NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2040

ANALYSIS OF AN INDUCTION BLOWDOWN SUPERSONIC TUNNEL

By Jerold M. Bidwell

Langley Aeronautical Laboratory
Langley Air Force Base, Va.

Washington
April 1950

NATIONAL ADVISORY COMMITTEE FOR AERONAUTI

TECHNICAL NOTE 2040

ANALYSIS OF AN INDUCTION BLOWDOWN SUPERSONIC TUNNEL

By Jerold M. Bidwell

SUMMARY

General ejector equations and certain assumptions with regard to flow conditions and pressure losses are used to calculate the running times for one type of induction blowdown supersonic tunnel for Mach numbers of 1 to 2 and for reservoir pressures of 2 to 4 atmospheres. For a given reservoir and for given test-section size and conditions, the running time has a maximum with respect to the area and static pressure or stagnation temperature of the inducing jet.

The results show that for a given test-section area, the running time increases with test-section Mach number up to about 1.35, after which the running time decreases rapidly. A Reynolds number comparison of the induction tunnel with a direct-discharge tunnel shows that for the same running time the Reynolds number of the induction tunnel will be greater than that of the direct-discharge tunnel at Mach numbers up to 1.93 for a reservoir pressure of 4 atmospheres.

INTRODUCTION

Wind tunnels with large test sections for low supersonic speeds ($M = 1.0$ to $M = 2.0$) are in demand. When short running times are satisfactory and cost is a primary objective, it may be desirable to consider a blowdown type of wind tunnel. The feasibility of such an arrangement is specially enhanced if a large high-pressure reservoir is already available. If, in the interest of economy, a small power plant is used, a great deal of time will be required between runs to pump up the reservoir. Blowdown tunnels, of course, require special equipment if moisture condensation is serious.

Blowdown tunnels are generally of two types: the direct-discharge tunnel and the induction tunnel. The direct-discharge type of blowdown tunnel uses air from a high-pressure reservoir to provide the mass flow through the test chamber and to provide energy for the losses in the system. This type is relatively simple, but very wasteful in that, near the beginning of the run, the pressure of the reservoir air is far in excess of that required to overcome the losses in the tunnel. Throttling over such a large pressure drop results in relatively large degradation of energy. The induction type of blowdown tunnel uses air from a high-pressure reservoir only to provide energy to overcome the losses. Since higher pressures are used in the induction process, energy degradation due to throttling occurs to a much smaller degree. Actually, however, the induction process itself involves inherent mixing losses, and certain of the basic blowdown losses are also retained, so that no simple statement on the relative efficiencies of the two types can be made.

Analyses have been made of high-speed induction tunnels (for example, references 1 to 3), but these have been concerned primarily with ejector performance and little attention has been given to blowdown operation. Unpublished British results indicate that a great deal of development work has been done on a particular type of induction blowdown supersonic tunnel. (Reservoir pressures of 8 to 25 atm were used.)

In the present paper, general ejector equations (see reference 4 or 5) and certain assumptions with regard to flow conditions and pressure losses have been utilized to calculate the running times of induction blowdown supersonic tunnels operating in the mentioned Mach number range. The solution of the equations is found graphically. Calculations have been made for reservoir pressures from 2 to 4 atmospheres but the equations are valid for higher pressures. The results obtained are believed to be adequate to point out the Mach number range where use of the induction blowdown supersonic tunnel may be desirable, to show the effects of the important variables, and to indicate the approximate proportions and running time of a particular design. The results are compared with a similarly arranged direct-discharge blowdown supersonic tunnel.

SYMBOLS

A	cross-sectional area, square feet
a	velocity of sound, feet per second
B	gas constant for air (53.3 ft/°F absolute)
g	acceleration of gravity (32.17 ft/sec^2)
H	total pressure, pounds per square foot

K running-time coefficient $\left(\frac{tA_aT_i}{V}\sqrt{\frac{Bg}{T_\Delta}}\right)$

l typical length, feet $\left(\sqrt{A}\right)$

M Mach number

m mass of gas, slugs

n exponent of polytropic expansion in reservoir

P absolute static pressure, pounds per square foot

Q rate of mass flow of gas leaving reservoir, slugs per second

R Reynolds number

T absolute temperature, °F absolute

t wind-tunnel running time, second

V volume of reservoir, cubic feet

v local velocity, feet per second

X jet-area ratio (A_a/A_j)

Y, Z parameters in equations (1) and (2)

γ ratio of specific heats (γ = 1.40 for air)

η pressure-recovery factor of subsonic diffuser

μ coefficient of dynamic viscosity, slugs per foot-second

ρ mass density of air, slugs per cubic foot

Subscripts:

a exit of induced-air nozzle (induction tunnel); exit of test section (direct-discharge tunnel)

D direct-discharge tunnel

e exit of mixing tube

I induction tunnel

i initial conditions of reservoir

j exit of inducing-air nozzle

S settling chamber

T throat of diffuser

α minimum section of inducing-air nozzle (induction tunnel); first minimum section in nozzles forming test section (direct-discharge tunnel)

Δ reference condition (atmosphere)

Superscripts:

condition after normal shock

METHOD OF ANALYSIS

Induction Blowdown Supersonic Tunnel

The arrangement of the induction blowdown supersonic tunnel used in the analysis is shown schematically in figure 1. High-pressure air from the reservoir is throttled by a pressure-regulating valve into the settling chamber. The air is heated at constant pressure and allowed to pass through the nozzle into the mixing chamber. The air passing through j induces a flow at a. The two streams of air mix in the constant-area mixing tube and pass through the diffuser to the exit.

Certain assumptions are necessary to formulate completely the problem. Assumptions that pertain to the working medium are:

(1) Both the inducing and the induced jets are perfect diatomic gases ($P = \rho gBT$, $\gamma = 1.40$).

(2) Expansion in the reservoir takes place polytropically. $P = \text{Constant}(\rho^n)$. The mode of expansion in the reservoir varies both with running time and with type of installation. The exponent of polytropic expansion ($1.0 \leqq n < 1.40$; for the calculations, $n = 1.2$) approximately takes into account the heat transfer between the reservoir walls and the contained air during the blowdown. (Choice of the exponent determines the quantity of air remaining in the reservoir at the end of a run, for a given settling-chamber pressure and temperature.)

(3) The temperature and pressure in the settling chamber are constant throughout the run. A constant settling-chamber pressure during the run may be maintained by a pressure-regulating valve installed between the reservoir and the settling chamber, and a constant temperature may be maintained similarly by heat addition.

Assumptions that pertain to the losses in the system are:

(4) The flow throughout the system is one-dimensional except in the mixing tube where the flow is necessarily one-dimensional only at the exit (complete mixing of the jets to a uniform stream).

(5) Flow through the nozzles is isentropic.

(6) The walls of the mixing tube and diffuser are perfectly insulated.

(7) Pressure losses due to friction in the mixing tube and in the supersonic diffuser are neglected.

(8) A normal shock stands in the throat of the diffuser.

(9) The pressure-recovery factor of the subsonic diffuser is taken as 80 percent in the calculations.

The losses in the system greatly affect the running time of the tunnel inasmuch as they determine the lowest acceptable settling-chamber pressure. The only system losses are in the normal shock and in the subsonic diffuser. Although pressure losses due to friction in the mixing tube and in the supersonic diffuser are neglected to facilitate the computation, it should not be inferred that these losses are negligible. It should be noted that the throat of a diffuser is an unstable position for a normal shock. In practice a small increase in pressure would be required to move the shock downstream to a stable position. In applying the results to a particular design, caution should also be taken with regard to the pressure recovery, inasmuch as smaller values of pressure-recovery factor are quite probable. The assumed normal-shock loss is, however, conservative in that an adjustment of the throat area after the flow has started might appreciably reduce the required settling-chamber pressure.

The ejector-diffuser system.- The two inlet nozzles and the mixing tube comprise an ejector. In reference 4 Ellerbrock gives equations for the pressure and Mach number of the flow at e in terms of the flow at a and j. The equations require only assumptions (1), (4), (5), and (6). In accordance with assumption (7), the pressure losses due to friction in the mixing tube are neglected. Equations (12), (13), (14), and (15) of reference 4 with the friction term omitted and the symbols revised are for the supersonic case:

$$\frac{P_e}{H_\Delta} = \left(\frac{P_a}{H_\Delta}\right)\left(\frac{\gamma}{\gamma + 1}\right)\left(\frac{1}{X + 1}\right)\left[Y - \sqrt{\gamma^2 Y^2 - 2(\gamma + 1)Z}\right] \qquad (1)$$

$$M_e^2 = \frac{\gamma Y + \sqrt{\gamma^2 Y^2 - 2(\gamma + 1)Z}}{\gamma Y - \gamma\sqrt{\gamma^2 Y^2 - 2(\gamma + 1)Z}} \qquad (2)$$

where

$$Y = XM_a^2 + M_j^2 \frac{P_j}{P_a} + \frac{1}{\gamma}\left(X + \frac{P_j}{P_a}\right)$$

and

$$Z = \left(XM_a + \frac{P_j}{P_a}\sqrt{\frac{T_a}{T_j}}\, M_j\right)\left[XM_a\left(1 + \frac{\gamma - 1}{2} M_a^2\right) + \frac{P_j}{P_a}\sqrt{\frac{T_j}{T_a}}\, M_j\left(1 + \frac{\gamma - 1}{2} M_j^2\right)\right]$$

From isentropic relationships,

$$\frac{A_e}{A_T} = \frac{M_T}{M_e}\left(\frac{\frac{\gamma - 1}{2} M_e^2 + 1}{\frac{\gamma - 1}{2} M_T^2 + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (3a)$$

$$\frac{H_S}{H_\Delta} = \frac{P_j}{P_a}\left(\frac{\frac{\gamma - 1}{2} M_j^2 + 1}{\frac{\gamma - 1}{2} M_a^2 + 1}\right)^{\frac{\gamma}{\gamma-1}} \qquad (3b)$$

$$\frac{T_S}{T_\Delta} = \frac{T_j}{T_a}\left(\frac{\frac{\gamma - 1}{2} M_j^2 + 1}{\frac{\gamma - 1}{2} M_a^2 + 1}\right) \qquad (3c)$$

In the starting condition the shock is ahead of the diffuser in the vicinity of section e. This condition determines the minimum area of the diffuser A_T. The contraction ratio of the diffuser is taken to be the maximum possible for starting the supersonic flow. (See reference 6.) The equation for the maximum contraction ratio is given by

$$\frac{A_e}{A_T} = \frac{A_a + A_j}{A_T}$$

$$= \left(\frac{\frac{\gamma - 1}{2}}{\frac{\gamma - 1}{2} M_e^2 + 1}\right)^{1/2} \frac{\left(\frac{\gamma + 1}{\gamma - 1} M_e^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{\left(\frac{2\gamma}{\gamma - 1} M_e^2 - 1\right)^{\frac{1}{\gamma-1}}} \tag{4}$$

The rise in static pressure through the diffuser in terms of the difference between the total pressure and static pressure in the subsonic portion of the stream is defined as the pressure-recovery factor η.

$$\eta = \frac{H_\Delta - P_T'}{H_T' - P_T'}$$

From normal-shock and isentropic relationships, the ratio of mixing-tube static pressure to the static pressure required after diffusion is P_e/H_Δ,

$$\frac{P_e}{H_\Delta} = \frac{P_e}{P_T} \frac{P_T}{P_T'} \frac{P_T'}{H_\Delta}$$

$$= \left[\left(\frac{\frac{\gamma - 1}{2} M_T^2 + 1}{\frac{\gamma - 1}{2} M_e^2 + 1}\right)^{\frac{\gamma}{\gamma-1}}\right] \left(\frac{\frac{\gamma + 1}{2\gamma}}{M_T^2 - \frac{\gamma - 1}{2\gamma}}\right) \left(\frac{1}{\eta\left\{1 + \frac{\gamma - 1}{2}\left[\left(\frac{\gamma + 1}{2\gamma}\right)^2 \frac{1}{M_T^2 - \frac{\gamma - 1}{2\gamma}} + \frac{\gamma - 1}{2\gamma}\right]\right\}^{\frac{\gamma}{\gamma-1}} + (1 - \eta)}\right) \tag{5}$$

Equations (1) to (5) define the flow in the ejector-diffuser system. A solution of these equations is of the form:

$$\frac{H_S}{H_\Delta} = f\left(X, M_a, \frac{T_S}{T_\Delta}, \frac{P_j}{P_a}, \eta\right)$$

If η = Constant = 0.8 (assumption (9)),

$$\frac{H_S}{H_\Delta} = f_1\left(X, M_a, \frac{T_S}{T_\Delta}, \frac{P_j}{P_a}\right) \tag{6}$$

Although this function cannot be expressed algebraically, H_S/H_Δ can be found as a function of these variables by graphical solution. The quantity H_S/H_Δ when related to the reservoir pressure determines the running time of the system.

Running-time analysis.- The running time can be expressed as the running-time coefficient. (See appendix A.)

$$\frac{tA_aT_i}{V}\sqrt{\frac{Bg}{T_\Delta}} = K = \frac{X\sqrt{\frac{T_S}{T_\Delta}}\left(\frac{H_i}{H_\Delta}\right)\left(\frac{\gamma - 1}{2} M_a{}^2 + 1\right)^{\frac{1}{\gamma-1}}\left[1 - \left(\frac{\frac{H_S}{H_\Delta}}{\frac{H_i}{H_\Delta}}\right)^{\frac{1}{n}}\right]}{\left(\frac{P_j}{P_a}\right)^{\frac{1}{\gamma}}\left(\frac{H_S}{H_\Delta}\right)^{\frac{\gamma-1}{\gamma}}\sqrt{\frac{2\gamma}{\gamma - 1}\left[1 - \left(\frac{\frac{P_j}{P_a}}{\frac{H_S}{H_\Delta}}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{1}{\frac{\gamma - 1}{2} M_a{}^2 + 1}\right)\right]}} \tag{7}$$

In functional form, where γ and n are constants, equation (7) becomes

$$K = f\left(X, M_a, \frac{T_S}{T_\Delta}, \frac{H_i}{H_\Delta}, \frac{H_S}{H_\Delta}, \frac{P_j}{P_a}\right) \tag{8}$$

Substitution of H_S/H_Δ from equation (6) yields, finally, the following equation for the running-time coefficient as a function of five independent variables:

$$K = f\left(X, M_a, \frac{H_i}{H_\Delta}, \frac{P_j}{P_a}, \frac{T_S}{T_\Delta}\right)$$

The value of H_S/H_Δ corresponding to a specified set of these variables is given by equation (6). In general, for a given problem, M_a and H_i/H_Δ will be specified, so that it will be desired to determine the values of the three remaining variables such that the running-time factor will be a maximum. Actually, the amount of work involved in determining such an absolute maximum for three independent variables (and for a range of values of M_a and H_i/H_Δ) is prohibitive, and the analysis was therefore limited to two sets of computations. In the first, T_S was assumed equal to T_Δ, and the maximum running-time coefficient with respect to both X and P_j/P_a was determined. In the second, P_j was assumed equal to P_a, and the maximum running-time factor with respect to X and T_S/T_Δ was determined.

Direct-Discharge Blowdown Supersonic Tunnel

A direct-discharge blowdown tunnel is shown schematically in figure 1. High-pressure air from the reservoir R is throttled through a pressure-regulating valve into the settling chamber S. The air is heated at constant pressure and allowed to pass through the nozzle to the test chamber a and thence through the supersonic-subsonic diffuser to the atmosphere. The assumptions used are comparable to those of the induction-tunnel analysis.

The maximum contraction ratio for starting the supersonic flow is

$$\frac{A_a}{A_T} = \left(\frac{\frac{\gamma - 1}{2}}{\frac{\gamma - 1}{2} M_a^2 + 1}\right)^{\frac{1}{2}} \frac{\left(\frac{\gamma + 1}{\gamma - 1} M_a^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{\left(\frac{2\gamma}{\gamma - 1} M_a^2 - 1\right)^{\frac{1}{\gamma-1}}} \tag{9}$$

(Compare equation (4).)

From normal-shock and isentropic relationships,

$$\frac{H_S}{H_\Delta} = \frac{H_S}{P_T}\frac{P_T}{P_T'}\frac{P_T'}{H_\Delta}$$

$$= \left(\frac{\gamma - 1}{2} M_T^2 + 1\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{\frac{\gamma + 1}{2\gamma}}{M_T^2 - \frac{\gamma - 1}{2\gamma}}\right) \left(\frac{1}{\eta\left\{1 + \frac{\gamma - 1}{2}\left[\left(\frac{\gamma + 1}{2\gamma}\right)^2 \frac{1}{M_T^2 - \frac{\gamma - 1}{2\gamma}} + \frac{\gamma - 1}{2\gamma}\right]\right\}^{\frac{\gamma}{\gamma-1}} + (1 - \eta)}\right) \tag{10}$$

The running-time coefficient becomes

$$\frac{tA_aT_1}{V}\sqrt{\frac{Bg}{T_\Delta}} = K = \frac{\sqrt{\frac{T_S}{T_\Delta}}}{\sqrt{\gamma}M_a}\frac{H_i}{H_\Delta}\frac{H_\Delta}{H_S}\left(\frac{\gamma - 1}{2} M_a^2 + 1\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left[1 - \left(\frac{\frac{H_S}{H_\Delta}}{\frac{H_i}{H_\Delta}}\right)^{\frac{1}{n}}\right] \tag{11}$$

(See appendix B.)

RESULTS AND DISCUSSION

Induction Tunnel with $T_S = T_\Delta$

The running-time coefficient, maximum with respect to X and P_j/P_a, obtained for the induction tunnel for the case $T_S = T_\Delta$ is plotted in figure 2 as a function of Mach number and reservoir pressure. The corresponding values of settling-chamber pressure, jet-area ratio, and jet static-pressure ratio are presented in figures 3 and 4. The calculations show that these optimum sets of values X, P_j/P_a, and, hence, H_S/H_Δ are not well defined. For example, the broad maximum in the variation of running-time coefficient, maximum with respect to X, with P_j/P_a is shown in figure 5. Because of these broad maximums, inclusion of pressure losses due to friction might considerably alter the design values.

The running-time coefficient increases with Mach number up to about $M_a = 1.35$. This variation merely reflects the decreased mass flow in the main stream with increasing Mach number. When M_a is increased beyond about $M_a = 1.35$, the shock losses become increasingly important and the running-time coefficient decreases rapidly.

Induction Tunnel with $P_j = P_a$

For the induction blowdown tunnel with $P_j = P_a$, values of K (maximum with respect to X) and the corresponding values of X and $H_S/H_\triangle$ are given in figures 6, 7, and 8, respectively, as functions of $T_S/T_\triangle$ with $H_i/H_\triangle$ and M_a as parameters.

There is an optimum value of $T_S/T_\triangle$ but the increase in K with increasing $T_S/T_\triangle$ is small compared with the increase in K with increasing $H_i/H_\triangle$. (See fig. 6.) When $H_i/H_\triangle$ is increased, X and $H_S/H_\triangle$ are also increased as for the case of $T_S = T_\triangle$. (See figs. 7 and 8.)

Direct-Discharge Tunnel

The solution of equations (9), (10), and (11) for $T_S = T_\triangle$ for the direct-discharge tunnel are shown as the dashed lines in figure 2, which gives the running-time coefficient K as a function of test-section Mach number M_a with reservoir pressure $H_i/H_\triangle$ as a parameter.

A comparison of the curves for the induction tunnel and the direct-discharge tunnel shows that at low supersonic Mach numbers the running-time coefficient of the induction tunnel is much greater. The Mach numbers for which the running-time coefficients of the induction tunnel are higher increase as the reservoir pressure increases.

Reynolds Number Comparison of Induction Tunnel

and Direct-Discharge Tunnel

One of the reasons for employing large test sections is to permit tests at high Reynolds numbers. The density of the air in the test section is higher in the direct-discharge tunnel than in the induction tunnel. Over a certain range of Mach number, however, the induction tunnel permits a much larger test section than the comparable direct-discharge tunnel for a given running time. It is of interest, therefore, to determine which arrangement would permit the highest Reynolds number.

The ratio of the Reynolds number of the induction tunnel to the Reynolds number of the direct-discharge tunnel is R_I/R_D,

$$\frac{R_I}{R_D} = \left(\frac{V_{a_I}\rho_{a_I}l_I}{\mu_I}\right)\left(\frac{\mu_D}{V_{a_D}\rho_{a_D}l_D}\right) \tag{12}$$

As long as this ratio is greater than 1.0, the induction tunnel will offer higher Reynolds numbers than the direct-discharge tunnel.

Consider the case where $T_S = T_\Delta$. Because the test-section Mach numbers are equal, the velocities in the two test sections are equal. Viscosity is a function of the temperature; hence, the viscosity of the flow in the two test sections is equal. Equation (12) becomes

$$\frac{R_I}{R_D} = \frac{P_{a_I}}{P_{a_D}}\frac{l_I}{l_D} \tag{13}$$

From isentropic relationships,

$$\frac{P_{a_I}}{H_\Delta} = \left(1 + \frac{\gamma - 1}{2} M_a{}^2\right)^{-\frac{\gamma}{\gamma-1}} = \frac{P_{a_D}}{H_\Delta}\left(\frac{H_\Delta}{H_S}\right)_D$$

or

$$\frac{P_{a_I}}{P_{a_D}} = \left(\frac{H_\Delta}{H_S}\right)_D$$

If the typical length l is taken as the square root of the test-section area,

$$\frac{l_I}{l_D} = \sqrt{\frac{A_{a_I}}{A_{a_D}}}$$

or

$$\frac{l_I}{l_D} = \sqrt{\frac{\left(\frac{tA_aT_i}{V}\sqrt{\frac{Bg}{T_\Delta}}\right)_I}{\left(\frac{tA_aT_i}{V}\sqrt{\frac{Bg}{T_\Delta}}\right)_D}} = \sqrt{\frac{K_I}{K_D}}$$

since $\frac{tT_i}{V}\sqrt{\frac{Bg}{T_\Delta}}$ is assumed to be the same for both tunnels.

For any test-section Mach number, the running-time factors K can be obtained from figure 2 and the pressure ratio $(H_S/H_\Delta)_D$ from the isentropic relationships (equation 3(a)) and equations (9) and (10).

Equation (13) becomes finally

$$\frac{R_I}{R_D} = \left(\frac{H_\Delta}{H_S}\right)_D \sqrt{\frac{K_I}{K_D}} \tag{14}$$

Equation (14) has been evaluated and plotted against M_a for $T_S = T_\Delta$ in figure 9. The induction tunnel is definitely advantageous when the Reynolds number at low supersonic Mach numbers is considered. The highest Mach number at which the induction tunnel is more favorable increases with reservoir pressure and becomes 1.93 for a reservoir pressure of 4 atmospheres. The Reynolds number ratio increases at a decreasing rate with increasing reservoir pressure although no limit is indicated.

When T_S/T_Δ is greater than one, the running-time coefficient of the induction tunnel increases without affecting the flow in the test section. When the tunnels are considered with T_S/T_Δ greater than one in the induction tunnel, therefore, the Reynolds number ratio becomes larger.

CONCLUSIONS

An analysis has been made of the running time of one type of induction blowdown supersonic tunnel based on the following assumptions: (1) Both the inducing and the induced jets are perfect diatomic gases, (2) expansion in the reservoir takes place polytropically, (3) the temperature and pressure in the settling chamber are constant throughout the run, (4) the flow throughout the system is one-dimensional except in the mixing tube where the flow is necessarily one-dimensional only at the exit, (5) flow through the nozzles is isentropic, (6) the walls of the mixing tube and diffuser are perfectly insulated, (7) pressure losses due to friction in the mixing tube and in the supersonic diffuser are neglected, and (8) a normal shock stands in the throat of the diffuser. Calculations of the running times for the induction tunnel were made for Mach numbers of 1 to 2 and for reservoir pressures of 2 to 4 atmospheres with a pressure-recovery factor of 80 percent in the subsonic diffuser and with a value of the polytropic index of expansion in the reservoir of 1.2.

For given reservoir and test-section size and conditions, the running time has a maximum with respect to jet area and jet static pressure or jet stagnation temperature.

For a given test-section area, the running time coefficient increases with test-section Mach number M_a up to about 1.35. When M_a is increased beyond this value, the running-time coefficient decreases rapidly.

When the induction tunnel is compared with the direct-discharge blowdown supersonic tunnel, for the same running time the Reynolds number of the induction tunnel will be greater than that of the direct-discharge tunnel at Mach numbers up to 1.93 for a reservoir pressure of 4 atmospheres.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Air Force Base, Va., November 3, 1949

APPENDIX A

DERIVATION OF THE RUNNING-TIME COEFFICIENT FOR INDUCTION BLOWDOWN SUPERSONIC TUNNEL

The mass of air in the reservoir

$$m = \rho V \tag{A1}$$

where

m air mass

ρ air density

V volume of reservoir

The rate of change of mass Q is equal to the rate of discharge through the throat α of the supersonic effuser j. Then,

$$dt = \frac{-V\ d\rho}{Q} = \frac{-V\ d\rho}{A_\alpha a_S \rho_S \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \tag{A2}$$

By using assumption (2) that the expansion in the reservoir takes place polytropically and integrating between the initial condition and the condition where the reservoir pressure is equal to the settling-chamber pressure H_S,

$$\frac{tA_\alpha T_i}{V}\sqrt{\frac{Bg}{T_\Delta}} = \frac{\sqrt{\frac{T_S}{T_\Delta}}}{\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\sqrt{\gamma}}\left[\frac{H_i}{H_S} - \left(\frac{H_i}{H_S}\right)^{\frac{n-1}{n}}\right] \tag{A3}$$

By using isentropic relationships, A_α can be obtained in terms of other quantities as

$$A_\alpha = \frac{A_a}{X}\left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left(\frac{\frac{P_j}{P_a}}{\frac{H_S}{H_\Delta}}\right)^{\frac{1}{\gamma}}\left(\frac{1}{\frac{\gamma - 1}{2} M_a^2 + 1}\right)^{\frac{1}{\gamma-1}}\sqrt{\frac{2}{\gamma - 1}\left[1 - \left(\frac{\frac{P_j}{P_a}}{\frac{H_S}{H_\Delta}}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{1}{\frac{\gamma - 1}{2} M_a^2 + 1}\right)\right]} \qquad \text{(A4)}$$

By substituting equation (A4) in equation (A3), equation (7) is obtained.

$$K = \frac{X\sqrt{\frac{T_S}{T_\Delta}}\left(\frac{H_i}{H_\Delta}\right)\left(\frac{\gamma - 1}{2} M_a^2 + 1\right)^{\frac{1}{\gamma-1}}\left[1 - \left(\frac{\frac{H_S}{H_\Delta}}{\frac{H_i}{H_\Delta}}\right)^{\frac{1}{n}}\right]}{\left(\frac{P_j}{P_a}\right)^{\frac{1}{\gamma}}\left(\frac{H_S}{H_\Delta}\right)^{\frac{\gamma-1}{\gamma}}\sqrt{\frac{2\gamma}{\gamma - 1}\left[1 - \left(\frac{\frac{P_j}{P_a}}{\frac{H_S}{H_\Delta}}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{1}{\frac{\gamma - 1}{2} M_a^2 + 1}\right)\right]}}$$

APPENDIX B

DERIVATION OF THE RUNNING-TIME COEFFICIENT FOR DIRECT-DISCHARGE BLOWDOWN SUPERSONIC TUNNEL

The mass of air in the reservoir

$$m = \rho V \tag{B1}$$

The rate of change of mass Q is equal to the rate of discharge through the throat α of the supersonic effuser. Then,

$$dt = \frac{-V\ d\rho}{Q} = \frac{-V\ d\rho}{A_\alpha a_S \rho_S \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \tag{B2}$$

By assuming that the expansion in the reservoir takes place polytropically and integrating between the initial condition and the condition where the reservoir pressure is equal to the settling-chamber pressure H_S,

$$\frac{tA_\alpha T_i}{V}\sqrt{\frac{Bg}{T_\Delta}} = \frac{\sqrt{\frac{T_S}{T_\Delta}}}{\sqrt{\gamma}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}\left[\frac{H_i}{H_S} - \left(\frac{H_i}{H_S}\right)^{\frac{n-1}{n}}\right] \tag{B3}$$

By using isentropic relationships,

$$A_\alpha = A_a\left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{M_a}{\left(\frac{\gamma - 1}{2} M_a{}^2 + 1\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \tag{B4}$$

By substituting equation (B4) in equation (B3), equation (11) is obtained.

$$K = \frac{\sqrt{\frac{T_S}{T_\Delta}}}{\sqrt{\gamma}M_a}\left(\frac{\gamma - 1}{2}M_a^2 + 1\right)^{\frac{\gamma+1}{2(\gamma-1)}}\frac{H_i}{H_\Delta}\frac{H_\Delta}{H_S}\left[1 - \left(\frac{\frac{H_S}{H_\Delta}}{\frac{H_i}{H_\Delta}}\right)^{\frac{1}{n}}\right]$$

REFERENCES

1. Crocco, Luigi: High-Speed Wind Tunnels. Translation No. 366, Materiel Command, U. S. Army Air Corps, July 30, 1943.

2. Ackeret, J.: High-Speed Wind Tunnels. NACA TM 808, 1936.

3. Winter, H.: On the Use of Jet Drives for Wind Tunnels of High Velocity. Translation 219, David Taylor Model Basin, Navy Dept., April 1947.

4. Ellerbrock, Herman H., Jr.: General Treatment of Compressible Flow in Ejectors and Example of Its Application to Problem of Effect of Ejector Addition on Thrust Jet-Propulsion Units. NACA RM L6L23, 1947.

5. Propulsion Branch: A Theoretical and Experimental Investigation of Flow Induction Systems Applicable to Wind Tunnels. Rep. No. AD-526-A-1, Cornell Aero. Lab., Feb. 18, 1948.

6. Kantrowitz, Arthur, and Donaldson, Coleman duP.: Preliminary Investigation of Supersonic Diffusers. NACA ACR L5D20, 1945.

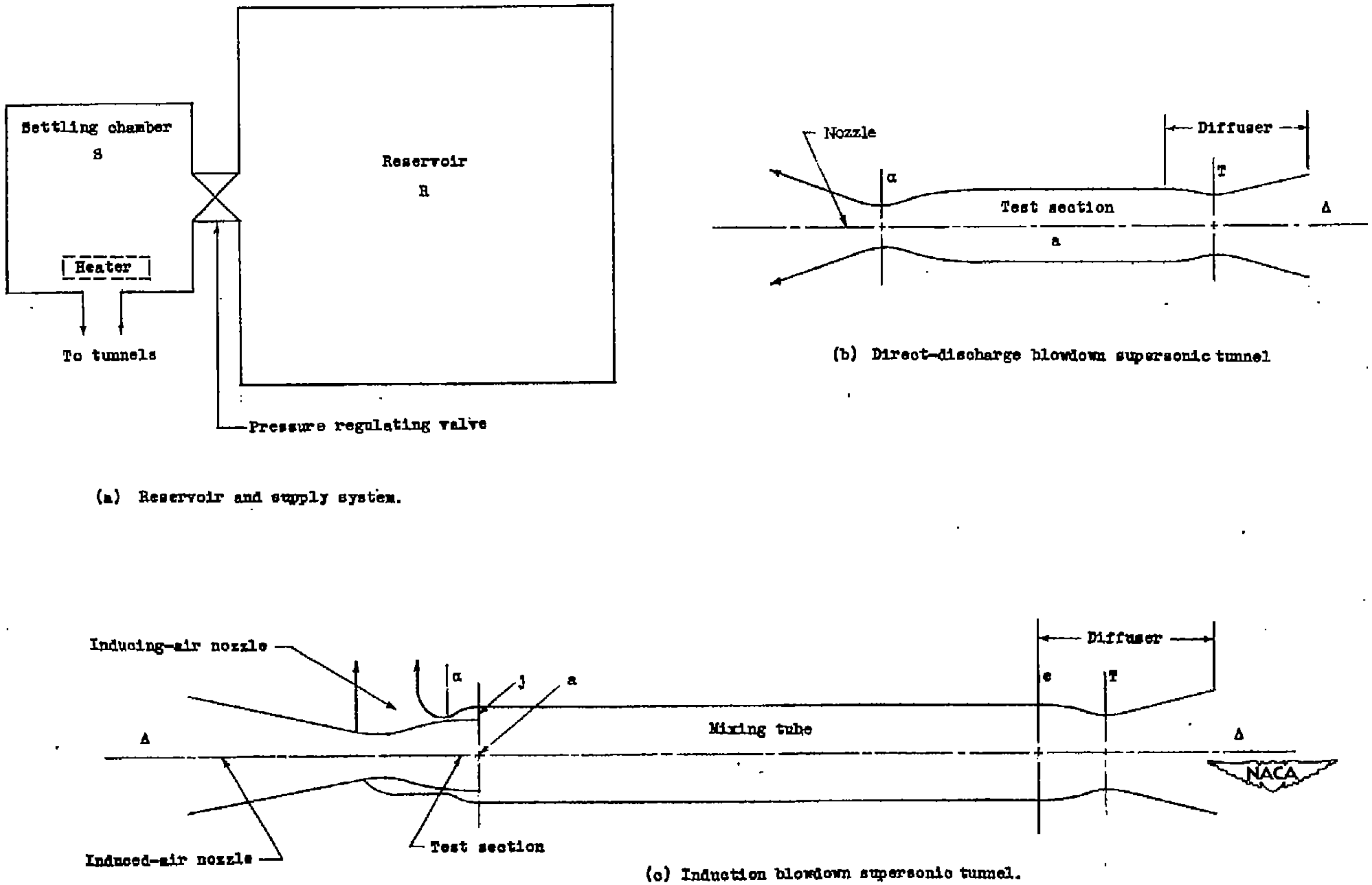

Figure 1.- Schematic drawings of the induction blowdown supersonic tunnel and a direct-discharge blowdown supersonic tunnel.

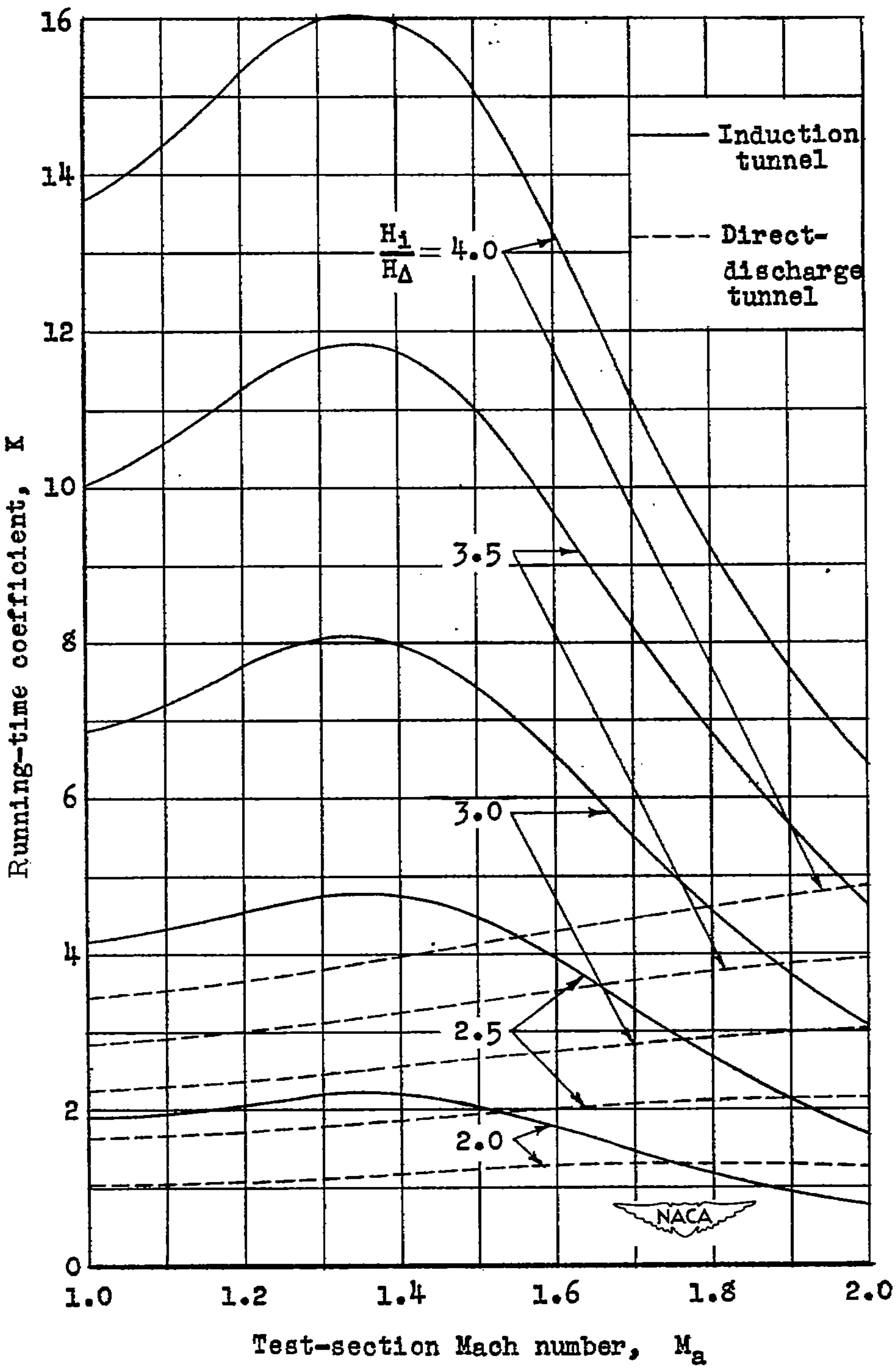

Figure 2.- Variation of running-time coefficient with test-section Mach number for several values of reservoir pressure. $\frac{T_S}{T_\Delta} = 1.0$. Running-time coefficient is a maximum with respect to X and P_j/P_a.

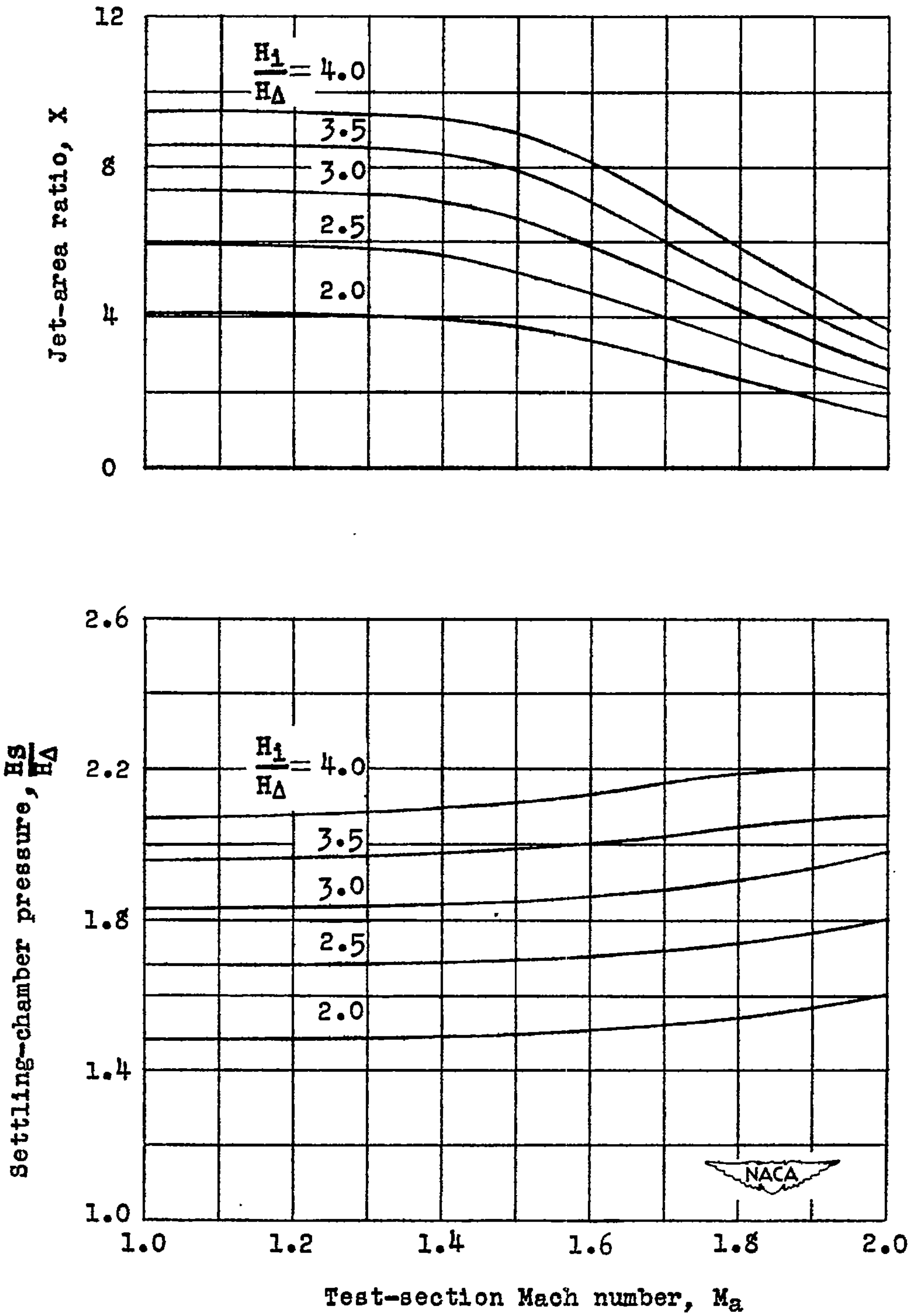

Figure 3.- Variation with test-section Mach number of the jet-area ratio and settling-chamber pressure that correspond to running-time coefficient (maximum with respect to X and P_j/P_a) for several values of reservoir pressure. $\frac{T_S}{T_\Delta} = 1.0$.

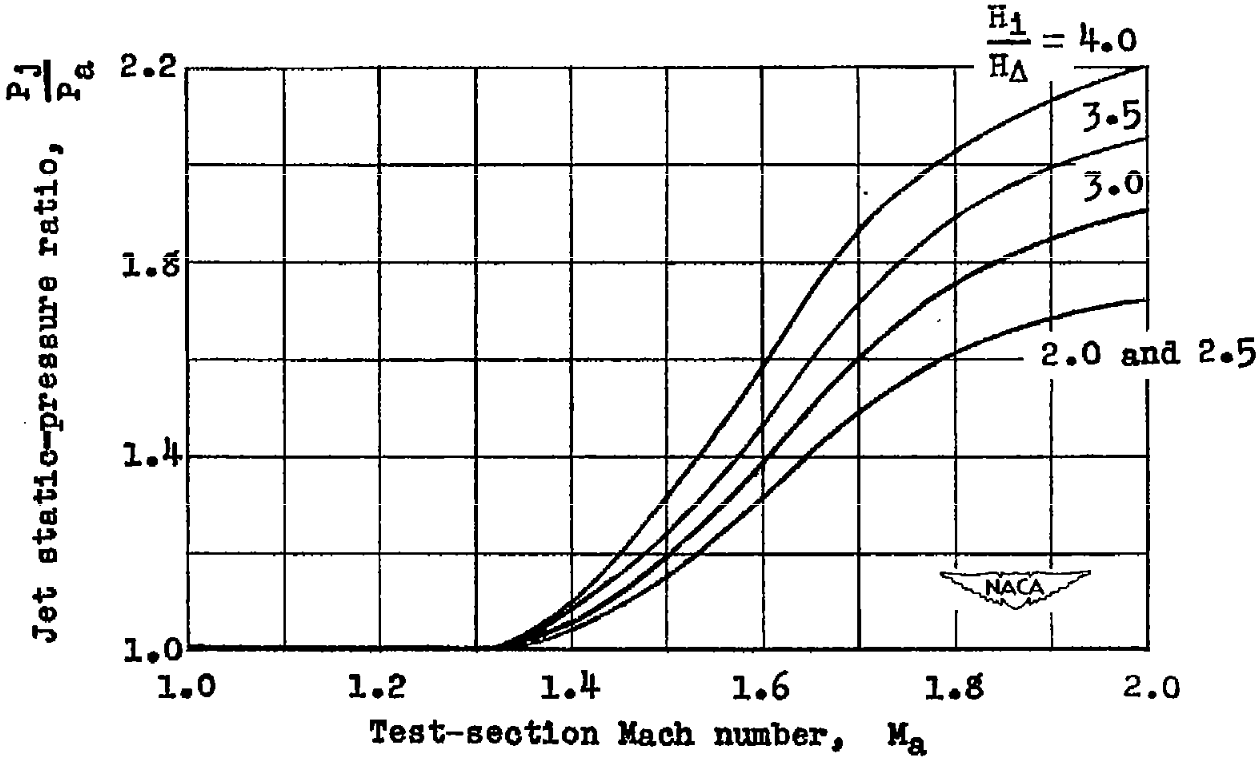

Figure 4.- Variation with test-section Mach number of jet static-pressure ratio that corresponds to running-time coefficient (maximum with respect to X and P_j/P_a) for several values of reservoir pressure. $\frac{T_S}{T_\Delta} = 1.0$.

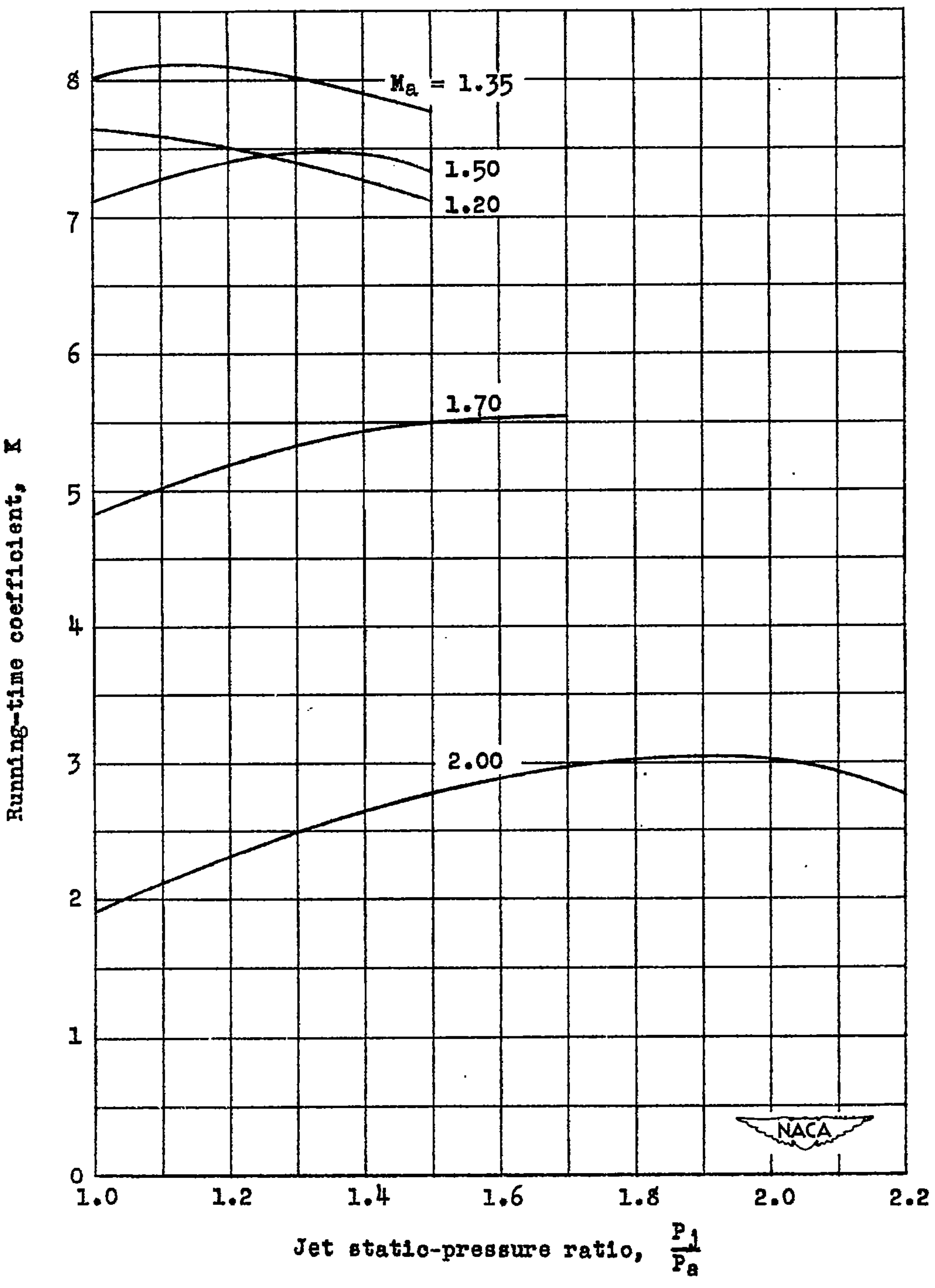

Figure 5.- Variation of running-time coefficient with jet-static-pressure ratio for several values of test-section Mach number. $\frac{T_S}{T_\Delta} = 1.0$; $\frac{H_i}{H_\Delta} = 3.0$. Running-time coefficient is a maximum with respect to X.

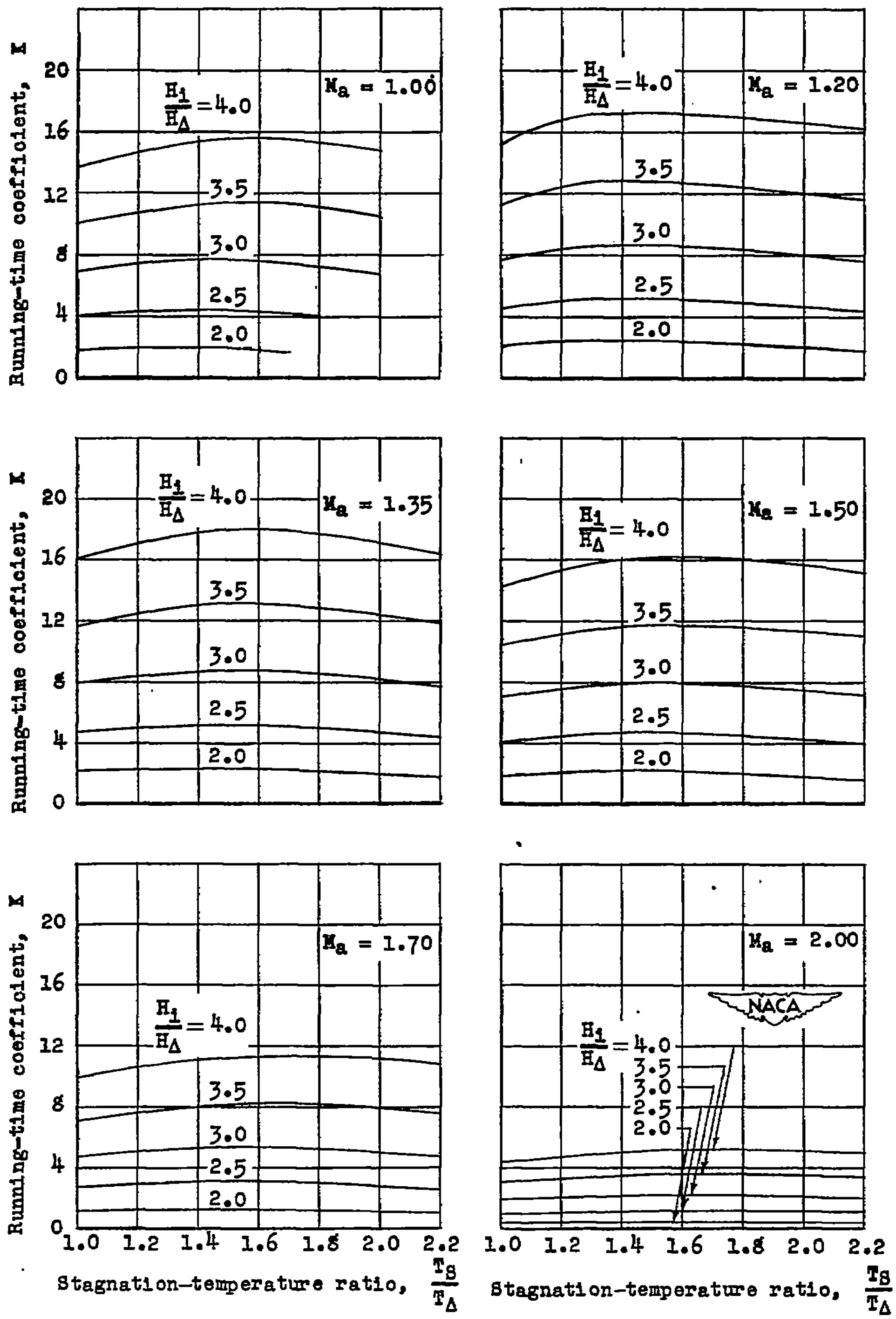

Figure 6.- Variation of the running-time coefficient with stagnation-temperature ratio for several values of reservoir pressure and test-section Mach number. $\frac{P_j}{P_a} = 1.0$. Running-time coefficient is a maximum with respect to X.

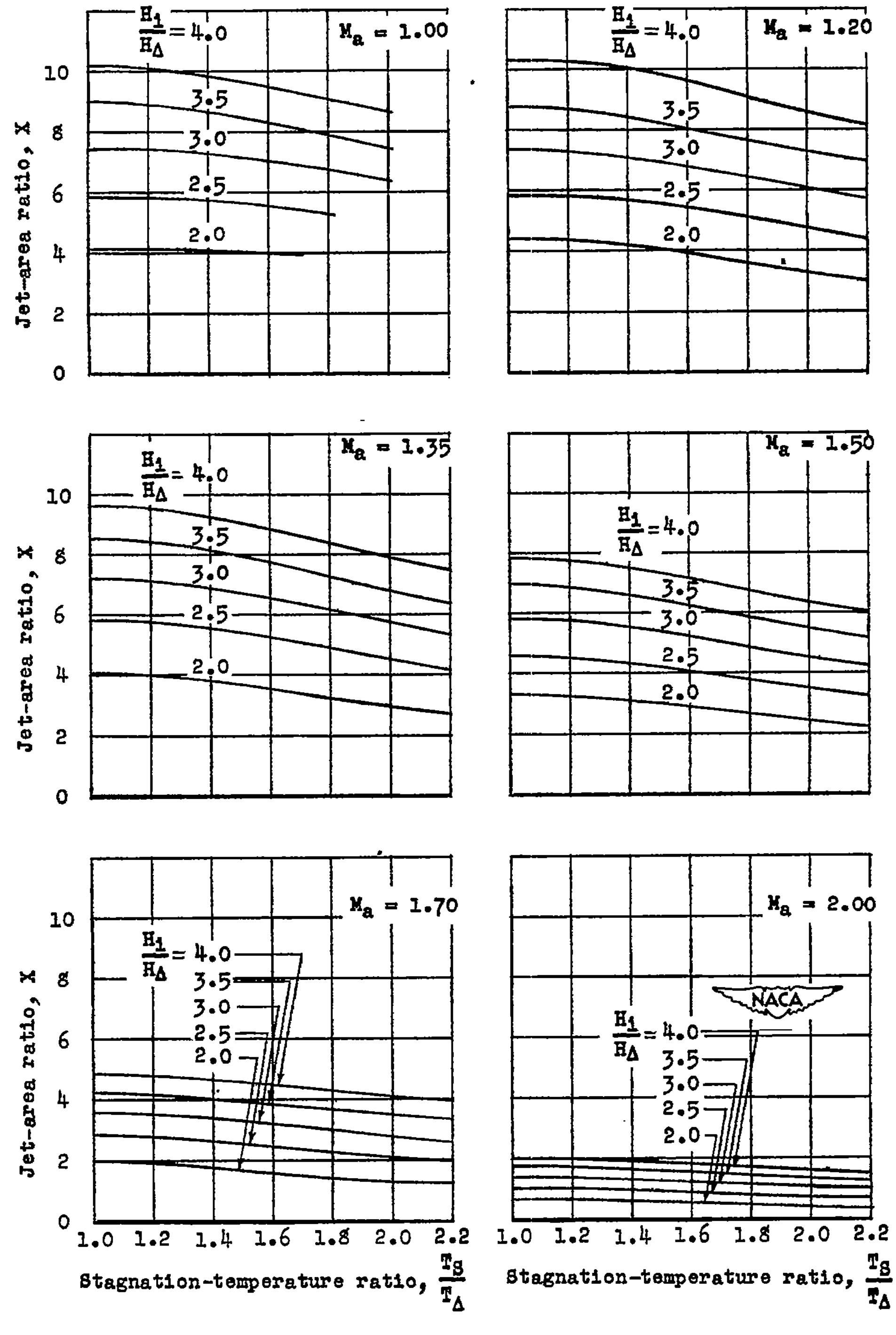

Figure 7.- Variation of the jet-area ratio with stagnation-temperature ratio for several values of reservoir pressure and test-section Mach number. $\frac{P_j}{P_a} = 1.0$.

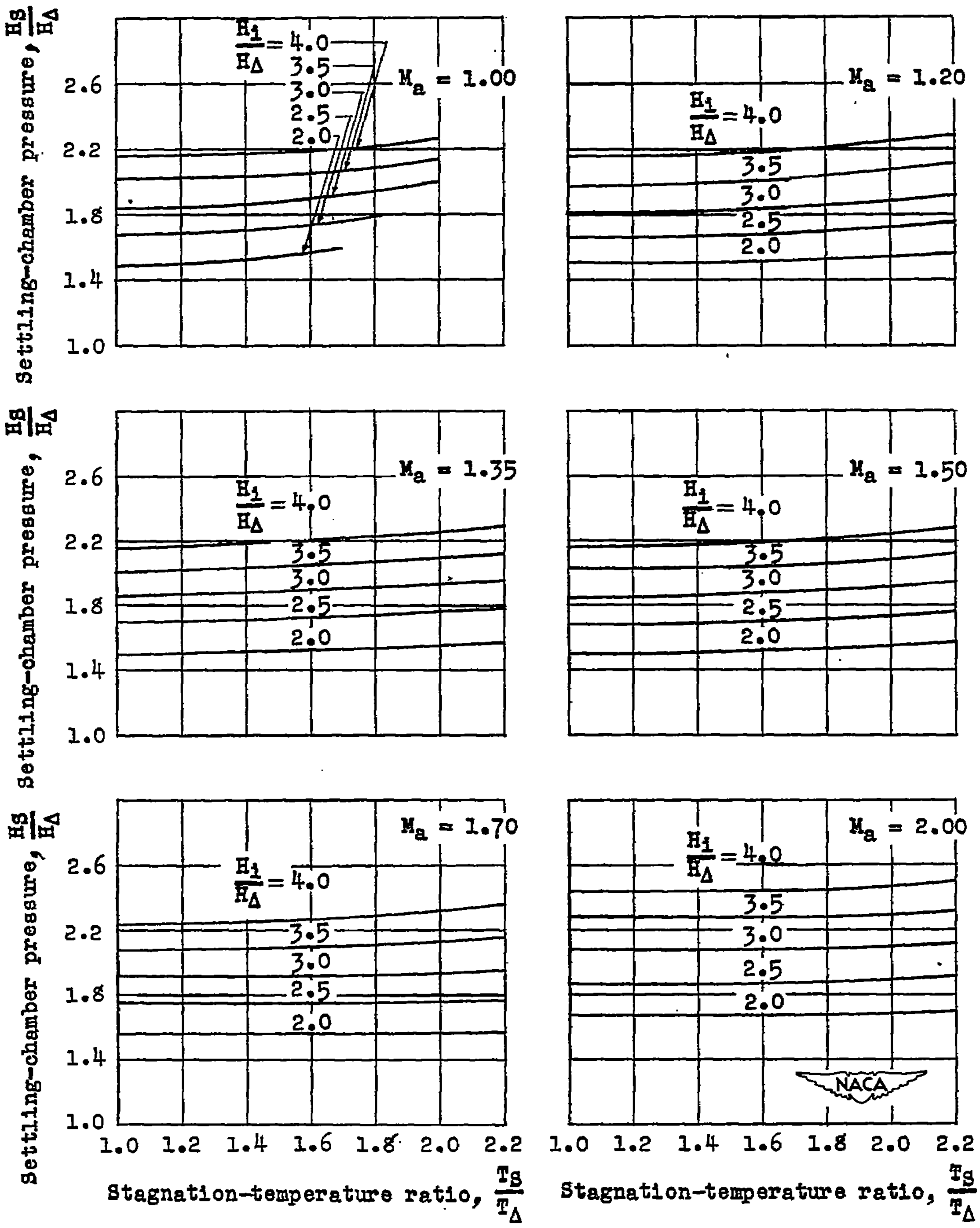

Figure 8.- Variation of the settling-chamber pressure with stagnation-temperature ratio for several values of reservoir pressure and test-section Mach number. $\frac{P_j}{P_a} = 1.0$.

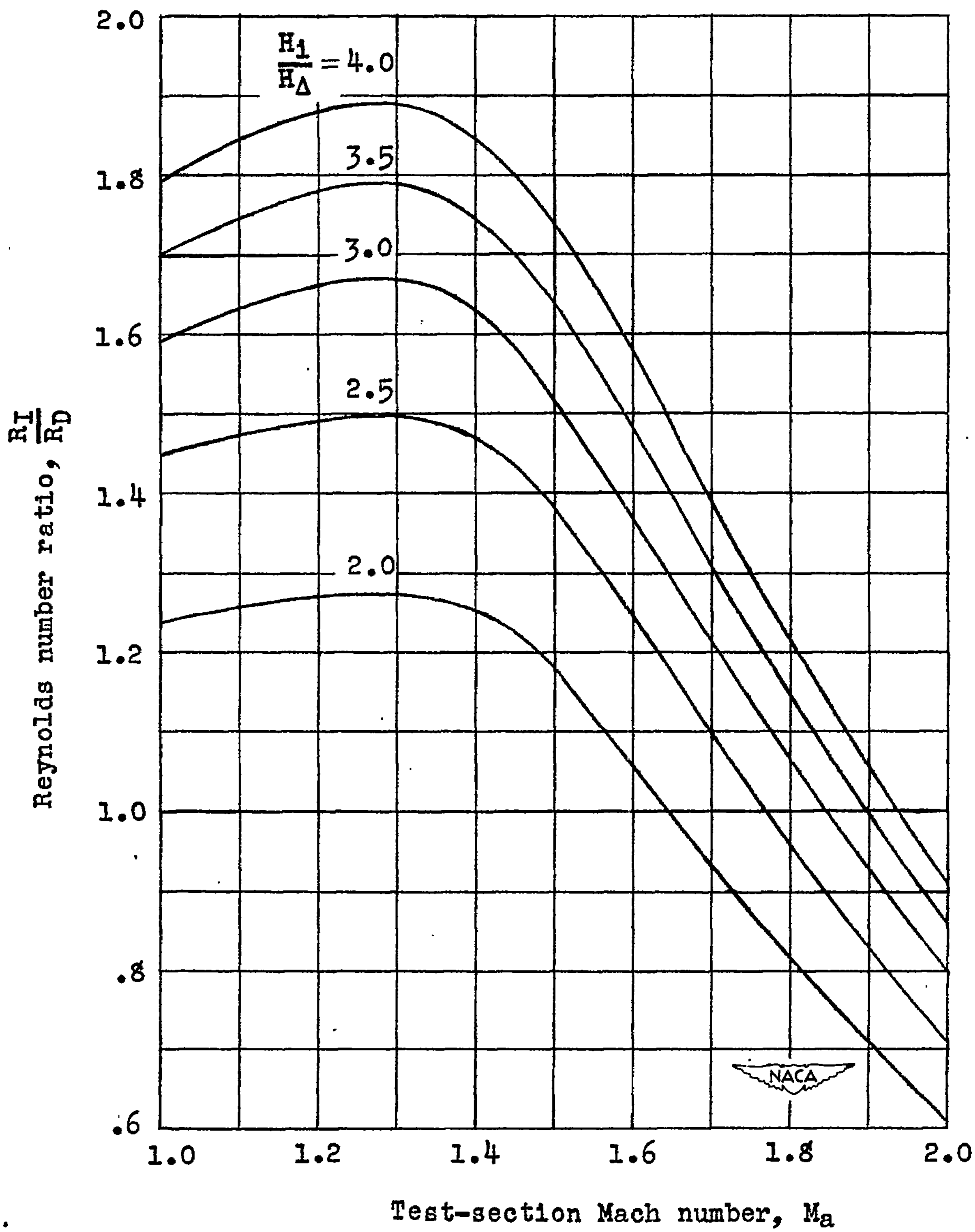

Figure 9.- Reynolds number comparison of an induction blowdown supersonic tunnel and a direct-discharge blowdown supersonic tunnel. $\frac{T_S}{T_\Delta} = 1.0$.

NACA-Langley - 4-14-50 - 950