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EFFECT OF AEROFOIL ASPECT RATIO ON THE SLOPE OF
THE LIFT CURVE.

By

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Introduction.

One of the most important characteristics of an aerofoil is the rate of change of lift with angle of attack, \( \frac{dC_l}{d\alpha} \). This factor determines the effectiveness of a tail plane in securing static longitudinal stability. The following application of the Göttingen formulas in calculating the variation of \( \frac{dC_l}{d\alpha} \) with aspect ratio should therefore be of interest to many aeronautical engineers.

Variation of \( \frac{dC_l}{d\alpha} \) with Aspect Ratio.

The relation between the angles of attack at which a given lift coefficient obtains for two aerofoils of the same section but of different aspect ratio is expressed by the equation:

\[
(\alpha_1 - \alpha_2) = \frac{C_{L_1}}{U} \left( \frac{S_1}{b_1} - \frac{S_2}{b_2} \right) \times 57.3
\]

where \( S \) is the area, \( b \) the span and \( C_L \) the absolute lift coefficient defined by \( L = C_L S \frac{1}{2} \rho S V^2 \). This formula, due to Dr. Prandtl and Dr. Munk of Göttingen University, has been checked by tests and found reliable. A verification by Dr. Prandtl may be
found in "Ergebnisse der Versuchsanstalt zu Göttingen" (1921, p.51 et seq.

If the value of \( \frac{dC_L}{d\alpha} \) be known for an aerofoil section at a given aspect ratio the value for any other aspect ratio may be calculated from (1) by the method illustrated in Fig. 1. For the average aerofoil \( \frac{dC_L}{d\alpha} \) is substantially constant over an angular range of, say 10°, or more. Assuming \( \frac{dC_L}{d\alpha} \) to be constant with the value thus defined, the angular range corresponding to an increase in the lift coefficient from zero to any value \( C_L \) is

\[
\alpha_1 = C_L / \left( \frac{dC_L}{d\alpha} \right)
\]

For the same section in other aspect ratio, \( C_L \) will obtain at the angle \( \alpha_2 \) defined by equation (1) or

\[
\alpha_2 = \alpha_1 - (\alpha_1 - \alpha_2)
\]

The value of \( \frac{dC_L}{d\alpha} \) corresponding to this aspect ratio is

\[
\left( \frac{dC_L}{d\alpha} \right)_2 = \frac{C_L}{\alpha_2}
\]

Illustration of Method.

Assume that it is desired to find \( \frac{dC_L}{d\alpha} \) for an aerofoil of aspect ratio 2.5 when from test data it is known that \( \frac{dC_L}{d\alpha} = 0.72 \) for the same section at aspect ratio 6. For convenience, take \( \Delta C_L = 0.10 \), then

\[
\alpha_1 = \frac{\Delta C_L}{\left( \frac{dC_L}{d\alpha} \right)_1} = \frac{0.10}{0.72} = 1.390^\circ
\]
\[(a_1 - a_2) = \frac{C_L}{\pi} \left[ \frac{S_1}{b_2} - \frac{S_2}{C_2} \right] \times 57.3 \]

\[= \frac{0.10}{\pi} \left[ \frac{1}{6} - \frac{1}{2.5} \right] \times 57.3 \]

\[= -0.427^\circ \]

\[c_2 = a_1 - (a_1 - a_2) \]

\[= 1.390^\circ + 0.427^\circ \]

\[= 1.817^\circ \]

\[
\frac{dC_L}{d\alpha}
\]

\[
\frac{0.1C}{1.817} = 0.055
\]

**Application.**

For the convenience of the engineer, a set of curves calculated by this method are given in Fig. 1. Also, the observed value of \(\frac{dC_L}{d\alpha}\) for a few standard aerofoils are given in Table 1. It is of interest to note that the observed values of \(\frac{dC_L}{d\alpha}\) for the same aerofoil at various aspect ratios follow the calculated curves closely. For application, reference is made to N.A.C.A. Report #96, "Statical Longitudinal Stability of Airplanes," in which the effect of \(\frac{dC_L}{d\alpha}\) is treated.
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