 2 | 2.840 | 30.57 | 31.84 | 11.21 | .481 | 30.40 | 29.40 | 29.60 | 1.04 |
| 3 | 3.090 | 32.68 | 34.04 | 11.02 | .477 | 29.84 | 29.46 | 29.60 | 1.11 |
| 4 | 1.427 | 26.15 | 27.24 | 19.10 | .565 | 28.70 | 28.70 | 28.60 | .91 |
| 5 | 1.540 | 31.26 | 32.56 | 21.13 | .580 | 29.40 | 28.60 | 28.60 | 1.09 |
| 6 | 2.840 | 34.29 | 35.72 | 12.58 | .500 | 28.00 | 29.40 | 29.60 | 1.22 |

[1]For upright failure.

[2]For web failure.

[3]For failure as an equal-strength beam (see fig. 3).

[4]P' is the lowest one of the predicted loads $P_1$, $P_2$, or $P_3$.

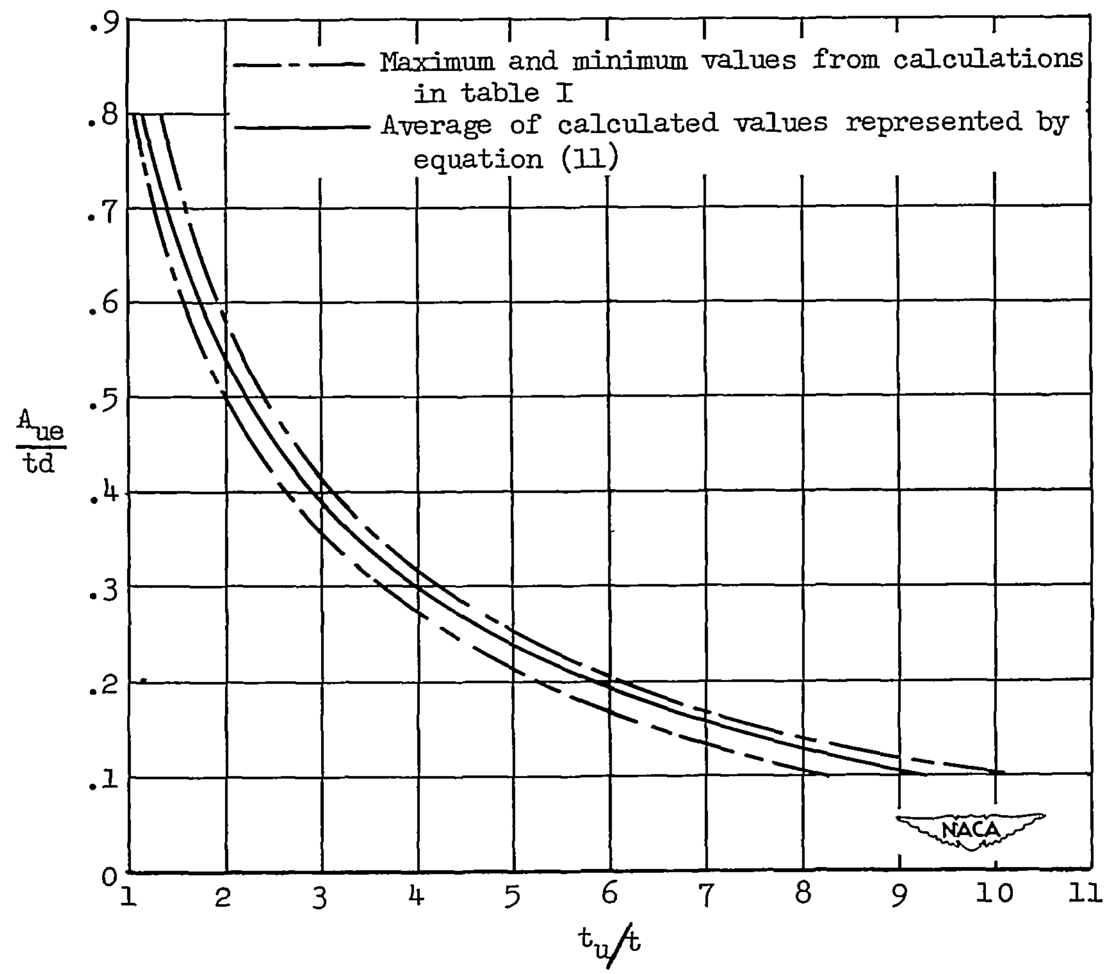



Figure 1.- Dimensionless design parameters for thin-web beams of equal-strength design. $0.4 \leqq (d/h)_c \leqq 0.8$; $400 \leqq h_c/t \leqq 800$.

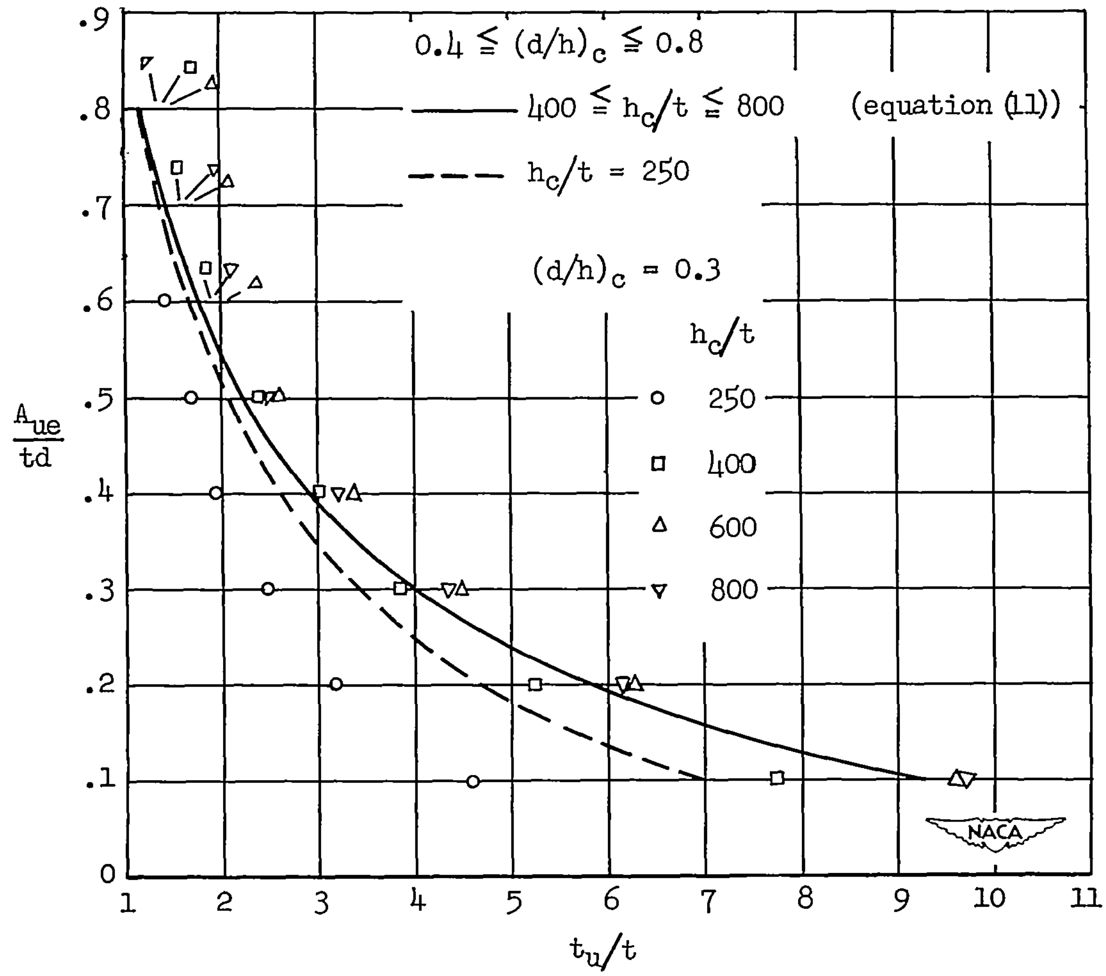
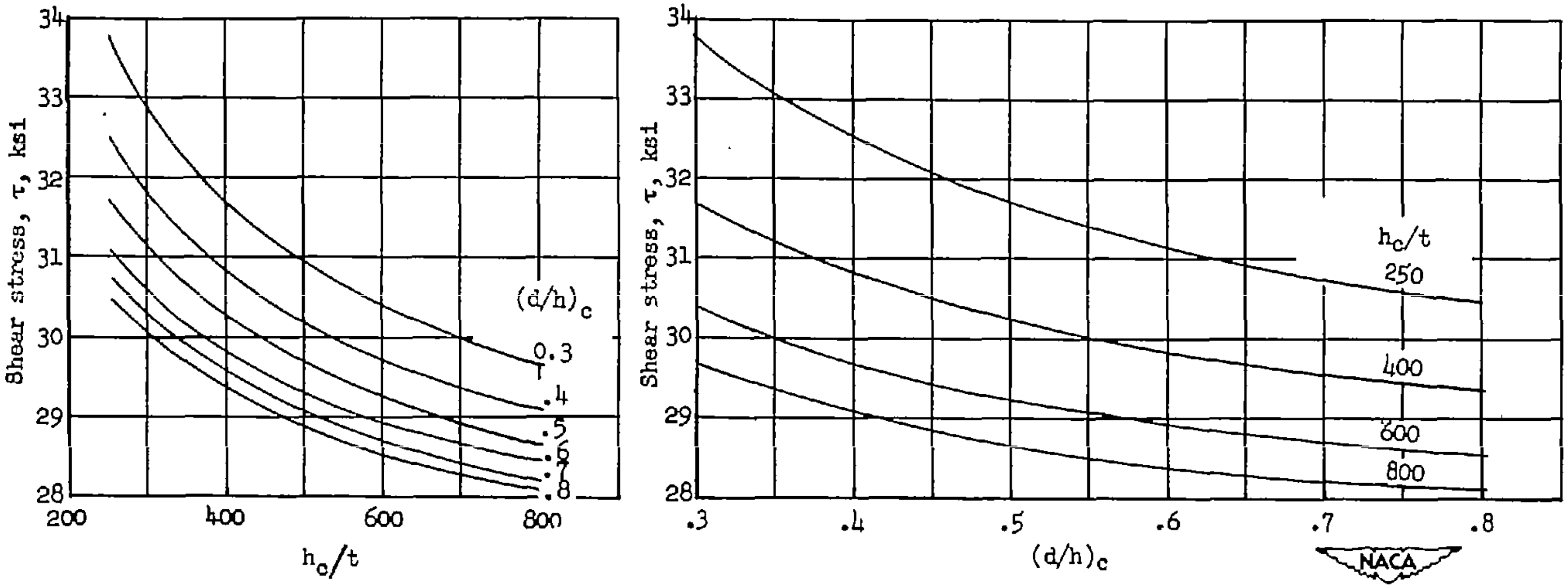


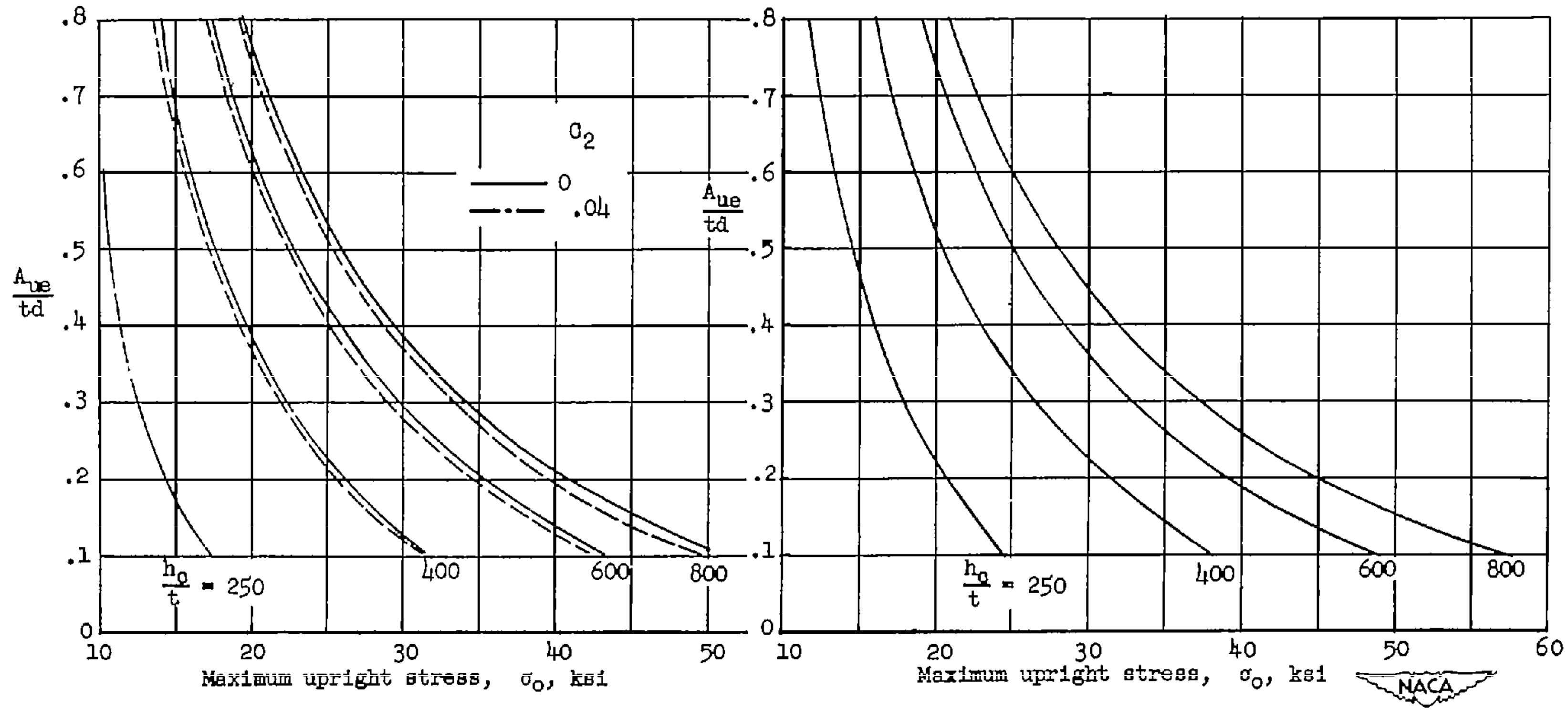

Figure 2.- Summary of computations of dimensionless design parameters for thin-web beams of equal-strength design.




(a) Stress plotted against $h_c/t$.

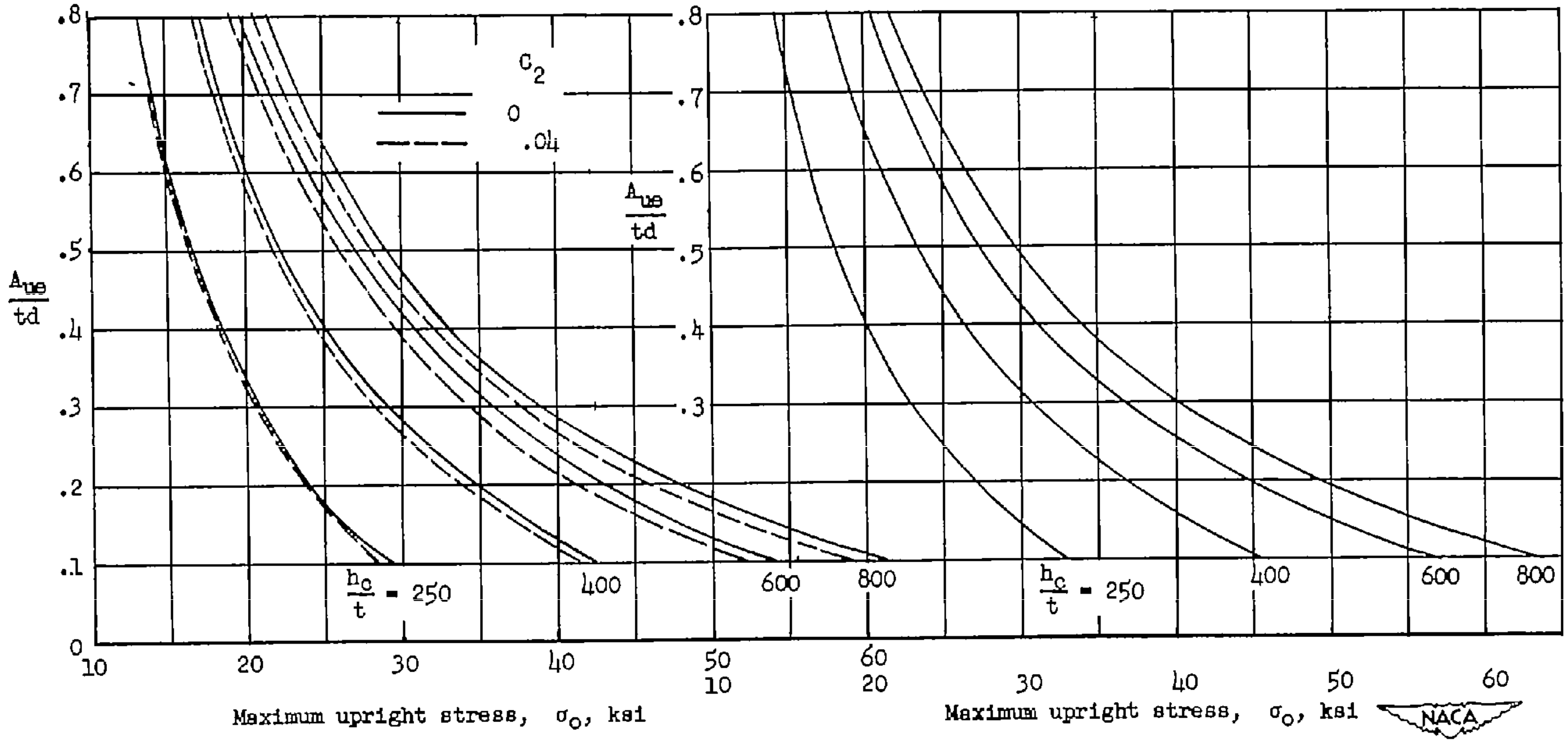
(b) Stress plotted against $(d/h)_c$.

Figure 3.- Average shear stress in web of equal-strength beam at failure.



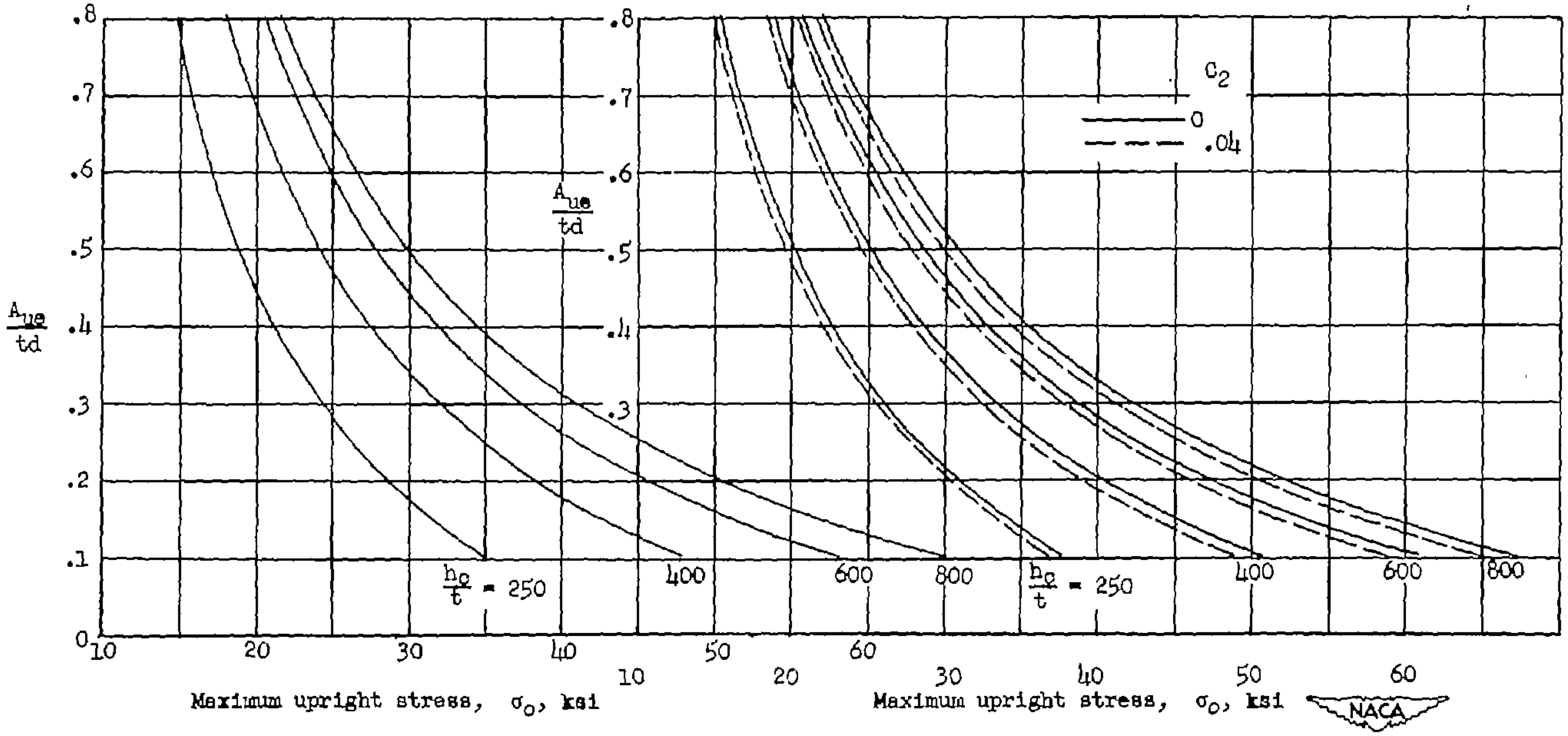

(a) $(d/h)_c = 0.3$ (b) $(d/h)_c = 0.4$

Figure 4.- Maximum upright stress in beams of equal-strength design.




(c) $(d/h)_c = 0.5$. (d) $(d/h)_c = 0.6$.

Figure 4.- Continued.




(e) $(d/h)_c = 0.7$.

(f) $(d/h)_c = 0.8$.

Figure 4.- Concluded.

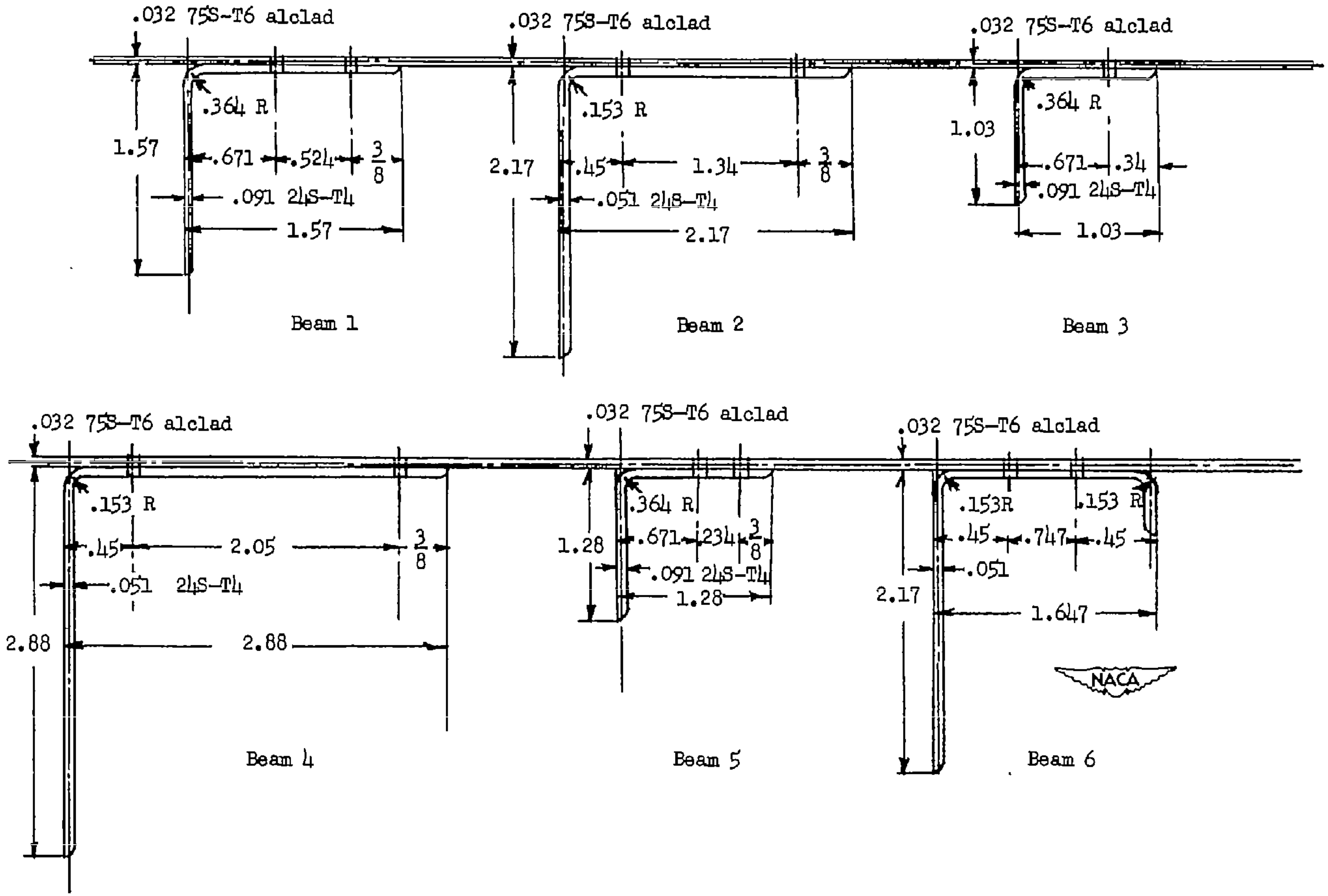



Figure 5.- Dimensions of uprights of test beams. Dimensions are in inches.

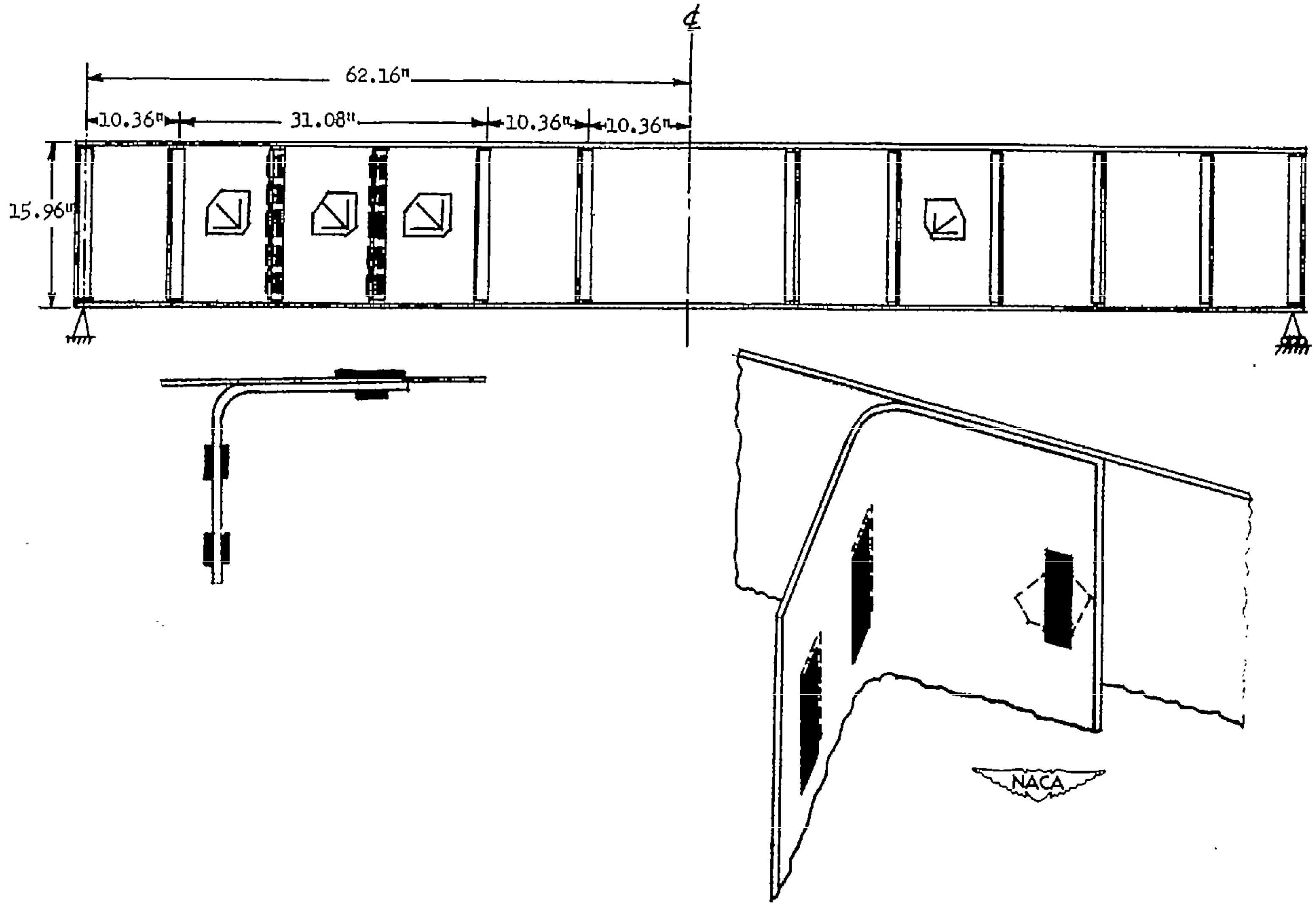



Figure 6.- Strain-gage locations and over-all beam dimensions.

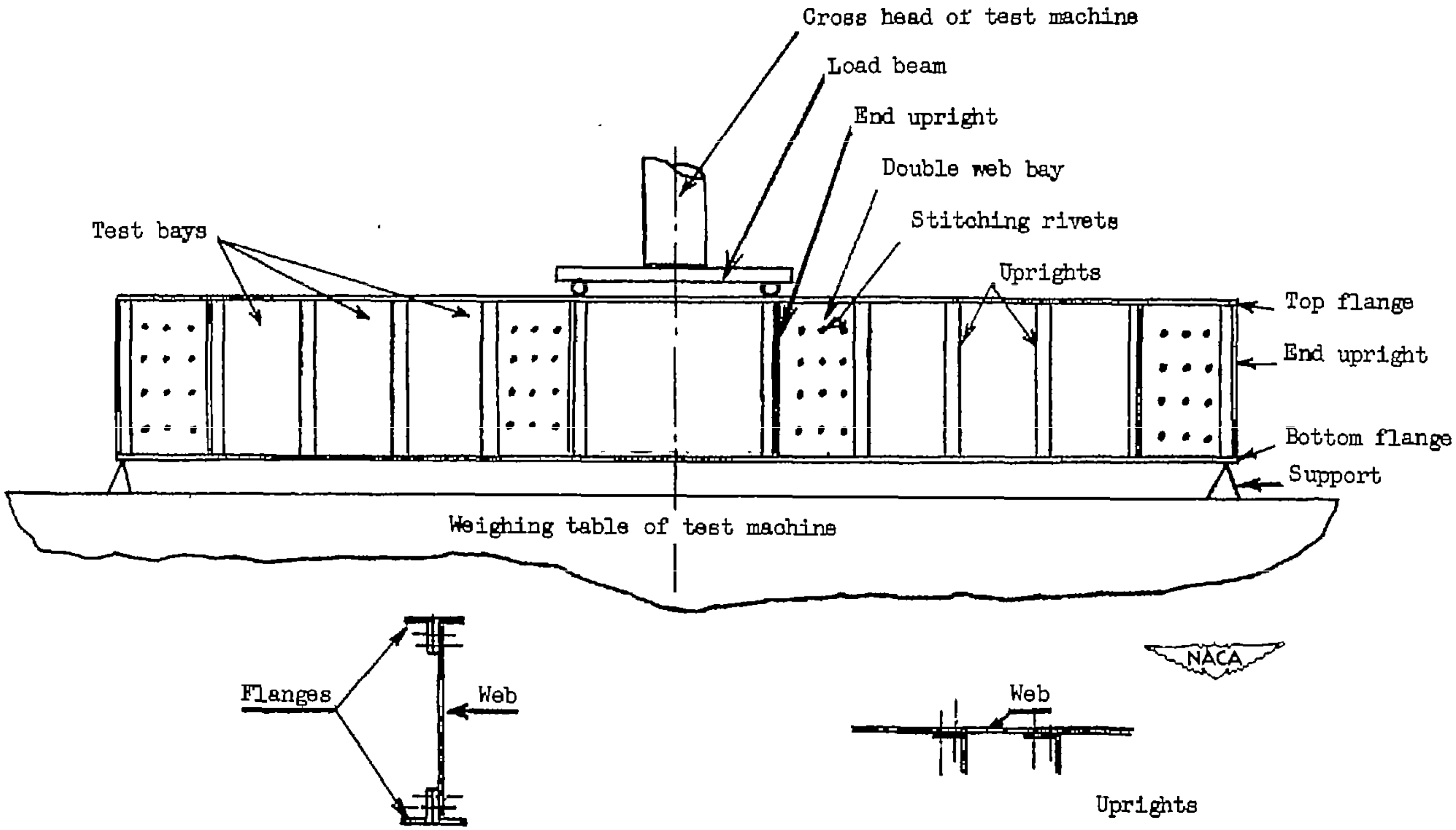



Figure 7.- Arrangement of test beam 1 in universal testing machine.


Figure 8.- Beam 2 at failure.


Figure 9.- Beam 4 at failure.


Figure 10.- Web of beam 6 after failure.

Figure 11.- Beam 3 at failure.

Figure 12.- Beam 5 at failure.

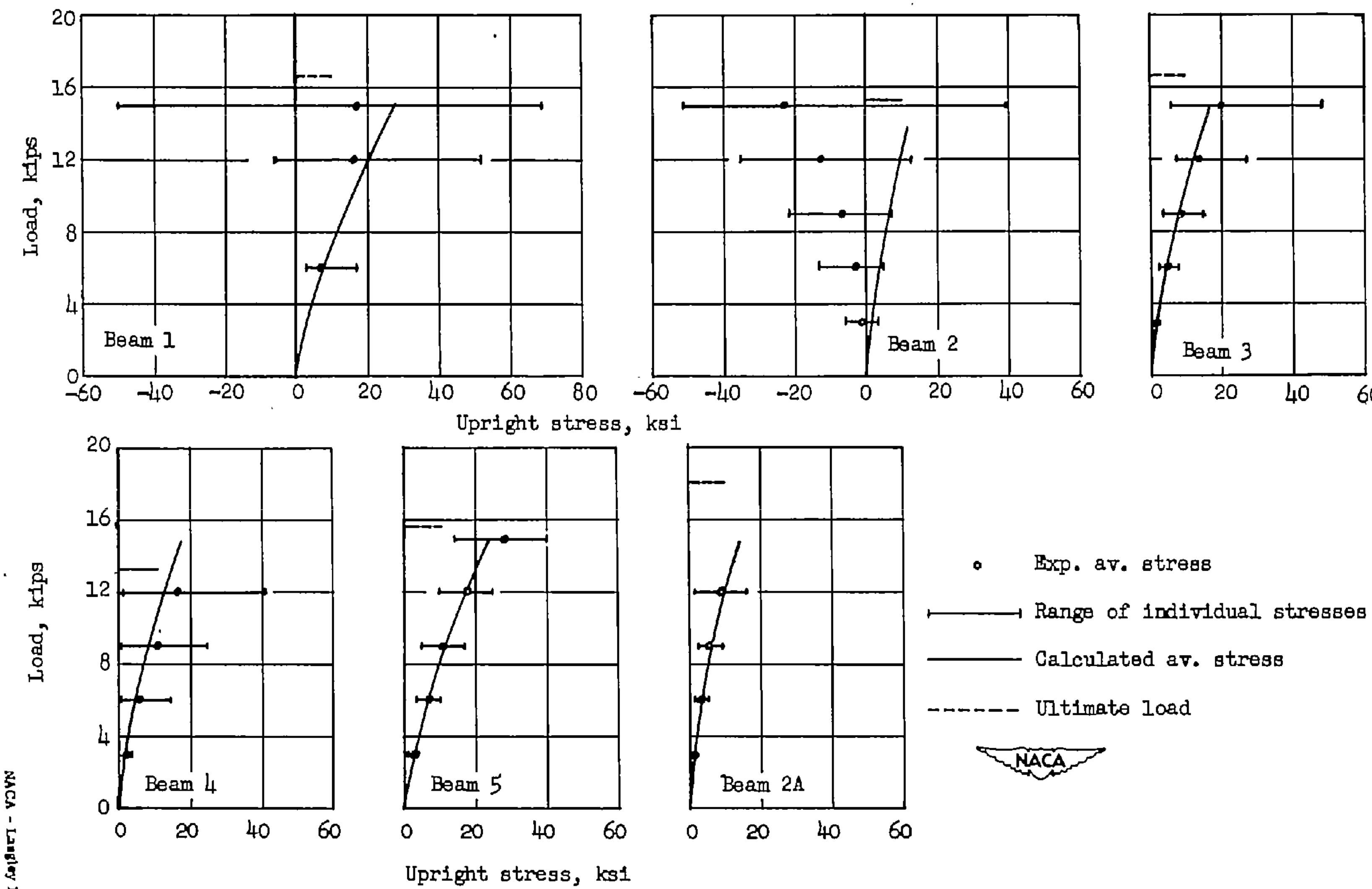



Figure 13.- Stresses in uprights of test beams.

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