TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 599

CHARTS EXPRESSING THE TIME, VELOCITY, AND ALTITUDE RELATIONS FOR AN AIRPLANE DIVING IN A STANDARD ATMOSPHERE

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SUMMARY

In this report charts are given showing the relation between time, velocity, and altitude for airplanes having various terminal velocities diving in a standard atmosphere. The range of starting altitudes is from 8,000 to 32,000 feet, and the terminal velocities vary from 150 to 550 miles per hour. A comparison is made between an experimental case and the results obtained from the charts. Examples pointing out the use of the charts are included.

INTRODUCTION

The velocity-altitude relations for airplanes in a dive have been treated in the past by several writers. Diehl (reference 1) assumed a constant density atmosphere. Wilson (reference 2) and Becker (reference 3), who have taken the variation of density into account, though attacking the problem differently, have given no expression for the time to dive. Regardless of the manner in which the density is taken into account, the velocity-altitude equations become too lengthy and complicated for general use.

Because of this fact, and also because the time variable has previously been omitted, it is the purpose of this paper to present in a readily usable form charts expressing all three relations with density variation taken into account.

CHARTS

The charts shown in figures 1 to 9 cover a range of terminal velocities from 150 to 550 miles per hour by 50-mile-per-hour increments. The starting altitudes vary from 8,000 to 16,000 feet by 2,000-foot intervals, and from 16,000 to 32,000 feet by 4,000-foot increments. This latitude in both speed and altitude will, it is believed, cover all cases of interest from the extremely slow airplane up to the fastest airplane available.

The terminal velocity U by which each chart is designated, is that which would be obtained in an atmosphere of constant sea-level density. The abscissas of the curves are the <u>true</u>, not the indicated, velocities.

In the establishment of the velocity-altitude curves, the equation developed in reference 2 was used with slight modifications to make it agree with the now recognized "standard atmosphere" of reference 4. The modified equation was similar in form to that given in reference 2 except that the factors 3 and 1,200 occurring in the original were replaced by the factors 2.7 and 1,254, respectively. The new equation was:

$$V^2 = 2g\left(1 + \frac{2.7h}{64000}\right)^{\left(\frac{1254}{U}\right)^2} \int_h^H \left(1 + \frac{2.7h}{64000}\right)^{-\left(\frac{1254}{U}\right)^2} dh \qquad (1)$$

where V is the true air speed in f.p.s.

g, gravity constant, 32.2

h, altitude in feet above sea level

H, starting altitude in feet above sea level

U, terminal velocity of airplane in air at standard sea-level density, f.p.s.

The placing of the time network on the charts was accomplished by using three methods: (1) by the use of the vacuum formula, (2) by use of an equation developed in the appendix which takes variation of density into account, and (3) by use of a step-by-step process of integration. Each of the foregoing methods had its particular region in which it was more easily applied than the others for the same degree of accuracy.

The acceleration of an airplane in a vertical dive, at any instant, is given by

$$a = 32.2\left(1 - \frac{q}{q_u}\right)$$

where a is the acceleration in ft./sec.^{2}

q, dynamic pressure

q_u, dynamic pressure at terminal velocity

It is obvious from this equation that during the early part of the dive the acceleration differs only slightly from g; hence the vacuum formula is applicable, within the plotting accuracy, for this range of the charts. This range increases with increase in starting altitude and terminal velocity. All timing lines below 6 seconds are computed by using the vacuum formula and in some cases the range has been extended to 8 seconds. Above these time values, use was made of a step-by-step integration of the velocity-altitude curves when the terminal velocities were 300 miles per hour, or more. Below a terminal velocity of 300 miles per hour, the time-altitude formula developed in the appendix was used. Application of this formula showed that it was necessary to carry a large number of terms when the terminal velocity was greater than 300 miles per hour, and also when the altitude loss was small, regardless of the value of the terminal velocity. Consequently, its practical use is limited to altitude losses of more than 2,000 feet and limiting speeds of less than 250 miles per hour. The foregoing limits are, of course, perfectly flexible depending both upon the combination of U and $H-h$ and upon the degree of accuracy desired.

The charts, although derived for a vertical dive starting from rest, may be used to include various diving angles and starting velocities. If U is the terminal velocity of an airplane in a vertical dive, its terminal velocity in a dive where the flight path makes an angle γ with the ground is

$$U\sqrt{\sin\gamma}$$

The vertical component of this velocity, which is the one used in choosing the appropriate chart, is

$$U' = U (\sin \gamma)^{3/2}$$

These factors will be found tabulated on each of the charts, except figure 1. The effect of an initial diving speed is taken into account in the charts simply by considering that the airplane started to dive from a higher altitude.

PRECISION

As far as the charts are concerned, the only errors are those due to plotting, to neglecting terms in the time equation, and to the discrepancies that will occur in any step-by-step integration. The error in the time lines due to the last two sources is believed to be within 2 percent; the plotting error in the velocity-altitude curves is negligible.

In the application of the charts to diving airplanes, however, several uncontrollable sources of error will exist; namely, those due to:

1. Variation of atmosphere from "standard."
2. Scale and compressibility effect on the airplane drag.
3. Manner of entry into the dive.
4. Dive-angle variation from that assumed.
5. Propeller effect.

The error due to the first source is negligible and need not be considered. Present knowledge indicates that the error due to source 2 is small up to 500 miles per hour with small angles of attack (reference 5). The manner of entry into the dive will be relatively unimportant in the longer dives but may be important in the shorter ones; however, in practice, its effect may be obviated if desirable. The effect of errors in the dive angle may become considerable in the shallower dives.

Probably the largest individual source of error is that due to the propeller effect. In the computation of the values of U with which to enter the charts this effect should, if possible, be taken into account.

In order to determine the cumulative effect of some of the above-listed errors a comparison is made in figure 10 between an experimental case and the results given by the charts. The initial air speed is taken as 120 miles per hour in order to eliminate the entry into the dive from level flight. It can be seen that a satisfactory agreement exists between experiment and the results given by the charts. If the dive had been longer and the starting altitude greater, the agreement would have been much better.

EXAMPLES

1. Given:

   a. Airplane with U of 496 m.p.h.

   b. Altitude at start H, 16,000 ft.

   c. Velocity at start, 100 m.p.h.

   d. Dive angle 60° to the ground.

   Required:

   a. Velocity at 6,000 ft. in m.p.h.

   b. Time to dive, sec.

   Solution:

$$U' = U\,(\sin\rho)^{3/2} = 496\,(0.866)^{3/2} = 400 \text{ m.p.h.}$$

Using figure 6, point A is located at 100 miles per hour and 16,000 feet. An interpolated curve is drawn between the existing curves down to B at 6,000 feet. Projecting from B to C, the velocity is found to be 406 miles per hour (an indicated velocity of 372 m.p.h.). The time is 28.3 - 4.6 = 23.7 seconds, found by subtracting the time corresponding to A from that at B.

2. Given:

   a. Airplane with U of 500 m.p.h.

   b. Altitude at start H, 14,000 ft.

   c. Velocity at start, 0 m.p.h.

   d. Starts pull-out at 3,000 ft.

   Required:

   a. The velocity at pull-out.

   Solution:

   Using figure 8 and following 14,000-foot curve down to 3,000-foot altitude, the true velocity is 449 miles per hour or the indicated velocity is 430 miles per hour.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., May 2, 1934.

APPENDIX

Derivation of Time-Altitude Formula

As stated in the introduction, previous investigators have not included an equation connecting time and altitude taking density variation into account. Consequently, the following derivation is made and is based upon Wilson's velocity-altitude equation (reference 2), modified to conform to the standard atmosphere of reference 4.

$$V = \frac{dh}{dt}; \quad \text{hence,} \quad V^2 = \left(\frac{dh}{dt}\right)^2$$

In equation (1) of the text, let $\frac{2.7}{64000} = a$, and

$$\left(\frac{1254}{U}\right) = c.$$

Then $$dt = \frac{dh}{\sqrt{2g\ (1 + ah)^{c^2} \int_h^H (1 + ah)^{-c^2}\, dh}}$$

from whence

$$t = \int \frac{dh}{\sqrt{2g\ (1 + ah)^{c^2} \int_h^H (1 + ah)^{-c^2}\, dh}} + C_1$$

If $t = 0$ when $h = H$, C_1 disappears. After integrating that portion under the radical, the foregoing expression is

$$t = \int_h^H \frac{dh}{\sqrt{2g\ (1+ah)^{c^2} \frac{1}{a} \frac{1}{1-c^2} \left[(1+aH)^{1-c^2} - (1+ah)^{1-c^2}\right]}}$$

Now, letting $\frac{2g}{a\ (1-c^2)} = -k^2$ and $(1+aH)^{1-c^2} = K$

(Note: $\frac{2g}{a\ (1-c^2)}$ is negative for $U < 1254$)

then $$t = \frac{1}{k} \int_h^H \frac{dh}{\sqrt{(1 + ah) - K(1 + ah)^{c^2}}}$$

Letting $u = \sqrt{1 + ah}$, then $dh = \frac{2}{a}\sqrt{1 + ah}\, du$

Substituting, there results

$$t = \frac{2}{ak} \int_{\sqrt{1+ah}}^{\sqrt{1+aH}} \frac{du}{\sqrt{1 - K\, u^{2(c^2-1)}}}$$

This expression cannot be integrated directly but may be put into a form that can, by expanding binomially the expression

$$\left[1 - K\, u^{2(c^2-1)}\right]^{-\frac{1}{2}}$$

The resulting series converges when $K\, u^{2(c^2-1)} \lesseqgtr 1$, which is the case here.

When the foregoing expansion is made the expression for t becomes

$$t = \frac{2}{ak} \int_{\sqrt{1+ah}}^{\sqrt{1+aH}} \left[1 + \frac{1}{2} Ku^{2(c^2-1)} + \frac{3}{8} K^2\, u^{4(c^2-1)} + \frac{5}{16} K^3\, u^{6(c^2-1)} + \ldots\right] du$$

which becomes upon integrating

$$t = \frac{2}{ak}\left[u + \frac{1}{2} K \frac{u^{2c^2-1}}{2c^2 - 1} + \frac{3}{8} K^2 \frac{u^{4c^2-3}}{4c^2 - 3} + \frac{5}{16} K^3 \frac{u^{6c^2-5}}{6c^2 - 5} + \ldots\right]_{\sqrt{1+ah}}^{\sqrt{1+aH}}$$

Remembering that $u = \sqrt{1 + ah}$ and $K = (1 + aH)^{1-c^2}$, the upper limit is

$$\frac{2}{ak} \sqrt{1 + aH} \left[1 + \frac{1}{2(2c^2 - 1)} + \frac{3}{8} \frac{1}{4c^2 - 3} + \frac{5}{16} \frac{1}{6c^2 - 5} + \ldots \frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n} \frac{1}{2nc^2 - (2n-1)} \right]$$

In the same manner the lower limit becomes

$$- \frac{2}{ak} \sqrt{1 + ah} \left[1 + \frac{1}{2} \frac{1}{2c^2 - 1} \left(\frac{1 + aH}{1 + ah} \right)^{1-c^2} + \frac{3}{8} \frac{1}{4c^2 - 3} \left(\frac{1 + aH}{1 + ah} \right)^{2(1-c^2)} + \ldots \frac{1.3.5\ldots 2n - 1}{2.4.6\ldots 2n} \frac{1}{2nc^2 - (2n-1)} \left(\frac{1 + aH}{1 + ah} \right)^{n(1-c^2)} \right]$$

Combination of the two limits gives an equation for the time in terms of the altitude,

$$t = \frac{2}{ak} \left[\sqrt{1 + aH} \sum_{n=0}^{\infty} \frac{[2n - 1]}{[2n]} \frac{1}{2nc^2 - (2n-1)} - \sqrt{1 + ah} \sum_{n=0}^{\infty} \frac{[2n - 1]}{[2n]} \frac{1}{2nc^2 - (2n-1)} \left(\frac{1 + ah}{1 + aH} \right)^{n(c^2-1)} \right]$$

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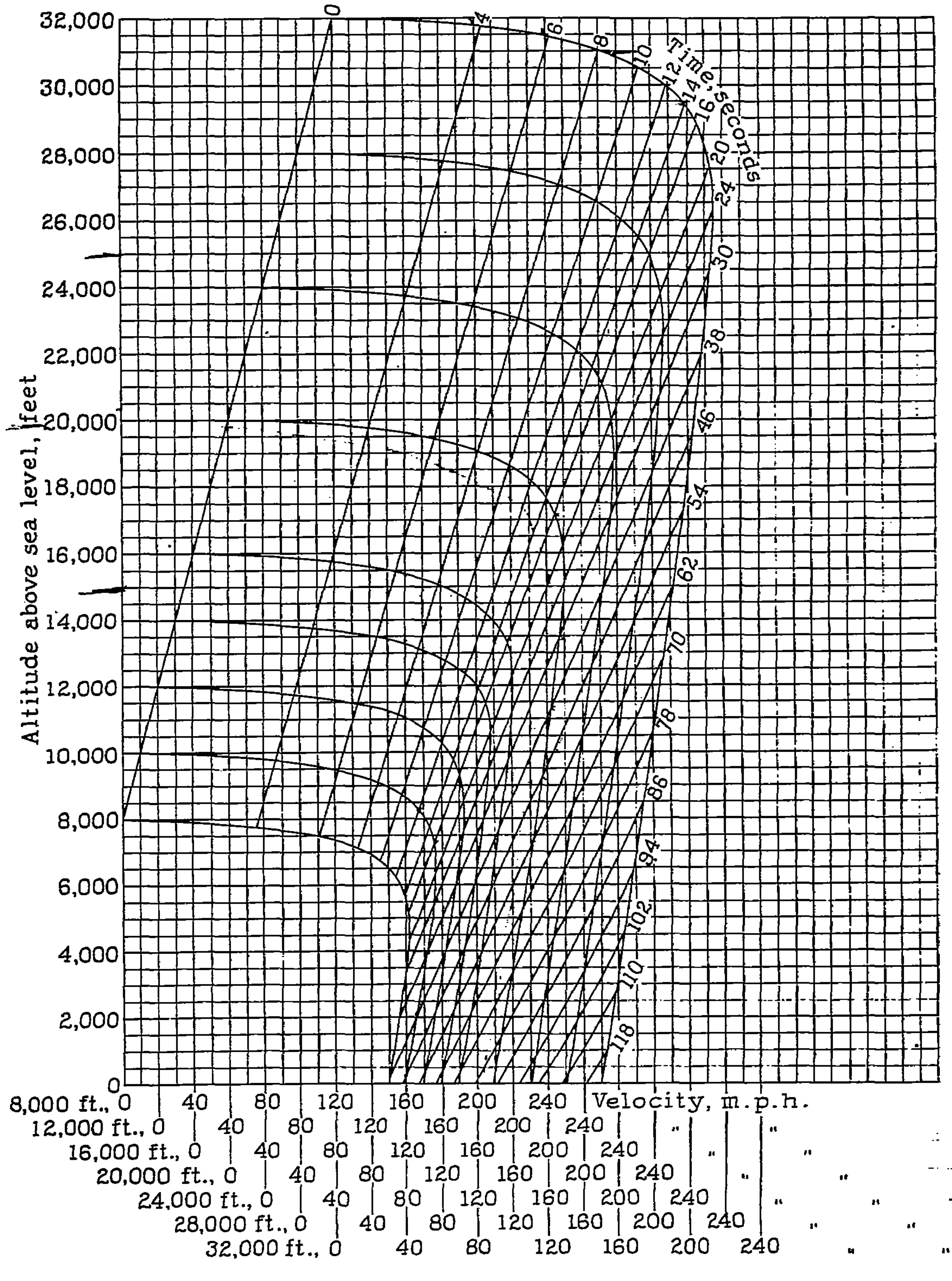
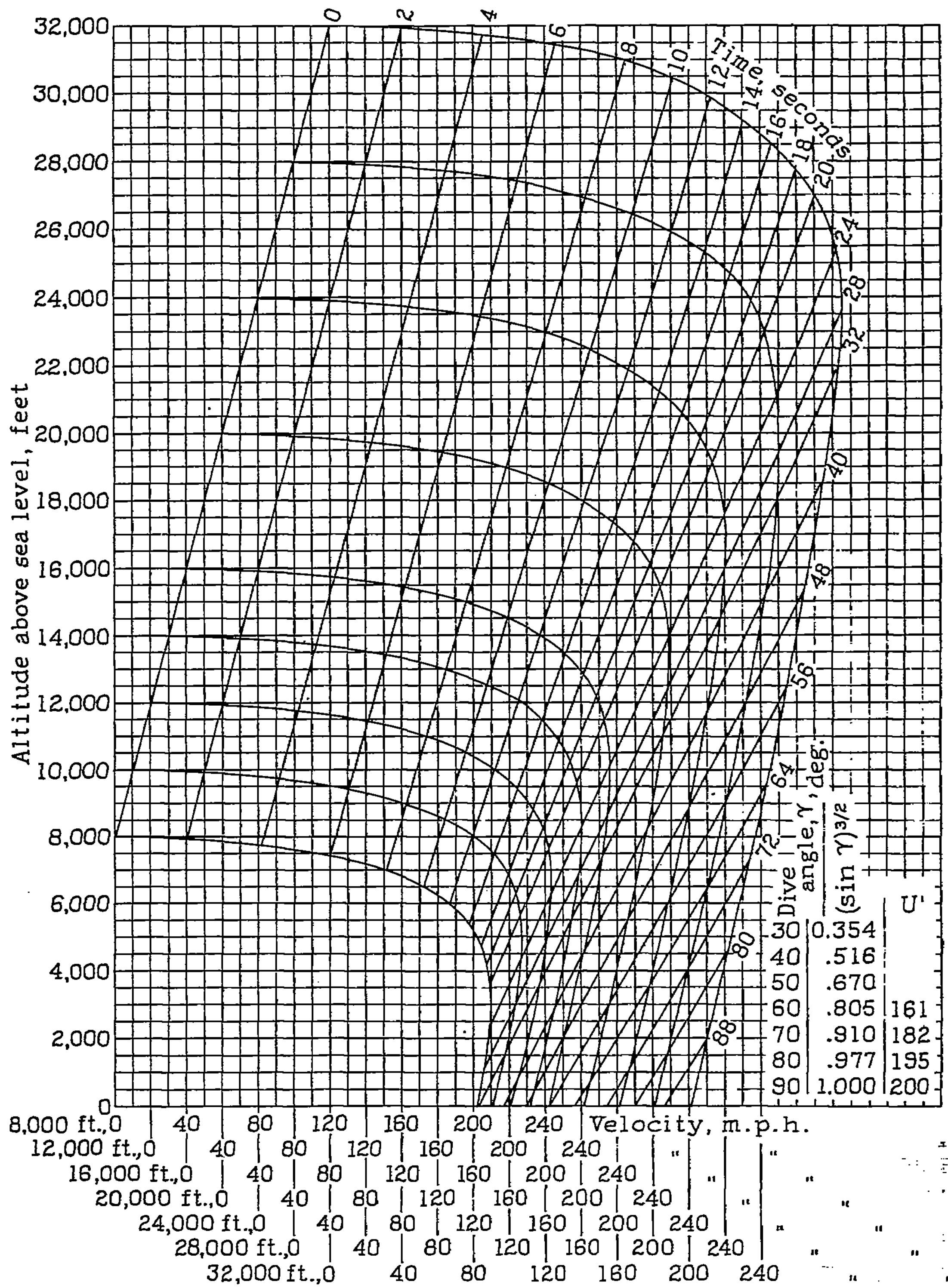


Figure 1. - Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 150 m.p.h.



Dive angle, γ, deg.	$(\sin\gamma)^{3/2}$	U'
30	0.354	
40	.516	
50	.670	
60	.805	161
70	.910	182
80	.977	195
90	1.000	200

Figure 2. - Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 200 m.p.h.

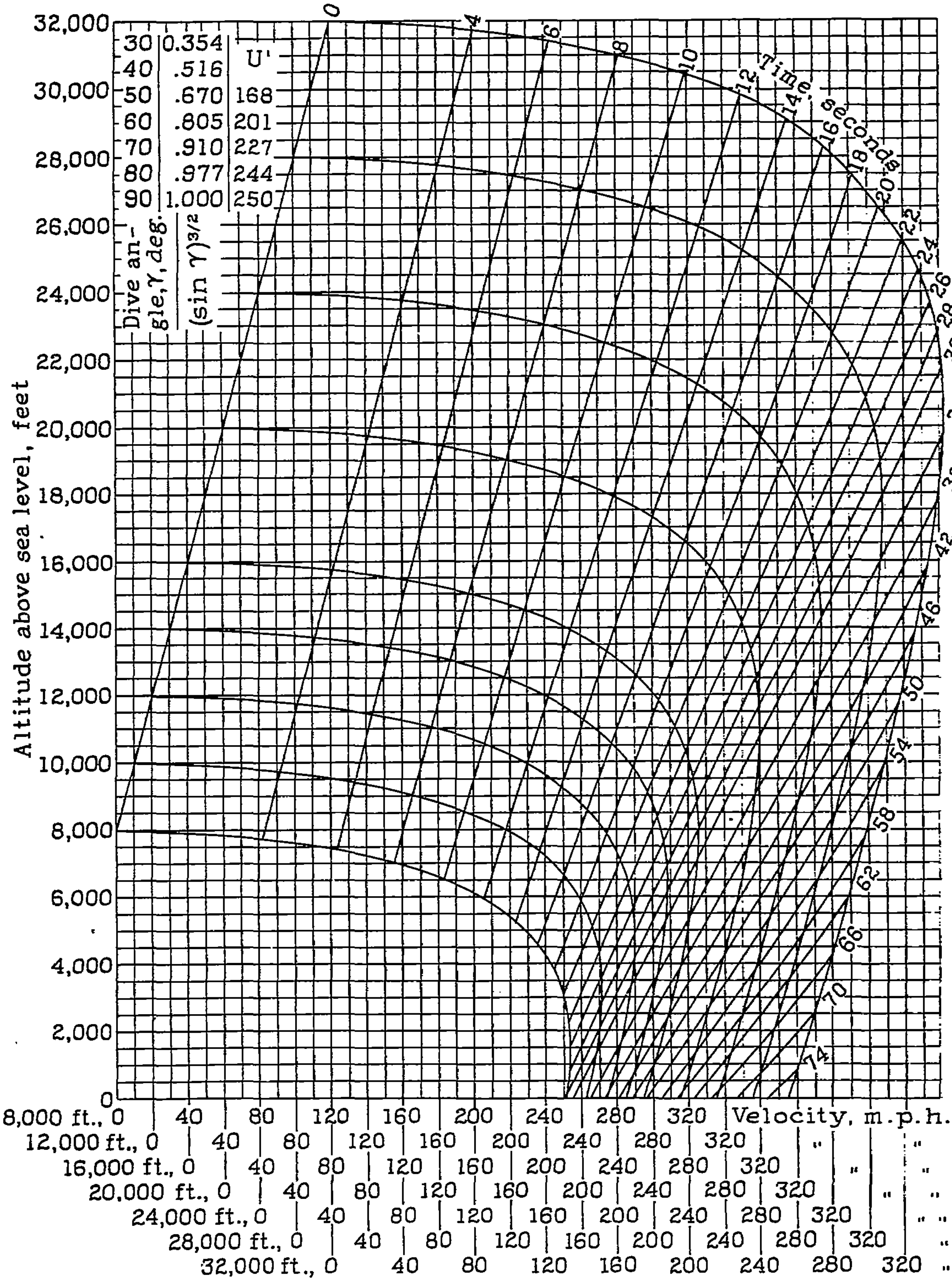


Figure 3.- Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 250 m.p.h.

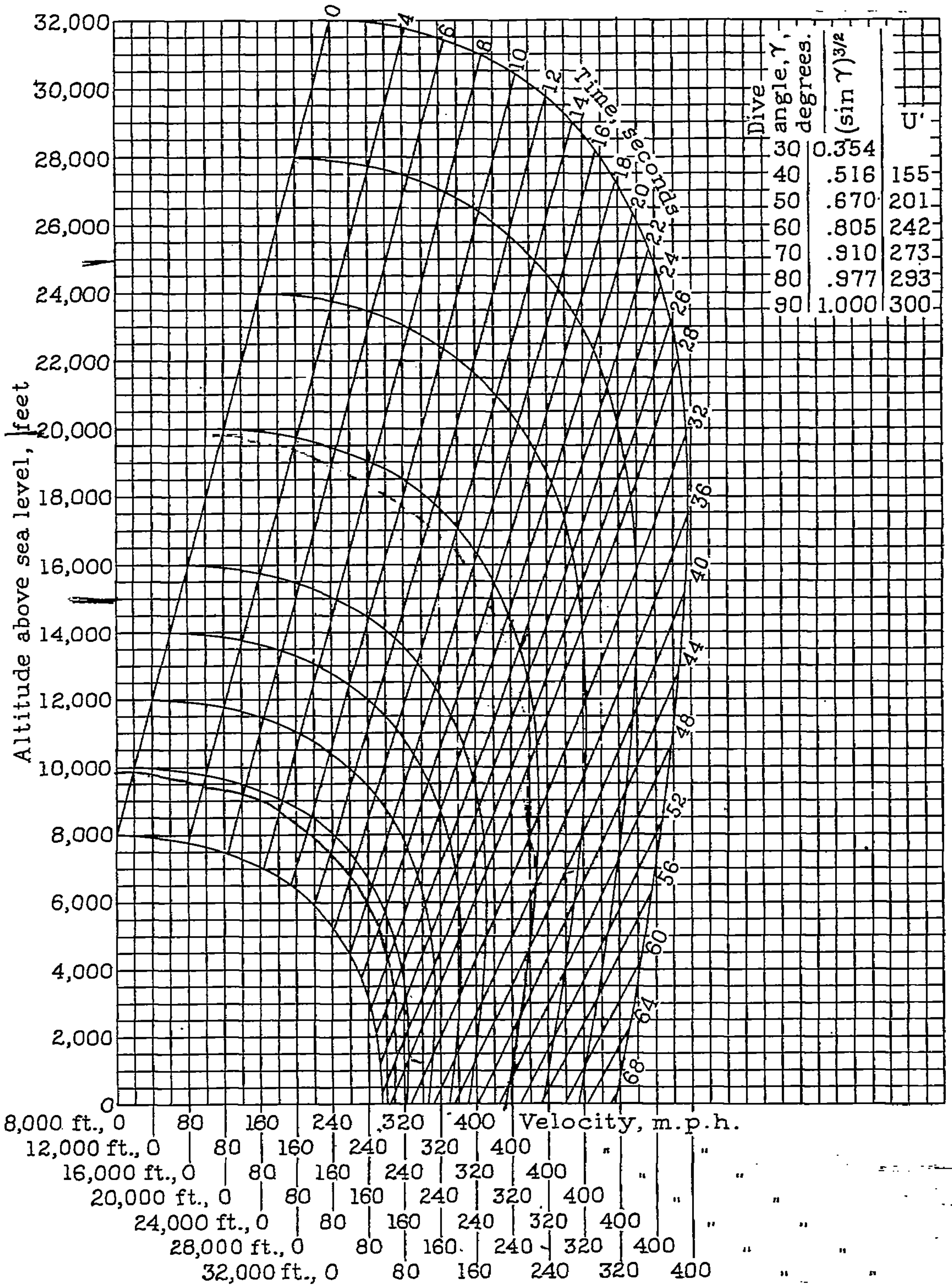


Figure 4. - Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 300 m.p.h.

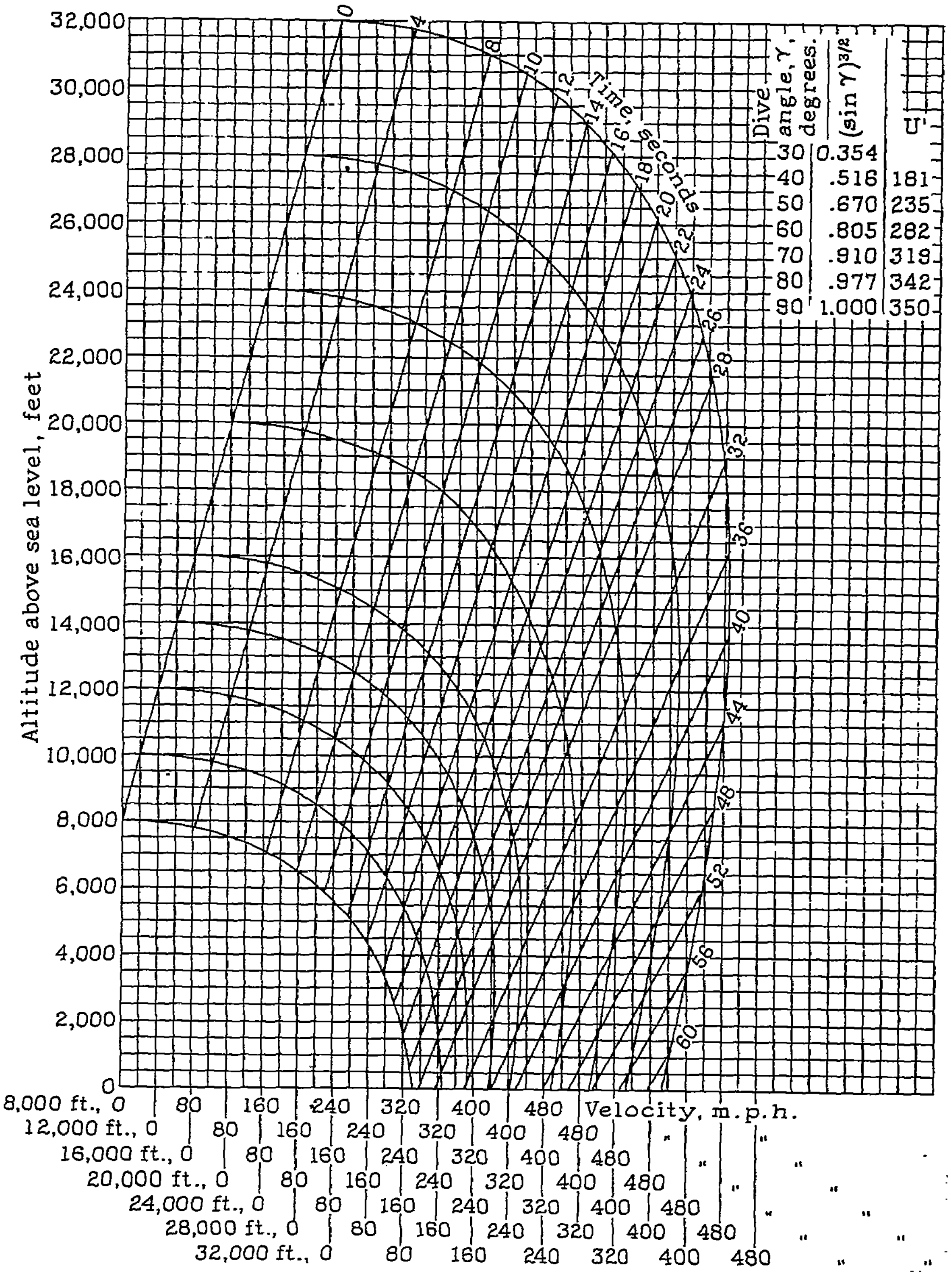


Figure 5. - Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 350 m.p.h.

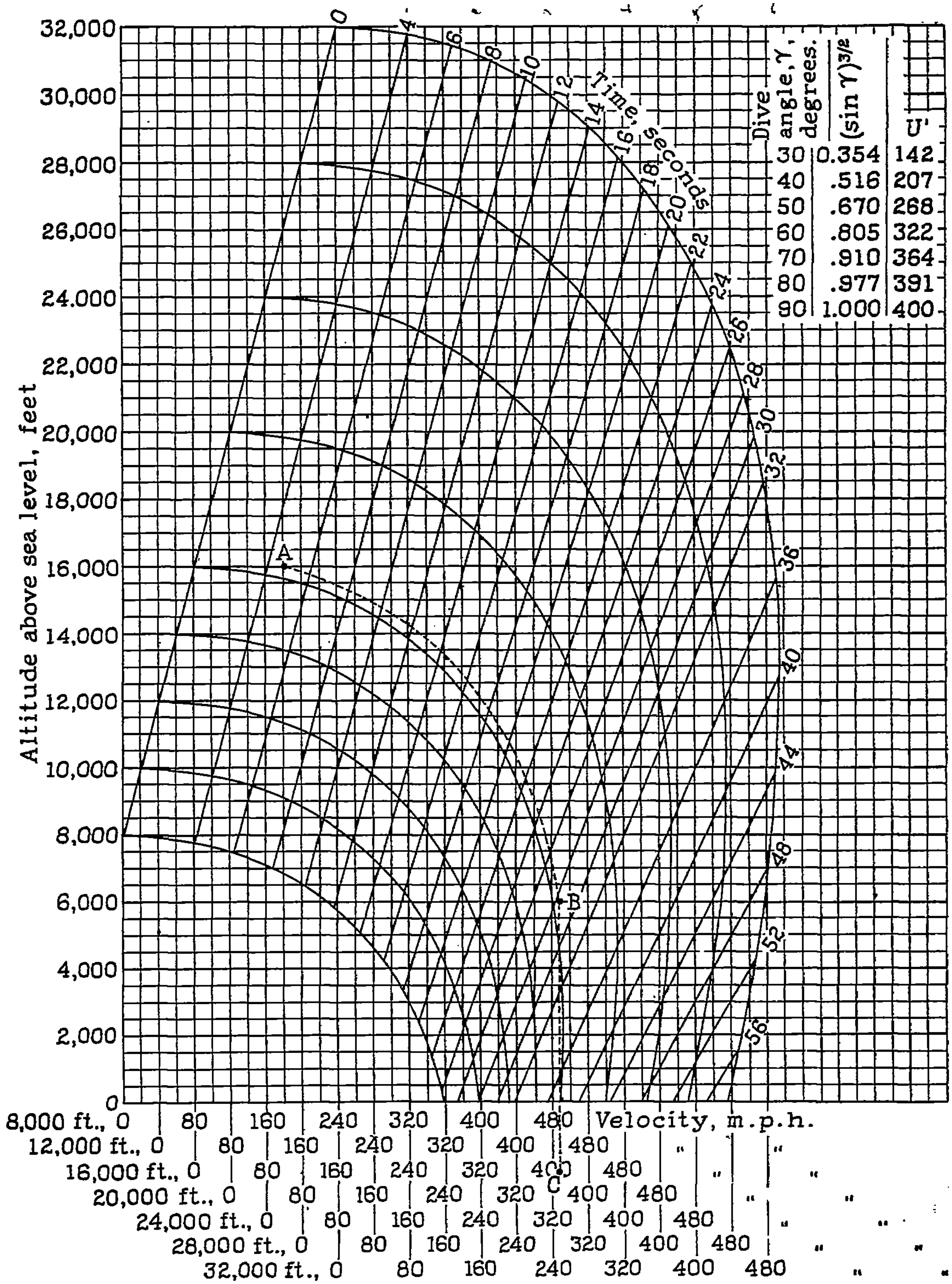


Figure 6. - Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 400 m.p.h.

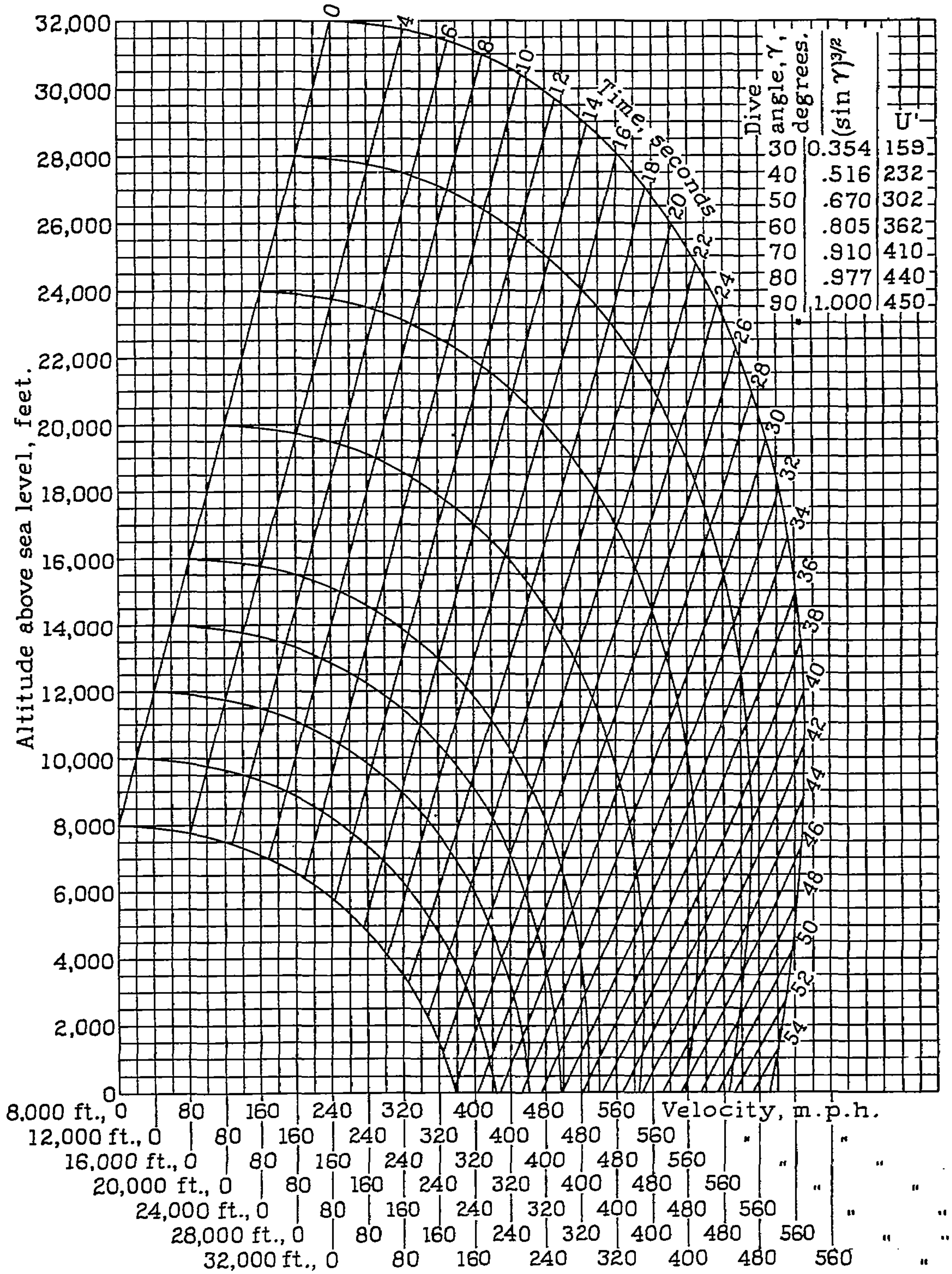


Figure 7. – Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 450 m.p.h.

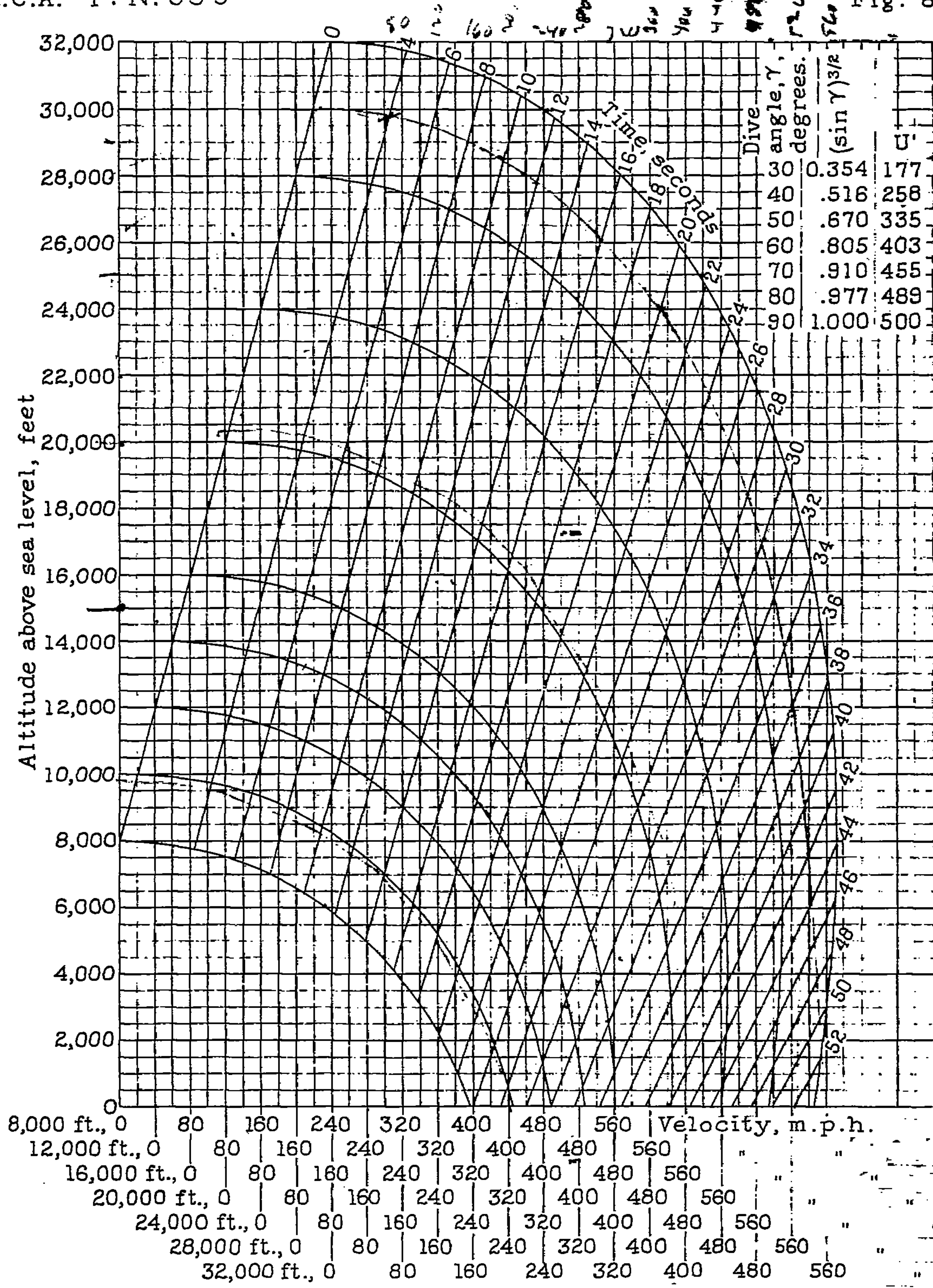
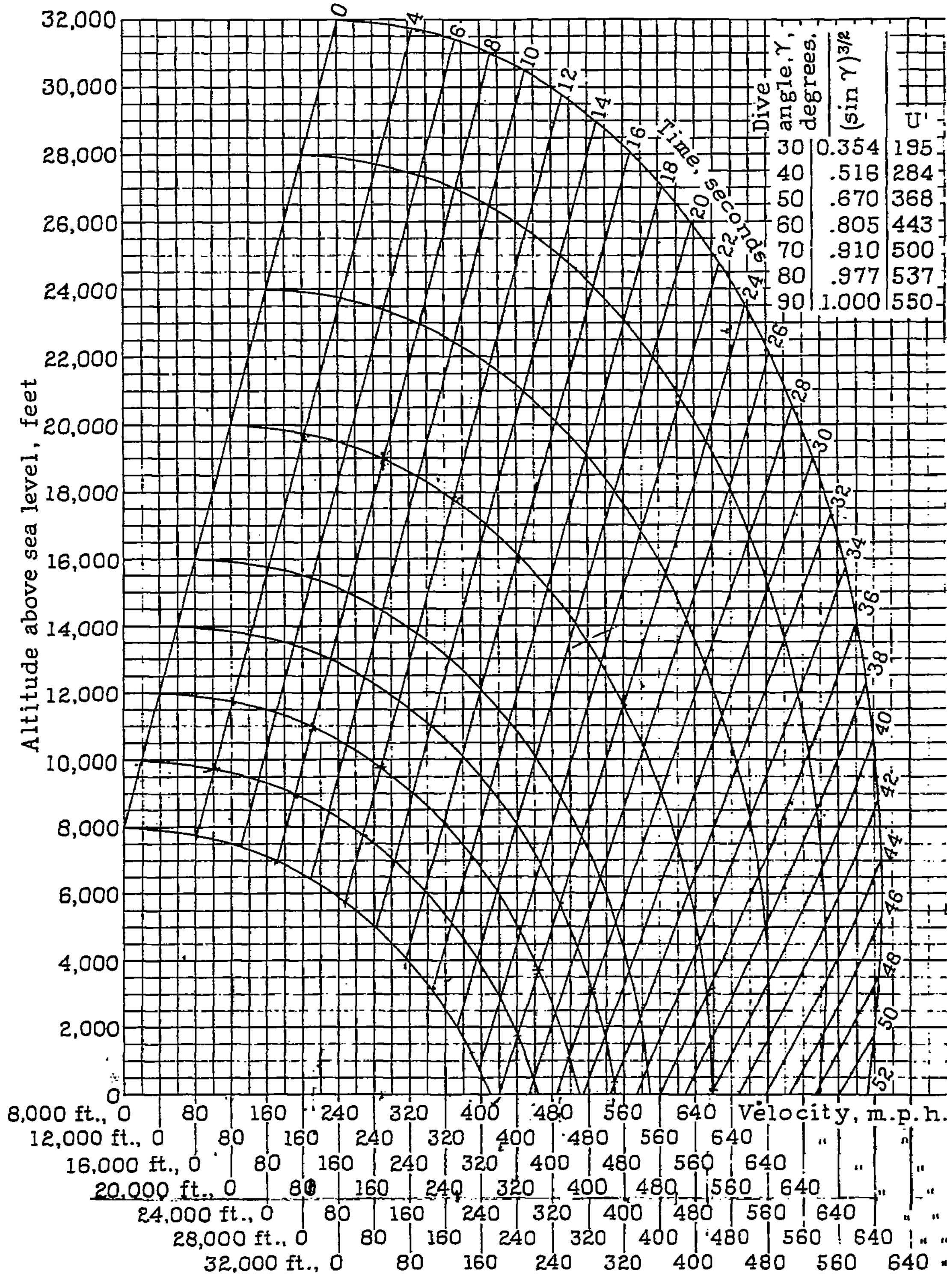


Figure 8. - Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 500 m.p.h.



Dive angle, γ, degrees.	$(\sin \gamma)^{3/2}$	U'
30	0.354	195
40	.516	284
50	.670	368
60	.805	443
70	.910	500
80	.977	537
90	1.000	550

Figure 9. - Time-altitude-velocity relations for airplanes diving in a standard atmosphere. U = 550 m.p.h.

1, Experimental, for vertical dive on F 6C-4. Initial velocity 120 m.p.h., initial altitude 12,000 ft.
2, Calculated, assuming a standard atmosphere.

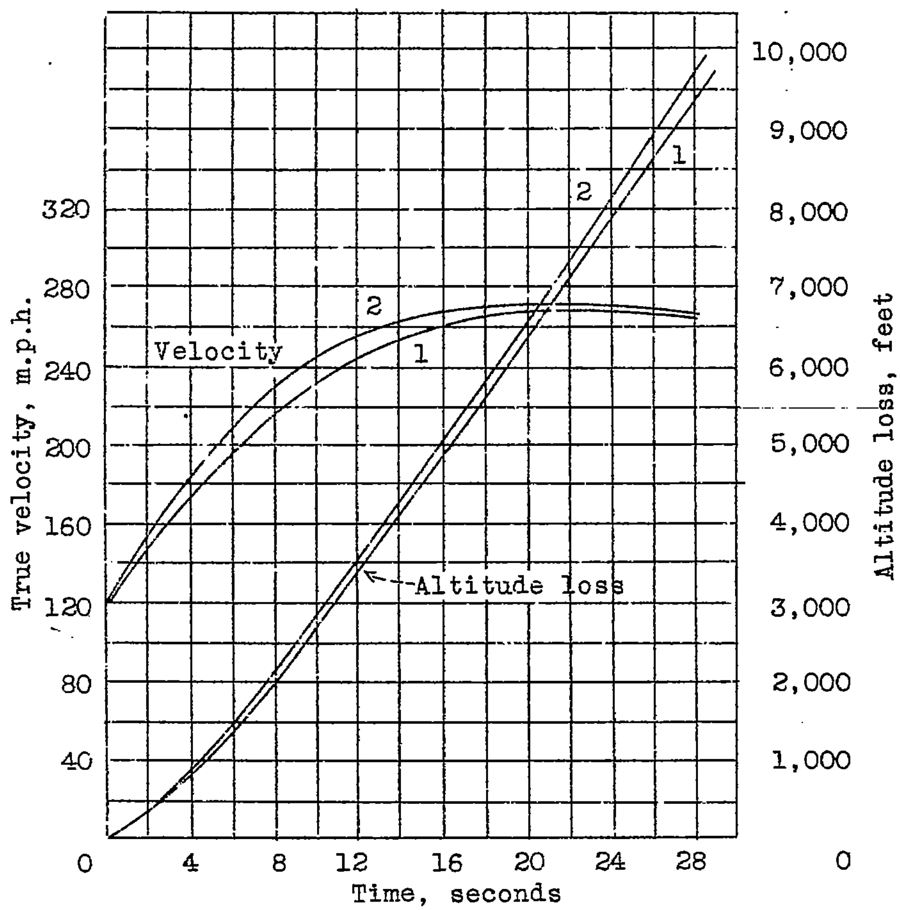


Figure 10.- Comparison between experimental and calculated results.