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CURVILINEAR FLIGHT OF AIRPLANES.

By E. Salkowski.

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I. Horizontal Curvilinear Flight.

Simultaneously with, but independently of, the foregoing investigation by Kann, work was in progress on the same subject and was carried out by the mutual cooperation of Messrs. Hoff, Hopf and the writer. The problem was placed on a somewhat wider basis, without any assumption in regard to the variation of engine power with the altitude and only with the assumption that this relation is, in fact, empirically known from measurements taken in altitude tests. If this is the case, the following method of calculation is as easily applicable as that of Kann, which latter assumes a proportionate decrease of engine power with altitude and is always applied when an approximately correct theoretical estimate of the ceiling must be made without knowing the results of engine tests. Since the two methods of calculation supplement each other, the results of our investigations are also given here.

Our method concerns the same special case of the curvilinear flight of an airplane, viz.: steady horizontal flight in a circle in which no side slip occurs. The questions of the most common steady turning-flight curves (i.e., of such curves as are

* From Technische Berichte, Volume III, No. 7, pp. 267-274.

traversed without change of acceleration or velocity) would be considerably broader. This problem might be treated exhaustively, but the requisite preliminary work is not yet finished. It is not yet settled as to whether steady turns are of any practical significance, since it may be that all turns are to be regarded as displacements from a position of unstable equilibrium. On theoretical grounds, Reissner had come to the conclusion before the war, that an airplane at that time could not fly in steady turns. If this were the case, then the practical importance of experiments in this direction would be greatly limited. The question cannot be settled as yet since, from the observation of pilots, such turns appear quite possible. In that case, a knowledge of steady turning conditions is not superfluous, because it is by this means, in the first place, that a suitable idea can be formed on the stability of the actual motions.

The investigations to be considered rest on two basic assumptions, which restrict the simple statement of the equations. The first is that the airplane lies correctly in the turn, so that the resultant of gravity and of the centrifugal force falls in its plane of symmetry. This stipulation is not a restriction, since it must be fulfilled in ordinary flight. The second assumption is more essential. The airplane must so lie in the turn that its axis coincides with the direction of motion, i.e., so that there will be no slipping sidewise, a condition which cannot always be fulfilled in practice. As a matter of fact, most turns are flown with an outward slip (skidding) while any inward slip (side-slipping) is avoided on account of the danger of falling into a spin. Hence, it is not justifiable to conclude, from the sensibly correct bank of the airplane, that its position is tangential to the line of flight, since a difference of a few degrees alters the bank but very slightly, while it considerably increases or decreases the danger of spinning. Among the turns calculated from the equations of motion, certain limiting values are of special interest, since the practical use of an airplane depends on their magnitude. The turn is of special importance in which the airplane changes its direction of motion most quickly, i.e., the turn which it describes with the greatest angular velocity. The curvature of this turn is a measure of the turning power, or maneuverability, of the airplane. On the other hand, it may be important for the airplane to fly the sharpest possible turns, i.e., those in which the radius of curvature of the flight path is as small as possible. The two turns are not identical, since a turn of greater radius can be described with so much greater velocity, that the time required to fly it is shorter than that required to traverse the sharper turn. The sharpest turn is actually flown with a different angle of attack than that of the quickest turn. For the range of the angles of attack under consideration, the change in the radius of curvature is vanishingly small, while the velocity along the flight path and the angular velocity change appreciably.

1. Equations of Motion.- The motion of a body in space is

determined by six equations, which can be written in the form of equilibrium conditions. Of these, the three equations which express moments do nott at first come into question, since they can always be satisfied by a suitable rudder deflection. It is, therefore, only necessary to bring the active forces into equilibrium. According to our assumptions, the air resistance D always acts in the direction of the tangent to the path of flight and in the opposite direction to the propeller thrust T. The two forces must be in equilibrium with each other. The weight of the airplane and the centrifugal force Z act in a plane perpendicular to the path. They have a resultant $R = \sqrt{W^2 + Z^2}$ which is in equilibrium with the lift L. Since the resultant force makes, with the vertical, an angle increasing with the centrifugal force, the airplane must be banked in the turn in order to maintain equilibrium, the wings forming, with the horizontal, the angle of bank given by the equation $\tan \Phi = \frac{Z}{W}$.

The propeller thrust for a velocity V of the airplane, output P of the engine in HP and efficiency of the propeller, is $T = \frac{75\, P\, \eta}{V}$. The centrifugal force (for a radius of curvature r) is $Z = \frac{W}{g} \frac{V^2}{r}$. The resistance and lift for the air density ρ ; surface area S, of the wings; and the coefficients C_D and C_L, corresponding to the profile, are

$$D = C_D \frac{\rho}{2g} S V^2$$

$$L = C_L \frac{\rho}{2g} S V^2$$

The conditions for the equilibrium of the forces are therefore

$$\frac{75\ P\ \eta}{V} = C_D \frac{\rho}{2g} S\ V^2 \qquad (1)$$

$$W \sqrt{1 + \left(\frac{V^2}{g r}\right)^2} = C_L \frac{\rho}{2g} S\ V^2 \qquad (2)$$

while, for the angle of bank Φ, we have

$$\tan \Phi = \frac{V^2}{g r} \qquad (3)$$

If the curve of quickest turning flight is to be obtained, then the radius of curvature must be replaced in equation (2) by the angular velocity ω. Now $\omega = \frac{V}{r}$, so that equation (2) assumes the form

$$W \sqrt{1 + \frac{\omega^2 V^2}{g^2}} = C_L \frac{\rho}{2g} S\ V^2 \qquad (2)$$

If V is eliminated from equations (1) and (2), we obtain a relation between the power output and the angular velocity, which, after introducing the non-dimensional auxiliary quantities

$$x = \left[(75\ P\ \eta)^2 \frac{S}{W^3} \frac{\rho}{2g}\right]^{1/3} \qquad (4)$$

$$y = 2 \frac{W}{S} \frac{\omega^2}{g \rho} \qquad (5)$$

may be written in the form

$$y = \frac{{C_L}^2}{{C_D}^{2/3}}\ x - \frac{{C_D}^{2/3}}{x} \qquad (6)$$

This is a relation, the coefficients of which are only dependent on the polar diagram. The numbers x and y have an easily recognized physical significance. x increases with the engine output and with ρ, but decreases with increasing altitude. y, on the other hand, increases with the altitude and angular velocity and depends only on the loading of the wing surface. Equation (6) holds good for a perfectly arbitrary turn. For each angle of attack there is a definite angular velocity, which, at a given altitude, corresponds to a definite engine output.

2. Curve of Quickest Flight.- For the quickest turn, the angular velocity and also y for a constant engine output must be a maximum, i.e., we must have

$$\frac{dy}{dC_D} = 0 \tag{7}$$

if C_L is considered dependent on C_D, as is the case for the polar diagram.

The calculation gives

$$0 = \frac{2\ C_L\ C_L'\ C_D^{2/3} - 2/3\ C_L^2\ C_D^{-1/3}}{C_D^{4/3}}\ x - \frac{\frac{2}{3} C_D^{-\frac{1}{3}}}{x}$$

that is,

$$x = \frac{C_D^{2/3}}{\sqrt{C_L\ (3\ C_D\ C_L' - C_L)}} \tag{8}$$

If this value is substituted in equation (5) we get for the quickest turn

$$y = (2\ C_L - 3\ C_D\ C_L')\sqrt{\frac{C_L}{3\ C_D\ C_L' - C_L}} \tag{9}$$

Since y from its definition, can only have a positive value, we must have

$$2\ C_L - 3\ C_D\ C_L' \geqq 0$$

that is,

$$3\ C_D^2\ C_L^2\ C_L' - 2\ C_L^3\ C_D \leqq 0.$$

The exoression on the left is

$$\left[C_D^4 \left(\frac{C_L^3}{C_D^2}\right)\right]'$$

that is, for the quickest turn, only such angles of attack (α) come into question, as are greater than the angle corresponding to the maximum value of $\frac{C_L^3}{C_D^2}$. If the angle α is so chosen, that $\frac{C_L^3}{C_D^2}$ is a maximum, this corresponds to a zero angular velocity, that is, to straight flight. As α increases, x increases and becomes ∞, i.e., the engine output would exceed all limits, when

$$3\ C_D\ C_L' - C_L = 0$$

that is, when

$$\frac{C_L^3}{C_D^2}$$

attains its maximum value. Hence the limits of the angle of attack between which all curves of quickest turning must lie, are fixed.

3. Sharpest Curve.- To determine the smallest turn which an airplane can describe under steady conditions of flight, we start

with the original equations (1) and (2) and eliminate V. By this means, a relation is obtained between the engine power and the radius of the turn, which can be written non-dimensionally, as before, if we use the auxiliary quantities

$$x = \left[(75\ P\ \eta)^2\ \frac{S}{W^3}\ \frac{\rho}{2g}\right]^{1/3} \tag{4}$$

and

$$z = \left(\frac{2\ \frac{W}{S}}{\rho r}\right)^2 \tag{10}$$

We then have the following relation between them:

$$z = C_D^2 - \frac{C_D^{4/3}}{x^2} \tag{11}$$

which (if we make use of the quantity y defined by equation (5) can be written in the form

$$z = \frac{yC_D^{2/3}}{x} \tag{11'}$$

Equations (11) and (11'), connecting the radius of the turn and the engine output, hold good for any angle of attack and any altitude. For the sharpest turn which can be flown under steady conditions with constant engine power and, therefore, with constant value of x, $\frac{dz}{dC_D}$ must equal 0; that is,

$$0 = 2\ C_L\ C_L' - \frac{\frac{4}{3}C_D^{1/3}}{x^2}\ ;$$

The angle of attack must, therefore, be so chosen that

$$x = \sqrt{\frac{2/3\ C_D^{1/3}}{C_L\ C_L'}} \tag{12}$$

If this value is substituted in equation (11), we have

$$z = C_D \left(C_L - \frac{3}{2} C_D C_L' \right) \qquad (13)$$

The case $z = 0$ occurs at the same time as $y = 0$, if the airplane cannot make a turn, but can only fly horizontally in a straight line. In this case, along with equation (12),

$$x = \frac{C_D^{2/3}}{C_L} \quad \text{or} \quad \frac{C_L^3}{C_D^2}$$

is a maximum.

With increasing x, the angle of attack increases and approaches a limit, for which $C_L' = 0$, that is, the lift reaches a maximum.

The sharpest turn is flown with an angle of attack, which is determined by the limits $\left(\frac{C_L^3}{C_D^2}\right)_{max}$ and $C_{L_{max}}$.

<u>4. Calculation of an Example</u>.- The calculations hold for the polar diagram (Fig. 1), which belongs to a C type airplane (Dfw C V). Since C_L and C_D must be differentiated in a region where C_L varies very little and the graphical determination of C_L' is consequently very inaccurate, it appears advisable to substitute a parabola. The equation for this curve may be found with sufficient accuracy by the Lagrange interpolation formula. This is then differentiated and evaluated.

Figure 2 presents, on the basis of equation (8), the relation between x and C_D for the quickest turn and also, with the help of equation (9), the relation between x and y. If x and y, as functions of the angle of attack, have been found in this way, we can obtain immediately from equations (4) and (5), for a particular airplane of given engine power, the quantities determining the quickest turn at any altitude. For a normal decrease of air density with altitude, we obtain for an airplane with wing surface $S = 42.16\ m^2$ and weight $S = 1540$ kg the following relation between the altitude h and the coefficient x.

Table 1.

h	P	ρ	x
0	220	1.25	0.462
1000	204	1.129	0.425
2000	183	1.016	0.382
3000	165	0.912	0.342
4000	146	0.818	0.306
5000	126	0.731	0.264
6000	106	0.652	0.229

Thereby we have taken an engine, of which the decrease in power with the altitude is known from experiment, and a propeller efficiency, $\eta = 70\%$.

If the dependence of the angle of attack on the altitude, for the case of the quickest turn, has been determined, the greatest angular velocity is calculated from equation (6)

$$\omega = \sqrt{\frac{\rho g y S}{2 W}} \tag{14}$$

and the velocity

$$V = \left(\frac{75 P y}{C_D S \frac{\rho}{2g}}\right)^{1/3} \tag{15}$$

From this we can calculate the radius of the curve of quickest turning

$$r = \frac{V}{\omega}$$

as well as the increased loading, which is determined as the resultant of the weight and centrifugal force

$$R = W\sqrt{1 + \frac{V^2 \omega^2}{g^2}} \tag{17}$$

The same process of calculation serves for the determination of the sharpest turn. From equation (12) we determine the relation between x and the angle of attack corresponding to the sharpest turn. Then the value of y from equation (13) and finally, with the help of Table 1, the relation between the altitude and the angle of attack corresponding to the sharpest turn.

Table 2.

	Quickest turn.				Sharpest turn.		
h	y	ω	V	r	z	r	V
0	1.28	0.494	29.6	60.0	0.95	60	28.4
1000	1.17	0.421	29.8	71.0	0.83	71	28.8
2000	0.87	0.345	30.1	87.3	0.68	87	29.2
3000	0.62	0.276	30.4	110.0	0.53	110	29.6
4000	0.45	0.222	30.8	138.8	0.42	138	30.2
5000	0.02	0.044	30.8	600(?)	0.02	600	30.8
6000	-0.23						

The radius of curvature of the sharpest turn is then given by equation (10)

$$r = \frac{2 \frac{W}{S}}{\rho z^{1/2}} \tag{18}$$

and the velocity is determined, as above, from equation (14), in which the coefficient of resistance or drag corresponding to the angle of attack for the sharpest turn, is to be put and the increased load is determined by the resultant force

$$R = W \sqrt{1 + \left(\frac{V^2}{\rho g}\right)\left(\frac{V}{g\rho}\right)^2} \tag{19}$$

The results of the calculations are contained in Table 2 and Figures 3 and 4.

From these figures it follows that the radius of the sharpest turn coincides with that of the quickest turn within the limits of accuracy of the calculation; while the velocity of the quickest turn is appreciably greater than that of the sharpest turn. This remarkable result is explained by the fact that the change of the radius of curvature for the angle of attack under consideration

$$\left[\left(\frac{C_L^3}{C_D^2}\right)_{max} \cdots \cdot C_{L_{max}}\right]$$

is very small, while the speed changes rapidly with the angle of attack. In practice, the sharpest turn is always flown when the angle of attack lies within the given limits. For the quickest flown turns are characterized by equations (14) to (17). The difference in speeds, however, is not great, since the speed depends only on the cube root of C_D. In the most extreme case, it does not exceed 10%. For rough calculations, it is only necessary to choose an angle of attack between the limits mentioned and there is no advantage in choosing the value corresponding to the maximum $\frac{C_L^3}{C_D^2}$.

The work necessary for a complete turn of the airplane is always greater than the kinetic energy and increases with the altitude. In the preceding example, the work required for reversal of direction is

At an altitude of	0	1000	2000	3000	4000	m
	0	3281	6562	9842	13123	ft
About	20	25	36	43	56	%

greater than the corresponding kinetic energy, when the airplane flies in the sharpest turn. This reversal of direction requires much more work than the perfectly inelastic impulse, in which only the kinetic energy is destroyed. The question of the turn, for which the work required is least, leads also to the quickest turn. The work required for a semicircle of radius r is $T \pi r = \frac{75 P \eta}{V} \pi r$, in which T is the propeller thrust. This expression must be a minimum for the turn sought. This occurs, for a constant engine power, when $\frac{r}{V} = \frac{1}{\omega}$ is as small as possible, i.e., when the value of ω is a maximum.

The quickest turn is, therefore, also the turn of smallest consumption of energy.

II. Gliding in a Turn.

The motion of an airplane descending in a glide may be simply treated with the aid of a method which is closely connected with that for horizontal flight. If V_z is the velocity of descent, i.e., the vertical component of the velocity of the airplane along its path, then θ, the inclination of the line of flight to the horizontal, is given by

$$\frac{V_z}{V} = \sin \theta \tag{1}$$

The air resistance is in equilibrium with the component of gravity acting in the direction of the tangent to the path

$$W \sin \theta = C_D \frac{\rho}{2g} V^2 S \tag{2}$$

The lift is the resultant of the centrifugal force and the component of gravity perpendicular to the tangent to the path.

$$\sqrt{W^2 \cos^2\theta + \frac{W^2}{g^2}\left(\frac{V^2}{r}\right)^2} = C_L \frac{\rho}{2g} V^2 S \qquad (3)$$

If θ is eliminated from equations (2) and (3), by means of equation (1), we have

$$V_z = C_D \frac{\rho}{2g} V^3 \frac{S}{W} \qquad (4)$$

$$1 - \left(\frac{V_z}{V}\right)^2 + \left(\frac{V^2}{gr}\right)^2 = C_L^2 \frac{\rho}{2g} V^4 \left(\frac{S}{W}\right)^2 \qquad (5)$$

In the following, the steady velocity of descent V_z is assumed and the unknown velocity V can be eliminated from equations (4) and (5) and the relation between the radius of curvature r_z of the turn and the angle of attack obtained. It is, however, convenient so to arrange the equations beforehand, that only non-dimensional quantities appear. If we put for this purpose

$$\frac{\rho}{2g} \frac{S}{W} V_z^2 = \frac{1}{\xi} \qquad (6)$$

$$\frac{V_z}{V} = \sin\theta = \eta \qquad (7)$$

$$\frac{V_z^2}{gr} = y \qquad (8)$$

$$\eta^3 = \frac{C_D}{\xi} \qquad (10)$$

$$y^2 = \eta^6 - \eta^4 + \frac{C_L^2}{\xi^2} \qquad (11)$$

After the elimination of η with the aid of equation (10), equation (11) can be written

$$y^2 = \frac{C_L^2 + C_D^2}{\xi^2} - \frac{C_D^{4/3}}{\xi^{4/3}} \qquad (11a)$$

<u>1. The Straight Glide</u>.- Equation (11) shows that

$$\eta^6 - \eta^4 + \frac{C_L^2}{\xi^2}$$

is always greater than zero and only vanishes when $y = 0$, i.e., when the radius of curvature of the path is infinite. For the straight glide

$$\xi_o^{2/3} = \frac{C_L^2 + C_D^2}{C_D^{4/3}} \tag{12}$$

while, for finite r, we must always have $\xi < \xi_o$. In Figure 5 $\xi_o^{2/3}$ there is shown as a function of C_D for a polar diagram corresponding to the Dfw C V airplane, ξ is greatest and the velocity of descent least, when $\frac{C_L^2 + C_D^2}{C_D^{4/3}}$ attains a maximum, i.e., a glide with minimum descent is always straight. It is flown with an angle of attack which corresponds to a maximum value of $\frac{C_L^2 + C_D^2}{C_D^{4/3}}$.

An essentially different question is that of the flattest possible glide, but it is easily seen, that the gliding angle can never be diminished by curved flight. From equations (2) and (3), in fact, it follows that

$$W^2 \sin^2\theta = C_D^2 \left(\frac{\rho}{2g} S\right)^2 V^4$$

$$W^2 \cos^2\theta = C_L^2 \left(\frac{\rho}{2g} S\right)^2 V^4 - \left(\frac{W}{g}\right)^2 \frac{V^4}{r^2}$$

Hence,

$$\cot^2\theta = \frac{C_L^2 - \frac{W^2}{\left(\frac{\rho}{2} r S\right)^2}}{C_D^2} \tag{13}$$

θ will be a minimum, when $\cot\theta$ is a maximum, or when $r = \infty$. Straight flight is always flatter than a turn flown with the same angle of attack; the flattest glide is a straight line, the inclination of which is given by $\cot\theta = \left(\frac{C_L}{C_D}\right)_{max}$.

2. The Sharpest Turn.- For the sharpest turn which the airplane can describe with a given velocity of descent, V_z, y attains its maximum value in equation (11). The corresponding angle of attack is obtained by putting $\frac{d(y^2)}{dC_D} = 0$, so that

$$0 = -\frac{2}{3} C_D^{2/3} \xi^{2/3} + C_L C_L' + C_D$$

whence the relation of ξ to C_D is given by

$$\xi^{2/3} = \frac{C_D + C_L C_L'}{\frac{2}{3} C_D^{1/3}} \qquad (14)$$

In this, C'_L is derived from C_L with respect to C_D. This relation is also given in Figure 5.

3. The Quickest Turn.- For the determination of this turn, which is characterized by the maximum angular velocity, equation (5) or the equivalent equation (11) is first slightly transformed.

If the angular velocity*

$$\omega = \frac{V}{r} \qquad (15)$$

* It is to be observed that the angular velocity in the turn is chosen, not with a view to changing the position of the airplane in a glide as quickly as possible relatively to the earth, but with a view to extricating it as quickly as possible from the enemy's fire. This corresponds, however, to the assumptions in the text.

is put in equation (8), we have

$$y = \frac{V_z \omega}{g} \frac{V}{V_z} ,$$

or, if we put

$$\frac{V_z \omega}{g} = z \tag{16}$$

so that

$$y = z \eta \tag{17}$$

Equation (11) now becomes

$$z^2 = \eta^4 - \eta^2 + \frac{C_L^2}{\xi^2 \eta^2} \tag{18}$$

or, after eliminating

$$z^2 = \frac{C_L^2 + C_D^2}{\xi^{4/3} C_D^{1/3}} - \frac{C_D^{2/3}}{\xi^{2/3}} \tag{18a}$$

The angular velocity reaches its maximum value at the same time as z, that is, when $\frac{d(z^2)}{d\,C_D} = 0$, or

$$0 = -\frac{2}{3}\frac{C_D^{-1/3}}{\xi^{2/3}} + \frac{4}{3}\frac{C_D^{1/3}}{\xi^{4/3}} + \frac{2C_L C_L'}{C_D^{2/3}\xi^{4/3}} - \frac{2}{3}\frac{C_L^2}{C_D^{5/3}\xi^{4/3}}$$

It follows that, for the quickest turn,

$$\xi^{2/3} = 2C_D^{2/3} + \frac{3C_L C_L'}{C_D^{1/2}} - \frac{C_L^2}{C_D^{4/3}} \tag{19}$$

The quickest turn coincides with the sharpest turn, only when $\xi^{2/3} = \frac{C_L^2 + C_D^2}{C_D^{4/3}}$ and, at the same time

$$\left(\frac{C_L^2 + C_D^2}{C_D^{4/3}}\right)' = 0 \tag{20}$$

The turn then passes into a straight line corresponding to a glide with the slowest descent. The calculation of velocity presents no difficulties in any of the cases considered. Since V_z is known, $V = \frac{1}{\eta} V_z$, where $\frac{1}{\eta} = \frac{C_D^{1/3}}{\xi^{1/3}}$. In Figure 5, this expression is shown as a function of $\xi^{2/3}$. The figure shows that the sharpest turn and the quickest turn do not differ appreciably from each other, but both turns have lost speed considerably as compared with a straight glide. It cannot be said, however, that a minimum velocity is attained in the sharpest turn. This minimum velocity occurs again with a given rate of descent in a straight line flight, as shown by Figure 5, but in an indirect way.

4. Turn with Constant Excess Load.- In a straight glide, the load on the wings is always equal to the weight of the airplane. It increases when the airplane flies in a turn, since the resultant of the weight and the centrifugal force must be in equilibrium with the effective air force. Hence, we may define the increased load by the ratio of the total air force to the weight. A curve of n-fold loading will then be given by an equation of the form $n^2 W^2 = (C_L{}^2 + C_D{}^2) \left(\frac{\rho}{2g} S\right)^2 V^4$. This can be written by introducing the auxiliary quantities ξ, η

$$n = (C_L{}^2 + C_D{}^2)^{1/2} \frac{1}{\xi \eta^2} \tag{21}$$

If η is eliminated by the use of equation (10), then

$$\xi^{2/3} = \frac{1}{n^2} \frac{C_L{}^2 + C_D{}^2}{C_D{}^{4/3}} \tag{22}$$

or

$$\xi^{2/3} = \frac{1}{n^2} \xi_0^{2/3} \tag{22a}$$

in which, according to equation (12), $\xi^{2/3}$ corresponds to straight flight.

The curve representing equation (22) geometrically, may also be obtained from the curve of equation (12), by reducing the ordinates in the ratio 1 : n. All these curves are orthogonal projections of the curve of straight flight, the curve being at right angles to the C_D axis.

In Figure 5, equation (22) is represented for $n = 2$.

5. Relation of Radius of Curvature to Altitude.- The radius of the sharpest turn and of the quickest turn varies with the altitude at which the airplane is flying, since y and z depend on ξ and therefore on the density of the air. If the airplane is so controlled that it always describes the smallest turn, the radius of curvature varies with the altitude and the path of flight is obviously not a steady turn. In the first approximation, however, the motion may be considered steady at any instant. The question in what sense the radius of curvature varies with the altitude, if V_z remains constant, can be deduced from the analysis in the following way. With increasing altitude, the air density decreases and ξ increases. As ξ increases, it is seen from Figure 5 that C_D decreases, as also

$\frac{C_L{}^2 + C_D{}^2}{C_D{}^{4/3}}$ and $\frac{C_D{}^{4/3}}{\xi^2}$ decrease, both for the quickest turn and the sharpest turn. Consequently

$$y^2 = \frac{C_D{}^{4/3}}{\xi^2} \left(\frac{C_L{}^2 + C_D{}^2}{C_D{}^{4/3}} - \xi^{4/3}\right) \qquad (11b)$$

decreases with the altitude and increases with the radius of curvature. If an airplane descends by gliding in its sharpest (quickest) turn, it describes a spiral, which becomes narrower during descent.

This conclusion is not applicable to the curve of constant excess loading, as soon as the angle of attack becomes smaller than that corresponding to the maximum value of $\frac{C_L{}^2 + C_D{}^2}{C_D{}^{4/3}}$. With the aid of equation (11), the radius of curvature is shown in Figure 6 as a function of $\xi^{4/3}$.

Translated by
National Advisory Committee
for Aeronautics.

A = $C_L$max. Upper limit for sharpest turn.

B = $\left(\frac{C_L^3}{C_D}\right)$max. Upper limit for quickest turn.

C = $\left(\frac{C_L^3}{C_D^2}\right)$max. Steepest climb. Lower limit of the two turns.

D = $\left(\frac{C_L}{C_D}\right)$max. Smallest gliding angle.

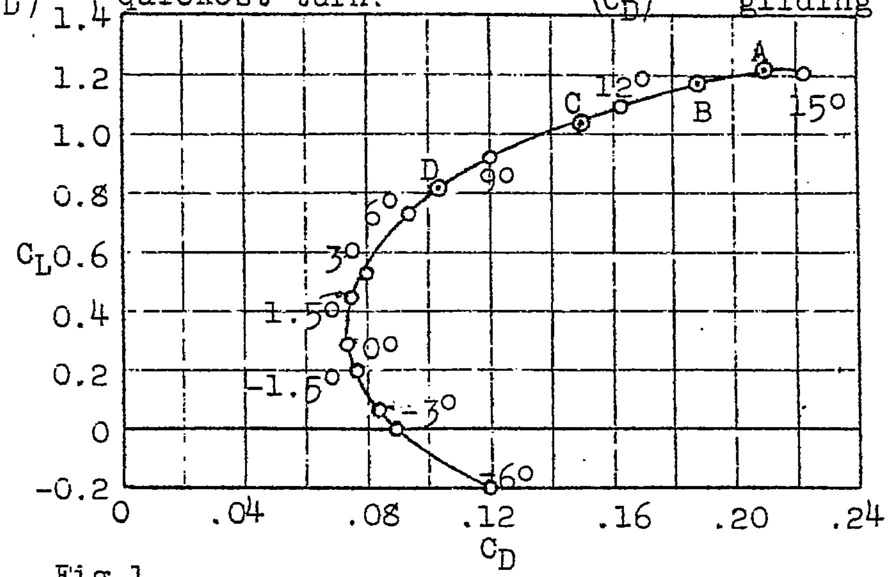

Fig.1

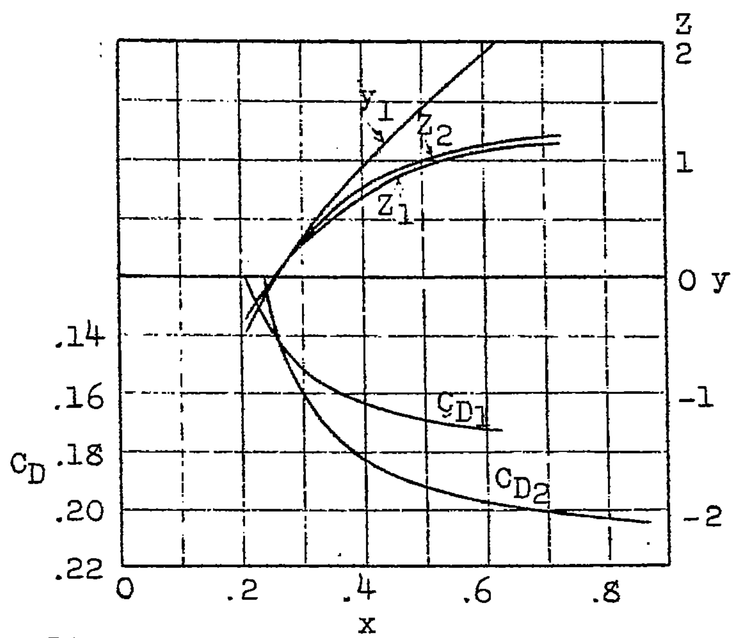

Fig.2

Figs. 3, 4.

A = Velocity
B = Overload
C = Radius of turn, r
D = Power output, P
E = Period of revolution, t

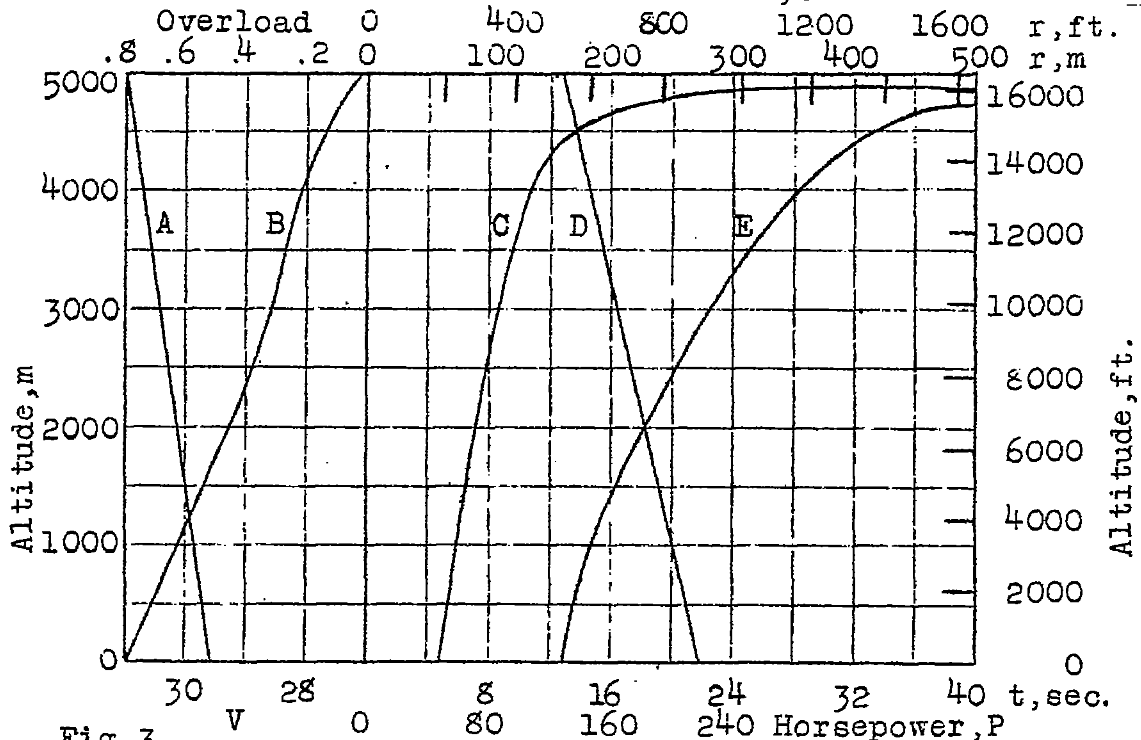

Fig. 3

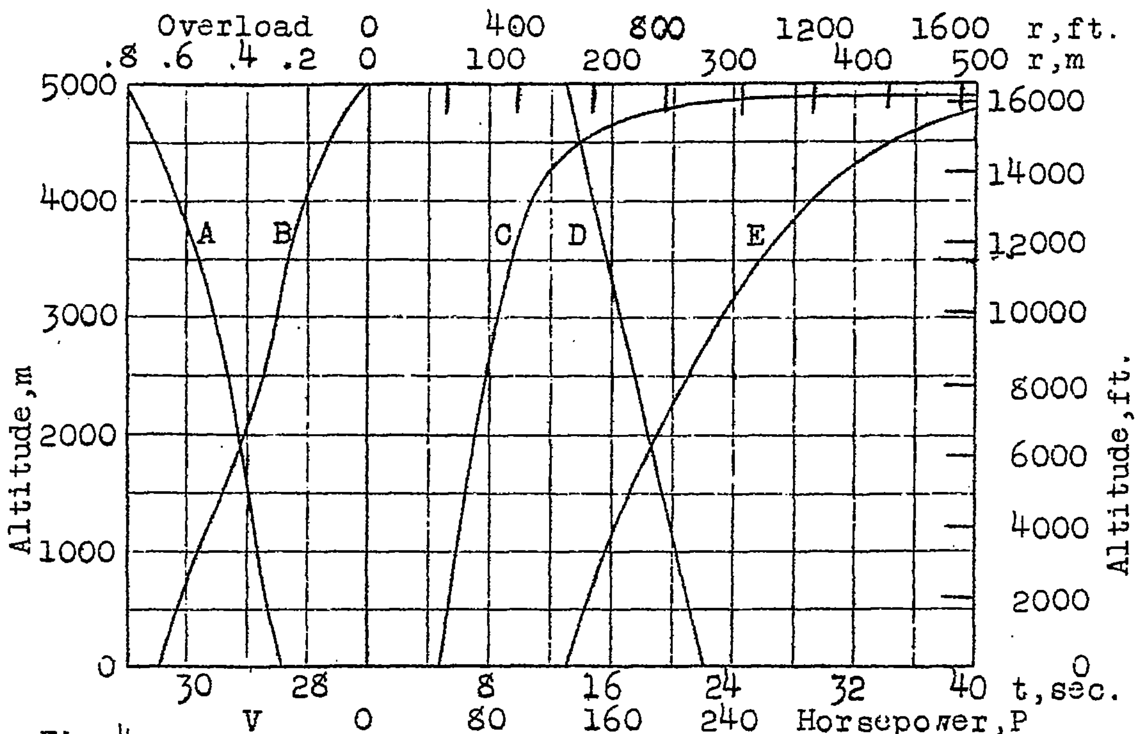

Fig. 4

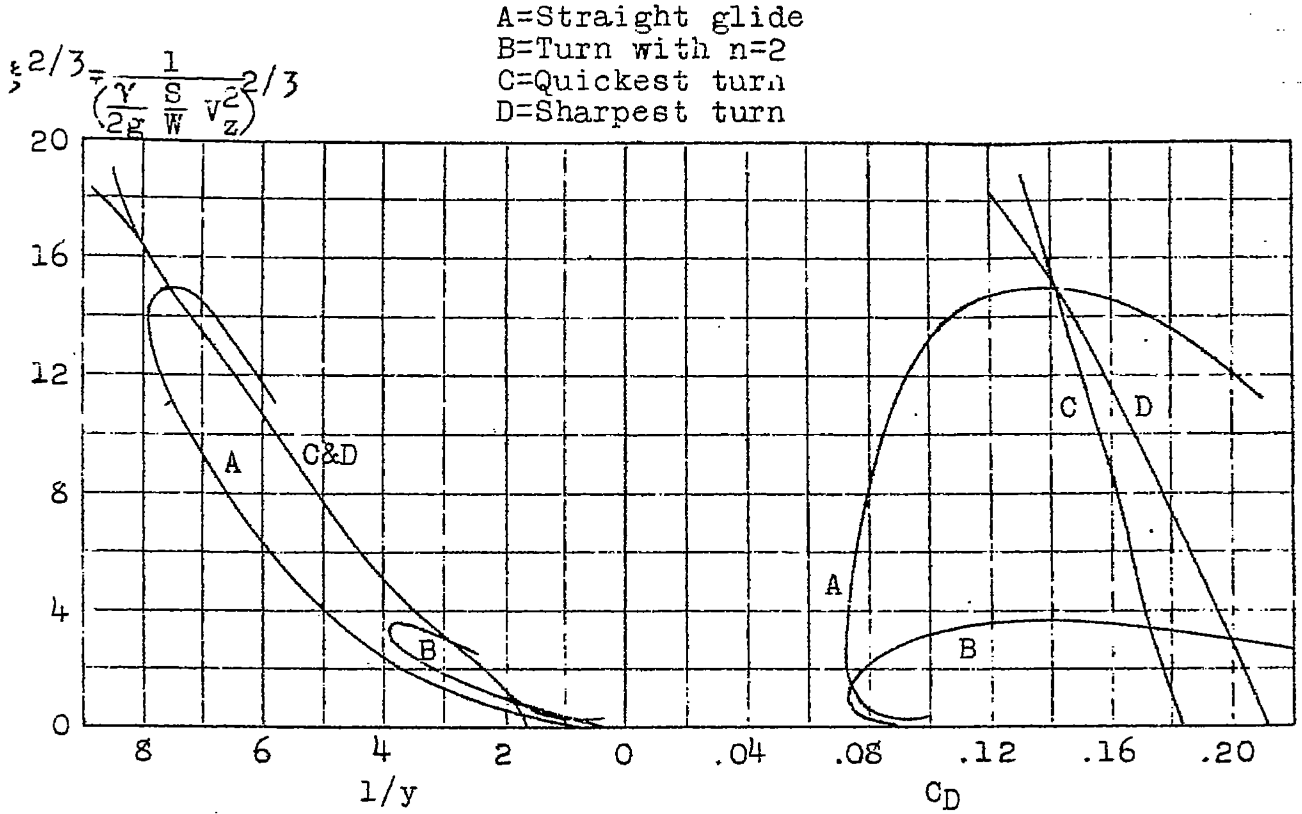
A=Straight glide
B=Turn with n=2
C=Quickest turn
D=Sharpest turn
$\xi^{2/3} = \frac{1}{\left(\frac{\gamma}{2g}\frac{S}{W}V_z^2\right)^{2/3}}$
20
16
12
8
4
0
8
6
4
2
0
.04
.08
.12
.16
.20
1/y
C_D
A
C&D
B
A
B
C
D

Fig.5

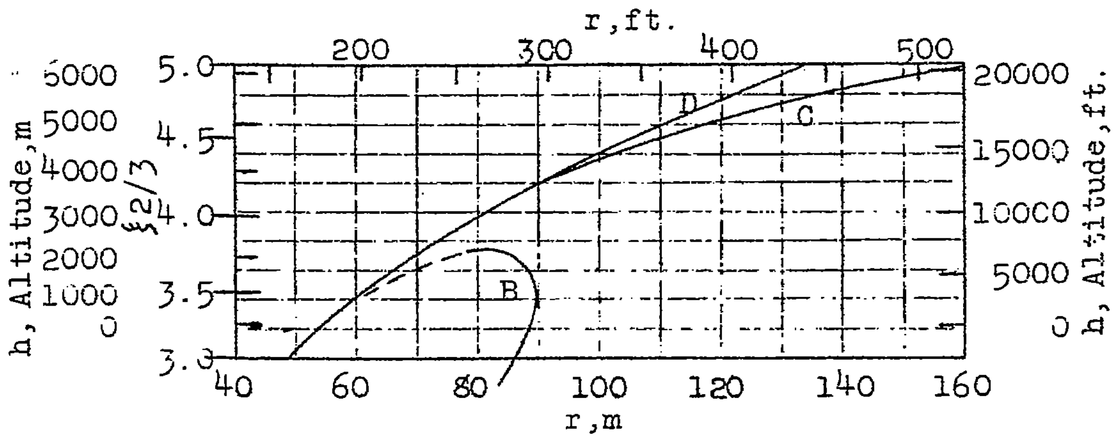
r,ft.
200
300
400
500
6000
5000
4000
3000
2000
1000
0
h, Altitude,m
$\xi^{2/3}$
5.0
4.5
4.0
3.5
3.3
20000
15000
10000
5000
0
h, Altitude,ft.
D
C
B
40
60
80
100
120
140
160
r,m

Fig.6