NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1076

A SIMPLIFIED METHOD FOR DETERMINING FROM FLIGHT DATA THE RATE OF CHANGE OF YAWING-MOMENT COEFFICIENT WITH SIDESLIP

C_{n_β}

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Washington
June 1946

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SUMMARY

A method is presented by which the directional stability derivative C_{n_β}, the rate of change of yawing-moment coefficient with sideslip angle, can be evaluated for a conventional airplane from flight records of a lateral or directional oscillation. For the method shown, the calculation of C_{n_β} for a particular high-speed-flight condition reduces to the determination of only the moment of inertia about the Z-axis and the period of a sideslipping, yawing or rolling oscillation.

When applied to conventional airplanes flying at low to moderate lift coefficients, the assumptions involved in this simplified method produce negligible error. A comparison of C_{n_β} as determined in flight and in the wind tunnel shows good agreement for the four conventional airplanes considered.

INTRODUCTION

During preliminary flight tests of experimental pursuit-type airplanes an accurate estimation of maximum sideslip angles attainable in rolling pull-outs or other dynamic conditions may be required before such critical tests are undertaken. Existing theoretical methods for calculating the maximum sideslip angle of an airplane in dynamic flight require an accurate knowledge of C_{n_β}. Methods now used for computation of C_{n_β}, when wind-tunnel data are unavailable, involve estimates of tail effectiveness which, because of fuselage interference, are subject to considerable error. Therefore a method has been developed for measuring in flight values of C_{n_β} which may be used to verify or correct the design computations.

The derivation and application of equations and the correlation of flight results with wind-tunnel data for the four airplanes shown in figure 1 are presented in this report.

COEFFICIENTS AND SYMBOLS

Coefficients and symbols defined herein are referred to the wind system of coordinates in which the origin is fixed at the center of gravity of the airplane, the Z-axis is in the plane of symmetry and perpendicular to the relative air stream, the X-axis is in the plane of symmetry and parallel to the relative air stream, and the Y-axis is perpendicular to the Z- and X-axes.

b wing span, feet

S wing area, square feet

m mass of airplane, slugs

l distance from center of gravity to rudder hinge line, feet

g acceleration due to gravity, feet per second per second

I_X moment of inertia about X-axis, slug-feet square

I_Z moment of inertia about Z-axis, slug-feet square

ρ air density, slugs per cubic foot

V velocity of airplane along flight path, feet per second

q free-stream dynamic pressure $\left(\frac{1}{2}\rho V^2\right)$, pounds per square foot

s operator

λ_1 root of the stability quartic

β angle of sideslip (positive when right wing is forward), radians

β^{o}	$180\ \beta/\pi$, degrees
P	period of oscillation, seconds
p	rate of roll, radians per second
r	rate of yaw, radians per second
v	component of flight velocity along the Y-axis, feet per second
Y	force along Y-axis, pounds
L	moment about X-axis, foot-pounds
N	moment about Z-axis, foot-pounds
C_Y	lateral-force coefficient (Y/qS)
C_l	rolling-moment coefficient (L/qSb)
C_n	yawing-moment coefficient (N/qSb)
C_{Y_β}	$(\partial C_Y/\partial\beta)$
C_{l_β}	$(\partial C_l/\partial\beta)$
C_{n_β}	$(\partial C_n/\partial\beta)$
C_{l_r}	$[\partial C_l/\partial(rb/2V)]$
C_{n_r}	$[\partial C_n/\partial(rb/2V)]$
C_{l_p}	$[\partial C_l/\partial(pb/2V)]$
C_{n_p}	$[\partial C_n/\partial(pb/2V)]$
Y_v	$(qS/mV)C_{Y_\beta}$
L_β	$(qSb/I_X)C_{l_\beta}$
N_β	$(qSb/I_Z)C_{n_\beta}$

N_p $(qSb/I_Z)\ (b/2V)\ C_{n_p}$

N_r $(qSb/I_Z)(b/2V)C_{n_r}$

L_p $(qSb/I_X)\ (b/2V)C_{l_p}$

L_r $(qSb/I_X)(b/2V)C_{l_r}$

THEORY

The stability quartic arising from the consideration of small lateral disturbances from steady horizontal flight can be written (reference 1)

$$s^4 + A_3\ s^3 + A_2\ s^2 + A_1\ s + A_0 = 0$$

where

$$A_3 = -L_p - Y_v - N_r$$

$$A_2 = L_p\ (Y_v + N_r) + N_r\ Y_v + N_\beta - L_r\ N_p$$

$$A_1 = -Y_v\ (N_r\ L_p - L_r\ N_p) - L_p\ N_\beta + L_\beta\ [N_p - (g/V)]$$

$$A_0 = (g/V)\ (L_\beta\ N_r - L_r\ N_\beta)$$

The real roots and the real parts of the complex roots of the preceding quartic determine the damping or divergence of the airplane motion, and the imaginary parts determine the period of oscillation. Normally one of the real roots is small enough so that it can be approximated by neglecting terms in the quartic of higher order than the first. Denoting this root by λ_1, it follows that

$$\lambda_1 \approx -(A_0/A_1)$$

By factoring this root out of the quartic the following cubic is obtained:

$$s^3 + (A_3 + \lambda_1)\ s^2 + (A_2 + A_3\ \lambda_1 + \lambda_1^2)\ s$$

$$+ (A_1 + A_2\ \lambda_1 + A_3\ \lambda_1^2 + \lambda_1^3) = 0$$

For conventional airplanes traveling at relatively high speeds, the values of λ_1, N_p, $\frac{1}{4}(Y_v - N_r)^2$, and g/V are small enough to be neglected in comparison with the terms L_p and N_β. By these assumptions the cubic may be factored giving

$$(s-L_p)\quad\left[s - \frac{1}{2}(Y_v + N_r) - \sqrt{-N_\beta}\right]\left[s - \frac{1}{2}(Y_v + N_r) + \sqrt{-N_\beta}\right] = 0$$

Since N_β is positive, $\sqrt{N_\beta}$ is the magnitude of the imaginary part of the complex roots and therefore the frequency of oscillation. It follows that the period of oscillation of the laterally or directionally disturbed airplane is given by the formula

$$P = \frac{2\pi}{\sqrt{N_\beta}}.$$

Hence

$$C_{n_\beta}{}^{o} = \frac{4\pi^2\ I_Z}{57.3\ qSb\ P^2} = 0.688\ \frac{I_Z}{qSb\ P^2}$$

The magnitude of the error in the period introduced by the assumption that the period of oscillation equals 2π divided by $\sqrt{N_\beta}$ depends upon both the relative and the absolute magnitude of the neglected stability derivatives of the airplane. Therefore, caution should be used when the assumption is applied to an unconventional design, such as a tailless airplane, where the relative magnitudes of the derivatives may differ considerably from those of the airplanes of this report.

In order to illustrate the accuracy of the approximation $P = 2\pi/\sqrt{N_\beta}$, the variations with indicated airspeed of the period as computed by three methods are shown in figure 2 for a representative modern airplane. The three methods used are: (1) the method of reference 1 in which (within the limitations of the initial assumptions) the theoretically true period is given as 2π divided by the magnitude of the complex part of an imaginary root of the stability quartic, (2) the method of the present report in which the period is equal to $2\pi/\sqrt{N_\beta}$, and (3) the method of reference 2 in which an alternative simplification and factorization of the quartic gives a period equal to

$$2\pi \Bigg/ \sqrt{\frac{L_\beta\ (N_p - g/V)\quad -N_\beta\ L_p}{-N_r - L_p}}$$

The comparisons of figure 2 indicate that, for the airplane considered, the approximation of this report (method (2)) gives results that agree within 5 percent with those obtained using the theoretically true method (method (1)), and are closer than those of method (3). At low speeds (and high lift coefficients), where the neglected stability derivatives assume larger values relative to C_{n_β}, the approximation presented herein tends to become less reliable and others should be used.

FLIGHT PROCEDURE

The period of motion necessary for calculation of C_{n_β} may be obtained by placing the airplane in a steady sideslip while trimmed for straight and zero-sideslip flight, then abruptly returning the control to trim and recording the resulting motion. A typical time history of the yawing and rolling oscillation that follows such a disturbance is shown in figure 3. From records similar to those shown in figure 3 the period of the oscillations may be determined and inserted in the equation given for C_{n_β}. However, it is not essential in the evaluation of C_{n_β} to obtain time histories of the airplane motions; any accurate means of measuring the period alone would be sufficient.

The value of C_{n_β} obtained by this method actually represents an average value for the range of sideslip angles covered. If it is desired to establish a curve of C_{n_β} against β, runs can be made with varying amounts of applied disturbance or with different initial trim positions.

The procedure may be used for either a control-fixed or a control-free configuration. In the interpretation of control-free results consideration should be given to the control-system friction, which is sometimes sufficient to hold the rudder fixed in a rudder-free maneuver.

PRECISION

The accuracy with which C_{n_β} can be evaluated from flight tests depends largely on the precision with which the period can be measured and the moment of inertia I_Z can be estimated. If ϵ represents the error in the flight-measured period, the percent error in C_{n_β} resulting from the error ϵ is given by

$$\text{Percent error in } C_{n_\beta} = \pm 100 \times 2\epsilon/P$$

It is estimated on the basis of available flight data that the value of ϵ is of the order of 0.10 second. In order to reduce the percentage error from this source to less than 6 percent in any given reading, a period greater than about 3.5 seconds would be required. Since the period is inversely proportional to the speed, the experimental error is reduced for lower speeds. The error likely to be incurred in the computation of I_Z is greatly dependent upon the method used for its determination and with care may be held within 1 percent.

The gain in experimental accuracy at lower speeds tends to be offset by the decreased accuracy in the theory due to the approximations. It is expected that the greatest over-all accuracy will be obtained between 200 and 300 miles per hour.

COMPARISON OF FLIGHT DATA WITH WIND-TUNNEL DATA

To check the validity of the simplified method developed in this report a comparison has been made of the values of C_{n_β} obtained from wind-tunnel and flight tests. Four airplanes were chosen (fig. 1) which exhibited a wide range of directional stability and for which there existed the necessary wind-tunnel and flight data.

The value of C_{n_β} was obtained from flight data using

$$C_{n_{\beta o}} = 0.688 \frac{I_Z}{qSb\,P^2}$$

The moments of inertia were obtained from reports of the Langley Spin Tunnel, and the values of P were determined from directional oscillations where the amplitude of the sideslip oscillations was $\pm 5^\circ$. The flight and wind-tunnel results in the form of C_n against β° are compared in figure 4. Since no values of intercept can be assigned to this curve, the flight curves were arbitrarily placed so that zero intercepts coincided with those of the wind-tunnel data. Good agreement between wind-tunnel and flight-test results for all four airplanes indicates that results at least as accurate as those obtainable in a wind tunnel may be anticipated if the approximate method of this report is used.

CONCLUDING REMARKS

The simplified method for the determination of C_{n_β} by flight test requires the measurement of only the period of oscillation of the airplane when disturbed laterally or directionally. For conventional airplanes flying at low to moderate lift coefficients, the assumptions involved in this simplified method produce negligible error; hence the method gives results which are more accurate than those obtained using the approximate method of reference 2 and which are in good agreement with wind-tunnel results. For unconventional airplanes (such as tailless) or for airplanes flying at high lift coefficients the error may become appreciable and the method should be used with caution.

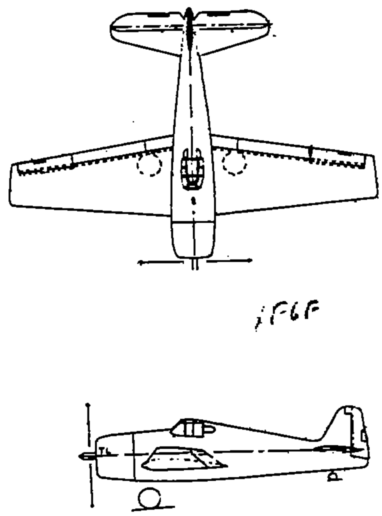
Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif., March 1946.

REFERENCES

1. Jones, Robert T.: A Simplified Application of the Method of Operators to the Calculation of Disturbed Motions of an Airplane. NACA Rep. No. 560, 1936.

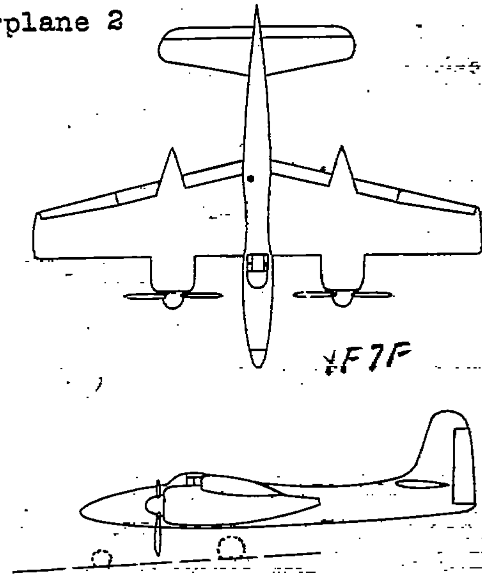
2. Zimmerman, Charles H.: An Analysis of Lateral Stability in Power-Off Flight with Charts for Use in Design. NACA Rep. No. 589, 1937.

Airplane 1



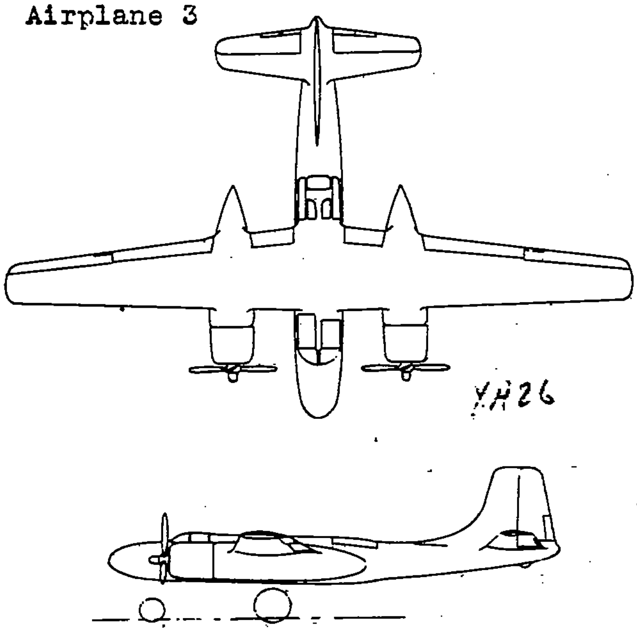
Wing area, 334 sq ft
Wing span, 42.83 ft
Weight, 11,500 lb
Length, 33.83 ft

Airplane 2



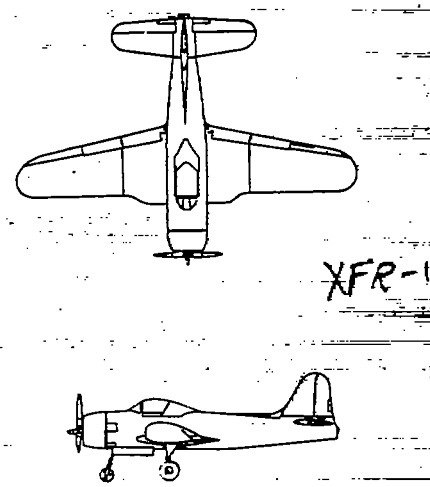
Wing area, 455.0 sq ft
Wing span, 51.5 ft
Weight, 20,000 lb
Length, 45.38 ft

Airplane 3



Wing area, 540.5 sq ft
Wing span, 70.0 ft
Weight, 30,000 lb
Length, 50.75 ft

Airplane 4



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Wing area, 275.0 sq ft
Wing span, 40.0 ft
Weight, 9,000 lb
Length, 32.1 ft

Figure 1.- Two-view drawings and pertinent specifications of the airplanes tested in flight.

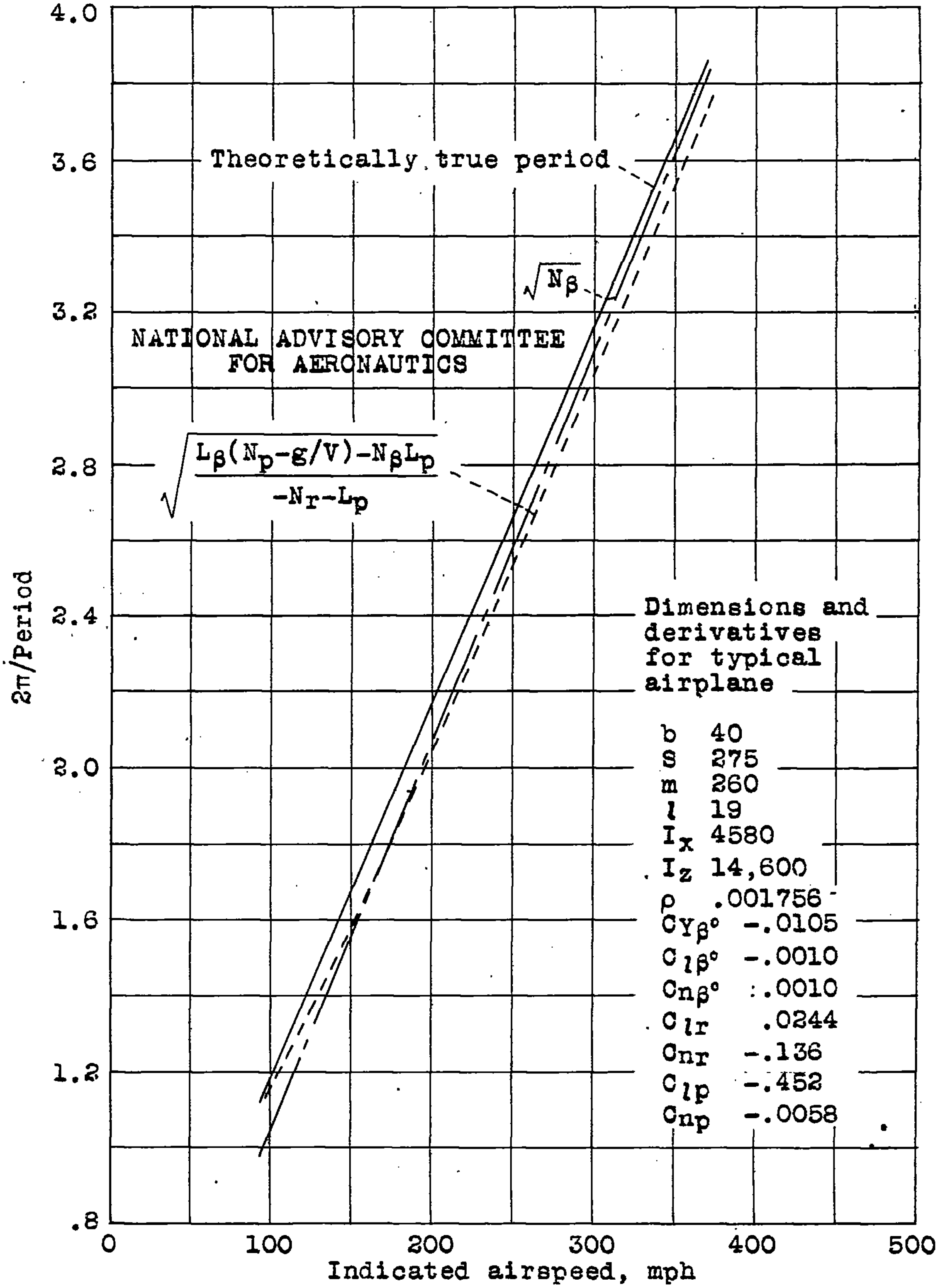

Figure 2.- Comparison of the true and approximate methods for determining the theoretical period of a representative airplane. Stability derivatives were computed for a lift coefficient of 0.132.

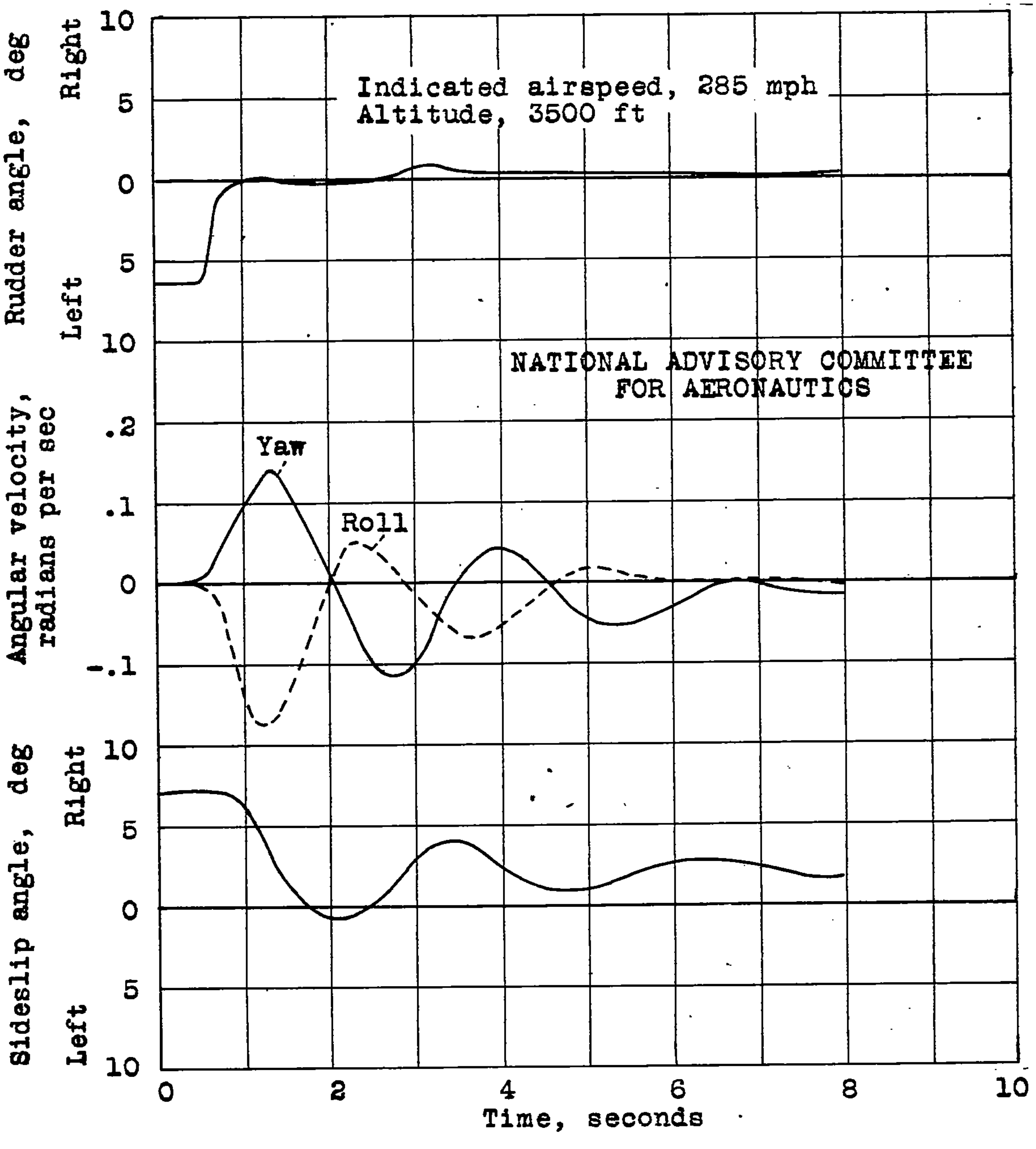

Figure 3.- Time history of a typical rudder-fixed lateral oscillation from which a value of C_{n_β} may be determined.

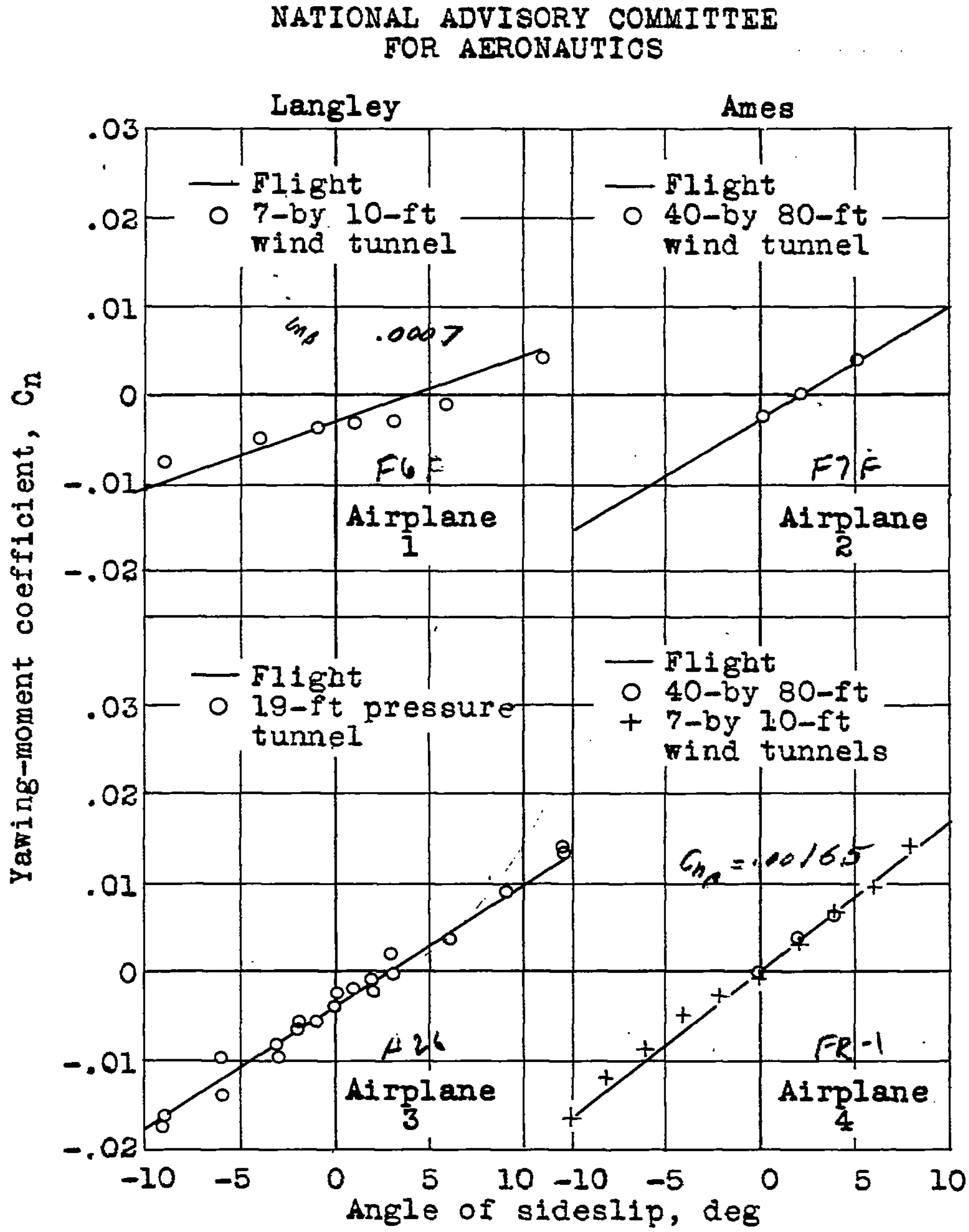

Figure 4.- Comparison of flight and wind-tunnel values of rate of change of yawing-moment coefficient with sideslip angle for four airplanes.