NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

TECHNICAL NOTE
No. 1241
MAR 31 1947

EXPERIMENTAL INVESTIGATION OF THE STRESS DISTRIBUTION AROUND REINFORCED CIRCULAR CUT-OUTS IN SKIN-STRINGER PANELS UNDER AXIAL LOADS

By Daniel Farb
Langley Memorial Aeronautical Laboratory
Langley Field, Va.

Washington
March 1947
Results are presented of strain surveys around four reinforced circular cut-outs successively made in an axially loaded skin-stringer tension panel. These surveys were made in the elastic range. In each case the reinforcing rings were of rectangular cross section, and the cross-sectional area of the reinforcement approximately equaled the cross-sectional area of the skin and stringers removed at the transverse center line of the cut-out.

Curves are presented which show the distribution of stringer and shear stresses in the panel and bending stresses in the reinforcing rings. Empirical methods are given for estimating the maximum stringer stress in the panel and for approximating the longitudinal stresses in the stringers and rings at the transverse center line of the cut-out.

INTRODUCTION

As a result of a lack of analytical and experimental information, the design of reinforced circular cut-outs in skin-stringer combinations at present depends largely upon judgment and experience. This paper gives experimental information which may be used to determine the suitability of present design practices and should be useful for comparison with the results of future analyses.

Strain surveys in the elastic range were made around reinforced circular cut-outs in a flat skin-stringer panel. The rings reinforcing the cut-outs were of rectangular cross section, and the cross-sectional area of reinforcement approximately equaled the cross-sectional area of skin and stringers removed at the transverse center line of the cut-out. Curves are presented to show the
experimental distributions of stringer and shear stresses in the panel and the bending stresses in the rings. Empirical relationships are presented to enable the designer to approximate quickly the maximum stringer stress in the panel and the stringer stresses and longitudinal ring stresses at the transverse center line of the cut-out.

TEST SPECIMENS AND PROCEDURE

The test specimen was a 16-stringer 24S-T aluminum alloy panel 144 inches long and approximately 48 inches wide, with a cross section as shown in figure 1. A schematic diagram of the panel is shown in figure 2.

Four circular cut-outs of progressively increasing diameter were made at the center of the panel. Each was tangent to continuous stringers at the transverse center line. The cut-outs had diameters of approximately 10, 22, 34, and 46 inches; for the smallest diameter, only the two center stringers of the panel were cut, and for the largest diameter only the two outside stringers remained uncut. Each cut-out was reinforced by a pair of circular rings of rectangular cross section, one on each side of the panel. Ring dimensions are given in the following table:

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<tr>
<td>10</td>
<td>15.81</td>
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</tr>
<tr>
<td>14</td>
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The rings were riveted to the sheet as well as to the stringers. Filler plates were placed on the sheet under the rings and were fastened in such a manner as to be as ineffective as possible in carrying load. In each case at the transverse center line of the cut-out the cross-sectional area of the rings approximately equaled the cross-sectional area of the skin and stringers removed.
Shear and stringer stresses in the panel as well as bending stresses in the reinforcing rings were experimentally determined for each cut-out condition. Before the first cut-out was made, tests were conducted to determine the elastic properties of the panel. An effective value of Young's modulus of $10.43 \times 10^3$ ksi was derived from measurements of the strains in all stringers at three stations along the span. The effective modulus may be considered as including corrections for the effect of rivet holes, average gage calibration factor, and dynamometer calibration factor. The average transverse strain indicated an effective Poisson's ratio of 0.302. The effective shear modulus was found to be $4.01 \times 10^3$ ksi. The preceding effective moduli were used to convert measured strain into stress.

The test setup is shown in figure 3. The total external load was uniformly distributed to the ends of the stringers by means of whippletrees. Stringer and shear strains were measured by Tuckerman optical strain gages with gage lengths of 2 inches. The shear strains in the sheet were obtained from strains measured at angles of $45^\circ$ and $135^\circ$ to the direction of the stringers. The gages were used in pairs, one gage on each side of the test panel. Strains were measured at corresponding points in all four quadrants of the doubly symmetrical panel, and the final figures are drawn as for one quadrant. Each plotted point represents, therefore, the average of four stations. Bending stresses in the rings were determined from resistance-type wire-strain-gage measurements. A typical arrangement of wire strain gages on the rings is shown in figure 4. The gages were placed on the inner and outer edges of each ring at regular intervals along the perimeters. The plotted experimental values represent the average stresses in each pair of rings reinforcing a cut-out.

The load was applied in three equal increments. A load-stress plot was made for each gage, and a straight line was drawn through the plotted points. If the line through the plotted points did not pass through the origin, a line parallel to it was drawn through the origin. The line through the origin was used to determine the measured strain at maximum load. If the necessary shift was more than 0.2 ksi, however, a new set of readings was taken.

RESULTS AND DISCUSSION

Figures 5 to 8 show the spanwise distribution of experimental stringer stresses. It may be noted from the curves that the maximum
stringer stress for each cut-out condition occurred in either the continuous stringer bounding the cut-out or in the cut stringer adjacent to this continuous stringer, and that, in general, the maximum stresses in both stringers were approximately equal. The maximum stress occurred at a transverse station which intersected the median line of the reinforcing rings at approximately 45° from the transverse center line.

Chordwise distributions of stringer stresses are shown in figure 9. At a distance of one radius from the transverse center line, the chordwise stress distribution was nearly uniform from the outer edge of the panel to some point close to the first cut stringer and then decreased approximately linearly to zero near the longitudinal center line. At the transverse center line the chordwise distribution was approximately uniform except in those panels with few uncut stringers.

The spanwise-distribution curves for the experimental shear stresses are shown in figures 10 to 13. Chordwise distributions of shear stresses are shown in figure 14.

The distribution of bending stresses in the reinforcing rings is plotted in figure 15 for each cut-out condition. It has been suggested in estimating these bending stresses that the stringer loads which existed at a large distance from the cut-out should be applied directly to the rings, without allowing for a redistribution of the forces through the skin-stringer combination. The maximum stresses resulting from this type of analysis were computed for the four cut-out conditions tested and were compared with the maximum bending stresses computed from the test data. In these calculations only the actual area of the rings was used to determine the moment of inertia. The calculations were made for a load of 15 kips. The results of this comparison are given in the following table. It may be observed that for the four cut-outs under consideration this method is unduly conservative.

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SUMMARY

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The distribution of bending stresses in the reinforcing rings is plotted in figure 15 for each cut-out condition. It has been suggested in estimating these bending stresses that the stringer loads which existed at a large distance from the cut-out should be applied directly to the rings, without allowing for a redistribution of the forces through the skin-stringer combination. The maximum stresses resulting from this type of analysis were computed for the four cut-out conditions tested and were compared with the maximum bending stresses computed from the test data. In these calculations only the actual area of the rings was used to determine the moment of inertia. The calculations were made for a load of 15 kips. The results of this comparison are given in the following table. It may be observed that for the four cut-outs under consideration this method is unduly conservative.

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The strain measurements on the reinforcing rings indicated that at the transverse center line the average longitudinal stress in the rings was approximately 58 percent of the average stress in the skin and stringers. Since the chordwise distribution of experimental stringer stress was found to be approximately uniform, the stringer stresses and the longitudinal stresses in the rings at the transverse center line may be approximated in the following manner:

\[
\sigma_s = \frac{P}{A_s + 0.58 A_r} \quad (1)
\]

\[
\sigma_r = 0.58 \sigma_s \quad (2)
\]

where \( \sigma_s \) is the average stringer stress, \( P \) is the external load on the panel, \( A_s \) is the cross-sectional area of skin and stringers, \( A_r \) is the cross-sectional area of rings, and \( \sigma_r \) is the average longitudinal stress in the rings. The experimental average stringer stresses and the average stringer stresses calculated by the empirical relationship of equation (1) at the transverse center line for a load of 15 kips are given in the following table:

<table>
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<th>Stringers cut</th>
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<th>Calculated ( \sigma_s ) (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3.9</td>
<td>3.9</td>
</tr>
<tr>
<td>6</td>
<td>4.5</td>
<td>4.4</td>
</tr>
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</tr>
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The experimental longitudinal stresses and the longitudinal stresses calculated by the empirical relationship of equation (2) in the reinforcing rings at the transverse center line are given in the following table for a load of 15 kips:

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<tr>
<td>2</td>
<td>2.6</td>
<td>2.3</td>
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<td>3.4</td>
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The maximum stringer stresses may be approximated by the empirical formula

\[ \sigma_{\text{max}} = (1 + \frac{D}{b}) \sigma_s \]  

(3)

where \( \sigma_{\text{max}} \) is the maximum stringer stress in the panel, \( D \) is the median diameter of the reinforcing rings, \( b \) is the width of the panel, and \( \sigma_s \) is the average stringer stress from equation (1).

The experimental maximum stringer stresses and the maximum stringer stresses calculated by the empirical relationship of equation (3) for a load of 15 kips are given in the following table:

<table>
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<tr>
<th>Stringers cut</th>
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The empirical relationships should be applied only to cut-outs with ring reinforcements similar to those of the test panel.

An analysis was made by means of the numerical procedure of reference 1, but it was discovered that because of the complex interaction of the reinforcing rings with the skin-stringer combination near the cut-out, an impracticable number of displacements would have to be calculated before satisfactory results could be obtained.

CONCLUSIONS

Results were presented of strain surveys around four reinforced circular cut-outs of varying diameter successively made in an axially loaded skin-stringer panel. These surveys were made in the elastic range and showed in the form of curves the distribution of stringer and shear stresses in the panel and bending stresses in the reinforcing rings. In each case the reinforcing rings were of rectangular cross section, and the cross-sectional area of the reinforcement approximately equaled that of the panel removed at the transverse center line of the cut-out. The following conclusions were indicated:
1. The maximum stringer stress occurred at a transverse station which intersected the median line of the reinforcing rings at approximately 45° from the transverse center line of the cut-out. The maximum stringer stress occurred in either the continuous stringer bounding the cut-out or in the cut stringer adjacent to this continuous stringer.

2. In estimating the bending stresses in the reinforcing rings, the practice of assuming the stringer loads at a large distance from the cut-out applied directly to the rings was found to be unduly conservative.

3. Empirical relationships were determined for evaluating the maximum stringer stress in the panel and for approximating the longitudinal stresses in the stringers and rings at the transverse center line of the cut-out.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., Dec. 27, 1946

REFERENCE

Figure 1.- Cross section of test panel.

Figure 2.- Schematic diagram of one quadrant of test panel. Dashed lines indicate median lines of reinforcing rings.
Figure 3.- Test set-up.
Figure 4. - Typical arrangement of wire strain gages on rings.
Figure 5.- Spanwise distribution of stringer stresses in panel with two stringers cut. Dashed lines indicate extrapolation. Load = 15 kips.
Figure 6.- Spanwise distribution of stringer stresses in panel with six stringers cut. Dashed lines indicate extrapolation. Load = 15 kips.
Figure 7.— Spanwise distribution of stringer stresses in panel with ten stringers cut. Dashed lines indicate extrapolation. Load = 15 kips.
Figure 8.- Spanwise distribution of stringer stresses in panel with fourteen stringers cut. Dashed lines indicate extrapolation. Load = 15 kips.
Figure 9.- Chordwise distribution of stringer stresses in one quadrant of panel. Load = 15 kips.
Figure 10.— Spanwise distribution of shear stresses in panel with two stringers cut. Load = 15 kips.
Figure 11.— Spanwise distribution of shear stresses in panel with six stringers cut. Load = 15 kips.
Figure 12. - Spanwise distribution of shear stresses in panel with ten stringers cut. Load = 15 kips.
Figure 13.—Spanwise distribution of shear stresses in panel with fourteen stringers cut. Load = 15 kips.
Figure 14. Chordwise distribution of shear stresses in one quadrant of panel. Load = 15 kips.
Figure 15.- Distribution of bending stresses in reinforcing rings. Load = 15 kips.