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CRITICAL SHEAR STRESS OF INFINITELY LONG, SIMPLY SUPPORTED PLATE WITH TRANSVERSE STIFFENERS

By Manuel Stein and Robert W. Fralich

Langley Aeronautical Laboratory
Langley Air Force Base, Va.

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SUMMARY

A theoretical solution is given for the critical shear stress of an infinitely long, simply supported, flat plate with identical, equally spaced, transverse stiffeners of zero torsional stiffness. Results are obtained by means of the Lagrangian multiplier method and are presented in the form of design charts. Experimental results are included and are found to be in good agreement with the theoretical results.

INTRODUCTION

The design of shear web beams and nonwrinkling skin surfaces requires a knowledge of the critical shear stress of stiffened plates. The purpose of the present paper is to give the theoretical critical shear stress of an infinitely long, simply supported, flat plate reinforced with identical, equally spaced, transverse stiffeners.

The results are found by means of the Lagrangian multiplier method. The stiffeners are assumed to have bending stiffness but no torsional stiffness and are assumed to be concentrated along transverse lines in the middle plane of the plate. The assumption that the stiffeners have no torsional stiffness applies with little error in the case of many open section stiffeners. The assumption that the stiffeners are concentrated along transverse lines in the middle plane of the plate is applicable whenever the width of the attached flange is small in comparison with the stiffener spacing.

The theoretical analysis of the problem is given in the appendixes. For completeness, an energy solution for the plate with relatively weak stiffeners is given in appendix A. The solution for a plate with stiffeners of intermediate or higher bending stiffness is given in appendix B. The results are presented in the form of nondimensional curves which cover the complete range of stiffener stiffness and various stiffener spacings and in a table giving values from which the curves were drawn (table I). Experimental results are presented for 20 panels. Comparison of these results with the present theory indicates good agreement between theory and experiment.

SYMBOLS

τ	critical shear stress
k_s	critical shear-stress coefficient $\left(\frac{\tau t b^2}{D\pi^2}\right)$
t	thickness of the plate
b	width of plate
d	stiffener spacing
b/d	panel aspect ratio
D	flexural stiffness of the plate $\left(\frac{E_p t^3}{12(1 - \mu^2)}\right)$
E_p	Young's modulus for plate
E	Young's modulus for stiffener
I	effective moment of inertia of stiffener
μ	Poisson's ratio for material
$\frac{EI}{Dd}$	ratio of stiffener stiffness to plate stiffness
λ	half wave length of buckles
w	deflection of the plate
$\left(w_s\right)_i$	deflection of the i'th stiffener
x,y	reference axes
$\left.\begin{matrix} m, n, q, \\ r, i \end{matrix}\right\}$	integers

$\left.\begin{matrix} a_n, b_n, \\ a_{mn}, b_{mn}, \\ \Delta_{ni}, \Delta_n \end{matrix}\right\}$ coefficients of deflection function

γ_n undetermined Lagrangian multipliers

V internal energy of bending of the plate

V_s internal energy of bending of stiffeners

T external work of the stresses

BACKGROUND

The problem of the buckling of stiffened plates in shear has been treated by many authors by the use of both theoretical and semi-empirical methods. In 1930 Schmieden (reference 1) solved the differential equation for an infinitely long plate stiffened by closely spaced transverse stiffeners (equivalent to orthotropic plate) and found exact stability criterions for shear buckling of plates with simply supported edges and with clamped edges. By making certain simplifying modifications of the stability criterions, Schmieden obtained approximate values of the critical shear stresses. Later in 1930 Seydel (reference 2) obtained exact solutions for infinitely long orthotropic plates with simply supported or clamped edges. With the use of the proper parameters Seydel's results can be readily applied to plate-stiffener combinations. The values of the stresses obtained from Schmieden's theory lie slightly below the exact values of Seydel. In 1947 T. K. Wang (reference 3) used the energy method to obtain an approximate solution for plate-stiffener combinations with simply supported edges. Wang's results lie above the exact values of Seydel. All the foregoing solutions are applicable only to the case of weak stiffeners, where the stiffening effect of the stiffeners can be considered to be uniformly distributed over the plate.

Solutions are also available for plates reinforced by rigid stiffeners. In 1936 Timoshenko (reference 4) treated the case of simply supported rectangular plates reinforced with one or two stiffeners. By means of the energy method Timoshenko found the stiffener flexural rigidity necessary to prevent buckling across stiffeners with the conservative assumption that the stiffeners act as simple supports. In 1948, Budiansky, Conner, and Stein (reference 5) found the critical shear stress for an infinitely long, clamped plate divided into square panels by nondeflecting intermediate supports which correspond to rigid stiffeners. They also considered the case of a plate of infinite length and width having nondeflecting intermediate supports that form an array of square panels.

Kuhn has written a number of papers on related subjects in which he presents semiempirical results for the critical shear stress of stiffened plates. (See, for example, reference 6.)

The available theoretical solutions treat the relatively unimportant case of weak or closely spaced stiffeners and the case of rigid stiffeners that divide a plate into square panels. None of the theoretical solutions presents results for the practical range of intermediate stiffener stiffness and very little theory is presented for the practical range of spacing of rigid stiffeners. Also, it is felt that the semiempirical results for transverse stiffened plates cannot be extended to all stiffener spacings and stiffnesses without a sound theoretical basis. The theoretical results of the present paper cover the complete range of stiffener stiffness and the practical range of stiffener spacing.

RESULTS AND DISCUSSION

The critical shear stress for a plate-stiffener combination is given by the formula

$$\tau = k_s \frac{\pi^2 D}{b^2 t}$$

Curves are presented in figure 1 giving corresponding values of k_s and the stiffness parameter $\frac{EI}{Dd}$ for simply supported, transversely stiffened plates with panel aspect ratios of 1, 2, and 5. These results are replotted in logarithmic form in figure 2 for comparison with experimental results.

The points of discontinuity of the slopes in the curves of figure 1 represent changes in buckle patterns. The present results for an orthotropic plate agree with the exact results of reference 2. The derivation of the buckling criterion for an orthotropic plate (a plate stiffened by stiffeners of low bending stiffness) is given in appendix A. The derivation of the buckling criterion for plates stiffened by stiffeners of higher bending stiffness is given in appendix B.

In previous solutions, values of k_s were found by using the orthotropic-plate curve and a cut-off at the value of k_s for simply supported panels. (See fig. 1.) These figures show that the present solution yields values of k_s that are considerably below those given by the orthotropic-plate curve in the intermediate range of stiffener stiffness. Also, the present solution for more rigid stiffeners yields a curve that is higher than the cut-off, which is obtained by assuming the stiffeners to have the effect of simple supports. Since the continuity of the plate across the stiffeners of higher bending stiffness certainly adds a constraint to the plate, a higher buckling stress than that corresponding to a simply supported edge is obtained.

In figure 2, experimental results are compared with the theoretical curves. These results are from two sources. The first set of experimental data is taken from NACA tests on shear webs of 24S-T aluminum alloy attached to torsion boxes. Drawings of a shear web and torsion box and the method of loading are given in reference 7. Buckling loads were obtained from the stiffener load-deflection curves which were taken from the original data. Each of the buckling loads given in the present paper is the average load at which the stiffeners start to deflect. The properties of the specimens and the buckling data are given in table II.

The second set of experimental data is taken from NACA tests on thick web beams described in reference 8. The beams were made of 24S-T aluminum alloy with heavy flanges and with joggled stiffeners riveted to the flanges. The open spaces in the joggles were filled with soft metal. A picture of a failed beam is shown in figure 3. The load was applied at the center and the reactions were at the ends of the beams. Lateral deflections were prevented by lateral supports. The load, when strain was first observed in the stiffeners, was taken as the buckling load. The properties of the specimens and the buckling data are given in table III.

The stiffener spacings for the test results are not the same as those for the theoretical results. All the test results fall in the expected regions among the theoretical curves. Only the group of test results for which $\frac{b}{d} = 2.4$ fall in the range which serves to verify the present theory over previous theory which considered the orthotropic-plate curve to hold up to the cut-off at which the stiffeners are assumed to act as simple supports. The other groups of test results agree with the present theory, but they do not cover the range in which an appreciable difference exists between the present theory and previous theory. More experimental results are required to confirm the present theory fully.

CONCLUDING REMARKS

Charts are presented from which the theoretical critical shear stresses can be obtained for infinitely long, simply supported plates stiffened with identical, equally spaced, transverse stiffeners of zero torsional stiffness. The theoretical results are based on the Lagrangian multiplier method. Previous theory considered the orthotropic curve to hold up to a cut-off value corresponding to the stiffener stiffness at which the buckling load was equal to the buckling load of a simply supported panel the size of each bay. Comparison of the present theory and previous theory shows that previous theory gives unconservative results for stiffeners of intermediate stiffness and conservative results for stiffeners of high stiffness. Test results of 20 panels are presented which are in good agreement with the present theory. For a conclusive check additional test results are required.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Air Force Base, Va., January 28, 1949

APPENDIX A

THEORETICAL SOLUTION OF CRITICAL SHEAR STRESS OF PLATES

WITH TRANSVERSE STIFFENERS OF LOW BENDING STIFFNESS

If the stiffener bending stiffness is low and the stiffeners are fairly closely spaced, the buckle pattern may be considered independent of the stiffener spacing, and the plate stiffener combination can then be analyzed as a plate with different bending properties in each direction, that is, an orthotropic plate. In this appendix buckling in shear of an orthotropic plate is analyzed by means of the energy method.

The buckling configuration of the plate shown in figure 4 is represented by the trigonometric series

$$w = \sin\frac{\pi x}{\lambda}\sum_{n=2,4,\ldots}^{\infty} a_n \sin\frac{n\pi y}{b} + \cos\frac{\pi x}{\lambda}\sum_{n=1,3,\ldots}^{\infty} b_n \sin\frac{n\pi y}{b} \tag{A1}$$

which satisfies the boundary conditions of simple support term by term. The internal bending energy of the plate V, the internal bending energy of the stiffeners V_s, and the external work of the shear stresses T are given by the expressions

$$\left.\begin{aligned}
V &= \frac{D}{2}\int_0^b\int_0^\lambda\left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\mu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right]\right\}dx\,dy \\
V_s &= \frac{EI}{2d}\int_0^b\int_0^\lambda\left(\frac{\partial^2 w}{\partial y^2}\right)^2 dx\,dy \\
T &= -\tau t\int_0^b\int_0^\lambda\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\,dx\,dy
\end{aligned}\right\} \tag{A2}$$

Substitution of the expansion for w (equation (A1)) into these energy integrals gives

$$V = \frac{D\lambda\pi^4}{8b^3}\left[\sum_{n=2,4,\ldots}^{\infty} a_n^2\left(\frac{b^2}{\lambda^2} + n^2\right)^2 + \sum_{n=1,3,\ldots}^{\infty} b_n^2\left(\frac{b^2}{\lambda^2} + n^2\right)^2\right]$$

$$V_s = \frac{EI\lambda\pi^4}{8db^3}\left(\sum_{n=2,4,\ldots}^{\infty} a_n^2 n^4 + \sum_{n=1,3,\ldots}^{\infty} b_n^2 n^4\right)$$

$$T = 2\tau t\pi \sum_{n=1,3,\ldots}^{\infty}\ \sum_{q=2,4,\ldots}^{\infty} a_q b_n \frac{nq}{n^2 - q^2}$$

Then

$$\begin{aligned}(V + V_s - T)\frac{8b^3}{D\lambda\pi^4} &= \sum_{n=2,4,\ldots}^{\infty} a_n^2\left[\left(\frac{b^2}{\lambda^2} + n^2\right)^2 + n^4\frac{EI}{Dd}\right] \\ &\quad + \sum_{n=1,3,\ldots}^{\infty} b_n^2\left[\left(\frac{b^2}{\lambda^2} + n^2\right)^2 + n^4\frac{EI}{Dd}\right] \\ &\quad - \frac{16bk_s}{\pi\lambda}\sum_{n=1,3,\ldots}^{\infty}\ \sum_{q=2,4,\ldots}^{\infty} a_q b_n \frac{nq}{n^2 - q^2} \qquad \text{(A3)}\end{aligned}$$

where

$$k_s = \frac{\tau t b^2}{D\pi^2}$$

According to the energy method the potential energy $(V + V_s - T)$ must be minimized with respect to the unknown coefficients a_n and b_n. By minimizing $(V + V_s - T)$ with respect to the coefficients a_n and b_n, the following set of equations is obtained:

$$a_n\left[\left(\frac{b^2}{\lambda^2} + n^2\right)^2 + n^4 \frac{EI}{Dd}\right] - \frac{8bk_s}{\pi\lambda} \sum_{q=1,3,\ldots}^{\infty} b_q \frac{nq}{\left(q^2 - n^2\right)} = 0 \qquad (n=2,4,6,\ldots) \tag{A4}$$

$$b_n\left[\left(\frac{b^2}{\lambda^2} + n^2\right)^2 + n^4 \frac{EI}{Dd}\right] - \frac{8bk_s}{\pi\lambda} \sum_{q=2,4,\ldots}^{\infty} a_q \frac{nq}{\left(n^2 - q^2\right)} = 0 \qquad (n=1,3,5,\ldots) \tag{A5}$$

The coefficients a_n can be found in terms of b_r from equation (A4). Substitution of the resulting expression for a_n in equation (A5) results in the following equations:

$$b_n\left[\left(\frac{b^2}{\lambda^2} + n^2\right)^2 + n^4 \frac{EI}{Dd}\right] - \left(\frac{8bk_s}{\pi\lambda}\right)^2 \sum_{q=2,4,\ldots}^{\infty} \sum_{r=1,3,\ldots}^{\infty} b_r \frac{rnq^2}{\left(n^2 - q^2\right)\left(r^2 - q^2\right)\left[\left(\frac{b^2}{\lambda^2} + q^2\right)^2 + q^4 \frac{EI}{Dd}\right]} = 0$$

$$(n=1,3,5,\ldots) \tag{A6}$$

A solution to equations (A6) exists if the following stability determinant vanishes:

$$\begin{vmatrix} C_{11} & C_{13} & C_{15} & \cdots \\ C_{31} & C_{33} & C_{35} & \cdots \\ C_{51} & C_{53} & C_{55} & \cdots \\ \vdots & \vdots & \vdots & \end{vmatrix} = 0 \tag{A7}$$

where

$$C_{nn} = \left[\left(\frac{b^2}{\lambda^2} + n^2\right)^2 + n^4\frac{EI}{Dd}\right] - \left(\frac{8bk_s}{\pi\lambda}\right)^2 \sum_{q=2,4,\ldots}^{\infty} \frac{n^2q^2}{\left(n^2 - q^2\right)^2\left[\left(\frac{b^2}{\lambda^2} + q^2\right)^2 + q^4\frac{EI}{Dd}\right]}$$

$$C_{nr} = C_{rn} = -\left(\frac{8bk_s}{\pi\lambda}\right)^2 \sum_{q=2,4,\ldots}^{\infty} \frac{rnq^2}{\left(n^2 - q^2\right)\left(r^2 - q^2\right)\left[\left(\frac{b^2}{\lambda^2} + q^2\right)^2 + q^4\frac{EI}{Dd}\right]}$$

(where $n \neq r$)

A solution including all the a_n's and b_1 can be obtained by setting equal to zero the first approximation of the determinant equation (A7)

$$C_{11} = 0$$

Similarly the second approximation includes all the a_n's, b_1, and b_3

$$C_{11}\, C_{33} - C_{13}^{\;2} = 0$$

Higher approximations are found in a similar manner. A second approximation was found to give satisfactory results. For a given approximation it is necessary to try values of b/λ and find the corresponding values of k_s until a minimum value of k_s with respect to b/λ is found for each $\frac{EI}{Dd}$. The results are given in table I and in figure 1.

APPENDIX B

THEORETICAL SOLUTION OF CRITICAL SHEAR STRESS OF PLATES WITH TRANSVERSE STIFFENERS OF HIGHER BENDING STIFFNESS

In appendix A a theoretical solution for a plate stiffened by stiffeners of low bending stiffness is presented where the buckle pattern was taken as sinusoidal in the longitudinal direction. The buckle pattern of plates with stiffeners of higher bending stiffness is no longer sinusoidal in the longitudinal direction. It is then necessary to consider deflection functions which are either symmetric or antisymmetric about the midpoint of each bay and are periodic over an integral number of bays. The critical shear stress of plates with transverse stiffeners of higher bending stiffness is analyzed by means of the Lagrangian multiplier method.

Deflection functions.- The correct buckle configuration for any given plate-stiffener combination is that which corresponds to the lowest buckling load. Several types of configurations are investigated. These buckling configurations are represented by the following two-dimensional trigonometric series (the coordinates are given in fig. 4). Symmetric buckling, periodic over each bay:

$$w = \sum_{m=2,4,\ldots}^{\infty} \sum_{n=2,4,\ldots}^{\infty} a_{mn} \sin \frac{m\pi x}{d} \sin \frac{n\pi y}{b}$$

$$+ \sum_{m=0,2,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} b_{mn} \cos \frac{m\pi x}{d} \sin \frac{n\pi y}{b} \qquad \text{(B1a)}$$

Antisymmetric buckling, periodic over each bay:

$$w = \sum_{m=2,4,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} a_{mn} \sin \frac{m\pi x}{d} \sin \frac{n\pi y}{b}$$

$$+ \sum_{m=0,2,\ldots}^{\infty} \sum_{n=2,4,\ldots}^{\infty} b_{mn} \cos \frac{m\pi x}{d} \sin \frac{n\pi y}{b} \qquad \text{(B1b)}$$

Symmetric buckling, periodic over two bays:

$$w = \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} a_{mn} \sin \frac{m\pi x}{d} \sin \frac{n\pi y}{b}$$

$$+ \sum_{m=1,3,\ldots}^{\infty} \sum_{n=2,4,\ldots}^{\infty} b_{mn} \cos \frac{m\pi x}{d} \sin \frac{n\pi y}{b} \qquad \text{(B1c)}$$

Antisymmetric buckling, periodic over two bays:

$$w = \sum_{m=1,3,\ldots}^{\infty} \sum_{n=2,4,\ldots}^{\infty} a_{mn} \sin \frac{m\pi x}{d} \sin \frac{n\pi y}{b}$$

$$+ \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} b_{mn} \cos \frac{m\pi x}{d} \sin \frac{n\pi y}{b} \qquad \text{(B1d)}$$

Symmetric buckling, one bay; antisymmetric buckling, next bay; periodic over four bays:

$$w = \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} a_{mn} \left[\sin \frac{m\pi x}{2d} + (-1)^{\frac{m-1}{2}} \cos \frac{m\pi x}{2d} \right] \sin \frac{n\pi y}{b}$$

$$+ \sum_{m=1,3,\ldots}^{\infty} \sum_{n=2,4,\ldots}^{\infty} b_{mn} \left[\sin \frac{m\pi x}{2d} - (-1)^{\frac{m-1}{2}} \cos \frac{m\pi x}{2d} \right] \sin \frac{n\pi y}{b} \qquad \text{(B1e)}$$

Careful study has shown that other buckle patterns would require higher buckling loads and that only the five buckle patterns given need be considered.

These deflection functions all satisfy term by term the conditions of simply supported edges at $y = 0, b$ and continuity of the plate across the stiffeners at $x = 0, d, 2d, \ldots$. The condition that stiffener deflection equal plate deflection at the stiffeners is introduced by means of Lagrangian multipliers.

The deflection functions (B1d) and (B1e) are found to be the governing ones for the aspect ratios investigated; the others lead to unconservative solutions. Buckling criterions for the critical shear stress are derived for the deflection functions (B1d) and (B1e).

Antisymmetric buckling, periodic over two bays.- The deflection of the plate is given by equation (B1d) as

$$w = \sum_{m=1,3,\ldots}^{\infty} \sum_{n=2,4,\ldots}^{\infty} a_{mn} \sin \frac{m\pi x}{d} \sin \frac{n\pi y}{b}$$

$$+ \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} b_{mn} \cos \frac{m\pi x}{d} \sin \frac{n\pi y}{b}$$

The deflection of the i'th stiffener is taken as

$$\left(w_s\right)_i = \sum_{n=1,3,\ldots}^{\infty} \Delta_{ni} \sin \frac{n\pi y}{b} \tag{B2}$$

where, since the interval to be considered includes two stiffeners, $i = 1$ and 2. The boundary conditions that stiffener deflection equal plate deflection are

$$w(id,y) - (w_s)_i = 0 \qquad (i=1,2)$$

or upon substitution,

$$\sum_{m=1,3,\ldots}^{\infty} b_{mn} + \Delta_{n1} = 0 \qquad (n=1,3,\ldots)$$

$$\sum_{m=1,3,\ldots}^{\infty} b_{mn} - \Delta_{n2} = 0 \qquad (n=1,3,\ldots)$$

These equations show that $\Delta_{n1} = -\Delta_{n2}$. If Δ_{n1} is redefined as Δ_n the boundary conditions become

$$\sum_{m=1,3,\ldots}^{\infty} b_{mn} + \Delta_n = 0 \qquad (n=1,3,\ldots) \qquad \text{(B3)}$$

These boundary conditions will be satisfied in the energy expression by means of Lagrangian multipliers.

The internal bending energy of the plate V, the internal bending energy of the stiffeners V_s, and the external work of the shear stresses T are given by the expressions

$$\left.\begin{aligned}
V &= \frac{D}{2}\int_0^b \int_0^{2d} \left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-u)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right]\right\} dx\,dy \\
V_s &= \sum_{i=1,2,\ldots}^{2} \frac{EI}{2}\int_0^b \left[\frac{\partial^2 (w_s)_i}{\partial y^2}\right]^2 dy \\
T &= -\tau t \int_0^b \int_0^{2d} \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\, dx\,dy
\end{aligned}\right\} \tag{B4}$$

Substitution of the deflection functions of the plate and stiffeners into these energy integrals gives

$$\left.\begin{aligned}
V &= \frac{Dd\pi^4}{4b^3}\left[\sum_{m=1,3,\ldots}^{\infty}\ \sum_{n=2,4,\ldots}^{\infty} a_{mn}^{2}\left(m^2\frac{b^2}{d^2} + n^2\right)^2 \right. \\
&\qquad \left. + \sum_{m=1,3,\ldots}^{\infty}\ \sum_{n=1,3,\ldots}^{\infty} b_{mn}^{2}\left(m^2\frac{b^2}{d^2} + n^2\right)^2\right] \\
V_s &= \frac{EI\pi^4}{2b^3}\sum_{n=1,3,\ldots}^{\infty} \Delta_n^{2} n^4 \\
T &= 4\tau t\pi \sum_{m=1,3,\ldots}^{\infty}\ \sum_{n=2,4,\ldots}^{\infty}\ \sum_{q=1,3,\ldots}^{\infty} a_{mn} b_{mq}\frac{mnq}{\left(q^2 - n^2\right)}
\end{aligned}\right\} \tag{B5}$$

The energy method requires that the potential energy $(V + V_s - T)$ be minimized with respect to the a's, b's, and Δ's. Since the a's, b's, and Δ's are, however, bound by equations (B3), the minimization is performed by the Lagrangian multiplier method by minimizing the following function F with respect to the a's, b's, and Δ's:

$$F = \frac{V + V_s - T}{\left(\frac{\pi^4 D\, d}{4b^3}\right)} + \sum_{n=1,3,\ldots}^{\infty} \gamma_n \left(\sum_{m=1,3,\ldots}^{\infty} b_{mn} + \Delta_n\right) \tag{B6}$$

where the γ's are the Lagrangian multipliers. When this minimization is performed, the following set of equations is obtained:

$$\left.\begin{aligned}
&\frac{\partial F}{\partial a_{mn}} = 0 = 2a_{mn}\left(m^2 \frac{b^2}{d^2} + n^2\right)^2 + \frac{16k_s}{\pi}\frac{b}{d}\sum_{q=1,3,\ldots}^{\infty} b_{mq}\frac{mnq}{\left(n^2 - q^2\right)} \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (m=1,3,\ldots) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (n=2,4,\ldots) \\
&\frac{\partial F}{\partial b_{mn}} = 0 = 2b_{mn}\left(m^2 \frac{b^2}{d^2} + n^2\right)^2 + \frac{16k_s}{\pi}\frac{b}{d}\sum_{q=2,4,\ldots}^{\infty} a_{mq}\frac{mnq}{\left(q^2 - n^2\right)} + \gamma_n \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (m=1,3,\ldots) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (n=1,3,\ldots) \\
&\frac{\partial F}{\partial \Delta_n} = 0 = \frac{4EI}{Dd} n^4 \Delta_n + \gamma_n \qquad\qquad\qquad\qquad (n=1,3,\ldots)
\end{aligned}\right\} \tag{B7}$$

When the equations (B7) are combined, the following equations are obtained:

$$b_{mn}\left(m^2\frac{b^2}{d^2}+n^2\right)^2-\frac{2EI}{Dd}n^4\Delta_n$$

$$-\left(\frac{8k_sb}{\pi d}\right)^2\sum_{q=2,4,\ldots}^{\infty}\sum_{r=1,3,\ldots}^{\infty}b_{mr}\frac{m^2q^2rn}{\left(n^2-q^2\right)\left(r^2-q^2\right)\left(m^2\frac{b^2}{d^2}+q^2\right)^2}=0 \qquad \text{(B8a)}$$

$$(m=1,3,\ldots)$$
$$(n=1,3,\ldots)$$

Equations (B8a) written in matrix form are

$$\begin{bmatrix} C_{m1} & C_{m13} & C_{m15} & \cdots \\ C_{m31} & C_{m3} & C_{m35} & \cdots \\ C_{m51} & C_{m53} & C_{m5} & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}\begin{bmatrix} b_{m1} \\ b_{m3} \\ b_{m5} \\ \vdots \end{bmatrix}=\frac{EI}{Dd}\begin{bmatrix} 2\Delta_1 \\ 162\Delta_3 \\ 1250\Delta_5 \\ \vdots \end{bmatrix} \qquad \text{(B8b)}$$

$$(m=1,3,\ldots)$$

where

$$C_{mn}=\left(m^2\frac{b^2}{d^2}+n^2\right)^2-\left(\frac{8k_sb}{\pi d}\right)^2\sum_{q=2,4,\ldots}^{\infty}\frac{m^2n^2q^2}{\left(q^2-n^2\right)^2\left(m^2\frac{b^2}{d^2}+q^2\right)^2}$$

$$C_{mnr}=C_{mrn}=-\left(\frac{8k_sb}{\pi d}\right)^2\sum_{q=2,4,\ldots}^{\infty}\frac{m^2q^2rn}{\left(n^2-q^2\right)\left(r^2-q^2\right)\left(m^2\frac{b^2}{d^2}+q^2\right)^2}$$

A solution including all the a_{mn}'s and b_{m1}'s can be obtained by the first approximation of the matrix equation (B8b)

$$C_{m1}b_{m1} = 2\frac{EI}{Dd}\Delta_1 \qquad (m=1,3,\ldots) \qquad \text{(B9)}$$

Substitution of b_{m1} from equation (B9) into the boundary equation (B3) yields

$$\left(\sum_{m=1,3,\ldots}^{\infty}\frac{1}{C_{m1}}+\frac{1}{2\frac{EI}{Dd}}\right)\Delta_1 = 0 \qquad \text{(B10)}$$

The following stability criterion is obtained by setting equal to zero the coefficient of Δ_1:

$$\sum_{m=1,3,\ldots}^{\infty}\frac{1}{C_{m1}}+\frac{1}{2\frac{EI}{Dd}} = 0 \qquad \text{(B11)}$$

Similarly, the second approximation includes all the a_{mn}'s, b_{m1}'s, and b_{m3}'s. Two simultaneous equations result from which b_{m1} and b_{m3} can be found. Substitution of these values into the boundary equation (B3) yields two linear homogeneous equations in Δ_1 and Δ_3. If the determinant of the coefficients of these two equations is set equal to zero, the following stability criterion is obtained:

$$\left(\sum_{m=1,3,\ldots}^{\infty} \frac{C_{m1}}{C_{m1}C_{m3} - C_{m13}^{2}} + \frac{1}{162\,\frac{EI}{Dd}}\right)\left(\sum_{m=1,3,\ldots}^{\infty} \frac{C_{m3}}{C_{m1}C_{m3} - C_{m13}^{2}} + \frac{1}{2\,\frac{EI}{Dd}}\right)$$

$$-\left(\sum_{m=1,3,\ldots}^{\infty} \frac{C_{m13}}{C_{m1}C_{m3} - C_{m13}^{2}}\right)^{2} = 0 \tag{B12}$$

Higher approximations are found in a similar manner. A second approximation was found to give satisfactory results. For each of these approximations, it is necessary to find the lowest value of k_s for each value of $\frac{EI}{Dd}$. The results are given in table I and in figure 1.

Buckling periodic over four bays.- The deflection of the plate is given by equation (B1e) as

$$w = \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} a_{mn} \left[\sin \frac{m\pi x}{2d} + (-1)^{\frac{m-1}{2}} \cos \frac{m\pi x}{2d}\right] \sin \frac{n\pi x}{b}$$

$$+ \sum_{m=1,3,\ldots}^{\infty} \sum_{n=2,4,\ldots}^{\infty} b_{mn} \left[\sin \frac{m\pi x}{2d} - (-1)^{\frac{m-1}{2}} \cos \frac{m\pi x}{2d}\right] \sin \frac{n\pi y}{b}$$

The deflection of the i'th stiffener is taken as

$$\left(w_s\right)_i = \sum_{n=1,2,\ldots}^{\infty} \Delta_{ni} \sin \frac{n\pi y}{b} \tag{B13}$$

where $i = 1, 2, 3$, and 4, since the interval considered includes four stiffeners. The boundary conditions

$$w(id,y) - (w_s)_i = 0 \qquad (i=1,2,3,4)$$

become

$$\sum_{m=1,3,\ldots}^{\infty} a_{mn} (-1)^{\frac{m-1}{2}} - \Delta_{n1} = 0 \qquad (n=1,3,\ldots)$$

$$\sum_{m=1,3,\ldots}^{\infty} b_{mn} (-1)^{\frac{m-1}{2}} - \Delta_{n1} = 0 \qquad (n=2,4,\ldots)$$

$$\sum_{m=1,3,\ldots}^{\infty} a_{mn} (-1)^{\frac{m-1}{2}} + \Delta_{n2} = 0 \qquad (n=1,3,\ldots)$$

$$\sum_{m=1,3,\ldots}^{\infty} b_{mn} (-1)^{\frac{m-1}{2}} - \Delta_{n2} = 0 \qquad (n=2,4,\ldots)$$

$$\sum_{m=1,3,\ldots}^{\infty} a_{mn} (-1)^{\frac{m-1}{2}} + \Delta_{n3} = 0 \qquad (n=1,3,\ldots)$$

$$\sum_{m=1,3,\ldots}^{\infty} b_{mn} (-1)^{\frac{m-1}{2}} + \Delta_{n3} = 0 \qquad (n=2,4,\ldots)$$

$$\sum_{m=1,3,\ldots}^{\infty} a_{mn} (-1)^{\frac{m-1}{2}} - \Delta_{n4} = 0 \qquad (n=1,3,\ldots)$$

$$\sum_{m=1,3,\ldots}^{\infty} b_{mn} (-1)^{\frac{m-1}{2}} + \Delta_{n4} = 0 \qquad (n=2,4,\ldots)$$

These equations show that

$$\Delta_{n1} = -\Delta_{n2} = -\Delta_{n3} = \Delta_{n4} \qquad (n=1,3,\ldots)$$

$$\Delta_{n1} = \Delta_{n2} = -\Delta_{n3} = -\Delta_{n4} \qquad (n=2,4,\ldots)$$

If Δ_{n1} is redefined as Δ_n, the boundary conditions become

$$\left.\begin{aligned} &\sum_{m=1,3,\ldots}^{\infty} a_{mn} (-1)^{\frac{m-1}{2}} - \Delta_n = 0 \qquad (n=1,3,\ldots) \\ &\sum_{m=1,3,\ldots}^{\infty} b_{mn} (-1)^{\frac{m-1}{2}} - \Delta_n = 0 \qquad (n=2,4,\ldots) \end{aligned}\right\} \tag{B14}$$

These boundary conditions will be satisfied in the energy expression by means of Lagrangian multipliers.

The energy integrals are the same as the energy integrals (B4), except that in the present problem the upper limit of integration $2d$ is replaced by $4d$ and the upper limit of the summation 2 is replaced by 4.

The deflection functions of the plate (equation (Ble)) and stiffeners (equation (B13)) are substituted into these energy integrals and result in the following expressions:

$$V = \frac{D d \pi^4}{b^3} \left[\sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} a_{mn}^2 \left(\frac{m^2 b^2}{4d^2} + n^2 \right)^2 + \sum_{m=1,3,\ldots}^{\infty} \sum_{n=2,4,\ldots}^{\infty} b_{mn}^2 \left(\frac{m^2 b^2}{4d^2} + n^2 \right)^2 \right]$$

$$V_s = \frac{EI\pi^4}{b^3} \sum_{n=1,2,\ldots}^{\infty} \Delta_n^2 n^4$$

$$T = 8\tau t \pi \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} \sum_{q=2,4,\ldots}^{\infty} a_{mn} b_{mq} (-1)^{\frac{m-1}{2}} \frac{mnq}{\left(n^2 - q^2\right)}$$

The minimization of $(V + V_s - T)$ is performed by the Lagrangian multiplier method by minimizing the following function F with respect to the a's, b's, and Δ's.

$$F = \frac{V + V_s - T}{\frac{\pi^4 D d}{b^3}} + \sum_{n=1,3,\ldots}^{\infty} \gamma_n \left[\sum_{m=1,3,\ldots}^{\infty} a_{mn} (-1)^{\frac{m-1}{2}} - \Delta_n \right] + \sum_{n=2,4,\ldots}^{\infty} \gamma_n \left[\sum_{m=1,3,\ldots}^{\infty} b_{mn} (-1)^{\frac{m-1}{2}} - \Delta_n \right] \qquad \text{(B15)}$$

where the γ's are the Lagrangian multipliers.

When the minimization is performed and the resulting equations are combined, the following set of equations is obtained:

$$\left.\begin{aligned}
&A_{mn}a_{mn} - \Gamma_m \sum_{q=2,4,\ldots}^{\infty} b_{mq}\frac{nq}{\left(n^2 - q^2\right)} + 2\frac{EI}{Dd}n^4\Delta_n(-1)^{\frac{m-1}{2}} = 0 \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (m=1,3,\ldots) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (n=1,3,\ldots) \\
&A_{mn}b_{mn} - \Gamma_m \sum_{q=1,3,\ldots}^{\infty} a_{mq}\frac{nq}{\left(q^2 - n^2\right)} + 2\frac{EI}{Dd}n^4\Delta_n(-1)^{\frac{m-1}{2}} = 0 \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (m=1,3,\ldots) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (n=2,4,\ldots)
\end{aligned}\right\} \quad \text{(B16a)}$$

where

$$A_{mn} = 2\left(\frac{m^2b^2}{4d^2} + n^2\right)^2$$

$$\Gamma_m = \frac{8k_sb}{\pi d}m(-1)^{\frac{m-1}{2}}$$

Equations (B16a) in matrix form are

$$\begin{bmatrix} A_{m1} & \frac{2}{3}\Gamma_m & 0 & \frac{4}{15}\Gamma_m & 0 & \frac{6}{35}\Gamma_m & \cdots \\ \frac{2}{3}\Gamma_m & A_{m2} & -\frac{6}{5}\Gamma_m & 0 & -\frac{10}{21}\Gamma_m & 0 & \cdots \\ 0 & -\frac{6}{5}\Gamma_m & A_{m3} & \frac{12}{7}\Gamma_m & 0 & \frac{2}{3}\Gamma_m & \cdots \\ \frac{4}{15}\Gamma_m & 0 & \frac{12}{7}\Gamma_m & A_{m4} & -\frac{20}{9}\Gamma_m & 0 & \cdots \\ 0 & -\frac{10}{21}\Gamma_m & 0 & -\frac{20}{9}\Gamma_m & A_{m5} & \frac{30}{11}\Gamma_m & \cdots \\ \frac{6}{35}\Gamma_m & 0 & \frac{2}{3}\Gamma_m & 0 & \frac{30}{11}\Gamma_m & A_{m6} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix} \begin{bmatrix} a_{m1} \\ b_{m2} \\ a_{m3} \\ b_{m4} \\ a_{m5} \\ b_{m6} \\ \vdots \end{bmatrix} = (-1)^{\frac{m+1}{2}} \frac{EI}{Dd} \begin{bmatrix} 2\,\Delta_1 \\ 32\,\Delta_2 \\ 162\,\Delta_3 \\ 512\,\Delta_4 \\ 1250\,\Delta_5 \\ 2592\,\Delta_6 \\ \vdots \end{bmatrix} \qquad (m=1,3,\ldots) \qquad \text{(B16b)}$$

A first approximation of k_s is found by considering all the a_{m1}'s and b_{m2}'s in equation (B16b).

$$\left.\begin{aligned} A_{m1}a_{m1} + \frac{2}{3}\Gamma_m b_{m2} &= (-1)^{\frac{m+1}{2}}\, 2\,\frac{EI}{Dd}\Delta_1 \qquad (m=1,3,\ldots) \\ \frac{2}{3}\Gamma_m a_{m1} + A_{m2}b_{m2} &= (-1)^{\frac{m+1}{2}}\, 32\,\frac{EI}{Dd}\Delta_2 \qquad (m=1,3,\ldots) \end{aligned}\right\} \qquad \text{(B17)}$$

Substitution of a_{m1} and b_{m2} from equations (B17) into the boundary equations (B14) yields

$$\left.\begin{aligned}
&-\left(\sum_{m=1,3,\ldots}^{\infty} \frac{A_{m2}}{A_{m1}A_{m2} - \frac{4}{9}\Gamma_m^{\,2}} + \frac{1}{2\frac{EI}{Dd}}\right)\Delta_1 \\
&\qquad + \left(\frac{32}{3}\sum_{m=1,3,\ldots}^{\infty} \frac{\Gamma_m}{A_{m1}A_{m2} - \frac{4}{9}\Gamma_m^{\,2}}\right)\Delta_2 = 0 \\
&\left(\frac{1}{24}\sum_{m=1,3,\ldots}^{\infty} \frac{\Gamma_m}{A_{m1}A_{m2} - \frac{4}{9}\Gamma_m^{\,2}}\right)\Delta_1 \\
&\qquad - \left(\sum_{m=1,3,\ldots}^{\infty} \frac{A_{m1}}{A_{m1}A_{m2} - \frac{4}{9}\Gamma_m^{\,2}} + \frac{1}{32\frac{EI}{Dd}}\right)\Delta_2 = 0
\end{aligned}\right\} \tag{B18}$$

If the determinant of the coefficients of the linear homogeneous equations (B18) is set equal to zero, the following stability criterion is obtained:

$$\left(\sum_{m=1,3,\ldots}^{\infty} \frac{A_{m2}}{A_{m1}A_{m2} - \frac{4}{9}\Gamma_m^{\,2}} + \frac{1}{2\frac{EI}{Dd}}\right)\left(\sum_{m=1,3,\ldots}^{\infty} \frac{A_{m1}}{A_{m1}A_{m2} - \frac{4}{9}\Gamma_m^{\,2}} + \frac{1}{32\frac{EI}{Dd}}\right)$$

$$-\frac{4}{9}\left(\sum_{m=1,3,\ldots}^{\infty} \frac{\Gamma_m}{A_{m1}A_{m2} - \frac{4}{9}\Gamma_m^{\,2}}\right)^2 = 0 \tag{B19}$$

Similarly, from the second approximation, including all the a_{m1}, b_{m2}, and a_{m3} terms, the following stability criterion is obtained:

$$\begin{vmatrix} \displaystyle\sum_{m=1,3,\ldots}^{\infty} \frac{A_{m2}A_{m3} - \frac{36}{25}\Gamma_m^{\;2}}{B_m} + \frac{1}{2\frac{EI}{Dd}} & \displaystyle\frac{2}{3}\sum_{m=1,3,\ldots}^{\infty} \frac{\Gamma_m A_{m3}}{B_m} & \displaystyle\frac{4}{5}\sum_{m=1,3,\ldots}^{\infty} \frac{\Gamma_m^{\;2}}{B_m} \\ \displaystyle\frac{2}{3}\sum_{m=1,3,\ldots}^{\infty} \frac{\Gamma_m A_{m3}}{B_m} & \displaystyle\sum_{m=1,3,\ldots}^{\infty} \frac{A_{m1}A_{m3}}{B_m} + \frac{1}{32\frac{EI}{Dd}} & \displaystyle\frac{6}{5}\sum_{m=1,3,\ldots}^{\infty} \frac{\Gamma_m A_{m1}}{B_m} \\ \displaystyle\frac{4}{5}\sum_{m=1,3,\ldots}^{\infty} \frac{\Gamma_m^{\;2}}{B_m} & \displaystyle\frac{6}{5}\sum_{m=1,3,\ldots}^{\infty} \frac{\Gamma_m A_{m1}}{B_m} & \displaystyle\sum_{m=1,3,\ldots}^{\infty} \frac{A_{m1}A_{m2} - \frac{4}{9}\Gamma_m^{\;2}}{B_m} + \frac{1}{162\frac{EI}{Dd}} \end{vmatrix} = 0 \qquad \text{(B20)}$$

where

$$B_m = A_{m1}A_{m2}A_{m3} - \frac{36}{25}\Gamma_m^{\;2}A_{m1} - \frac{4}{9}\Gamma_m^{\;2}A_{m3}$$

In a similar manner, a stability criterion is obtained from a third approximation, which includes all the a_{m1}, b_{m2}, a_{m3}, and b_{m4} terms. A second approximation was found to give satisfactory results for most cases. For certain cases noted in table I for a panel aspect ratio of 5, however, third approximations should be used. The terms used in these approximations are given in table I. For each of these approximations, it is necessary to find the lowest value of k_s for each value of $\frac{EI}{Dd}$. The results are given in table I and in figure 1.

REFERENCES

1. Schmieden, C.: The Buckling of Stiffened Plates in Shear. Translation No. 31, U. S. Experimental Model Basin, Washington Navy Yard, June 1936.

2. Seydel, Edgar: Wrinkling of Reinforced Plates Subjected to Shear Stresses. NACA TM No. 602, 1931.

3. Wang, Tsun Kuei: Buckling of Transverse Stiffened Plates under Shear. Jour. Appl. Mech., vol. 14, no. 4, Dec. 1947, p. A-269.

4. Timoshenko, S.: Theory of Elastic Stability. McGraw-Hill Book Co., Inc. 1936, p. 382.

5. Budiansky, Bernard, Conner, Robert W., and Stein, Manuel: Buckling in Shear of Continuous Flat Plates. NACA TN No. 1565, 1948.

6. Kuhn, Paul, and Peterson, James P.: Strength Analysis of Stiffened Beam Webs. NACA TN No. 1364, 1947.

7. Sandlin, Charles W., Jr.: Strength Tests of Shear Webs with Uprights Not Connected to the Flanges. NACA TN No. 1635, 1948.

8. Levin, L. Ross, and Sandlin, Charles W., Jr.: Strength Analysis of Stiffened Thick Beam Webs. NACA TN No. 1820, 1949.

TABLE I. – SHEAR-STRESS COEFFICIENTS FOR STIFFENED PLATES

WITH PANEL ASPECT RATIOS OF ONE, TWO, AND FIVE

Plates with stiffeners of low bending stiffness[a]		Plates with stiffeners of higher bending stiffness				
		Aspect ratio	Antisymmetric buckling periodic over two bays		Buckling periodic over four bays	
$\frac{EI}{Dd}$	k_s		$\frac{EI}{Dd}$	k_s	$\frac{EI}{Dd}$	k_s
0	5.34	1	0	5.53	0	6.08
2	10.34		2.91	7.85	7.09	10.0
5	16.07		7.78	9.82	19.03	10.5
20	37.14		22.29	11.78	∞	10.86
50	68.99		∞	13.86		
100	112.2	2	0	9.65	0	5.54
200	184.6		3.35	12.0	5.475	15.0
			14.50	16.0	11.93	20.0
			22.99	18.0	26.37	23.0
			33.11	20.0	36.29	24.5
			45.77	22.0	68.92	26.0
			61.97	24.0	145.4	27.0
			82.92	26.0	625	28.0
			112.3	28.0	∞	28.2
			605	35.0		
			∞	37.05		
		5	0	42.5	0	13.37
			18.02	70	49.19	60
			90.99	90	112.8	[b]100
			176.8	100	220	[c]140
			444.7	120	∞	[c]143
			704.4	140		

[a]Independent of aspect ratio.

[b]All the a_{m1}, b_{m2}, a_{m3}, and b_{m4} coefficients used.

[c]All the a_{m3}, b_{m4}, a_{m5}, and b_{m6} coefficients used.

TABLE II.- EXPERIMENTAL BUCKLING DATA OF SHEAR WEBS

WITH UPRIGHTS NOT CONNECTED TO THE FLANGES

Specimen (a)	d (in.)	b (in.)	t (in.)	Uprights (nominal size) (in.)	$\frac{EI}{Dd}$	τ (ksi)	k_s
2-D-0	5.0	23.5	0.0397	1/2 × 1/2 × 1/16	221	2.66	101
3-D-0	5.0	23.5	.0394	3/4 × 3/4 × 1/16	680	3.08	116.5
4-D-0	5.0	23.5	.0405	3/4 × 3/4 × 3/32	946	3.295	117.5
5-D-0	10.0	23.5	.0404	1/2 × 1/2 × 1/16	98.3	1.21	43.3
6-D-0	10.0	23.5	.0408	3/4 × 3/4 × 1/16	306	1.54	54.2
7-D-0	10.0	23.5	.0410	3/4 × 3/4 × 3/32	456	1.47	51.3
8-S-0	5.0	23.5	.0394	1/2 × 1/2 × 0.064	95.8	2.895	109
9-S-0	5.0	23.5	.0399	3/4 × 3/4 × 3/32	456	3.01	111
10-S-0	10.0	23.5	.0410	1/2 × 1/2 × 1/16	41.4	.82	28.6
11-S-0	10.0	23.5	.0398	3/4 × 3/4 × 1/16	151.5	1.357	50.1
12-S-0	10.0	23.5	.0405	3/4 × 3/4 × 3/32	217	1.41	50.3

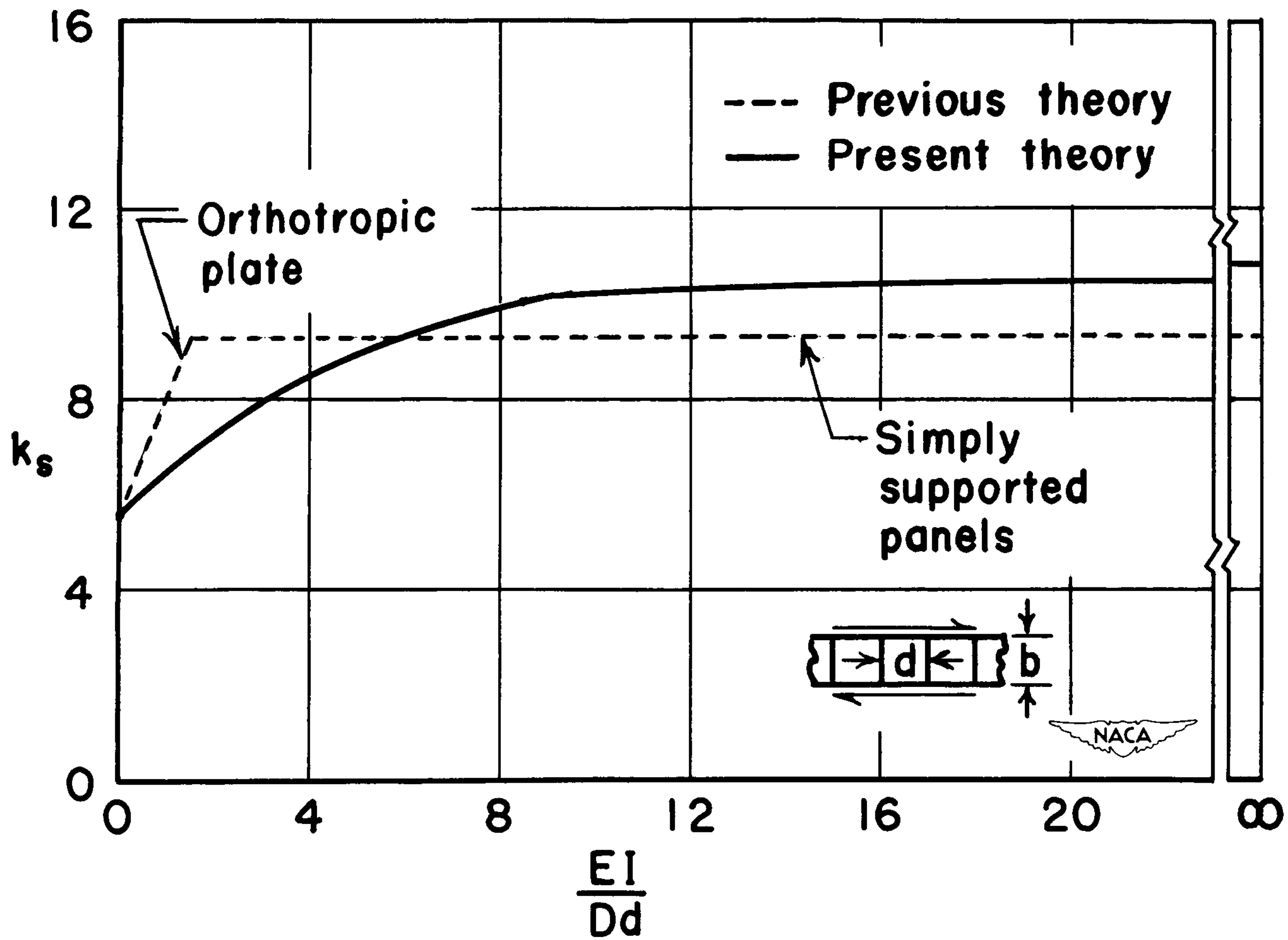
[a]S, stiffeners on one side of plate.
D, stiffeners on both sides of plate.

TABLE III. – EXPERIMENTAL BUCKLING DATA OF THICK WEB BEAMS

WITH UPRIGHTS CONNECTED TO THE FLANGES

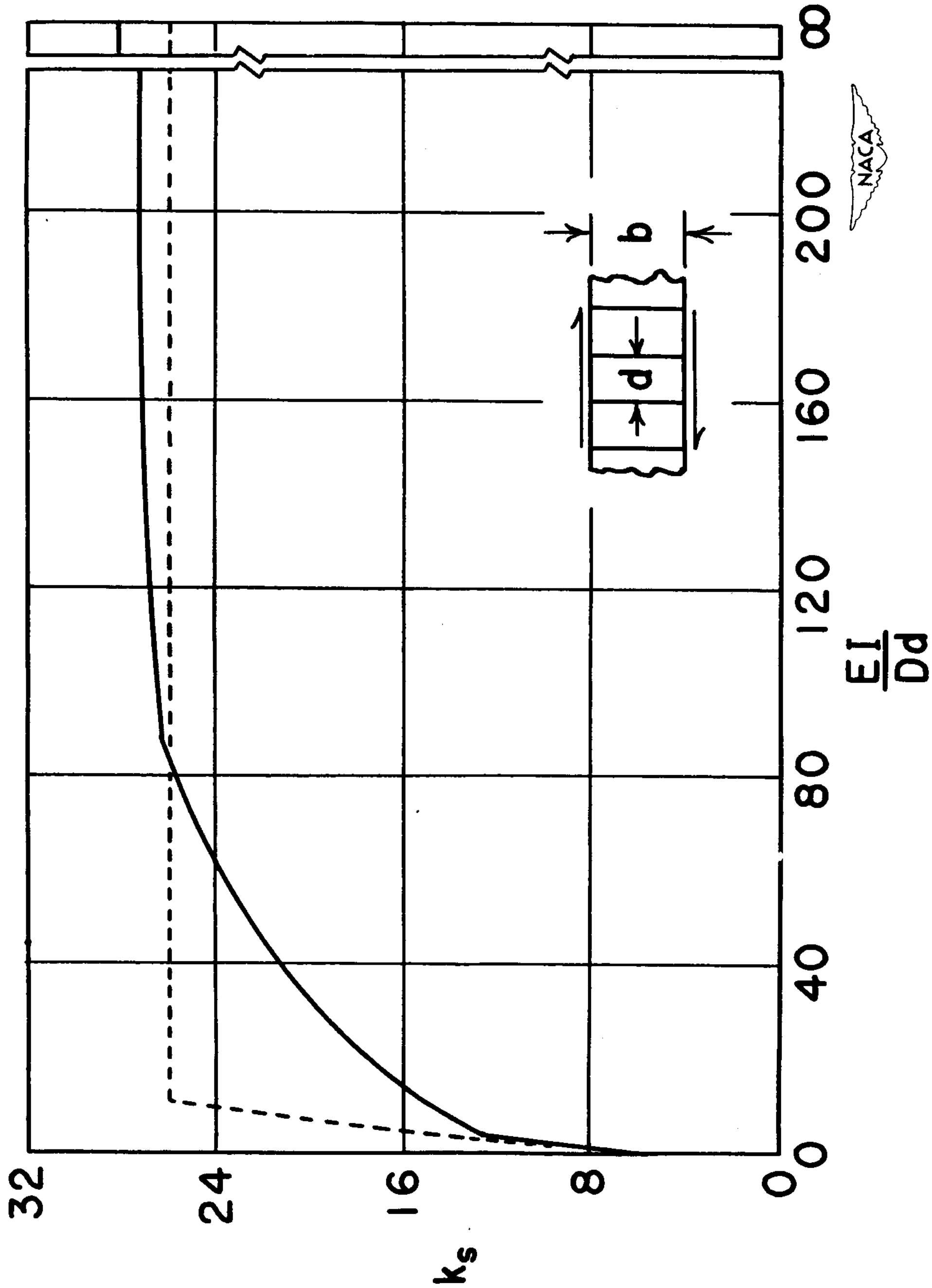
Specimen (a)	b (in.)	d (in.)	t (in.)	Uprights (nominal size) (in.)	$\frac{EI}{Dd}$	τ (ksi)	k_s
V-12-7S	9.88	7.00	0.1005	$1\frac{1}{8} \times 1\frac{1}{8} \times \frac{1}{8}$	91.0	15.5	15.4
V-12-8S	9.88	7.00	.1044	3/4 × 3/4 × 1/8	25.8	15.4	14.15
V-12-9D	9.13	7.00	.1025	5/8 × 5/8 × 1/8	40.4	16.8	13.65
V-12-10S	9.88	7.00	.1043	5/8 × 5/8 × 1/8	14.5	16.3	15.0
V-12-11D	9.13	7.00	.1025	5/8 × 5/8 × 3/32	30.3	17.2	14.0
V-12-12S	9.88	7.00	.0987	1/2 × 1/2 × 1/16	4.1	12.3	12.7
V-12-13D	9.13	7.00	.1000	1/2 × 1/2 × 1/16	11.3	13.1	11.15
V-12-14S	9.88	7.00	.1007	5/8 × 5/8 × 3/32	11.2	13.2	13.1
V-12-15D	9.13	7.00	.1057	5/8 × 5/8 × 1/16	18.8	15.7	12.0

[a] S, stiffeners on one side of plate.
D, stiffeners on both sides of plate.



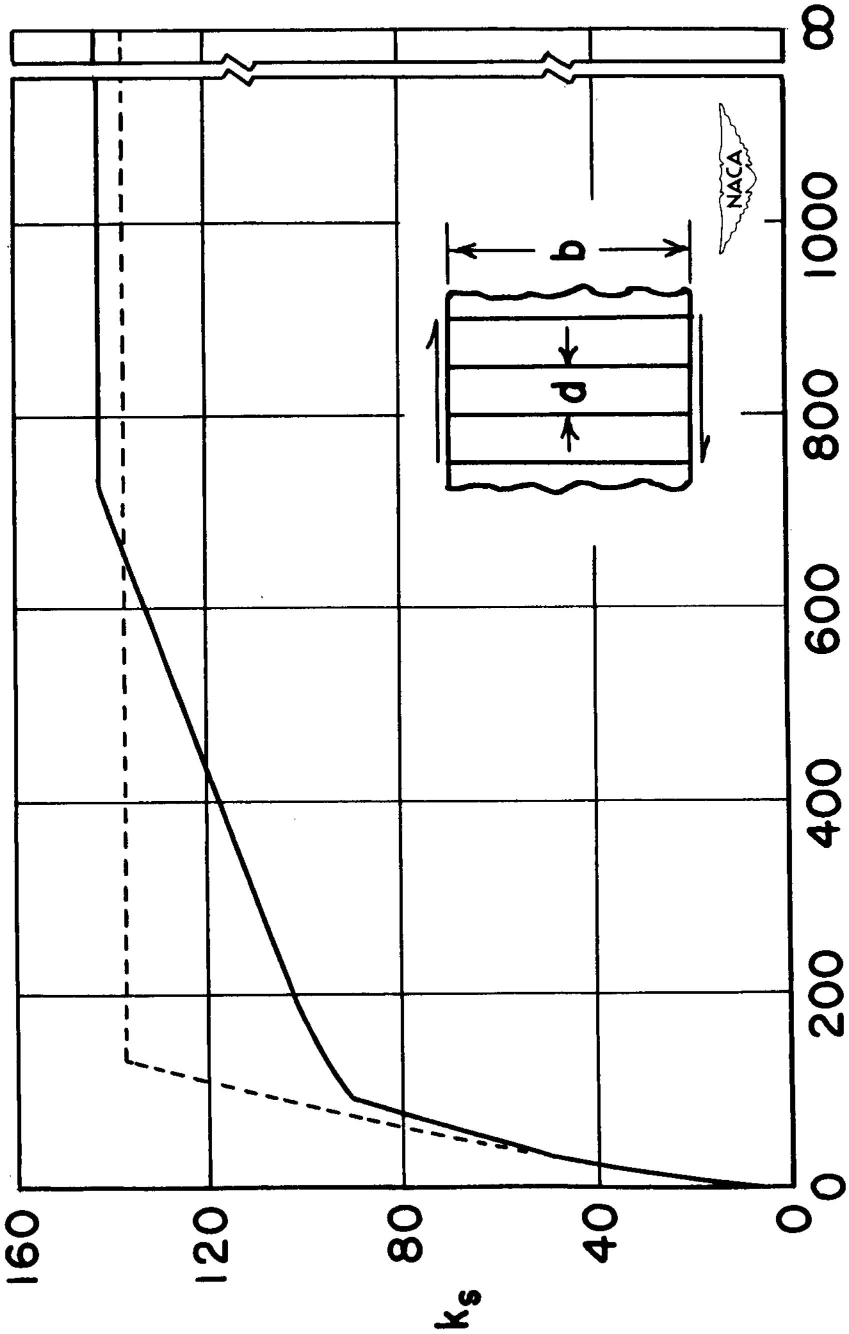
(a) Panel aspect ratio $\frac{b}{d} = 1$.

Figure 1.- Critical shear-stress coefficient $\left(k_s = \frac{\tau t b^2}{D\pi^2}\right)$ for infinitely long, transversely stiffened plates.



(b) Panel aspect ratio $\frac{b}{d} = 2$.

Figure 1.- Continued.



(c) Panel aspect ratio $\frac{b}{d} = 5$.

Figure 1.- Concluded.

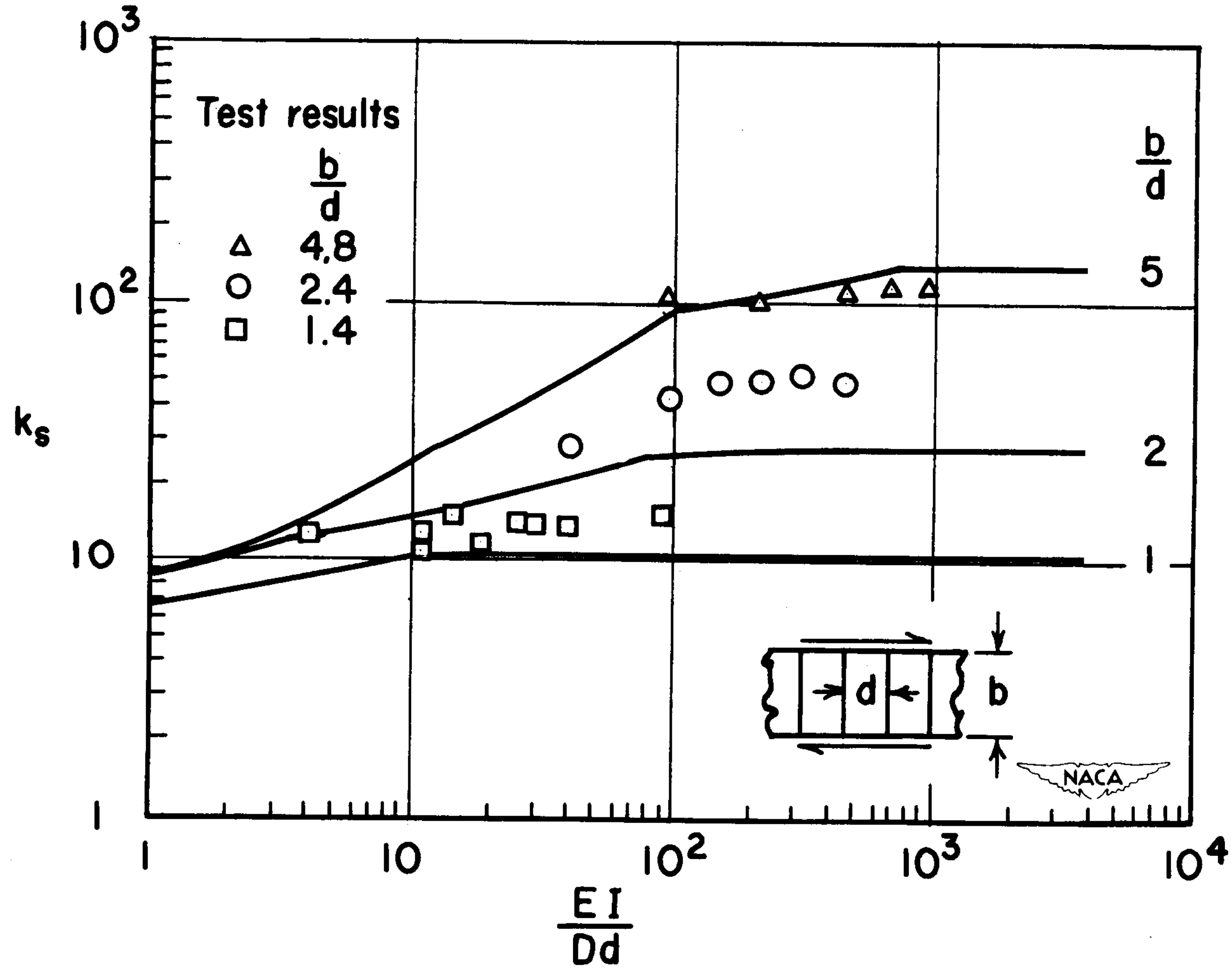


Figure 2.- Theoretical critical shear-stress coefficient $\left(k_s = \frac{\tau t b^2}{\pi^2 D}\right)$ compared with experimental results for infinitely long, transversely stiffened plates with various panel aspect ratios $\frac{b}{d}$ and stiffness ratios $\frac{EI}{Dd}$.

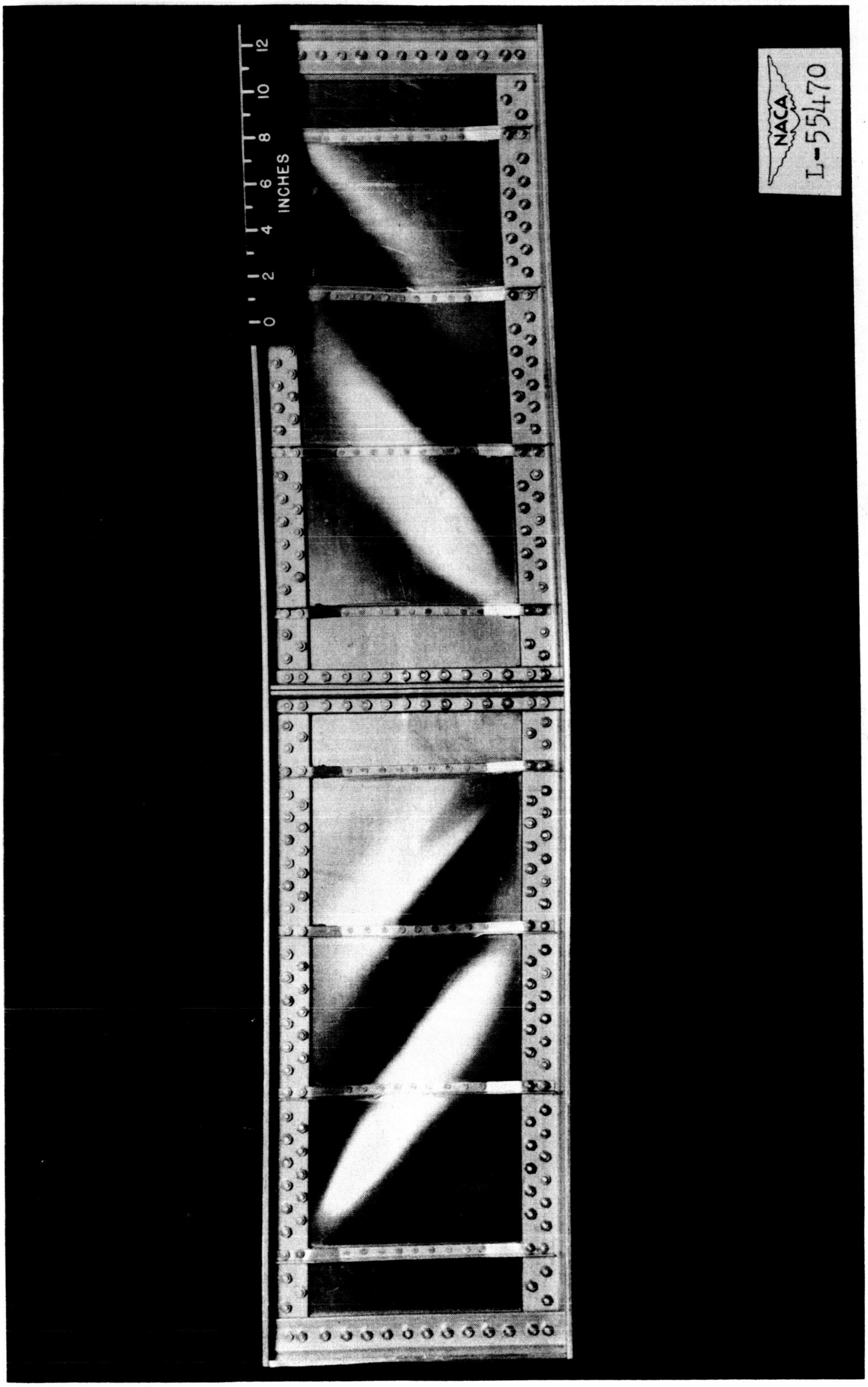


Figure 3.- Thick web beam (with uprights connected to flanges) after failure.

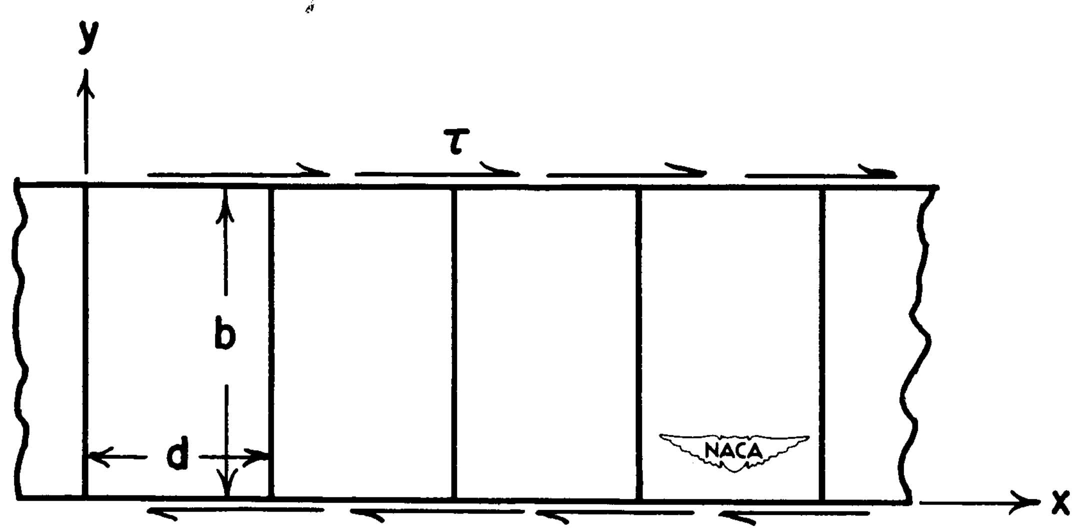


Figure 4.- Infinitely long, simply supported plate, with transverse stiffeners, under shear.