EXPERIMENTAL DETERMINATION OF THE EFFECTS OF DIHEDRAL,
VERTICAL-TAIL AREA, AND LIFT COEFFICIENT ON LATERAL
STABILITY AND CONTROL CHARACTERISTICS

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SUMMARY

The effects of wide variations of dihedral, vertical-tail area, and lift coefficient on lateral stability and control and on general flying characteristics have been determined by flight tests of a model in the Langley free-flight tunnel. In order to vary the effective dihedral and directional stability of the model, the geometric dihedral angle was varied from -20° to 18° and the vertical-tail area, from 0 to 35 percent of the wing area. The tests were made over a range of lift coefficient from 0.5 to 1.8.

The best general flight behavior was obtained when the effective dihedral angle was small (approx. 2°). Increasing the effective dihedral above 2° caused the flying characteristics to become worse because of the reduction in oscillatory stability and the increased effect of adverse yawing due to rolling and ailerons. As the effective dihedral was decreased to -15°, the model became increasingly difficult to fly because of an increasing rate of spiral divergence. Increasing the directional stability improved the general flight characteristics by increasing the oscillatory stability and reducing the adverse yawing for positive effective dihedral angles and by reducing the sideslipping and spiral instability for negative effective dihedral angles. Increasing the lift coefficient had a slightly detrimental effect on general flight behavior, particularly for low values of directional stability.

It is believed that the results of the tests can be interpreted to indicate that an airplane with a wing loading less than 35 pounds per square foot and with
rolling and yawing radii of gyration not exceeding 0.2 and 0.3 of the wing span, respectively, will have the best general flying characteristics if the effective dihedral is greater than zero but not so great that the value of the effective-dihedral parameter \(-C_{l\beta}\) exceeds one-half the value of the directional-stability parameter \(C_{n\beta}\) providing the value of \(C_{n\beta}\) is greater than 0.0020.

INTRODUCTION

Tests of modern military airplanes have indicated that large changes in effective dihedral may occur over the speed range of an airplane operating under various power conditions. This change in effective dihedral may cause an airplane that has a normal amount of positive effective dihedral in the high-speed condition to have large negative effective dihedral in a flaps-down, low-speed, high-power condition (wave-off or landing-approach condition). If an attempt is made to satisfy the requirements of reference 1 that the airplane have positive effective dihedral at all speeds, it may have excessive positive effective dihedral in the high-speed condition. Negative effective dihedral at low speeds and high positive effective dihedral at high speeds are known to cause poor flying characteristics. Unless the directional stability is very high or some device is employed that will give the airplane approximately the same effective dihedral at all speeds and power conditions, however, most high-powered airplanes must have poor flying characteristics at one or the other of the extreme speed conditions or must incorporate some compromise that will probably not provide good flying characteristics at either extreme condition.

The data of references 2 to 4 show the effect of variation of effective dihedral angle on the flying characteristics. The range of dihedral angle covered in these investigations was rather small in comparison with the range of effective dihedral angle that may be encountered with modern, high-powered airplanes. A comprehensive investigation of the effects of effective dihedral, directional stability, and lift coefficient on lateral stability and control and on general flying characteristics has therefore been conducted in the Langley
free-flight tunnel. The objects of this investigation were to determine the optimum combinations of dihedral and directional stability over a wide range of lift coefficient and to provide data that would aid in the selection of the proper dihedral angles for airplanes that must experience large changes of effective dihedral over the speed and power range. The results of the investigation are presented herein. Some of these results (negative dihedral at high lift coefficients) are reported in reference 5.

The present investigation consisted in power-off flight tests of a model on which changes in effective dihedral were obtained by varying the geometric dihedral angle. The tests were made over a range of geometric dihedral angle from -20° to 18° for vertical-tail areas from 0 to 35 percent of the wing area and for lift coefficients of 0.5 and 1.0 with flaps up and 1.0, 1.4, and 1.8 with flaps down. Sufficient combinations of dihedral angle and vertical-tail area were tested at each of the lift coefficients to determine the effect of dihedral, vertical-tail area, and lift coefficient on lateral stability and control and general flying characteristics over the range of the variables.

The results of the flight tests of the model are presented in the form of qualitative ratings of the spiral stability, oscillatory stability, and general flight behavior of the model for each test condition. From these qualitative flight ratings the range of good flight behavior was established.

SYMBOLS

The forces and moments are referred to the stability axes, which are defined as an orthogonal system of axes intersecting at the center of gravity in which the Z-axis is in the plane of symmetry and perpendicular to the relative wind, the X-axis is in the plane of symmetry and perpendicular to the Z-axis, and the Y-axis is perpendicular to the plane of symmetry. A diagram of these axes showing the positive direction of forces and moments is presented as figure 1.

The symbols and coefficients used in the present report are defined as follows:
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>mass of model, slugs</td>
</tr>
<tr>
<td>S</td>
<td>wing area, square feet</td>
</tr>
<tr>
<td>( S_t )</td>
<td>vertical-tail area, square feet</td>
</tr>
<tr>
<td>b</td>
<td>wing span, feet</td>
</tr>
<tr>
<td>V</td>
<td>free-stream velocity, feet per second</td>
</tr>
<tr>
<td>q</td>
<td>dynamic pressure, pounds per square foot ((\frac{1}{2} \rho V^2))</td>
</tr>
<tr>
<td>T</td>
<td>time to damp to one-half amplitude, seconds; negative values indicate time to increase to double amplitude</td>
</tr>
<tr>
<td>P</td>
<td>period of lateral oscillation, seconds</td>
</tr>
<tr>
<td>( k_X )</td>
<td>radius of gyration of model about longitudinal axis, feet</td>
</tr>
<tr>
<td>( k_z )</td>
<td>radius of gyration of model about vertical axis, feet</td>
</tr>
<tr>
<td>R</td>
<td>Routh's discriminant</td>
</tr>
<tr>
<td>D, E</td>
<td>coefficients in stability quartic equation, given in reference 6</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>roots of stability quartic equation</td>
</tr>
<tr>
<td>( r )</td>
<td>yawing angular velocity, radians per second</td>
</tr>
<tr>
<td>( \rho )</td>
<td>rolling angular velocity, radians per second</td>
</tr>
<tr>
<td>( \rho )</td>
<td>mass density of air, slugs per cubic foot</td>
</tr>
<tr>
<td>( \beta )</td>
<td>angle of sideslip, degrees except where otherwise specified</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>flight-path angle, degrees; positive for climb</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>geometric dihedral angle of mean-thickness line, degrees</td>
</tr>
<tr>
<td>( \mu )</td>
<td>airplane relative-density factor ((\frac{m}{\rho S b}))</td>
</tr>
</tbody>
</table>
\( \tau \) time-conversion factor \( \left( \frac{m}{\rho SV} \right) \)

\( C_L \) lift coefficient \( \left( \frac{\text{Lift}}{qS} \right) \)

\( C_Y \) lateral-force coefficient \( \left( \frac{\text{Lateral force}}{qS} \right) \)

\( C_\ell \) rolling-moment coefficient \( \left( \frac{\text{Rolling moment}}{qSb} \right) \)

\( C_n \) yawing-moment coefficient \( \left( \frac{\text{Yawing moment}}{qSb} \right) \)

\( C_{Y\beta} \) rate of change of lateral-force coefficient with angle of sideslip, per radian \( \left( \frac{\partial C_Y}{\partial \beta} \right) \)

\( C_{\ell\beta} \) rate of change of rolling-moment coefficient with angle of sideslip, per degree except where otherwise specified \( \left( \frac{\partial C_\ell}{\partial \beta} \right) \)

\( C_{n\beta} \) rate of change of yawing-moment coefficient with angle of sideslip, per degree except where otherwise specified \( \left( \frac{\partial C_n}{\partial \beta} \right) \)

\( C_{\ell p} \) rate of change of rolling-moment coefficient with rolling-angular-velocity factor \( \left( \frac{\partial C_\ell}{\partial \frac{p_b}{2V}} \right) \)

\( C_{\ell r} \) rate of change of rolling-moment coefficient with yawing-angular-velocity factor \( \left( \frac{\partial C_\ell}{\partial \frac{r_b}{2V}} \right) \)

\( C_{n p} \) rate of change of yawing-moment coefficient with rolling-angular-velocity factor \( \left( \frac{\partial C_n}{\partial \frac{p_b}{2V}} \right) \)

\( C_{n r} \) rate of change of yawing-moment coefficient with yawing-angular-velocity factor \( \left( \frac{\partial C_n}{\partial \frac{r_b}{2V}} \right) \)

**APPARATUS AND MODEL**

The investigation was conducted in the Langley free-flight tunnel, which is equipped for testing free-flying dynamic airplane models. A complete description of the tunnel and its operation is given in reference 7. Force
tests to determine the static lateral-stability derivatives of the model were made on the Langley free-flight-tunnel six-component balance, which is described in reference 8. This balance rotates in yaw so that all forces and moments are measured with respect to the stability axes. Free-oscillation tests were made to determine the rotary-damping derivative \( C_{mr} \) by the method described in reference 9.

The control used on free-flight-tunnel models is a "flicker" (full-on or full-off) system. During any one particular flight the control deflections in the full-on position are constant and the amount of control applied to the model is regulated by the length of time the controls are held on rather than by the magnitude of the deflections used.

A three-view drawing of the model used in the tests is shown as figure 2 and a photograph of the model is presented as figure 3. Figure 4 is a photograph of the model, with flaps down and a geometric dihedral of \(-15^\circ\), flying in the test section of the tunnel. Although the model used in the tests was not a scale model of any particular airplane, it approximately represented a \( \frac{1}{10} \)-scale model of any conventional fighter airplane.

The model was equipped with a duplex flap arrangement in order to obtain high lift coefficients. These flaps consisted of a 40-percent-chord double slotted flap located inboard over 40 percent of the semispan and a 20-percent-chord balanced split flap located outboard over 42 percent of the semispan. The front and rear parts of the double slotted flap were deflected \( 30^\circ \) and \( 70^\circ \), respectively, with respect to the wing chord line. The balanced split flap was deflected \( 40^\circ \) with its leading edge located 0.05 wing chord below the lower surface of the wing and 0.10 wing chord ahead of the trailing edge of the wing.

As previously mentioned, the effective dihedral was changed by altering the geometric dihedral angle of the wing, as indicated in figure 2. Four geometrically similar vertical tails and two end-plate vertical tails were used on the model to produce changes in directional stability.
The relative-density factor and radii of gyration for the model varied during the test program between the following limits:

\[
\begin{align*}
\rho &= 8.10 \text{ to } 8.92 \\
\frac{k_x}{b} &= 0.161 \text{ to } 0.181 \\
\frac{k_z}{b} &= 0.241 \text{ to } 0.290 
\end{align*}
\]

The data presented in references 4, 5, and 10 indicate that changes in weight and moment of inertia of the magnitude involved in the present investigation would make no pronounced difference in the stability or flying characteristics of the model.

**TESTS**

**Scope of Tests**

Flight tests of the model were made with the combinations of dihedral angle and vertical-tail area and at the lift coefficient shown in table I. The values of \( C_{l_b} \) and \( C_{n_b} \) corresponding to the various configurations tested are shown in figures 5 and 6. These data show that the tests covered a range of \( C_{l_b} \) from 0.0032 to \(-0.0042 \) (-16° to 21° effective dihedral) and a range of \( C_{n_b} \) from 0 to 0.0066. This range is considered representative of present limits for airplanes as shown by the data given in figure 7. These data show that three high-powered airplanes over their speed ranges fall within the range of values covered by the present tests, except at extremely high lift coefficients.

**Testing Procedure**

The model was flown at each test condition by means of ailerons alone and ailerons coupled with rudder. The rudder travels used were selected by visual observation of flight tests as the amount necessary to eliminate the yawing due to aileron deflection and rolling. For tests in which the rudder control was crossed (left rudder applied with right aileron and right rudder applied with left aileron), the rudder travel used was the same as
that used for coordinated rudder and aileron control at the same test condition. For the tail-off condition the ailerons were rigged up 12° in order to eliminate the adverse yawing due to aileron deflection. The stability and general flying characteristics of the model were noted by the pilot from visual observation and each test condition was assigned graduated ratings for spiral stability, oscillatory stability, and general flight behavior. Motion-picture records for later study were made to supplement the pilot's observations.

The spiral stability of the model was determined by the pilot from the rate at which the model, with controls fixed, sideslipped and rolled from level flight. An increasing rate of rolling and inward sideslip was judged as spiral instability.

The general oscillatory-stability characteristics were judged by the pilot from the damping of the lateral oscillations of the model after a disturbance. A model could never be allowed to fly with controls fixed for sufficient time to allow measurement of the period and damping from the motion-picture records.

The general flight-behavior ratings are based on the over-all flying characteristics of the model. The ratings indicate the ease with which the model can be flown, both for straight and level flight and for performance of the mild maneuvers possible in the Langley free-flight tunnel. Any abnormal characteristics of the model are generally judged as unsatisfactory general flight behavior, inasmuch as they are disconcerting to the free-flight-tunnel pilots. In effect, then, the general flight-behavior ratings are much the same as the pilot's opinion of an airplane and indicate whether stability and controllability are properly proportioned.

CALCULATIONS

Boundaries for neutral spiral stability \(E = 0\), neutral oscillatory stability \(R = 0\), and neutral directional stability \(D = 0\) were calculated over the test range by means of the stability equations of reference 6 and are shown in figures 8 to 12.
Lines of constant damping of the spiral mode were also calculated for the model by determining the root of the stability quartic \( \lambda \) that would give the desired value of damping by the following formula (reference 6):

\[
\lambda = \frac{-0.693T}{T}
\]

and determining various values of \( C_{l\beta} \) and \( C_{n\beta} \) that would give this root \( \lambda \) by substitution of the root in the stability quartic. The calculated lines of constant damping are shown in figures 8 to 12.

Lines of constant period and damping of the oscillatory mode were calculated from the following approximate relations given in reference 6:

\[
P = \frac{2\pi T}{\sqrt{D/B}}
\]

and

\[
T = \frac{-0.693T}{\frac{1}{2} \left( \frac{C}{B} \cdot \frac{D}{B^2} - \frac{E}{D} \right)}
\]

The calculated lines of constant period and damping of the lateral oscillation are shown in figures 8 to 12.

Values of the static-lateral-stability derivative \( C_{Y\beta} \) and the variation of \( C_{Y\beta} \) with \( C_{n\beta} \) used in the calculations were determined from force tests of the model. As was previously mentioned, the values of the rotary derivative \( C_{n\eta} \) were obtained from free-oscillation tests of the model by the method described in reference 9. The other rotary derivatives \( C_{l\rho} \), \( C_{n\rho} \), and \( C_{l\eta} \) were estimated from the charts of reference 11 and the formulas of reference 12. The values of the mass characteristics \( m \), \( k_x \), and \( k_z \) were measured for the model. Values of the stability derivatives used in the calculations are given in table II.
RESULTS AND DISCUSSION

The variations of effective-dihedral parameter $C_{T\beta}$ and directional-stability parameter $C_{n\beta}$ were obtained in the present investigation by changing the geometric dihedral angle and the vertical-tail area. Flying characteristics, however, depend on the values of the stability derivatives, not on the method by which they are obtained; hence, the flying characteristics of the model may be applied to conditions in which changes in the stability derivatives were obtained by some other means, such as power.

The principal results of the present investigation are given in figures 8 to 14 in the form of ratings of the general flight behavior of the model. All flight ratings not in parentheses were obtained with a total aileron deflection of $70^\circ$; those in parentheses were obtained with a total aileron deflection of $50^\circ$. The maximum values of $pb/2V$ corresponding to aileron deflections of $30^\circ$ and $50^\circ$ were determined to be about 0.08 and 0.12, respectively, from roll-offs at a geometric dihedral angle of $0^\circ$, with the vertical tail having $S_t/S = 0.15$ and with coordinated rudder. These values of $pb/2V$ were approximately constant over the range of lift coefficient covered in the tests.

The results of the tests are believed to be directly applicable to airplanes having moderate wing loadings (approx. 35 lb/sq ft or less) and rolling and yawing radii of gyration not exceeding 0.2 and 0.3 of the wing span, respectively.

Spiral Stability

In general, the tests showed that reducing the effective dihedral or increasing the lift coefficient caused a reduction in spiral stability. The changes in spiral stability over most of the range tested were slight, although the spiral divergences were rapid enough at large negative effective dihedral angles ($-C_{T\beta} < -0.002$) and high lift coefficients ($C_L > 1.0$) to be considered dangerous.
These results are in qualitative agreement with the calculated spiral-stability characteristics of the model presented in figures 8 to 12 as lines of constant damping of the spiral mode. These theoretical results, like the test results, show that reducing the effective-dihedral parameter \(-C_{z\beta}\) or increasing the lift coefficient caused an increase in the time for the spiral mode to damp to one-half amplitude or a decrease in the time to increase to double amplitude over the range of conditions tested. Similarly, the theoretical and experimental results show that increasing the directional-stability parameter \(C_{n\beta}\) caused a slight reduction in spiral stability for positive effective dihedral angles and a slight increase in spiral stability for negative effective dihedral angles with very little effect of varying the directional stability for effective dihedral angles near zero.

No quantitative check of theory with tests could be obtained inasmuch as a spiral divergence could not be allowed to develop far enough in the confines of the tunnel to permit measurement of the rate of spiral convergence. A reasonably good check of the calculated spiral-stability boundary \((E = 0)\) was obtained, however, when the nature of flight in the free-flight tunnel is considered. Very low rates of spiral stability cannot be detected in the tunnel because the model cannot be allowed to fly without application of controls in the rather gusty air of the tunnel for sufficient time to allow low rates of spiral divergence to be detected.

Oscillatory Stability

Accurate quantitative measurements of the damping could not be obtained for all conditions. The results are therefore presented in the form of qualitative ratings for damping at each test condition. The approximate quantitative equivalents of these ratings are:
<table>
<thead>
<tr>
<th>Rating</th>
<th>Qualitative rating</th>
<th>Approximate quantitative equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Stable</td>
<td>Damps to one-half amplitude in less than 2 cycles</td>
</tr>
<tr>
<td>B</td>
<td>Slightly stable</td>
<td>Damps to one-half amplitude in more than 2 cycles</td>
</tr>
<tr>
<td>C</td>
<td>Neutral</td>
<td>Zero damping</td>
</tr>
<tr>
<td>D</td>
<td>Slightly unstable</td>
<td>Builds up to double amplitude in more than 1 cycle</td>
</tr>
<tr>
<td>E</td>
<td>Dangerously unstable</td>
<td>Builds up to double amplitude in less than 1 cycle</td>
</tr>
</tbody>
</table>

The ratings in figures 8 to 12 show that, although increasing the lift coefficient reduced the oscillatory stability for virtually all model configurations having positive effective dihedral, the magnitude of the reduction varied for the different combinations of effective dihedral and directional stability. In general, the effects of lift coefficient on the oscillatory damping were more pronounced with high effective dihedral and low directional stability. This variation in the magnitude of lift-coefficient effects was in good agreement with the variation shown by the shifting of the theoretical oscillatory-stability boundaries and lines of constant damping shown in figures 8 to 12.

A comparison of the theoretical oscillatory-stability boundaries \((R = 0, \ T = \infty)\) in figures 8 to 12 with the ratings for damping of the oscillation obtained in flight tests of the model indicates good agreement between theoretical and test results for the part of the boundary within the positive dihedral range. Detection of a lateral oscillation is difficult when the spiral instability is great. Apparently, however, the part of the oscillatory-stability boundary within the negative dihedral range had no significance or was in error inasmuch as no lateral oscillation could be detected at test conditions near the boundary.
Lateral Control

Increasing the effective dihedral caused a reduction in the effectiveness of the ailerons for roll-offs from a zero-bank condition and an increase in the effectiveness of the ailerons for recoveries because of the sideslips involved in these maneuvers when the controls were coordinated in a normal manner. No measurements of this effect of dihedral on rolling velocity were made but the pilot's comments indicated that recoveries were more rapid than roll-offs at large positive effective dihedral angles, whereas the roll-offs were much more rapid than recoveries at all negative effective dihedral angles. Roll-offs and recoveries appeared to be equally rapid at small or moderate positive effective dihedral angles. The over-all effect of dihedral on lateral control was adverse inasmuch as the slow recoveries at the negative dihedral angles were objectionable when the pilot attempted to prevent the model from falling off into a spiral and the slow roll-offs at high positive dihedral were objectionable for maneuvering.

Use of only ailerons for lateral control caused the flying characteristics at large positive dihedral angles to become worse as may be seen from a comparison of the general flight-behavior ratings of figures 13 and 14 with those of figures 8 to 12. The adverse yawing in aileron rolls caused an appreciable reduction in the rolling velocities in roll-offs, which the pilots considered objectionable. At negative effective dihedral angles, however, use of ailerons alone caused the rolling velocities in recoveries to be slightly more rapid than if both ailerons and rudder were used. Much of this favorable effect of adverse yawing was lost, however, inasmuch as the pilots considered the yawing motion objectionable. The differences between the rolling response of the model when controlled by ailerons alone or by ailerons and rudder were, of course, increased at higher values of lift coefficient, which caused an increase in the adverse yawing. The effect of use of ailerons alone for control with flaps deflected might be expected to be greater for most airplanes than was indicated by the present tests inasmuch as the ailerons used on the model give less adverse yawing moment than the types of aileron generally used on full-scale airplanes.

Control by means of rudder alone was generally fairly good for test configurations having an effective
dihedral angle greater than 10° \((-C_{\text{\beta}} > 0.002)\). When the effective dihedral angle was less than 10° but greater than 0°, it was possible to pick up a low wing by means of rudder alone although control by rudder alone was not satisfactory.

**General Flight-Behavior**

The results of the tests are best summarized by the general flight-behavior ratings. Spiral stability, oscillatory stability, and controllability are all considered desirable but a proper balance of these factors, with consideration of their relative importance, is necessary to give satisfactory flying characteristics. The general flight-behavior ratings, for which the overall flying characteristics have been considered, are therefore thought to be the most significant results of the tests.

**Effect of dihedral.** The general effect of variations of effective dihedral on the general flight behavior is evident from the ratings of figures 8 to 14. Increasing or decreasing the effective dihedral from a moderate positive value \((-C_{\text{\beta}} = 0 \text{ to } 0.001)\) caused the general flight behavior to become worse, particularly when the directional stability was low. The causes of the undesirable general flight behavior in both the positive and negative effective dihedral ranges were quite different.

The oscillatory stability seemed to be the predominant factor affecting the general flight behavior within the range of positive effective dihedral. This conclusion is fairly well borne out by the general flight-behavior ratings of figures 8 to 14. These ratings show that the boundary regions of good, fair, or poor general flight behavior are roughly similar in shape to the oscillatory-stability boundary and lines of constant damping of the oscillatory mode, whereas these ratings in the spirally unstable regions show no pronounced adverse effect of spiral instability for positive effective-dihedral regions.

Oscillatorily unstable configurations were generally considered to have poor general flight behavior although the model was never so oscillatorily unstable as to be unflyable when the directional stability was positive.
The oscillatory-stability characteristics, however, were not the only factors affecting the general flight behavior in the positive effective-dihedral region. Increasing the effective dihedral angle caused the flying characteristics to become worse because of the abrupt rolling and lateral oscillations that followed each gust disturbance in the normally rough air of the tunnel and because of the adverse effects of high dihedral angles on the lateral control. The rolling oscillations resulting from gusts were particularly objectionable at high airspeeds, whereas the control characteristics were the more predominant cause of the poor flying characteristics at low speeds.

The rate of spiral divergence for the test conditions at which the model had positive effective dihedral was observed to be small for the range of lift coefficient covered in the present investigation, and the controls-fixed lateral motion was characterized by a slow gentle roll-off and sideslip from the steady state. The divergence could be controlled readily by occasional application of a total aileron deflection of 30°. Under these conditions, the model was as easy to fly as if it had been spirally stable and because of the gusty air in the tunnel did not seem to require more frequent control than if it had been slightly spirally stable.

Within the negative effective-dihedral range, however, the spiral stability was the predominant factor affecting the general flight behavior, and the effects of the oscillatory stability were hardly discernible.

At small values of negative effective dihedral, flight characteristics were not much worse than those at small values of positive effective dihedral and the slow spiral divergences were readily controlled by application of the aileron and rudder controls. The rate of spiral divergence, however, was found to become increasingly rapid with negative effective dihedral until, at an effective dihedral of about -15°, the divergence was quite violent for lift coefficients of 1.0 and over. As in the case of small positive effective dihedral, the motions were characterized by a roll-off and sideslip from steady flight. As the negative effective dihedral was increased, the rate of spiral divergence increased until, for the largest negative dihedral angles, the motion appeared to be as rapid as a fast aileron roll.
The controls had to be applied almost immediately after the divergence was noticed because, when there was only a slight lag in the application of corrective control following a disturbance, the unstable moments resulting from spiral instability became sufficiently large to overpower the moments of the controls so that return to straight flight was impossible.

It was generally found impossible to fly the model with negative effective dihedral angles greater than about \(-10^\circ\) \((-C_{l\theta} \approx -0.002\) with a total aileron deflection of 30\(^\circ\). The rate of spiral divergence apparently had become great enough by the time the pilot applied opposite control to make recovery impossible. Aileron application retarded but did not stop the divergence.

In order to obtain data for the whole test range, the total aileron deflection was increased from 30\(^\circ\) to 50\(^\circ\) for almost all test conditions for which \(-C_{l\theta} < -0.002\). It was therefore possible to control the spiral divergence over the complete range of negative dihedral angle. Flight was difficult, however, when \(-C_{l\theta} < -0.002\), because constant attention to the controls was required.

The largest negative effective dihedral angles \((-C_{l\theta} \approx -0.003\) seemed to be the maximum for which the model could be flown with a total aileron deflection of 50\(^\circ\), inasmuch as even slight delays in applying lateral control allowed the model to continue to diverge. Many crashes, therefore, occurred during the tests at values of \(-C_{l\theta}\) of about \(-0.003\).

The model was found to be unflyable at low lift coefficients \((C_L \approx 0.5)\) with large negative effective dihedral angles and low directional stability. Such a condition is probably only of academic interest inasmuch as theory indicates that the spiral instability is not so great as for some conditions at which the model has been flown at higher lift coefficients; however, the cause of the bad flying characteristics seems to be worth mentioning. The tests agreed with theory in that the spiral instability was not so great at low lift coefficients as at higher lift coefficients. The yaw
of the model due to gust disturbances appeared to be the cause of the trouble. When the model yawed around due to a gust disturbance the leading wing dropped very rapidly, because of the high airspeed, and the roll had developed so far by the time the controls were applied that no recovery was possible.

The general flight-behavior ratings in figures 8 to 12 were given when the rudder was coordinated with the ailerons in the normal manner (right rudder with right ailerons). The flight tests, however, showed that when the ailerons alone were used or even when the rudder control was crossed the flying characteristics of the model were improved throughout the negative dihedral range and the model was slightly easier to fly. This improvement evidently occurred because the sideslip resulting from adverse yawing opposed the inward angle of sideslip caused by the spiral divergence and, in spite of the adverse effect of rolling due to yawing, reduced the rolling divergence. This reduction of inward sideslip improved the response to the controls. The large amplitude of the yawing motions caused by crossing the rudder control, however, was objectionable to the free-flight-tunnel pilots. Application of opposite rudder with ailerons would probably be objectionable to the pilot of an airplane because it is an unnatural motion and would cause a loss of altitude. In a crucial moment, the pilot would probably react by applying coordinated rudder and aileron control rather than thinking to apply rudder opposite to the ailerons. A pilot might, however, be trained to apply no rudder with aileron control when flying an airplane in conditions that are known to give negative dihedral effect. Thus improvement in the control response for recovery may be obtained.

The wave-off, take-off, and landing-approach conditions are believed to be dangerous for airplanes that have large negative effective dihedral because, when these conditions are encountered, there is only a limited altitude in which to apply corrective control. Flight with as much negative effective dihedral as was encountered in the present tests should be possible if the airplane ailerons are as powerful as those of the model tested and careful attention is given to controlling the airplane. Flight with greater negative effective dihedral angles than were encountered in the present tests might be possible inasmuch as the rate of divergence of the airplane would be \( \sqrt{N} \) times as fast as
that of the model, where \( N \) is the scale of the model as 1/10, 1/15, and so forth. No information is available, however, concerning the relative reaction time and the time to deflect the controls for free-flight and airplane pilots. Because no correlation has been made of time to damp with the boundaries of the region in which flight is impossible in the Langley free-flight tunnel, an extension of the results to more negative dihedral angles is difficult. Inasmuch as the rate of spiral divergence of full-scale airplanes is slower than that of the model, however, it is believed that the amount of negative effective dihedral that would constitute a dangerous condition would be greater for airplanes than for the model.

The results of the tests have been summarized in figure 15 as boundaries of the region within which good general flight behavior of the model was obtained. These results, as shown in figure 15, are believed to be directly applicable to airplanes having mass characteristics similar to the model. This criterion, however, should be modified to take into consideration differences in the mass characteristics of airplanes from those of the model. The data of references 3, 4, and 10 may be used to interpret the present data for the effects of wing loading, altitude, and mass distribution. The results of the present tests may be applied directly to airplanes having moderate wing loadings and radii of gyration to indicate that the effective dihedral should be greater than 0° and that the ratio of \(-C_{\ell_{\phi}}\) to \(C_{n_{\phi}}\) should not exceed 1/2. The data of references 3, 4, and 10 considered together with the present data indicate that airplanes having high wing loadings and/or high radii of gyration should have an effective dihedral angle greater than 0° and that the ratio of \(-C_{\ell_{\phi}}\) to \(C_{n_{\phi}}\) should not exceed 1/4.

**Effect of directional stability.** Increasing the directional stability improved the general flight behavior of the model over the range of dihedral angle and lift coefficient tested, as shown in figures 8 to 14.

The tests showed that for the range of small positive effective dihedral angles, adequate directional stability was more desirable than the slightly lower rate of spiral divergence associated with lower directional stability, because excessive yawing was encountered
with low directional stability. The rates of spiral divergence within the positive effective dihedral range were, as previously discussed, quite slow even with a high degree of directional stability.

For higher positive values of effective dihedral, at which the oscillatory stability is an important factor affecting the general flight behavior, increasing the directional stability caused a great improvement in the general flight behavior by increasing the oscillatory stability as well as reducing the rolling and yawing due to gusts and improving the control characteristics as was previously discussed. The detrimental effect on general flight behavior of the slight decrease in spiral stability with increasing directional stability was thus heavily overbalanced by the improvement of the oscillatory characteristics and lateral control.

When the effective dihedral was negative, increasing the directional stability caused a slight reduction in the spiral instability as well as a reduction in the yawing due to gusts and aileron control and resulted in an improvement in the general flight behavior.

The motions of the model with tails off, geometric dihedral angle of -20°, and at lift coefficients of 1.4 and 1.8 appeared to be directional divergences. Immediately after taking off, the model commenced a divergence in yaw that was followed by rolling in the opposite direction caused by the negative dihedral. No other indications of directional divergence were observed in the tests with tails off although several tests were made at values of $C_{n\beta}$ and $C_{n\phi}$ that were below the directional divergence boundary.

The minimum values of the directional-stability parameter $C_{n\phi}$ required to obtain good general flight characteristics are shown in figure 15 for the range of values of lift coefficient and effective-dihedral parameter covered in the present tests. If an airplane has the optimum value of effective dihedral and can attain a maximum lift coefficient of about 1.8, and if the critical control condition is considered to be control by ailerons alone, figure 15 shows that a value of $C_{n\phi} > 0.002$ is required to obtain good general flight behavior.
Effect of lift coefficient.—Figure 16 was prepared by interpolation from figures 8 to 12 to show the effects of lift coefficient on the general flight behavior inasmuch as the effect of lift coefficient was slight and could not readily be ascertained from an inspection of the separate figures. Figure 16 shows that increasing the lift coefficient caused the general flight behavior of the model to become slightly worse for the range of effective dihedral angle presented except for the condition of negative effective dihedral and low directional stability, which has previously been discussed. The effect of lift coefficient was slightly greater at low values of the directional-stability parameter $C_{n\phi}$. The detrimental effect of increasing the lift coefficient was greater when the ailerons were used as the sole means of control, as may be determined from figures 13 to 16, because of the increase in adverse yawing due to rolling and ailerons at the higher lift coefficients.

CONCLUSIONS

Tests were made in the Langley free-flight tunnel to determine the effects of effective dihedral, vertical-tail area, and lift coefficient on the lateral stability and control and general flying characteristics of a free-flying dynamic model. The following conclusions are believed to be directly applicable to airplanes having moderate wing loadings (approx. 35 lb/sq ft or less) and rolling and yawing radii of gyration not exceeding 0.2 and 0.3 of the wing span, respectively:

1. In order to obtain the best flying characteristics over the range of lift coefficient tested the following conditions should be satisfied:

   (a) The effective dihedral parameter $C_{l\phi}$ should be positive ($-C_{l\phi} > 0$).

   (b) The directional-stability parameter $C_{n\phi}$ should be greater than 0.002.

   (c) The ratio $-C_{l\phi}/C_{n\phi}$ should be less than 1/2.
These criterions are believed to be applicable to airplanes having mass characteristics similar to those of the model tested.

2. The model was found to be flyable over the range of positive effective dihedral angle tested, provided it was directionally stable. As the effective dihedral was increased from an optimum value of approximately 2°, however, the flying characteristics became worse and more critically dependent upon the use of the correct amount of rudder control in conjunction with the ailerons. At high speeds the use of large rudder travels caused unnaturally rapid rolling, and at low speeds the use of too little rudder caused serious adverse yawing with accompanying reduction in rolling.

3. The model was found to be flyable for effective dihedral angles as low as -15° for lift coefficients of 1.0 or greater. As the effective dihedral was decreased from 0° to -15°, however, the model became increasingly difficult to fly. With an effective dihedral of -15° \((-C_{Lf} < 0.003)\) the flying characteristics were considered to be dangerous because when there was only a slight lag in the application of corrective control following a disturbance, the unstable moments resulting from spiral instability became sufficiently large to overpower the moments of the controls so that return to straight flight was impossible. Inasmuch as full-scale airplanes because of their greater size will diverge at a slower rate than free-flight-tunnel models, the amount of negative effective dihedral that would constitute a dangerous condition is expected to be greater for full-scale airplanes.

4. Increasing the directional stability improved the general flight behavior over the entire dihedral range in spite of reduction in spiral stability with increasing directional stability within the positive effective dihedral range.

5. Increasing the lift coefficient had a slightly detrimental effect on the general flight behavior, particularly when the ailerons were used as the sole lateral
control because the adverse yawing due to rolling and ailerons was increased by an increase in lift coefficient.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., January 18, 1946
REFERENCES


Figure 1. - System of stability axes. Arrows indicate positive directions of moments, forces, and control-surface deflections.
Figure 2 - Three-view sketch of model tested in Langley free-flight tunnel showing range of dihedral adjustment and alternate vertical-tail arrangements.
Figure 3.—Three-quarter front view of the variable-dihedral model with vertical tails 30 percent of wing area. Flaps down.
Figure 4.- Variable-dihedral model flying in the Langley free-flight tunnel with -15° dihedral.
Figure 5.— Values of $C_{lB}$ and $C_{nB}$ for the model with various combinations of dihedral angle and vertical-tail area. Flaps up.
Figure 6.—Values of $C_\theta$ and $C_{\phi b}$ for the model with various combinations of dihedral angle and vertical-tail area, Flaps down.
Figure 7.—Values of $C_{1b}$ and $C_{2b}$ for three modern high-powered airplanes as compared with the range tasted.
Figure 8.— Stability and general flight-behavior ratings for the model. Control by ailerons and rudder; flaps up; $C_L = 0.5$. 

Lines of constant damping of the spiral mode and spiral-stability ratings:

- A Stable
- B Slightly stable
- C Neutral
- D Slightly unstable
- E Dangerously unstable

Lines of constant period and damping of the oscillatory mode and oscillatory-stability ratings:

- A Stable
- B Slightly stable
- C Neutral
- D Slightly unstable
- E Dangerously unstable

Stability boundaries and general flight-behavior ratings:

- A Good
- B Fair
- C Poor
- D Flight impossible
Figure 9: Stability and general flight-behavior ratings for the model. Control by ailerons and rudder; flaps up; $C_L=1.0$. 

Lines of constant damping of the spiral mode and spiral-stability ratings:

- **A**: Stable
- **B**: Slightly stable
- **C**: Neutral
- **D**: Slightly unstable
- **E**: Dangerously unstable

Lines of constant period and damping of the oscillatory mode and oscillatory-stability ratings:

- **A**: Stable
- **B**: Slightly stable
- **C**: Neutral
- **D**: Slightly unstable
- **E**: Dangerously unstable

Stability boundaries and general flight-behavior ratings:

- **A**: Good
- **B**: Fair
- **C**: Poor
- **D**: Flight impossible

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Figure 10.—Stability and general flight-behavior ratings for the model. Control by ailerons and rudder; flaps down; $C\eta=1.0$. 

Lines of constant damping of the spiral mode and spiral-stability ratings

- A Stable
- B Slightly stable
- C Neutral
- D Slightly unstable
- E Dangerously unstable

(1) Total aileron travel increased, 30° to 60°

Lines of constant period and damping of the oscillatory mode and oscillatory-stability ratings

- A Stable
- B Slightly stable
- C Neutral
- D Slightly unstable
- E Dangerously unstable

(1) Total aileron travel increased, 30° to 60°

Stability boundaries and general flight-behavior ratings

- A Good
- B Fair
- C Poor
- D Flight impossible

(1) Total aileron travel increased, 30° to 60°
Figure 11.—Stability and general flight-behavior ratings for the model, Control by ailerons and rudder; flaps down; $C_1 = 1.4$. 

**Stability boundaries and general flight-behavior ratings**

- **A** Good
- **B** Fair
- **C** Poor
- **D** Flight impossible

(1) Total aileron travel increased, 30° to 60°
Figure 12.— Stability and general flight-behavior ratings for the model. Control by ailerons and rudder; flaps down; $C=1.8$. 
Figure 13.—General flight-behavior ratings for the model. Control by ailerons alone; flaps up.
Figure 14.—General flight-behavior ratings for the model. Control by ailerons alone; flaps down.
Figure 15. — Region of good general flight behavior as determined by tests of a model in the Langley free-flight tunnel.
General flight-behavior ratings

A  Good
B  Fair
C  Poor
D  Flight impossible

Figure 16.- Effect of lift coefficient on general flight behavior. Ailerons and rudder used for control.