TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 483

CHARTS FOR DETERMINING THE PITCHING MOMENT OF TAPERED WINGS WITH SWEEPBACK AND TWIST

By Raymond F. Anderson
Langley Memorial Aeronautical Laboratory

Washington
December 1933

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SUMMARY

This report presents a convenient method for calculating the pitching-moment characteristics of tapered wings with sweepback and twist. The method is based on the fact that the pitching-moment characteristics of a wing may be specified by giving the value of the pitching moment at zero lift and the location of the axis about which the moment is constant. Data for calculating these characteristics are presented by curves which apply to wings having a linear distribution of twist along the span and which cover a large range of aspect ratios. The curves are given for wings having straight taper and distorted elliptical plan forms. The characteristics of wings of other shapes may be determined by interpolation.

INTRODUCTION

The use of tapered wings on airplanes has led to the development of methods of calculating the characteristics of such wings from the known characteristics of the sections. Two methods for calculating the pitching moment of tapered wings are given in references 1 and 2. In reference 1, formulas are given which apply to wings without twist and which have a lift distribution closely approaching an ellipse. Factors for use in the formulas are given for a few plan forms and a few variations of section pitching moment across the span. In reference 2, formulas are given for straight-taper wings with twist and with constant section moment across the span. Factors for use in the formulas are tabulated for a few aspect ratios and taper ratios. The information given in reference 1 is limited in scope, and that of reference 2 is presented in such form that it cannot be readily applied by airplane designers.

The purpose of this report is to present a method by which airplane designers may readily calculate the pitching-moment characteristics of tapered wings with sweepback and twist. Formulas are given for calculating the pitching-moment characteristics and factors for use in the formulas are presented on charts. The data on the charts are given for a large range of aspect ratios and apply to wings having a linear distribution of twist along the span. Curves for wings having straight taper and distorted elliptical plan forms are presented in a form which facilitates the determination of the characteristics of wings of other shapes.

METHOD OF DETERMINING THE PITCHING-MOMENT CHARACTERISTICS

In the method used in this report for the determination of the pitching-moment characteristics of wings, the wings are conceived to consist of a series of wing sections each of which has a constant pitching moment about a point called the aerodynamic center (approximately the quarter-chord point) and a lift force acting through that point. The locus of the aerodynamic centers of the wing sections is the wing axis. For the wings considered in this paper the axis of each half wing is straight (figs. 1(a), 1(b), and 1(c)). The angle between the wing axis and the lateral axis is the angle of sweepback (β) of the wing. (See fig. 1(a).) The plan form is obtained by superposing on the wing axis any desired distribution of chord lengths across the span. Two basic chord distributions are used in this report. One is linear and results in a straight-taper wing, which is shown in figure 1(a). The taper of the wing is defined by the ratio c_t/c_c. The other distribution is elliptical and results in a distorted elliptical wing, which is shown by figure 1(b). The distorted elliptical wing is obtained from an ellipse by displacing each chord in the line of its original position until the wing axis is straight. A third chord distribution, for which partial data are given, is the straight-taper wing modified to have rounded tips (fig. 1(c)). The trailing edge of the tip of this wing is formed by a radius with center on the wing axis and the leading edge is determined by the condition that the wing axis remains straight to the tip. (See fig. 1(c).)

The angle of attack of each section is the angle through which the section has been rotated from its attitude of zero lift. The twist of the wing is the variation in angle of attack along the span and is considered positive when the angle of attack increases from the center to the tip. In this paper the distribution of twist along the span is considered to be linear and the angle of twist from center to tip is ϵ.

The method of determining the pitching-moment characteristics is given only in outline here; a detailed explanation of the method including the derivation of the formulas will be found in the appendix.

The aerodynamic properties of a wing with sweepback and twist that contribute to the pitching moment may be regarded as the pitching moments of the sections and the lift forces distributed along the wing axis. To be strictly accurate, the drag should be considered as well as the lift. However, as the moment contributed by the drag is small compared to the moment contributed by the lift, sufficient accuracy may be obtained by considering only the lift. The moment contributed by the sections depends on the distribution of the chord and the section moment across the span. It is designated M_S and is expressed as a coefficient based on the mean chord in the form

$$C_{m_S} = \frac{M_S}{q\ S\ \bar{c}},\ \text{where}\ \bar{c} = \frac{S}{b}$$

To determine C_{m_S}, M_S is found by integrating across the span. When the section moment coefficient $C_{m_{c/4}}$ has a linear variation from C_{m_c} at the center to C_{m_t} at the tip, C_{m_S} may be expressed in the form

$$C_{m_S} = E\ C_{m_c} + F\ (C_{m_t} - C_{m_c})$$

The factors E and F depend on the plan form and are given in a chart (fig. 2) for wings having various plan forms. As the pitching-moment coefficient of the individual sections is considered independent of the angle of attack, C_{m_S} is independent of the angle of attack and is not affected by twist or sweepback.

The effect of the lift forces distributed along the axis of a wing with sweepback and twist may be explained by considering the lift distribution as consisting of two distinct parts. One part is the initial distribution that the wing has at the attitude of zero lift. If the twist of the wing is negative, as is usual, the forces near the tips will be directed downward and those near the center will be directed upward. The shape of this distribution is dependent primarily on the twist and is the same for all angles of attack. The forces composing this distribution produce a pitching moment that does not change with angle of attack. This moment is designated "M_T", the moment due to twist.

The moment M_T depends on the angle of twist ϵ, the angle of sweepback β, the aspect ratio A, and the plan form and is expressed as a coefficient C_{m_T} based on the mean chord in the form

$$C_{m_T} = - G \epsilon a_0 A \tan \beta$$

where a_0 is the lift curve slope for infinite aspect ratio and the factor G, which is given by charts (figs. 3 and 4), takes into account the effects of plan form and aspect ratio on the shape of the lift distribution.

The second part of the lift distribution is produced when the angle of attack is increased from the attitude of zero lift. This distribution is proportional to the angle of attack but has a fixed centroid. The net lift of the wing may be regarded as acting at this centroid, which is therefore called the aerodynamic center of the wing by analogy with the term for an airfoil section. Its location is given by its distance d behind the aerodynamic center of the central section. This distance depends on the angle of sweepback, the aspect ratio, and the plan form but is independent of the twist and the airfoil section. The location of the aerodynamic center is expressed as a fraction of the mean chord in the form

$$\frac{d}{\bar{c}} = H A \tan \beta$$

where the factor H, which is given by a chart (fig. 5), takes into account the effects of plan form and aspect ratio on the shape of the lift distribution.

The total moment coefficient of a wing may be found as the sum

$$C_{m_{ac}} = C_{m_S} + C_{m_T}$$

where $C_{m_{ac}}$ is constant about an axis through the aerodynamic center. The pitching-moment characteristics of a wing may accordingly be specified by the location of the aerodynamic center and the constant value of the pitching moment about an axis through that point.

CONSTRUCTION OF THE CHARTS

The values of C_{m_S}, C_{m_T}, and $\frac{d}{c}$ are easily found for any wing when the factors E, F, G, and H are known. The factors have been calculated and plotted for the plan forms previously described. For use in the formula $C_{m_S} = E\ C_{m_c} + F\ (C_{m_t} - C_{m_c})$ values of E and F, as calculated for the straight-taper wing for the complete range of taper ratios, are plotted on figure 2 against taper ratio. Values of E as determined for the wing with rounded tips for the cases of 2:1 taper and the rectangular wing are also plotted on figure 2 for aspect ratios of 3, 6, and 9. In addition the value of E for an elliptical wing is plotted on the same curve to aid in finding E for wings lying between the straight-taper and the elliptical plan forms.

The values of G in the formula $C_{m_T} = -G\,\epsilon\, a_o\, A \tan\beta$ as calculated for the straight-taper wing for certain aspect ratios and taper ratios, are plotted against taper ratio on figure 3, together with intermediate values found from a cross-plot against aspect ratio. The values of G for the straight-taper wing are also given for various aspect ratios in figure 4, together with values computed for the distorted elliptical wing. This chart permits interpolation to find G for a wing lying between the straight-taper and the distorted elliptical plan forms.

For use in the formula $\frac{d}{c} = H\ A \tan\beta$ values of H as determined for the wing with straight taper of aspect ratios 3, 6, 9, and 20 are plotted against taper ratio on figure 5. The value for the distorted elliptical wing of any aspect ratio is also shown.

USE OF THE CHARTS

The total pitching-moment coefficient $C_{m_{ac}}$ of a wing is determined from the equation previously given

$$C_{m_{ac}} = C_{m_S} + C_{m_T}$$

where C_{m_S} is given by $C_{m_S} = E\ C_{m_c} + F\ (C_{m_t} - C_{m_c})$. The values of C_{m_t} and C_{m_c} may be determined from wind-tunnel data. If reliable test data are not available the pitching-moment coefficient may be estimated from reference 3 which gives $C_{m_{c/4}}$ for a large number of related airfoil sections. The values of E and F for the particular plan form are given in figure 2.

The value of C_{m_T} is given by $C_{m_T} = -\ G\ \epsilon\ a_0\ A\ \tan\ \beta$. The value of a_0 may be determined from a wind-tunnel test of the airfoil or may be estimated from reference 3 if test data are not available. When a_0 varies along the span its average value should be used. The factor G for the particular plan form may be obtained from figure 3 or figure 4.

The location of the aerodynamic center is determined from the relation

$$\frac{d}{\bar{c}} = H\ A\ \tan\ \beta$$

in which H for the particular plan form is given in figure 5.

A few notes on calculating the characteristics of wings having plan forms not given on the charts may be useful. For any wing having a rounded tip an approximate value of c_t/c_c may be found by determining c_t for the tip extended, as shown by the dotted lines in figure 1(c). The angle of twist ϵ and the moment coefficient C_{m_t} of a wing with a rounded tip may be found by extrapolation from values of twist and moment coefficient determined near the tip. If the axis of a wing is curved, a straight line may be drawn through the axis to estimate the angle of sweepback. When finding the factors from the charts for a wing with rounded tips it is convenient to regard the wing as intermediate between the straight-taper wing and the distorted elliptical wing which may be considered as one having an extreme form of rounded tip.

An example of one of the many uses of the charts is the solution of the problem of determining how a change of the sweepback of a twisted wing affects the pitching-moment characteristics. The effect on the location of the aerodynamic center and the value of the pitching moment may be found as follows. The curve of pitching-moment coefficient $C_{m_{ac}}$, based on the mean chord and referred to the aerodynamic center of the wing, is determined from wind-tunnel data. The coefficient C_{m_T} for the wing is then calculated and subtracted from the curve of $C_{m_{ac}}$ to obtain the curve of C_{m_S}. The coefficient C_{m_T} is calculated for the new sweepback and added to the curve of C_{m_S} to obtain the curve of $C_{m_{ac}}$ for the new sweepback. The determination of the characteristics of the altered wing is completed by finding $d/\bar{c}$ for the new sweepback.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., November 16, 1933.

APPENDIX

List of Symbols

c, chord at any point along the span.

c_c, chord at the center.

c_t, chord at the tip.

b, span.

S, area.

$\bar{c}$, mean chord $= \frac{S}{b}$.

A, aspect ratio $= b^2/S$.

β, angle of sweepback (positive for sweepback).

ϵ, angle of twist from the center to the tip of the wing (positive when the angle of attack increases from the center to the tip).

M, pitching moment (a stalling moment is positive).

$C_m = \frac{M}{q\,S\,\bar{c}}$, pitching-moment coefficient.

$C_{m_{c/4}}$, section pitching-moment coefficient about the quarter-chord point.

C_{m_c}, $C_{m_{c/4}}$ at the center.

C_{m_t}, $C_{m_{c/4}}$ at the tip.

C_{m_S}, pitching-moment coefficient of a wing due to the sections.

C_{m_T}, pitching-moment coefficient of a wing due to the twist.

$C_{m_{ac}}$, pitching-moment coefficient of a wing about an axis through the aerodynamic center.

α, angle of attack at any point along the span.

α_c, angle of attack at the center; these angles are measured from the attitude of zero lift for the sections.

a_o, $= dC_L/d\alpha_o$, slope of the lift curve for infinite aspect ratio.

w, induced velocity.

K, circulation.

E, F, G, H, factors presented on the charts.

d, the distance from the aerodynamic center of the central section to the aerodynamic center of the wing measured along the chord of the central section.

Derivation of Formulas

The method used for obtaining the formulas is substantially the same as the method used by Glauert (reference 2), but because of a different viewpoint and different coefficients the derivation of the required formulas is given.

As stated previously, the aerodynamic properties that contribute to the pitching moment of a wing may be regarded as the pitching moments of the sections and the lift forces distributed along the wing axis. The contribution of these properties to the pitching moment will be considered separately.

The Pitching Moment Due to the Sections

The pitching moment of a wing element of width dy of any wing is

$$dM_S = C_{m_{c/4}}\ q\ c\ d\ S = C_{m_{c/4}}\ q\ c^2\ dy$$

The total moment of the wing due to the sections is then given by

$$M_S = 2q \int_0^{\frac{b}{2}} C_{m_{c/4}}\ c^2\ dy \qquad (1)$$

where $C_{m_{c/4}}$ and c are functions of y in general. The moment M_S due to the sections may be expressed as a coefficient based on the mean chord by

$$C_{m_S} = \frac{M_S}{q\ S\ \bar{c}} = \frac{2}{S\ \bar{c}} \int_0^{\frac{b}{2}} C_{m_{c/4}}\ c^2\ dy \qquad (2)$$

where $\bar{c} = \frac{S}{b}$

The coefficient C_{m_S} may be obtained from (2) by introducing the values of $C_{m_{c/4}}$ and c as a function of y and integrating. For the straight-taper wing (using the right half) $C_{m_{c/4}}$ was expressed as a linear function of y by

$$C_{m_{c/4}} = C_{m_c} + \frac{y}{b/2}\ (C_{m_t} - C_{m_c})$$ and c was expressed

$$c = c_c\ (1 - \frac{y}{b/2}\ r) \quad \text{where} \quad \frac{c_t}{c_c} = 1 - r.$$ C_{m_S} was obtained in the form

$$C_{m_S} = E\ C_{m_c} + F\ (C_{m_t} - C_{m_c})$$

where $E = \dfrac{1-r+\frac{r^2}{3}}{1-r+\frac{r^2}{4}}$, $F = \dfrac{\frac{1}{2}-\frac{2}{3}r+\frac{r^2}{4}}{1-r+\frac{r^2}{4}}$

For the wing with rounded tips and the distorted elliptical wing, $C_{m_{c/4}}$ was considered constant across the span and C_{m_S} was obtained in the form $C_{m_S} = E\, C_{m_{c/4}}$.

The Pitching Moment Produced by the Lift Forces

In the following calculation of the pitching moment produced by the lift forces for a wing having a straight wing axis a familiarity with wing theory is assumed. From the theory the lift produced by a wing section of width dy may be expressed $dL = \rho VKdy$ where K is the circulation around the section. The lift acts at the aerodynamic center of the section (which is assumed to be the quarter-chord point) and referring to figure 1(a), will produce a moment about the lateral axis of

$$d\,M_L = y\,(\tan\beta)\,\rho VKdy$$

The moment of the entire wing may be found from the integral

$$M_L = 2\,\rho V\,\tan\beta \int_{-\frac{b}{2}}^{0} K\,y\,dy \qquad (4)$$

By use of the method of reference 4 (p. 138) the coordinate y is replaced by the angle θ by the relation

$$y = -\frac{b}{2}\cos\theta \qquad (5)$$

so that θ varies from 0 to π from left to right across the span of the wing. Also the circulation is represented by the Fourier series

$$K = 2b\,V \sum_{n=1}^{n=\infty} A_n \sin n\,\theta \qquad (6)$$

where n has only odd integral values. Equation (4) may now be expressed in terms of θ, using the values for y and K given by (5) and (6).

$$M_L = -2b^3 \frac{\rho V^2}{2} \tan\beta \int_0^{\frac{\pi}{2}} \Sigma A_n \sin n\theta \sin\theta \cos\theta \, d\theta$$

After integrating, M_L is found to be

$$M_L = -2b^3 \frac{\rho V^2}{2} \tan\beta \left(\frac{A_1}{3} + \frac{A_3}{5} - \frac{A_5}{21} + \frac{A_7}{45} - \ldots\right)$$

The moment produced by the lift forces is expressed as a coefficient $C_{m_L} = \dfrac{M_L}{q\, S\, \bar{c}}$ as in the case of the moment due to the sections. Then in terms of the aspect ratio A, the pitching-moment coefficient of the lift forces about the lateral axis becomes

$$C_{m_L} = -2A^2 \tan\beta \left(\frac{A_1}{3} + \frac{A_3}{5} - \frac{A_5}{21} + \frac{A_7}{45} - \ldots\right) \qquad (7)$$

For any wing the coefficients A_n may be expressed in the form $A_n = B_n \alpha_c + C_n \epsilon$, as will be shown later. The angle α_c is the angle of attack of the central section of the wing and ϵ is the increase in angle of attack from the center to the tip. The coefficients B_n and C_n are functions of the plan form and of A/a_o. The coefficients may also be expressed in terms of the lift coefficient of the wing from the relation $C_L = \pi A A_1 = \pi A(B_1 \alpha_c + C_1 \epsilon)$ (reference 4). Then

$$\alpha_c = \frac{1}{B_1}\left(\frac{C_L}{\pi A} - C_1 \epsilon\right)$$

and finally

$$A_n = \frac{B_n}{B_1}\left(\frac{C_L}{\pi A} - C_1 \epsilon\right) + C_n \epsilon$$

By introducing this value of A_n in (7) and rearranging the terms, the pitching-moment coefficient of any wing about the lateral axis may be expressed

$$C_{m_L} = -\frac{2A\ \tan\beta}{\pi\ B_1}\left[\frac{B_1}{3} + \frac{B_3}{5} - \frac{B_5}{21} + \frac{B_7}{45} - \dots\right] C_L$$
$$-2A^2 \tan\beta\left[\left(\frac{C_3}{5} - \frac{C_5}{21} + \frac{C_7}{45} - \dots\right) - \frac{C_1}{B_1}\left(\frac{B_3}{5} - \frac{B_5}{21} + \frac{B_7}{45} - \dots\right)\right]\epsilon \qquad (8)$$

From the form of the above equation it is apparent that C_{m_L} consists of a part which is proportional to the lift coefficient of the wing C_L, and a part which is proportional to the twist ϵ. The lift may be regarded as acting at a point in the plane of the central section of the wing. This point is named "the aerodynamic center of the wing," and is located by its distance d behind the aerodynamic center of the central section. (See fig. 1(a).) Then the part of the C_{m_L} that is proportional to the lift coefficient is equal to $-\frac{d}{\bar{c}}\ C_L$. The part that is proportional to the twist is the moment coefficient about the lateral axis when the net lift is zero, as may be seen by putting $C_L = 0$ in (8). The distribution of lift forces that produce this moment coefficient depends primarily on the twist hence it is designated "C_{m_T}", the moment coefficient due to twist. Then C_{m_L} may be written

$$C_{m_L} = -\frac{d}{\bar{c}}\ C_L + C_{m_T} \qquad (9)$$

From (8) and (9) $\frac{d}{\bar{c}}$ and C_{m_T} may be expressed in the forms

$$\frac{d}{\bar{c}} = H\ A\ \tan\beta \qquad (10)$$

where

$$H = \frac{2}{\pi\ B_1}\left(\frac{B_1}{3} + \frac{B_3}{5} - \frac{B_5}{21} + \frac{B_7}{45} - \dots\right)$$

and

$$C_{m_T} = -G\ \epsilon\ a_o\ A\ \tan\beta \qquad (11)$$

where

$$G = \frac{2A}{a_o}\left[\left(\frac{C_3}{5} - \frac{C_5}{21} + \frac{C_7}{45} - \dots\right) - \frac{C_1}{B_1}\left(\frac{B_3}{5} - \frac{B_5}{21} + \frac{B_7}{45} - \dots\right)\right]$$

Reference to the foregoing equations shows that the location of the aerodynamic center of a wing is independent of the twist. Also, as the lift acts at the aerodynamic center, C_{m_T} is constant about an axis through that point. Then the total moment coefficient of the wing is

$$C_{m_{ac}} = C_{m_S} + C_{m_T}$$ where $C_{m_{ac}}$ is the

moment coefficient about an axis through the aerodynamic center. The pitching-moment characteristics of any wing may accordingly be specified by giving the location of the aerodynamic center and the value of the pitching moment about an axis through that point. The location of the aerodynamic center and the pitching-moment coefficient due to twist are found for any wing having a straight wing axis from (10) and (11), by the substitution of the values of the coefficient B_n and C_n of the wing.

Determination of the coefficients.- The coefficients for any wing may be found from the following equation (reference 4, p. 139):

$$\alpha = \Sigma A_n \sin n\theta \left(\frac{4b}{a_0 c} + \frac{n}{\sin \theta}\right) \qquad (12)$$

The angle of attack at any point along the span α may be expressed as a function of θ by

$$\alpha = \alpha_c + f(\theta)\epsilon \qquad (13)$$

where α_c is the angle of attack at the center of the wing, and ϵ is the increase in angle of attack from the center to the tip. In determining the coefficients A_n from (12) only values of θ between 0 and $\pi/2$ need be considered, as the curve of lift distribution is symmetrical about the midpoint. Any number of the coefficients A_n may be obtained by putting the corresponding number of values of θ successively in (12) to obtain a series of equations. When the value of $\frac{4b}{a_0 c}$ for the particular wing is substituted and the equations are solved simultaneously (expressing α by (13)), the coefficients A_n are obtained in the form $A_n = B_n \alpha_c + C_n \epsilon$.

Wing with straight taper and linear twist.- When the twist has a linear variation along the span the angle of attack at any point of the left half of the wing may be written

$$\alpha = \alpha_c + \varepsilon \cos \theta,$$

following the form given by (13). The chord at any point of the left half of the wing may be expressed

$$c = c_c \left(1 + \frac{y}{b/2} r\right), \text{ where } c_t/c_c = 1 - r.$$

Then in terms of θ

$$c = c_c \; (1 - r \cos \theta) \tag{14}$$

Also $b = A \bar{c}$ and expressing $\bar{c}$ in terms of c_c and r, b becomes

$$b = A \frac{c_c}{2} (2 - r)$$

Then

$$\frac{4b}{a_o\ c} = \frac{2\ A\ (2 - r)}{a_o (1 - r \cos \theta)}$$

The slope of the lift curve a_o does not vary greatly for the wing sections in common use and may therefore be considered constant. The value used will be 5.79 per radian, which is the value for a good wing section as determined from tests in the variable-density wind tunnel. Then, substituting the above values of α and $\frac{4b}{a_o\ c}$ in (12) the equation for determining the coefficients A_n for a wing with straight taper and linear twist may be written

$$\alpha_c + \varepsilon \cos \theta = \Sigma A_n \sin n \theta \left[\frac{2A(2 - r)}{5.79(1 - r \cos \theta)} + \frac{n}{\sin \theta}\right] \tag{15}$$

From this equation four values of the coefficient A_n were determined by putting $\theta = 22\text{-}1/2°$, $45°$, $67\text{-}1/2°$, and $90°$ successively to obtain four equations. These equations were then solved simultaneously, using several values of A and r. The values of A and r and the corresponding values of B_n and C_n are given in table I. The values of H and G were then calculated by substituting the values of the coefficients in the formulas under (10) and (11).

<u>Distorted elliptical wing with linear twist.</u>- The location of the aerodynamic center and the moment due to twist

for the distorted elliptical wing may be found by a simpler method than that used for the straight-taper wing. As stated previously, the location of the aerodynamic center is independent of twist. When the wing is without twist ($\epsilon = 0$), $C_{m_T} = 0$, and from (9) $C_{m_L} = -\frac{d}{\bar{c}} C_L$.

Also, when the wing is without twist the lift distribution is in the form of a half ellipse and all the coefficients in the series for the circulation are zero except A_1. (See reference 4, p. 143.) Then from equation (7) we may write

$$C_{m_L} = -\frac{2}{3} A_1 A^2 \tan\beta = -\frac{d}{\bar{c}} C_L$$

but $A_1 = \frac{C_L}{\pi A}$, hence

$$\frac{d}{\bar{c}} = \frac{2}{3\pi} A \tan\beta$$

Referring to equation (10) it will be seen that for the distorted elliptical wing H has the constant value

$$H = \frac{2}{3\pi}$$

The coefficient of moment due to twist, C_{m_T}, is equal to C_{m_L} when $C_L = 0$, as may be seen by referring to equation (9). When $C_L = 0$, $A_1 = 0$; therefore, using (7) we may write

$$C_{m_T} = -2 A^2 \tan\beta \left(\frac{A_3}{5} - \frac{A_5}{21} + \frac{A_7}{45} - \ldots \right) \tag{16}$$

The coefficients A_n may be found from (12) as before. The chord in terms of θ is

$$c = c_c \sqrt{1 - \cos^2\theta} = c_c \sin\theta$$

Also

$$b = A\,\bar{c}$$

and

$$\bar{c} = \frac{S}{b} = \frac{\pi c_c}{4}$$

then

$$b = \frac{\pi c_c A}{4}$$

and
$$\frac{4b}{a_0\ c} = \frac{\pi\ A}{a_0\ \sin\ \theta}$$

This wing was considered to have a linear distribution of twist, as in the case of the wing with straight taper. Substituting the value of α for linear twist and the above value of $\frac{4b}{a_0\ c}$ in (12)

$$\alpha_c + \epsilon\ \cos\ \theta = \frac{\Sigma\ A_n\ \sin\ n\ \theta}{\sin\ \theta}\left(\frac{\pi\ A}{a_0} + n\right) \tag{17}$$

The coefficients A_n which occur in equation (16) for C_{m_T} could be obtained from the above equation by the method used in the determination of the coefficients for the wing with straight taper, that is, by solving a set of simultaneous equations for each value of A. However, owing to the simpler form of the equation for the distorted elliptical wing (17), C_{m_T} may be expressed in terms of A and a new series of coefficients which may be found by the solution of only one set of simultaneous equations as follows:

Equation (17) is expanded and reduced to the form

$$\cos\ \theta = k_3\left(1 + \frac{\sin\ 3\ \theta}{\sin\ \theta}\right) - k_5\left(1 - \frac{\sin\ 5\ \theta}{\sin\ \theta}\right) + k_7\left(1 + \frac{\sin\ 7\ \theta}{\sin\ \theta}\right) - \ldots \tag{18}$$

The coefficients A_n are obtained in the form

$$A_n = \frac{k_n\ \epsilon}{\frac{\pi A}{a_0} + n}$$

and (16) may therefore be written

$$C_{m_T} = -\ 2\ A^2\ \tan\ \beta\left[\frac{k_3\ \epsilon}{5\left(\frac{\pi A}{a_0} + 3\right)} - \frac{k_5\ \epsilon}{21\left(\frac{\pi A}{a_0} + 5\right)} + \frac{k_7\ \epsilon}{45\left(\frac{\pi A}{a_0} + 7\right)} - \ldots\right]$$

and using the form given by (11), Θ becomes

$$G = \frac{2\,k_3}{5\pi\left(1 + \frac{3}{\pi A}\frac{a_0}{}\right)} - \frac{2\,k_5}{21\pi\left(1 + \frac{5\,a_0}{\pi A}\right)} + \frac{2\,k_7}{45\pi\left(1 + \frac{7\,a_0}{\pi A}\right)} - \ldots$$

In the calculation of the values of G for the distorted elliptical wing given in figure 4, three values of k_n were used. They were determined by solving simultaneously the three equations obtained by putting $\theta = 22\text{-}1/2^\circ$, 45°, and $67\text{-}1/2^\circ$, successively, in equation (18). The coefficients were found to have the following values:

$$k_3 = 0.2724, \quad k_5 = -0.0811, \quad k_7 = 0.0542$$

Also a_0 was taken as 5.79 per radian as in the case of the straight-taper wing.

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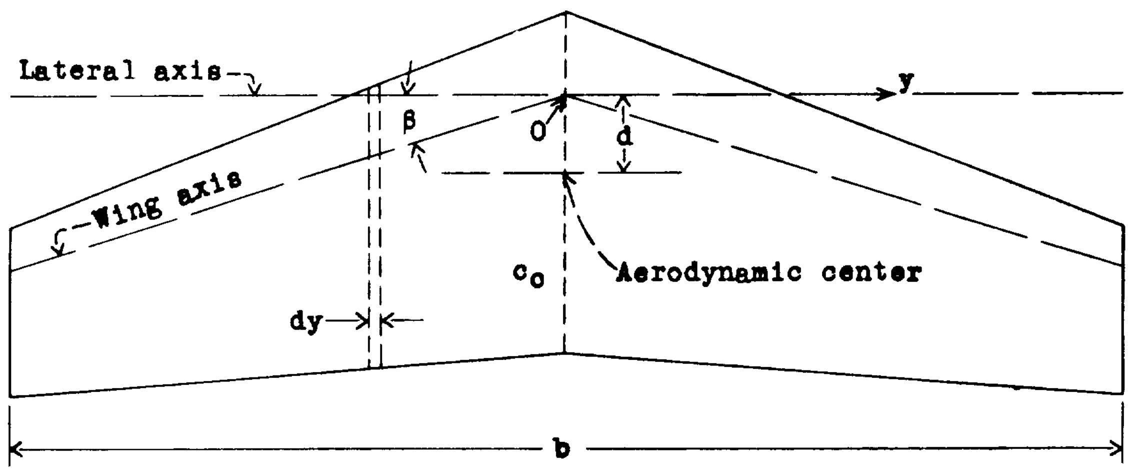
TABLE I

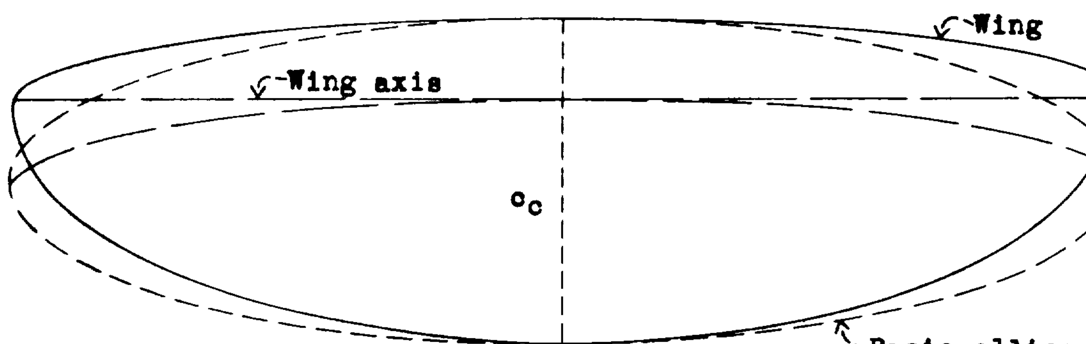
Values of B_n and C_n - Straight Taper Wing

$(c_t/c_c = 1 - r)$ $a_c = 5.79$

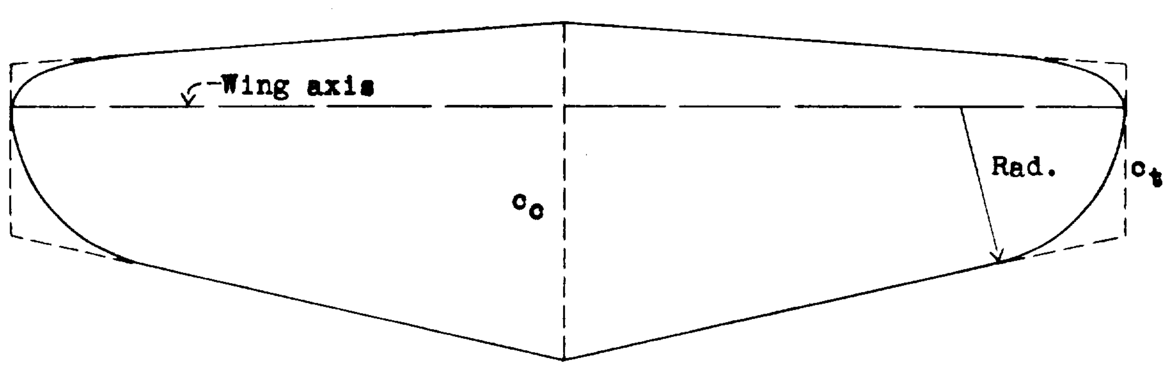
r	B_1	B_3	B_5	B_7	C_1	C_3	C_5	C_7
				Aspect ratio 3				
0	0.3660	0.0298	0.0046	0.0007	0.1569	0.0745	-0.0065	0.0065
1/2	.3777	.0075	.0094	-.0009	.1553	.0645	-.0067	.0069
1	.3620	-.0584	-.0015	-.0044	.1317	.0267	-.0197	.0051
				Aspect ratio 6				
0	.2258	.0284	.0058	.0010	.0999	.0596	-.0024	.0057
1/4	.2297	.0195	.0078	.0001	.0984	.0554	-.0029	.0059
1/2	.2333	.0072	.0096	-.0008	.0962	.0495	-.0037	.0062
3/4	.2347	-.0116	.0100	-.0017	.0916	.0395	-.0062	.0064
1	.2265	-.0490	.0021	-.0045	.0786	.0166	-.0160	.0048
				Aspect ratio 20				
0	.0818	.0170	.0053	.0012	.0376	.0302	.0015	.0037
1/2	.0840	.0044	.0064	-.0003	.0349	.0237	-.0006	.0040
1	.0838	-.0244	.0039	-.0035	.0270	.0067	-.0033	.0035
Aspect ratio				r = 1/2				
4	.3131	.0075	.0097	-.0009	.1289	.0586	-.0054	.0067
5	.2673	.0074	.0097	-.0008	.1101	.0537	-.0045	.0064
8	.1860	.0067	.0092	-.0007	.0758	.0428	-.0027	.0058
10	.1546	.0062	.0087	-.0006	.0640	.0377	-.0020	.0054



(a) Straight taper.



(b) Distorted elliptical.



(c) Rounded tip.

Figure 1.- Plan forms of the wings.

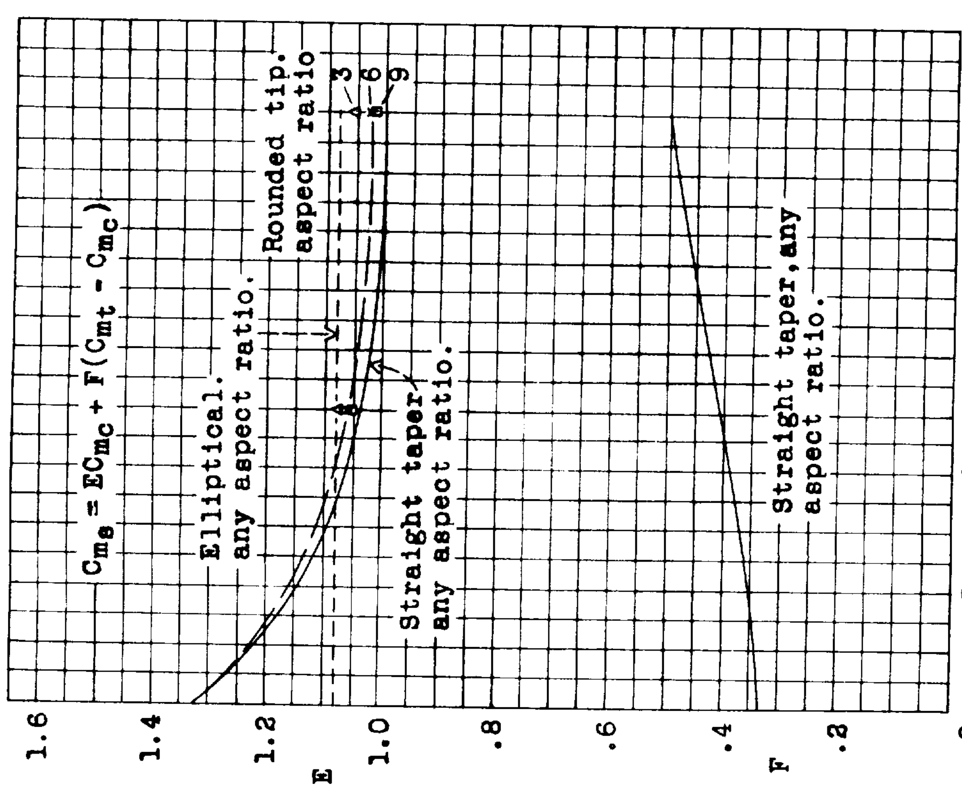


Figure 2.- Chart for calculating the pitching moment due to the sections, for wings of various plan forms.

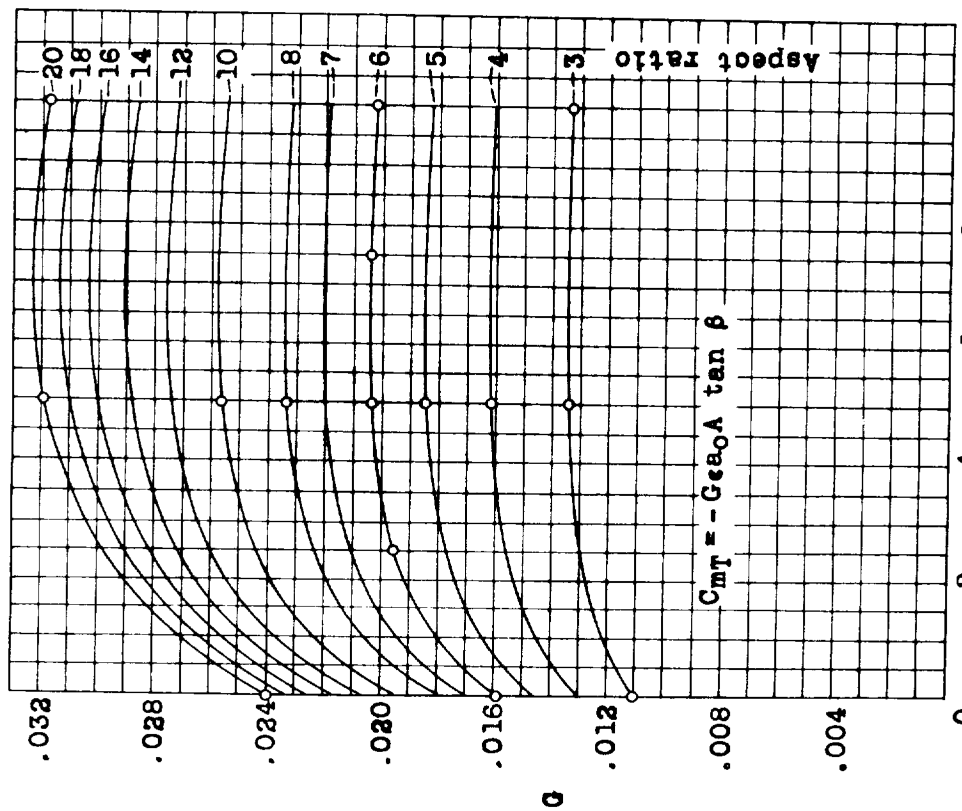


Figure 3.- Chart for calculating the pitching moment due to twist for wings with straight-taper plan forms.

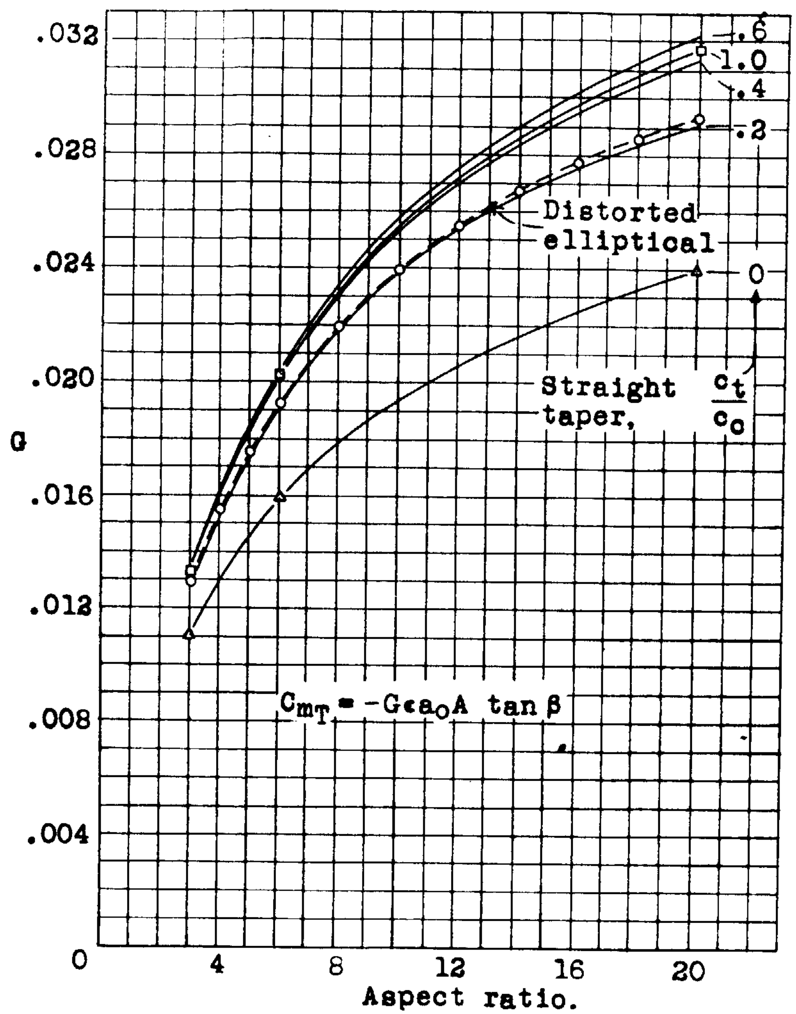


Figure 4.- Chart for calculating the pitching moment due to twist, for wings of various plan forms.

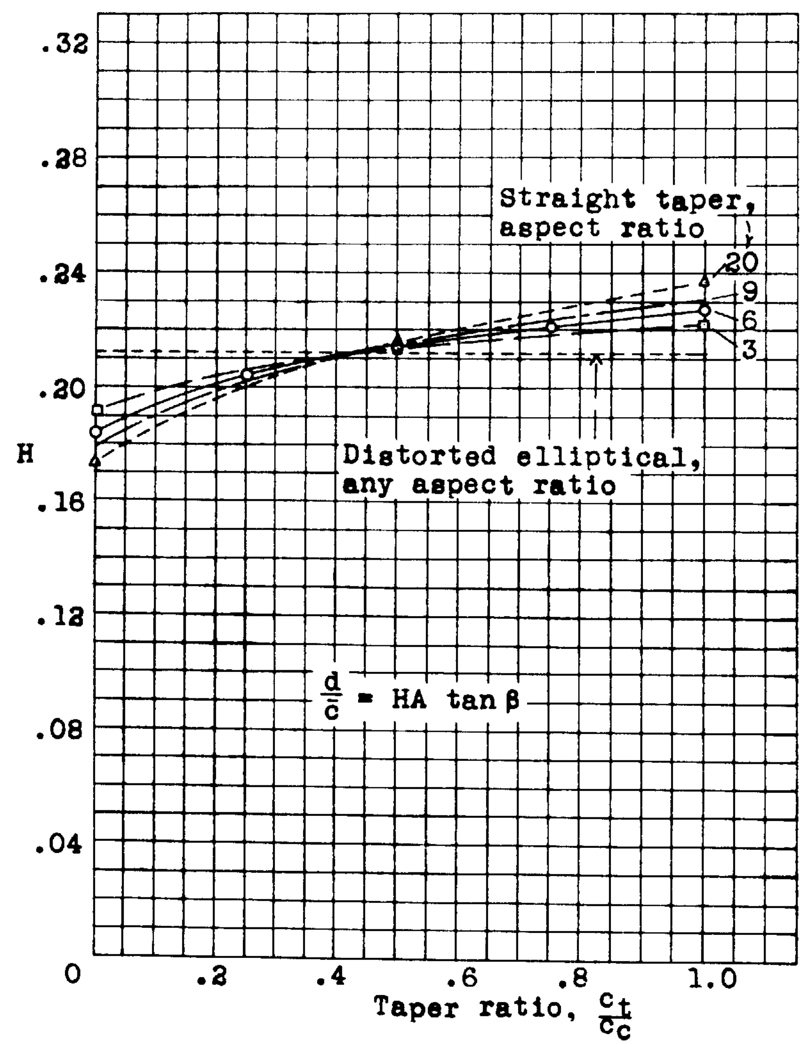


Figure 5.- Chart for calculating the location of the aerodynamic center, for wings of various plan forms.