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TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 217

NOTE ON THE AIR FORCES ON A WING CAUSED BY PITCHING.

By Max M. Munk.

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Summary

The following Note, prepared for the National Advisory Committee for Aeronautics, contains information on the air forces on a wing produced by its pitching at a finite rate of angular velocity. The condition of smooth flow at the region of the trailing edge is maintained and the fluid supposed to be perfect. The wing then experiences the same lift as if moving with the momentary velocity of the rear edge.

When a wing - in addition to its translatory motion - has assumed an angular pitching velocity, it will experience special additional air forces caused thereby. In the following Note I wish to give an account of the air forces to be expected. I suppose the velocity of any point of the wing due to the pitching rotation to be small when compared with the velocity V of translation. I further confine the consideration to the case of a two-dimensional flow, that is, the one of infinite aspect ratio. The magnitude of the forces set up by the pitching cannot be computed exactly at the present state of knowledge. I have, therefore, to introduce certain simplifying assumptions in order to obtain at least the main portion of the forces produced. This approximate or theoretical solution should be compared with the results of careful measurements of the same quantity. The comparison would constitute the right interpretation of such tests. The differences between the test results and the computed results would throw light on the general physical questions involved in the special problem in question. Its theme is the unsteady motion of fluids, more especially of air.

The research work of the past has been chiefly confined to the investigation of steady motion only. Unsteady motion is of equally great practical importance to aeronautical technics, and its study should be taken up exhaustively. Let us begin by determining the air force on a wing which is in the process of changing its angle of attack at a finite rate of pitching angular velocity.

The simplified treatment to which I have to take recourse for want of proper empirical information, is based on the assumptions used with success and with better justification for wings in a steady translatory motion. This is Kutta's condition. Kutta assumes that each wing of infinite span gives rise to that potential flow that closes itself at the point of the sharp rear edge, rather than to any other potential flow that gives rise to infinite velocities in the neighborhood of the rear edge. This condition is plausible for steady motions and leads to a good agreement with the facts observed. With an unsteady motion, like the one under consideration, Kutta's condition is in contradiction with a very broad principle of mechanics. If the flow is not in agreement with Kutta's description, it will change gradually and it will take some time until it is. In the meantime, the angle of attack (for instance) has changed again and hence the actual flow will lag behind the theoretical one. I do not try to dwell longer on this lag, on its causes, or on its probable or approximate magnitude. The first step necessary is to compute the theoretical flow, or rather the air forces caused thereby. The lag should then be investigated by empirical research and it may be advisable afterwards to consider the result critically.

Since the pitching angular velocity is supposed to be small, its effects on the air pressure and on the air forces can be treated like those of a small angle of attack, of a small camber, or of a small thickness, namely, it can be superposed by mere addition to the effects of the latter. (Reference 1.) We have to concentrate our attention on the pitching rotation of a wing, say, of a straight line of the chord length $c = 2a$, rotating in a perfect fluid with the angular velocity ω around its middle. The problem is two-dimensional. The potential flow caused by the angular velocity alone is steady and hence does not give rise to a resultant force or moment. However, this potential flow in combination with the translatory flow gives rise to a resultant lift. Moreover, the flow is not in accordance with Kutta's condition, having infinite components when flowing around the rear edge. Hence in combination with the translatory flow a superposed circulation flow around the straight line is set up. This flow again gives rise to a resultant lift. The lift caused by the combination of the rotation and of the translation can be computed in the following way. The component of translation at right angles to the line has the magnitude $V \sin\alpha$, where α is the momentary angle of attack. Let α be zero at the time $t = 0$, then $\alpha = \omega t$ and hence the lateral component $\mu = V \omega t$. This lateral flow sets up a "transverse" flow around the line discussed in the paper referred to and having the potential

$$\Phi = \pm aV \omega t \sqrt{1 - \left(\frac{x}{a}\right)^2}$$

at both sides of the line, x denoting the distance from the middle and $2a$ the chord. The rate of change per unit of turn is
time

$$\frac{d\Phi}{dt} = \pm V \omega \sqrt{1 - \left(\frac{x}{a}\right)^2}$$

and this multiplied by the density ρ is equal to the pressure due to the unsteadiness in question. The integration

$$\rho \int V \omega \sqrt{1 - \left(\frac{x}{a}\right)^2} \, dx$$

around the wing gives

$$V \omega a^2 \pi \rho = V \omega c^2 \frac{\pi}{4} \rho = L_1 \qquad (1)$$

This is the lift desired.

I now turn to the computation of the lift caused by the circulation flow. Without this circulation flow there would be no accordance with Kutta's condition.

It is much easier to reduce the problem to one known before than to solve it directly. I proceed to show that the boundary conditions at all points of the straight lines are exactly the same as in the case of determining the potential flow around a circular or parabolic arc of small camber. Let x denote the distance of a point from the middle. Then the transverse component of velocity at this point is $-x\,\omega$. With a parabolic arc

$$\xi = A\left(1 - \left(\frac{x}{a}\right)^2\right)$$

of the same length $2a$ of the chord, the slope at each point is $-\frac{2A}{a^2}x$ and hence its lateral velocity component is

$$-\frac{2A\,x\,V}{a^2}$$

Hence, choosing the camber equal to

$$A = \frac{\omega\,a^2}{2V}$$

the boundary conditions agree. Hence the flows agree and so do the forces caused therefrom. The effective angle of attack of the rotating wing will be $\frac{\omega\,a}{2\,V}$ and hence the lift is

$$L_2 = V\,\omega\,c^2\,\frac{\pi}{4}\,\rho$$

The entire lift of the wing is therefore

$$L = L_1 + L_2 = V\,\omega\,c^2\,\frac{\pi}{2}\,\rho$$

The result can be expressed in a more convenient way by determining that point of the wing around which a rotation would not produce any lift. Such point is necessarily existing, for by choosing any point, a, other than the middle as point of reference, the middle has a transverse motion $a\,\omega$ corresponding to an angle of attack $\frac{x\,\omega}{V}$. Hence the point results from putting this equal to the effective angle of attack of the rotation. It appears at once

$$x = \frac{c}{2}$$

Hence we obtain: <u>A straight line rotating about the rear edge of the chord does not give rise to any resultant air force</u>. Or, otherwise expressed: <u>The resulting air force of a wing while pitching is equal to its resultant air force while in a mere translatory motion, if the magnitude and direction of this motion relative to the air is equal to that of the rear edge</u>.

Reference

1. Max M. Munk: Elements of the Wing Section Theory and of the Wing Theory. N.A.C.A. Technical Report No. 191. 1924.