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THEORETICAL TURBOJET THRUST AUGMENTATION BY EVAPORATION OF WATER DURING COMPRESSION AS DETERMINED BY USE OF A MOLLIER DIAGRAM

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THEORETICAL TURBOJET THRUST AUGMENTATION BY EVAPORATION OF WATER DURING COMPRESSION AS DETERMINED BY USE OF A MOLLIER DIAGRAM

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SUMMARY

A Mollier diagram for air saturated with water vapor and charts showing thermodynamic properties of various air - water vapor and exhaust gas - water vapor mixtures are presented as aids in calculating the thrust augmentation theoretically possible by the evaporation of water during compression.

Curves are presented that show the theoretical performance of this augmentation method and the effects of varying flight Mach number, altitude, ambient-air temperature, ambient relative humidity, compressor pressure ratio, and inlet-diffuser efficiency. Numerical examples illustrating the calculation of compressor-outlet conditions when water is evaporated during compression are presented.

For a typical turbojet engine at sea-level zero Mach number conditions, the maximum theoretical ratio of augmented to normal thrust, when a nominal decrease in compressor efficiency with water injection was assumed, was 1.29. The weight ratio of augmented to normal liquid flow for these conditions was 5.01 and the specific liquid consumption ratio was 3.88. At a sea-level flight Mach number of 1.5, the maximum thrust ratio was 2.07 with a liquid-flow ratio of 10.66 and specific-liquid-consumption ratio of 5.15. The percentage increase in thrust for maximum augmentation was approximately one-half as great at an altitude of 35,300 feet as it was at sea level for the same flight Mach number. An increase in ambient-air temperature caused a decrease in both normal and augmented thrust but, in general, caused an increase in thrust ratio. An increase in ambient relative humidity caused a decrease in augmented thrust and a corresponding decrease in thrust ratio, especially at high ambient-air temperatures. The ratio of maximum augmented thrust to normal thrust increased at an increasing rate as the flight Mach number was increased.

INTRODUCTION

One method of augmenting the thrust of turbojet engines is to inject water into the air as it enters the engine. The evaporation of this water cools the air and results in a lower mixture temperature and a higher compressor pressure ratio than for the same engine rotor speed without evaporation. An increase in compressor pressure ratio increases the jet velocity and the mass flow of gases through the engine, which results in increased thrust.

Water injection can be divided into two phases: evaporation before mechanical compression, and evaporation in the compressor. A discussion of the theoretical thrust augmentation that may be obtained by the evaporation of water before compression and a method of calculating this increase in thrust is presented in reference 1.

An analysis of thrust augmentation by evaporation of water during the compression process was made at the NACA Lewis laboratory and is reported herein. The process of evaporation during mechanical compression greatly complicates calculations of compressor-outlet conditions. No equations are available by which pressure, temperature, and amount of water vapor present at the end of a saturated compression process may be directly calculated. These conditions may be found by trial-and-error methods (for example, reference 2), but they are involved and tedious. In order to investigate the possibilities of thrust augmentation by evaporation of water during compression, a Mollier diagram for air saturated with water vapor was developed as a means of calculating the conditions of the mixture at the end of the saturated compression process. This Mollier diagram is similar to the one presented in reference 3, but the range of temperatures and pressures is extended to values suitable for use in compressor problems.

A chart of thermodynamic properties of air and water-vapor mixtures, which may be used in connection with the Mollier diagram in calculating compressor performance when less water is injected than is necessary to keep the air saturated during the entire compression process, and a similar chart for use in turbine and exhaust-gas calculations are also included. The use of the charts in calculating compressor performance with evaporation during compression is illustrated by numerical examples.

The performance of this thrust-augmentation method over a range of flight Mach numbers at two altitudes was calculated using the Mollier diagram to obtain the compressor-outlet conditions and step-by-step calculations to determine conditions for the rest of the engine cycle. The effect on thrust augmentation of varying ambient-air temperature and relative humidity, compressor pressure ratio, and inlet-diffuser efficiency were also calculated.

METHODS OF ANALYSIS

Calculations for the Mollier diagram, which was required for obtaining the temperature and the pressure after evaporation and the amount of water evaporated, were made using the following assumptions:

1. All values for the diagram are for a saturated mixture containing 1 pound of air.

2. The water vapor and the air in the mixture are at the same temperature.

3. Dalton's law of partial pressures is valid for the mixture.

4. Air is a perfect gas.

The symbols used in the calculations and the equations used in plotting the Mollier diagram, which is enclosed in a pocket at the back of the report, are given in appendixes A and B, respectively.

For a centrifugal-flow compressor, the assumption may be made that all the water to be evaporated during compression is added at the compressor inlet, and the compressor power may be calculated per unit mass flow of the mixture of air and water vapor at the compressor outlet. This power per unit mass flow must then be converted to power per unit air flow in order to use the Mollier diagram. If calculations for an axial-flow compressor with interstage water injection are desired, the total power input must be divided among the sections of the compressor between injection points and calculations similar to the case for the centrifugal-flow compressor made for each section in order to allow for the change in mass flow from section to section and the corresponding change in power per unit air flow in using the Mollier diagram.

When less water is injected than is necessary to saturate the air at the compressor outlet, the compression process may be divided into two parts: a saturated compression, which may be calculated using the Mollier diagram, and an adiabatic compression of the resulting mixture of air and water vapor.

In order to simplify the calculation of the adiabatic compression process, a chart of thermodynamic properties for various air - water vapor mixtures was plotted. The equations used to compile this chart are given in appendix C.

In order to determine the theoretical augmentation possible by evaporation of water during compression, step-by-step calculations of normal and augmented thrust for a typical turbojet engine operating under various flight conditions were made. The Mollier diagram was used to calculate compressor-outlet conditions with water injection and an illustration of these calculations is given in appendix D. The other equations, which were used in calculating the rest of the engine cycle, are listed in appendix E.

The component efficiencies chosen were: inlet-diffuser adiabatic efficiency, (except where otherwise noted) 0.85 to a flight Mach number of 1, linearly decreasing to 0.75 at a flight Mach number of 2.0; compressor adiabatic efficiency, 0.80; turbine adiabatic efficiency, 0.85; and exhaust-nozzle efficiency, 0.95. Because experimental water-injection data show an apparent decrease in compressor efficiency when compared with the theoretical saturated compression cycle, the compressor efficiency for the calculations was arbitrarily assumed to decrease according to the relation $\eta_c = 0.80 - \Delta X_c$, where ΔX_c is the change in water vapor - air ratio in the compressor. This relation yields augmentation results in closer agreement with actual experimental data at sea-level zero Mach number conditions than does a constant compressor efficiency and was assumed to apply to all conditions of altitude and flight Mach number. A single-stage centrifugal-flow compressor with a tip speed of 1500 feet per second and a slip factor of 0.95 that produces a pressure ratio of 4.61 at sea-level zero Mach number conditions with no water injection was assumed for all of the calculations except when it was desired to show the effect of varying the normal compressor pressure ratio. A combustion-chamber pressure loss of 3 percent and a turbine-inlet temperature of 2000° R were assumed. The ambient relative humidity was assumed to be 0.50 except when it was desired to show the effect of varying ambient relative humidity. All liquid water was assumed injected at the compressor inlet at a temperature of 519° R and the air saturated as long as any liquid water was present. Calculations were made for standard sea-level conditions and for an altitude of 35,300 feet.

In order to facilitate the calculations for the turbine and the exhaust nozzle, a chart of thermodynamic properties for various mixtures of exhaust gases and water vapor for a temperature of 1650° R was prepared. Inasmuch as the turbine-inlet temperature was held constant, the average temperature for the expansion process is approximately constant for all flight conditions and the temperature of 1650° R was chosen as an average temperature for the expansion process in the turbine and the jet. The equations used to compile this chart are given in appendix C.

DISCUSSION OF CHARTS

Mollier Diagram

The Mollier diagram is so constructed that each point represents values for 1 pound of air plus the water vapor necessary for saturation. The sections into which the Mollier diagram has been divided are shown in figure 1. The abscissa is entropy in Btu per pound air per °R and the ordinate is enthalpy in Btu per pound air. The solid slanting lines with positive slope are lines of constant pressure in pounds per square inch. The broken lines with positive slope are lines of constant temperature in °R. The lines of negative slope are lines of constant weight ratio of water vapor to air. The range of pressures is from 3 to 500 pounds per square inch and the temperature ranges from 440° to 790° R. The entropy and the enthalpy for liquid water were considered as zero at 519° R, and the bases for enthalpy and entropy of air were so fixed that at a temperature of 519° R and a pressure of 14.696 pounds per square inch the enthalpy of the saturated mixture is 100 Btu per pound of air and the entropy is 0.10 Btu per pound air per °R. When liquid water is present that is not at a temperature of 519° R, appropriate corrections for enthalpy and entropy must be made. These corrections are discussed in appendix B and a chart of enthalpy and entropy for liquid water at various temperatures to be used for these corrections is inserted in the Mollier diagram.

In order to utilize the Mollier diagram to calculate compressor-outlet conditions with evaporation of water during compression, knowledge of the conditions of the saturated air at the compressor inlet and the actual and isentropic work of the compression process is necessary. The conditions at the end of the saturated compression process may be obtained as follows: A point is found on the Mollier diagram corresponding to the conditions of saturated air at the compressor inlet. These conditions may be found by using reference 1 or by the procedure used in Sample Calculation II (appendix D). An isentropic compression process is then followed on the diagram and a second point determined by moving vertically along a line of constant entropy until a value of enthalpy equal to the original enthalpy plus the enthalpy of the isentropic work of compression per pound of air (actual work per pound of air multiplied by adiabatic efficiency) is reached. The pressure read from the diagram at this second point is the pressure at the end of the compression process. The final temperature and water-air ratio for the compression process are read from the diagram at a third point, which is obtained by following a constant pressure line from the second point until a value of enthalpy equal to the original enthalpy plus the enthalpy of the actual work of compression per pound of air is reached.

Mixture Properties for Compressor

The chart for thermodynamic properties of air - water vapor mixtures is shown in figure 2(a). The variation of specific heat at constant pressure c_p with temperature and pressure is very small in the range of interest for compressor calculations; figure 2(a) was therefore based on an average temperature of 600° R and a pressure of 0 pound per square inch absolute. Values of the ratio of specific heats γ_m, gas constant R_m, and specific heat at constant pressure $c_{p,m}$ are plotted for various values of the water-air ratio X_2. The use of this chart in calculating the compressor-outlet conditions when less water is injected during compression than the amount necessary to saturate the air at the compressor outlet is illustrated in Sample Calculation III (appendix D), which includes a case in which a desired amount of water is injected at the compressor inlet.

Mixture Properties for Turbine and Jet

The chart of thermodynamic properties of exhaust gas - water vapor mixtures at a temperature of 1650° R is shown in figure 2(b). Values for the ratio of specific heats γ_e, gas constant R_e, and specific heat at constant pressure $c_{p,e}$ are plotted for various values of the water-air ratio X_3 and the fuel-air ratio f/a. (A hydrogen-carbon ratio of 0.175 was used.) This chart may be used to determine average values of γ, R, and c_p for the expansion process in the turbine and exhaust nozzle when water vapor is present because of water injection.

DISCUSSION OF THEORETICAL AUGMENTATION RESULTS

The results of the calculations that were made for the engine described in METHODS OF ANALYSIS are plotted in figures 3 and 4.

Effect of Water-Injection Rate

In figure 3, the ratio of augmented thrust to normal thrust is plotted as a function of the weight ratio of total liquid flow (water plus fuel) to normal fuel flow for three different operating conditions: flight Mach numbers of 0 and 1.5 at sea level and a flight Mach number of 1.5 at an altitude of 35,300 feet. The points on the curves indicated by a circle represent the injection of only sufficient water to saturate the air at the compressor inlet with no further evaporation during compression. The end points on the curves represent the condition when the air is saturated with water vapor at the compressor outlet and indicate the maximum amounts of augmentation theoretically possible by evaporation of water before and during compression in the engine previously described for the given flight conditions. As the amount of water injected is increased, the thrust ratio increases, but at a slightly decreasing rate. At sea-level zero Mach number conditions, the thrust ratio for saturation only up to the compressor inlet is 1.035 with a liquid-flow ratio of 1.18 and a ratio of augmented to normal specific liquid consumption of 1.13. (Values for specific-liquid-consumption ratio are not shown.) As the liquid-flow ratio is increased by evaporating water during mechanical compression, the thrust ratio increases but at a decreasing rate to a maximum of 1.29 with a liquid-flow ratio of 5.01 for saturated compressor discharge. The corresponding ratio of augmented to normal specific liquid consumption is 3.88. Increasing the flight Mach number for the same altitude increases the compressor-inlet temperature, which permits the evaporation of more water and increases the maximum-thrust and liquid-flow ratios (for saturation at the compressor outlet) as well as increasing the thrust ratio for a given liquid-flow ratio. At a sea-level Mach number of 1.5 for saturation at the compressor outlet, the thrust ratio, the liquid-flow ratio, and the specific-liquid-consumption ratio are 2.07, 10.66, and 5.15, respectively. Increasing the altitude for the same flight Mach number decreases the compressor-inlet temperature, which reduces the amount of water that can be evaporated and has the opposite effect of increased flight Mach number. At an altitude of 35,300 feet and a flight Mach number of 1.5, the thrust ratio, the liquid-flow ratio, and the specific-liquid-consumption ratio are 1.53, 6.85, and 4.48, respectively.

Effect of Operating Conditions at Maximum Injection Rate

The effects of flight conditions, ambient-air temperature and relative humidity, compressor pressure ratio, and inlet-diffuser efficiency on thrust and augmentation are shown in figure 4. The augmented thrust in this figure is defined as the maximum amount obtainable, that is, the air is saturated at the compressor inlet and saturation is maintained during the entire compression process.

Effect of flight Mach number and altitude. - The normal and augmented thrust per unit turbine-nozzle area and the thrust ratio are shown in figure 4(a) as a function of flight Mach numbers from 0 to 2.0 at sea level and at 35,300 feet. The normal thrust at sea level is a maximum at a flight Mach number of 0. As the flight Mach number is increased, the normal thrust first decreases to a flight Mach number of about 0.7 then increases slightly to a flight Mach number of 1.5 after which it again decreases. This variation in thrust is the result of the combined effects of inlet-air momentum, inlet-diffuser-efficiency assumptions, and a constant turbine-inlet temperature. If more optimistic values of inlet-diffuser efficiency had been assumed, the thrust at high flight speeds would be considerably greater. The effect of inlet-diffuser efficiency on thrust and augmentation is discussed later. The augmented thrust at sea level decreases slightly as flight Mach number is increased from 0 to about 0.5 after which it increases with increased flight Mach number. Augmented thrust at an altitude of 35,300 feet increases as flight Mach number is increased, whereas normal thrust remains essentially constant for the range of flight Mach numbers shown. The thrust ratio at both altitudes increases at an increasing rate as the flight Mach number is increased. At sea level for a compressor efficiency equal to $0.80-\Delta X_c$, the thrust ratio is 1.29 for a flight Mach number of 0 and increases with increasing flight Mach number, reaching a value of 1.64 at a flight Mach number of 1.0 and 3.05 at a flight Mach number of 2.0. At an altitude of 35,300 feet, the thrust ratio increases from 1.14 to 2.00 as the flight Mach number is increased from 0.25 to 2.0. The percentage increase in thrust [100 (thrust ratio-1)] at 35,300 feet is approximately one-half of the percentage increase in thrust at sea level for the range of flight Mach numbers shown. Also shown in figure 4(a) is the thrust ratio at sea level for a constant compressor efficiency of 0.80. If no decrease in compressor efficiency with water injection is assumed, a marked increase in the thrust ratio is obtained. Additional calculations made for a constant compressor efficiency of 0.85 gave thrust ratios approximately equal to those for a constant efficiency of 0.80, showing that the magnitude of compressor efficiency has little effect on thrust augmentation if the efficiency for the normal and the augmented case are assumed to be the same.

Effect of ambient-air temperature and ambient relative humidity. - The effects of ambient-air temperature and ambient relative humidity on normal thrust, augmented thrust, and thrust ratio for sea-level altitude and flight Mach numbers of 0 and 0.85 are shown in figure 4(b). Normal and augmented thrust per unit turbine-nozzle area are plotted as functions of ambient-air temperature from

492° to 610° R for inlet relative humidities of 0, 0.50, and 1.00 and sea-level ambient pressure. The decrease in the normal thrust with increased ambient relative humidity at high ambient-air temperatures is due to the change in thermodynamic properties of the working fluid when large amounts of water vapor are present. Both normal and augmented thrust decrease as ambient-air temperature is increased for both flight Mach numbers shown. This decrease in thrust is due to the lower compressor pressure ratios obtained at high ambient-air temperatures. The decrease in augmented thrust with increasing ambient-air temperature is greater for high ambient relative humidities than for low humidities because the amount of evaporative cooling before compression for low ambient relative humidities increases as the ambient-air temperature increases, whereas this cooling is unavailable at high ambient relative humidities. At a flight Mach number of 0, the ambient-air temperature has little effect on the thrust ratio for an ambient relative humidity of 1.00. For an ambient relative humidity of 0.50, the thrust ratio increases slightly with an increase in ambient-air temperature, and for zero ambient relative humidity a marked increase in the thrust ratio occurs with increased ambient-air temperature. At a flight Mach number of 0 with an ambient-air temperature of 492° R, the thrust ratio decreases from 1.28 to 1.22 as the ambient relative humidity increases from 0 to 1.00. At 610° R, the decrease in thrust ratio is from 1.67 to 1.21 as the ambient relative humidity increases from 0 to 1.00. For a sea-level flight Mach number of 0.85, the thrust ratio increases with increased ambient-air temperature for all ambient relative humidities except for a humidity of 1.00 above an ambient-air temperature of about 570° R where the thrust ratio decreases with increasing ambient-air temperature.

Effect of normal compressor pressure ratio. - Normal thrust and augmented thrust per unit turbine-nozzle area as a function of the normal (unaugmented) compressor pressure ratio for sea-level and zero flight Mach number conditions are shown in figure 4(c). The values of augmented thrust for any given normal compressor pressure ratio were obtained by using the same work input per unit mass flow in the compressor during the saturated compression as was used for calculating the normal thrust. The normal thrust increases at a decreasing rate as the compressor pressure ratio is increased. The augmented thrust increases approximately linearly with increased normal compressor pressure ratio. The ratios of augmented to normal thrust, liquid flow, and specific liquid consumption for the same range of normal compressor pressure ratios are also shown in figure 4(c). Both the thrust ratio and liquid-flow ratio increase approximately linearly with increasing normal compressor pressure ratio.

The ratio of augmented to normal specific liquid consumption increases with increasing normal compressor pressure ratio, but at a decreasing rate. The thrust ratio for maximum augmentation increases from 1.08 to 2.36 as the normal compressor pressure ratio is increased from 2 to 20. The corresponding ratio of augmented to normal liquid flow increases from 2.2 to 23.6 and the specific-liquid-consumption ratio increases from 2.0 to 10.0. The increase in thrust ratio with increased normal compressor pressure ratio is due to the greater amount of evaporative cooling possible at the higher temperatures associated with high compressor pressure ratios and to the leveling-off of the normal thrust at high compressor pressure ratios caused by the limitation of the turbine-inlet temperature to 2000° R.

Effect of inlet-diffuser efficiency. - At high flight Mach numbers, the inlet-diffuser efficiency has a marked effect on compressor-inlet pressure and therefore on thrust. Figure 4(d) shows normal and augmented thrust for inlet-diffuser adiabatic efficiencies of 0.60, 0.80, and 1.00 at sea level and 35,300 feet plotted as a function of flight Mach numbers from 0 to 2.0; also shown is the thrust ratio for these conditions. The thrust ratio increases more rapidly with increased flight Mach number for low inlet-diffuser efficiencies than for high efficiencies at both altitudes shown. At sea level, the thrust ratio is 1.29 for a flight Mach number of 0 and as the flight Mach number is increased to 2.0 the thrust ratio increases to 4.10 for an inlet-diffuser efficiency of 0.60 and to 2.42 for an inlet-diffuser efficiency of 1.00. This increase in thrust ratio at low inlet-diffuser efficiencies over high diffuser efficiencies does not indicate an increase in augmented thrust, but is due instead to the decrease in the normal thrust with decreased diffuser efficiency. The thrust curves in figure 4(d) show that both the normal thrust and the increase in thrust are greater for the high inlet-diffuser efficiencies.

SUMMARY OF RESULTS

The following results were obtained from a theoretical analysis of the thrust augmentation obtained by evaporation of water during compression in a typical turbojet engine having a normal compressor pressure ratio of 4.61 at sea-level altitude and zero flight Mach number when a nominal decrease in compressor efficiency with water injection was assumed:

1. Increasing the amount of evaporated water for constant inlet conditions increased the ratio of augmented to normal thrust, the ratio of augmented to normal liquid flow, and the ratio of augmented

to normal specific liquid consumption. Increasing the amount of water injected at sea level and flight Mach number of 0 from that required for saturation at the compressor inlet to saturation at the compressor outlet increased the thrust ratio from 1.035 to 1.29, the liquid-flow ratio from 1.18 to 5.01, and the specific-liquid-consumption ratio from 1.13 to 3.88.

2. Increasing the flight Mach number to 1.5 at sea level with saturation up to the compressor outlet increased the thrust ratio to 2.07 with a liquid-flow ratio of 10.66 and a specific-liquid-consumption ratio of 5.15.

3. At an altitude of 35,300 feet, the percentage increase in thrust [100(thrust ratio - 1)] was approximately one-half of that for sea level at any given flight Mach number.

4. An increase in ambient-air temperature at sea level caused a decrease in both normal and maximum augmented thrust but the ratio of augmented to normal thrust increased for atmospheric relative humidities of 0.50 and lower. The thrust ratio was greater for low ambient relative humidities than for high ambient relative humidities, especially at high ambient-air temperatures.

5. As the normal compressor pressure ratio was increased, the ratio of augmented thrust to normal thrust increased almost linearly. For sea level zero Mach number conditions, the thrust ratio for maximum augmentation increased from 1.08 to 2.36 as the normal compressor pressure ratio was increased from 2 to 20, the corresponding ratio of augmented to normal liquid flow increased from 2.2 to 23.6 and the specific-liquid-consumption ratio increased from 2.0 to 10.0.

6. Both normal and augmented thrust decreased as inlet-diffuser efficiency decreased. The ratio of augmented thrust to normal thrust, however, increased more rapidly with increasing flight Mach number for low inlet-diffuser efficiencies than for high inlet-diffuser efficiencies. At sea level, the thrust ratio for an inlet-diffuser efficiency of 0.60 increased from 1.29 to 4.10 as the flight Mach number was increased from 0 to 2.0. For an inlet-diffuser efficiency of 1.00, the thrust ratio increased from 1.29 to 2.42 for the same increase in flight speed.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, November 21, 1949.

APPENDIX A

Symbols

The following symbols are used in the calculations and the figures:

A	area, sq in.
c_p	specific heat at constant pressure, Btu/(lb)(°R)
F	net thrust, lb
f/a	fuel-air ratio, lb fuel/lb air
g	acceleration due to gravity, 32.2 ft/sec^2
H	enthalpy, Btu/lb
H_{corr}	corrected enthalpy for a saturated mixture of air and water vapor plus liquid water not at 519° R, Btu/lb air
ΔH	enthalpy rise, Btu/lb
h_a	enthalpy of air as given in reference 4, Btu/lb
h_f	enthalpy of liquid water as given in reference 5, Btu/lb
h_g	enthalpy of water vapor as given in reference 5, Btu/lb
J	mechanical equivalent of heat, 778 ft-lb/Btu
K	compressor slip factor
M	flight Mach number
P	total pressure, lb/sq in. absolute
p	static or partial pressure, lb/sq in. absolute
q	ambient relative humidity
R	gas constant, ft-lb/(lb)(°R)
S	entropy, Btu/(lb)(°R)
S_{corr}	corrected entropy for saturated mixture of air and water vapor plus liquid water not at 519° R, Btu/(lb air)(°R)

s_f entropy of liquid water as given in reference 5, Btu/(lb)($^{\circ}$R)

s_g entropy of water vapor as given in reference 5, Btu/(lb)($^{\circ}$R)

T total temperature, $^{\circ}$R

ΔT total-temperature rise, $^{\circ}$R

t static temperature, $^{\circ}$R

Δt static-temperature rise, $^{\circ}$R

V flight velocity, ft/sec

V_j jet velocity, ft/sec

V_T compressor-rotor tip velocity, ft/sec

v specific volume, cu ft/lb

v_g specific volume of water vapor as given in reference 5, cu ft/lb

W mass flow, slug/sec

X ratio of weight of water vapor to weight of air in mixture, lb water/lb air

ΔX total amount of injected liquid water, lb water/lb air

γ ratio of specific heats

η adiabatic efficiency

θ $t_0/519$

ϕ_a $\int c_p dt/t$ for air, as given in reference 4, Btu/(lb)($^{\circ}$R)

Subscripts:

a air

b mixture of air and products of combustion

c compressor

d diffuser

e	exhaust gas plus water vapor
f	liquid water
g	water vapor
i	ideal (isentropic)
j	jet
m	air - water vapor mixture
n	exhaust nozzle
T	compressor-rotor tip
t	turbine
0	ambient air
1	compressor inlet
2	end of portion of compression during which air remains saturated and after which no liquid water is present
3	compressor outlet
4	turbine inlet
5	turbine outlet
6	exhaust-nozzle outlet
(-)	between stations enclosed

Primed symbols are approximate values.

APPENDIX B

DERIVATION OF EQUATIONS FOR MOLLIER DIAGRAM

In a saturated mixture of air and water vapor, the enthalpy per pound of air is

$$H = H_a + XH_g \tag{B1}$$

and the entropy per pound of air is

$$S = S_a + XS_g \tag{B2}$$

For a compression of 1 pound of air saturated with water vapor with excess water present at the start of compression and for a saturated condition with no excess liquid at the end of compression, the enthalpy at the end must equal the enthalpy at the start plus the work of compression added. Thus

$$H_{a,1} + X_1H_{g,1} + (X_2 - X_1)\ H_{f,1} + \Delta H_c = H_{a,2} + X_2H_{g,2} \tag{B3}$$

For ideal conditions the entropy remains constant therefore

$$S_{a,1} + X_1S_{g,1} + (X_2 - X_1)\ S_{f,1} = S_{a,2} + X_2S_{g,2} \tag{B4}$$

If the enthalpy and the entropy of the excess liquid water are assigned values of zero at a particular temperature and the liquid water is introduced at this temperature, equations (B3) and (B4) become

$$H_{a,1} + X_1H_{g,1} + \Delta H_c = H_{a,2} + X_2H_{g,2} \tag{B5}$$

and

$$S_{a,1} + X_1S_{g,1} = S_{a,2} + X_2S_{g,2} \tag{B6}$$

If equations (B1) and (B2) are plotted for a range of temperatures and pressures, the conditions of the mixture resulting from a compression of 1 pound of air that is continually saturated with water vapor may be determined. Equations (B1) and (B2) are the basis for the Mollier diagram presented. The base temperature for enthalpy and entropy of liquid water was chosen as 519° R. Thus, if any liquid water present is assumed to be at 519° R, no correction need be made for the enthalpy and the entropy of the liquid when using the Mollier diagram. The values of enthalpy and entropy were so adjusted that at standard sea-level conditions of temperature of 519° R and pressure of 14.696 pounds per square inch the enthalpy of the saturated mixture of air and water vapor is 100 Btu per pound of air and the entropy is 0.10 Btu per pound of air per °R. The modified versions of equations (B1) and (B2), which were used in plotting the Mollier diagram, are

$$H = h_a + 60.24 + X(h_g - 27.06) \tag{B7}$$

and

$$S = \phi_a - \frac{R_a}{J} \log_e \frac{(p_m - p_g)}{18.189} + X(s_g - 0.0536) \tag{B8}$$

Values for h_a and ϕ_a were obtained from reference 4 and are for a base temperature of 400° R; values for h_g and s_g were obtained from reference 5 and are for a base temperature of 32° F (492° R).

The weight ratio of water vapor to air in a saturated mixture may be determined by the relation

$$X = \frac{v_a}{v_g} \tag{B9}$$

When the equation of state and Dalton's law of partial pressures are used, equation (B9) may be written

$$X = \frac{R_a t_a}{(p_m - p_g)(144\ v_g)} \tag{B10}$$

Equation (B10) was used for determining values of X for equations (B7) and (B8).

If liquid water is injected at any temperature other than 519° R, the enthalpy and the entropy of the water will not be zero and an error will be introduced when using the Mollier diagram. This error can be corrected by adjusting the initial enthalpy and entropy, as read from the diagram, to include the enthalpy and the entropy of the injected liquid water. The corrected enthalpy and entropy for the initial condition will be

$$H_{corr} = H + (h_f - 27.06)\ \Delta X \tag{B11}$$

and

$$S_{corr} = S + (s_f - 0.0536)\ \Delta X \tag{B12}$$

A similar correction, made by subtracting the enthalpy and the entropy of the liquid, can be performed at the end of the process if any liquid water remains. Values of H_f, which is $h_f - 27.06$, and S_f, which is $s_f - 0.0536$, are plotted in an insert on the Mollier diagram for various temperatures.

If liquid water is injected at a different temperature than the initial temperature of the saturated air, a slight increase in entropy occurs because of the mixing of the constituents as a new temperature equilibrium is reached. This increase in entropy, however, is very small and difficult to calculate, requiring many trial-and-error calculations, and may be neglected without introducing appreciable error when using the Mollier diagram.

APPENDIX C

PROPERTIES OF AIR - WATER VAPOR AND EXHAUST GAS - WATER VAPOR MIXTURES

Air - Water Vapor Mixtures

If less water is injected at the compressor inlet than is necessary to keep the air saturated for the complete compression process, the final condition of the mixture after compression may be found by using the Mollier diagram to evaluate the conditions of the mixture at the end of the portion of compression during which evaporation occurs and then calculating the rest of the compression process by adiabatic temperature and pressure relations.

In order to evaluate the compression of the mixture after evaporation has been completed, knowledge of the thermodynamic properties of the mixture is necessary. Because these properties are not only functions of the proportion of air and water vapor present in the mixture but also of temperature and pressure, they are very difficult to ascertain. Average values of these properties that are reasonably accurate for purposes of calculating the compression process may be obtained, however, if the values of specific heat at constant pressure for air and water vapor are taken at an arbitrary average temperature of 600° R and a pressure of zero and values of the gas constants R_a and R_g of 53.35 and 85.78 foot-pounds per pound per °R, respectively, are used.

The change in thermodynamic properties of a water vapor - air mixture due to the change in the amount of water present is much greater than changes due to varying pressure and temperature. Although the specific heat at constant pressure changes slightly as the temperature varies, the range of temperatures encountered in aircraft-engine compressors is limited so that a constant value may be used with small error. This error is reduced to a great extent in calculating compressor work because any change in value of c_p is accompanied by a corresponding effect on the value of γ, which nearly compensates for the change in c_p if R is constant. The specific heat at constant pressure also varies only slightly in the range of temperature being considered as the pressure is increased from zero. The effects of temperature and pressure on the specific heat of air are discussed in reference 6.

The gas constant R_m for the compressor per pound of mixture is

$$R_m = R_a \left(\frac{1}{1+X}\right) + R_g \left(\frac{X}{1+X}\right) \tag{C1}$$

or

$$R_m = 53.35 \left(\frac{1}{1+X}\right) + 85.78 \left(\frac{X}{1+X}\right) \text{ ft-lb/(lb)(}^{\circ}\text{R)} \tag{C2}$$

The value of $c_{p,a}$ at 600° R from reference 4 is

$$c_{p,a} = 0.2406 \text{ Btu/(lb)(}^{\circ}\text{R)}$$

The value of $c_{p,g}$ at 600° R from reference 5 is

$$c_{p,g} = 0.4461 \text{ Btu/(lb)(}^{\circ}\text{R)}$$

The specific heat at constant pressure for the mixture is

$$c_{p,m} = c_{p,a} \left(\frac{1}{1+X}\right) + c_{p,g} \left(\frac{X}{1+X}\right) \tag{C3}$$

or

$$c_{p,m} = 0.2406 \left(\frac{1}{1+X}\right) + 0.4461 \left(\frac{X}{1+X}\right) \text{ Btu/(lb)(}^{\circ}\text{R)} \tag{C4}$$

The value of γ_m for the mixture may be found by means of the relation

$$\gamma_m = \frac{1}{1 - \frac{R_m}{Jc_{p,m}}} \tag{C5}$$

Values of R_m, $c_{p,m}$, and γ_m are plotted in figure 2(a) for a range of values of X_2.

Exhaust Gas - Water Vapor Mixtures

In order to facilitate turbine and exhaust-gas calculations, figure 2(b) was prepared to give the thermodynamic properties of various exhaust gas - water vapor mixtures. The equations used to calculate figure 2(b) are

$$R_e = R_b \left(\frac{1 + f/a}{1 + X + f/a}\right) + R_g \left(\frac{X}{1 + X + f/a}\right)$$

$$c_{p,e} = c_{p,b} \left(\frac{1 + f/a}{1 + X + f/a}\right) + c_{p,g} \left(\frac{X}{1 + X + f/a}\right)$$

and

$$\gamma_e = \frac{1}{1 - \frac{R_e}{Jc_{p,e}}}$$

A temperature of 1650° R was chosen as an average temperature for the expansion process and all values were taken at this temperature. Values of R_b and $c_{p,b}$ at a hydrogen-carbon ratio of 0.175, and $c_{p,g}$ were obtained from reference 7. A value of R_g of 85.78 foot-pounds per pound per °R was used.

APPENDIX D

SAMPLE CALCULATIONS

The use of the Mollier diagram in calculating a saturated compression process is illustrated by the following sample calculations.

Sample Calculation I

Saturated compressor-inlet air, and sufficient water injected at 519° R to keep air saturated at all times, and the following compressor conditions are assumed:

Inlet total temperature, T_1, °R 519
Inlet total pressure, P_1, lb/sq in. absolute 14.7
Enthalpy rise, ΔH_c, Btu/lb air 80
Adiabatic efficiency, η_c 0.80

Find P_2, T_2, X_2, and the amount of water evaporated during compression.

From the Mollier diagram at a temperature of 519° R and a pressure of 14.7 pounds per square inch, values of H, S, and X are

$$H_1 = 100 \text{ Btu/lb air}$$

$$S_1 = 0.10 \text{ Btu/lb air}$$

$$X_1 = 0.0107$$

By increasing the enthalpy at constant entropy by an amount equal to the ideal work of compression, the actual pressure at the compressor outlet may be read from the diagram. The ideal work of compression is

$$\Delta H_{c,i} = \eta_c \Delta H_c = 0.80 \times 80 = 64 \text{ Btu/lb air}$$

and

$$H_{2,i} = H_1 + \Delta H_{c,i} = 100 + 64 = 164 \text{ Btu/lb air}$$

The value of pressure read from the diagram at an enthalpy of 164 Btu per pound air and an entropy of 0.10 Btu per pound air per $^{\circ}$R is

$$P_2 = 70.7 \text{ lb/sq in.}$$

The enthalpy at the compressor outlet is equal to the enthalpy at the inlet plus the actual enthalpy change or

$$H_2 = H_1 + \Delta H_c = 100 + 80 = 180 \text{ Btu/lb air}$$

From the Mollier diagram at a pressure of 70.7 pounds per square inch and an enthalpy of 180 Btu per pound air, the values of temperature and water-air ratio at the compressor outlet are

$$T_2 = 630^{\circ} \text{ R} \quad \text{and} \quad X_2 = 0.0583$$

The amount of water evaporated during compression is

$$X_2 - X_1 = 0.0583 - 0.0107 = 0.0476 \text{ lb water/lb air}$$

Sample Calculation II

In calculating a compression process for an actual engine, the work input to the compressor is usually obtained on the basis of 1 pound of working fluid passing through the compressor. In order to use the Mollier diagram for a saturated compression process, the work input must be found on a basis of 1 pound of air (the unit on which the Mollier diagram is based). Corrections must also be made for the enthalpy and the entropy of the liquid water when the water injected is not at a temperature of 519° R if it is desired to maintain a high degree of accuracy in using the Mollier diagram. In order to illustrate these refinements, the following example is presented for an engine with a single-stage centrifugal compressor. The following conditions are assumed:

Ambient static temperature, t_0, $^{\circ}$R	519
Ambient static pressure, p_0, lb/sq in.	14.7
Flight Mach number, M	0.85
Flight velocity, V, ft/sec	949
Diffuser adiabatic efficiency, η_d	0.85
Ambient relative humidity, q_0	0.50
Compressor-rotor tip velocity, V_T, ft/sec	1500
Compressor slip factor, K	0.95
Compressor adiabatic efficiency, η_c	0.80

and that enough liquid water is injected at the compressor inlet at a temperature of 540° R to saturate the air at this point and to keep the air saturated during the entire compression process. Find P_2, T_2, X_2, and the amount of water evaporated. The water-air ratio for saturated conditions at an ambient temperature of 519° R and a pressure of 14.7 pounds per square inch is, from the Mollier diagram,

$$X = 0.0107$$

For a relative humidity of 0.50, the value of X for the ambient air is

$$X_0 = \frac{0.0107}{2} = 0.00535$$

The static-temperature rise in the inlet diffuser due to ram is (value of $c_{p,m}$ obtained from fig. 2(a))

$$\Delta t_{(0-1)} = \frac{V_0^{\,2}}{2gJc_{p,m}} = \frac{949^2}{2 \times 32.2 \times 778 \times 0.2416} = 74.4^\circ \text{ R}$$

and

$$T_1 = 519 + 74.4 = 593^\circ \text{ R}$$

When equation (B7) is evaluated for 593° R, the enthalpy at the compressor inlet without taking into account the enthalpy of the liquid water that is injected at this point is

$$\begin{aligned} H_1' &= h_a + 60.24 + X(h_g - 27.06) \\ &= 46.31 + 60.24 + 0.00535\ (1119.1 - 27.06) \\ &= 112.5 \text{ Btu/lb air} \end{aligned}$$

The compressor-inlet pressure is (γ_m obtained from fig. 2)

$$\begin{aligned} P_1 &= p_0 \left[\frac{\eta_d \Delta t_{(0-1)}}{t_0} + 1\right]^{\frac{\gamma_m}{\gamma_m - 1}} \\ &= 14.7 \left(\frac{0.85 \times 74.4}{519} + 1\right)^{\frac{1.398}{0.398}} \\ &= 22.0 \text{ lb/sq in.} \end{aligned}$$

When liquid water is injected at the compressor inlet, the temperature decreases but the pressure remains constant as does the enthalpy if the enthalpy of the injected water is neglected. From the Mollier diagram at an enthalpy of 112.5 Btu per pound air and a pressure of 22.0 pounds per square inch, the entropy at the compressor inlet without considering the entropy of the liquid water is

$$S_1' = 0.0953 \text{ Btu/(lb air)}(^{\circ}\text{R})$$

This point on the Mollier diagram could also have been determined from P_1 and T_1 where the value of T_1 is obtained by the method given in reference 1.

In order to obtain the enthalpy of the mixture in Btu per pound air at the compressor inlet, correction must be made for the enthalpy of the liquid water that is injected at 540° R, which requires that the amount of water injected for saturation at the compressor outlet be known. Because X_2 is unknown, an approximate value must be assumed. The following approximation was obtained by inspection of calculations for a wide range of ambient-air temperatures, flight Mach numbers, and compressor pressure ratios at sea-level pressure and may be used to determine X_2' in order to eliminate or to simplify successive approximation methods:

$$X_2' = X_0 + 0.00065\ \Delta H_{m,(0-2)}\theta^2$$

The enthalpy rise in the compressor is

$$\Delta H_{m,(1-2)} = \frac{KV_T^2}{gJ}$$

$$= \frac{0.95 \times 1500^2}{32.2 \times 778}$$

$$= 85.4 \text{ Btu/lb mixture}$$

and the enthalpy rise in the inlet diffuser is

$$\Delta H_{(0-1)} = c_{p,d}\Delta t_{(0-1)}(1 + X_0) = 0.2416 \times 74.4 \times 1.00535$$

$$= 18.1 \text{ Btu/lb air}$$

Therefore

$$X_2' = 0.00535 + 0.00065(18.1 + 85.4) = 0.0727$$

and

$$\Delta X'_{(0-2)} = 0.0727 - 0.00535 = 0.06735$$

These values will be used for X_2 and $\Delta X_{(0-2)}$ for the rest of the calculations.

From the insert on the Mollier diagram, the enthalpy and the entropy of the liquid water at 540° R are

$$H_f = 21.0 \text{ Btu/lb water}$$

and

$$S_f = 0.04 \text{ Btu/(lb water)(}^{\circ}\text{R)}$$

The enthalpy and the entropy due to the liquid water are then

$$H_{f,1} = \Delta X_{(0-2)} H_f = 0.06735 \times 21.0 = 1.4 \text{ Btu/lb air}$$

and

$$S_{f,1} = \Delta X_{(0-2)} S_f = 0.06735 \times 0.04 = 0.0027 \text{ Btu/(lb air)(}^{\circ}\text{R)}$$

The values for enthalpy and entropy at the compressor inlet corrected for the liquid water at a temperature of 540° R are

$$H_{1,corr} = H_1' + H_{f,1} = 112.5 + 1.4 = 113.9 \text{ Btu/lb air}$$

and

$$S_{1,corr} = S_1' + S_{f,1} = 0.0953 + 0.0027 = 0.0980 \text{ Btu/(lb air)(}^{\circ}\text{R)}$$

When the enthalpy rise per pound of mixture in the compressor is known, it must be converted to enthalpy rise per pound of air in order to use the Mollier diagram. This conversion also requires a value for the water-air ratio at the compressor outlet. The ideal enthalpy rise per pound of air in the compressor is

$$\Delta H_{i,(1-2)} = \Delta H_{m,(1-2)}(1 + X_2)\ \eta_c = 85.4 \times 1.0727 \times 0.80$$

$$= 73.3 \text{ Btu/lb air}$$

and the final enthalpy for an isentropic compression is

$$H_{2,i} = H_{1,corr} + \Delta H_{i,(1-2)} = 113.9 + 73.3 = 187.2 \text{ Btu/lb air}$$

The compressor-outlet pressure P_2 read from the Mollier diagram at an enthalpy of 187.2 Btu per pound of air and an entropy of 0.0980 Btu per pound of air per $^{\circ}R$ is

$$P_2 = 118.7 \text{ lb/sq in.}$$

The actual enthalpy per pound air at the compressor outlet is

$$H_2 = H_1 + \Delta H_{m,(1-2)}(1 + X_2) = 113.9 + (85.4 \times 1.0727) = 205.5 \text{ Btu/lb air}$$

When the Mollier diagram is read at an enthalpy of 205.5 Btu per pound air and a pressure of 118.7 pounds per square inch the temperature is

$$T_2 = 664^{\circ} \text{ R}$$

and

$$X_2 = 0.0732$$

This value of X_2 is very close to the approximate value that was used for X_2 and therefore no further correction is necessary. If this value of X_2 differed greatly from the assumed approximate value, it would be advisable to repeat the calculations using the value of X_2 just obtained.

The amount of water injected at the compressor inlet is

$$\Delta X_{(0-2)} = 0.0732 - 0.00535 = 0.06785 \text{ lb water/lb air}$$

In order to show the effect of corrections for liquid-water temperatures other than 519° R, the following table presents values of P_2, T_2, and X_2, which were obtained for liquid-water injection temperatures of 519° R (no correction), 540° R (the sample calculation just completed), and for 620° R; all other conditions remained the same as in the sample calculation:

Temperature of injected water ($^\circ$R)	P_2 (lb/sq in.)	T_2 ($^\circ$R)	X_2
519	119.2	663	0.0722
540	118.7	664	.0732
620	118.0	666	.0786

This table shows that for most limits of accuracy the enthalpy and the entropy of the liquid water may be neglected if the temperature is not too far from 519° R.

SAMPLE CALCULATION III

The following example illustrates the calculation of compressor-outlet conditions when a desired amount of water is injected at the compressor inlet.

The following conditions are assumed:

Ambient static temperature, t_0, $^\circ$R 519
Ambient static pressure, p_0, lb/sq in. 14.7
Flight velocity, V, ft/sec 0
Ambient relative humidity, q_0 . . . . 0.50
Compressor adiabatic efficiency, η_c 0.80
Compressor enthalpy rise, ΔH_c, Btu/lb mixture 85.4
Air weight flow, W_a, slug/sec 2.50
Water-injection rate at 519° R, lb/sec 4.0

From Sample Calculation II,

$$X_0 = 0.0053$$

and by evaluating equation (B7) for 519° R

$$H_0 = h_a + 60.24 \quad X(h_g - 27.06)$$

$$= 28.53 + 60.24 + 0.00535(1087.6 - 27.06)$$

$$= 94.4 \text{ Btu/lb air}$$

From the Mollier diagram at an enthalpy of 94.4 Btu per pound air and a pressure of 14.7 pounds per square inch, the entropy is S_0 = 0.0890 Btu/(lb air)($^\circ$R).

The change in water-air ratio ΔX due to the addition of the liquid water is

$$\Delta X_{(0-2)} = \frac{4}{2.50 \times 32.2} = 0.0497$$

and

$$X_2 = X_0 + \Delta X_{(0-2)} = 0.00535 + 0.0497 = 0.0551$$

If the approximation from Sample Calculation II is used,

$$\Delta H'_{m(0-2)} = \frac{\Delta X_{(0-2)}}{0.00065\ \theta^2} = \frac{0.0497}{0.00065 \times 1} = 76.4 \text{ Btu/lb mixture}$$

Using this approximate value as $\Delta H_{m(0-2)}$,

$$\Delta H_{(0-2)} = \Delta H_{m(0-2)}\ (1 + X_2) = 76.4 \times 1.0551 = 80.6 \text{ Btu/lb air}$$

Then

$$H_2 = H_0 + \Delta H_{(0-2)} = 94.4 + 80.6 = 175.0 \text{ Btu/lb air}$$

and

$$H_{2,i} = H_0 + \Delta H_{(0-2)}\eta_c = 94.4 + (80.6 \times 0.80) = 158.9 \text{ Btu/lb air}$$

From the Mollier diagram at an enthalpy of 158.9 Btu per pound and an entropy of 0.0890 Btu per pound per $^{\circ}$R,

$$P_2 = 73.3 \text{ lb/sq in.}$$

From the Mollier diagram at a pressure of 73.3 pounds per square inch and an enthalpy of 175.0 Btu per pound air,

$$X_2 = 0.0542$$

This value of X_2 is lower than the desired value. If the value of $\Delta H'_{m(0-2)}$ is increased to 77.9 Btu per pound air and the calculation repeated, the values obtained are

$$P_2 = 75.3 \text{ lb/sq in.}$$

$$X_2 = 0.0551$$

and

$$T_2 = 631^{\circ} \text{ R}$$

The rest of the compression process may be calculated as an adiabatic compression of the mixture of air and water vapor using the thermodynamic properties given in figure 2(a). From figure 2(a) at $X = 0.0551$

$$c_{p,m} = 0.2513 \text{ Btu/(lb)(}^{\circ}\text{R)}$$

and

$$\gamma_m = 1.3916$$

Then

$$\Delta T_{(2-3)} = \frac{\Delta H_{m(2-3)}}{c_{p,m}} = \frac{(85.4-77.9)}{0.2513} = 29.8^{\circ} \text{ R}$$

and

$$\frac{P_3}{P_2} = \left[\left(\frac{\Delta T_{(2-3)}}{T_2}\right) \eta_c + 1\right]^{\frac{\gamma_m}{\gamma_m - 1}} = \left[\left(\frac{29.8}{631}\right) 0.80 + 1\right]^{\frac{1.3916}{0.3916}} = 1.141$$

The conditions at the compressor outlet are

$$P_3 = \left(\frac{P_3}{P_2}\right) P_2 = 1.141 \times 75.3 = 85.9 \text{ lb/sq in.}$$

$$T_3 = T_2 + \Delta T_{(2-3)} = 631 + 29.8 = 661\ ^{\circ}\text{R}$$

and

$$X_3 = X_2 = 0.0551$$

For a comparison of the effect of the amount of water evaporated on pressure and temperature at the compressor outlet, the results of Sample Calculation II (air saturated up to the compressor outlet for a water-injection temperature of 519° R), and for the cases of air saturated during one-half of the work input to the compressor, saturated up to the compressor inlet, and for no injection are shown in the following table:

($M = 0.85$; $\Delta H_c = 85.4$ Btu/lb mixture; $\eta_c = 0.80$)

Condition	Amount of water evaporated (lb water/lb air)	P_3 (lb/sq in.)	T_3 (^{o}R)
No injection	0	86.5	947
Air saturated up to compressor inlet	0.0113	95.4	895
Air saturated through one-half of work input	0.0384	108.0	778
Air saturated up to compressor outlet	0.0668	119.2	663

APPENDIX E

EQUATIONS USED IN AUGMENTATION CALCULATIONS

The method of calculating compressor-outlet conditions when water is evaporated during compression is given in the section DISCUSSION OF CHARTS and illustrated in Appendix D. The other equations used for the step-by-step calculations of engine performance for figures 3 and 4 are:

Inlet diffuser,

$$\frac{P_1}{p_0} = \left(\frac{\eta_d V^2}{t_0 2Jgc_{p,m}} + 1\right)^{\frac{\gamma_m}{\gamma_m - 1}}$$

Compressor (for normal-thrust calculations),

$$\frac{P_3}{P_1} = \left(\frac{\eta_c K V_T^{\ 2}}{T_1 gJc_{p,m}} + 1\right)^{\frac{\gamma_m}{\gamma_m - 1}}$$

Turbine,

$$\frac{P_5}{P_4} = \left[1 - \frac{\Delta H_{m,c}(1+X_3)}{\eta_t T_4 (1+X_3+f/a) c_{p,e}}\right]^{\frac{\gamma_e}{\gamma_e - 1}}$$

Thrust (for sonic jet velocity),

$$\frac{F}{A_t} = \frac{V_j W_t}{A_t} + \frac{A_n}{A_t}(p_6 - p_0) - \frac{V W_a}{A_t}(1+X_0)$$

where

$$V_j = \sqrt{g\gamma_e R_e t_6}$$

$$t_6 = T_5\left(\frac{2}{\gamma_e + 1}\right)$$

$$\frac{W_t}{A_t} = \left(\frac{P_4}{\sqrt{T_4 R_e g/\gamma_e}}\right)\left(\frac{2}{1+\gamma_e}\right)^{\frac{\gamma_e+1}{2(\gamma_e-1)}}$$

$$\frac{A_n}{A_t} = \frac{W_t}{A_t}\,\frac{gR_e t_6}{V_j p_6}$$

$$\frac{p_6}{P_5} = \left[1 - \left(\frac{\gamma_e-1}{\gamma_e+1}\right)\frac{1}{\eta_n}\right]^{\frac{\gamma_e}{\gamma_e-1}}$$

and

$$\frac{W_a}{A_t} = \frac{W_t}{A_t}\left(\frac{1}{1+X_3+f/a}\right)$$

Thrust (for subsonic jet velocity),

$$\frac{F}{A_t} = V_j\,\frac{W_t}{A_t} - \frac{V\,W_a}{A_t}\,(1+X_0)$$

where

$$V_j = \sqrt{\eta_n 2gR_e\left(\frac{\gamma_e}{\gamma_e-1}\right)T_5\left[1 - \left(\frac{p_0}{P_5}\right)^{\frac{\gamma_e-1}{\gamma_e}}\right]}$$

Values for $c_{p,m}$ and γ_m were obtained from figure 2(a) and values of $c_{p,e}$, R_e, and γ_e from figure 2(b).

Values for f/a were obtained from reference 8.

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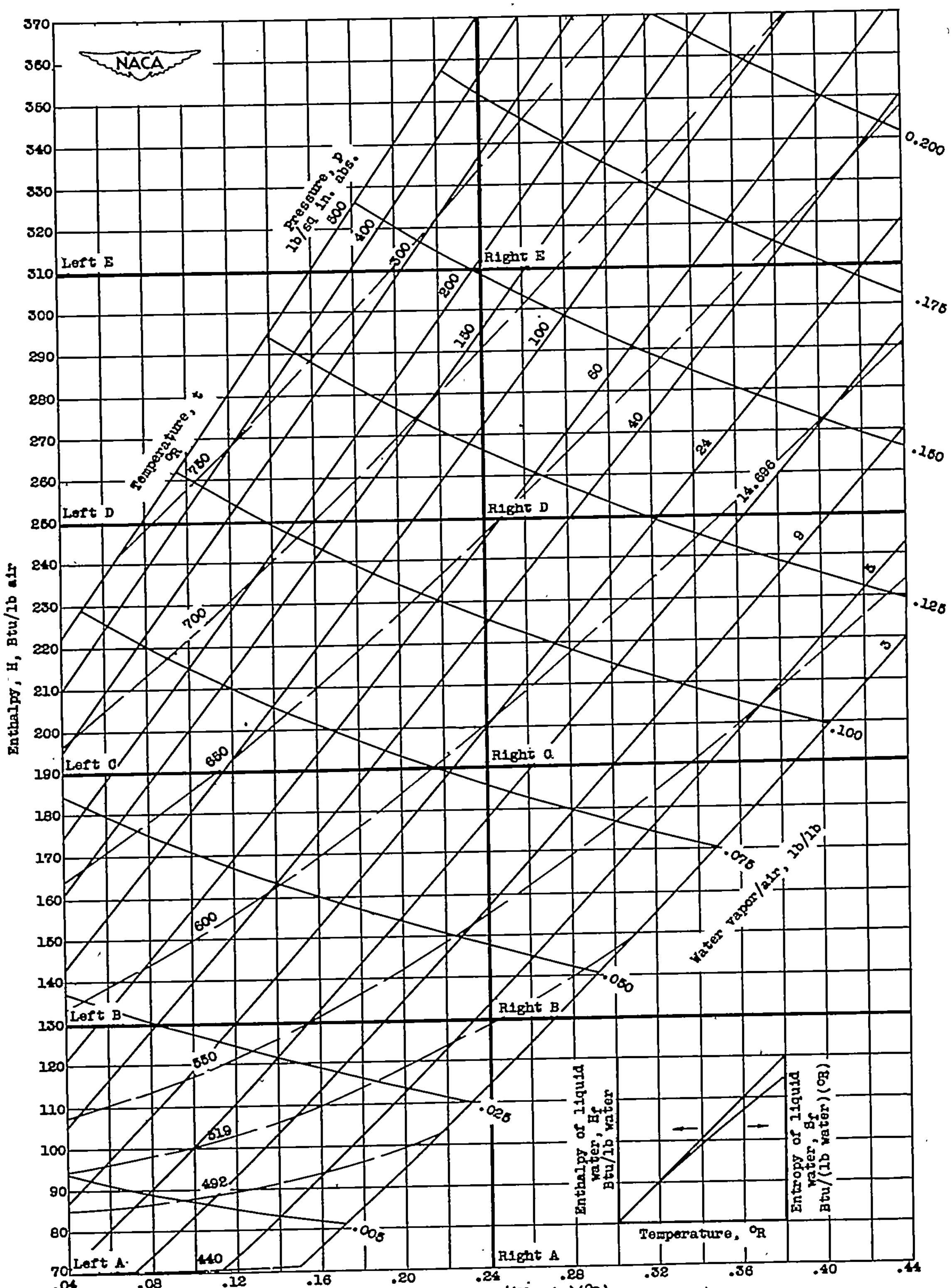
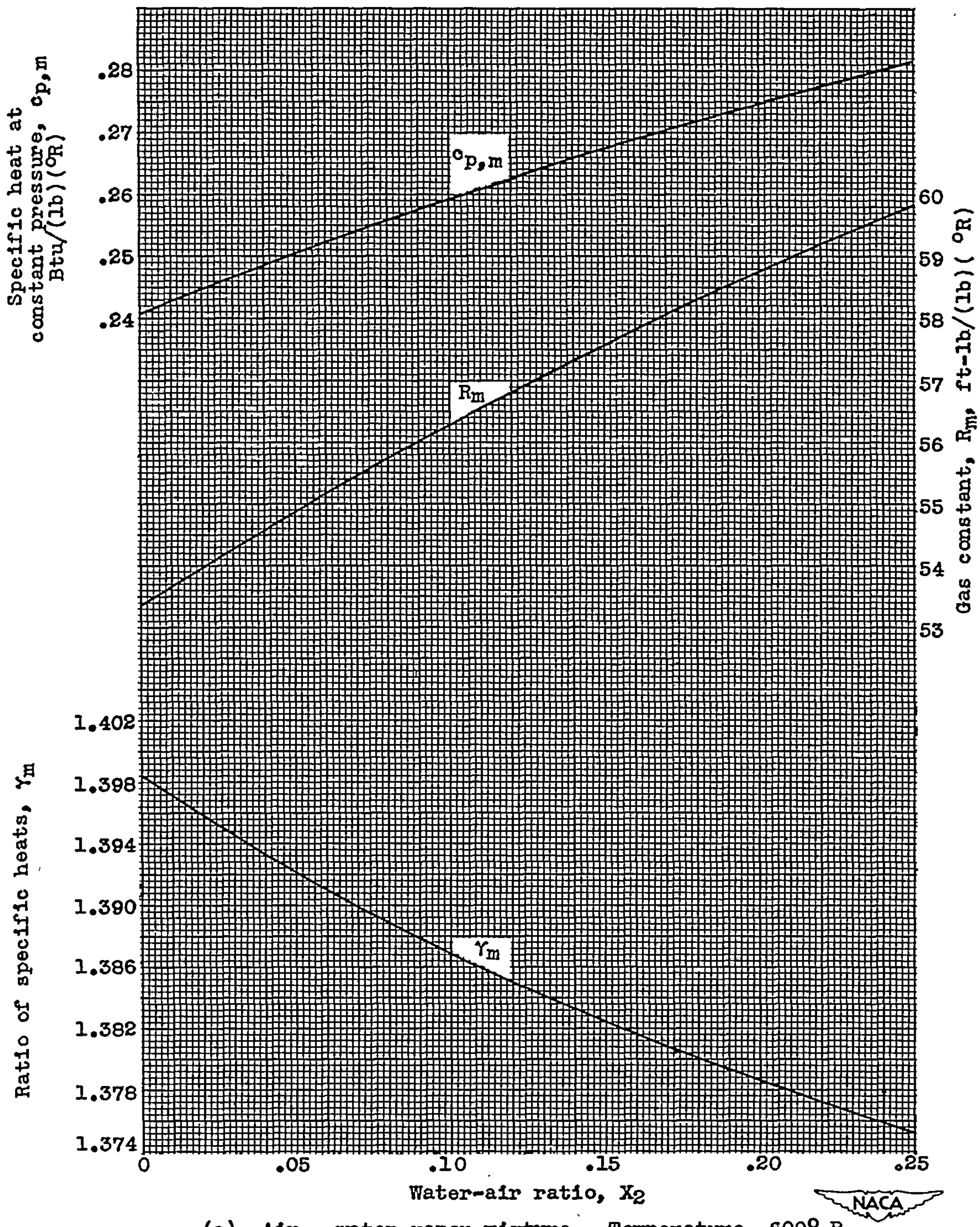
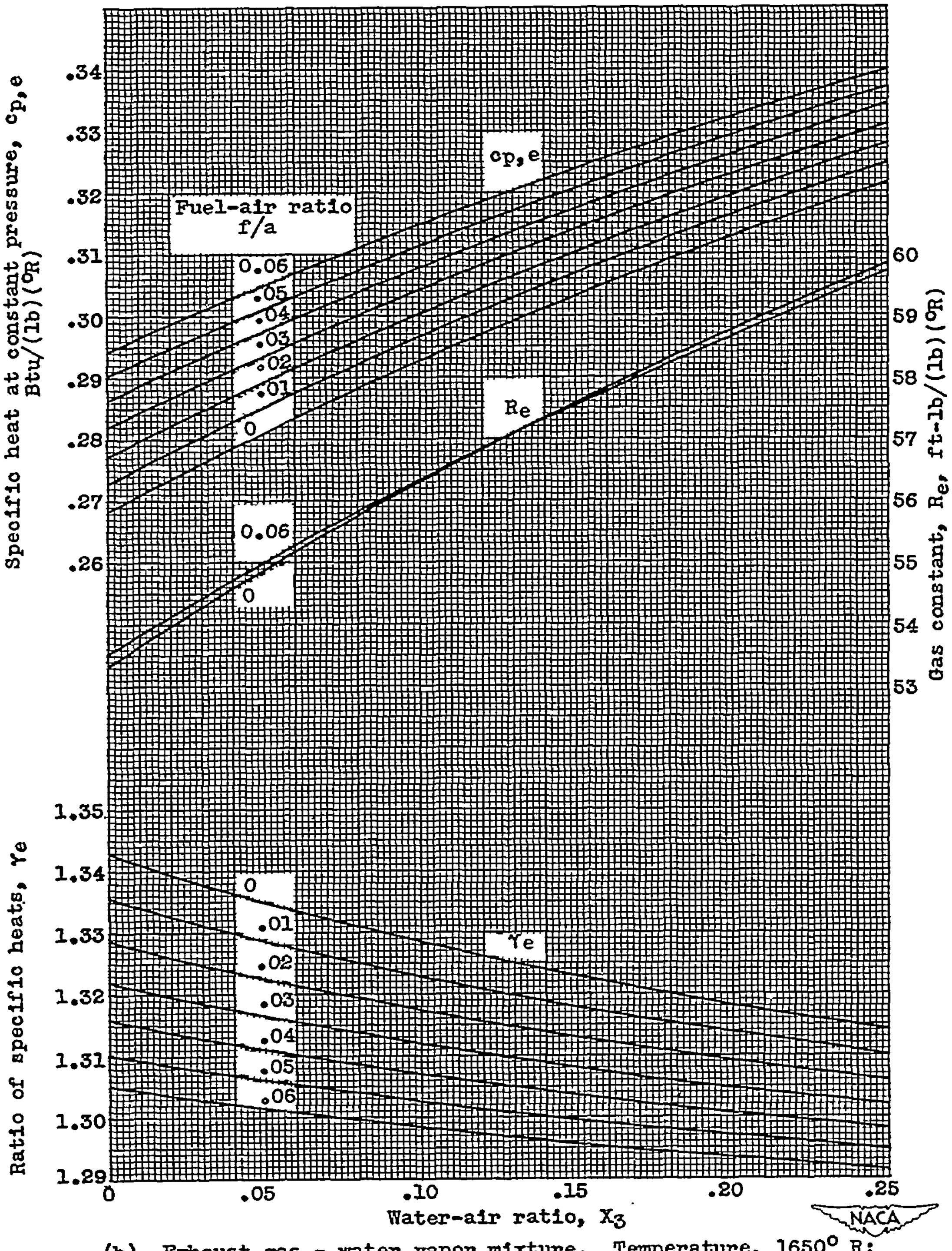


Figure 1. - Key to sections of Mollier diagram.



(a) Air - water vapor mixture. Temperature, 600° R.

Figure 2. - Thermodynamic properties of mixtures at pressure of 0 pound per square inch absolute.



(b) Exhaust gas - water vapor mixture. Temperature, 1650° R; hydrogen-carbon ratio, 0.175.

Figure 2. - Concluded. Thermodynamic properties of mixtures at pressure of 0 pound per square inch absolute.

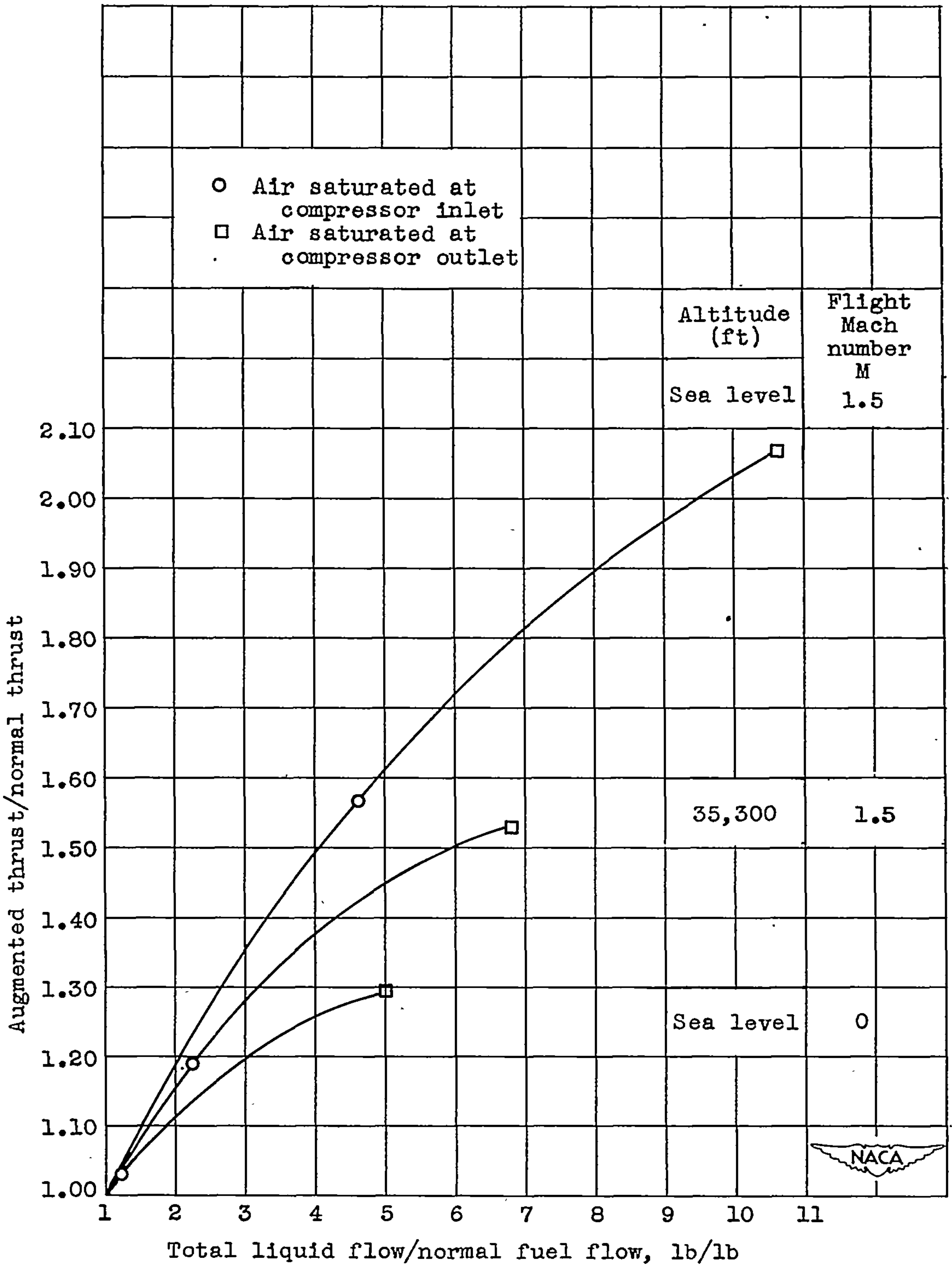
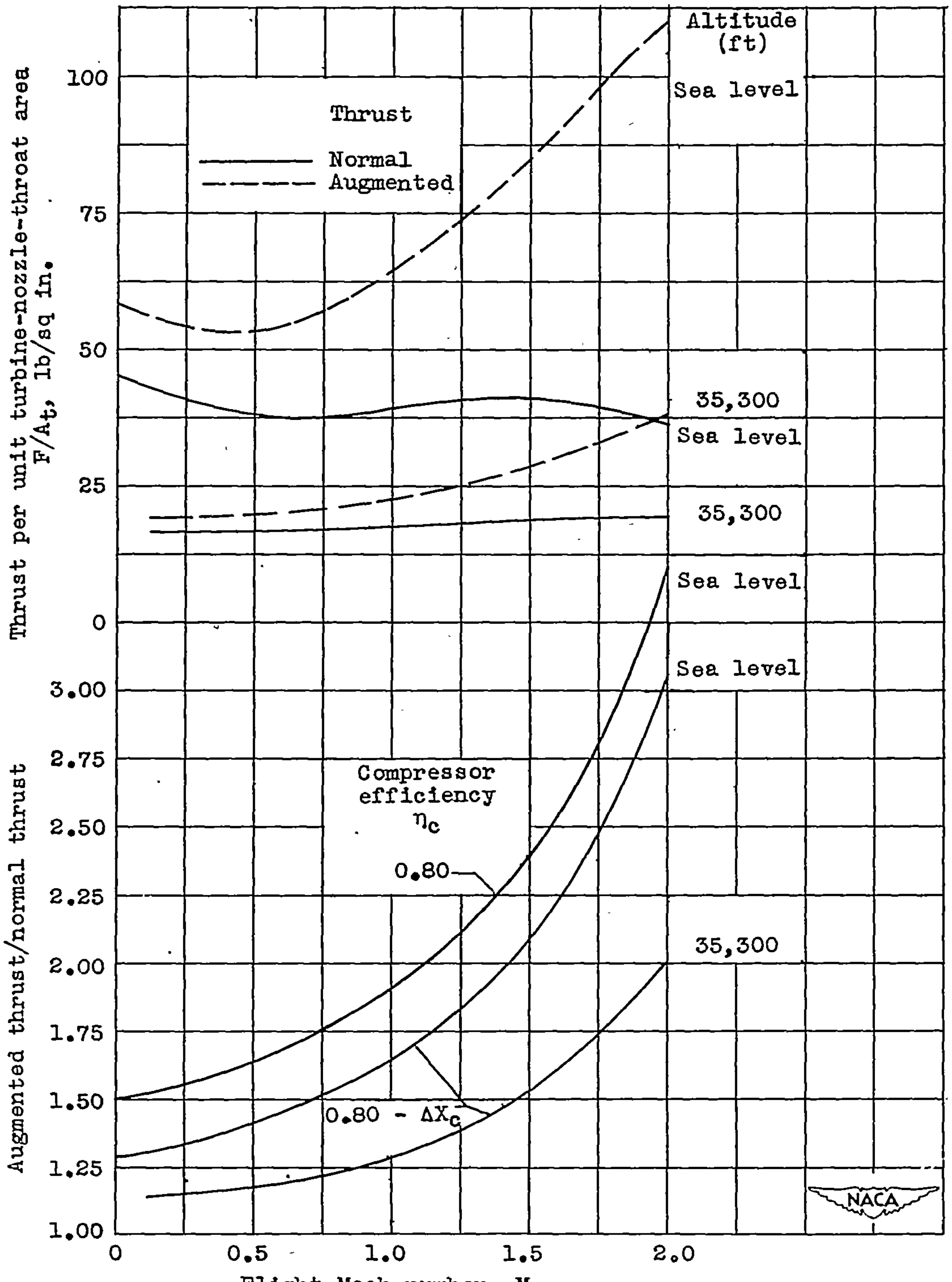
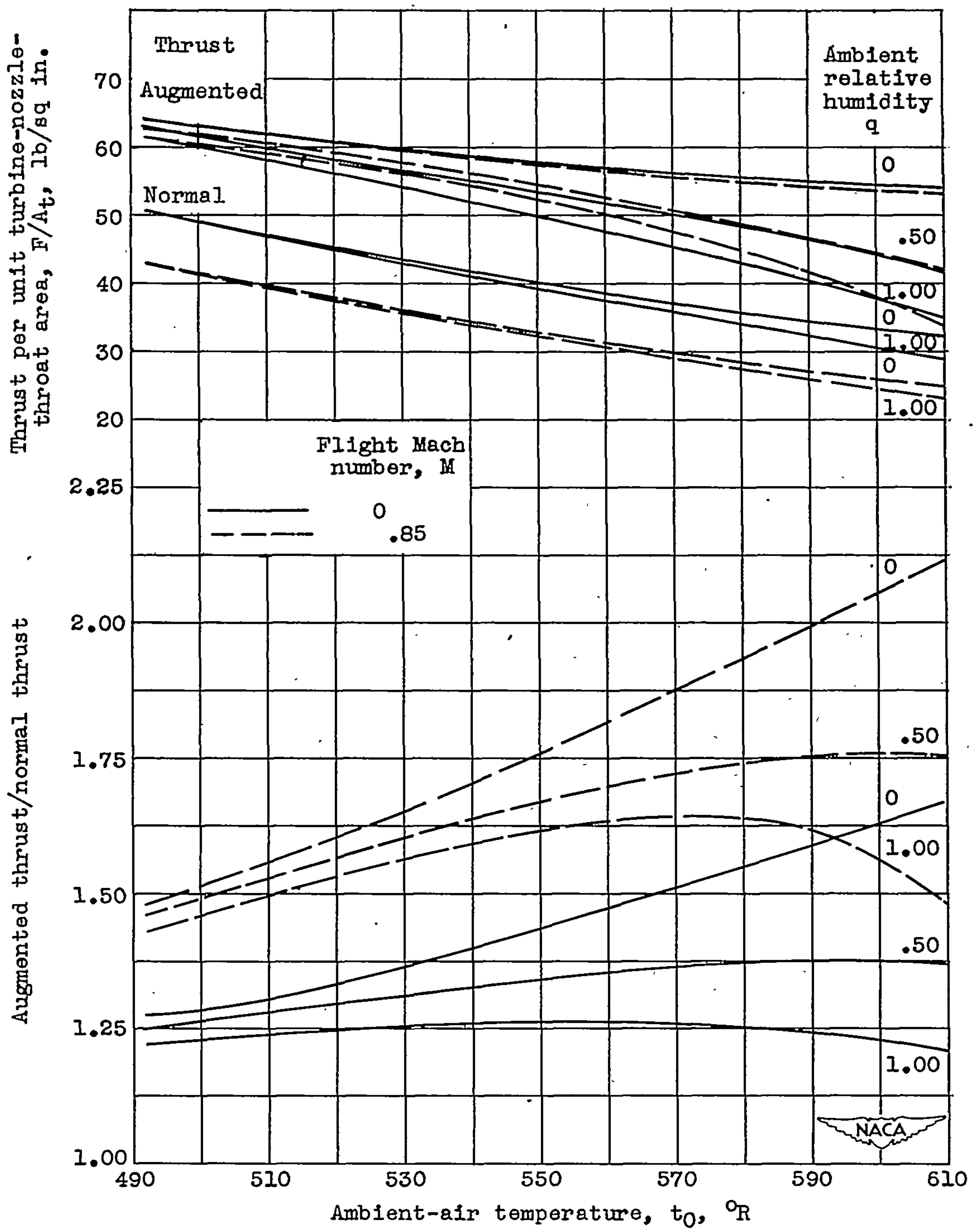


Figure 3. - Effect of flight conditions and ratio of total to normal liquid flow on thrust augmentation.



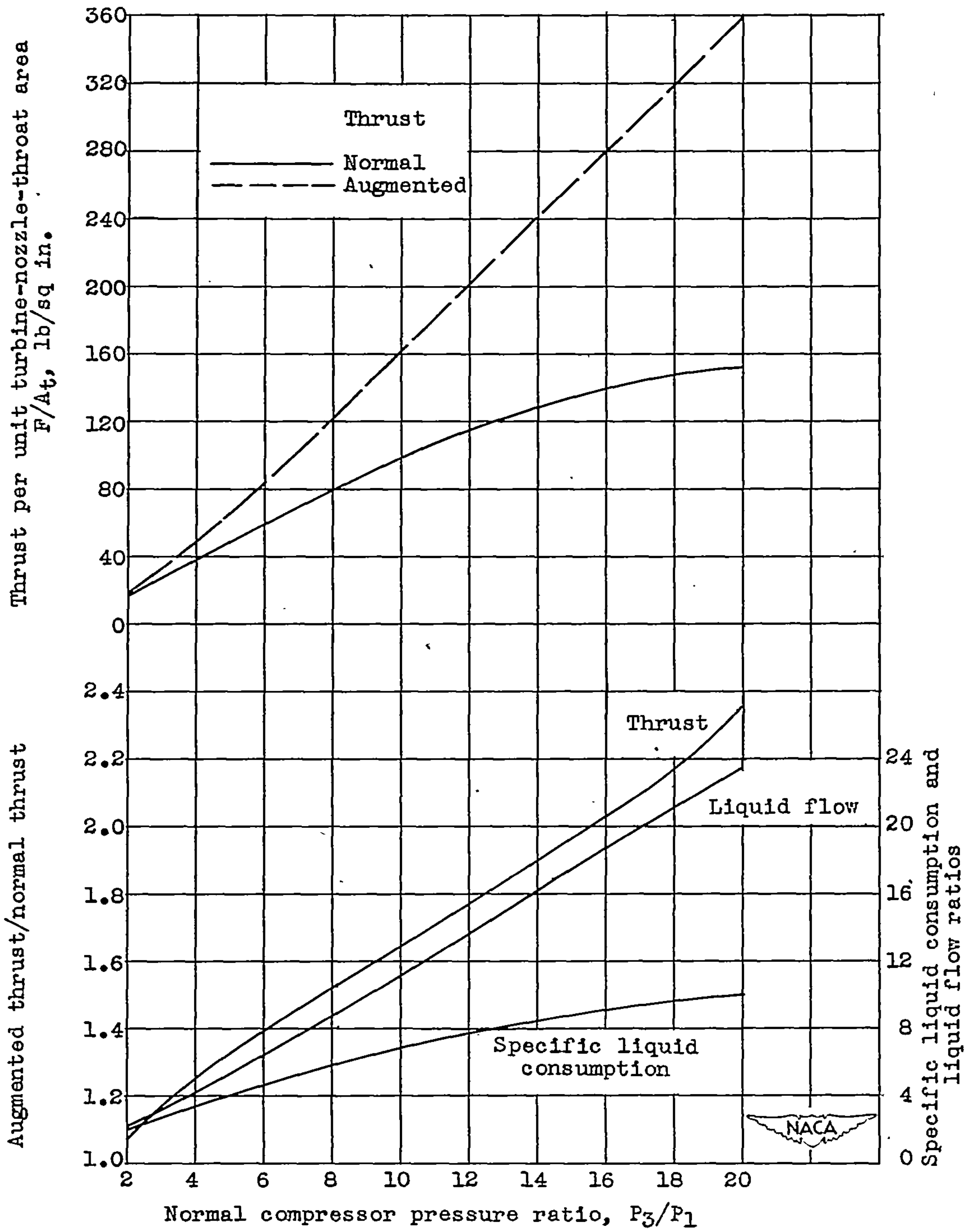
(a) Altitude and flight Mach number.

Figure 4. - Effect of operating conditions on performance.



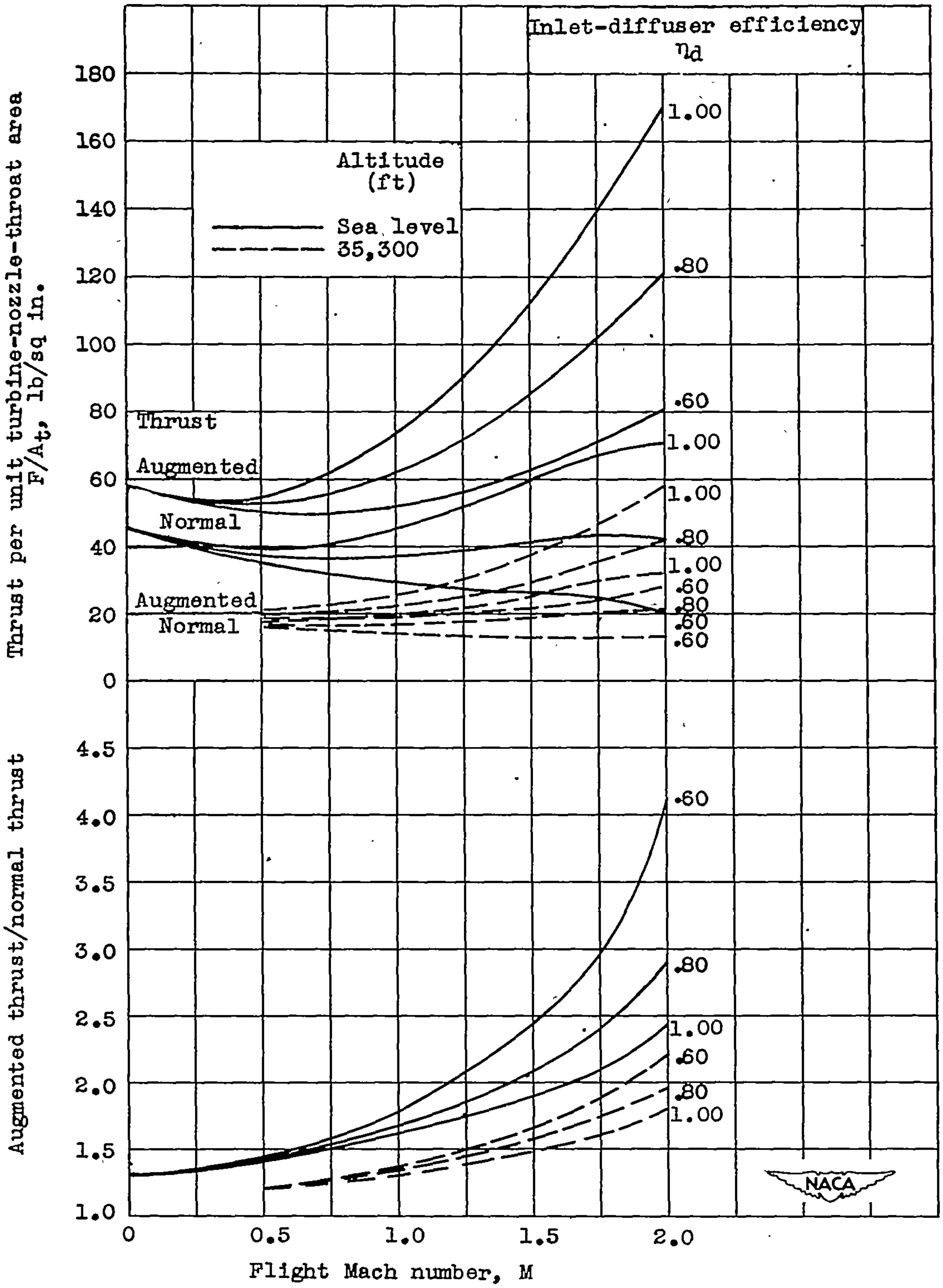
(b) Ambient-air temperature and ambient relative humidity at sea level.

Figure 4. - Continued. Effect of operating conditions on performance.



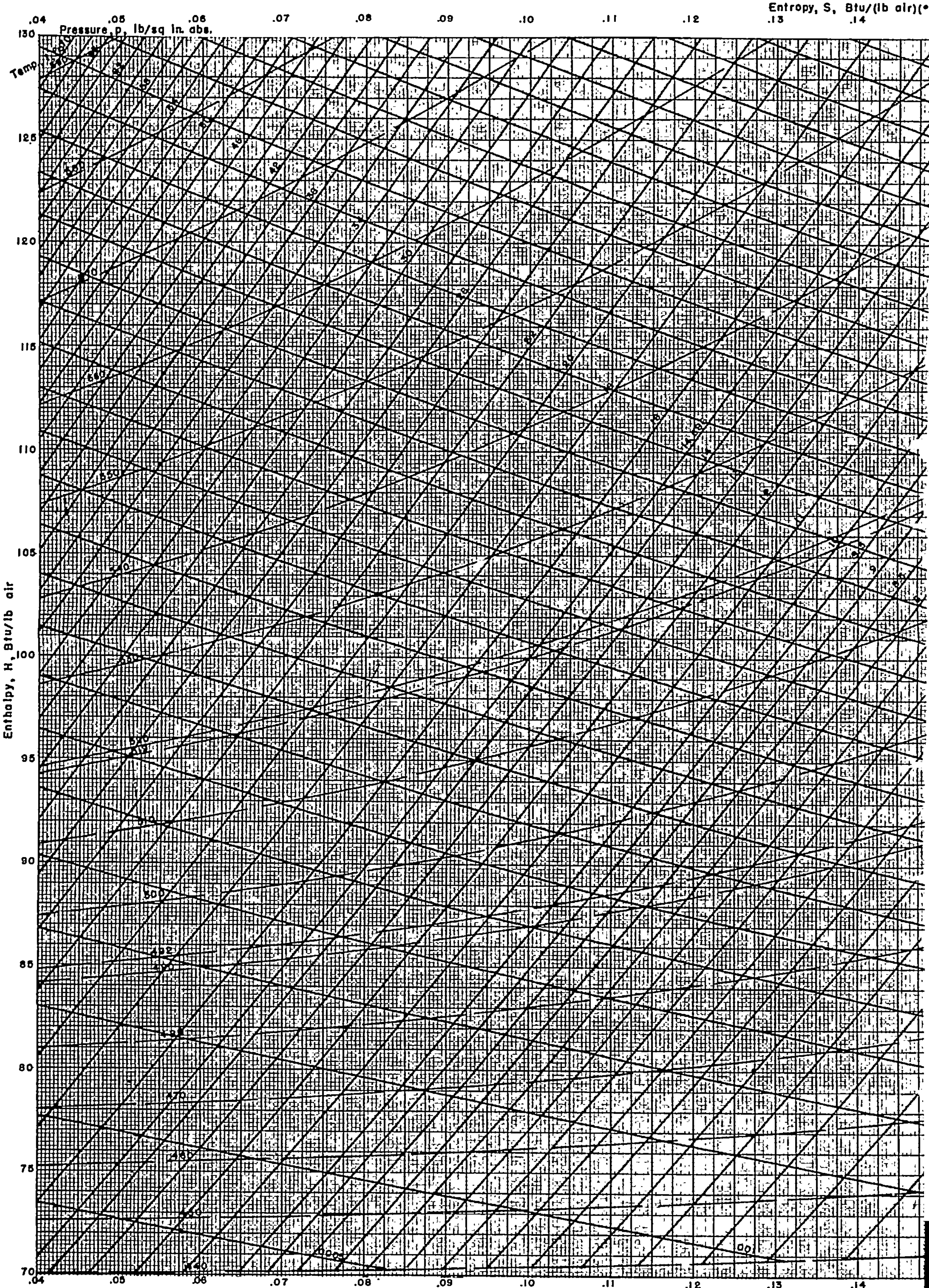
(c) Compressor pressure ratio at sea level and flight Mach number of 0.

Figure 4. - Continued. Effect of operating conditions on performance.

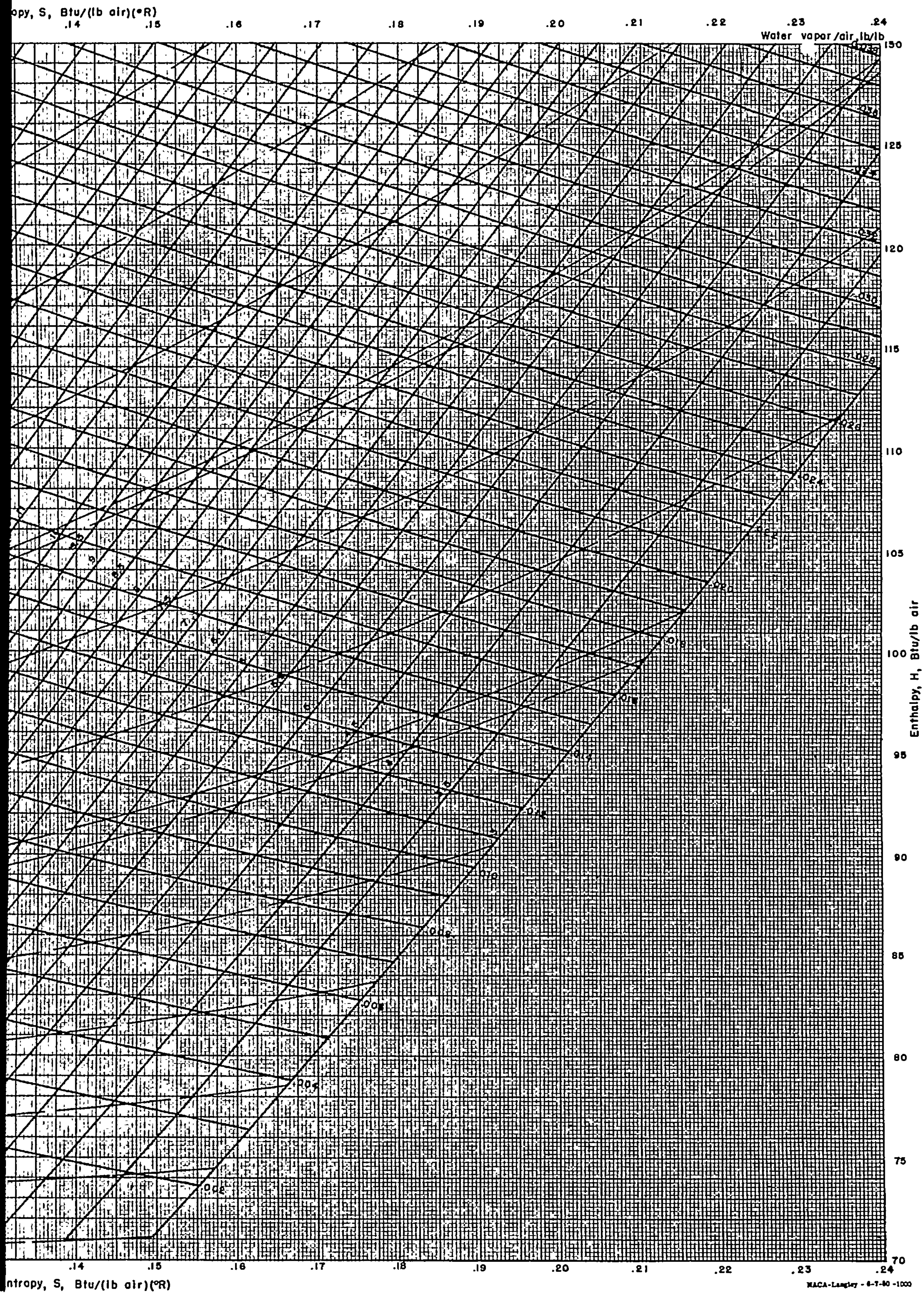


(d) Inlet-diffuser efficiency at altitudes of sea level and 35,300 feet and various flight Mach numbers.

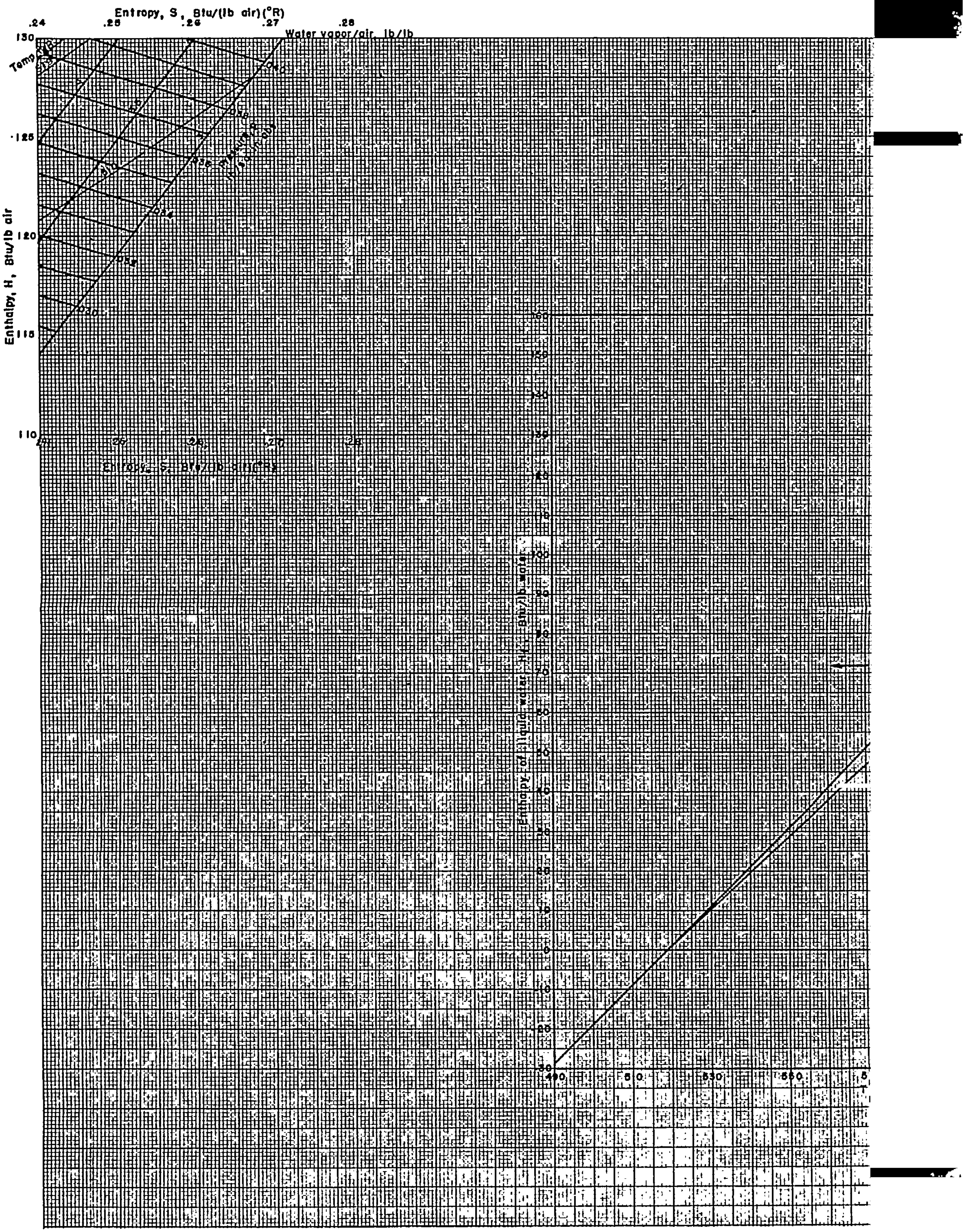
Figure 4. - Concluded. Effect of operating conditions on performance.


Entropy, S, Btu/(lb air)(°
Pressure, p, lb/sq in. abs.
Temp
Enthalpy, H, Btu/lb air
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
.14
130
125
120
115
110
105
100
95
90
85
80
75
70
Entropy, S, Btu/(lb air

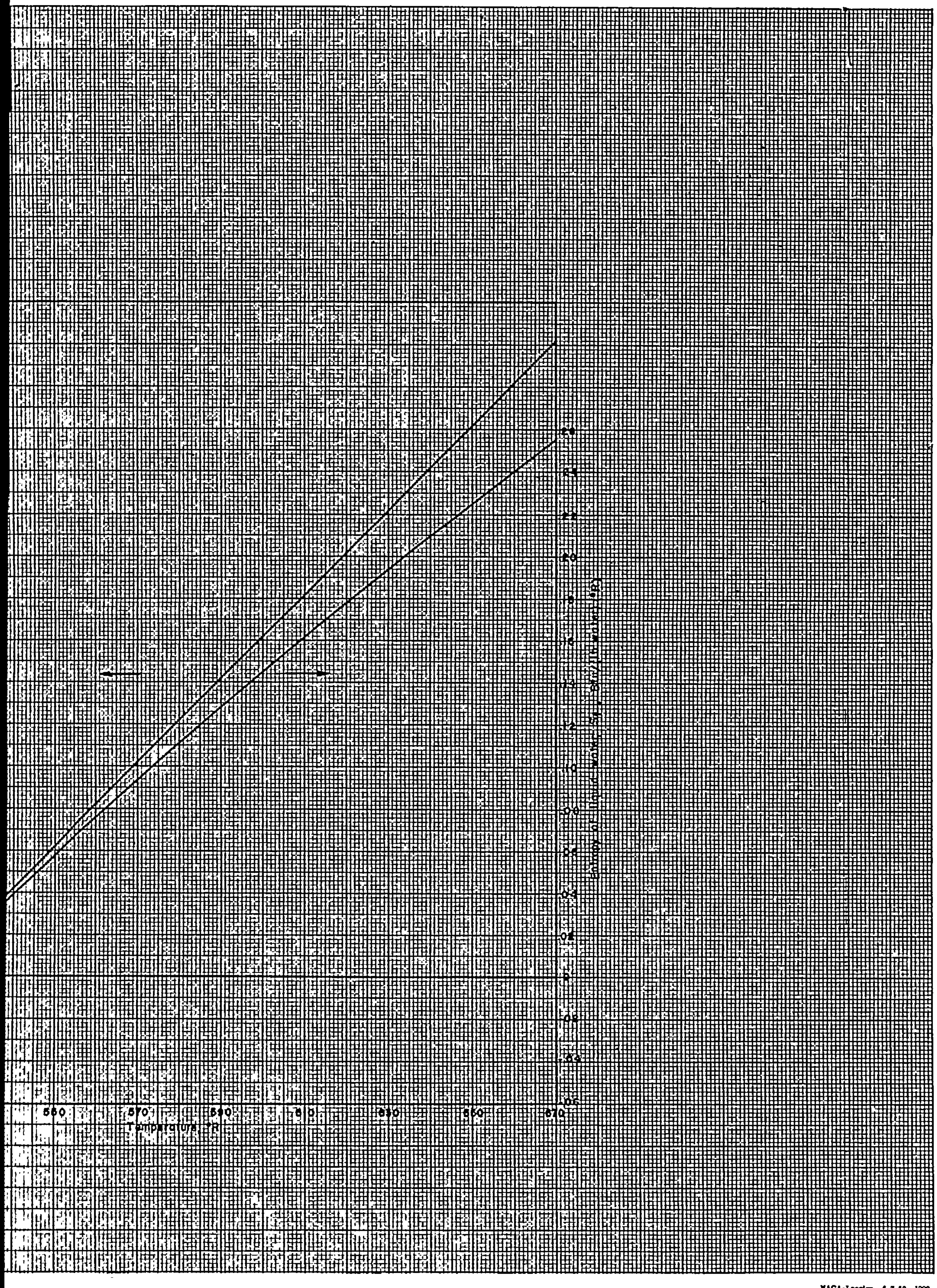
Left A

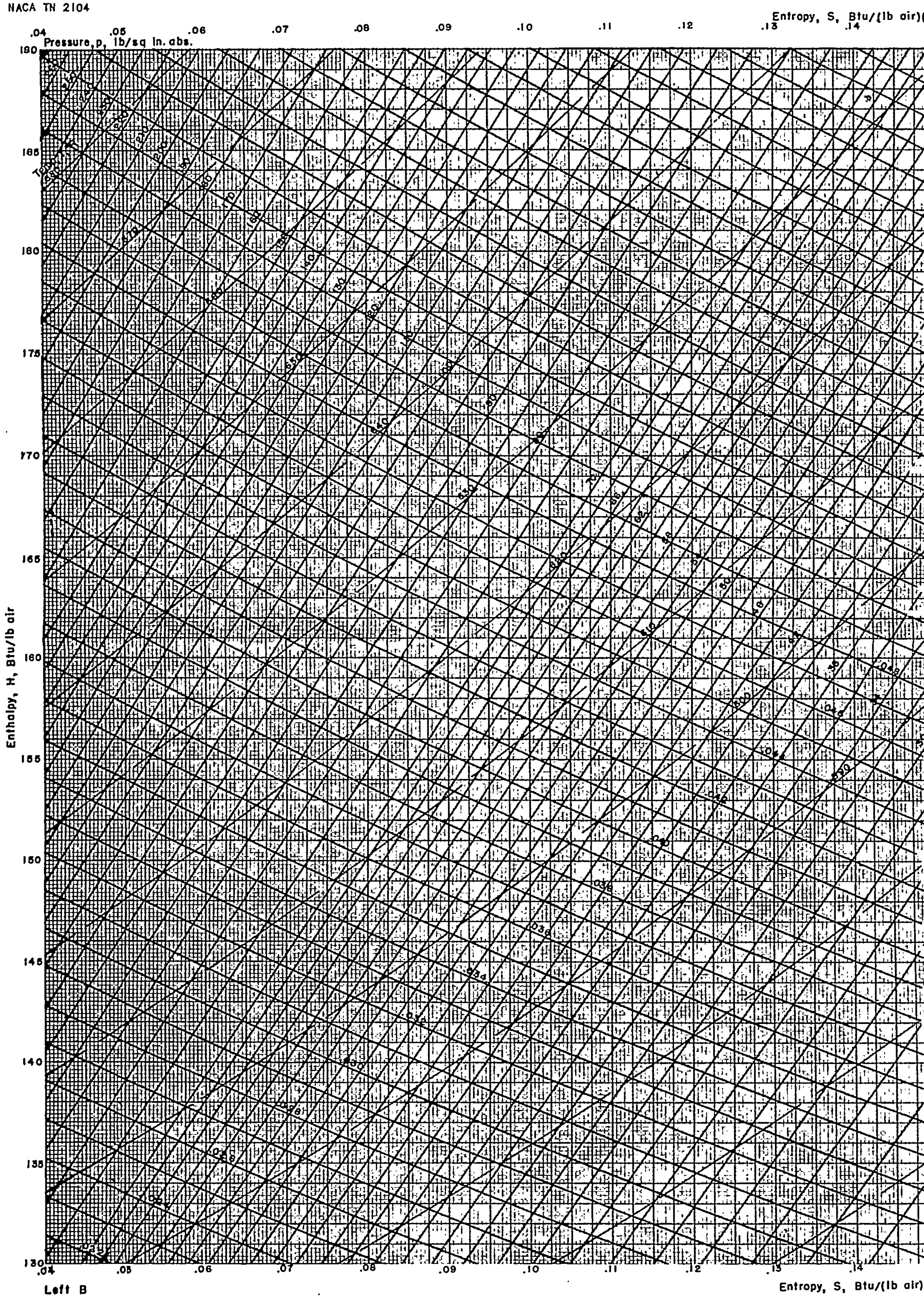


opy, S, Btu/(lb air)(°R)
Water vapor/air, lb/lb
Enthalpy, H, Btu/lb air
ntropy, S, Btu/(lb air)(°R)
.14
.15
.16
.17
.18
.19
.20
.21
.22
.23
.24
70
75
80
85
90
95
100
105
110
115
120
125
130

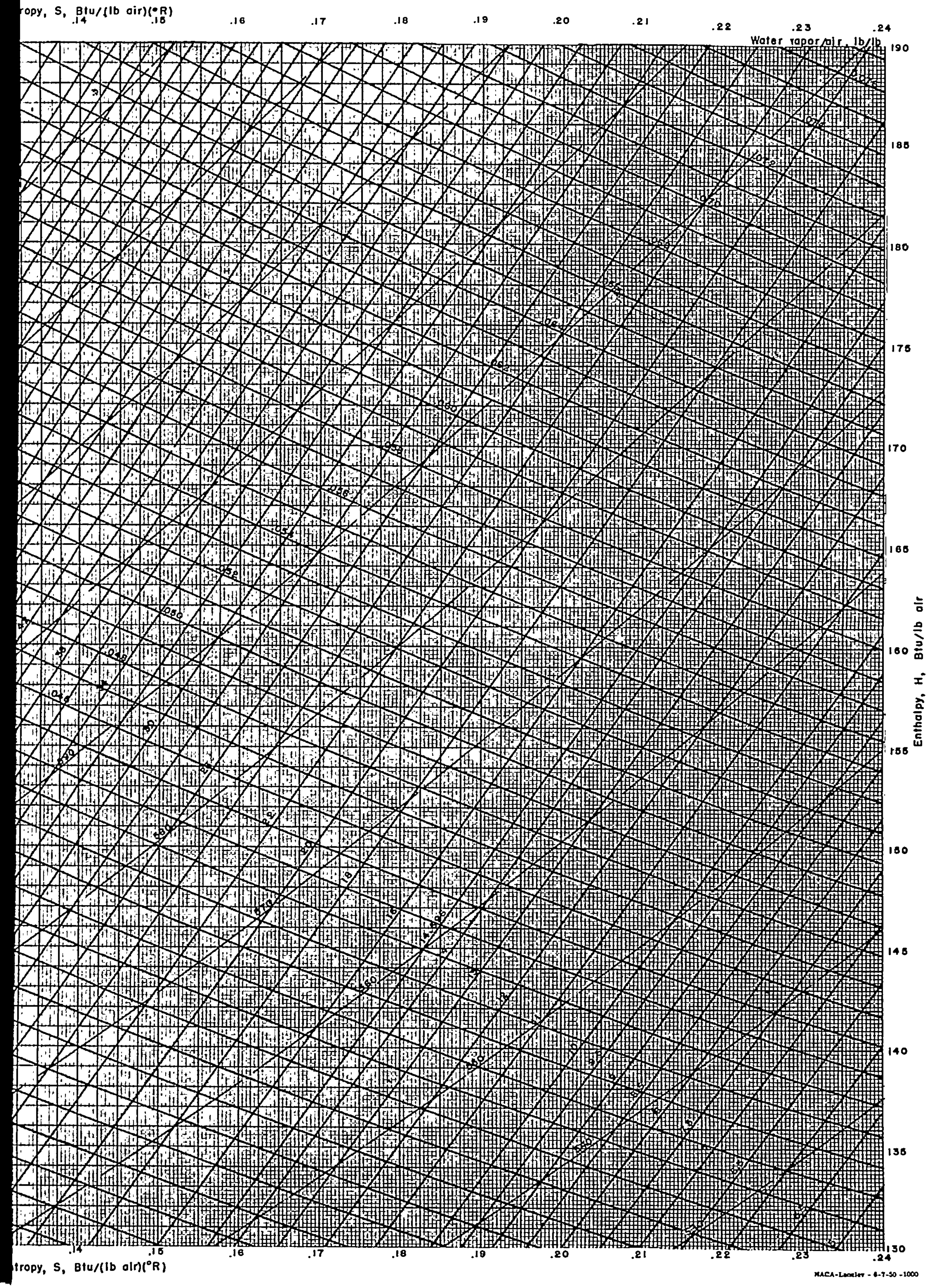

Entropy, S, Btu/(lb air)(°R)
Water vapor/air, lb/lb
Temp (°R)
Enthalpy, H, Btu/lb air
Entropy, S, Btu/(lb air)(°R)
Enthalpy of liquid water, H, Btu/lb water

Right A

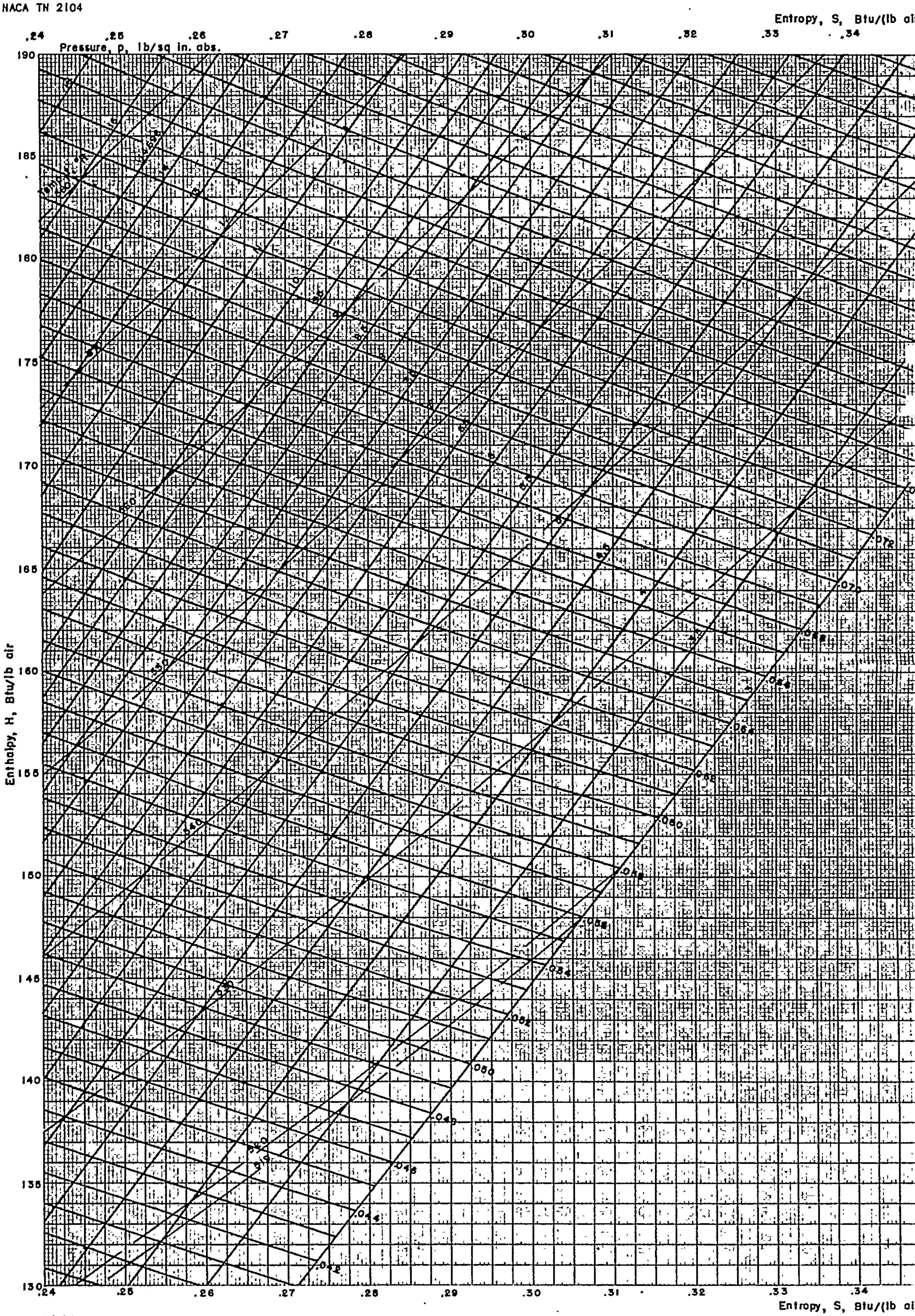



Entropy, S, Btu/(lb air)(
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
.14
Pressure, p, lb/sq in. abs.
Enthalpy, H, Btu/lb air
130
135
140
145
150
155
160
165
170
175
180
185
190

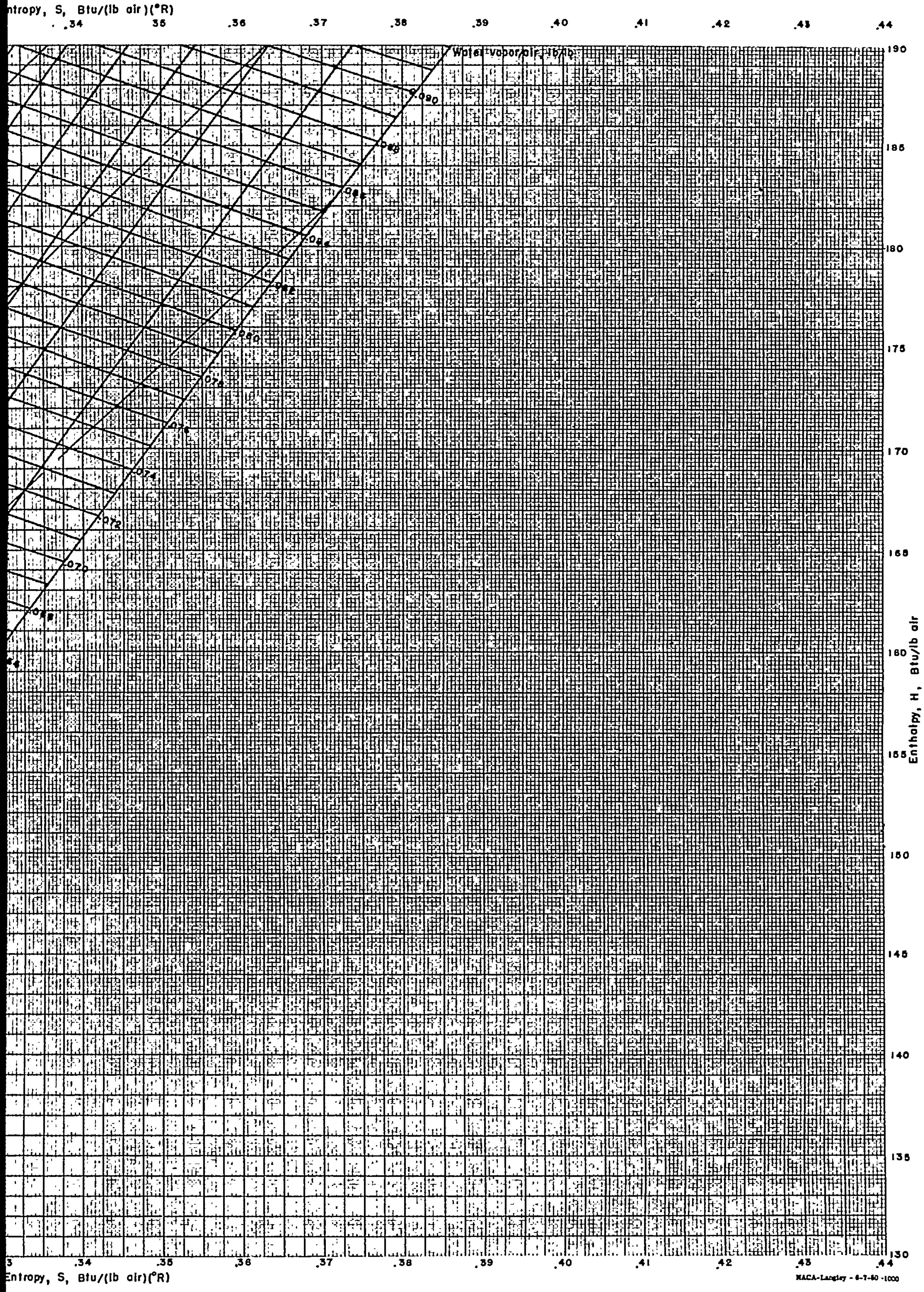
Left B



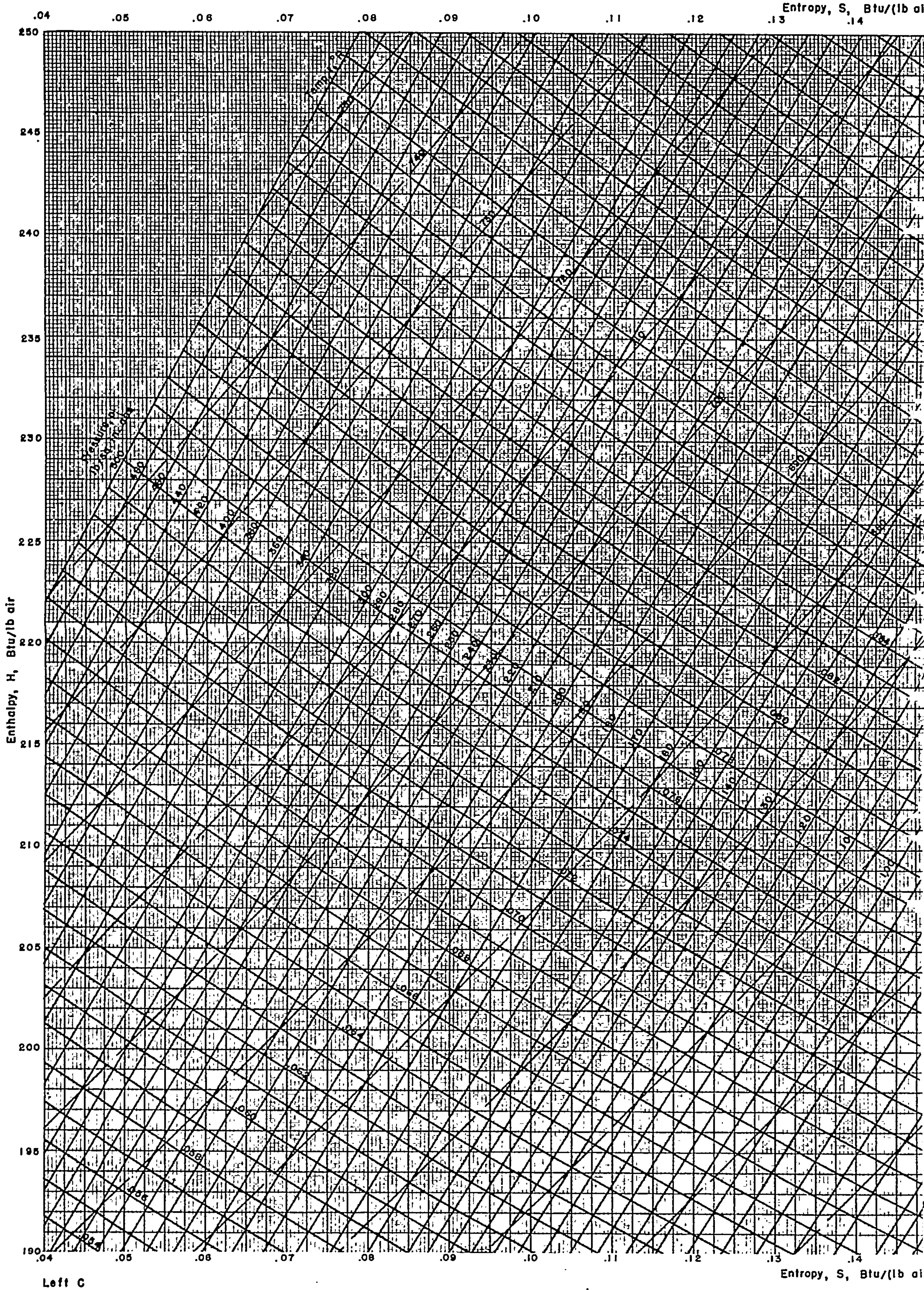
NACA-Langley - 8-7-50 -1000

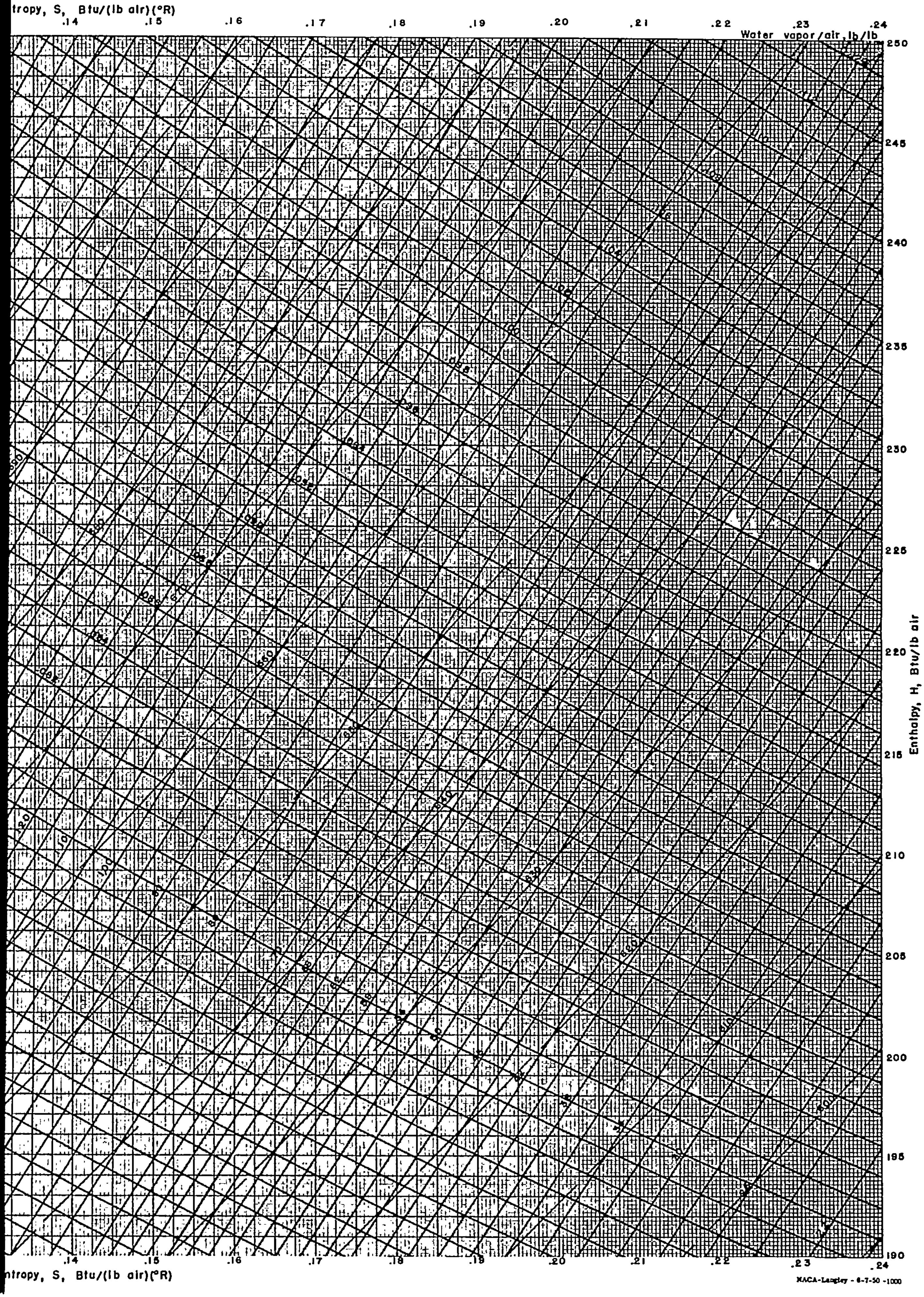

Entropy, S, Btu/(lb air
Pressure, p, lb/sq in. abs.
Enthalpy, H, Btu/lb air
.24
.25
.26
.27
.28
.29
.30
.31
.32
.33
.34
190
185
180
175
170
165
160
155
150
145
140
135
130

Right B

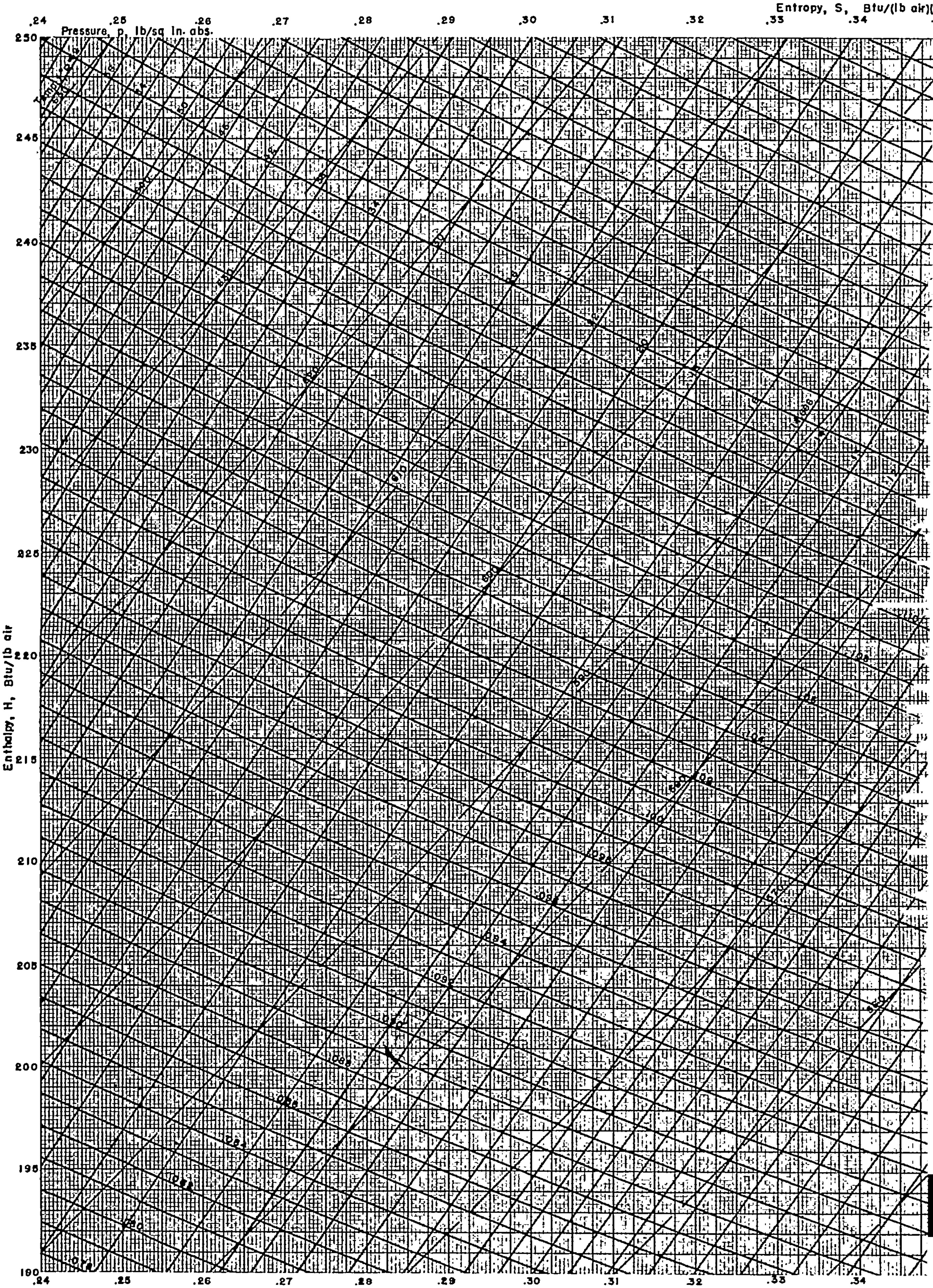


NACA-Langley - 6-7-50 -1000

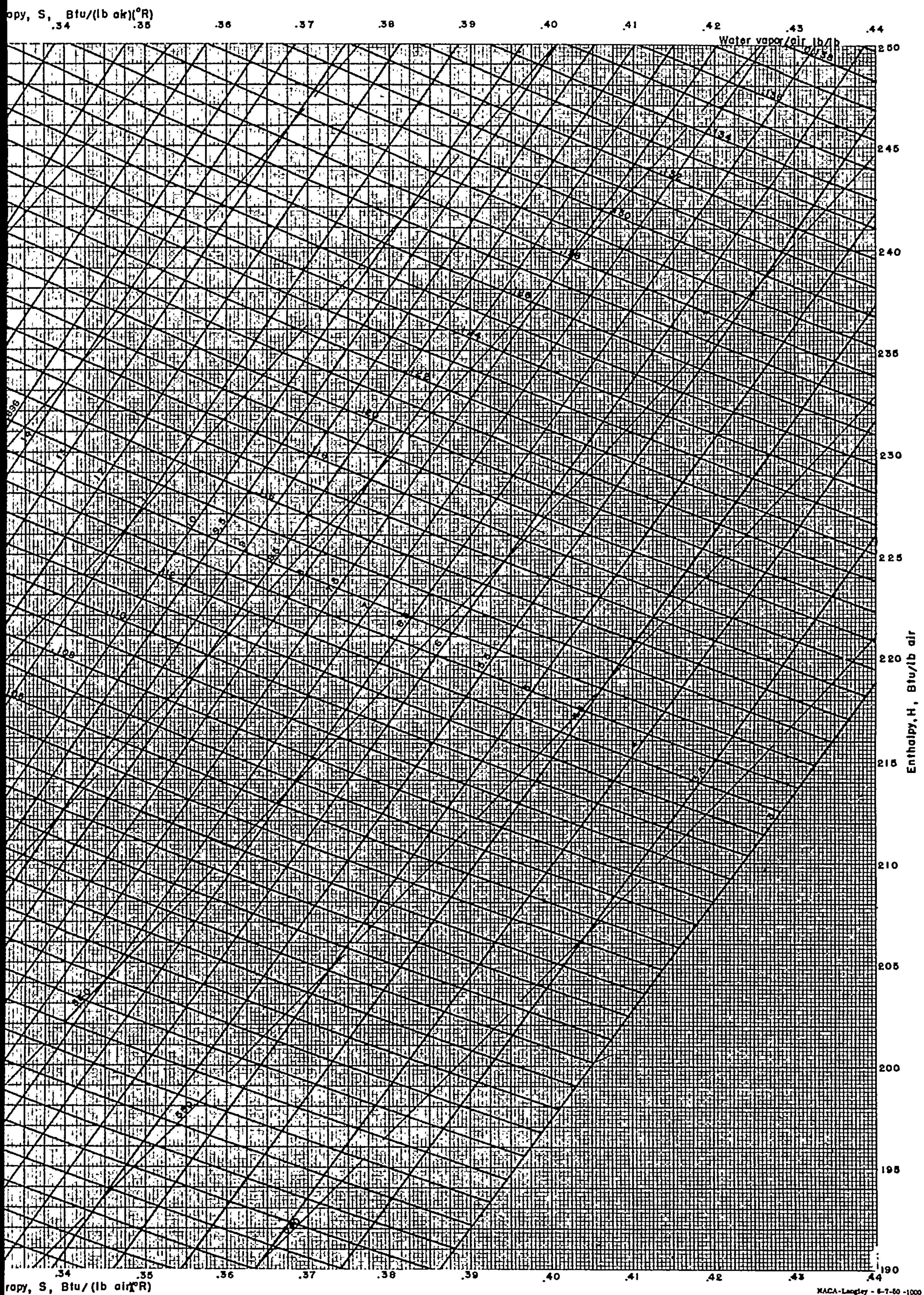

Entropy, S, Btu/(lb air
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
.14
Enthalpy, H, Btu/lb air
250
245
240
235
230
225
220
215
210
205
200
195
190
Left C



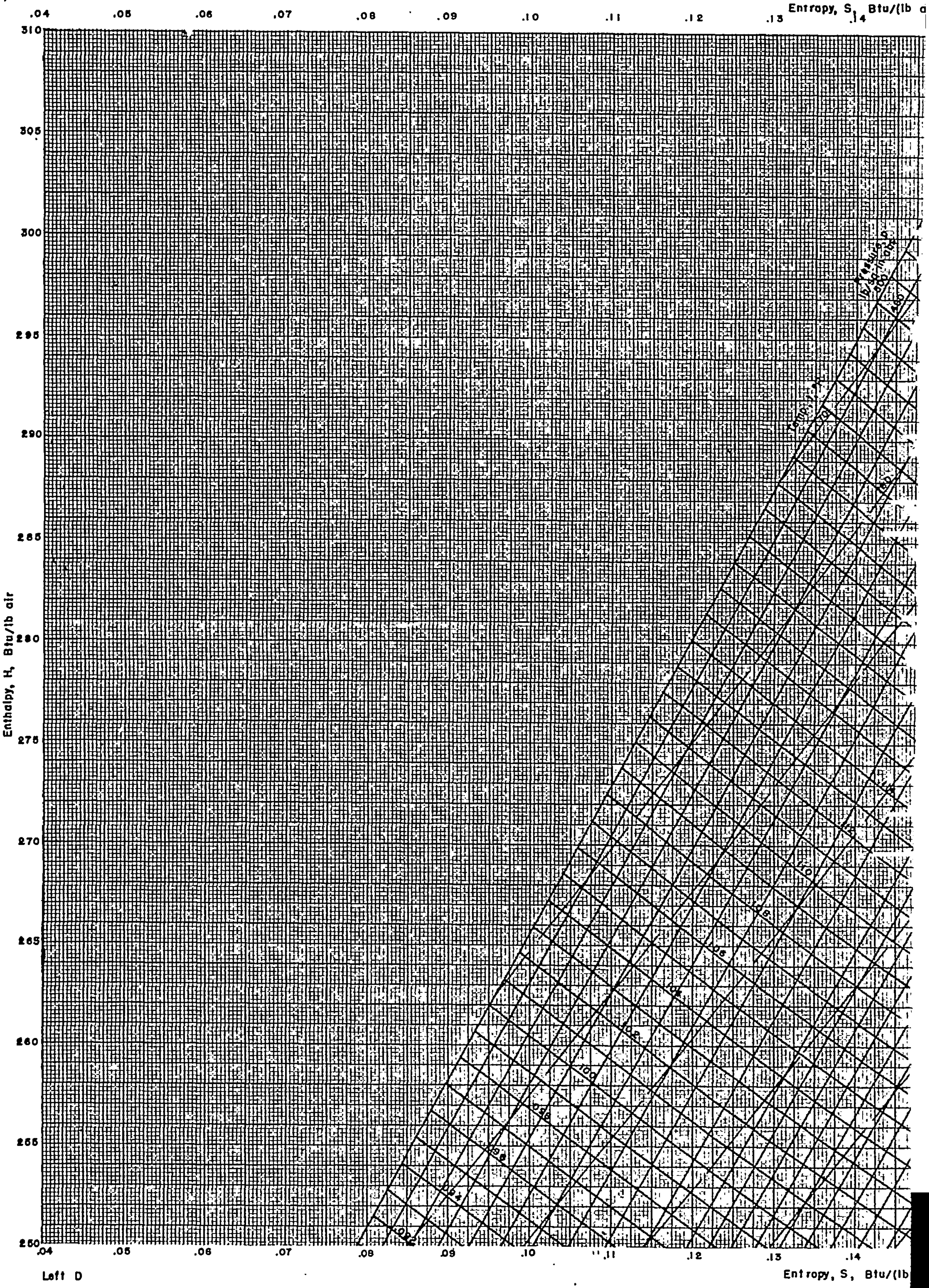
NACA-Langley - 8-7-50 -1000

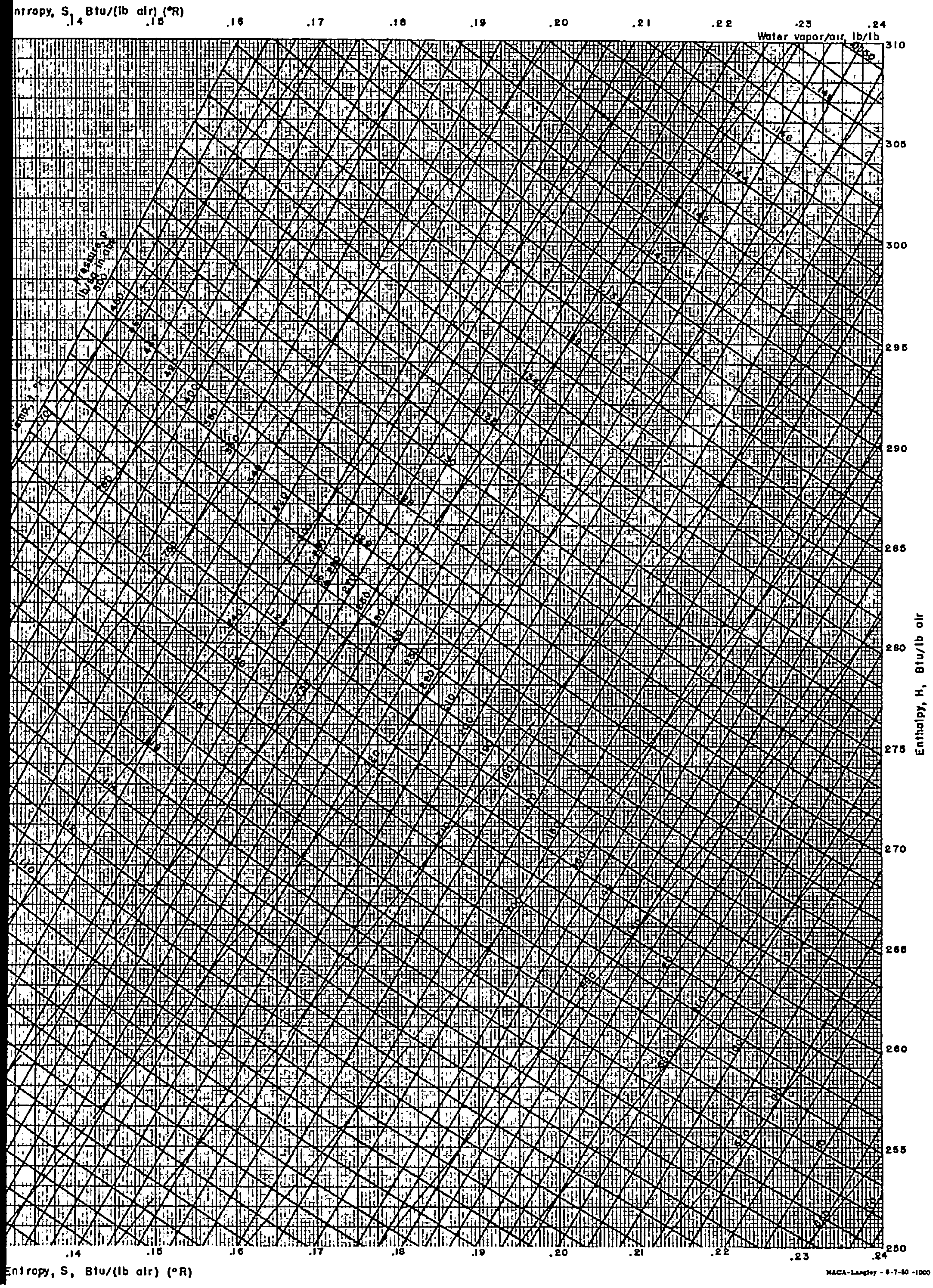

Entropy, S, Btu/(lb air)(
Pressure, p, lb/sq in. abs.
Enthalpy, H, Btu/lb air
Entropy, S, Btu/(lb air)
.24
.25
.26
.27
.28
.29
.30
.31
.32
.33
.34
250
245
240
235
230
225
220
215
210
205
200
195
190

Right C

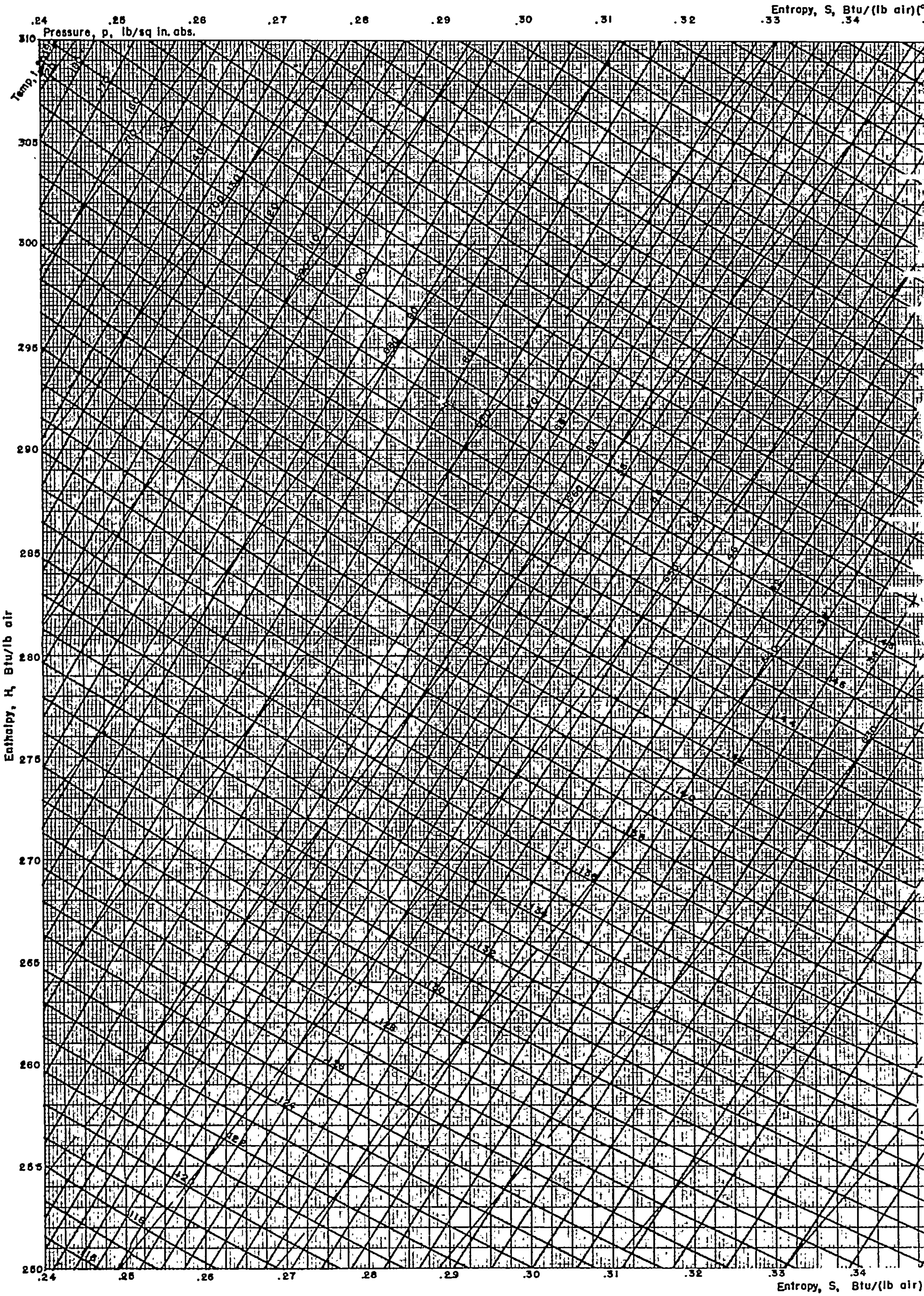


NACA-Langley - 8-7-50 -1000

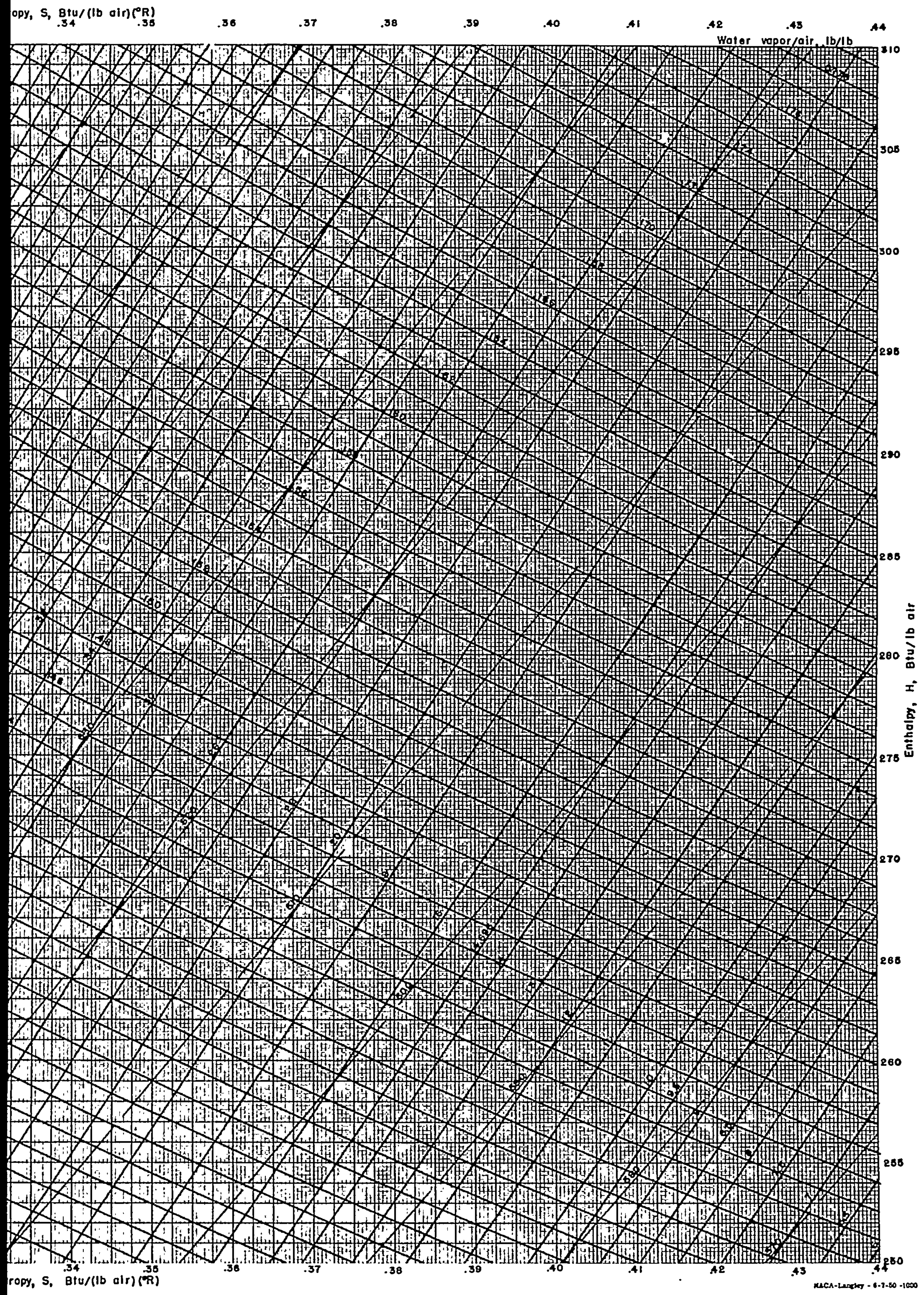

Entropy, S, Btu/(lb a
Enthalpy, H, Btu/lb air
Pressure, p, lb/sq in. abs
Temp, °R
Left D
Entropy, S, Btu/(lb



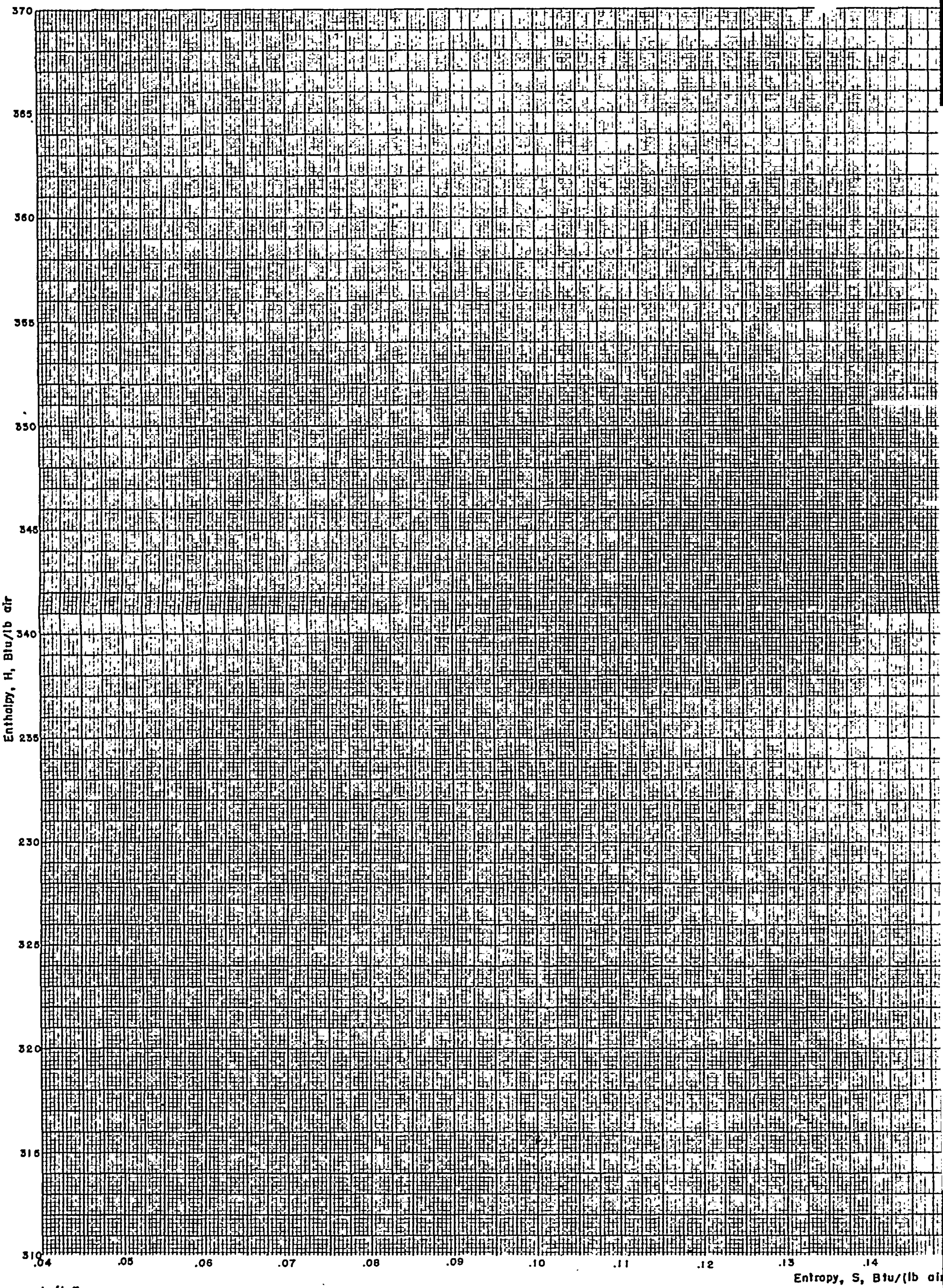
NACA-Langley - 8-7-50 -1000


Entropy, S, Btu/(lb air)(°
Pressure, p, lb/sq in. abs.
Temp, °R
Enthalpy, H, Btu/lb air
.24
.25
.26
.27
.28
.29
.30
.31
.32
.33
.34
310
305
300
295
290
285
280
275
270
265
260
255
250
Entropy, S, Btu/(lb air)

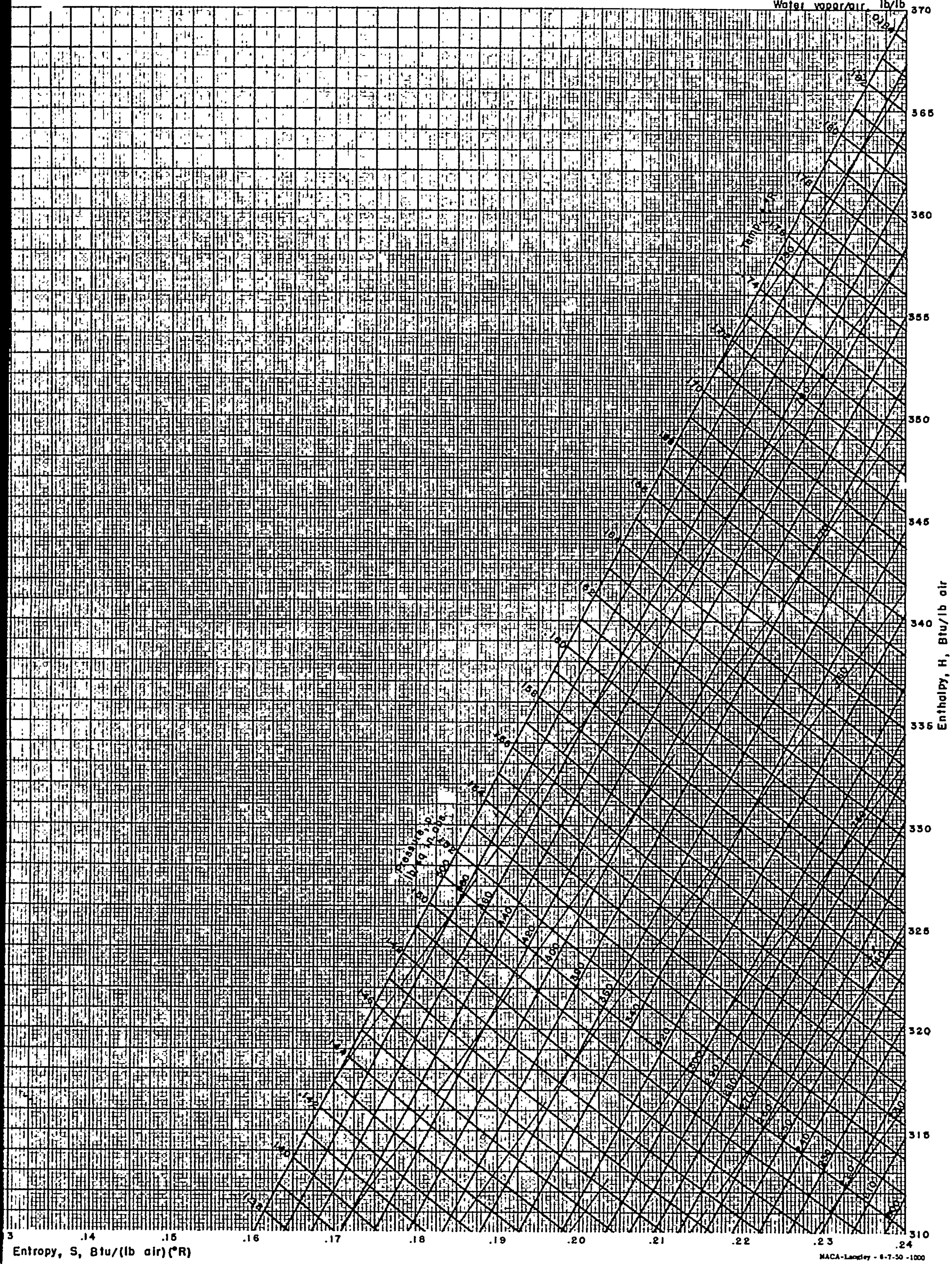
Right D



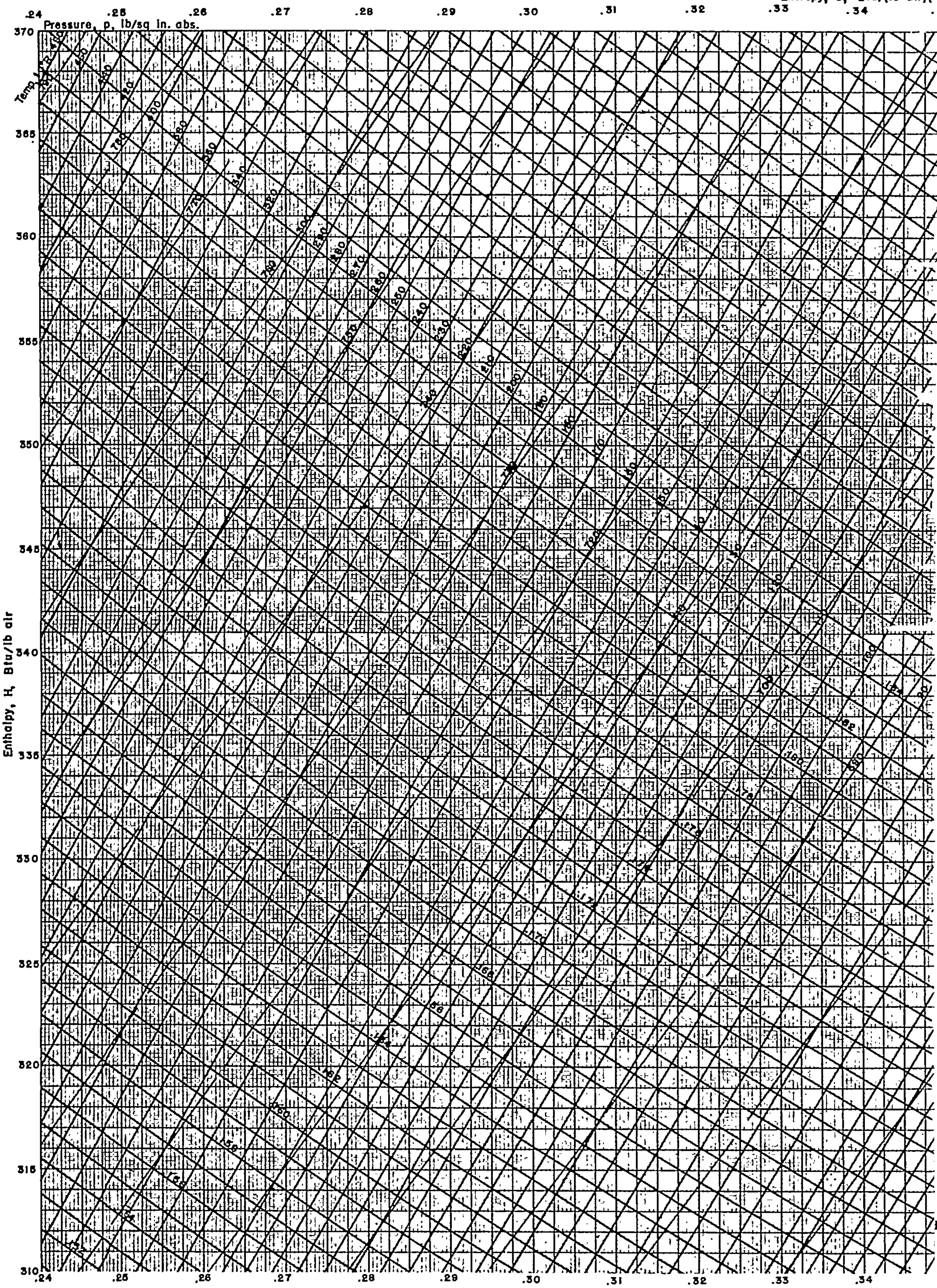
NACA-Langley - 6-7-50 -1000


370
365
360
355
350
345
340
335
330
325
320
315
310
Enthalpy, H, Btu/lb air
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
.14
Entropy, S, Btu/(lb ai

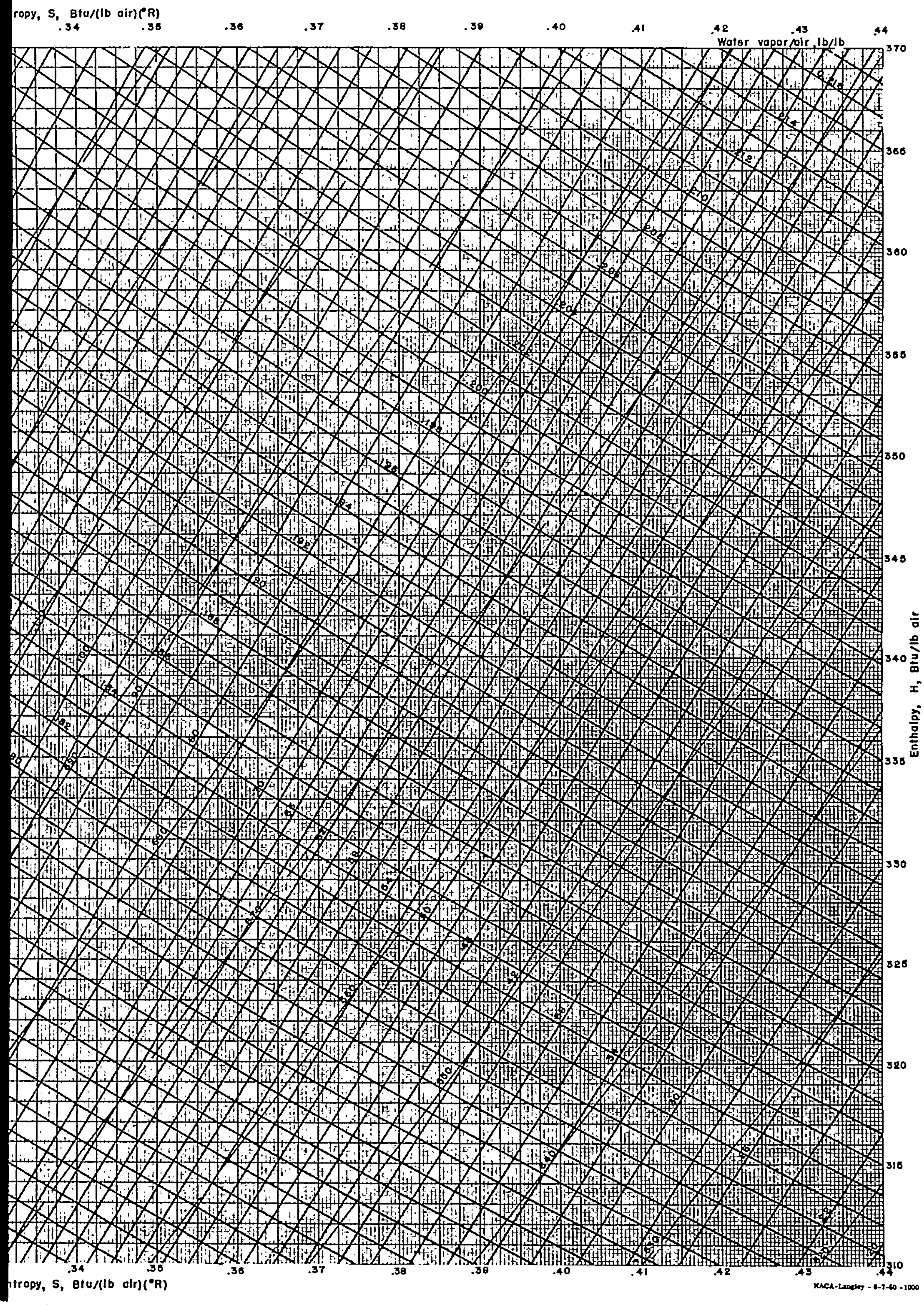
Left E



NACA-Langley - 6-7-50 -1000


Entropy, S, Btu/(lb air)(
Pressure, p, lb/sq in. abs.
Temp
Enthalpy, H, Btu/lb air
.24
.25
.26
.27
.28
.29
.30
.31
.32
.33
.34
310
315
320
325
330
335
340
345
350
355
360
365
370
Entropy, S, Btu/(lb air)(

Right E


tropy, S, Btu/(lb air)(°R)
.34
.35
.36
.37
.38
.39
.40
.41
.42
.43
.44
Water vapor/air, lb/lb
370
365
360
355
350
345
340
335
330
325
320
315
310
Enthalpy, H, Btu/lb air
ntropy, S, Btu/(lb air)(°R)