NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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CALCULATION OF THE EFFECT OF THRUST-AXIS INCLINATION ON PROPELLER DISK LOADING AND COMPARISON WITH FLIGHT MEASUREMENTS

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SUMMARY

An analysis based on simple geometry has been made of the effect of thrust-axis inclination on propeller disk loading. Calculations are in excellent agreement with available flight measurements, so that the analysis is indicated to be adequate for predicting the primary effects of inclination. For accuracy in the general case, however, a more complete treatment may be necessary. Fuselage and wing interference effects may be large and, for accuracy, surveys for determining flow angles are suggested.

Consideration of factors involved indicates that, for a given blade design, the simplest means for reducing the fluctuating blade stresses due to inclination is suitable thrust-axis setting and/or restriction of airplane angle-of-attack range through use of flaps.

INTRODUCTION

Published papers (references 1 and 2) concerning propeller flight tests have shown that inclination of the thrust axis to the air stream causes large variations in propeller disk loading. With the trend toward high-solidity, large-diameter propellers for high-speed, long-range operation, this effect is now becoming important in the structural design of propeller blades. Consequently, it is desirable to be able to predict the magnitude of this effect.

In this paper a simple analysis is made of the effect of angle of attack on thrust distribution. Calculations based on the analysis are compared with some unpublished data from previous flight tests in which the thrust distributions on the right and left sides of the propeller disk and the thrust-axis angle of attack were measured.

SYMBOLS

b	blade chord
B	number of blades
c_{l_α}	section lift-curve slope
C_P	power coefficient $\left(\frac{\text{Power}}{\rho n^3 D^5}\right)$
C_T	thrust coefficient $\left(\frac{\text{Thrust}}{\rho n^2 D^4}\right)$
D	propeller diameter
h	blade-section maximum thickness
J	advance ratio (V/nD)
L	load
M	Mach number of advance
M_x	Mach number based on resultant velocity at station x
n	propeller rotational speed
q	dynamic pressure $\left(\frac{1}{2}\rho V^2\right)$
r	radius to a blade element
V	forward velocity
W	resultant velocity at blade section
x	fraction of propeller tip radius $\left(\frac{r}{R}\right)$
α	angle of attack of blade section
α_{ta}	thrust-axis angle of attack
β	blade angle at $x = 0.75$
ϵ	inflow angle
η	efficiency
θ	blade angle

ρ density

σ solidity $\left(B\frac{b}{D}/\pi x\right)$

ϕ aerodynamic helix angle

ϕ_o geometric helix angle $\left(\tan^{-1}\frac{J}{\pi x}\right)$

Primed symbols indicate quantities affected by inclination.

ANALYSIS

The analysis, as presented, is developed for determining the maximum effects of angle of attack. At a positive angle of attack, the down-traveling blade will experience an increased load and the up-traveling blade on the other side of the propeller disk will experience a decreased load. Under normal circumstances, therefore, the maximum effects of inclination will occur along the horizontal diameter.

This analysis should also apply for angles of yaw or combinations of angle of attack and yaw. In any case, the maximum effects should be experienced along a propeller diameter perpendicular to the plane of resultant angle variation.

In figure 1 is presented a vector diagram of the blade-section operating conditions pertinent to this analysis.

When not inclined to the air stream the blade section angle of attack is given by

$$\alpha = \theta - \phi_o - \epsilon$$

where

$$\phi_o = \tan^{-1}\frac{V}{\pi nDx}$$

and ϵ is the inflow angle determined by conventional propeller theory.

The primary effect of inclination will be to rotate the forward velocity vector V through twice the thrust-axis angle of attack α_{ta}

during each revolution. As a result, the blade angle of attack on one side of the propeller disk will be increased and that on the other side will be decreased.

The new blade angles of attack are given by

$$\alpha' = \theta - \phi_o' - \epsilon$$

where

$$\phi_o' = \tan^{-1} \frac{V \cos \alpha_{ta}}{\pi nDx \pm V \sin \alpha_{ta}}$$

$$= \tan^{-1} \frac{\frac{J}{\pi x} \cos \alpha_{ta}}{1 \pm \frac{J}{\pi x} \sin \alpha_{ta}}$$

If ϵ is assumed to remain unchanged,

$$\Delta\alpha = \alpha - \alpha'$$

$$= \phi_o' - \phi_o$$

or

$$\tan \Delta\alpha = \tan \left(\phi_o' - \phi_o\right)$$

$$= \frac{\tan \phi_o' - \tan \phi_o}{1 + \tan \phi_o' \tan \phi_o}$$

Substituting the values for ϕ_o and ϕ_o' yields

$$\tan \Delta\alpha = \frac{\frac{J}{\pi x} \cos \alpha_{ta} - \frac{J}{\pi x} \mp \left(\frac{J}{\pi x}\right)^2 \sin \alpha_{ta}}{1 \pm \frac{J}{\pi x} \sin \alpha_{ta} + \left(\frac{J}{\pi x}\right)^2 \cos \alpha_{ta}}$$

For small values of α_{ta}, it may be assumed that $\cos \alpha_{ta} = 1$ and that $\frac{J}{\pi x} \sin \alpha_{ta}$ is small with respect to $1 + \left(\frac{J}{\pi x}\right)^2$ and the foregoing equation reduces to

$$\tan \Delta\alpha \approx \frac{\pm \left(\frac{J}{\pi x}\right)^2}{1 + \left(\frac{J}{\pi x}\right)^2} \sin \alpha_{ta}$$

or finally,

$$\Delta a \approx \pm \alpha_{ta} \sin^2 \phi_o$$

Assuming a section lift-curve slope c_{l_α} equal to $\frac{0.10}{\sqrt{1 - M_x^2}}$ per degree and using simple blade-element theory results in the following expression for the thrust-gradient coefficients resulting from inclination of the thrust axis:

$$\left(\frac{dC_T}{dx^2}\right)' = \left(\frac{dC_T}{dx^2}\right)_o \pm \frac{\sigma\pi^3 x^2 \sin^2\phi_o}{8 \cos \phi} \frac{0.10\alpha_{ta}}{\sqrt{1 - M_x^2}}$$

where $\left(\frac{dC_T}{dx^2}\right)_o$ is the thrust-gradient coefficient at zero-thrust-axis angle of attack and $\phi = \phi_o + \epsilon$.

COMPARISON WITH FLIGHT TESTS

In connection with the problem of the effect of angle of attack on propeller efficiency, a few tests were made of the Hamilton Standard No. 6507A-2 four-blade propeller, on an airplane of the type shown in figure 2, in which the thrust distributions on the right and left sides of the propeller disk and the thrust-axis angle of attack were measured. The blade-form curves of the 13-foot-diameter test propeller are given in figure 3.

In figure 4 are presented the thrust distributions, as determined from wake surveys, for three angles of attack. Other pertinent operating conditions are listed in table I. On these figures are also plotted the thrust-gradient coefficients as calculated according to the foregoing analysis.

The values of $\left(\dfrac{dC_T}{dx^2}\right)_0$ used in the calculations were determined from tests made under the same operating conditions but at substantially zero thrust-axis angle of attack obtained by appropriate use of the airplane flaps.

DISCUSSION

Examination of figure 4 shows that the effects of thrust-axis inclination on blade loading as calculated by the simple analysis presented are in excellent agreement with experiment. This circumstance, however, is possibly fortuitous since some factors are neglected in these calculations.

The change in interference angle ϵ due to the change in local disk loading was neglected (in these calculations ϵ was assumed constant at 2°). If the change in ϵ had been taken into account, the change in blade loading would have been underestimated by perhaps 10 to 15 percent for the test conditions. The exact amount cannot be determined since the procedure for calculating ϵ is based on a uniform disk loading and cannot be expected to apply in the present instance of nonuniform loading.

Also neglected in the calculations is the effect of upflow ahead of the wing. While this effect is believed to be small, it presumably is sufficient to compensate for the effect of the change in ϵ.

For operating conditions other than those for which data are available, or for different airplane configurations, the effects of the neglected factors may not be compensating. With the information available at this time, however, the simple analysis seems adequate for predicting the primary effects of inclination.

The fluctuating blade bending moments and stresses and the vibratory exciting forces are all proportional to the fluctuating load dL at any blade section. It is seen that,

$$dL = \left(c_{l_\alpha}\Delta\alpha\right)\left(\frac{1}{2}\rho W^2\right)b\ dr$$

$$= \left(c_{l_\alpha}\alpha_{ta}\ \sin^2\phi_o\right)\left(\frac{1}{2}\rho\frac{V^2}{\sin^2\phi_o}\right)b\ dr$$

$$= \alpha_{ta}qc_{l_\alpha}b\ dr$$

This equation indicates that the blade stresses are a function only of thrust-axis angle and the dynamic pressure of flight and are independent of the propeller operating conditions. The stresses cannot, for example, be reduced by a reduction in propeller speed since the reduction in blade-section speed is counterbalanced by the increase in the section angle-of-attack increment due to inclination.

Aside from the obviously complicated procedure of using articulated blades, the only evident means of limiting or reducing stresses for a given blade is by suitable thrust-axis orientation or by control of the range of thrust-axis angles.

For one airplane angle of attack the fluctuating blade stresses may be reduced to zero by proper choice of thrust-axis inclination. Similarly, in many instances the stresses may be reduced by a suitable compromise thrust-axis setting. However, as airplane speed ranges are increased and particularly for long-range operations in which the variation of airplane weight is large, it becomes increasingly difficult to effect any appreciable reduction in blade stresses by this means alone. Restriction of the thrust-axis angle-of-attack range may then be desirable. This result may be accomplished, particularly at low speeds, by use of the airplane flaps.

Not all the reduction, by use of flaps, of the airplane angle of attack is necessarily realized at the propellers. The increase in lift over the flapped portion of the wing will be accompanied by an increase in upwash ahead of the wing and a propeller located in this region will experience only part of the gross angle reduction.

Other interference effects may also seriously aggravate the problem. It may be anticipated, for example, that the inboard propellers on multiengine airplanes will be particularly susceptible. If located too close to the fuselage, the propeller tips will be affected by the flow around the fuselage and these effects, added to those already existing due to angle of attack, may become critical. Because even small angles of inclination are important at high speeds, it is probable that surveys to determine air-flow angles are necessary if accurate results are desired.

At low speeds, the effect of thrust-axis angle of attack may cause the blades to alternately stall and unstall. Similarly at high speeds shock stalling may occur. Normally, it is expected that these effects will cause no serious blade stresses and may, in fact, reduce the vibratory stress since any stall will reduce the positive peak load. It is possible though, that the fluctuating drag load associated with periodic stalling may produce chordwise vibration. Lack of appropriate airfoil data will, in any event, make estimates of stresses difficult; and neglect of the effects of stall should at least be conservative.

CONCLUDING REMARKS

An analysis based on simple geometry seems adequate for predicting the primary effects of thrust-axis inclination on disk loading. For accuracy in the general case, however, a more complete treatment may be necessary.

For a given propeller, the least complicated methods for reducing blade stresses due to inclination appear to be (a) compromise thrust-axis angle setting, and (b) restriction of the thrust-axis angle-of-attack range by use of flaps.

Because of the possible large effects of wing and fuselage interference, flow angles at the propeller should be determined by survey for most accurate results.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., April 19, 1948

REFERENCES

1. Vogeley, A. W.: Climb and High-Speed Tests of a Curtiss No. 714-1C2-12 Four-Blade Propeller on the Republic P-47C Airplane. NACA ACR No. L4L07, 1944.

2. Gardner, J. J.: Effect of Blade Loading on the Climb and High-Speed Performance of a Three-Blade Hamilton Standard No. 6507A-2 Propeller on a Republic P-47D Airplane. NACA MR No. L5G09a, 1945.

TABLE I.- FLIGHT TEST CONDITIONS

Figure	J	C_T	C_P	η	V (mph)	M	α_{ta} (deg)
3(a)	1.48	0.109	0.184	0.875	230	0.306	3.6
3(b)	1.43	.110	.182	.866	224	.303	4.5
3(c)	1.36	.116	.180	.876	212	.293	6.1

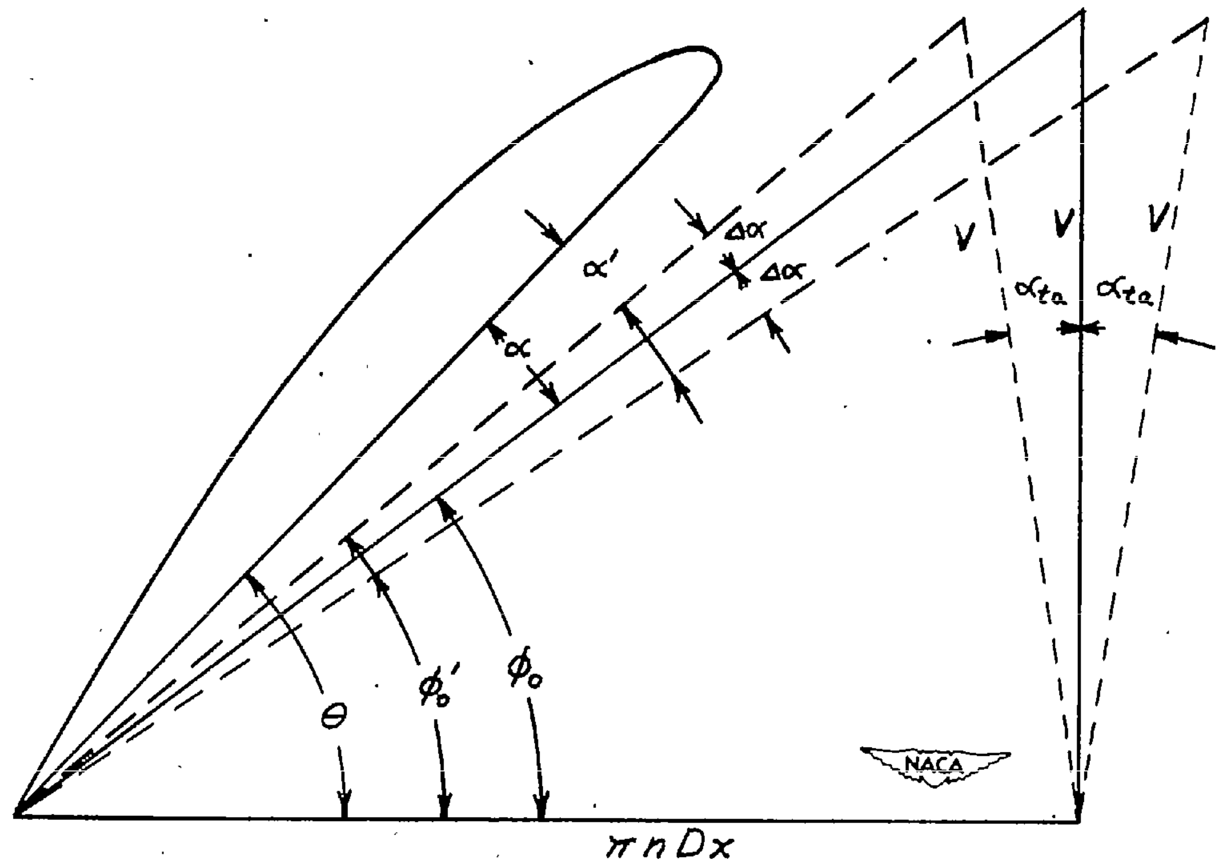


Figure 1.- Blade operating conditions as affected by thrust-axis inclination. (Interference angle σ omitted to avoid confusion.)

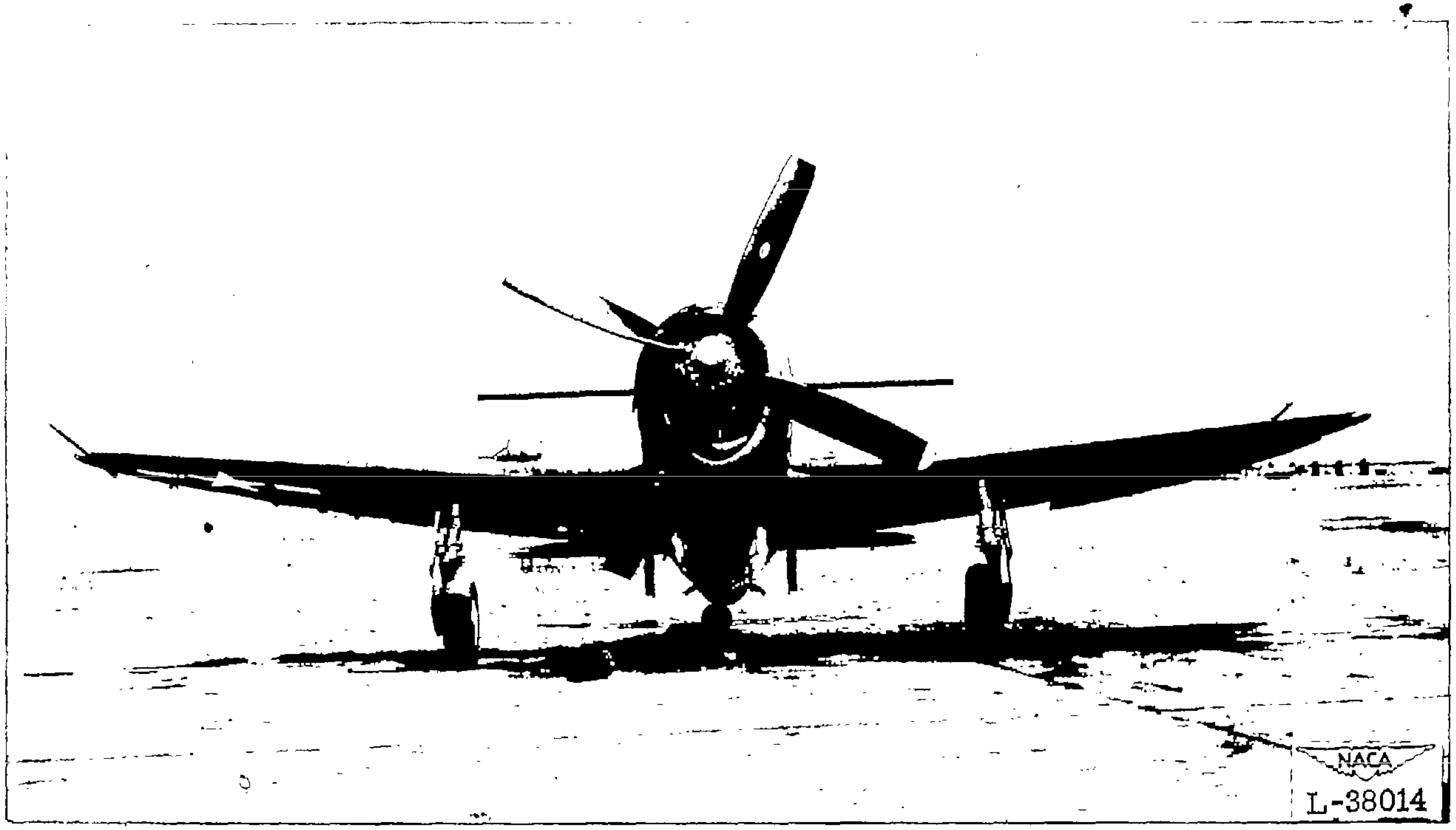


Figure 2.- Airplane with test propeller.

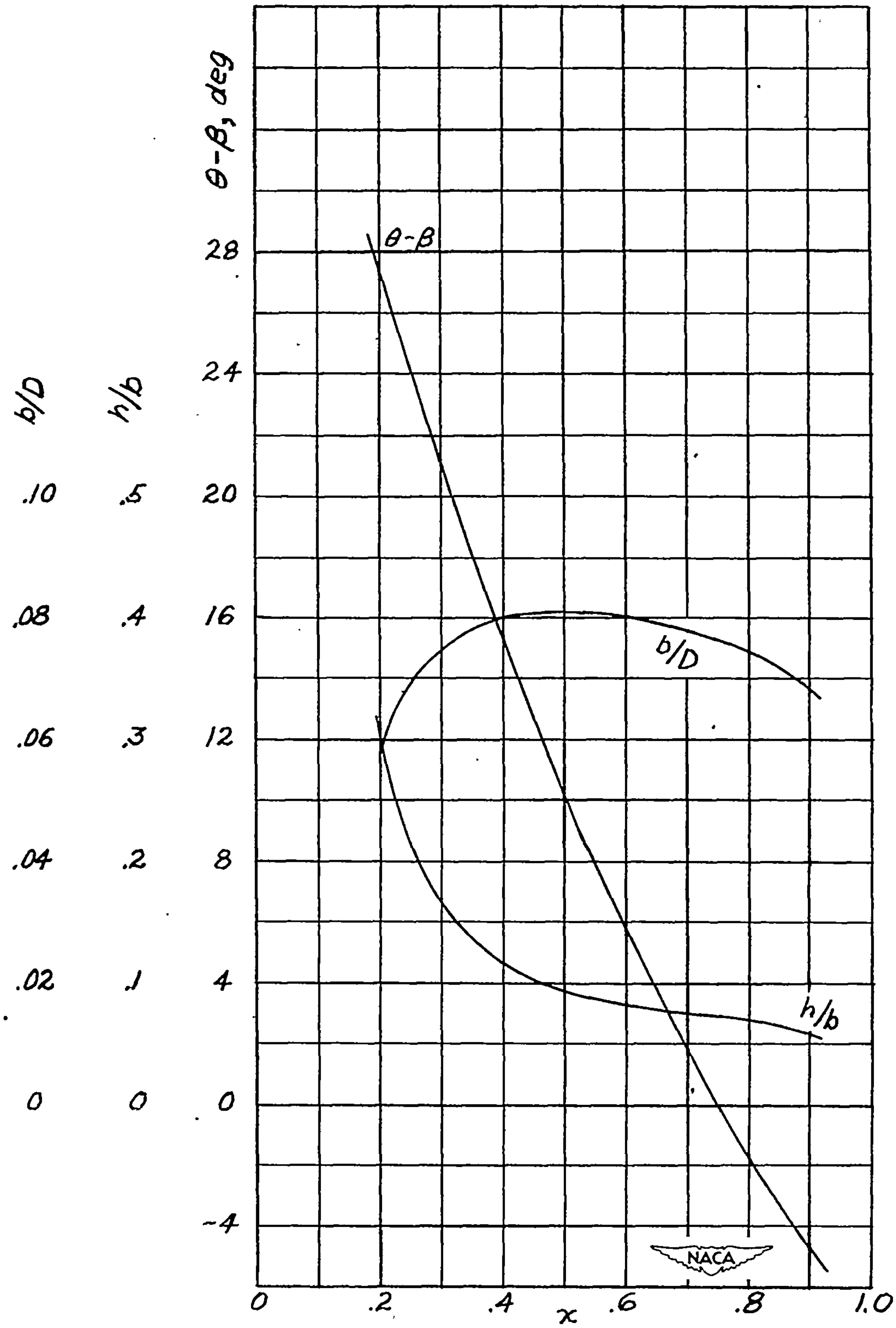


Figure 3.- Blade-form curves. Hamilton Standard No. 6507A-2 four-blade propeller. Diameter, 13 feet.

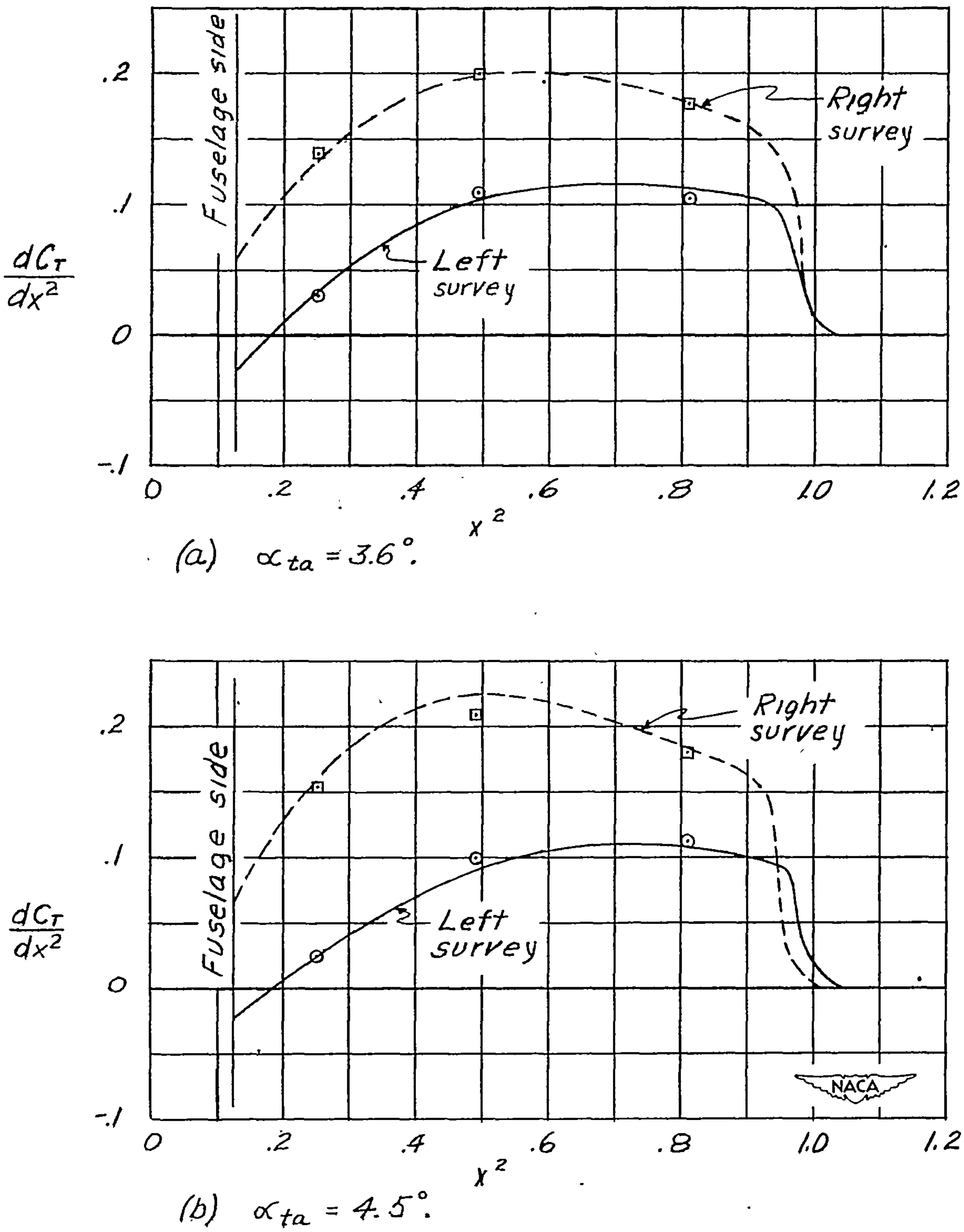
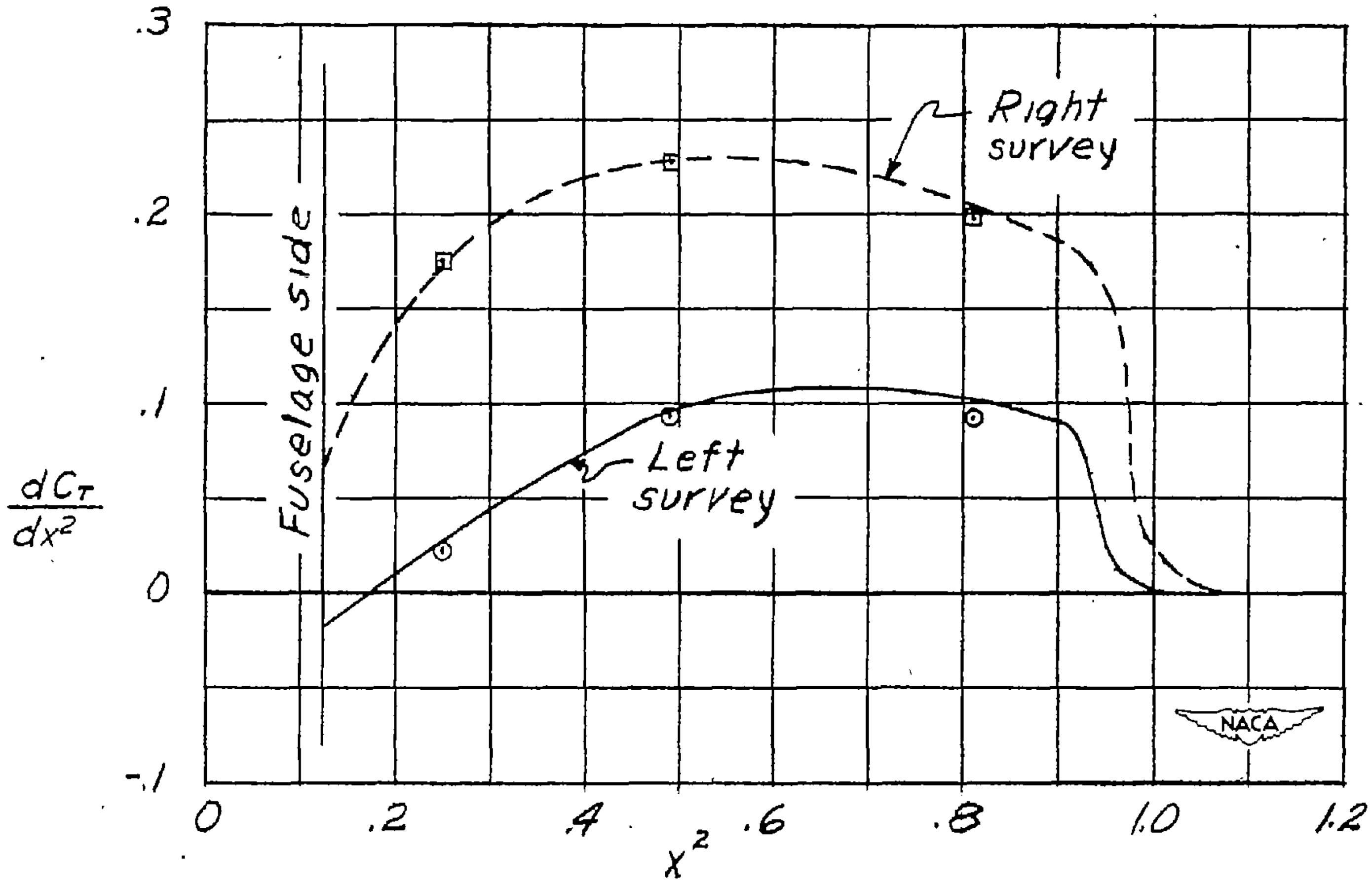


Figure 4.- Comparison of representative points calculated by present theory with experimental thrust distributions obtained by wake surveys.



(c) $\alpha_{ta} = 6.1°$.

Figure 4.- Concluded.